UNIVERSITÀ DEGLI STUDI DI TRIESTE

XXVI CICLO DEL DOTTORATO DI RICERCA IN
NEUROSCIENZE E SCIENZE COGNITIVE- INDIRIZZO PSICOLOGIA

Children and mathematics: Beyond the role of cognitive abilities in early math achievement

Settore scientifico-disciplinare:
Psicologia dello Sviluppo e dell'Educazione

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TITOLO DELLA TESI

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Ever tried. Ever failed.
No matter.
Try again. Fail again.
Fail better.

Samuel Beckett

SUMMARY

The field of mathematical learning has received in recent years increasing attention in research, clinical and educational settings. The leading research line is dedicated to the investigation of the cognitive abilities fundamental for the acquisition and application of math concepts. Among general abilities, there is a wealth of evidence in favor of the recruitment of both working memory and short-term memory, despite there is no agreement concerning the involvement of the related subcomponents. Conflicting results pertain also the role of intelligence. Even major debate concerns more specific cognitive abilities, therefore those closely related to mathematics. In particular, it has not been elucidated the involvement of basic skills entailing approximate judgments about magnitudes and identified with ANS. Moreover, it is not yet clear how the recruitment of all these abilities can vary in dependence on stage of development and level of instruction.

The other research line, almost independent from the previous, is dedicated to the evaluation of constructs non-cognitive in nature, for instance affective and motivational factors but also self-perceptions, in relation to academic achievement. Mainly studied are constructs such as self-efficacy and anxiety, with particular reference to a subtype of anxiety that is specific to math. Other relevant aspects are represented by constructs such as self-concept and self-esteem, but also depression. Nevertheless, this kind of studies is usually conducted on old children, typically in those attending secondary school or college, whereas less attention is dedicated to younger students.

Starting from these considerations, the purpose of the current dissertation has been that of elucidating which are the factors, both cognitive and non-cognitive, that can assume a greater relevance at the beginning of schooling, i.e., in the first grades of primary school. These factors have been inspected both separately and by trying to find a possible interrelation between them.

In CHAPTER 1, the topics that are object of the present work are illustrated by delineating the state-of-the-art pertinent to each of them. CHAPTER 2 is dedicated to the description of Study 1, where a broad range of cognitive abilities including memory, intelligence and ANS has been investigated just at the beginning of formal instruction and therefore in relation to early math competence. Having proved the significant involvement of all tested skills, the consequent aim was that of exploring to which extent the same are suitable in the prediction of math performance in following grades. This investigation has represented the topic of Study 2, illustrated in CHAPTER 3. In this study, children were longitudinally followed from first to third grade, observing that the tested abilities can successfully predict future math learning, but with a leading role of working memory. Once having shed light on the involvement of cognitive abilities, a second purpose was the investigation of the possible involvement in young students of non-cognitive factors. These constructs were thus assessed in Study 3, reported in CHAPTER 4. The sample was represented by second graders and more relevant aspects resulted to be self-efficacy and general

anxiety. Contrary to expectations, anxiety specific to math appeared be non-significantly related to math performance. For this reason, Study 4, described in CHAPTER 5, was dedicated to an extensive evaluation of this constructs in third graders, in order to inspect when it could become relevant. Results suggested the association with math performance to establish in third grade, with particular impact of anxiety related to learning math rather than that associated to the math testing condition. The main findings emerging from overall studies and limitations, future directions and implications of the research are finally discussed in CHAPTER 6.

RIASSUNTO

Negli ultimi anni, lo studio dell'apprendimento della matematica ha iniziato a ricevere crescente attenzione nel campo della ricerca, ma anche in quello clinico ed educativo. Maggiore interesse è dedicato allo studio delle abilità cognitive che sottostanno all'apprendimento e all'applicazione dei concetti matematici. Tra le abilità a carattere generale, in letteratura esiste un forte consenso sul ruolo cruciale della memoria, sia di lavoro che a breve termine, nonostante non sia del tutto chiarito il coinvolgimento relativo delle varie componenti della stessa. Dibattito sussiste anche in merito al ruolo dell'intelligenza. Ancora maggiori divergenze permangono in merito al ruolo di abilità più specifiche, ovvero strettamente pertinenti alla matematica. In particolare non c'è accordo sul ruolo di abilità molto di base, indicate come ANS, e che consistono nel fornire giudizi approssimati in merito a grandezze e quantità. In aggiunta, non è chiaro il coinvolgimento relativo delle sopraccitate abilità in relazione a determinati stadi dello sviluppo o livelli di istruzione.

Il secondo filone di ricerca, perlopiù indipendente dal precedente, è rappresentato dalla valutazione di aspetti prettamente non-cognitivi, quali quelli affettivi e motivazionali, ma anche percezioni che gli individui formano in merito a se stessi e alle proprie capacità. I costrutti maggiormente indagati sono quelli dell'auto-efficacia e dell'ansia, sia generale che specifica per la matematica. Altri aspetti rilevanti sono rappresentati dal concetto di sé, dall'autostima e dalla depressione. Questi fattori sono tuttavia tipicamente valutati in studenti a partire dalla scuola secondaria, mentre minore attenzione viene dedicata a quelli frequentanti i primi anni del percorso scolastico.

A partire da queste considerazioni, l'obiettivo primario del presente lavoro di tesi è consistito nella valutazione di quali fattori, sia cognitivi che non, hanno una maggiore rilevanza nell'ambito della prestazione matematica all'inizio della scolarità, più precisamente nelle prime classi della scuola primaria. Si è voluto valutare questi fattori sia indipendentemente, sia esplorandone la possibile influenza reciproca.

Il CAPITOLO 1 è quindi dedicato alla discussione degli argomenti trattati in modo da fornire una panoramica sullo stato dell'arte attuale in merito alle ricerche condotte e ai relativi risultati. Il CAPITOLO 2 è centrato sulla descrizione dello Studio 1, in cui è stato testato un ampio spettro di abilità cognitive quali memoria, intelligenza e ANS, in bambini appena avviati all'istruzione formale e pertanto valutando il ruolo di queste abilità in relazione ad abilità matematiche precoci, prettamente informali. Verificato il coinvolgimento significativo di abilità tanto generali quanto specifiche all'inizio della scolarità, l'obiettivo conseguente è stato quello di verificare in che modo tali abilità siano in grado di predire l'apprendimento matematico negli anni successivi della scuola primaria. Questo obiettivo ha caratterizzato lo Studio 2, descritto nel CAPITOLO 3. Un campione di bambini è stato seguito longitudinalmente dalla classe prima alla classe terza, riscontrando che tutte le abilità indagate hanno un significativo impatto anche sull'apprendimento formale della matematica, ma con un ruolo primario assunto dalla memoria di lavoro.

Una volta delineato il quadro delle abilità cognitive cruciali nei primi anni scuola, la volontà è stata quello di esplorare se anche costrutti non-cognitivi possano avere un impatto significativo. Lo Studio 3, illustrato nel CAPITOLO 4, si è quindi focalizzato anche sulla valutazione di questi aspetti in bambini di classe seconda, riscontrando un diretto coinvolgimento di auto-efficacia ed ansia generale. Contrariamente alle aspettative, l'ansia specifica per la matematica non è risultata avere alcun legame significativo con la prestazione matematica. A partire da questo risultato, l'obiettivo dello Studio 4, riportato nel CAPITOLO 5, è consistito nella valutazione più approfondita di questo costrutto in bambini di classe terza, in modo da esplorare quando lo stesso possa diventare rilevante ai fini della prestazione matematica. I risultati hanno dimostrato un ruolo significativo a questo livello, in particolare per quanto concerne l'ansia da apprendimento, piuttosto che di valutazione, della disciplina. Il CAPITOLO 6 è quindi dedicato alla discussione generale dell'elaborato in cui sono riassunti i principali risultati emersi e discusse le limitazioni, prospettive future ed implicazioni pratiche della ricerca.

TABLE OF CONTENTS

CHAPTER 1

General introduction

Nowadays there are increasing evidences that mathematics assumes an essential role in many aspects of our life, starting from professional engagement (National Mathematics Advisory Panel, 2008). Mastering mathematics, however, is fundamental not only from the perspective of career success, but it involves many other aspects, such as psychological well-being (e.g., Paglin & Rufolo, 1990; Rivera-Batiz, 1992) and social activities (McCloskey, 2007). Nevertheless, in the last decades difficulties concerning mathematics have received little attention in relation to other learning disabilities like dyslexia, although they occur with comparable frequency rates (Mazzocco & Thompson, 2005). Only in the very recent years, mathematical learning and related issues have begun to acquire grater consideration in both research and educational settings.

The act of exploring the processes regarding mathematical learning and performance is a challenging task, since various factors also different in nature have been proven to have an influence. In fact, beside the cognitive and intellectual abilities, also social dimensions such as income level (Griffin, Case, & Siegler, 1994), home learning environment (Melhuish et al., 2008), and teaching (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006) but also personality and affective (e.g., Ma, 1999) traits can have a remarkable impact.

Past research has been dedicated on prevalence to the investigation of cognitive processes and abilities at the basis of math learning. These studies have contributed in shedding light on how tricky is the development of math learning and on how much cognitive effort is needed at every stage. Mathematical skills are indeed observed to develop hierarchically, from basic to increasingly more complex concepts and procedures (Dowker, 1998). As a matter of fact, difficulties affecting math bases reverberate to subsequent skills and permeate overall math aspects. Being math learning so multifaceted, it is easily deducible how can be difficult to achieve a global view of all the aspects, cognitive but not only, subserving this process.

Keeping in mind these considerations, the global purpose of the studies illustrated in the present dissertation has been that of trying to give an outline of math achievement, by combining the investigation of both cognitive and non-cognitive aspects which are, in general, separately treated in literature. In particular, the aim was to help clarifying the role of these aspects with particular attention to the initial stages of formal math learning. Hence, the final goal has been the identification of the factors appearing to be the most salient in the context of early math proficiency and on which therefore intervene in case of possible difficulties in this field.

1.1 Development of mathematical learning

1.1.1 Mathematics as an innate ability

Are math abilities innate or are merely the result of formal education? This question has fretted many researchers in the past, since children seemed to be naturally predisposed to manipulate quantities and learn numbers. Anyway, it wasn't clear if this disposition could be completely innate or be driven by external stimuli. Pioneering experiments were therefore set up to explore if even new-born infants can somewhat master number-related capacities. Classical experiments were based on the *habituation-dishabituation paradigm* (see Figure 1.1), according to which children are expected to gaze longer on new or unexpected stimuli (dishabituation), while losing interest for long-lasting or expected stimuli (habituation). Well, by observing 4- to 6-month-old infants looking at the progressively increasing number of dots appearing on a screen, Starkey and Cooper (1980) noticed raised attention for each change of numerosity from one to four (whereas higher numerosities changes weren't already "caught" and pupils rapidly habituated). Other similar fascinating experiments were conducted by Karen Wynn (e.g., 1992), who demonstrated that infants spontaneously develop arithmetic expectancies, thereby figuring out how the elements of a set can vary after having added or subtracted some of them.

By answering to the question posited at the beginning of the paragraph, mathematics seems to be somehow an inborn capacity to which we are naturally predisposed. In line with this consideration, Butterworth (1999) claimed the existence of an innate *number module* predisposed to process exact quantities and affirmed that our brain is "born to count".

Figure 1.1 Habituation-dishabituation Paradigm

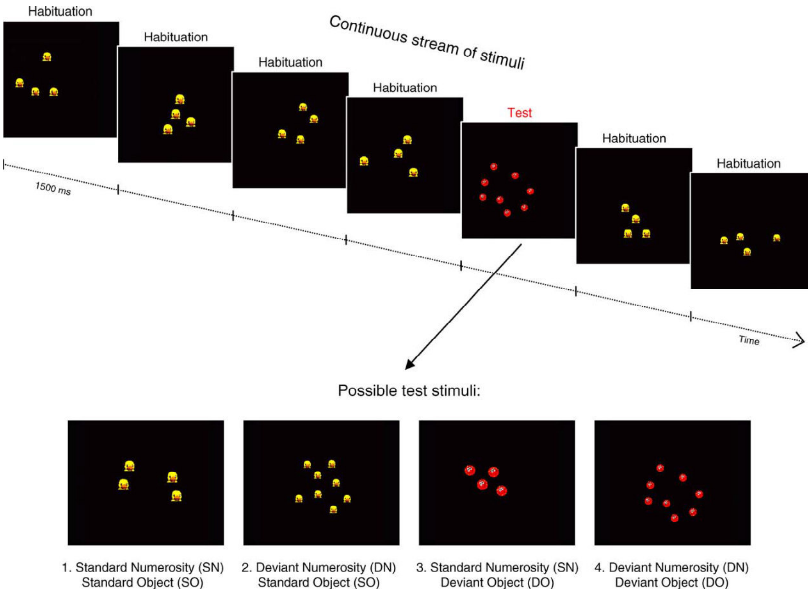

Figure 1.1 Example of habituation-dishabituation paradigm. From Izard, Dehaene-Lambertz, & Dehaene, 2008.

1.1.2 From the concept of numerosity to basic arithmetic

The previously illustrated studies hinted that a fundamental ability at the basis of math learning is represented by the capacity to grasp the number of elements in a set, in other terms, to identify their *numerosity*. It is widely accepted that possessing and mastering the concept of numerosity is somehow crucial in arithmetical performance. Despite Piaget (1952) stated that the this concept can't be acquired before the age of 7, empirical evidences suggested that even preschoolers can master it.

Deep numerosity understanding is tightly related to counting ability. Learning how to count is indeed supposed to be the key factor allowing to link the innate ability of discriminating numerosities with the acquisition of more formal math concepts. It indeed represents the first child's experience with the world of numbers.

Direct and indirect experience proves that when children begin to manipulate small numerosities heavily rely on counting to perform basic arithmetic. Probably everyone of us has in mind children attempted at performing a simple addition by counting in succession the told numerosities on their fingers. Children rely on counting to execute basic arithmetic in a differential way according to the developmental stage (i.e., from counting all numbers to counting on from the larger). Only with experience and practice, children finally learn to associate number combinations to the correspondent result and retrieve it automatically from memory, without relying on counting anymore (Geary, Brown, & Samaranayake, 1991).

1.1.3 Formal mathematical learning

1.1.3.1 Models of number processing

As previously forecasted, math learning is a complex and multifaceted process which consists of different but intermingled capacities. With the aim of shedding light on empirical data collected on numerical cognition and of clarifying the structural relationships among these competences, different numerical models have been proposed. Two of the most widely accepted and empirically validated are the one proposed by McCloskey (McCloskey, 1992; McCloskey, Caramazza, & Basili, 1985) and the *Triple-Code model* by Dehaene (Deahene, 1997; Dehaene & Cohen, 1995).

McCloskey's model is described from a cognitive perspective and posits the presence of three separate stores which however make reference to a common internal representation of numbers:

- *calculation* mechanisms, in turn subdivided into arithmetic signs, procedures, arithmetical facts. Recognition of the *arithmetic signs* is the first step when performing computation, since they remind which arithmetic operation has to be carried out. The other aspect is that of *procedures* which regard the steps that have to be executed in a precise order to perform computation; finally, *arithmetic facts* represent a sort of automatism, being them the results of common operations (e.g., multiplication tables); throughout exercise and frequent rehearsal, they are stored in memory and then easily recalled without applying every time the computation procedures;

- *number comprehension* mechanisms, which entail understanding numbers in both written and oral formats;
- *number production* mechanisms, that consist in producing numbers in both written and oral formats.

The last two stores can be in fact distinguished on the basis of format-specific number codes, either Arabic or verbal (number names), and transcoding procedures are necessary to shift from one representation to the other, throughout processes such as reading or writing.

On the other hand, the *Triple-Code model*, depicted in Figure 1.2, has been defined on the bases of both cognitive and neuroanatomical evidences and is term in this way for the reason that it encompasses three specific codes:

- *magnitude representation* in an analogue (semantic) format, which includes abilities such as magnitude comparison and estimation or approximate computation. From the neuroanatomical viewpoint, it has been proven to lie bilaterally in a region of the parietal cortex termed intraparietal sulcus (IPS);
- *visual system* that provides number representation in a symbolic way, more specifically in the Arabic format, and is involved in reading and writing numbers. It is represented as a sort of workspace where performing multi-digit operations. It seems to lie bilaterally in the fusiform gyrus;
- *verbal system*, responsible of number representation in an auditory verbal form. This component is dedicated to the comprehension and production of spoken number names and to the storage of arithmetic facts. It is assumed to lie in the left angular gyrus and it is therefore tightly associated with language.

Contrary to McCloskey's model, the current one posits that it is possible to produce verbal numbers from visual numbers and vice versa, without necessarily having access to the central number meaning. The two described models differ also in other aspects, first of all the representation of numbers. Indeed, although both assume a central semantic number store, different is the internal structure of this representation. While, according to the McCloskey's model, numbers are represented as base-10 units, in the Triple-Code model numbers are expected to be mentally disposed along a continuum in the form of an analogue *mental number line* which is logarithmically compressed towards lager numbers.

The mental number line is supposed to be modeled as overlapping Gaussian distributions (one for each number) which are logarithmically compressed along a continuum. The logarithmic compression is the result of how we mentally visualize great numbers, viewed as closer one to the other than smaller numbers. We can try to think about two couples of numbers and mentally figure them. If we mentally imagine the couple 9-10 and then 99-100, we can probably notice that the last two numbers are visualized in our mind more closely than the first two, despite the fact that both couples differ only by one unit.

This representation appears to have important implications on our capacity to distinguish and compare close numbers. Discrimination abilities can however refine. In parallel with development and education, in fact, our internal mental representation of numbers progressively shifts from logarithmic to linear, at least for numbers we easily master, and this transition is associated to a better math proficiency (e.g., Siegler & Opfer, 2003).

Figure 1.2 The Triple-code Model

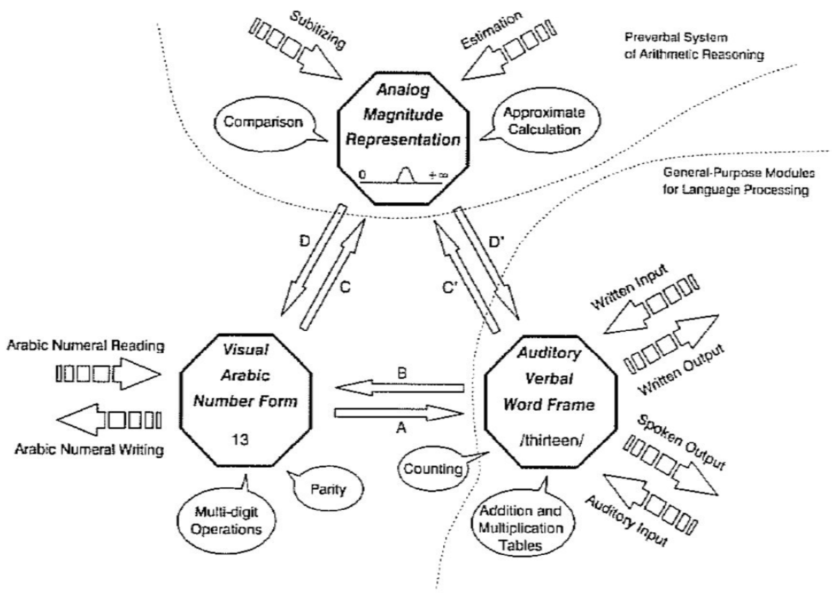

Figure 1.2 Schematic representation of the Triple-code model. From Dehaene, 1992.

1.1.3.2 Steps for the acquisition of formal math competences

According to von Aster and Shalev (2007), formal numerical abilities develop along four consecutive and hierarchically organized stages:

1. numerical cognition evolving from an innate component and coinciding with subitizing (immediate identification of arrays of elements up to four without counting) and approximation;
2. linguistic number representation;
3. number representation by means of symbols (e.g., Arabic format);

4. semantic number representation with numbers representation along a mental number line.

This last step, in particular, is viewed as a representational redefinition of the first one after the intermediate development of the other specific abilities.

As highlighted, a crucial and necessary phase in learning formal arithmetic is the acquisition of number symbols, which principally occurs thanks to the intervention of language. This represents the necessary ability for achieving subsequent math competences by inducing important changes in the mental visualization of numbers. As hinted before, number representation becomes more precise (Dehaene, 2007) and shifts from approximate and logarithmic to exact and linear.

Apart from the visual recognition of the specific symbol (e.g., digits written in Arabic format), understanding numbers in their symbolic aspect entails the combination of three processes: *semantic*, regarding the quantity associated to each number, thus its value or meaning (in other terms, its position along the just mentioned mental number line); *syntactic*, a sort of number inner grammar which determines the positional value of a certain digit in dependence of its position within the number; *lexical*, concerning the ability of naming numbers, where each digit assumes a certain nominal value.

1.1.3.3 Written arithmetic operations

Almost everyone of us probably remembers personal school experience in learning how to compute the four arithmetic operations, that are our first significant approach with the arithmetic word. A lot of time and effort we have probably dedicated to master operations and to be able to solve also the most complicated equations. While, as described in previous sessions, children approach to simple arithmetic even before the entrance into primary school and perform simple calculations by counting with their fingers, learning how to perform written, more complex, multi-digit operations takes almost all grades of primary school. It is undeniable that complex written arithmetic operations pose a huge cognitive load and effort to be carried out, by the moment that require the simultaneous application of different rules and concepts.

Actually, in accordance with McCloskey's model, computation requires three fundamental steps: the *algorithm selection*, meaning the correct association of the arithmetic sign to the correspondent operation; secondarily, the knowledge of the appropriate *procedures*; finally, the correct execution of the *operation*. Main difficulties are usually associated to the proper acquisition and application of procedures which are the major cause of arithmetic failure (Dehaene, 1997; Semenza, Miceli, & Girelli, 1997). Once learned, wrong computation procedures are indeed difficult to eradicate, since the constant application of the same error strengthens its acquisition and will always lead to the same incorrect result.

In addition, written multi-digit calculations require a close connection between understanding multi-digit numbers and the correspondent computational procedures. The earlier and better this connection takes place, the higher will be the arithmetic proficiency. It has been in fact noticed that first graders who rapidly and efficiently understand place-value and base-10 concepts will perform at higher levels in written computation in third grade (Hiebert & Wearne, 1996). Nevertheless, dissociations can be found within different arithmetic operations, which require different processes to be performed (e.g., multiplication seems to rely mainly on verbal than semantic processes which instead typically characterize subtraction, see Lemer, Dehaene, Spelke, & Cohen, 2003).

Independently to how arithmetic operations are effectively dissociated, these and other math tasks have in common certain cognitive abilities they recruit in order to be carried out. As further deeply illustrated, the consequence of deficiencies at this level can be responsible of widespread obstacles increasing in parallel with task complexity and demands.

1.1.3.4 Word problems

Learning how to make computations and store some of the related results are fundamental because are the first necessary step for the resolution of various kinds of problems occurring in the everyday life. We can merely think to when we go shopping and have to calculate the change the tradesman has to give us back. The aim of learning mathematics, even if sometimes misunderstood, is also that of instructing people to successfully challenge these daily-occurring practical problems.

Children begin to exercise with such kind of problems at school, usually starting from second grade, when some computation skills are mastered and reading abilities are adequate. Indeed, in the school context, problems assume the form of story or word problems, where a “story” is construed in order to create a suitable context. However, reading and catching the story can place additional difficulties to children. Actually, solving a word problem isn’t just the execution of the proper arithmetical operation, but involves a lot of previous fundamental actions.

In order to successfully solve a problem, students have indeed to fulfill five abilities (Mayer & Wittrock, 2006):

- *concepts*, which include knowledge of principles, models and categories;
- *facts*, that represent a basic knowledge related to elements or event features (e.g., the amount of centimeters in a meter);
- *procedures*, that indicate the correct and proper way to proceed (as for computation);
- *strategies*, that concern the fastest and most proficient choice concerning the way of proceeding to achieve the solution;
- *beliefs*, last but not the least, which regard the solvers’ cognition about the problem and about their capability to successfully succeed in its resolution.

In relation to the way of proceeding and the steps that have to be carried out in a precise, logical order, Mayer and collaborators (Mayer, 1983, 1987, 1998; Mayer, Larkin, & Kadane, 1984) have identified, within the theoretical frame of *Human Information Processing*, four key cognitive processes. Specifically, the authors posited the solving procedure to begin with the problem *encoding* phase, which consists of two processes:

1. *problem translation*, which involves the translation of the problem text into a solver's own internal representation. This step requires the individual to possess linguistic capabilities, but also adequate semantic knowledge through which infer the implications of a certain problem sentence. In other terms, each statement of the problem, if relevant for the problem itself, has to be put in relation to an arithmetic equation and thus undergoes an appropriate arithmetical transposition;
2. *integration*, which requires the solver to put in a coherent relation all the problem sentences, hence obtaining an integrated and overall view of the problem itself.

