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SUBMITTED ARTICLES

MODELING OF MULTIAXIAL STATE OF STRESS AND DETERMINE THE FATIGUE LIFETIME FOR ALUMINUM ALLOY DURING CYCLIC LOADING UNDER IN-AND-OUT OF PHASE SHIFT Φ = 0° AND Φ = 90°

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Abstarct: *This article deals with determining of fatigue lifetime of structural materials during by multiaxial cyclic loading. The theoretical part deals with the fatigue and with the criterions for evaluation of multiaxial fatigue lifetime. The experimental part deals with modeling of combined bending - torsion loading and determining the number of cycles to fracture in region low-cycle and high-cycle fatigue and also during of loading with the sinusoidal wave form under in phase φ = 0° and out phase φ = 90°.*

Keywords: *sinusoidal cyclic loading, multiaxial fatigue, stress, structural material*

1. INTRODUCTION

Fatigue failures in metallic structures are a well-known technical problem. In a specimen subjected to a cyclic load, a fatigue crack nucleus can be initiated on a microscopically small scale, followed by crack grows to a macroscopic size, and finally to specimen failure in the last cycle of the fatigue life. Understanding of the fatigue mechanism is essential for considering various technical conditions which affect fatigue life and fatigue crack growth, such as the material surface quality, residual stress, and environmental influence. This knowledge is essential for the analysis of fatigue properties of an engineering structure [1, 2].

Fatigue under combined loading is a complex problem. A rational approach might be considered again for fatigue crack nucleation at the material surface [3]. The state of stress at the surface is two-dimensional because the third principal stress perpendicular to the material surface is zero [4]. Another relatively simple combination of different loads is offered by an axle loaded under combined bending and torsion. This loading combination was tested in our and also in many others experiments [5, 6, 7]. In spite of this fact, fatigue mechanisms are still not fully understood. This is partly due to the complex geometrical shapes and also complex loadings of engineering components and structures which result in multiaxial cyclic stress-strain states rather than uniaxial.

2. CRITERIA

Criteria valid for the fatigue lifetime calculation can be classified in three different categories: strain based methods, strain-stress based methods and energy based approaches.

Goodman used main stresses for evaluating the fatigue under multiaxial loading. Normal stresses are calculated for each plane and their ranges are used for calculation of fatigue lifetime. If the point of the combined stress is below the relevant Goodman line then the component will not fail. This is a less conservative criteria based on the material ultimate strength yield point Sut. To establish the factor of safety relative to the Goodman´s criteria can be written as:

$$\frac{K_f \times \sigma_{amp}}{S_e} + \frac{\sigma_{mean}}{S_{ut}} = \frac{1}{f_f} \qquad (1)$$

Sines published his works throughout the fifties of the last century. His criteria are very much alike, utilizing the amplitude of second invariant of stress tensor deviator (which corresponds to the von Mises stress) as the basis. Another term is added to the equation in order to cope with the mean stress effect – while Sines prefers the mean value of first invariant of stress tensor (i.e. hydrostatic stress σh). His resulting failure criterion can be expressed as:

$$\frac{\Delta\tau_{oct}}{2} + \alpha \times (3 \times \sigma_h^{mean}) == \tau_f' \times (N_f)^b \qquad (2)$$

Findley criterion is the first critical plane criterion. He suggested that the normal stress σn, acting on a shear plane might have a different linear influence on the allowable alternating shear stress, Δτ/2. Criterion has the following form:

$$\frac{\Delta\tau}{2} + k \times \sigma_n = \tau_f' \times (N_f)^b \qquad (3)$$

Minimum circumscribed ellipse (MCE) – The origin of this method goes out from minimum circumscribed circle method (MCCM). This method was first presented by Papadopoulos. Its major feature is its explicitness in determination of mean shear stress. Papadopoulos later shows that such minimum circumscribed circle can be obtained by a search through all pairs and triads of points in the shear stress path, but such an approach can be very lengthy. The contrast in comparison with MCCM is clear – it should offer a better solution of phase shift effect problems. Nevertheless, as regards the definition of mean shear stress, it does not offer any new approach. For proportional loading this will always be a straight line and for non-proportional loading histories will have some complex shape.

