Tests on the Combined Bending and Torsional Strength of Cast Iron.

By

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Many experiments made recently on the elastic failure of mild and harder steels, show results not in close conformity with the principal stress theory of Rankine or with the principal strain theory of St. Vénant. They rather support a new hypothesis proposed by Mohr in 1900.[1] Mohr's hypothesis may be stated as follows: "The elastic failure of a ductile material or the breaking of a brittle material takes place when the shearing stress, acting at a point along a certain plane, attains a limiting value, which is a function of the normal stress at that point." According to this law and assuming the limiting stress as the linear function of the normal stress, a formula for the combined bending and torsional strength of the following form can be deduced:[2]

$$M_0=(1-A)M+A\sqrt{M^2+T^2} \qquad (1)$$

in which M is the bending moment, T the torsional moment, A a constant and M_0 the equivalent bending moment, that is

$$M_0=\frac{\pi}{32}d^3K,$$

if d=diameter of the rod subjected to the combined action and K=bending strength (elastic bending strength for a ductile material.)

1) See Mohr's paper: "Welche Umstände bedingen die Elastizitätsgrenzen und Bruch eines Materials."—Zeitschrift des Vereins deutscher Ingenieure, 1900.

2) See Prof. Ono's paper: "On the Combined Stress Problem."—Journal of the Society of the Mechanical Engineers, Tokyo, Vol. XVI, No. 29.

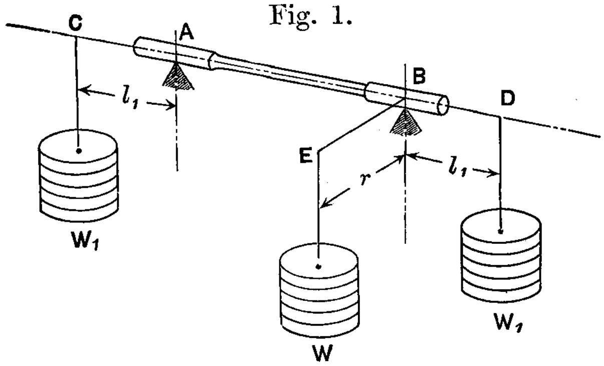
Eq. (1) is of the same form as the usual formula, resulting from Rankine's theory or from St. Vénant's. But in the usual formula A is the constant of a certain definite value, while in eq. (1) it is an empirical constant having different values for different materials. For mild steel, for example, eq. (1) with $A = \sim 1$ agree well with results of experiments.

As regards the combined strength of a brittle material as cast iron, little is known. As far as the writers are aware, no regular test has hitherto been made. For this reason they carried out the experiments on the combined strength of cast iron, whose results will hereinafter be reported.

The Testing Machine.

The testing machine was designed by the writers, according to the principle of Coker's testing machine,[1] and was made in the college workshop. Fig. 1 shows the scheme of the machine. AB is the test piece, supported at the ends. The torsional moment applied to the piece is resisted at the section A, without impeding thereby the free swivel of the piece on the support A. AC, BD and BE are lever arms attached to the piece, each 10 inches in length, on the extremities of which are hung weights. Thus the part between A and B is subjected to the uniform bending momert $M = W_1 l_1$ and the uniform torsional moment $T = Wr$.