The second essential phase is represented by the process of *solution research*, characterized by two sequential steps:

3. *solution planning*, where the solver needs to design a solution plan, figuratively but also graphically (if helpful), and generate different aims that lead, one after the other, to the solution. If the problem is similar to problems whose solution procedure is known, it can be traced back to them in order to recall from memory an already applied solution planning;
4. *solution execution*, which is the last step to be carried out and consists in the determination and execution of the arithmetical operations that are required. In the academic setting frequently happens that the greatest attention is dedicated to this last step, whereas all the previously mentioned processes, fundamental for a successful solving, are frequently ignored (as, consequently, possible difficulties at that level).

More recent studies have empirically tested Mayer's model, with a partial revision of the same (see Lucangeli, Tressoldi, & Cedron, 1998). For instance, it has been identified an additional crucial component that is represented by that of *categorization*, which entails the identification of the deep problem structure and schemata. This process seems to represent the core of problem solving and has been observed to be frequently deficient in poor problem solvers (Hinsley, Hayes, & Simon, 1977; Passolunghi, 1999; Passolunghi, Lonciari, & Cornoldi, 1996).

As easily deducible, problem solving is one of the mathematical competences to pose greater demands on students, since a variety of even different abilities is required at each step. As a result, if more basic skills aren't mastered, problem solving can't be successfully performed. Therefore, in order to have a global idea of the possible difficulties emerging at different levels within the field of math, the next session will briefly examine what happens when learning mathematics encounters some obstacles.

1.1.4 When learning mathematic is too difficult: Mathematical learning disability

It is not infrequent to find, within a typical classroom, children struggling a lot with math. It is indeed estimated that in the Italian setting about 20% of children (4-5 children per classroom) experience difficulties in learning calculation skills (Lucangeli, Dupuis, Genovese, & Rulli, 2006). Some of these children, through effort, work and external support can overcome possible obstacles, but others can surrender in face of continuous failure. But what distinguishes the former from the latter students' categories? In the first case, the condition is more likely to be represented by a *difficulty* affecting math learning. This term indicates a transient, reversible condition often due to causes that are external to children, for instance socio-economic status, cultural background, or problems attributable to teaching (Cornoldi, 1999). Normally, with the appropriate education methods, children can recover the learning delay and achieve an adequate performance level.

These temporal difficulties have however to be distinguished from a more serious condition, whose characteristic feature is the resistance to common treatments. That is, despite students can invest great effort, they don't succeed in overcoming their learning obstacles. In this case, rather than of a difficulty, we talk about a real disability, commonly termed *Developmental Dyscalculia* (DD) or *Mathematical Learning Disability* (MLD).

As defined by the neuropsychologist Temple (1992), with developmental dyscalculia we referred to a disease regarding arithmetic and numerical abilities which manifests in children with adequate intelligence level and without neurological damages. The word "developmental" suggests this condition to arise along with child's cognitive development and to be not the consequence, for instance, of a brain damage (in that case it is indicated as *acquired dyscalculia*). The definition given by Temple partially coincides with those included in the International Classification of Diseases (ICD-10, WHO, 2010) and in the Diagnostic and Statistical Manual of Mental Disorders (DSM-V, American Psychiatric Association, 2013). Only in its last edition, DSM provided a direct reference to problem solving capacities, whereas previously the main focus was represented by the obstacles in the field of computation (as also hinted by the term *dys-calculia*).

Despite usually not detected or underestimated, this condition is very represented in the school population, with prevalence comparable to that of the other learning disabilities (LD, see dyslexia). In fact, while previously reported prevalence ranged from 1% to 6% (see Kavale & Forness, 1996), more recent data support an increasing prevalence of the phenomenon, presented in the 6-10% of students (Mazzocco, 2007), even if the identification varies according to definition and diagnostic criteria (Murphy, Mazzocco, Hanich, & Early, 2007).

Output difficulties depend on to the severity and subtype of the disease, but generally encompass weak mastery in learning math concepts, applying procedures and strategies, retrieving math facts or formulas, remembering data and solving word problems, particularly when a lot of information has to be handled. More pervasive deficits regard also the inability to count, correctly write or read Arabic numbers (number dysgraphia and dyslexia, respectively) and to understand the concept of number and related semantic meaning.

The early detection of this condition is fundamental and derives from surveying typical warning signs (e.g., drawbacks in classifying objects by shape or other physical features, in comparing objects by size or numerosity or in learning the counting sequence) or by monitoring the cognitive abilities at the basis of this learning process. The earlier the detection, the better will be the improvement success if the proper intervention procedures are applied.

In fact, in addition to the negative repercussions in academic performance, the consequences of a learning disability can be detrimental also from other viewpoints. From the perspective of psychopathological diseases, disabled children are observed to manifest internalizing behavior characterized by anxiety, panic attacks and social phobia, or externalizing behavior, for instance oppositional-defiant or conduct disorders, which favor school maladjustment and abandonment, but also addiction and delinquency. As a result, the impact of such disabilities spreads wide to embrace all aspects of children’s life. In line with this considerations, a section of this thesis has been dedicated also to non-cognitive aspects supposed to be related to math learning proficiency, because research and clinical aims haven’t to be merely the recovery of learning difficulties *per se*, but rather the global children’s well-being.

1.2 Math learning precursors

A consistent branch of research within the mainstream of mathematical learning is dedicated to the investigation of the cognitive abilities being significantly involved in this process. These abilities are also defined in terms of *math learning precursors*, since they allow the acquisition of math competences and, measured even before the beginning of formal instruction, are suitable in the early prediction of future math performance.

Generally, the main subdivision within cognitive precursors is that between domain-general and domain-specific abilities. *Domain-general* mechanisms are termed in this way for their extensive recruitment in relation to learning processes in general. In fact, they include, among the others, cognitive skills as working memory, intelligence, and processing information speed which are shared also by other learning processes such as reading and writing (e.g., Ackerman, 1998; Lubinski, 2000). *Domain-specific* precursors, on the other hand, are specific to a particular domain of learning, mathematics in this case. Actually, they include abilities pertaining to understanding and manipulation of quantities and numbers. Domain-general and domain-specific abilities (hereinafter indicated respectively as *general* and *specific*) have been observed to interact in order build academic competences (Ferrer, Shaywitz, Holahan, Marchione, & Shaywitz, 2010).

1.2.1 General precursors

1.2.1.1 Memory

One of the mainly studied cognitive abilities in the field of learning processes is represented by *working memory*. The word "memory" suggest this ability to be involved in retaining information, but working memory is something more than this. Every time we have to hold in mind a particular information and somehow manipulate the same, we are making good use of our working memory. Now it would be easy to capture this exhaustive (but not unique) definition of working memory, expressed by Swanson (2006) as:

> "*a processing resource of limited capacity, involved in the preservation of information while simultaneously processing the same or other information.*"

This cognitive capacity permeates a lot of aspects of our everyday life and we recruit it often without being aware of that. But let imaging how deleterious will be the consequences of lacking working memory. This is partly the topic of the present paragraph, where is described the influential role working memory in the learning context but also problems arising when the same doesn't work in the proper manner.

1.2.1.1.1 From the first memory operationalizations to Baddeley's model

Starting from the 1970s, we have assisted to an increasing necessity to operatively define memory. One of the first organic models was that proposed by Atkinson & Shiffrin (1968), which referred specifically to *short-term memory* and *long-term memory*. Their model includes, in fact, three subsequent stores in which a certain information is firstly received from the sensory system in the first store and, if sufficiently salient to the subject, is then shifted to the short-term memory store. Here the information is destined to rapid decay if additional processes (such as information reiteration and recall) don't occur. If this instead happens, information reaches the long-term memory store, where it is maintained for several years (when not for the entire life) and from which it can be recalled when requested.

However, this pioneering model had leaved opened some questions and appeared not exhaustive in terms of different modality and encoding processes, since it treaded memory as a unitary component. In order to overcome these limitations, Baddeley (Baddeley, 1986; Baddeley & Hitch, 1974), one of the most prominent memory experts, proposed and subsequently revised (Baddely, 2000, 2007) a new working memory model, which nowadays is still one of the most known and cited (see Figure 1.3). This author recently (2007) defined working memory has a capacity subjected to the control of attentional resources and having an active role also in complex thought, thus in higher order thinking and reasoning. In this new definition, working memory role is even more exalted.

The original Baddeley's working memory model consists of three main components. *Central executive* is the key component associated to higher processes and responsible of the attentional control of the whole system. This component, beside a certain storage capacity (not contemplated in the last model revisions), is thought to allocate resources between all memory components by focusing, dividing or switching attention. Central executive controls indeed the other two components, defined, for this reason, *slave subsystems* and dedicated to temporary storage of the information. Here, the memory trace is momentarily held but subjected to rapid decay. More in detail, the component responsible for storing verbal information is the *phonological loop*, in turn divided in two subcomponents: *phonological store*, which temporary holds verbal material, and the *articulatory rehearsal* mechanism, that recites the verbal information in order to slow down its decay. The other component, the *visuo-spatial sketchpad*, is responsible of the maintenance of both visual and spatial information, demonstrated to be reliant on separate mechanisms that don't selectively interfere with each other (see Logie & Pearson, 1997). Within various interpretations on the nature of visuo-spatial memory, Logie (1995) proposed the existence of a *visual cache*, retained to hold visual information such as color and shape, and of an *inner scribe*, assumed to maintain information about movement and to carry out rehearsal in order to reduce time-related decay.

This suggested tripartition of working memory found robust empirical support not only in adults, but also in children aged 6 to 15 (Alloway, Gathercole, & Pickering, 2006; Gathercole, Pickering, Ambridge, & Wearing, 2004), suggesting that even early in life the three main components can be dissociated and partially work in an independent manner. However, memory, as other cognitive resources, develops and strengthens as children grow up and reach maturation, and it is therefore easy to imagine how tasks that appear low-demanding for adults can instead determine a huger cognitive load in children. Therefore, when developing or selecting task addressed to children, attention has to be dedicated to item selection.

Before concluding, also the fourth working-memory component introduced in a more recent revision of the model (Baddeley, 2000) has to be mentioned. This newly-introduced component is represented by the *episodic buffer* and is labeled as a multimodal temporary store not specialized for a specific kind of information, but dedicated to bind information from different sources. Its capacity seems to be limited to the number of chunks or episodes that have to be simultaneously held.

Figure 1.3 Baddeley's Model

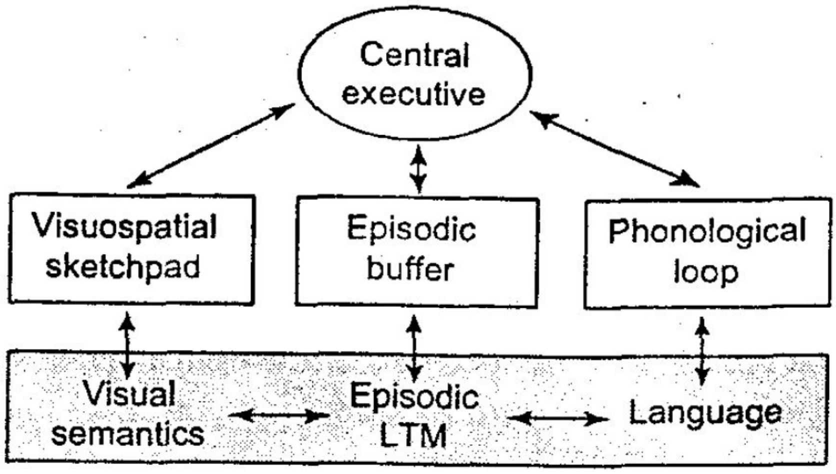

Figure 1.3 Schematic representation of the revised Baddeley's working memory model. From Baddeley, 2000.

1.2.1.1.1.1 Executive functions

Concerning central executive, researchers are not in agreement in relation to the extent with which it can be identified with the so called *executive functions*. These are defined as "processes that control and regulate thought and action" (Friedman et al., 2006) and are assumed to ascribe distinct but related sub-skills, like information switching and inhibition, planning, problem solving and fluency. According to Pennington and Ozonoff (1996) who identified, among the others, different abilities supposed to pertain to executive functions, also working memory can be included in the spectrum of such abilities. Moreover, empirical evidences highlighted that Baddeley's central executive overlaps with working memory expressed in the form of inhibitory and attentional processes, so it is proposed the interchangeability of the two terms, being conceptually the same (Conway & Engle, 1994).

Similarly to adults, also in children distinct executive functions have been distinguished (Lehto, Juujärvi, Kooistra, & Pulkkinen, 2003). Nevertheless, in children a less sharp dissociation between different abilities is observable. Factorial analyses have however revealed that a strong component is represented by central executive, which is quite stable across development, in particular between 5-6 years of age when important cognitive changes occur (Usai, Viterbori, Traverso, & De Franchis, 2013).

Central executive has been also associated to the *Supervisory Attentional System* (SAS), put forward by Norman & Shallice (1986). This contributes to direct behavior when planning and reasoning in novel situations are needed and recruits attention to direct choice selection when competing solutions are envisaged.

1.2.1.1.2 Cornoldi and Vecchi's model

Among the various working memory models that have been proposed, the one theorized by Cornoldi and Vecchi (2003) deserves to be mentioned. This model, also known as model of *continua*, has been developed by the authors in order to fill the too sharp distinction between the different Baddeley's components. Moreover, there was the necessity to diversify the wide spectrum of stimuli and elaboration processes, reduced to a few in Baddeley's model.

Figure 1.4 Cornoldi and Vecchi's Model

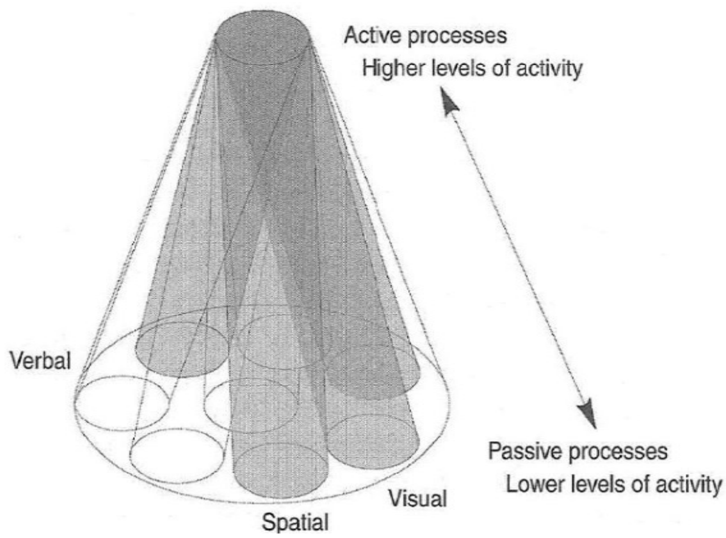

Figure 1.4 Schematic representation of the working memory model proposed by Cornoldi & Vecchi. From Cornoldi & Vecchi, 2003.

This model thereby assumes information to be process along a *continuum*. The graphical representation (illustrated in Figure 1.4) is that of a cone, where a horizontal and a vertical continuum can be identified. The *horizontal continuum* is dependent on the kind of information to be processed. In particular, there is a diversification according to the type of material to be processed, so to the information format. For example, within verbal modality, different can be the representation of either words or numbers. The second axis is represented as a *vertical continuum*, along which information is progressively more actively elaborated and thus requires increasing cognitive resources. In other terms, higher control processes have to be applied as information elaboration raises. This implies tasks at the base of the cone to require low-order, almost passive processes, roughly identifiable with the slave systems of Baddeley's model, whereas more active tasks placed in upper positions along the vertical axis involve higher control and can be therefore traced back to central executive. Proceeding along this axis, the nature of information, so important for low-order processes, becomes progressively irrelevant for data elaboration. That is, more complex activities are less anchored to the nature of information.

Despite some conceptual differences between the two illustrated models, that of Baddeley and of Cornoldi and Vecchi, both have found strong empirical support and have received great consensus. Some authors (e.g., Bull et al., 2008; Gathercole & Alloway, 2006; Passolunghi & Siegel, 2001), by taking as reference Baddeley's model and also the identification of low and high control in that of Cornoldi and Vecchi, have suggested also a terminological distinction within working memory. In fact, just because the two slave systems, phonological loop and visuo-spatial sketchpad, are dedicated to passive retention processes, they can be more properly expressed in terms of *short-term memory*, whilst the label working memory should be maintained only for more active processes closely pertaining to central executive. This distinction has been maintained also in the studies we conducted and that are reported in the current dissertation. Anyway, since the original Baddeley's model is that to which the studies we cited made reference, for sake of simplicity this terminological definition has been maintained in describing such studies, without making distinction between working memory and short-term memory.

1.2.1.1.3 Memory and mathematical learning

As previously hinted, memory is fundamental almost in every learning process and it is particularly crucial for academic learning. Children progressively acquire new concepts and capabilities in a way that is strictly dependent on the cognitive capacities they possess in that particular stage of development. Despite also other general abilities, such as *processing speed* (meaning the ease with which cognitive tasks are performed, so the efficiency of the underlying cognitive processes), have been observed to be significantly related to math achievement (e.g., Fuchs et al., 2006; Hecht, Torgesen, Wagner, & Rashotte, 2001; Passolunghi & Lanfranchi, 2012), memory has been proven to be especially crucial at various levels in a wider extent than other general abilities. For this reason, the current thesis has been specifically focused on its handling and evaluation.

Actually, a wealth of evidence supports the significant involvement of working memory in math learning (e.g., Bull & Scerif, 2001; Gathercole & Pickering, 2000a; Holmes & Adams, 2006). Moreover, its fundamental role is corroborated by findings on children affected by MLD who are observed to manifest severe working memory deficits (e.g., Bull, Johnston, & Roy, 1999; Gathercole & Pickering, 2000b; Geary, Hamson, & Hoard, 2000; Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001).

This section examines the role of working memory by specifically referring to Baddeley's model and related components. Firstly, the global role of working memory components in math learning processes is treated and then the contribution in more specific math competences is examined.

From the introductive description of the central executive and of the abilities pertaining to the same, it can be easy to envisage the essential recruitment it can hold in relation to math skills acquisition. As a matter of fact, various researchers have identified this component to contribute even uniquely to individual math performance differences at various levels (e.g., Bull & Scerif, 2001; De Smedt et al., 2009; Gathercole & Pickering, 2000a; Passolunghi, Vercelloni, & Schadee, 2007), given its involvement in allocating attentional resources and contributing to the selection of the most appropriate strategy to be applied (e.g., Bull et al., 1999).

On the other hand, more contrasting results linger on the significant contribution of either slave systems. The phonological loop, in particular, was shown to have a significant impact in primary school math achievement (Hecht et al., 2001; Swanson & Sachse-Lee, 2001); in some studies, anyway, it lost significant involvement when controlling for other factors, such as central executive abilities (Holmes & Adams, 2006), or seemed to have an influence only in the extent to which it contributed to reading capacity (Swanson & Beebe-Frankenberger, 2004).

Furthermore, a large body of literature reported the recruitment of the phonological component to be limited in comparison to that of the visual-spatial sketchpad thought to assume a major role (Bull et al., 2008; Jarvis & Gathercole, 2003; Rasmussen & Bisanz, 2005), in particular when considering the spatial rather than the visual subcomponent (e.g., Passolunghi & Mammarella, 2010, 2012). The importance of the visuo-spatial aspect is such that a sub-type of dyscalculia is defined in terms of spatial deficiencies (Geary, 1993) which are responsible, among all, of difficulties in interpreting the positional, base-10 meaning of numbers (Russell & Ginsburg, 1984).

Anyway, it is important to stress again that the involvement of either working memory components has to be deeply explored by considering the developmental perspective. Before the age of 7, in fact, children were observed to possess visuo-spatial capacities stronger than the verbal ones. Starting from this age, verbal abilities and spontaneous verbal rehearsal strengthen and consequently also the reliance on these newly-mastered capacities. It is therefore likely for younger children to mainly rely on visuo-spatial representations (McKenzie, Bull, & Gray, 2003) and process also numerical data by means of abstract visuo-spatial representations (Temple & Posner, 1998). On the contrary, the greater role of the verbal component can emerge in the following grades (see De Smedt et al., 2009; Holmes & Adams, 2006; McKenzie et al., 2003).

Opposite outcomes however emerged from Meyer et al. (2010), where the phonological loop was a strong predictor in second grade and visuo-spatial sketchpad in the third, and from Reuhkala (2001) founding a consistent visuo-spatial memory involvement in older children, in particular during the performance of geometrical exercises.

In general, the involvement of the two slave components appeared to be more limited and selective with respect to that of central executive. Impairments affecting this component were indeed confirmed to lead to math learning disabilities in general (Geary, Hoard, Bryd-Craven, Nugent, & Numptee, 2007), whereas deficiencies affecting either slave systems cause more selective deficits (Geary, 1993).

Results are anyway inconclusive. In fact, it is fundamental to point also out that the major role of one component with respect to the other can tightly depend on administered tasks tapping both precursor and math skills. For instance, in Meyer et al. (2010) the phonological loop could be observed to contribute to poor math reasoning for the reason that the selected tasks required a significant verbal processing. More difficult memory tasks can indeed involuntary involve also central executive, despite developed to specifically tap slave systems.

Another important consideration pertains the kind of information children have to process. For example, within verbal memory, some authors proposed a significant prediction of math learning given not by the overall component, but specifically by the capacity to retain numerical information. Memory for digits, which nature is obviously highly pertinent to math, was frequently significant associated to math competence when general verbal memory did not (e.g., Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Passolunghi & Cornoldi, 2008; Siegel, Pinkerton, Ford, & Anderson, 2012; Temple & Sherwood, 2002). Digit span scores were also found to be correlated with math performance in MLD children (Andersson & Lyxell, 2007).

In line with this, a criticism concerning the involvement of working memory in math learning has been raised by Landerl, Bevan and Butterworth (2004), who rebutted a significant involvement of this precursor, affirming its significant role to be found out only when related tasks involve numerals. Anyway, a lot of studies, hereinafter cited, demonstrated the significant role of working memory even when assessed by tasks not involving number-related information.

1.2.1.1.3.1 Relevance of memory in computation and word problem solving

Not all aspect of mathematics require the same abilities to be performed or share the same precursor skills. For instance, procedural calculation and word problem solving, despite joining some key features, pose different challenges since the former is already sets up for solution, whereas the latter requires preliminary actions before achieving the result. In this sense, it is a higher-demanding procedure, which requires more active processes and more intense cognitive load to be carried out.
Anyway, it can be generally stated, in line with von Aster and Shalev (2007), that general and specific precursors are both involved in math achievement, but give differential contribution in relation to the kind of math skills that is considered.

More in detail, working memory seems to be essential in both computation and word problem solving (e.g., Fuchs et al., 2010; Meyer et al., 2010; Swanson, Cooney, & Brock, 1993).

Central executive is indeed required during complex operations involving carrying procedures (Ashcraft, 1992; Geary, Frensch, & Wiley, 1993) and, in relation to problem solving, appears to be engaged in translating word problem sentences into the correspondent arithmetical equations and in performing all the subsequent steps required for the solution.

Phonological loop, on the other hand, is recruited in the basic processes of computation, for instance in counting, transcoding from symbolic to verbal formats, and rehearsal of partial results (Heathcote, 1994). Maintenance of phonological information favors the storage of arithmetic facts in long-term memory (Siegler & Shrager, 1984) and their subsequent recall.

Finally, visuo-spatial sketchpad is recruited during written multi-digit computation (Heathcote, 1994), from the correct identification and interpretation of the arithmetic sign to the association between problem statements and related equations (Oakhill & Johnson-Laird, 1984). Visuo-spatial systems, particularly in children, are also supposed to be engaged in accessing numerical information (Temple & Posner, 1998).

By more deeply exploring the contribution of the working memory components one with respect to the other, significant outcomes emerged. For instance, Fuchs et al. (2010) extensively examined, by means of a complete working memory battery, which of these cognitive abilities predict calculation and problem solving skills in first graders. The study revealed that central executive predicted both competences. In relation to calculation, this component appeared essential in allowing the access to number combination solutions stored in memory while performing other required intermediate steps. In relation to problem solving, central executive has been observed to be fundamental at different levels, culminating with the model construction and the solution strategy selection.

Developmental changes have also observed to occur. Some studies showed that in both first and second grades central executive is the stronger predictor, at least in math reasoning and also when accounting for the two slave components (Swanson, 2006). Other findings suggest arithmetic reasoning to be significantly influenced by central executive capacities in younger but not in older children (Henry & McLean, 2003). In the latter, a great role seems to be assumed by the phonological loop, since articulatory suppression has been observed to impair children's solving capacities (Adams, Hitch, & Donlan, 1998).

Meyer and colleagues (2010) instead highlighted a decreased role from second to third grade of central executive and phonological loop in both computation and problem solving, whereas the visuo-spatial sketchpad seemed to remain as the unique significant predictor.