$$\tau_a = \sqrt{R_A^2 + R_B^2} \qquad (4)$$

Brown and Miller [8] observed that the fatigue life prediction could be performed by considering the strain components normal and tangential to the crack initiation plane. Moreover, the multiaxial fatigue damage depends on the crack growth direction. Different criteria are required if the crack grows on the component surface or inside the material. In the first case they proposed a relationship based on a combined use of a critical plane approach and a modified Manson-Coffin equation, where the critical plane is the one of maximum shear strain amplitude. Criterion, which was created, has the following form:

$$\frac{\Delta\gamma_{max}}{2} + S \times \Delta\varepsilon_n == A \times \frac{\sigma_f' - 2 \times \sigma_{n,mean}}{E} \times (2 \times N_f)^b + B \times \varepsilon_f' \times (2 \times N_f)^c \qquad (5)$$

Smith, Watson and Topper (SWT) created a parameter for multiaxial load, which is based on the main deformation range Δε1 and maximum stress σn,max to the main plane. Criterion has the following form:

$$\sigma_{n,max} \times \frac{\Delta\varepsilon_1}{2} == \frac{\sigma_f'^2}{E} \times (2 \times N_f)^{2b} + \sigma_f' \times \varepsilon_f' \times \times (2 \times N_f)^{b+c} \qquad (6)$$

Fatemi and Socie [9] observed that the Brown and Miller's idea could be successfully employed even by using the maximum stress normal to the critical plane, because the growth rate mainly depends on the stress component normal to the fatigue crack. Starting from this assumption, he proposed two different formulations according to the crack growth mechanism: when the crack propagation is mainly MODE I dominated, then the critical plane is the one that experiences the maximum normal stress amplitude and the fatigue lifetime can be calculated by means of the uniaxial Manson-Coffin curve; on the other hand, when the growth is mainly MODE II governed, the critical plane is that of maximum shear stress amplitude and the fatigue life can be estimated by using the torsion Manson-Coffin curve [9]. Criterion has the following form:

$$\frac{\Delta\gamma}{2} \times \left(1 + k \times \frac{\sigma_{n,max}}{\sigma_y}\right) == \frac{\tau_f'}{G} \times (2 \times N_f)^{b_\gamma} + \gamma_f' \times (2 \times N_f)^{c_\gamma} \qquad (7)$$

Liu created a virtual model of the deformation energy, which is a generalization of the axial energy on the basis of prediction of fatigue life. Criterion has the following form:

$$\Delta W = 4 \times \sigma_f' \times \varepsilon_f' \times (2 \times N_f)^{b+c} + \frac{4 \times \sigma_f'^2}{E} \times (2 \times N_f)^{2b} \qquad (8)$$

Where γ_f' is the fatigue ductility coefficient in torsion; ε_f' is the fatigue ductility coefficient; σ_f' is the fatigue strength coefficient; σ_h^{mean} is the mean hydrostatic stress; σ_n is the normal stress; $\sigma_{n,max}$ is the maximum stress; $\sigma_{n,mean}$ is the mean stress; σ_y is the stress in the direction of the axis y; τ_a is the equivalent shear stress; τ_f' is the fatigue strength coefficient in torsion; $\Delta\gamma_{max}$ is the maximum shear strain range; $\Delta\varepsilon_1$ is the principal strain range; $\Delta\varepsilon_n$ is the normal strain range; $\Delta\tau/2$ is the alternating shear stress; $\Delta\tau_{oct}$ is the octahedral shear stress; ΔW is the virtual strain energy; N_f is the number of cycles to fracture; S_e is the modified fatigue strength; S_{ut} is the ultimate tensile strength; f_f is the factor of safety applicable the fatigue; E is the elasticity modulus in tension; G is the elasticity modulus in torsion; R_A is the major axis of the ellipse; R_B is the maximum distance of stress point; b is the fatigue strength exponent; b_γ is the fatigue strength exponent in torsion; c is the fatigue ductility exponent; c_γ is the fatigue ductility exponent in torsion; A, B, S, k, α are material parameters.

3. NUMERICAL CALCULATIONS AND RESULTS

In ANSYS software was created the model of the test bar. The real geometry of this component is shown in Fig.1. The rod bar had a circular shape with a defined section, in which was expected an increased concentration of stress and creation a fatigue fracture [10, 11].