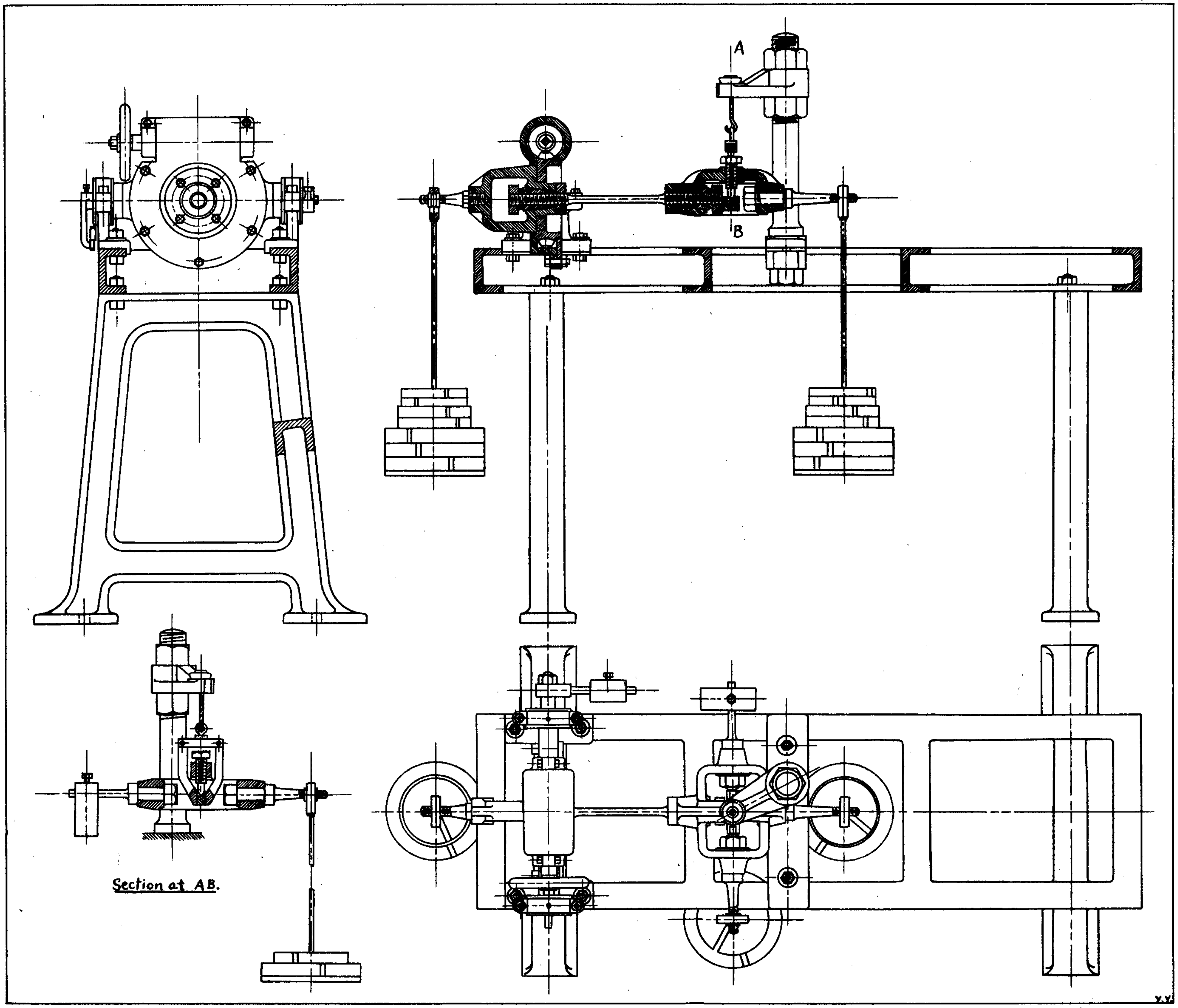
Fig. 1.



The section A is made capable of rotating and when, on account of the twisting of the test piece, the lever arm BE inclines from the horizontal, the section is rotated to bring the arm back to the level.

1) See Philosophical Magazine, Jan.-June 1909, p. 496.

Fig. 2.



In the test, W is kept constant for one test piece and W_1 is gradually increased until it breaks, except when the test piece is to be subjected to mere torsion.

Fig. 2 shows a working drawing of the testing machine with the test piece mounted on it.

The Test Piece.

The test piece was made of the form and the dimensions shown in Fig. 3. The diameter d was taken at $\frac{11}{16}$ inch. for ordinary cast iron and at $\frac{5}{8}$ inch for high grade cast iron.