In another research (Swanson, 2006), the predictive power of central executive assessed in first grade has been recorded in relation to problem solving skills, while visuo-spatial sketchpad abilities measured one year later were observed to predict calculation skills. it could be noted that the visuo-spatial sketchpad better predicted number combinations, while central executive problem solving mainly. Anyway, in some studies, the association between working memory and problem solving capacities almost disappeared when controlling for mastery in reading (Fuchs et al., 2006; Swanson et al., 1993).

1.2.1.2 Intelligence

1.2.1.2.1 Definition and evaluation

Intelligence is a very multifaceted construct, complex to be defined, since it involves a broad spectrum of even different capacities. One of the possible definitions is that proposed by an *ensemble* of researches who collected all definitions emerged in the field (see Mainstream Science on Intelligence, 1994). They delineated intelligence as:

> "*A very general mental capability that, among other things, involves the ability to reason, plan, solve problems, think abstractly, comprehend complex ideas, learn quickly and learn from experiences. […] it reflects a broader and deeper capability for comprehending our surroundings.*"

Intelligence results therefore to be the key ability that allows us to adapt in difference and new situations, to solve different kinds of problems, understand situations, cope with difficulties, reason and plan. Similarly, and by considering the developmental perspective, Piaget (Piaget & Inhelder, 1973) defined this construct as the capability to adapt to reality both external, by modifying the own pre-existing mental models, and internal, by integrating and coordinating the different mental structures.

But intelligence is also more than this. More recent views have, for instance, introduced the concept of *emotional intelligence* (Goleman, 1995), corresponding to the capacity to understand ourselves but also feel sympathetic with others, to understand their behavior. That is, intelligence is a multiform construct that embraces abilities even very different in nature. In line with this, the leading psychologist David Wechsler, who developed one of the widely used test battery for the assessment of intelligence (named WISC test), in his last definition (1981) expressed this capability as a global capacity that predisposes the individual to comprehend the world and to face up challenges. However, he was also aware, after many ages of studies on the topic, of the fact that intelligence is a function of the whole personality that is also sensitive to factors that go beyond the cognitive domain. Anyway, for the purposes of studies investigating the role of intelligence abilities in academic learning processes, intelligence is mainly treated by considering only its cognitive nature, but it has to be kept in mind that this is only one aspect of the overall intelligence.

In learning and school settings, as in other contexts, defining the individual's intellective capacities can be a crucial step also from the clinical viewpoint, for the reason that both low and high intelligence level can help in explaining possible problems emerging in students struggling at school.

1.2.1.2.2 Theories of intelligence

Discussing theories of intelligence is beyond the scope of the current dissertation. Therefore, only a few hints will be provided in order to capture the essential information necessary to understand the involvement of this construct in the learning processes.

Almost one century ago, emerged the idea of intelligence as a complex construct which included components or factors that could be differentiated but all referring to a unique and more general component. The first psychometric theory providing this factor identification came from Spearman (1923), who observed that scores achieved by children in different school subjects were highly correlated. For this reason, Spearman posited the existence of a single general factor, labeled as *g*, supposed to underlie a wider number of narrow task-specific factors, named *s*. Despite various criticisms this theory received (where *g* is viewed more likely as the result of a statistical artifact, see Gould, 1996), nowadays the concept of *g* is still widely used also in research settings.

A consequent intelligence theory, proposed by Cattell (1971), postulated a further distinction within general intellectual abilities, thus identifying the aspects of *crystallized* and *fluid* intelligence (and adding, in a second moment, a broad range of both general and more specific abilities, see Cattell-Horn-Carroll theory). *Crystallized* intelligence is retained to include abilities which are typically knowledge-based and rooted on previous learned concepts or experiences (e.g., mastering a rich lexicon is considered an intelligence aspect, but is a capacity resulting from teaching and learning), whereas *fluid* intelligence regards the capability to solve problems and face up with situations that are new and unfamiliar. In this case, it isn't possible to draw on previous experience, hence new solutions have to be developed and experienced. The identification of these two intelligence components is interesting since different studies have examined the differential role they assume in the context of learning, as afterwards pointed out.

1.2.1.2.3 Intelligence and learning

Intellectual capabilities, as highlighted by Ackerman (1988), are fundamental for different kinds of learning, in particular during initial phases, while the effect is likely to reduce with sufficient practice. Intelligence appears fundamental, for instance, in de-contextualization, transfer processes, thus when individuals gain knowledge of a certain rule in a specific context and have to learn how to generalize and apply it also in other contexts (Jensen, 1998). Actually, *g* is assumed to reflect individual differences in processing information, so the efficiency of the mental processes through which knowledge and skills are acquired.

The importance of intelligence in academic learning was clear already at the beginning of the previous century, when Alfred Binet (1905) was requested to develop a proper battery of test (used till nowadays in a revised form, see Stanford-Binet, first ed. 1916) suitable in the identification of gifted versus unfit students with the precise purpose of differentiating their academic activities. In particular, many researches accounted for a principal role of the fluid component in the prediction of school-related success, particularly in relation to mathematics (Kyttälä & Lehto, 2008; Passolunghi & Lanfranchi, 2012; Passolunghi, Mammarella, & Altoè, 2008). Nevertheless, also more crystallized aspects seem to have a significant role (e.g., Passolunghi & Lanfranchi, 2012), but more usually secondary to that of the fluid components (e.g., Passolunghi et al., 2008).

1.2.1.2.4 Relation between intelligence and other cognitive functions

A growing body of literature deals with the relation between intelligence and other cognitive abilities, in particular with memory. There is agreement among these studies in considering the two constructs as being closely related, with the link observed in both directions, meaning that intelligence is shown to predict memory but also vice versa. Fluid intelligence has been indeed observed to be linked to both short-term and working memory (Swanson & Jerman, 2007) and lower intelligence was proven to lead to inferior reading achievement via reduction in memory capacities (Swanson & Jerman, 2007).

Colom and colleagues (2004) found indeed that working memory was one of the constructs to be best predicted by *g*, with the noteworthy correlation value of .94. Others argued, on the other hand, that is the working memory capacity that contributes to individual differences in fluid intelligence (e.g., Engle, Tuholski, Laughlin, & Conway, 1999; Kyllonen & Christal, 1990). Interestingly, Engle et al. (1999) observed only working memory, but not short-term memory, to predict *g*. This finding seems to suggest that only more active, high-control processes are related to intelligence However, also opposite results emerged from Süß, Oberauer, Wittman, Wilhelm, & Schulze (2002), where a link between intelligence and short-term memory was pointed out.

Unsworth and Engle (2004) suggested the possible involvement of short-term memory to be dependent on the difficulty level of tasks tapping it (span task with numerous items to be remembered can require similar capacities that higher-difficulty spans). Other argued that removing common variance between short-term memory and working memory, residual short-term memory loses significant relation with *g* (Engle et al., 1999).

Nonetheless, the significant link between intelligence and memory is such that the latter is one of the components assessed by the last version of one of the tests more frequently used to evaluate intelligence (WISC-IV, Wechsler, 2003). In this perspective, memory is defined as one of the components composing global intelligence.

Before concluding this issue, it is important to highlight that despite the relation between intelligence and memory is widely accepted (even if with uncertainties regarding the direction of this association and the specific link between their components), that between intelligence and number sense, and particularly with more innate approximate abilities, is almost unexplored.

1.2.2 Specific precursors: Number sense

Griffin (2003) stated that mathematical competence arises from the proper integration between three mathematical "worlds": the world of real quantities, the world of counting numbers and finally the world of formal symbols. These "worlds" somehow embrace the specific abilities at the basis of math learning which comprise what it is generally indicated as *number sense*, delineated for the first time by Dantzig (1954) and then more systematically by Dehaene (1997, 2001). In recent years, number sense has received increased attention also from the educational point of view, as its vital role for math success has been recognized and recommendations about its refinement during instruction have been provided (National Council of Teachers of Mathematics, 2000, 2006).

Now try to have a look at the image below (Figure 1.5). This is a number line along which ideally allocate all numbers from 0 to 100. Without having recourse to the assistance of a straightedge or other external aids (and without spending much time on it), try to visualize where the number 38 can be collocated along this line. Do you think to have done a good approximation of the real position? Now you can check the accuracy of your approximation. If this results to be quite good, it is possible to infer that you probably possess a good sense of number.

Figure 1.5 Example of Number Sense Task

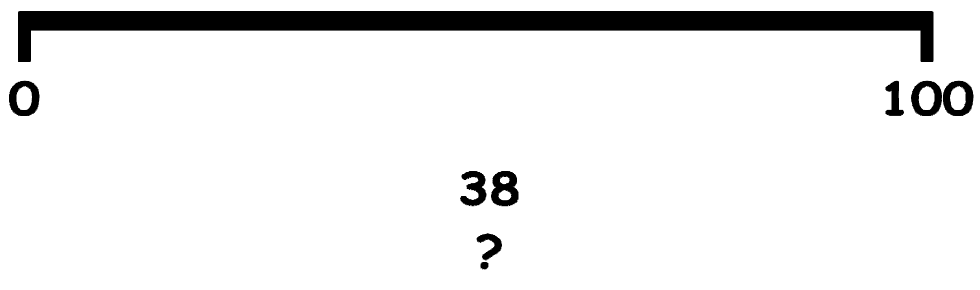

Figure 1.5 Example of number sense task in which it is required to estimate the approximate position of the target number along a number line.

Number sense is thought to emerge "spontaneously without much explicit instruction" (Deahene, 1997). It is indeed present already in newborns, but also shared with other animals, such as pigeons, rats, dolphins and non-human primates, indicating its importance from the evolutionary viewpoint. Having the capacity to grasp, for example, which is the heap with the larger amount of food implies, without any doubt, noteworthy benefits on survival possibilities. Anyway, despite being innate, these skills are supposed to progressively refine with age, cognitive development and instruction (Dehaene, 1997; Jordan et al., 2006).

A precise and unanimous definition of number sense is still lacking (Gersten, Jordan, & Flojo, 2005), for the reason that it includes a wide spectrum of different-order abilities and each author has his own specific interpretation of this concept. Case (1998) stated in fact that number sense is easy to recognize even if difficult to define. Despite Dehaene (2001) defines number sense, for sake of economy, as single and unified ability, it appears as a complex and many-sided construct (Berch, 2005). A possible distinction within number sense skills is that proposed by Berch (2005), who identified low-order abilities, which are related to a sort of perceptual sense of quantity (e.g., perception of numerosities or comparison of numerical magnitudes), and high-order abilities, which entail an acquired sense-making related to mathematics (meaning, for instance, the capacity to deeply understand math concepts and relations, or be confident with operations and procedures). For a wider and more complete list of number sense skills see Berch (2005).

A stricter definition of number sense disentangles it from math-related competences that are a consequence of formal, even if very basic teaching and that for instance include counting or understanding and performing simple arithmetic operations. These skills are better defined by some authors as *early numeracy* (Griffin, 2003; Ruijssenaars, van Luit, & van Lieshout, 2004), while maintaining the definition of number sense only for innate and spontaneous capacities.

Anyway, it turns out that making a clear distinction between number sense and math skills is a hard task, since the former is the base of the latter and can be viewed either as a precursor or part of math competence. Also the operationalization of number sense is therefore difficult, not only because it is represented by even different capacities, but also because there is usually a subtle boundary between what is measured as a precursor and what is defined as an outcome math measure.

1.2.2.1 The Approximate Number System

An important component characterizing number sense is represented by the so-termed *Approximate Number System* (ANS), which can be defined as an evolutionary ancient innate system entailing approximate number-related judgments. Within ANS, well represented are the abilities of estimating which can approximately be the numerosity of a certain set of elements (e.g., of the marbles on the table) or comparing arrays of elements in order to rapidly and approximately judge which is, for instance, the most numerous (e.g., Are there many green or red marbles?).

As can be inferred, ANS can be considered roughly independent of symbolic number representations. This account is also proven by the fact that ANS is developed also in populations with a reduced number lexicon, as evidenced in the study of Pica et al. (Pica, Lemer, Izard, & Dehaene, 2004) conducted on an Amazonian indigene group, who can perform approximate and non-symbolic comparison and addition with numerosities beyond their number naming range (contrary to exact arithmetic).

Despite double dissociations can be operated within number sense aspects, implying that both approximate and exact skills can be represented in both symbolic and non-symbolic aspects, in the present dissertation has been kept the subdivision between lower, more immediate capacities and the formal, instruction-based ones. That is, approximate skills are treated only in relation of their non-symbolic aspect, whereas the symbolic aspect is associated to exact skills. Keeping in mind this consideration, in the paragraph below are examined more in detail which are some of the key abilities pertaining to ANS.

1.2.2.1.1 Magnitude comparison and approximate addition

A common ability associated to ANS entails the capacity to catch a non-symbolic magnitude or numerosity (typically the number of dots in the experimental setting) and compare it with another one in order to judge, at a glance and without counting, which one is the most numerous. Comparison skills are also at the basis of simple, approximate arithmetic, performed without necessarily know numbers or formal arithmetical procedures. Performance in approximate comparison, addition and subtraction showed high correlation (Iuculano, Tang, Hall, & Butterworth, 2008).

Considering approximate computation, the most typical experimental setting involves approximate addition, where two arrays of dots appear in succession on the visual field and are then hidden in the same place. Individuals have, consequently, to mentally visualize the two numerosities together, thus to sum them in order to compare their whole amount with a third array of dots finally presented. The experiment conducted by Barth and collaborators (2005) proved that 5-year-olds can successfully perform above chance approximate addition in both visual and cross-modal formats (two visual arrays that sum together and have to be compared with a sequence of tones). Anyway, so young children seem to be not so smart in performing approximate subtraction, since performance is lower than for both comparison and addition, as highlighted in Barth et al. (Barth, Beckmann, & Spelke, 2008).

Quantities discrimination is dependent of the numerical ratio between the two quantities to be compared (Moyer & Landauer, 1967). This means that the closer the two numerosities, the smaller the ratio and the greater the difficulty to discriminate them. In other words, as the relative ratio reduces, judgments will become more noisy, with increased error rates and/or reaction times for correct responses. The capacity to discriminate close numerosities is also defined *numerical acuity*.

As easily surmised from the preliminary remarks, discrimination ability sharpens along with development and exercise (see Figure 1.6). The finest numerical discrimination ratios reached in adulthood are 7:8 or, utmost, 9:10. This means that adults can discern, rapidly and without counting, two arrays of 7 vs. 8 elements, or possibly, also 9 vs. 10 (and related multiples). These ratios are often expressed in terms of Weber fraction, which measures the finest numerical change that can be detected. By referring again to studies employing the habituation-dishabituation paradigm, these demonstrate that 6-month-old infants can discriminate arrays of dots in a 1:2 ratio (e.g., Xu & Spelke, 2000), but fail in presence of a 2:3 ratio, which begins to be discriminated at 9 months of age, in both auditory and visual modalities (e.g., Lipton & Spelke, 2003). Discrimination accuracy progressively refines till reaching, at the entrance to primary school, the ability to discriminate numerosities which instantiate a 6:7 ratio. Summing up, numerical acuity rapidly asymptotes up to the first year of life and usually increase more gradually throughout early childhood, with discrimination becoming more precise prior to the onset of language.

Figure 1.6 Developmental Curve of ANS Accuracy

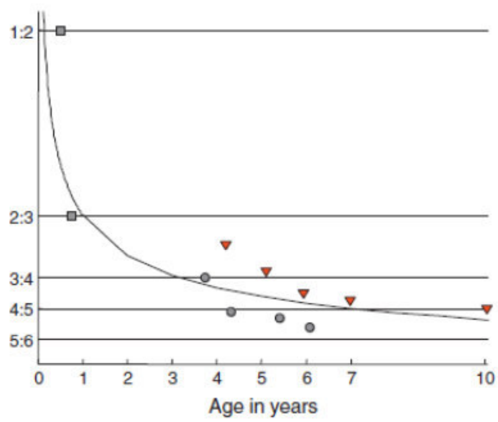

Figure 1.6 Developmental trajectory of the increasing discriminating abilities in children. On the *y* axis numerical ratios are reported. From Piazza, 2010.

1.2.2.2 Number sense and formal math learning

General agreement seems to exist on the involvement of number sense in the development of formal math. In fact, as previously argued, number sense is at the basis of subsequent formal math learning and various authors (e.g., Butterworth, 2005; Landerl et al., 2004; Wilson & Dehaene, 2007) support the theory that developmental dyscalculia origins from a "core deficit" affecting these specific capacities.

Anyway, research hasn't been conducted in the same ample extent of working memory. As a result, number sense has been more frequently observed in relation to basic formal math skills including arithmetic, whereas lacking are data exploring its involvement in more complex processes such as word problem solving. There is however consensus in assuming number sense to be crucial also at higher levels of math education, since it allows the application of sophisticated algorithms and procedures while solving complex numerical operations and problems (Berch, 2005).

Various are the studies reporting number sense to be able to predict math-related success in first grade, especially when measured at kindergarten (e.g., Jordan et al., 2007), and also computational fluency by second grade, by also successfully predicting the condition of risk (Locuniak & Jordan, 2008). Interestingly, number sense has been proven to influence math achievement even over and above general precursors such as intelligence and working memory (Jordan, Glutting, & Ramineni, 2010; Locuniak & Jordan, 2008).

Major results concern anyway symbolic abilities (e.g., De Smedt et al., 2009). For instance, Mazzocco and Thompson (2005) revealed that symbolic magnitude comparison, counting and reading and writing numerals assessed in kindergarten are the skills acting as strongest predictors of math performance in third grade.

1.2.2.2.1 ANS and formal math learning

Even if it is almost widely accepted that number sense, interpreted as a single and multifaceted ability, has a significant role in learning mathematics, major divergences stand in relation to the approximate skills associated to ANS. Actually, there is a strong debate on the topic and recent studies account for opposite outcomes. For instance, Desoete and collaborators (Desoete, Ceulemans, De Weerdt, & Pieters, 2012) proved that the capacity of comparing magnitudes measured in kindergartners is predictive of math competence in the first two grades of primary school, similarly to findings from Mazzocco and colleagues (Mazzocco, Feigenson, & Halberda, 2011a). In the same direction go the results on the ability of making approximate additions at the beginning of kindergarten which resulted to be highly correlated with math achievement several months later (Gilmore, McCarthy, & Spelke, 2010). The study of Libertus and collaborators (Libertus, Feigenson, & Halberda, 2011) was the first one that assessed ANS and math abilities concurrently in very young children (3- to 5-year-old) and proved ANS relevant role to be present even before formal instruction. ANS acuity, anyway, appeared to be fundamental not only at early stages of math learning, but also in older children, as demonstrated in the study of Halberda et al. (Halberda, Mazzocco, & Feigenson, 2008), where ANS assessed in 14-year-old students retrospectively predicted math performance in previous grades.

Results in favor of a significant ANS involvement are corroborated also by the fact that the same was documented to be deficient in MLD children (e.g., Mazzocco et al., 2011b; Mussolin et al., 2010; Piazza et al., 2010). For instance, Piazza et al. (2010) noticed that dyscalculic third graders had approximate abilities comparable to that of younger, 5-year-old children. ANS seemed, however, to be not affected in low achieving students (with math difficulties less severe than that of dyscalculic peers), suggesting that the deficit can be revealed only in presence of very pronounced, deep-rooted difficulties, and probably being the cause of (Mazzocco et al., 2011b).

Nevertheless, while the previous studies globally supported a significant involvement of approximate skills in formal math learning, in the opposite direction go findings from researches that lowered the role of ANS. In particular, the most recent works subject ANS to symbolic skills, meaning that a significant recruitment, if found, however disappears when symbolic abilities are taken into account. In support of this hypothesis, some studies reported MLD children to present impairments in making comparisons between quantities, but only when the same are represented by symbols (Iuculano et al., 2008; Rousselle & Noël, 2007). These authors indeed stated that the cause of a possible math disability hasn't to be attributed to poor ANS, but rather to a failed connection between the approximate representations of numbers and their symbolic depiction, process that normally takes places with formal education. It is therefore hypothesized that symbolic quantities can be mapped on the pre-existing core quantity representation associated to ANS (e.g., Verguts & Fias, 2004). Other authors instead retain that limited approximate skills can be at least the consequence, rather than the cause, of scarce math achievement and thus of deficient symbolic mastering (Le Corre & Carey, 2007).

A developmental hypothesis (e.g., Desoete et al., 2012), could possibly unify the two distant positions on ANS recruitment. In this perspective, basic approximate skills can hold a significant role in early math learning, but lose it in later math skill acquisition. Unfortunately, also this hypothesis appeared to be inconclusive, since various studies have documented the active role of ANS across the entire lifespan (Dehaene, 1997; Halberda et al., 2011). This system has been indeed recorded to activate even in adults during the performance of symbolic tasks as proven by neurophysiological data (Piazza et al., 2007) and can therefore provide the bases for more advanced and sophisticated mathematics (Gilmore et al., 2007).

1.2.2.3 Number sense and other cognitive skills

An intriguing question regards the association between number sense and more general cognitive resources. ANS, in particular, has been seen to develop gradually during childhood in parallel to executive functions (Diamond, 2002), but studies relating both general and specific math precursors are however limited. As a consequence, it is not clear if the development of number sense is related to that of other cognitive functions or if they evolve in a roughly independent way.

About this point, Dehaene (2001b) proposed representations within number sense to become connected with the other cognitive systems as a reflection of development and instruction, thus to be independent in early stages of development. According to Geary, Hoard, Nugent and Byrd-Craven (2008), instead, general precursors as working memory and fluid intelligence intervene in the process of shifting from the internal magnitude representation to the acquisition of formal numbers, and therefore have a crucial role number sense development.

1.3 Non-cognitive aspects relevant for math learning

Another branch of research is dedicated to the investigation of affective processes influencing academic learning and performance. Generally, *affect* concerns instinctual reactions occurring either before or after a certain cognitive process (see Lerner & Keltner, 2000) and includes valence, arousal and motivation. It refers, in fact, to a wide range of beliefs, feelings and moods that are beyond the domain of cognition and which can be either positive or negative. It is easy to image how children endowed with a good cognitive gear can be brilliant not only at school, but in different domains of their life, if they also possess positive personality and behavioral disposition. On the other hand, children potentially clever but characterized by low self-esteem, low confidence in their capacities and by anxious behavior are more prone to failure rather than to the expected success.

Very complex and intricate are the processes that intervene in children's growing as adults and many of these processes take place in the academic context. School is fundamental not only to instruct children but also to shape their whole personality and future behavior. Nowadays, there is a growing awareness about the fundamental role held by emotional and social competences in determining children academic success (Duncan et al., 2007; Raver & Knitzer, 2002), despite the fact that frequently, in the school environment, non-academic skills are not emphasized (Liew, McTigue, Barrois, & Hughes, 2008).

1.3.1 Perceptions about the Self

In quite recent years, many areas of psychology have dedicated increased attention to self-referent phenomena and even more importance begun to be attributed to the *Self* also in relation to selection and construction of the environment (Bandura, 1993). In fact, human action and motivation are thought to be closely linked to processes related to the Self and to be therefore unavoidably determined by it. Beliefs and perceptions about the Self are rooted in the past experience and this is true also in achievement settings. Actually, past experience of failure or success can mould the idea that people develop about their capabilities. Yet, these perceptions easily establish through subsequent reinforcements and considerably act on children's growth and development (Bandura, 1997).

1.3.1.1 Self-efficacy

"People's judgments of their capability to organize and execute courses of action required to obtain designated types of performance" is the definition given by Albert Bandura (1977) of *self-efficacy*, a kind of self-perception particularly relevant in academic contexts. This definition reveals a lot about how individuals stand in front of a given task to be performed, suggesting that the way they set about a certain task can effectively influence the possibilities to fulfill it. In fact, Bandura (1986) also stated that "the types of outcomes people anticipate depend largely on their judgments of how well they will be able to perform in given situations." Therefore, individuals steer their action on the basis of the expectancies of success they have developed.

The characterizing feature of self-efficacy is the task-dependence, meaning that it is specific to a certain task or domain and cannot be generalized in principle. Self-efficacy beliefs are therefore multidimensional and differ on the basis of a given domain of functioning. This construct, as a consequence, regards specific performance capabilities and is not a reflection of personal capacities on the whole.

1.3.1.1.1 Sources and effects of self-efficacy beliefs

But which are the factors responsible of shaping our self-efficacy beliefs? Bandura (1997) has identified four major sources at the onset of individuals' self-perceptions: enactive or mastery experience, vicarious experience, social persuasion, and physiological and emotional states.

As already mentioned, *mastery experience* is the result of past considerations about achieved results and the following interpretation of the same in relation to a certain task or domain. According to Bandura (1986, 1997), it is the most potent self-efficacy source and has a stronger positive effect when individuals successfully perform challenging tasks that are carried out with difficulty by other people. *Vicarious experience* is, instead, the source of self-efficacy beliefs that is influenced by others' performance: by observing other people performing a given task, in fact, individuals can derive information about what they can be able to do (with thoughts like "If my classmates succeed in doing this math exercise, probably I can too."). Also *social persuasion* has an important impact. It entails evaluative feedback given by significant others, such as peers, parents or teachers. Finally, *emotional states* such as fatigue, stress, anxiety, mood changes and related physiological reactions are often negatively interpreted as alarm signs of lack of ability. Bodily arousal can be so predictive, in low self-efficacious subjects, of potential failure, since these people think that they feel in that way (anxious, nervous, sweating, breathless and with accelerated heart rate) because they are not able to effectively face up that situation.