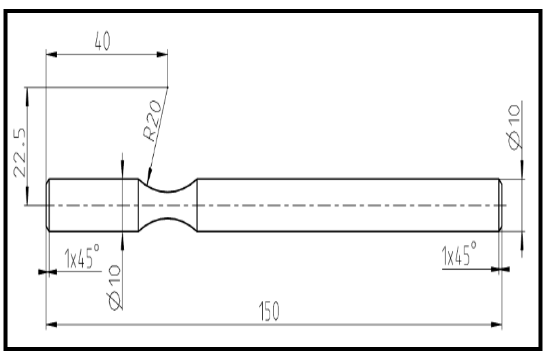

Figure. 1 Geometry of the test bar.

The ends of this model were loaded by reversed bending moment on the one side and by reversed torsion moment on the opposite site. The values of presented stresses and strains in the middle of the rod radius were taken from computational analysis using finite element method. We used the following parameters in finite element model: used material was aluminum alloy EN AW 2007.T3 (AlCu4PbMg) with Young's modulus E = 0,817.10^{11} Pa, Poisson number μ = 0,3 and with the strength limit R_m = 491 MPa. From computational analysis can be seen that the area with greatest concentration of stresses or eventually the place with the higher deformation was localized in the middle of the rod radius (see Fig.2).

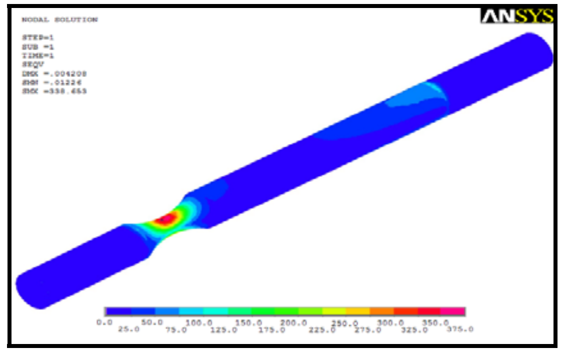

Figure. 2 Result of FEM analysis.

Obtained values of the stresses from finite element analysis were next computational analyzed using Fatigue Calculator software. This is a program which can quickly calculate fatigue lifetime of selected material. After starting the calculation, Fatigue Calculator displayed the number of cycles to failure for different models of damage. In our calculation we considered with all multiaxial criteria described above which can be applied to low-cycle and also to high-cycle fatigue region. All the tests were performed under controlled bending and torsion moments. Frequency of each analysis was equal to 30 Hz. It was first detected the number of cycles to fracture for multiaxial low-cycle fatigue with amplitudes in the phase shift 0° and then out of the phase shift 90° for stress. The same was done for multiaxial high-cycle fatigue.

The obtained number of cycles are processed into Wöhler curves $\sigma - \log N_f$ for multiaxial cyclic combined bending - torsion loading. For multiaxial low-cycle fatigue with phase shift 0°, Wöhler curves are shown in Fig.3. For multiaxial low-cycle fatigue with phase shift 90°, Wöhler curves are shown in Fig.4.

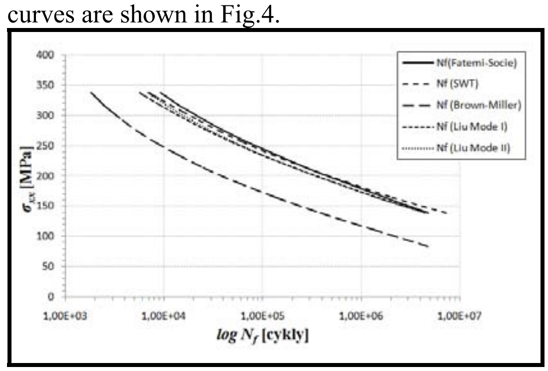

Figure. 3 Wöhler curves for multiaxial low-cycle fatigue with phase shift 0°.

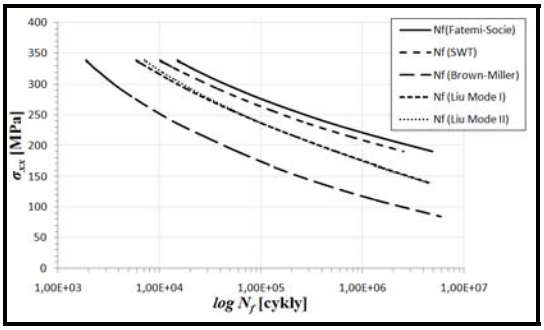

Figure. 4 Wöhler curves for multiaxial low-cycle fatigue with phase shift 90°.