Fig. 3.

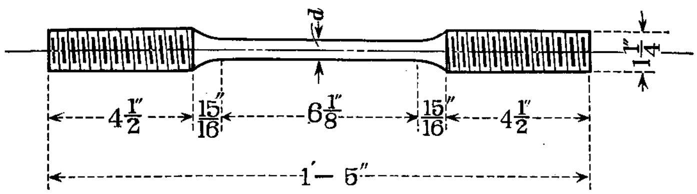


Fig. 4.

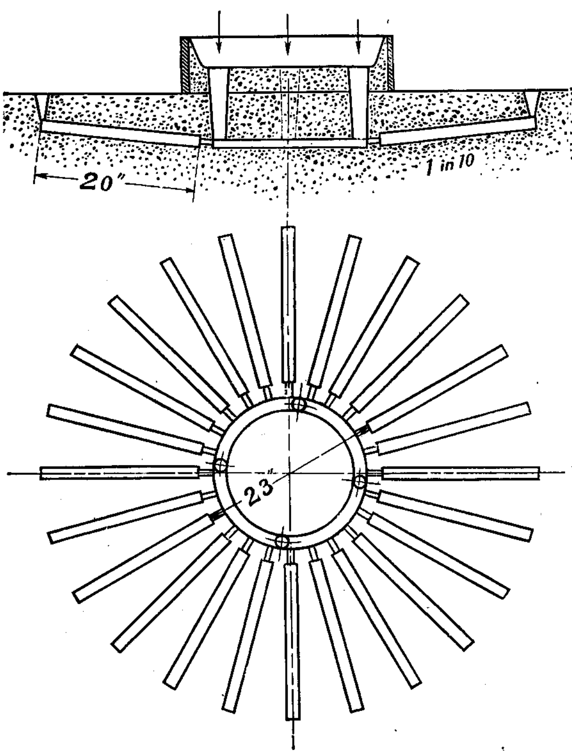
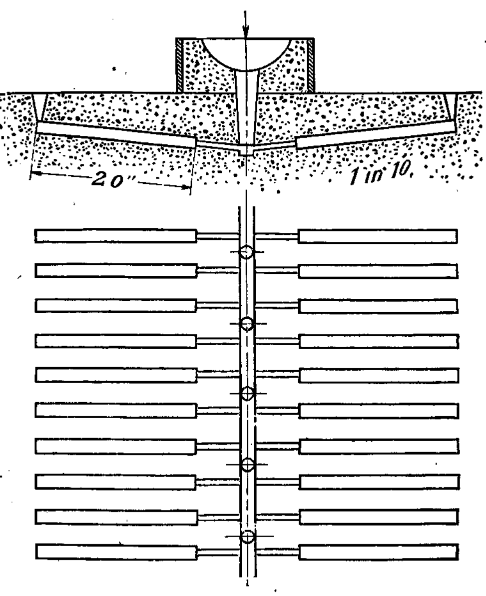


Fig. 5.



All the test pieces in a single series of tests were cast in one charge. Fig. 4 shows the method of moulding for series 1 and 2, and Fig. 5 that for series 3 and 4.

Results of Experiments.

Four series of tests were carried out and the results are given in tables I to IV. The values of M and T in one horizontal row, i.e. for one test piece, are the simultaneous values, for which the test piece broke.

TABLE I.—Series 1, Ordinary Cast Iron, $d=\frac{11}{16}''$.

Test piece No.	M in inchlbs	T in inchlbs	Test piece No.	M in inchlbs	T in inchlbs
1	1370	0	8	1100	950
2	1310	0	9	1120	1100
3	1310	200	10	1050	1250
4	1320	350	11	960	1400
5	1260	500	12	690	1550
6	1240	650	13	570	1700
7	1240	800	14	0	1970

TABLE II.—Series 2, Ordinary Cast Iron, $d=\frac{11}{16}''$.