Self-efficacy beliefs, arising from the combination of these sources, are proven to produce diverse effects by acting through four major processes: cognitive, motivational, affective, and selection processes (Bandura, 1992). Of particular interest are the cognitive effects associated to self-efficacy. In fact, although non-cognitive in nature, this construct is however known to significantly interact with cognitive processes. Self-efficacy actually operates by inducing the visualization of anticipatory success scenarios, by stimulating analytic thinking and by favoring self-regulation, strategy choice, goal selection, and cognitive effort.

Beliefs about own capacities are also strongly influenced by the conception individuals form in relation to the nature of these capabilities (e.g., Bandura & Dweck, 1988). In fact, if a certain ability is viewed as fixed, a deficiency at that level is perceived as stable and immutable, and often interpreted as a manifest sign of low intelligence. The consequent response is the choice of tasks which can minimize the error probability in order to avoid a failing performance which can undermine self-esteem.

Motivation is the second crucial aspect influenced by self-efficacy jointly with outcome expectancies, that represent what we expect from a given situation on the bases of the probability of related success. Both are indeed supposed to influence human motivation, one of the strongest aspects driving human action. This can be observed also in relation to learning processes, as put on evidence by Shell and colleagues (Shell, Murphy, & Bruning, 1989). They indeed observed how efficacy beliefs and outcome expectancy could predict, together, the 32% of reading achievement scores.

By influencing motivation, high self-efficacy beliefs drive the choice of activities, the tendency to persist, and the level of effort to be spent when task difficulty raises. Highly self-efficacious students, for instance, participate at activities more readily, select more challenging and difficult tasks, persist longer even in face of failure, work harder, and have less negative emotional reactions in response to encountered obstacles (Bandura, 1997). By maintaining dedication to tasks perceived as difficult (Salomon, 1984), these students significantly and positively influence their learning methods.

In fact, possessing higher self-efficacy leads to increased self-regulation and self-monitoring during learning (Buffard-Buchard, Parent, & Larivee, 1991). Self-efficacious students are more capable in holding and managing their working time, more rapidly establishing a plan to execute a certain task, selecting the best strategies, and self-evaluating their work. As a result, these learners succeed in learning more than their non-self-efficacious peers with the same potential capacities.

1.3.1.1.2 Self-efficacy and mathematics

Till this moment, self-efficacy has been treated on its whole, despite some hints to academic performance. Nevertheless, there is a specific subtype of self-efficacy is that associated to the academic competences and is therefore termed *academic self-efficacy*. This specific self-efficacy has been proven to predict children outcomes in different academic areas, from scientific to literal disciplines (Klassen & Usher, 2010; Pajares, 1996; Pajares & Urdan, 2006). Self-efficacy has been explored also with specific reference to math learning.

Various studies reported in fact the relation between self-efficacy and math achievement in various domains. It has however to be specified that the fact of perceiving efficacious in a given math task can't automatically determine the same perception of overall math competence (Pajares, 1996). Pajares and Miller (1994) found that math self-efficacy was a reliable predictor of problem solving capacities, even more than other constructs such as math self-concept (discussed in the following section). Math self-efficacy was further observed to act as a mediator between the effect of previous experiences (either positive or negative) and actual math performance and also to significantly predict future achievement. Mastery experience has indeed been defined as the most influential source of math self-efficacy and proven to account for a significant portion of variance above and beyond the influence of prior achievement (Joët, Usher, & Bressoux, 2011).

High self-efficacy beliefs pertaining to math have an important impact since they also direct the choice of activities, leading to the voluntary selection of arithmetic instead of other tasks, whilst low self-efficacy frequently induces an avoidance behavior (Bandura & Schunk, 1981). As previously reported, self-efficacy influences also self-regulated learning. Actually, by comparing children with low, intermediate and high math competence, Collins (1982) noticed that children who firmly believe in their capacities were able to more quickly reject defective strategies in favor of the most fruitful during word problem solving. In addition, students with higher self-efficacy level were observed to work more accurately than their peers with low self-efficacy, in a way that attitudes towards math were predicted more by self-efficacy beliefs than actual math competence.

1.3.1.2 Self-concept

Self-concept is a construct very similar to that of self-efficacy, even if important differences subsist. Firstly, self-concept differs from self-efficacy since the latter varies as a function of both specificity and correspondence to given tasks and contexts, whereas the former does not. Self-concept is, in effect, a more general self-descriptive construct incorporating various aspects of self-knowledge and self-evaluative feelings (Marsh & Shavelson, 1985).

Self-concept is a sort of picture that everyone of us depicts about oneself on the basis of how own characteristics are perceived and cognized (Damon & Hart, 1982). Therefore, it is a sort of mirror image that human beings have about themselves (Maslow, 1970) and which includes characteristics such as appearance, abilities, attitudes and beliefs that we think to possess and that are heavily influenced by evaluations given by significant others (Shavelson & Bolus, 1982). It has been proven that children, even when very young (at around 2 years of age), are already capable of making evaluative self-descriptions, despite the acquisition of language is the crucial step allowing judgments about the Self to be better articulated (Harter, 1998).

Despite self-concept had been defined, at the beginning, as a global perception of oneself, it was soon after clear that this unitary view couldn't be applied, as a whole, to some specific domains like student's academic performance. For this reason, a following re-conceptualization of the construct took place in the form of a hierarchical multifaceted construct whose related model was developed by Shavelson and colleagues (Shavelson, Hubner, & Stanton, 1976). Another influential and for some aspects similar self-concept model is that proposed by Harter (1986).

By focusing however to Shavelson's model and its following revisions (e.g., Marsh & Shavelson, 1985), it is possible to note that it entailed more specific domains to be subsumed under the more general ones. Specifically, a *general* self-concept could be represented at the apex of the hierarchy. This general self-perception was hypothesized to be represented by two main facets: *academic* and *non-academic* components of self-concept. These ones could be in turn further subdivided into more specific subcomponents (see Figure 1.7).

More in detail, academic self-concept is represented by the attitudes that people perceive with regard to school-related tasks and subjects, but it identifies essentially with the two main components of mathematics and verbal abilities (Marsh & Shavelson, 1985). On the other hand, non-academic self-concept includes the social, emotional and physical dimensions.

Figure 1.7 Academic Self-concept

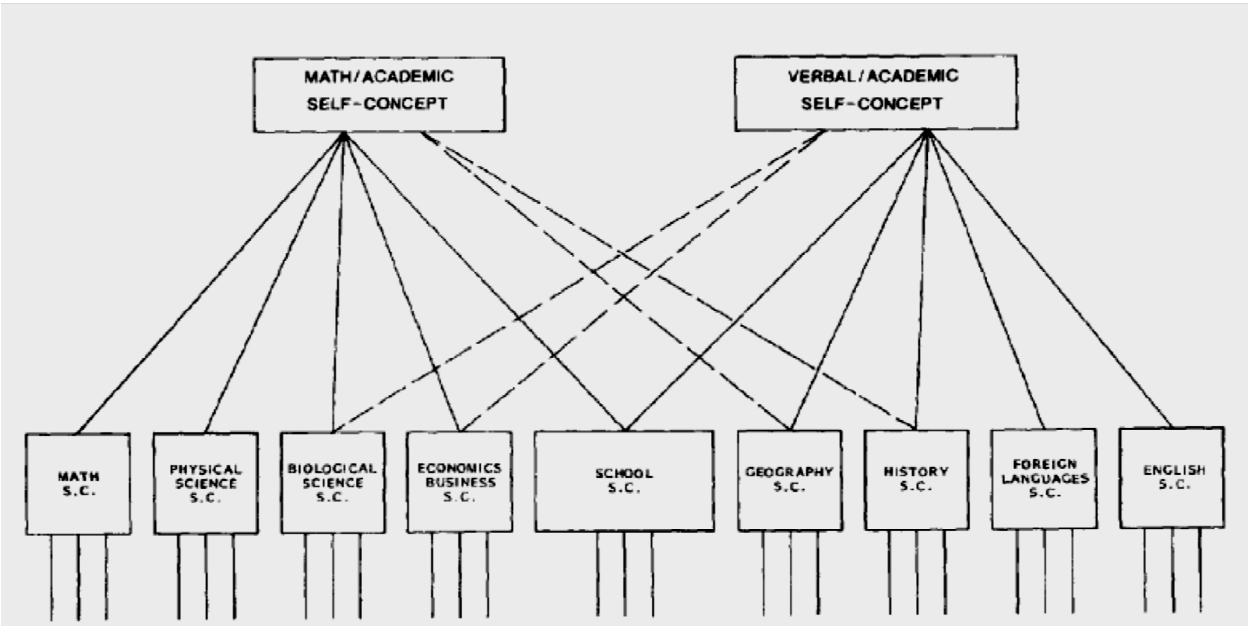

Figure 1.7 Schematic representation of academic self-concept. Note the two higher-order self-concepts concerning math and verbal competences. From Marsh et al., 1998.

1.3.1.2.1 Self-concept in academic settings

Academic self-concept, despite being highly related to academic self-efficacy, can be distinguished from it. In fact, academic self-concept more likely represents knowledge and perceptions that the individuals possess about themselves in achievement situations (Byrne, 1984), whereas self-efficacy, as already specified, regards the conviction about possible successful performance in a given task (Schunk, 1991). In other words, self-concepts is on prevalence based on past accomplishments, whereas self-efficacy on future expectancies, so they are respectively past- and future-oriented (e.g., Wigfield & Eccles, 2000). In this perspective they can overlap only very early in development, when the lack of much past experience leads also academic self-concept to be subjected to cross-situational variability (Markus, 1977).

Academic self-concept is deemed to be central in the learning processes either as an outcome or as an explanatory variable (Hattie & Marsh, 1996). Hence, it is generally thought to act on performance but also to be influenced by the same, in turn. The *self-enhancement model* posits, in fact, self-concept to be the primary determinant of academic achievement. On the contrary, the *skill-development model* asserts that self-concept is more likely to emerge as a consequence of academic achievement (Calsyn & Kenny, 1977). Marsh (1990; Marsh, Byrne, & Yeung, 1999), however, proposed a combination of the two models asserting the occurrence of “reciprocal effects” between self-concept and academic skills.

The effect of prior self-concepts was supposed to be mediated by factors like students’ characteristics in the form of different effort, persistence and motivation (Marsh et al., 1999). Students with higher ability perception will approach new tasks with more confidence. In this way they increment their probability of success which, in turn, bolsters the confidence in their capacities (Wigfield & Karpathian, 1991). Therefore, the higher the children’s academic self-concept, the better will be their academic achievement (Marsh, 1994).

This hypothesis is corroborated by many empirical data. Actually, academic self-concept has been proven to relate to teachers’ ratings about engagement and persistence in classroom activities (e.g., Skaalvik & Rankin, 1996). Marsh (1986) found out that math and verbal self-concepts were not related but both were significantly linked to the specific domain of learning, mathematic and verbal achievement, respectively (see also Chapman & Tunmer, 1997; Manger & Eikelan, 1998).

Actually, also in relation to mathematics, self-concept was identified among the precursors of math performance, in both conditions of low and high task familiarity (Norwich, 1987), and has been also observed to improve as a consequence of improvements in the achievement levels (Marsh, 1986). Nevertheless, when observing academic self-concept in learning disabled students, not all studies found its significant decrement (e.g., Cooley & Ayres, 1988; Crabtree & Rutland, 2001; Durrant, Cunningham, & Voelker, 1990; Vaughn, Haager, Hogan, & Kouzekanani, 1992). Some authors (e.g., Stone & May, 2002) further highlighted learning disabled students to overestimate their actual performance. Students affected by learning problems are in fact expected to derive their global self-conceptions from domains other than school. In line with these findings, other researches detected a decreased academic self-concept in LD students, but adequate levels of global self-evaluations (see Chapman, 1988).

A tricky point indeed concerns the involvement of either general or academic self-concept in relation to academic achievement (Byrne, 1984). However, despite academic self-concept and academic achievement have been proven to be strong predictors of each other (Muijs, 1997), there are controversial findings considering the role of global self-concept, since some studies reported its inflation in learning disabled students (e.g., Chapman, 1988), but others did not (e.g., Bear & Minke, 1996; Hagborg, 1999; Montgomery, 1994).

1.3.1.3 Self-esteem

Very close to the self-concept construct is viewed that of self-esteem. One of the possible definitions of this construct is that proposed by Coppersmith (1967), which delineates it as a personal judgment of worthiness, expressed in the attitudes of individuals towards themselves. Self-esteem is thus related to how people accept and like themselves. Despite self-concept and self-esteem are often defined as overlapping constructs, a distinction can be operated (see Damon & Hart, 1982). Self-esteem is more precisely considered as the result of self-concept, since different views about ourselves generate and maintain a different level of self-esteem.

The classical current of thought views self-esteem, in children as in adults, as typically leading to positive feelings when experienced, but to negative states when present in a limited extent (Dweck, 1999). This boosted the so called “Self-esteem movement” aimed at artificially enhancing children’s self-esteem by avoiding them personal failure. Nevertheless, other influent positions (e.g., Dweck, 1999), partially going in the opposite direction, suggested that a right amount of setbacks and difficulties are challenging and productive and help in strengthening self-appraisal.

1.3.1.3.1 Self-esteem and academic performance

It is quite well-established that self-esteem and academic achievement are closely linked, but the direction of this relationship isn’t so clear. Some authors indeed evidenced that high self-esteem led to successful achievement, for instance in reading (Patil, Saraswathi, & Padakannaya, 2009) and also writing (Kershner, 1994), and determined better academic performance also throughout the enhancement of intrinsic motivation (Gillis & Connell, 2003; Redden, 2000; Saracoglu, Minden, & Wilchesky, 2002). On the contrary, low self-esteem seemed to be associated to maladaptive learning strategies and to poor educational attainment and non-participation in scholastic activities (Lloyd & Sullivan, 2003).

In the same way, some researchers studied the effects of academic performance on self-esteem fluctuations. For instance, Emler (2001) contended the reliable influence of success and failure in determining self-esteem enhancement and drop respectively. Children with low reading skills were shown to develop a lack of confidence in themselves, in this way leading them to diminish the opportunities to improve (Galbraith & Alexander, 2005). A drop in self-esteem is also the result of frustration and emotional problems that can arise from repetitive failure in the context of learning problems (Lyon, 2000).

Many studies reported lowered self-esteem in children with learning disabilities (e.g., Rosenthal, 1973), but also in those who are not classified as LD but present however a poor achieving profile (e.g., Shaw, 1961; Strang, 1968). Nevertheless, as for self-concept, results are contradictory. Some researches support self-esteem advantages in LD children receiving special placement (Coleman, 1983; Ribner, 1980), other no differences at all when compared with the typically-achieving peers (e.g., Bear & Minke, 1996).

Actually, the fact of possessing a learning disability isn't considered, *per se*, a condition sufficient to impair self-esteem. LD children are often observed to dedicate much more attention to the non-academic domains such as physical appearance or social relationships and thus to have global positive views of themselves (Hagborg, 1996). In line with this, the study by Stevenson and Romney (1984) on a group of learning disabled students, evidenced that only the subgroup with higher depression was also characterized by low-self esteem, thus indicating the possible co-occurrence of both conditions in learning disabled children.

In addition, these children are sometimes not deeply conscious of their learning problems, as stated with reference to self-concept. The research conducted by Cosden and collaborators (1999) showed that third- to sixth-grade children who were less able in explaining the consequence of their disability had higher self-esteem levels than their peers who were more conscious of the condition. In this perspective, the fact of appearing unconscious of their disability seems to preserve LD children from undermining self-esteem.

1.3.2 Negative affect

1.3.2.1 Anxiety

Anxiety is the feeling that we easily experience in everyday life as a normal response to stress-inducing situations. Experiencing such anxiety frequently predispose us to positively react in front of problems. For instance, an adequate anxiety level during an examination is functional to a proper mood and arousal. Problems arise when stressors become excessive or enduring or when the reaction to the same is disproportioned and excessive and thus destabilizes the individual.

Research on anxiety has been fostered since the last quarter of the previous century. Anxiety has been defined as a state of emotion that underpins fear and dread (Lewis, 1970). It arises when people feel uncertainty in face of danger (May, 1977), but these reactions are generally disproportioned in relation to the threat. The consequences of this negative state are widespread, since anxious individuals frequently experience increased heartbeat, anticipation of punishment and self-esteem loss.

Anxiety is, in children, one of the most common forms of psychopathology. Between 8% and 12% of children experience anxiety symptoms which can interfere with daily activities (Costello, 1989). There is evidence that, if not treated, anxiety disorders persist till childhood and adolescence (Keller et al., 1992). It seems likely that innate vulnerabilities have an important contribution in anxiety development, but also that early life experiences can have a role in regulating the development of the same vulnerabilities (Chorpita & Barlow, 1998).

The occurrence of anxiety is crucial also in academic context in a so extensive way that results on studies conducted on children affected by learning disabilities confirmed that these children are usually affected by higher levels of anxiety. For instance, before performing the requested tasks, students aged 9 to 11 reported higher trait and state anxiety than their normally-achieving peers, even if both groups had comparable performance capacities (Fisher, Allen, & Kose, 1996).

1.3.2.1.1 Anxiety and interference with cognitive resources

Anxiety is known to impair performance also by decreasing the cognitive abilities recruitment. An excessive cognitive arousal, in fact, is hypothesized to impair the recruitment of the cognitive abilities necessary to perform the task at hand. Worrisome thoughts, that arise as a consequence of experiencing anxiety, are very hard to inhibit and therefore will absorb working memory and attentional resources, phenomenon that is known as *Deficient inhibition mechanism*, (see Hopko et al., 1998).

Two main theories have been proposed with the specific aim of explaining how cognitive mechanisms can be impaired:

- *Inhibition theory* (Hasher & Zacks, 1988) which postulates a decreased cognitive performance as a consequence of increasing difficulty in inhibiting distracting stimuli;
- *Processing efficiency theory* (Eysenck & Calvo, 1992), which argued that cognitive performance drop is the result of impaired working memory capacities because of anxiety interference. More in detail, this theory regards the relationship between the quality of task performance and the effort or cognitive resources spent in it.

This second theory has been recently extended by Eysenck and collaborators (Derakshan & Eysenck, 2009; Eysenck, Derakshan, Santos & Calvo, 2007) and reformulated in terms of *Attentional* *control theory*, that claims that anxiety as a personality trait is tightly related to individual differences in higher cognitive control. As a consequence of this, anxious people are mainly characterized by a *stimulus-driven* attentional behavior rather than *a goal-directed* one. Such unbalance between the two attentional systems is hypothesized to impair processing efficiency in terms of speed rather than accuracy, in particular when tasks require high attentional shifting or inhibition of distracting stimuli.

A wealth of evidence indeed reported high versus low anxious individuals to be more easily distracted by task-irrelevant stimuli, particularly if emotionally arousing (e.g., Bar-Haim, Lamy, Pergamin, Bakermans-Kranenburg, & van IJendoorn, 2007). This theory is in line with a previous one proposed by Easterbrook (1959), who hypothesized anxiety to narrow attention by creating a sort of "tunnel effect" causing enhanced focusing to certain stimuli but not to relevant others.

1.3.2.2 Math anxiety

Math anxiety is defined as a negative emotional response characterized by feelings of stress and avoidance behavior in situations involving mathematics (Ashcraft & Ridley, 2005) "in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972). Despite research in this field took major advance in the 1970s thanks to Richardson and Suinn (1972) investigations, already in the 1950s the first allusions to this feeling emerged (see "emotional difficulties with math", Gough, 1954).

The detrimental impact induced by math anxiety tends to be lifelong (e.g., Rubinsten & Tannock, 2010) and to increase in severity over time (Ma, 1999). These effects are so prominent that not only determine a negative attitude towards math (Krinzinger et al., 2007), but have repercussions also in career choice, employment and professional success (Ma, 1999). Moreover, these effects are so discomforting that situations involving math are perceived also to threat self-esteem (Cemen, 1997).

This specific subtype of anxiety is a very prevalent phenomenon, occurring in approximately the 4% of high-school students (Chin, 2009) and is detected even in absence of generalized anxiety (Ashcraft & Krause, 2007). The longitudinal study conducted by Hembree (1990) highlighted that anxiety levels peaked in middle school to be then leveled off in high school and college. Levels of math anxiety were found out to be somewhat higher in low and average achieving students.

Lang (1968) stated that, in the same way of other phobia, also math anxiety influences individuals on three levels: *cognitive*, for instance determining the onset of worrisome thoughts (see Richardson & Woolfolk, 1980); *behavioral*, by typically inducing an avoidance behavior (see Hembree, 1990; Krinzinger, Kaufmann, & Willmes, 2009); *physiological*, with bodily reactions as increased heart rate and sweating (see Faust, 1992).

Other authors instead investigated the sources of math anxiety and identified the three major ones, classified as cognitive, personal or environmental (Hadfield & McNeil, 1994). *Cognitive* causes generally involve low cognitive and intellectual abilities and poor mathematic proficiency. *Personal* reasons, instead, regard factors such as low self-esteem, lack of confidence or previous negative discomfort in the subject. *Environmental* causes, finally, include negative experiences collected on prevalence in the classroom setting and specifically with the math teacher.

As deducible, there is a strong support to the link between math anxiety and poor math achievement (e.g., Faust et al., 1996; Ma, 1999; Ma & Xu, 2004), although the direction of this relation isn't yet clear. It is however reasonable for math anxiety to arise from unpleasant recall of past failure in the field of mathematics (Ma & Xu, 2004), but also to be the cause of an *affective drop* in performance that origins when it is experienced at a high degree (Ashcraft & Moore, 2009). The influence of math anxiety is such that treatments properly developed to reduce it (including systematic desensitization or relaxation training) improved math performance which became comparable to that of the low-anxious peers with equivalent ability (Hembree, 1990).

Math anxiety, however, wasn't initially identified as a sub-construct well distinguishable from more general text anxiety, which is experienced when individuals are subjected to testing conditions, typically in the academic context (see Zeidner, 1998). Earliest findings proposed math anxiety to be viewed as no more than test anxiety (Brush, 1981), others considered math anxiety utmost a specification of more general test anxiety (e.g., Tryon, 1980). However, more recent findings are in favor of a high specificity of math anxiety, which can be present even in absence of more general test anxiety. In fact, Wu et al. (Wu, Barth, Amin, Malcame, & Menon, 2012) yielded math anxiety to negatively impair math but not reading proficiency, suggesting that it can't be generalized to test anxiety which is supposed to affect academic performance on the whole.

Generally, studies on math anxiety focused on students by secondary school and demonstrated that this condition interferes with solving two-column addition problems and particularly in carrying procedures (e.g., Ashcraft & Faust, 1994; Faust et al., 1996). Also the effects on performance are conflicting. Some studies reported, indeed, a greater impairment in the speed of solution, since arithmetical problems took three times longer to be solved in high-anxious subjects (e.g., Faust et al., 1996). Other authors, instead, observed a consistent increased error rates (e.g., Ashcraft & Kirk, 1998), in face of fast execution times. This result can be in favor of the tendency of math anxious people to speed up the resolution with the specific purpose of ending the painful condition as soon as possible. Avoidance is, in fact, the most common behavioral pattern of math-anxious individuals (Ashcraft & Moore, 2009)

Other researchers have nonetheless found out negative math-anxiety-induced effect also on simpler math task, as in basic numerical operations and also counting (e.g., Ashcraft & Ridley, 2005), but the same evidences were not collected when the same students were tested in untimed, low-pressure settings (Ashcraft, Krause, & Hopko, 2007; Faust et al., 1996). Furthermore, the absence of an impaired performance on simple arithmetic has been attributed to the fact that results of common arithmetic operations are retrieved automatically as they are stored as arithmetic facts and hence are not supposed to be dented by possible anxiety.

Even if not in an extensive way, this condition has been explored also in MLD children, showing that these usually manifest high levels of math anxiety. Actually, daily math lessons appear to be tremendously stressful for such children who effort much more than their peers in order to achieve even basic concepts (Ma & Xu, 2004). In fact, Rubinsten and Tannock (2010) observed that when MLD students have to solve arithmetic equations even math words were suitable in eliciting the same response induced by negative affective primes under other conditions.

1.3.2.2.1 Math anxiety in young children

Despite a consistent body of research has been dedicated to the assessment of math anxiety in college and undergraduate students, very few are the studies investigating math anxiety in younger children and especially in first grades of primary school (Krinzinger et al., 2007, 2009; Ramirez, Gunderson, Levine, & Beilock, 2013; Thomas & Dowker, 2000; Wu et al., 2012). Before these recent researches, no one study explored math anxiety in children below fifth grade.