For multiaxial high-cycle fatigue with phase shift 0° and with phase shift 90°, Wöhler curves are shown in Fig.5 and in Fig.6.

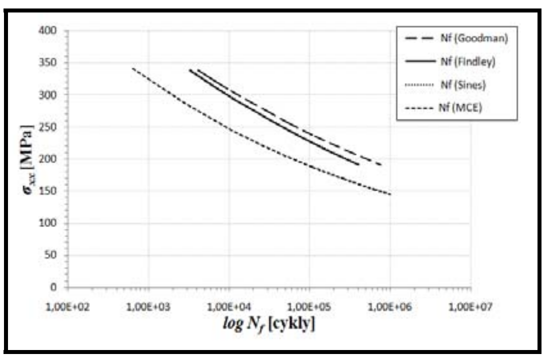

Figure. 5 Wöhler curves for multiaxial high-cycle fatigue with phase shift 0°.

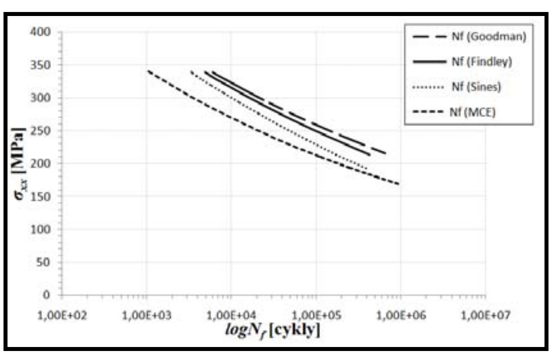

Figure. 6 Wöhler curves for multiaxial high-cycle fatigue with phase shift 90°.

4. CONCLUSION

All multiaxial models applied to fatigue lifetime calculation of aluminum alloy EN AW 2007.T3 increases with decreasing stress amplitude continuously in the cycles of number region.

Comparing Wöhler curves for low-cycle fatigue (see Fig.7), for amplitudes of the load with phase shift 0° (solid lines) and for amplitudes of the load with phase shift of 90° (blank lines), it can be seen that some models (such as Fatemi-Socie and SWT) give higher resistance to fatigue damage in the phase shift than the synchronized load amplitudes. This may be caused by, that the bending loading and neither torsion loading not active with the maximum value on the sample at the same time during the phase shift, but alternately. In this way, as if the sample was loaded by lower value of stress or deformation in a given time (phase shift of 90°). For other models, this shift of amplitudes did not cause any significant changes and the differences are minimal.

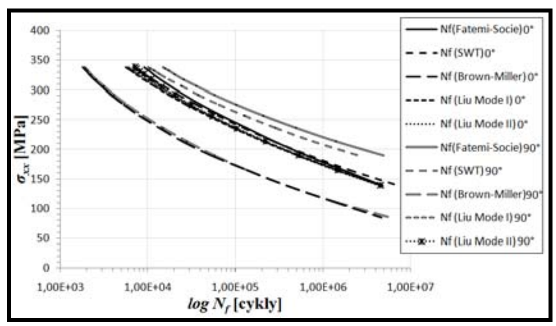

Figure. 7 Comparison of Wöhler curves for multiaxial low cycle fatigue.

Comparing Wöhler curves for high-cycle fatigue (see Fig.8), for amplitudes of the load with phase shift 0° (solid lines) and for amplitudes of the load with phase shift of 90° (blank lines), it can be seen that all models (except for Sines) gives a higher resistance against fatigue damage in the phase shift than in the synchronized amplitudes of loading. Probably the reason will be same as for low-cycle fatigue.

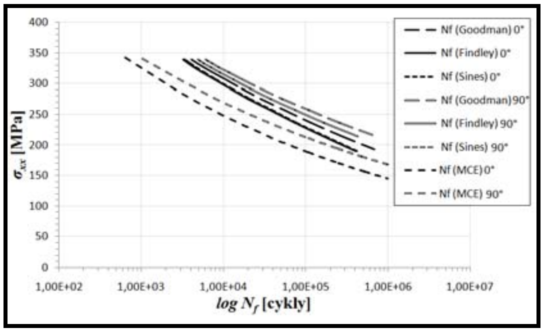

Figure. 8 Comparison of Wöhler curves for multiaxial high cycle fatigue.

It was observed that a phase shift 90° is the cause of "rotating" curves of fatigue life, which may have an impact on partial increase of fatigue life for the area of low-cycle and high-cycle fatigue.

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