Test piece No.	M in inchlbs	T in inchlbs	Test piece No.	M in inchlbs	T in inchlbs
1	1250	0	11	990	900
2	1260	0	12	1010	1000
3	1240	100	13	920	1100
4	1160	200	14	910	1200
5	1210	300	15	720	1300
6	1190	400	16	710	1400
7	1160	500	17	690	1500
8	1160	600	18	610	1600
9	1140	700	19	260	1800
10	1060	800	20	0	1890

TABLE III.—Series 3, Ordinary Cast Iron, $d=\frac{11}{16}''$.

Test piece No.	M in inchlbs	T in inchlbs	Test piece No.	M in inchlbs	T in inchlbs
1	1360	0	13	1040	1100
2	1370	0	14	960	1200
3	1340	100	15	770	1300
4	1340	200	16	890	1400
5	1310	300	17	810	1500
6	1290	400	18	740	1600
7	1240	500	19	590	1700
8	1270	600	20	460	1800
9	1140	700	21	360	1900
10	1220	800	22	0	1990
11	1120	900	23	0	2010
12	1170	1000			

TABLE IV.—Series 4, High-grade Cast Iron, $d=\frac{5}{8}''$.

Test piece No.	M in inchlbs	T in inchlbs	Test piece No.	M in inchlbs	T in inchlbs
1	1510	0	13	1170	1150
2	1400	0	14	1170	1300
3	1520	100	15	1090	1450
4	1420	200	16	970	1450
5	1540	300	17	760	1600
6	1410	400	18	890	1600
7	1400	550	19	690	1750
8	1360	700	20	550	1900
9	1290	850	21	490	2000
10	1310	850	22	300	2100
11	1340	1000	23	0	2120
12	1270	1150	24	0	2250

Figs. 6 to 9 show points plotted by taking the simultaneous values of M and T as coordinates.

The constant A and the bending strength K were calculated from the above values of M and T for each series of our cast iron, by the method of least squares and the results are given in Table V.

TABLE V.

	A	M_0	K
Series 1	0.6590	1362·2	42699
,, 2	0·6264	1240·8	38899
,, 3	0·6481	1345·5	42176
,, 4	0·6659	1494.4	62348

The curve in Figs. 6 to 9 represents the law of the combined strength, corresponding to eq. (1) with the respective values of A and M_0 given above,

The usual formula due to St. Vénant:

$$M_0 = \frac{m-1}{2m} M + \frac{m+1}{2m} \sqrt{M^2 + T^2}$$

reduces to

$$M_0 = 0.35M + 0{\cdot}65\sqrt{M^2 + T^2}$$

with $m = \frac{10}{3}$.

Comparing this with the foregoing results, it may be concluded that St. Vénant's theory applies to cast iron in the most satisfactory manner.

It may now be of some interest to observe the forms of fracture. Fig. 10 shows the form for series 2. It is remarkable that as the ultimate torsional moment is greater, the section of fracture is more inclined from the plane normal to the rod axis.

In conclusion the writers desire to express their thanks to Mr. S. Hasegawa, the director of the locomotive works "Kishaseizōkaisha", Osaka, for supplying the test pieces of high grade cast iron.