To date, the developmental increase in the relationship between math anxiety and math performance is no clearly established. Some relevant studies, such as those of Krinzinger et al. (2009) and Thomas and Dowker (2000), outlined the presence of a specific math anxiety already in earliest stage of schooling, but did not find a significantly negative relation with math proficiency. More in detail, Krinzinger and colleagues (2009) explored math anxiety in children from the end of first grade till middle of third grade. Results reported math anxiety to be significantly correlated with concurrent evaluation of mathematics given by students, but were inconclusive regarding the possible negative relation between math anxiety and math performance. The proposed interpretation was that a significant relation between these two variables can emerge only when both are extremely pronounced (very high math anxiety and very low math performance). Otherwise, it could be possible for this association to be not yet relevant in so young students.

Interestingly, anyway, in the study by Wu and colleagues (2012), a significant correlation between math anxiety and math proficiency in standardized tests emerged and has been seen to last even after having controlled for trait anxiety. In particular, math anxiety appeared to significantly affect the aspect of mathematical reasoning, while its effects on arithmetic operations didn't attain statistical significance. By trying to explain how in the previously mentioned researches the significant link with math performance hadn't been found, the authors noticed that administered math tests usually propose computations that are quite easy to perform and mainly regard addition and subtraction.

1.3.2.2.2 Math anxiety and memory

Only in the last two decades, the two independent research lines focused either on cognitive processes at the basis of math learning or on math anxiety have begun to intermingle. In fact, as for general anxiety, the attempt was to investigate which cognitive processes and abilities can be affected specifically by math anxiety.

The research carried out by Ashcraft and Kirk (2001), in fact, was aimed at investigating if, and with which extent, math anxiety could affect memory processes. The outcomes proved that high math anxiety corresponded to lower working memory. Moreover, particularly under high memory-load conditions, solution latencies and error rates in math exercises (chiefly in carrying procedures related to computation) were observed to dramatically increase in parallel to task difficulty (Ashcraft, Copeland, Vavro, & Falk, 1999).

The effects on math anxiety have been recorder on-line, meaning during the execution of math tasks, proving that impaired performance had to be attributed to the transitory disruption of working memory. It is suggested this disruption to affect math-related tasks in general, therefore also those associated to counting (Logan & Klapp, 1991).

By combining the two previously mentioned theories regarding the drop in cognitive performance (Inhibition and Processing deficit theories), Hopko et al. (1998) proposed low math performance to origin from the inability to focus away attention from worrisome thoughts. Working memory resources are indeed expected to be reduced since negative thinking associated to ruminative thoughts load on them and can't therefore be dedicated to math tasks performance.

1.3.2.2.3 Math anxiety and number sense

Very few are, on the other hand, the studies that investigated the relationship between math anxiety and number sense measures. Of these, that performed by Maloney and colleagues (Maloney, Risko, Ansari, & Fugelsang, 2010) is noteworthy since it explored math anxiety interference in enumeration abilities. Participants had to identify the correct number of 1 to 9 squares, either by subitizing (in the 1-4 range) or counting (for numerosities greater than 4). Findings revealed high-math-anxious individuals performing significantly worse than their non-anxious peers in the counting range enumeration (expected to entail greater cognitive resources than subitizing). Anyway, when controlling for working memory capacities, difference in performance disappeared.

The subsequent study (Maloney, Ansari, & Fugelsang, 2011) explored the aspect of symbolic comparison by investigating the numerical distance effect (NDE), which reflects the response times that are necessary to distinguish between two close numbers (a greater NDE reveals less mastery in number representation). As hypothesized, students characterized by high math anxiety levels had greater NDE and thus required a larger amount of time to compare closer numbers.

Observed from another point of view, these results seem to lead to the conclusion that high-math-anxious individuals have an impaired performance not only on complex math tasks, but also at the level of basic numerical processes. Deficits at higher math levels are therefore seen as a consequence of impairments at a low level of numerical processing. Unfortunately, both studies have been conducted on undergraduates, therefore no data are available for younger children and only inferences can be drawn.

1.3.2.3 Depression

Depressive disorders are quite common in youth, with up to 9% of children experiencing at least one major depression episode before the age of 14 (Lewinsohn, Rohde, Seeley, & Fisher, 1993). Depressed youths are normally listless, solitary, appear unable to enjoy life, judge themselves as ugly, stupid, inadequate or hopeless. Other common symptoms, as for adults, are irritability, restlessness or fatigue and sense of vigor loss, insomnia or excessive sleeping, and loss of interest in different activities. Care has to be taken since it is proven that early depressive vulnerability is highly associated to depression in adulthood (Petersen et al., 1993).

Stressful life events and interpersonal difficulties (e.g., Weisz, Thurber, Sweeney, Proffitt, & LeGagnoux, 1997; Wolchik et al., 1993) and low self-esteem are among the risk factors triggering depression (e.g., Nolen-Hoeksema, Girgus, Seligman, 1992). Also anxiety has been proposed to be one of the factors contributing to depression onset. Actually, anxiety is one of the most common disorder to be comorbid with depression, with estimates ranging from 30% to 75% in pre-adolescents and 25% to 50% in adolescent (e.g., Lowry-Webster, Barrett, & Dadds, 2001). Many studies therefore indicated a considerable overlap between anxiety and depressive symptoms in youth (e.g., Brady & Kendall, 1992; Seligman & Ollendick, 1998). Major evidence postulates anxiety to precede the onset of depression, rather than the reversal (e.g., Angst, Vollrath, Merikangas, & Ernst, 1990; Cole, Peeke, Martin, Truglio, & Seroczynski, 1998) and depressed youths are often observed to be also anxious, whereas the reverse condition is much less frequent (e.g., Chorpita & Daleiden, 2002; Finch, Lipovsky, & Casat, 1989; King, Ollendick, & Gullone, 1991). For the link between anxiety and depression it is possible to refer to the *Tripartite model* proposed by Clark and Watson (1991) or to that postulated by Alloy et al. (Alloy, Kelly, Mineka, & Clements, 1990).

One of the most acknowledged cognitive theories about depression is that developed by Beck (1967, 1983). According to this theory, subjects characterized by depressogenic schemata are more vulnerable to depression, with the consequent generation of dysfunctional reactions to negative stimuli. Such individuals are more likely to develop a cognitive triad represented by three different depressogenic cognitive patterns: negative views of the Self, negative views of the world, and negative views of the future. The consequence of this dysfunctional way of thinking will give rise to depressive symptoms. Relevant is the depressive reaction to stressors, experienced by both children and adults but with an intensity and duration that however don't necessary fall within clinical parameters.

1.3.2.3.1 Depression in academic contexts

Few are the studies which investigated the role of depression in learning settings. However, the collected results reported depressive symptoms to be exhibited by students encountering academic difficulties (Blechman, McEnroe, Carella, & Audette, 1986). For instance, 13- to 17-year-old students reporting academic difficulties were also observed to be characterized by depressive symptoms (Fröjd et al., 2008). The occurrence of depression was also observed in the in the 36% younger children aged 8 to 11 and affected by a learning disability. Anyway, parents' reports of their children's depression were not significant (Wright-Strawderman & Watson, 1992).

In the study of Fristad and colleagues (Fristad, Topolosky, Weller, & Weller, 1992), it was observed that the occurrence of a learning disability in 6- to 12-year-olds with major depressive disorder was seven times more frequent than in children without this symptomatology. This evidence seems to suggest a greater predisposition to learning disability in clinically depressed children. Further, results from Goldstein et al. (Goldstein, Paul, & Sanfilippo-Cohn, 1985), who verified the incidence of depression in students affected by learning disabilities, suggested this condition to be either the cause or the consequence of academic failure in dependence of the clinical profile of affected children. Anyway, it seem to be rather the consequence in children affected by the learning disability alone (so without other symptoms of clinical relevance).

The influence of poor school achievement as a potential risk factor for depression onset has been investigated also in relation to the so termed *learned helplessness*, widely detected also in children (see Marsh, 1975). This condition is characterized by a negative reaction in front of repetitive failure, reaction characterized by passivity, decreasing interest, perception of loss of control in given situations, resignation and lowered self-esteem. This condition has been primarily observed in individuals with maladaptive explanatory styles, who therefore attribute failure to internal, stable and global (rather than external, changeable and specific) causes (Abramson, Seligman, & Teasdale, 1978).

CHAPTER 2

Study 1

Relative contribution of general and specific math learning precursors in the determination of early math skills.

2.1 Theoretical background

To date, very few are the studies which have investigated the contribution of both general and specific math precursors in relation to early math proficiency. In fact, only in recent years researchers have begun to be interested in the investigation of the relative contribution of one kind of precursors with respect to the other, since sufficiently robust results have emerged in relation to the significant involvement of both of them.

One of the leading general precursors of math skills is represented by working memory which has a fundamental impact also in early stages of learning (e.g., Passolunghi & Lanfranchi, 2012), despite some authors minimize its crucial involvement (e.g., Landerl et al., 2004). Further, the role of more passive memory processes, better defined in terms of short-term memory, is not clear in the same extent. Many authors however indicated the visuo-spatial component to be more relevant than the verbal one at least in very young children (e.g., Bull et al., 2008).

On the other hand, specific precursors can be identified with the so termed number sense, represented by two main aspects: that of making approximate magnitude judgments referring to the Approximate number system (ANS), and that associated to exact and symbolic capacities. Regarding the latter, there is quite consensus about its involvement in math learning (e.g., Landerl et al., 2004; Mazzocco & Thompson, 2005), while major debate stands in relation to the former.

By examining the studies evaluating both kinds of precursors, it is possible to note that many of them focused mainly on one of these precursors, for instance on memory (e.g., Passolunghi & Lanfranchi, 2012) or number sense (e.g., Locuniak & Jordan, 2008). Moreover, in relation to the latter, symbolic or basic numeracy skills are the components usually examined as specific precursors, whereas less attention is dedicated to more innate, approximate aspects. In other studies, counting skills are the ability to be typically assessed as either specific precursor or outcome measure (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Koponen, Salmi, Eklund, & Aro, 2013; Kroesbergen et al., 2009) and therefore a broader range of competences is not investigated.

Among the studies broadly assessing both kinds of precursors, that of Fuchs et al. (2010) and Geary (2011) extensively examined memory components recruitment respectively in first graders and from first to fifth graders. They collected also number sense measures, but these were all related to the formal knowledge of numbers (recognition and manipulation of small, exact numerosities and symbolic number line estimations). At the date of the planning of the current research project, another study was noteworthy, also because it investigated the relationships among all precursors via structured equation models. The research was performed by Cirino (2011), on the basis of LeFevre et al. (2010), and explored general abilities and number sense skills, both symbolic and non-symbolic (approximate). Anyway, non-symbolic measures were spoiled in relation to the symbolic ones and precursors were observed in relation to basic but symbolic math capacities. The outcome measure was indeed represented by single-digit additions, which are further a quite limited index of overall early math competence.

2.2. The current study: Aims and hypotheses

The principal goal of the current study, hereinafter indicated as Study 1, was therefore to explore which general and specific cognitive abilities are primarily involved in the acquisition of math skills and act consequently as main precursors of mathematical learning. We selected to focus on the acquisition of early math skills and we therefore tested children at beginning of formal schooling.

More in detail, the principal aim was to investigate the relative contribution of general and specific math precursors and in particular of the approximate ANS skills, whose involvement in math competence is strongly debated. A consequent although not direct aim of the study was the identification of the key abilities that can be monitored prior to formal instruction with the specific purpose of precociously identifying children who can develop a math learning disability.

Considering the general precursors, we specifically explored the involvement of short-term memory (STM) and working memory (WM) considering both verbal and visuo-spatial aspects. In this way, we tried to disentangle opposite views considering the greater importance of one component over the other in that particular stage of development. The other general precursor to be assessed was intelligence, in both its verbal and performance aspects, to investigate with which extent it can possibly add a further contribution with respect to more relevant precursors. Regarding number sense and more specifically ANS, we investigated it by means of different tasks, hence trying to achieve a reliable and complete ANS indicator.

The intention was also that of evaluating the possible differential involvement of the assessed precursors in relation to different levels of skills within early math competences, by distinguishing the more basic from those requiring the knowledge of numbers.

Considering the related hypotheses, these were, first of all, that general abilities could act as the main precursor of early math learning. Memory was attended to have a fundamental role, but also intelligence was expected to give an additional, independent effect. More in detail, active processes associated to WM were supposed to assume a more crucial role than the passive ones associated STM. Considering related components, the visuo-spatial aspect of memory and the capacity of remembering digits rather than words were hypothesized to be more extensively involved. Finally, also the specific component represented by ANS was expected to have a significant involvement, estimated however to decrease by the end of the school year.

2.3 Method

2.3.1 Participants

A sample of 164 first graders was enrolled in the study. Children were recruited from seven classrooms across four primary schools in Northeastern Italy. From this sample, 2 children were excluded because diagnosed for neurological diseases, and other 5 children because of their limited proficiency in the Italian language. The final sample was represented by 157 students (80 males; mean age: 6 years, 3 months).

2.3.2 Procedure

After having obtained formal consent from the school headmaster, class math teachers and parents, students were tested at the beginning of their Grade 1. Data were collected individually for each child. Three separate sessions lasting approximately 30 minutes each were carried out in a silent room outside the classroom. A brief break was provided if needed by the child. The three sessions took place within two consecutive weeks for each child. All trials were preceded by the pertaining task instructions.

2.3.3 Measures

2.3.3.1 Intelligence

In order to measure intelligence in its verbal and performance components, two subtests were selected from the Italian edition of the Wechsler Intelligence Scale for Children or WISC-III (Wechsler, 1991; Italian edition, 2006). The choice has been motivated by the high correlation of the two subtests with the whole IQ index (Sattler, 1992). Both subtests yield age-referenced scaled scores between 1 and 19, based on a mean of 10 and standard deviation (SD) of 3.

The *Vocabulary* subtest involves words presented orally by the experimenter in order of increasing complexity and of which children have to give an exhaustive definition. For this age group, up to 30 items are presented. Responses are scored 0 to 2 points accordingly to their accuracy and thoroughness. Testing in case stops if children make four consecutive errors (four 0 point responses). The maximum achievable score is 60 points.

The *Block Design* subtest entails the reproduction of bi-dimensional pictorial configurations shown by the experimenter by means of 2 to 9 cubes with solid or two-tone diagonal sides. The subtest includes 12 items of increasing difficulty which have to be reproduced within a predetermined time limit. Scoring varies according to the accuracy and time needed to complete the construction. The test administration stops possibly after two consecutive failed block reconstructions. The maximum score is 69 points.

2.3.3.2 Short-term memory (STM)

Forward word recall (from Lanfranchi, Cornoldi, & Vianello, 2004). In this tasks, tapping verbal STM, children are asked to repeat sequences of increasing number of common bi-syllabic words in the exact order in which they are produced by the experimenter. The test is made up of 2 trials for each of the 4 levels of difficulty (2- to 5-word spans), giving a total of 8 trials, plus the example. For each word sequence reproduced in the correct order, 1 point is assigned. Testing ends before-hand if children fail in the exact reproduction of two sequences belonging to the same level of difficulty. The final maximum score is 8 points.

Forward digit recall (from WISC-III, Wechsler, 1991; Italian edition, 2006). This task, exploring verbal STM, entails the repetition of sequences with an increasing number of digits in the same order in which they are produced by the experimenter. The test is composed by 2 trials for each of the 8 levels of difficulty (2- to 9- digit spans), with a total of 16 trials preceded by an example. Responses are scored as correct, and thus receive 1 point, only if all digits are reproduced in the requested order. Testing possibly ends if children fail in correctly repeating both sequences of the same difficulty level. The maximum score is 16 points.

Path recall (from Lanfranchi et al., 2004). This task explores visuo-spatial STM and requires children to reproduce pathways of increasing length traced by the experimenter on a grid (with the assistance of a toy frog simulating an increasing number of jumps). Besides the example, the test is composed of 2 trials for each of the 4 levels of difficulty (2 to 4 frog jumps within 3x3 and 4x4 grids), giving a total of 8 trials. 1 point is assigned if children correctly reproduce the entire sequence of jumps. Testing ends if they fail in recalling both pathways of the same difficulty level. The maximum score is 8 points.

2.3.3.3 Working memory (WM)

Backward word recall (developed from Lanfranchi et al., 2004). This task assesses verbal WM and requires children to repeat sequences made of an increasing number of common bi-syllabic words in the inverted order (from the last to the first word) of that used by the experimenter when pronouncing them. The task includes 2 trails for each of the 4 increasing levels of difficulty (2- to 5-word spans), with a total of 8 trials plus the example. Correct responses, to which 1 point is assigned, need the exact repetition (in the inverted experimenter's order), of all words forming the sequence. Testing interrupts if children don't report in the exact order both word sequences of the same difficulty level. The maximum score is 8 points.

*Backward digit recall (*from WISC-III, Wechsler, 1991; Italian edition, 2006). Children have, in this task tapping verbal WM, to exactly reproduce sequences of increasing number of digits in the inverted order (from the last to the first digit) of that used by the experimenter. The test consists of 2 trials for each of the 7 levels of increasing difficulty (2- to 8- digit spans), determining a total of 14 trials, plus the example. 1 point is assigned when all digits making the sequence are correctly remembered in the inverted order. Task administration interrupts if both sequences of the same difficulty level are incorrectly reported. The maximum score is 14 points.

Path dual task (from Lanfranchi et al., 2004). In the present task, which explores visuo-spatial WM, the experimenter traces a pathway on a grid by using a toy frog. Children have to carry out a simple task (knock the hand on the desk) when the frog jumps on the target (red-colored) square, while keeping in memory the first square to which the frog has jumped. The test is composed by 2 items for each of the 4 increasing levels of difficulty (2 to 5 jumps per pathway on a 4x4 grid), for a total of 8 trials preceded by a preliminary training and the example trial. 1 point is assigned to correct responses entailing both remembering the first square and realizing the concomitant task. No interruption criteria are envisaged. The maximum score is 8 points.

2.3.3.4 Approximate number system (ANS)

ANS tasks have been reproduced following the indications given by the authors who originally developed them. Timing and numerosities of each task had been re-adapted on the bases of results achieved in preliminary pilot studies. Tasks reliability, measured by Cronbach's alpha, are from acceptable to fair (see Table 3.1). Before beginning testing, it has been checked if each child could understand the concepts of "less" and "more", by making simple examples with candies.

Magnitude comparison of intermixed quantities, hereinafter *Comparison-Intermixed* (adapted from Halberda, Mazzocco, & Feigenson, 2008). This task requires children to rapidly judge, without counting, which is the most numerous set of dots (represented as smiles). The two sets consist in intermixed blue and yellow smiles of different sizes on a gray background presented simultaneously on a pc screen for 600 ms. After this interval of time, smiles disappear and children have 5 sec to indicate if the most numerous set was that of the yellow or of the blue smiles.

Smiles in each trial vary in number between 3 and 15 and the proportion of the quantities being compared reflects the range proposed by Halberda et al. (2008): 1:2, 3:4, 5:6, 7:8 and their inverse (2:1, 4:3, 6:5, 8:7). Each of these 8 proportions is presented 5 times in different numerosity combinations, giving a total of 40 trials, plus 5 practice trials. There is an equal number of trials having the yellow and blue dots as most numerous array. Dots are controlled in size and total area. That is, half of the trials are size-controlled and the remaining are area-controlled. No interruption criteria are applied. 1 point is given to each correct response, with a maximum score of 40 points.

Magnitude comparison of separate quantities, hereinafter *Comparison-Separate* (adapted from Piazza et al., 2010). Also in this task children are asked to rapidly judge, without counting, which is the most numerous array of dots. These are represented by black dots included within two white disks on a black background and presented simultaneously one either horizontal side of a white fixation cross for 1,000 ms. After this interval of time, dot arrays disappear and children have 5 sec to indicate (typically pointing with their finger to the correct side of the screen, left vs. right) which was the most numerous array of dots.

In each trial one of the two disks contains the reference dot number (16), while the other contains a target number of dots varying from 8 to 24 (16 ± 8, 11 and 21 numerosities excluded). The task includes 4 trials for each of the 14 possible target numerosities, giving 56 total number of trials, plus 5 practice trials. There is an equal number of trials having the most numerous array in the left or in the right side of the screen. Dots within each disk are of the same size, but dot size can differ from the two disks. As a consequence, half of the trials are size-controlled and the remaining are area-controlled. None interruption criterion has been introduced. 1 point is assigned to each correct response, giving a maximum score of 56 points.

Approximate addition (adapted from Iuculano, Tang, Hall, & Butterworth, 2008). This task requires children to visually and approximately sum two arrays of blue dots and compare them with an array of red dots, all appearing on a yellow background. In each trial, two arrays of blue dots come into view in succession on the left side of the screen and are then hidden behind the same black rectangular occluder. Children have to mentally visualize the amount of dots resulting from the summation of the two arrays and compare this amount with the array of red dots appearing in the right side of the screen. After its appearance, each array of dots remains on the screen for 1,000 ms. In the end, also red dots disappear and children have 5 sec to indicate if it was most numerous the overall array of blue dots or that of red dots.

The total number of dots in each trial is between 8 and 25, with the same dot sizes within each trial.

The possible proportions between the total number of blue vs. red dots are: 4:5, 4:6, 4:7 and their inverse 5:4, 6:4, 7:4. The test consists of 4 trials for each of the 6 possible proportions, giving 24 total trials, plus 4 preliminary training trials. There is an equal number of trials having the blue and the red array of dots as the most numerous. Dots density varies according to the two possible sizes of the black occluder: 5 x 7 cm or 6 x 9 cm. No interruption criteria are applied. 1 point is given to each correct response, with a maximum score of 24 points.

2.3.3.5 Early math skills

Early math skills were assessed by means of a translated version of *Early Numeracy Test* (ENT, van Luit, van de Rijt, & Pennings, 1994), Form A. This test explores the 8 components in which the early mathematical abilities can be categorized and that have been defined by the authors as: Comparison, Classification, Correspondence, Seriation, Using counting words, Structured counting, Resultative counting and General knowledge of numbers. The first 4 components include tasks basically requiring innate quantitative abilities, while the remaining ones involve more knowledge-based skills (knowledge of numbers and performance of related tasks). Comparison requires children to compare objects on the basis of their qualitative or quantitative feature; Classification entails grouping objects by recognizing similarities (or dissimilarities) between them; in Correspondence, pupils have to make one-to-one relations between objects; in Seriation tasks, children are requested to rank in a predefined order a certain amount of objects on the basis of their features. Considering counting-based task, Using counting words requires counting forward, backward, counting on or using ordinal or cardinal numbers; Structured counting consists in synchronous and shortened counting also from the dice structure and children are allowed to point to the objects when counting; in Resultative counting, pupils have to count structured and unstructured arrays of objects and also to count hidden elements, while not allowed to use their fingers to indicate elements to be counted; General knowledge of numbers, in the end, tests the ability to use the knowledge of numbers to solve simple problem situations.

There are 5 questions/exercises for each component, with a total number of 40. None interruption criterion is applied. 1 point is assigned to each correct answer or correctly-made exercise, with a maximum score of 40 points. The final score is then converted into a competence score ranging from 0 to 100.

2.4 Results

2.4.1 Data Analysis and preliminary results

Preliminary statistical analyses were conducted by means of the *PAW Statistics 21* statistical package. AMOS 21 was used to perform path analyses. Descriptive and correlation analyses included the raw scores achieved in each test (with the exception of the intelligence indices for which age-referenced scales scores were reported). Path analyses were instead conducted by using standardized z-scores.

In Table 2.1 are reported descriptive and reliability results pertaining to each assessed variable.

Considering overall ENT performance, the same was observed to be possibly represented by two factors, as postulated in the studies of Aunio, Hautamäki, Sajaniemi and van Luit (2009) and of Schopman, van Luit and van de Rijt (1996), respectively for Finnish and Dutch ENT versions. More precisely, according to this perspective, the first four ENT sub-scales (Comparison, Classification, Correspondence and Seriation) require basic and spontaneously-developing capacities associated to the knowledge of quantities and relational aspects, contrary to the last four (Using counting words, Structured counting, Resultative counting and General knowledge of numbers). The latter indeed investigate more teaching-based skills and involve the ability to count and manipulate numerical information by means of concrete elements and reference points.

In order to check this two-factor structure, a factor analysis was conducted by Principal Axis extraction method and additional Varimax rotation. The rotated output matrix is reported in Table 2.2 and suggested the possibility to perform analyses by considering separately the two ENT factors. These two different early math domains were termed, in order, ENT-Relational and ENT-Counting, in accordance with Aunio et al. (2009).