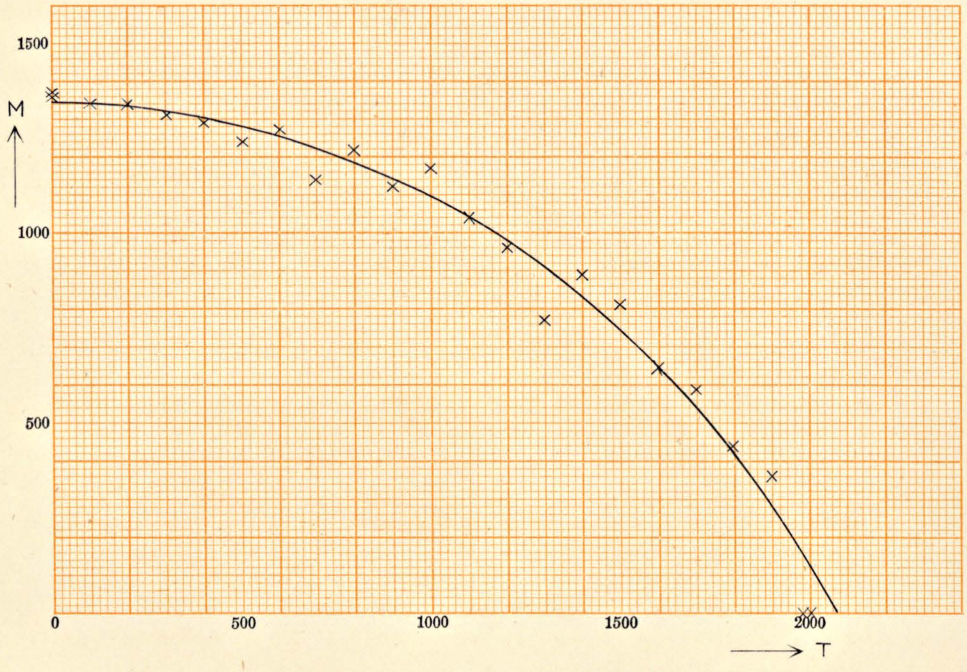
Fig. 6.

1500
1000
500
M
0
500
1000
1500
2000
T

Fig. 7.

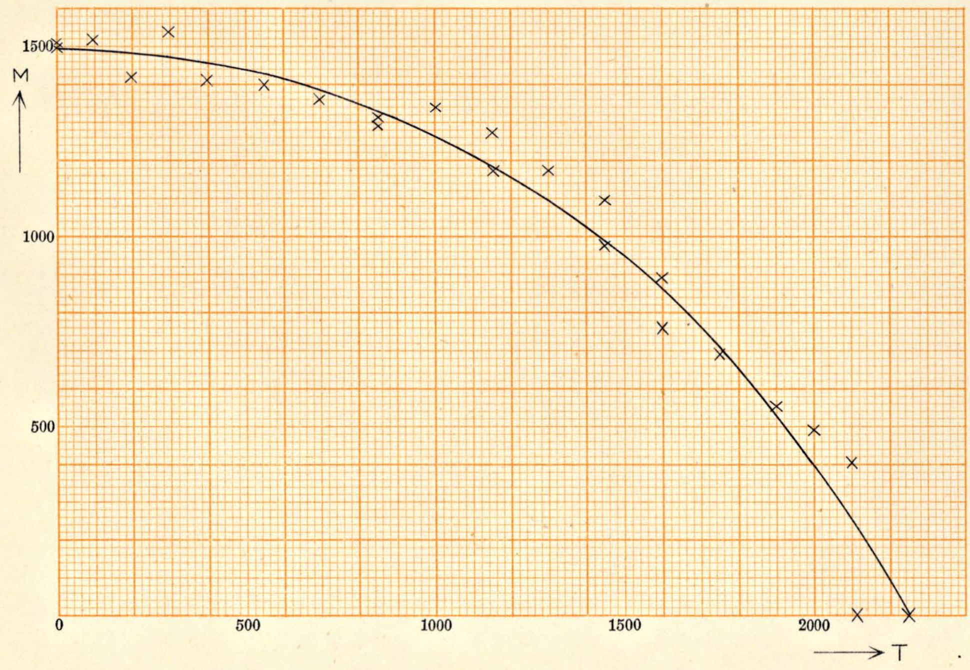
1500
1000
500
M
0
500
1000
1500
2000
T

Fig. 8.



1500
M
1000
500
0
500
1000
1500
2000
T

Fig. 9.