Table 2.1

Descriptive Statistics and Reliability Measures

	Task	Min	Max	Mean	SD	Skewness	Kurtosis	Reliability
Math measures	ENT	36.00	91.00	68.45	10.09	-.17	.24	.94
	ENT-Relational	7.00	20.00	15.88	2.46	-.77	.56	.87
	ENT-Counting	2.00	20.00	12.61	4.11	-.56	-.16	.90
Intelligence	Vocabulary	2.00	19.00	11.36	2.84	-.39	.81	.74 [a]
	Block design	3.00	18.00	11.19	2.98	-.54	.23	.80 [a]
STM	Forward word recall	3.00	8.00	5.77	0.96	-.10	.03	.88
	Forward digit recall	2.00	11.00	6.52	1.62	.32	.28	.87 [a]
	Path recall	3.00	8.00	6.24	1.06	-.68	.67	.70
WM	Backward word recall	.00	6.00	2.65	1.06	.54	.85	.86
	Backward digit recall	.00	6.00	2.50	1.09	.70	1.54	.85 [a]
	Path dual task	.00	8.00	4.68	1.99	-.18	-.51	.81
ANS	Comparison-Intermix	8.00	35.00	25.92	5.06	-.71	1.04	.70
	Comparison-Separate	21.00	47.00	37.65	4.00	-.73	1.73	.65
	Approximate addition	8.00	21.00	15.79	2.68	-.25	-.11	.62

Note. Min= minimum; Max= maximum; SD= standard deviation.

[a] Reliability values taken from the manual (standardized tests). The remaining were computed on the sample.

Table 2.2

Factor Loadings of ENT components

	Factor			
Sub-scale	1	2	Eigenvalues	R^2
1. Comparison	.10	.01	3.47	43.31
2. Classification	.36	.20	1.07	13.32
3. Correspondence	.56	.54	.92	11.49
4. Seriation	.50	.27	.72	8.98
5. Using counting words	.36	.66	.60	7.55
6. Structured counting	.14	.79	.45	5.65
7. Resultative counting	.21	.69	.42	5.21
8. General knowledge of numbers	.31	.58	.36	4.49

Note. The first for sub-scales are associated to Factor 1, named ENT-Relational. Last four sub-scales are more tightly associated to Factor 2, named ENT-Counting.

Table 2.3 reports zero-order correlations among variables. From its inspection, it is possible to highlight the significant association between ENT achievement and both intelligence scores: Block design (r=.51, p<.001) and Vocabulary (r=.32, p<.001). Correlations of comparable strength were globally observed between the ENT and WM measures: Backward word recall (r=.17, p<.05), Backward digit recall (r=.34, p<.001), and Path dual task (r=.35, p<.001), but also with STM tasks: Forward word recall (r=.28, p<.001), Forward digit recall (r=.32, p<.001), and Path recall (r=.21, p<.05). In relation to ANS, milder correlations emerged between ENT and the magnitude comparison tasks (for both tasks r=.20, p<.05), whereas the relation with Approximate addition didn't attain significance (r=.14, p=.09).

ENT-Relational and ENT-Counting resulted to be strongly correlated (r=.57, p<.001). However, they presented also differential correlations with the other variables. In fact, while the former was significantly correlated with Comparison-Intermixed (r=.23, p<.01), the latter had significant correlations with Backward word recall (r=.19, p<.01) and Comparison-Separate (r=.21, p<.05).

Table 2.3

Correlation Matrix between All Variables

		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	ENT	1.00													
2	ENT-Relational	.81***	1.00												
3	ENT-Counting	.93***	.57**	1.00											
4	Vocabulary	.32***	.28***	.28***	1.00										
5	Block design	.51***	.43***	.45***	.16*	1.00									
6	Forward word recall	.28***	.21**	.27***	.22*	.10	1.00								
7	Forward digit recall	.32***	.26***	.31***	.13	.26***	.52***	1.00							
8	Path recall	.21*	.19*	.22**	.00	.16*	-.01	.07	1.00						
9	Backward word recall	.17*	.07	.19*	.10	.20**	.14	.01	.06	1.00					
10	Backward digit recall	.34***	.18*	.34***	.06	.26***	.33**	.16*	.10	.43***	1.00				
11	Path dual task	.35***	.26***	.32***	.09	.30***	.03	.02	.18	.22***	.30***	1.00			
12	Comparison-Intermix	.20*	.23**	.15	.06	.10	.20**	.24**	-.01	.02	-.02	-.03	1.00		
13	Comparison-Separate	.20*	.11	.21**	.25***	.07	.14	-.02	-.03	.04	-.08	.00	.16*	1.00	
14	Approx addition	.14	.05	.15	.07	.22***	.16	.03	.03	.25**	.20**	.29**	.07	.16*	1.00

Note. *$p \leq .05$; **$p \leq .01$; ***$p \leq .001$.

Table 2.4

Correlation Matrix between Split Comparison Scores, ENT and Memory Measures

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	ENT	1.00														
2	ENT-Relational	.81***	1.00													
3	ENT-Counting	.93***	.57***	1.00												
4	Forward word recall	.28***	.21**	.27***	1.00											
5	Forward digit recall	.32***	.26***	.31**	.52***	1.00										
6	Path recall	.21**	.19*	.22**	-.01	.07	1.00									
7	Backward word recall	.17*	.07	.19*	.14	<.01	.06	1.00								
8	Backward digit recall	.34***	.18*	.34***	.36***	.16*	.10	.43***	1.00							
9	Path dual task	.35***	.26***	.32***	.03	.02	.18*	.30***	.22**	1.00						
10	Comparison-Intermix-congr	.10	.13	.09	.06	.17*	.08	.01	-.05	-.06	1.00					
11	Comparison-Intermix-neutral	.16*	.17*	.12	.17*	.20*	-.04	-.03	-.04	-.04	.32***	1.00				
12	Comparison-Intermix-incongr	.15	.19*	.12	.17*	.13	-.01	.08	-.01	.05	.34***	.34***	1.00			
13	Comparison-Separate-congr	.14	.09	.16*	.15	.01	-.04	-.06	-.12	-.05	.32***	.23**	.13	1.00		
14	Comparison-Separate-neutral	.17*	.13	.18*	.07	-.07	-.02	.02	-.09	.01	.15	.19*	.05	.59***	1.00	
15	Comparison-Separate-incong	-.02	-.06	-.02	<.01	.03	.02	.09	.09	.04	-.26**	-.20*	-.07	-.56***	-.42***	1.00

Note. *$p \leq .05$; **$p \leq .01$; ***$p \leq .001$.

2.4.2 More detailed analyses on ANS

Considering ANS tasks, it is important to firstly point out that all of them were performed above chance, that is above the 50% probability of giving the right response by chance: Comparison-Intermixed (*M*=25.92, *t*(156)=14.65, *p*<.001), Comparison-Separate (*M*=37.65, *t*(156)=30.23, *p*<.001), and Approximate addition (*M*=15.79, *t*(156)=15.79, *p*<.001).

For the reason that the two magnitude comparison tasks were developed in order to be balanced by total area and therefore present dots of different sizes, we split related scores in order to have separate scores for the three possible conditions: congruent (when the most numerous array is that with the bigger dots), neutral (when the two arrays have the same dot size) and incongruent trials (when the most numerous array has smaller dots). Also considering the split scores, responses were above chance for all Comparison-Intermixed trials: neutral (*M*=14.15, *t*(156)=11.82, *p*<.001), congruent (*M*=5.50, *t*(156)=12.83, *p*<.001), and incongruent (*M*=6.27, *t*(156)=7.18, *p*<.001). In relation to the Comparison-Separate task, responses were significantly above chance for neutral (*M*=18.61, *t*(156)=29.01, *p*<.001), and congruent (*M*=14.31, *t*(156)=38.82, *p*<.001) trials; anyway, performance was significantly below the 50% expected if children had responded by guessing (*M*=4.73, *t*(156)= -14.93, *p*<.001).

Since various recent studies stated that the significant relation between magnitude comparison and math performance has to be attributed solely to incongruent trials (which reflect WM and inhibition capacities), new exploratory correlation analyses were carried out. Considering ENT performance, by inspecting Table 3.4 it is possible to notice that neutral and incongruent Comparison-Intermixed trials were significantly related to ENT-Relational only (respectively, *r*=.17, *p*<.05; *r*=.19, *p*<.05), and the neutral ones also with overall ENT (*r*=.16, *p*<.05). For Comparison-Separate task, neutral trials were significantly related to both overall ENT and ENT-Counting (respectively, *r*=.17, *p*<.05; *r*=.19, *p*<.05), whereas congruent trials to ENT-Counting only (*r*=.16, *p*<.05). Shifting to the relation with memory measures, neutral and incongruent Comparison-Intermixed were significantly associated to Forward word recall only (for both, *r*=.17, *p*<.05). Regarding Comparison-Separate task, no one significant relation could be observed.

2.4.3 Path models

In order to explore the relationships standing between math precursors and early math performance, various path analyses have been conducted. This technique is an extension of multiple regression since it allows the assessment of the relationship between source variables, called *exogenous variables*, and the outcomes, defined *endogenous variables*. For models estimation, the Maximum Likelihood approach (Jöreskog & Sörbom, 1996) has been chosen, while performing an additional (Bollen-Stine) bootstrapping procedure.

On the bases of theoretical assumptions, different path models have been drawn. Related variables were represented by composite scores computed by averaging the z-scores of the selected tasks. The first proposed solution, termed Model 1.1, entailed the subdivision, within memory, between verbal and visuo-spatial aspects. As a consequence, STM and WM were defined by two variables each: one represented by verbal (Forward word and Forward digit recall for STM and Backward word and Backward digit recall for WM) composites scores, and the other by the standardized score achieved in the visuo-spatial tasks (Path recall for STM and Path dual task for WM). In relation to intelligence, the two standardized scores achieved in Vocabulary and Block design were treated as separate variables, representing respectively Verbal and Performance intelligence. Since it was not possible to determine which of the two aspects mainly influenced the other, the relation between these two variables was defined in terms of correlation. Finally, considering ANS, Approximate addition was considered as a single variable dissociated from the composite score deriving from the two magnitude comparison tasks (Comparison-Intermixed and Comparison-Separate).

Alternative models were however possible. For instance, on the basis of literature evidences, the direction of the link between memory and intelligence could be reversed. A new model was therefore designed, where intelligence was supposed to be predicted by both STM and WM, but also by ANS. Anyway, final model fit wasn't statistically acceptable. As a consequence, also by referring to similar studies reporting intelligence as a more background variable (see Passolunghi et al., 2007, 2008), all the subsequent models entailed relations directed from intelligence to the other cognitive abilities.

The final model is that reported in Figure 2.1.1 (and related model parameters in Table 2.5.1). As can be clearly noticed, this model was highly predictive of ENT, since R^2=.41. Moreover, model fit indices were good: CMIN=15.98, df=17, *p*=.53, CFI=1.000, NFI=.920, TLI=1.013, RMSEA<.001.

By inspecting Model 1.1, it can be observed that overall ENT was directly predicted by both Verbal and Performance intelligence (respectively *β*=.20, *p*<.01, *β*=.36, *p*<.001), both Verbal (*β*=.13, *p*<.05) and Visuo-spatial WM (*β*=.16, *p*=.01) and by Comparison (*β*=.19, *p*<.01). The predictive power of Visuo-spatial STM approached significance (*β*=.12, *p*=.06). Verbal STM acted only indirectly (*β*=.02) by influencing Verbal WM level.

Other effects are noteworthy. A significant link departed from both Verbal and Visuo-spatial WM to Approximate addition (respectively, *β*=.20, *p*=.01; *β*=.23, *p*<.01) and from Verbal to Visuo-spatial WM (*β*=.15, *p*<.01). This link was observed to be bidirectional, but a better statistical fit was achieved with the reported model.

Figure 2.1.1 Path Model 1.1

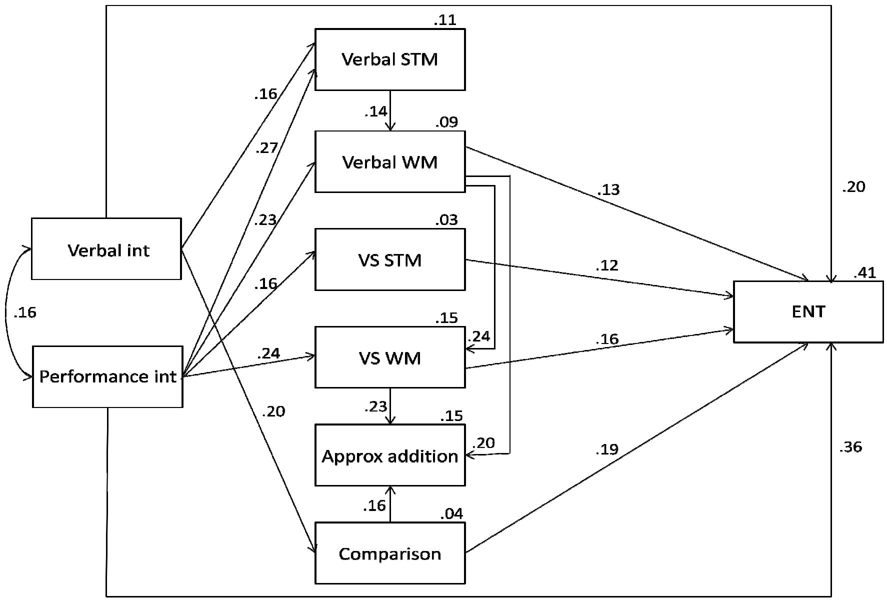

Figure 2.1.1. Standardized solution for Model 1.1 of the relationships between predictor variables and overall ENT performance. VS STM/WM= Visuo-spatial STM/WM. Model fit indices: CMIN=15.98, df=17, *p*=.53, CFI=1.000, NFI=.920, TLI=1.013, RMSEA<.001.

Table 2.5.1
Standardized Parameters of Path Model 1.1

Endogenous variable	Exogenous variable	Direct effect	Indirect effect	R^2
Verbal STM	Verbal intelligence	.16*		.11
	Performance intelligence	.27**		
Visuo-spatial STM	Performance intelligence	.16*		.03
Verbal WM	Performance intelligence	.23**		.09
	Verbal STM	.14 [a]		
Visuo-spatial WM	Performance intelligence	.24**		.15
	Verbal WM	.15**		
Comparison	Verbal intelligence	.20**		.04
Approximate addition	Verbal WM	.20**		.15
	Visuo-spatial WM	.23**		
	Comparison	.16*		
ENT	Verbal intelligence	.20**	.04	.41
	Performance intelligence	.36***	.13	
	Verbal STM		.02	
	Visuo-spatial STM	.12 [b]		
	Verbal WM	.13*	.04	
	Visuo-spatial WM	.16**		
	Comparison	.19**		

Note. Direct and indirect effects are expressed as *β* values. Indirect effects are reported only for ENT.
*$p \leq .05$; **$p \leq .01$; ***$p \leq .001$. [a] $p = .07$; [b] $p = .06$.

A second investigation was carried out by testing a partially different model, indicated as Model 1.2, which included the same intelligence and ANS variables of the previous, but differed in the organization of memory components. In fact, a distinction within memory can be operated also in reference to the nature of the information to be processed. In line with this, three new composite scores were calculated: Word memory (from Forward and Backward word recall), Digit memory (Forward and Backward digit recall) and Visuo-spatial memory (Path recall and Path dual task). Model 1.2 is illustrated in Figure 2.1.2 and pertaining parameters in Table 2.5.2.

Also in this case, except for Approximate addition, all the other variables were observed to predict, directly and/or indirectly, ENT scores, by explaining the 42% of its variance. Model fit was good: CMIN=16.39, df=16, *p*=.42, CFI=.998, NFI=.920, TLI=.995, RMSEA=.012.

Direct effects were provided by Verbal Intelligence (*β*=.21, *p*<.001), Performance intelligence (*β*=.36, *p*<001), Digit memory (*β*=.20, *p*<.01), Visuo-spatial memory (*β*=.20, *p*<.01), and Comparison (*β*=.19, *p*<.01). No significant relationships were observed between memory and ANS variables. On the contrary, the three memory measures resulted to be related with bidirectional links. Anyway, the reported relationships are those determining the best theoretical and statistical fit and no one effect on ENT varied by changing the directionality of these links (i.e., Word memory was predictive in no one case).

Figure 2.1.2 Path Model 1.2

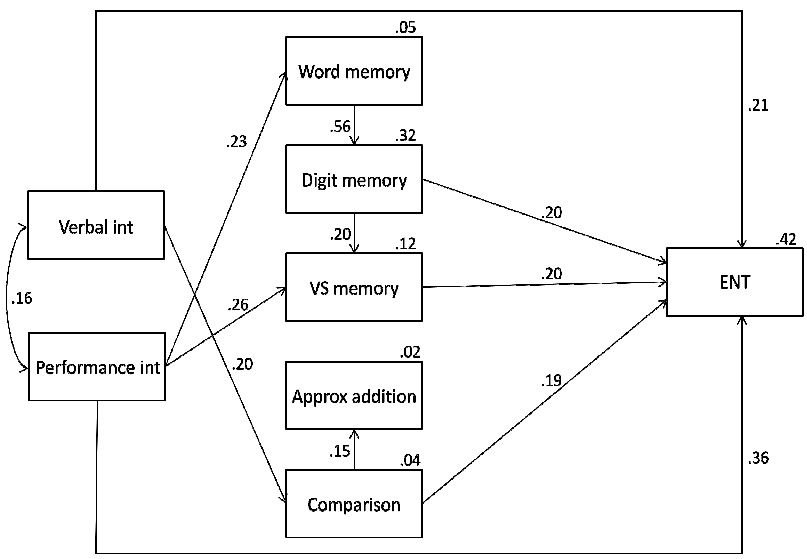

Figure 2.1.2. Standardized solution for Model 1.2 of the relationships between predictor variables and overall ENT performance. VS memory= Visuo-spatial memory. Model fit indices: CMIN=16.39, df=16, *p*=.42, CFI=.998, NFI=.920, TLI=.995, RMSEA=.012.

Table 2.5.2

Statistical Parameters of Path Model 1.2

Endogenous variable	Exogenous variable	Direct effect	Indirect effect	R^2
Word memory	Performance intelligence	.23**		.05
Digit memory	Word memory	.56***		.32
Visuo-spatial memory	Performance intelligence	.26***		.12
	Digit memory	.20**		
Comparison	Verbal intelligence	.20**		.04
Approximate addition	Comparison	.15 [a]		.02
ENT	Verbal intelligence	.21***	.08	.42
	Performance intelligence	.36***	.04	
	Word memory		.13	
	Digit memory	.20**	.04	
	Visuo-spatial memory	.20**		
	Comparison	.19**		

Note. Direct and indirect effects are expressed as *β* values. Indirect effects are reported only for ENT.
*$p \leq .05$; **$p \leq .01$; ***$p \leq .001$; [a] $p = .06$.

By summing up findings concerning overall ENT, global results suggested the same to be directly influenced by both aspects of intelligence, overall WM, Visuo-spatial STM and by Comparison within ANS. Concerning verbal memory, a significant role was mainly held by the aspect of digit recall. Having examined precursor influence on overall ENT, a related aim was to inspect how their predictive power could change in relation to different levels of assessed ENT skills. Pertaining results are illustrated in the next session.

2.4.3.1 Models for different levels of early math skills

In this section, the influence of precursor abilities is examined by taking into account the two main skill levels identifiable within early mathematics and represented by ENT-Relational and ENT-Counting. New models correspondent to Model 1.1 and 1.2 were so tested and termed respectively Model 2.1 and Model 2.2. In both models, ENT-Relational and ENT-Counting were associated in a way that the former was supposed to be at the basis, and therefore predict, the latter.

Model 2.1 in represented in Figure 2.2.1 and related parameters in Table 2.6.1. This model explained 26% of ENT-Relational variability and 42% of that of ENT-Counting. Fit indices reflected a good statistical model: CMIN=31.59, df=25, p=.17, CFI=.969, NFI=.881, TLI=.931, RMSEA=.041.

As the relations between precursors were the same of Model 1.1, only the direct influence of the same on ENT domains has been examined. In particular, ENT-Relational was directly influenced by Verbal and Performance intelligence (respectively, β=.19, p<.01; β=.35, p<.001), Visuo-spatial WM (β=.14, p=.05) and Comparison (β=.15, p<.05). ENT-Counting, on the other hand, besides the indirect influence of the just mentioned variables mediated by ENT-Relational (which significantly predicted it, β=.43, p<.001), was also significantly and directly predicted by Performance intelligence (β=.19, p<.01), Verbal WM (β=.20, p<.01) and Comparison (β=.13, p<.05).

Also the inspection of Model 2.2 showed interesting results (see Figure 2.2.2 and Table 2.6.2 for related parameters). This model explained 28% and 41% variance of ENT-Relational and ENT-Counting, respectively. Model fit indices were acceptable: CMIN=31.17, df=21, p=.07, CFI=.954, NFI=.884, TLI=.902, RMSEA=.050.

Taking into consideration only direct relations, it was possible to notice that ENT-Relational was significantly predicted by both Verbal and Performance intelligence (respectively, β=.19, p<.01; β=.33, p<.001), Visuo-spatial memory (β=.18, p=.01), and Comparison (β=.15, p<.05). Considering how the model has been set up, Word and Digit memory were observed to act only indirectly (respectively, β=.02; β=.04), via Visuo-spatial memory.

Figure 2.2.1 Path Model 2.1

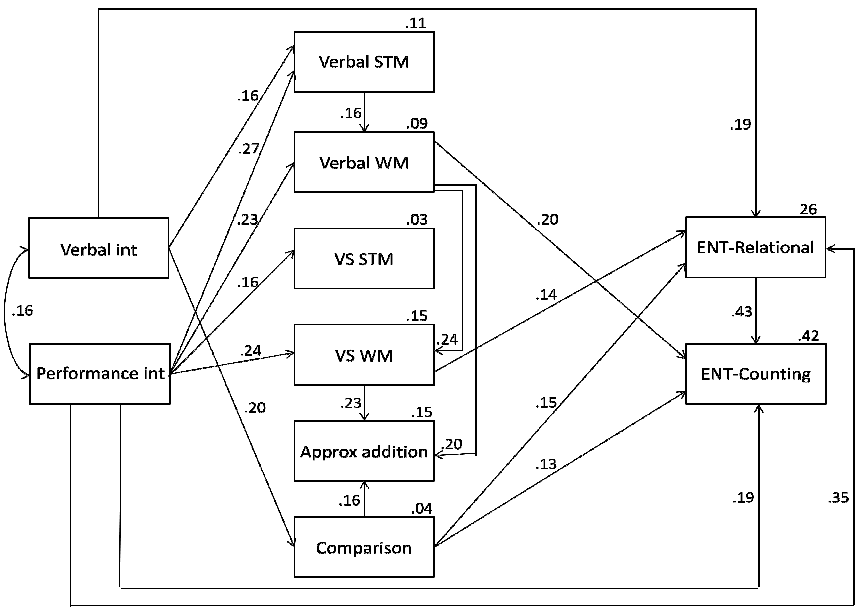

Figure 2.2.1. Standardized solution for Model 2.1 of the relationships between predictor variables and ENT-Relational and ENT-Counting performance. VS STM/WM= Visuo-spatial STM/WM. Model fit indices: CMIN=31.59, df=25, p=.17, CFI=.969, NFI=.881, TLI=.931, RMSEA=.041.

Table 2.6.1

Statistical Parameters of Path Model 2.1

Endogenous variable	Exogenous variable	Direct effect	Indirect effect	R^2
Verbal STM	Verbal intelligence	.16*		.11
	Performance intelligence	.27***		
Visuo-spatial STM	Performance intelligence	.16*		.03
Verbal WM	Performance intelligence	.23**		.09
	Verbal STM	.16 [a]		
Visuo-spatial WM	Performance intelligence	.24**		.15
	Verbal WM	.24**		
Comparison	Verbal intelligence	.20**		.04
Approximate addition	Verbal WM	.20**		.15
	Visuo-spatial WM	.23**		
	Comparison	.16*		
ENT-Relational	Verbal intelligence	.19**	.03	.26
	Performance intelligence	.35***	.04	
	Verbal STM		.01	
	Visuo-spatial STM			
	Verbal WM		.03	
	Visuo-spatial WM	.14*		
	Comparison	.15*		
ENT-Counting	Verbal intelligence		.13	.42
	Performance intelligence	.19**	.22	
	Verbal STM		.03	
	Visuo-spatial STM			
	Verbal WM	.20**	.01	
	Visuo-spatial WM		.06	
	Comparison	.13*	.06	
	ENT-Relational	.43***		

Note. Direct and indirect effects are expressed as *β* values. Indirect effects are reported only for ENT-Relational and ENT-Counting. *p≤.05; **p≤.01; ***p≤.001; [a] p=.06.

ENT-Counting, on the other hand, apart from the indirect influence received via ENT-Relational (*β*=.41, *p*<.001), is also directly predicted by Performance intelligence (*β*=.19, *p*<.01), Digit memory (*β*=.18, *p*<.01), Visuo-spatial memory (*β*=.14, *p*<.01), and Comparison (*β*=.15, *p*<.05).