1500
M
1000
500
0
500
1000
1500
2000
T

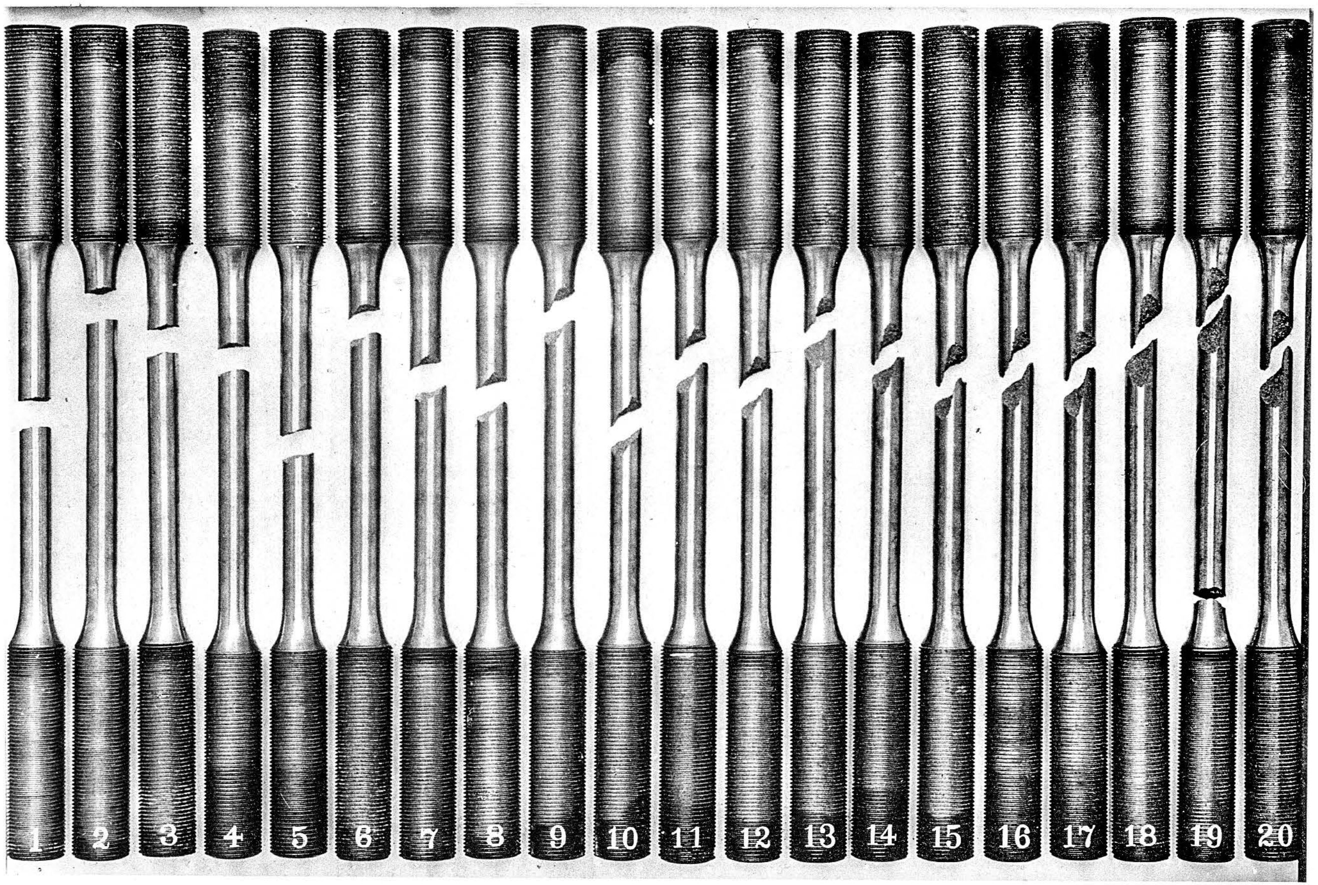


Fig. 10.