By concluding this section, it is possible to point out that ENT-Relational and ENT-Counting were both directly influenced by intelligence, Visuo-spatial memory and Comparison within ANS. Verbal memory, and specifically that associated to digit recall, instead projected uniquely toward ENT-Counting.

Figure 2.2.2 Path Model 2.2

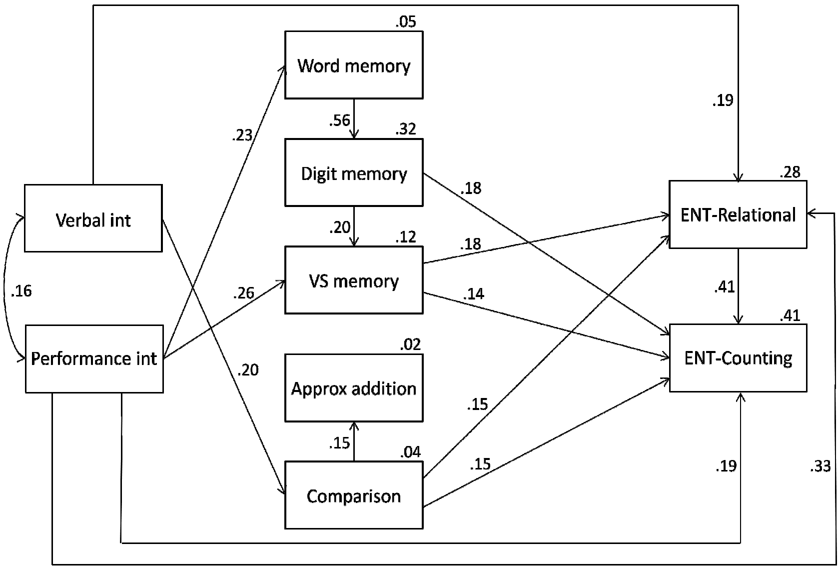

Figure 2.2.2. Standardized solution for Model 2.2 of the relationships between predictor variables and ENT-Relational and ENT-Counting performance. VS memory = Visuo-spatial memory. Model fit indices: CMIN=31.17, df=21, *p*=.07, CFI=.954, NFI=.884, TLI=.902, RMSEA=.050.

Table 2.6.2

Statistical Parameters of Path Model 2.2

Endogenous variable	Exogenous variable	Direct effect	Indirect effect	R^2
Word memory	Performance intelligence	.23**		.05
Digit memory	Word memory	.56***		.32
Visuo-spatial memory	Performance intelligence	.26***		.12
	Digit memory	.20**		
Comparison	Verbal intelligence	.20**		.04
Approximate addition	Comparison	.15 [a]		.02
ENT-Relational	Verbal intelligence	.19**	.05	.28
	Performance intelligence	.33***	.03	
	Word memory		.02	
	Digit memory		.04	
	Visuo-spatial memory	.18**		
	Comparison	.15*		
ENT-Counting	Verbal intelligence		.21	.41
	Performance intelligence	.19**	.12	
	Word memory		.12	
	Digit memory	.18**	.04	
	Visuo-spatial memory	.14*	.07	
	Comparison	.15*	.06	
	ENT-Relational	.41***		

Note. Direct and indirect effects are expressed as *β* values. Indirect effects are reported only for ENT-Relational and ENT-Counting.
*p≤.05; **p≤.01; ***p≤.001; [a] p=.06.

2.5 Discussion

The primary purpose of the current study was to investigate the relative role of both general and specific cognitive abilities in early math skills not involving symbolic aspects. In fact, very few were the researches having explored, before that date, the relative influence of all these variables. More in detail, the study was aimed at investigating, within general abilities, the involvement of memory in both its verbal and visuo-spatial aspects, by also distinguishing between short-term memory, STM (more passive processes) and working memory, WM (more active processes). Actually, only a few works have extensively examined the involvement of memory by considering all components (e.g., Fuchs et al., 2010; Passolunghi & Lanfranchi, 2012) and the investigation led sometimes to contradictory results. Further, regarding general abilities, also the role of intelligence (in both its verbal and performance aspects) has been explored and put in relation to that of the other variables.

Even more tricky was the evaluation, within specific abilities, of the approximate, very basic aspect of number sense, termed ANS and explored here by means of magnitude comparison and approximate addition tasks. In fact, while some studies reported its significant involvement in math learning (e.g., Desoete et al., 2012; Gilmore et al., 2010; Mazzocco et al., 2011), many others evidenced opposing results (Holloway & Ansari, 2009; Iuculano et al., 2008; Rousselle & Noël, 2007; Soltész, Szűcs, & Szűcs, 2010).

In order to achieve these goals, different path models have been designed and their validity tested. On the whole, our findings suggested that all the investigated abilities have a significant role in determining early math performance, even if in a different extent. Specifically, two different models were tested in relation to each math outcome: the first models operated a distinction between intelligence aspects (verbal and performance, corresponding roughly to crystallized and fluid aspects) and also, within both STM and WM, between verbal and visuo-spatial components. Regarding ANS, the aspect of approximate addition was considered separately from that of magnitude comparison. The second models differed only for subdivisions within memory and therefore separately considered the specific nature of data to be processed: words, digits and visuo-spatial patterns.

Considering Models 1.1 and 1.2, they explored precursor effects on overall ENT scores and both explained an appreciable quote of ENT variance (41% and 42% for Model 1.1 and Model 1.2, respectively). By considering the overall findings emerging from both models, it was possible to notice a significant contribution of almost all the assessed components. In some cases, standardized effects weren't particularly strong, but it is important to point out that they regarded individual components and thus that the whole effect of related constructs has been partitioned.

Considering intelligence, it could be evidenced a significant contribution, both direct and indirect, of both aspects and particularly of the performance one. In terms of strength, intelligence appeared to be the strongest precursors of early math achievement. With regard to memory, interesting outcomes emerged. Within verbal memory, a significant direct contribution of WM could be noticed, while STM acted only indirectly by influencing the WM level. However, in relation to visuo-spatial memory, both STM and WM gave a direct significant contribution.

Taken together, these results about memory confirmed the main role that it assumes, on the whole, in determining math achievement at the beginning of schooling. More in detail, our findings corroborate those reported in studies assuming a more consistent role of the more active processes associated to WM (e.g., Bull & Scerif, 2001; De Smedt et al., 2009; Passolunghi & Lanfranchi, 2012) and of the visuo-spatial component in particular (e.g., Jarvis & Gathercole, 2003; McKenzie et al., 2003). Nonetheless, also STM appeared to be significantly involved in early math achievement, in agreement with some research findings (e.g., Bull et al., 2008). In relation to the verbal component, the recall of a verbal material tightly associated to math (digits) was considered a stronger predictor of early math competences, as put forward in other works (e.g., Geary et al., 2004; Passolunghi & Cornoldi, 2008; Temple & Sherwood, 2002). In this perspective, children who are observed to master with greater facility number words (in a way that allows a better recall of the same) seem to be also more advantaged in math tasks. Seen from another point of view, precocious math difficulties can probably be rooted in a deficient number acquisition, causing consequent difficulties in retaining numerical information.

Finally, in relation to ANS, it was interesting to note that only the component of magnitude comparison contributed to ENT scores and in a comparable extent of that of memory components. A possible explanation for the absence of a significant approximate addition involvement (found in many studies, see Glimore et al., 2010) can be that this task has been developed in the Anglo-Saxon context where a formative period (kindergarten) occurs before entering primary school. Italian children begin primary school without notions of math except for those acquired in the family context. Hence, their performance on this more difficult task resulted to be not predictive of early math skills. This is also proven by the not-strong-as-expected relation between approximate addition and magnitude comparison (come out in Iuculano et al., 2008).

Approximate addition, in fact, showed different correlation patterns with respect to magnitude comparison. Actually, it was predicted by WM, both verbal and visuo-spatial, suggesting the cognitive load it entailed. That is, active memory resources can be recruited by so young children to perform this task (by visually manipulating the shown information, but also by elaborating it verbally to produce self-explanations concerning what observed).

In this sense, magnitude comparison can be considered a more basic and pure ANS task, at least in young children. By considering therefore only this component as representing ANS, it can be stated that it is almost dissociated from memory. This finding is interesting since allows to affirm ANS to contribute to math achievement roughly independently from memory, thus contradicting results arguing ANS involvement to disappear when taking into account memory resources. In fact, when developing magnitude comparisons tasks, obligatory constraints have to be followed in order to avoid perceptual confounding effects. As a consequence, the resulting tasks are characterized by features that are inevitably perceptually relevant and include the modification of dot size, array size and dot density. In this way a proper number of incongruent trials (where the information given by the size of dots is incongruent with that of their numerosity) needs to be set up to control for total area effects. In other terms, in some trials the most numerous array was made up by dots with dimensions smaller than that of the less numerous one.

Keeping in mind this, some studies (Gilmore et al., 2013; Soltész et al., 2010; Szűcs et al., 2007) affirmed that the significant ANS role can be the result of the performance achieved only in the incongruent trials, which is, in turn, affected by memory resources and by visuo-spatial features. In the current study, however, a relationship between these aspects and magnitude comparison didn't emerge (ANS was mildly influenced only by intelligence). Nevertheless, in order check more deeply this possible association, further analyses have been conducted by separately considering the relationships between ENT performance, memory measures and scores achieved in neutral, congruent and incongruent trials of the two magnitude comparison tasks. Correlation analyses reveled that the only association with memory was that between Comparison-Intermixed task, incongruent but also neutral trials, and Forward word recall and that this relationship wasn't particularly strong. Therefore, these findings seemed to lead to the conclusion that, at least in this stage of development, pure ANS (thus not the aspect involving also approximate addition) and memory are almost two independent abilities that don't significantly interact in the determination of early math attainment.

2.5.1 Predictive effects on different early math skills domains

A second aim of the current study was to explore possibly different recruitment of math precursors in dependence to tasks of increasing difficulty within early math skills. With this purpose, path models were also developed in order to compare precursors contribution with respect to the two components identified within ENT and named ENT-Relational and ENT-Counting (see also Aunio et al., 2009). The former explored more basic and immediate quantitative-numerical capacities, whereas the latter assessed higher skills requiring counting and deeper knowledge of numbers.

As hypothesized, interesting differences could be highlighted. For instance, intelligence mainly explained ENT-Counting than ENT-Relational performance. In relation to memory, it could be noted that visuo-spatial WM acted directly on ENT-Relational and indirectly on ENT-Counting. When taking overall visuo-spatial memory (without distinction between active or passive processes), this was shown to act directly on both ENT domains. Verbal WM, instead, positively affected only the latter. This finding can be explained by the fact that the acquisition of more complex early math abilities could require not simply the capacity of passively retaining new verbal information (such as counting sequence or enumeration) but also that of actively manipulating it, at least in early stages. Furthermore, it was specifically memory for digits to predict ENT-Counting, which indeed required the knowledge of numbers up to 20. By summing up, also considering different early math skills levels, the importance of active memory processes, and specifically of digits and visuo-spatial patterns, was further confirmed.

Finally, considering ANS, magnitude comparison held a significant direct role in predicting both ENT-Relational and ENT-Counting. This finding was interesting since it suggested that very basic number sense skills are not simply involved in very basic math abilities requiring possibly similar competences, but have also an additional significant involvement in the acquisition of more structured abilities.

2.5.2 Concluding remarks

The set up of the present study originated from the necessity to elucidate the involvement of different kinds of precursors at the basis of math learning, with particular attention to the early phases of this process. In fact, very few studies deeply examined the interrelation of both general and specific precursors in the determination of math achievement. Results emerging from the current study helped in providing a clearer picture of the process, by confirming not only the leading role of memory, but also of that of very basic specific precursors represented by the capacity of making approximate magnitude judgments. This work indeed demonstrated these skills to significantly contribute to early math learning, with a role that is almost disengaged from that of memory.

A crucial point however regarded the assessment of ANS. Actually, since approximate addition didn't result to be a suitable measure, other tasks have to be develop to better explore ANS. In addition, these approximate, non-symbolic skills have been evaluated by considering only accuracy and not reaction times. Additional information would probably emerge when taking into account also the time necessary to discriminate between different magnitudes. This index is likely to be more indicative by the end of first grade, when accuracy can become less discriminative and thus less informative.

Finally, having not the possibility to evaluate other factors such as socio-economic status, math income level couldn't be controlled for variables that could have had an impact on both early math achievement and cognitive abilities development. Therefore, as the gap deriving from a different background level is supposed to reduce as the result of schooling, the involvement of these precursors should be also subsequently monitored to more closely check their predictive role also in future, more formal math attainment.

CHAPTER 3

Study 2

Longitudinal monitoring of math precursors for the prediction of future math achievement.

3.1 Theoretical background

The investigation of the cognitive abilities fundamental for the acquisition of academic competences is one of the main research focus in the field of mathematical learning. Many researchers argued that fundamental abilities or precursors of math learning are represented by general factors such as memory. Nowadays, there is a quite strong agreement regarding the crucial role of this ability in almost every stage of development and instruction (e.g., Bull & Scerif, 2001; Gathercole & Pickering, 2000a; Holmes & Adams, 2006; Passolunghi et al., 2007, 2008). However, greater consensus has received the more active aspect of memory, represented by the central executive component of Baddeley's model and defined working memory (WM) in the strict sense. On the other hand, the role of more passive processes identifiable with short-term memory (STM) is not clear in the same extent. Some authors support the idea that the visuo-spatial processes (visuo-spatial sketchpad) have a prominent involvement (Bull et al., 2008; Jarvis & Gathercole, 2003), others argued in favor of the verbal component (phonological loop, e.g., Hecht et al., 2001; Swanson & Sachse-Lee, 2001). A third position postulates the visuo-spatial aspect to be crucial very early and the verbal one starting from the age of 7, when children begin to master a spontaneous verbal rehearsal (e.g., McKenzie et al., 2003). To further complicate the picture, the different memory components seem to have a different recruitment also in relation to the math task at hand, so in dependence to the nature of the task (e.g., computation or word problem solving), but also to its complexity.

Another debated (but not much studied) general precursor is represented by intelligence, known to be crucial in particular in learning new information, but by definition not lacking in individuals affected by learning disabilities (see also Alloway, 2009). Therefore, it appears quite difficult to identify with which extent it is crucial in learning math at a certain developmental stage.

Other studies, on the contrary, have been focused on the evaluation of the specific math learning precursors generally identifiable with number sense. Within this ability, a strong debate subsists on the involvement of very basic, informal, approximate skills referring to the Approximate number system (ANS). In fact, while many studies supported its significant involvement (e.g., Halberda et al., 2008; Libertus et al., 2011; Mazzocco et al., 2011), others did not (e.g., Iuculano et al., 2008; Rousselle & Noël, 2007). Specifically, some authors argued these abilities to lose their significant role when more formal, symbolic competences are acquired and therefore already in the first grades of primary school their predictive power seemed to be almost absent (e.g., Cirino, 2011; Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013).

Finally, very few are the studies investigating both general and specific (especially in the form of ANS) math precursors. Furthermore, while many authors inspected longitudinally the involvement of memory (e.g., De Smedt et al., 2009; Meyer et al., 2010; Passolunghi et al., 2008), the predictive power of ANS was often measured concurrently or at least some months later than its assessment (or retrospectively, as in Halberda et al., 2008). The result is that it is not clear if its early evaluation can be effectively predictive not only of concurrent math skills but also of those acquired in subsequent grades.

3.2 The current study: Aims and hypotheses

The fundamental purpose of the present study, hereinafter indicated as Study 2, was that of exploring the recruitment of selected math cognitive precursors in a longitudinal perspective. More precisely, the intention was that of following longitudinally children from first to third grade and investigate the predictive power across grades of the fundamental precursors assessed in Study 1: intelligence, STM, WM and ANS. Particularly intriguing was the investigation of ANS involvement in formal math skills, since in Study 1 an evaluation of these competences hasn't been carried out. Hence, the final aim was to evaluate the effective capability of these selected precursors to predict not only early math skills but also the subsequent, more formal ones. The intention was also to investigate the role these skills can hold in the prediction of specific math competences, by dedicating particular attention to the main aspects of computation and word problem solving.

A secondary aim consisted in a further evaluation of the same cognitive precursors in second grade in order to inspect how the extent of their impact can differ in dependence to the assessment time. In this perspective, it could be possible to identify which are the factors that measured at certain grade are more effective in the prediction of concurrent and future math learning.

General hypotheses were that precursors assessed in Grade 1 could hold a significant involvement in the prediction of math achievement in the following grades. More in detail, WM was expected to be a key predictor across all grades, whereas the role of STM was expected to be secondary but with a main impact of the visuo-spatial component. The verbal aspect was hypothesized to be more crucial in relation to problem solving. The involvement of intelligence and ANS was supposed to decrease. Intelligence, however, was hypothesized to maintain at least an indirect involvement.

Considering precursor investigation in the two different time points, assessment in Grade 1 was expected to predict performance in Grade 3 in a slightly different extent of Grade 2 assessment. Actually, the involvement of the different components of cognitive precursors has been proven to chance along with development and formal education.

3.3 Method

3.3.1 Participants

From the large sample of first-grade children recruited for Study 1, 85 were enrolled also for testing in following grades. These students attended four different classes across the three primary schools in Northeastern Italy that expressed their consent to participate in a longitudinal study. From the available sample, 2 children were excluded because diagnosed for neurological diseases, and other 3 of them because not mastering sufficiently the Italian language. From the sample 80 students, 78 (48 males; mean age: 6 years, 5 months at Grade 1; 7 years, 5 months at Grade 2; 8 years, 5 months at Grade 3) participated to all testing sessions and thus represented the final sample.

3.3.2 Procedure

Data collection took place before the end of first term of each school year from Grade 1 to Grade 3. Before proceeding, formal consent was obtained from the school headmaster, math teachers and parents.

In Grade 1, children had been tested for both cognitive abilities and early math skills. The assessment of the cognitive precursors occurred also in Grade 2 by using equivalent tasks. Tasks could differ from the two assessment times for the reason that it hadn't been possible to find all tests suitable for children of both ages. Anyway, when differing, tasks were selected in order to be equivalent and of a proper difficulty for children of that age. Math tests were administered, apart from Grade 1, in both Grades 2 and 3.

In Grade 2, as for Grade 1, cognitive precursor abilities were tested by evaluating each child individually in two different sessions lasting approximately 30 min. Testing took place in a quiet room outside the classroom and within two consecutive weeks for each child. All the administered tasks were preceded by related task instructions. Math tests in Grades 2 and 3 were instead administered to the whole class. Instructions were collectively given and testing began after having checked that children have understood them. Testing was on a time basis.

3.3.3 Measures

3.3.3.1 Grade 1 precursors

For detailed information about tasks administered in Grade 1, it is possible to refer to the Method section of Study 1. By summing up, tasks were the following:

Intelligence: *Vocabulary* and *Block design* (from WISC-III, Wechsler, 1991; Italian edition, 2006), respectively for the evaluation of verbal and performance intelligence. *Vocabulary* entails children to give exhaustive definitions of selected words; *Block design* requires the reproduction of bi-dimensional configurations by means of solid cubes. Maximum age-reference score is 19 points for both tasks.

Short-term memory (STM): *Forward word recall* (from Lanfranchi et al., 2004) and *Forward digit recall* (from WISC-III, Wechsler, 1991; Italian edition, 2006) for the verbal component; *Path recall* (from Lanfranchi et al., 2004) for the visuo-spatial component. These tasks require the reproduction of increasing number respectively of words, digits and moves on a grid in the same experimenter's presentation order. Maximum score for both Forward word recall and Path recall is 8 points, 16 for Forward digit recall.

Working memory (WM): *Backward word recall* (developed from Lanfranchi et al., 2004) and *Backward digit recall* (from WISC-III, Wechsler, 1991; Italian edition, 2006) for the verbal component; *Path dual task* (from Lanfranchi et al., 2004) for the visuo-spatial component. The first two tasks entail the repetition of increasing number of words and digits, respectively, in the inverted order of that used by the experimenter. The last task involves the selective recall of the first step of paths of increasing length traced on a grid and the simultaneous performance of a secondary task. Maximum score for both Backward word recall and Path recall is 8 points, 14 for Backward digit recall.

Approximate number system (ANS): *Magnitude comparison of intermixed quantities* or *Comparison-Intermixed* (adapted from Iuculano et al., 2008), *Magnitude comparison of separate quantities* or *Comparison-Separate* (adapted from Piazza et al., 2010), and *Approximate addition* (adapted from Iuculano, Tang, Hall, & Butterworth, 2008). The goal in the first two tasks is the identification, without counting, of the most numerous array of dots simultaneously presented. The goal is similar for the last task, but in this case dots are presented in succession in three different arrays, of which the first two are supposed to be summed and then compared to the third. Maximum score is 40, 56 and 24 points respectively.

3.3.3.2 Grade 2 precursors

The typology of tasks administered in Grade 2 is the same of that of Grade 1. Anyway, previous pilot studies suggested the adaptation or reselection of some of the tasks in order to be of a proper difficulty for second graders. Tasks are listed below and those differing from Grade 1 are briefly described.

Intelligence: *Vocabulary* and *Block design* (from WISC-III, Wechsler, 1991; Italian edition, 2006), respectively for the evaluation of verbal and performance intelligence.

Short-term memory (STM): *Forward word recall* (from Passolunghi & De Beni, 2001) and *Forward digit recall* (from WISC-III, Wechsler, 1991; Italian edition, 2006) for the verbal component; *Path recall* (adapted from Lanfranchi et al., 2004) for the visuo-spatial component. *Forward word recall* is made up of 2 trials for each of the 5 levels of difficulty (3- to 7-word spans), giving a total of 10 trials, plus 2 examples. 1 point is assigned to each correct recall, with a final maximum score of 10 points. *Path recall* differs from the previous version in task difficulty. In fact, the 4 difficulty levels are represented by 2- to 5-square pathways on 4x4 and 5x5 grids. The maximum score is always 8. *Forward digit recall* remained unchanged.

Working memory (WM): *Backward word recall* (from Passolunghi & De Beni, 2001) and *Backward digit recall* (from WISC-III, Wechsler, 1991; Italian edition, 2006) for the verbal component; *Path dual task* (adapted from Lanfranchi et al., 2004) for the visuo-spatial component. *Backward word recall* includes 2 example trials, followed by 2 trials for each of the 3 increasing levels of difficulty (3- to 5-word spans), with a total of 6 trials. 1 point is assigned to correct responses. The maximum score is 6 points. *Path dual task* differs from the previous version in items difficulty. There are always 4 level of increasing difficulty, but they include 3- to 5-square pathways on 4x4 and 5x5 grids. The maximum score is 8. *Backward digit recall* remained unchanged

Approximate number system (ANS): *Magnitude comparison of intermixed quantities* or *Comparison-Intermixed* (adapted from Iuculano et al., 2008), *Magnitude comparison of separate quantities* or *Comparison-Separate* (adapted from Piazza et al., 2010), and *Approximate addition* (adapted from Iuculano, Tang, Hall, & Butterworth, 2008). *Comparison-Separate* differs from the previous version in the number of trials and related difficulty. Numerosities to be compared with the target one (16 dots) are 10 to 22 (16 ± 6, 11 and 21 numerosities excluded). The task includes 4 trials for each of the 10 possible target numerosities, giving 40 total number of trials, plus 2 practice trials. The maximum achieving score is 40 points, 1 point for each correct answer. *Approximate addition* differs from that used in the previous study in the number of proposed ratios. The test is composed by 2 trials for the 4:7/7:4 ratios, 10 trials for the 4:6/6:4 ratios, and 12 trials for the 4:7/7:4 ratios, thus having a smaller number of trials for the higher ratios. The total number of trials is 24, plus 2 examples, with a maximum achieving score of 24 points, that is 1 point for each correct answer. *Comparison-Intermixed* remained unchanged.

3.3.3.3 Math skills

3.3.3.3.1 Grade 1

Early Numeracy Test (ENT, Van Luit et al., 1994) was the test that had been administered when children attended Grade 1. This test is comprehensive of 40 exercises exploring basic, mainly informal quantitative-numerical capacities. The maximum achievable competence score is 100. A detailed description of the test can be found in the Method section of Study 1.

3.3.3.3.2 Grade 2

Written computation, hereinafter *Computation_2*. This task has been appropriately developed for the purposes of the research project and comprises 16 arithmetic expressions (8 additions and 8 subtractions) to be solved in 10 min. Computations involve 2-digit numbers with results ranging from 20 to 100. For each correct solution, 1 point is assigned, giving a maximum of 16 points.

Word problems, hereinafter *Problem solving_2* (from Giovanardi Rossi & Malaguti, 1994). The task requires children to solve word problems involving addition and subtraction operations. There are 5 problems to be completed in a limit of time of 15 min. The last problem presents a double question, giving a total of 6 expressions to be set out and performed. To each correct answer is assigned up to 1.5 points (1 point for the correct set out of the expression to be solved, plus 0.5 if also the computation procedure is correctly executed). The maximum score is 9 points.

MAT-2, hereinafter *MAT-2_2* (from Amoretti, Bazzini, Pesci, & Reggiani, 2007). Of the whole test, we selected to administrate the module *Number*, which explores a wide spectrum of math abilities, from ranking numbers from smallest to greatest or decomposing numbers, to word problems. Testing involves 11 tasks to be performed in 20 min. Each correctly-solved exercise is scored 1 point, with a maximum score of 11 points.

3.3.3.3.3 Grade 3

Written computation hereinafter *Computation_3*. The current task has been developed properly for this study. It consists of 16 arithmetic expressions (4 for each arithmetic operation) to be solved in 10 min and with results up to 1,000 for addition and subtraction, and 100 for multiplication and division. Each expression that is correctly solved is scored 1 point, giving a maximum score of 16 points.

Word problems, hereinafter *Problem solving_3* (adapted from Giovanardi Rossi & Malaguti, 1994). In the present task, 6 word problems involving all the four arithmetic operations have to be solved in limit of time of 15 min. 2 of them present a double question, thus determining 8 expressions to be set out and performed. Some problem statements have been simplified since pilot studies had proven their difficulty. For each correct answer, up to 1.5 points are assigned (1 point for the correct set out of the expression to be solved, plus 0.5 if also the computation procedure is correctly executed). The maximum score corresponds to 12 points.

MAT-2, hereinafter *MAT-2_3* (adapted from Amoretti, Bazzini, Pesci, & Reggiani, 2007). Of the whole test, we selected to administer the module *Number*, tapping various math abilities including writing down numbers in the rank of thousands or solving problems involving the concepts of expenses and profits. Also in this case, some questions had been re-formulated. There are 13 exercises to be executed in 20 min; those correct in each part receive 1 point, thus determining a maximum score of 13 points.

3.4 Results

3.4.1 Data analysis and preliminary results

Statistical analyses were conducted by means of the *PAW Statistics 21* statistical package. Descriptive and correlation analyses included the raw scores achieved in each test (with the exception of the intelligence indices for which age-referenced scales scores were reported). Regression analyses were instead conducted by using standardized z-scores.

In performing regression, it has been selected to recur to the same subdivision between precursor components adopted in Study 1. In fact, being the sample size not so numerous, we tried to limit the number of inserted variables. For the same reason, path models were not drawn. In this case, in fact, the primary aim wasn't the inspection of the relationships among all variables, but rather the effects of precursors on math performance.

Variables were therefore the following: Verbal and Performance intelligence (measured respectively by Vocabulary and Block design), Verbal STM (composite score between Forward word recall and Forward digit recall), Visuo-spatial STM (Path recall), Verbal WM (composite score between Backward word recall and Backward digit recall), Visuo-spatial WM (Path dual task), Comparison (composite score between Comparison-Intermixed and Comparison-Separate), and Approximate addition (measured by the equally-named task). Alternative analyses entailing the subdivision of memory according to the nature of the information to be processed (words, digits or visuo-spatial patterns) were not tested, being less informative for our purposes. Outcome variables were represented by Computation, Problem solving and Global math (composite score between Computation, Problem solving and MAT-2. The latter was not chosen because less informative than the global measure). In order to differentiate measures collected in different grades, variable names are followed by a number indicating the correspondent grade (for instance, Computation_2 indicates scores achieved in the computation test performed in Grade 2). It is possible to refer to Table 3.1 to take hold of the three assessment times with the correspondent collected measures.

Table 3.1
Précis of the Study Design

Assessment time	Assessed variables
Grade 1	Math precursors Mathematics (ENT)
Grade 2	Math precursors Mathematics (Computation, Problem solving, MAT-2)
Grade 3	Mathematics (Computation, Problem solving, MAT-2)

Descriptive results are reported in Table 3.2. Two different correlation matrices were instead defined for Grade 1 and Grade 2 precursors assessments (respectively, Table 3.3.1 and 3.3.2). The rapid inspection of correlation results suggests the significant association between precursors assessed in each of the two grades and math performance in subsequent grades.

Regression analyses were carried out by putting in relation precursor variables tested at both Grades 1 and 2 to math outcomes collected in Grades 2 and 3. In this way it was possible also for precursors assessed in Grade 2 to have an indication of concurrent prediction. Finally, regressions were carried out for both Grades 1 and 2 assessments by taking into account also global math performance achieved in each grade, therefore controlling for prior achievement.

Results are displayed by comparing the predictive power of Grade 1 and Grade 2 precursors in relation to each math outcome. Hierarchical regressions were computed to assess the amount of unique variance provided for each predictor variable. Hence, all memory and ANS variables were entered simultaneously in a first step. Intelligence scores, known from Study 1 to predict the other precursors, were inserted in a second step. In this way we wanted to explore if they held a significant role even beyond that of the other precursors. Anyway, since memory and ANS entered simultaneously didn't provide significant R^2 variation, in a subsequent attempt they were entered in different steps in the following order: WM, STM and ANS. Intelligence was always inserted last. No substantial differences could be observed by changing variables entrance order.

Table 3.2

Descriptive Statistics and Reliability Measures

	Task	Min	Max	Mean	SD	Skewness	Kurtosis	Reliability
Math measures	ENT_1	36.00	91.00	69.34	10.77	-.16	.04	.94 [a]
	Computation_2	.00	16.00	11.65	4.30	-1.04	.30	.84
	Problem solving_2	.00	9.00	6.61	2.25	-1.21	1.16	.69
	MAT-2_2	.00	11.00	7.47	2.25	-.96	1.54	.74 [a]
	Computation_3	.00	15.00	10.06	2.58	.17	1.08	.88
	Problem solving_3	.00	12.00	5.65	3.19	.34	-.85	.79
	MAT-2_3	1.00	11.00	.13	2.31	.20	-.38	.80
Grade 1 precursors	Vocabulary_1	3.00	19.00	11.29	2.94	-.42	.89	.74 [a]
	Block design_1	3.00	17.00	11.76	2.79	-.54	.32	.80 [a]
	Forward word recall_1	4.00	8.00	5.75	.91	.20	.28	.88
	Forward digit recall_1	3.00	11.00	6.18	1.41	.57	1.01	.87 [a]
	Path recall_1	4.00	8.00	6.22	1.00	-.30	-.31	.70
	Backward word recall_1	2.00	6.00	2.91	1.09	.96	.10	.86
	Backward digit recall_1	2.00	6.00	2.83	1.04	1.16	.78	.85 [a]
	Path dual task_1	1.00	8.00	5.43	1.81	-.32	-.58	.81
	Comparison-Intermix_1	8.00	34.00	24.45	4.99	-1.23	1.71	.70
	Comparison-Separate_1	21.00	47.00	37.41	4.69	-.89	1.64	.65
	Approx addition_1	13.00	21.00	17.29	2.18	-.15	-.57	.62
Grade 2 precursors	Vocabulary_2	3.00	19.00	10.44	3.47	.19	-.28	.87 [a]
	Block design_2	5.00	17.00	11.86	2.91	-.34	-.28	.86 [a]
	Forward word recall_2	2.00	6.00	3.54	.71	-.35	1.18	.87
	Forward digit recall_2	5.00	11.00	6.99	1.46	.77	.27	.78 [a]
	Path recall_2	2.00	8.00	6.31	1.25	.06	-.06	.78
	Backward word recall_2	.00	4.00	1.85	1.19	.21	-.72	.83
	Backward digit recall_2	1.00	6.00	3.76	1.26	.31	-.55	.79 [a]
	Path dual task_2	1.00	8.00	4.89	1.72	-1.45	1.66	.86
	Comparison-Intermix_2	21.00	37.00	28.02	5.62	.76	1.49	.84
	Comparison-Separate_2	21.00	33.00	27.51	3.25	-.20	-.70	.62
	Approx addition_2	12.00	23.00	17.99	2.51	-.50	-.14	.65

Note. Min= minimum; Max= maximum; SD= standard deviation.

[a] Reliability values taken from the manual (standardized tests). The remaining were computed on the sample.

Table 3.3.1

Correlation Matrix between Grade 1 Precursors and Math Outcomes

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	ENT	1.00																	
2	Computation_2	.24*	1.00																
3	Problem solving_2	.37***	.58***	1.00															
4	MAT-2_2	.32**	.71***	.66***	1.00														
5	Computation_3	.53***	.26*	.31**	.17	1.00													
6	Problem solving_3	.54***	.34**	.35**	.35**	.43**	1.00												
7	MAT-2_3	.51***	.35**	.31**	.36**	.38**	.67***	1.00											
8	Vocabulary_1	.36***	.20	.15	.08	.19	.34**	.31**	1.00										
9	Block design_1	.51**	.33**	.36***	.24*	.20	.32**	.35**	.26*	1.00									
10	Forward word recall_1	.24*	.06	.01	.04	.08	.09	.18	.35**	.32	1.00								
11	Forward digit recall_1	.15	.05	.05	.10	.12	.11	.10	.14	.30**	.41***	1.00							
12	Path recall_1	.21*	.05	.14	-.01	.29**	.32**	.05	-.13	.10	.02	-.04	1.00						
13	Backward word recall_1	.13	-.07	-.01	-.13	.38**	.12	.21	-.02	.25***	.18	.06	.06	1.00					
14	Backward digit recall_1	.29**	-.05*	<.01	-.02	.30**	.11	.22	.09	.12	.43**	.14	.14	.34**	1.00				
15	Path dual task_1	.35***	.09	.12	.06	.30**	.34**	.37	-.13	.29**	.02	.14	.08	.04	.12	1.00			
16	Comparison-Interm_1	.36***	.32**	.23*	.33**	.09	.41**	.39***	.03	.31**	.18	.25*	.05	.04	-.02	.12	1.00		
17	Comparison-Separate_1	.25*	.09	.22	.11	.11	.33**	.19	.30**	.12	.23*	.06	-.09	.05	-.01	-.03	.24*	1.00	
18	Approx addition_1	.09	.35**	.22	.32**	-.10	.30**	.33**	.13	.27*	.25*	.18	.02	.06	.09	.09	.35**	.27*	1.00

Note. *p≤.05; **p≤.01; ***p≤.001.

Table 3.3.2

Correlation Matrix between Grade 2 Precursors and Math Outcomes

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	Computation_2	1.00																
2	Problem solving_2	.58***	1.00															
3	MAT-2_2	.71***	.66***	1.00														
4	Computation_3	.26*	.31**	.17	1.00													
5	Problem solving_3	.34**	.35**	.35**	.43***	1.00												
6	MAT-2_3	.35**	.31**	.36**	.38***	.67***	1.00											
7	Vocabulary_2	.34**	.25*	.14	-.01	.18	.13	1.00										
8	Block design_2	<.01	.04	.15	-.18	-.01	-.14	.34**	1.00									
9	Forward word recall_2	.21	.10	.25*	-.02	.06	-.05	.19	.23*	1.00								
10	Forward digit recall_2	.15	.05	.04	-.11	-.11	-.07	.04	.10	.14	1.00							
11	Path recall_2	-.02	.03	.12	-.07	.03	-.03	.03	.44***	.23*	.16	1.00						
12	Backward word recall_2	.11	.19	.09	.06	-.03	.02	-.01	.16	.16	.25*	.06	1.00					
13	Backward digit recall_2	.11	.19	.22*	-.04	.18	.12	.26*	.35**	.46***	.34**	.27**	.40***	1.00				
14	Path dual task_2	.10	.16	.28*	.12	.16	.16	-.05	.13	.28*	.15	.42***	-.02	.14	1.00			
15	Comparison-Interm_2	.23*	.12	.06	.13	.11	.03	-.07	-.05	-.04	.04	.10	.11	.05	.15	1.00		
16	Comparison-Separate_2	.21	.21	-.02	.14	.14	.01	-03.	-.04	.19	.18	.14	.02	.04	.05	.54***	1.00	
17	Approx addition_2	.27*	.53***	.30**	.24*	.22*	.19	-.01	.11	-.04	.09	-.06	.13	-.08	-.14	.21	.37***	1.00

Note. *p≤.05; **p≤.01; ***p≤.001.

3.4.2 Grade 2 math achievement

Firstly, we explored the predictive power of the assessed precursors in relation to overall math skills represented by Global math. Then, Computation and Problem solving were separately taken into account.

Table 3.4 details parameters of models achieved by inserting Global math_2 as dependent variable. Grade 1 parameters accounted, on the whole, for the 15% of total variance. Significant predictors were ANS (with Approximate addition contribution approaching significance, *β*=.21, *p*=.08) and Intelligence (Performance intelligence, *β*=.28, *p*<.05). Grade 2 precursors explained, on the other hand, the 26% of Global math_2 overall variance with WM (Visuo-spatial WM, *β*=.31, *p*<.01), ANS (Approximate addition, *β*=.48, *p*<.001), and Intelligence (Verbal intelligence, *β*=.32, *p*<.01) acting as unique predictors.

Table 3.4
Hierarchical Regression Models for Global math_2

Variable	*β*	*t*	Adjusted R^2	R^2 change
Grade 1				
1. Verbal WM_1	-.10	-.79	-.01	-.01
Visuo-spatial WM_1	<.01	.02		
2. Verbal STM_1	-.12	-.98	-.03	-.02
Visuo-spatial STM_1	.07	.60		
3. Comparison_1	.18	1.49	.10	.13**
Approximate addition_1	.21	1.70 [a]		
4. Verbal intelligence_1	.05	.45	.15	.05*
Performance intelligence_1	.28	2.21*		
Grade 2				
1. Verbal WM_2	.08	.64	.04	.04
Visuo-spatial WM_2	.31	2.75**		
2. Verbal STM_2	.06	.46	.04	.00
Visuo-spatial STM_2	-.04	-.35		
3. Comparison_2	-.04	-.37	.18	.14***
Approximate addition_2	.48	4.35***		
4. Verbal intelligence_2	.32	2.97**	.26	.08**
Performance intelligence_2	-.01	-.12		

Note. *$p≤.05$; **$p≤.01$; ***$p≤.001$; [a] $p=.08$.

Table 3.5 reports statistical parameters of regression models in relation to Computation_2. As can be noticed, the model for Grade 1 precursors accounted for the 12% of the outcome measure variance. Significant contribution was given by ANS (Approximate addition, *β*=.25, *p*<.05), and Intelligence (Performance intelligence, *β*=.25, *p*<.05). Grade 2 precursors, on the other hand, explained the 17% of Computation_2 variance. Significant predictors were ANS (Approximate addition, *β*=.24, *p*<.05) and Intelligence (Verbal intelligence, *β*=.41, *p*<.01). Visuo-spatial WM effects only approached significance (*β*=.18, *p*=.07).

Table 3.5

Hierarchical Regression Models for Computation_2

Variable		β	*t*	Adjusted R^2	R^2 change
Grade 1					
1.	Verbal WM_1	-.01	-.85	-.02	-.02
	Visuo-spatial WM_1	-.01	-.06		
2.	Verbal STM_1	-.12	-.93	-.03	-.01
	Visuo-spatial STM_1	.05	.45		
3.	Comparison_1	.10	.83	.08	.11**
	Approximate addition_1	.25	1.99*		
4.	Verbal intelligence_1	.10	.83	.12	.04 [a]
	Performance intelligence_1	.25	1.99*		
Grade 2					
1.	Verbal WM_2	-.12	-.89	-.01	-.01
	Visuo-spatial WM_2	.18	1.5 [a]		
2.	Verbal STM_2	.09	.66	-.03	-.02
	Visuo-spatial STM_2	-.05	-.41		
3.	Comparison_2	.12	1.07	.04	.07*
	Approximate addition_2	.24	2.03*		
4.	Verbal intelligence_2	.41	3.60**	.17	.13**
	Performance intelligence_2	-.11	-.89		

Note. *$p \leq .05$; **$p \leq .01$; ***$p \leq .001$; [a] $p = .07$.

Parameters regarding Problem solving_2 are reported in Table 3.6. In relation to this skill, the model for Grade 1 precursors accounted for the 12% of the outcome variance. Significant predictor variables were represented by ANS (Comparison approaching significance, *β*=.21, *p*=.08) and Intelligence (Performance intelligence, *β*=.32, *p*<.05). However, when entering Computation_2, the new model accounted for 31% of Problem solving_2 variance, but no additional variables despite Computation_2 (*β*=.48, *p*<.001) added a significant additional unique contribution. Precursors variables assessed during Grade 2 accounted for 34% of concurrent Problem solving_2 variance, with significant predictors represented by WM (Visuo-spatial WM, *β*=.26, *p*<.05), ANS (Approximate addition, *β*=.56, *p*<.001) and Intelligence (Verbal intelligence, *β*=.28, *p*<.01). When entering Computation_2 in a first step, the amount of explained variance raised up to 48% and additional unique contribution, apart from Computation_2 (*β*=.43, *p*<.001), was given only by ANS (Approximate addition, *β*=.45, *p*<.001).

Taken on their whole, findings concerning Grade 2 math achievement put forward a moderate predictive power of precursors assessed in Grade 1, with the leading role of Approximate addition and Performance intelligence. Grade 2 precursors better predict concurrent math proficiency and the greater predictive power was held by Approximate addition, Verbal intelligence and in a lesser extent by visuo-spatial WM. Now, we will inspect the contribution of the same precursors in relation to Grade 3 math attainment.

Table 3.6

Hierarchical Regression Models for Problem solving_2

Variable	*β*	*t*	Adjusted R^2	R^2 change
Grade 1_Model 1				
1. Verbal WM_1	-.01	-.15	-.01	-.01
Visuo-spatial WM_1	.02	.11		
2. Verbal STM_1	-.17	-1.30	-.02	-.01
Visuo-spatial STM_1	.13	1.14		
3. Comparison_1	.21	1.70 [a]	.05	.07*
Approximate addition_1	.07	.52		
4. Verbal intelligence_1	.07	.60	.12	.07*
Performance intelligence_1	.32	2.48*		
Grade 1_Model 2				
1. Computation_2	.48	4.34***	.31	.31***
2. Verbal WM_1	.03	.29	.30	-.01
Visuo-spatial WM_1	.02	.16		
3. Verbal STM_1	-.11	-.96	.29	-.01
Visuo-spatial STM_1	.11	1.05		
4. Comparison_1	.16	1.47	.30	.01
Approximate addition_1	-.05	-.47		
5. Verbal intelligence_1	.02	.23	.31	.01
Performance intelligence_1	.20	1.67		
Grade 2_Model 1				
1. Verbal WM_2	.02	.19	.02	.02
Visuo-spatial WM_2	.26	2.49*		
2. Verbal STM_2	.04	.35	-.01	-.03
Visuo-spatial STM_2	-.03	-.28		
3. Comparison_2	-.01	-.09	.28	.29***
Approximate addition_2	.56	5.47***		
4. Verbal intelligence_2	.28	2.81**	.34	.06*
Performance intelligence_2	-.01	-.01		
Grade 2_Model 2				
1. Computation_2	.43	4.45***	.34	.34***
2. Verbal WM_2	.07	.68	.35	.01
Visuo-spatial WM_2	.16	1.64		
3. Verbal STM_2	.01	.94	.33	-.02
Visuo-spatial STM_2	-.01	-.07		
4. Comparison_2	-.05	-.51	.48	.15***
Approximate addition_2	.45	4.71***		
5. Verbal intelligence_2	.10	1.04	.48	.00
Performance intelligence_2	.10	.47		

Note. **p*≤.05; ***p*≤.01; ****p*≤.001; [a] *p* =.08.

3.4.3 Grade 3 math achievement

Also in relation to Grade 3 math performance, Global math_3 was firstly taken into account. By examining the related Table 3.7, it is possible to observe Grade 1 precursors to contribute to the 41% of its variance, with WM (Verbal WM, *β*=.26, *p*<.01; Visuo-spatial WM, *β*=.30, *p*<.01), ANS (Comparison, *β*=.32, *p*<.01) and Intelligence (Verbal intelligence, *β*=.27, *p*<.01) as significant predictor variables. Grade 2 precursors, on their whole, accounted for the 13% of Global math_3 variance, mainly throughout the significant involvement of WM (Visuo-spatial WM, *β*=.29, *p*<.01), and ANS (Approximate addition, *β*=.29, *p*<.001). Despite overall Intelligence didn't provide a significant additional quote of variance, Verbal intelligence was observed to be a significant predictor variable (*β*=.18, $p<.01$).

Table 3.7

Hierarchical Regression Models for Global math_3

Variable	*β*	*t*	Adjusted R^2	R^2 change
Grade 1				
1. Verbal WM_1	.26	2.73**	.22	.22***
Visuo-spatial WM_1	.30	3.18**		
2. Verbal STM_1	-.16	-1.47	.21	-.01
Visuo-spatial STM_1	.16	1.76		
3. Comparison_1	.32	3.09**	.34	.13***
Approximate addition_1	<.01	.09		
4. Verbal intelligence_1	.27	2.72**	.41	.07**
Performance intelligence_1	.12	1.24		
Grade 2				
1. Verbal WM_2	.20	1.48	.07	.07*
Visuo-spatial WM_2	.29	2.40**		
2. Verbal STM_2	-.02	-.14	.06	-.01
Visuo-spatial STM_2	-.03	-.19		
3. Comparison_2	-.02	-.15	.12	.06*
Approximate addition_2	.29	2.33***		
4. Verbal intelligence_2	.18	1.52**	.13	.01
Performance intelligence_2	-.21	-1.57		

Note. *$p\leq.05$; **$p\leq.01$; ***$p\leq.001$.

Table 3.8

Hierarchical Regression Models for Computation_3

Variable	*β*	*t*	Adjusted R^2	R^2 change
Grade 1				
1. Verbal WM_1	.33	3.08**	.18	.18***
Visuo-spatial WM_1	.23	2.14*		
2. Verbal STM_1	-.05	-.45	.21	.03
Visuo-spatial STM_1	.25	2.46*		
3. Comparison_1	.18	1.52	.26	.05
Approximate addition_1	-.15	-1.22		
4. Verbal intelligence_1	.17	1.58	.27	.01
Performance intelligence_1	.07	.57		
Grade 2				
1. Verbal WM_2	.34	2.55*	.11	.11**
Visuo-spatial WM_2	.22	1.79 [a]		
2. Verbal STM_2	-.03	-.21	.10	-.01
Visuo-spatial STM_2	-.06	-.44		
3. Comparison_2	.06	.47	.14	.04 [a]
Approximate addition_2	.21	1.66		
4. Verbal intelligence_2	.02	.13	.14	.00
Performance intelligence_2	-.18	-1.37		

Note. *$p\leq.05$; **$p\leq.01$; ***$p\leq.001$; [a] $p=.08$.

In relation to Computation_3, whose parameters can be inspected in Table 3.8, Grade 1 precursor variables accounted for the 27% of its variance. Unique significant predictor was WM (Verbal WM, *β*=.33, *p*<.01; Visuo-spatial WM, *β*=.23, *p*<.05). Whereas overall STM didn't provide significant additional contribution with respect to WM, Visuo-spatial STM alone was a significant predictor of Computation_3 (*β*=.25, *p*<.05). With regard to Grade 2 precursors, these accounted for the 14% of Computation_3 variance, with the only significant prediction represented by WM (Verbal WM, *β*=.34, *p*<.05; Visuo-spatial WM approaching significance, *β*=.22, *p*=.08).

Table 3.9

Hierarchical Regression Models for Problem solving_3

Variable	*β*	*t*	Adjusted R^2	R^2 change
Grade 1_Model 1				
1. Verbal WM_1	.09	.84	.08	.08*
Visuo-spatial WM_1	.23	2.34*		
2. Verbal STM_1	-.19	-1.65	.07	-.01
Visuo-spatial STM_1	.13	1.27		
3. Comparison_1	.37	3.38***	.28	.21***
Approximate addition_1	.12	1.09		
4. Verbal intelligence_1	.26	2.54**	.34	.06*
Performance intelligence_1	.10	.94		
Grade 1_Model 2				
1. Computation_3	.28	2.49*	.16	.16***
2. Verbal WM_1	-.01	-.08	.18	.02
Visuo-spatial WM_1	.17	1.71 [b]		
3. Verbal STM_1	-.17	-1.58	.16	-.02
Visuo-spatial STM_1	.05	.53		
4. Comparison_1	.32	2.99**	.36	.20***
Approximate addition_1	.19	1.76		
5. Verbal intelligence_1	.21	2.11*	.39	.03 [a]
Performance intelligence_1	.09	.79		
Grade 2_Model 1				
1. Verbal WM_2	.04	.26	.01	.01
Visuo-spatial WM_2	.24	1.83 [a]		
2. Verbal STM_2	.03	.20	-.01	-.02
Visuo-spatial STM_2	-.01	-.05		
3. Comparison_2	.02	.15	.02	.03
Approximate addition_2	.24	1.83 [a]		
4. Verbal intelligence_2	.22	1.74 [b]	.04	.02
Performance intelligence_2	-.10	-.73		
Grade 2_Model 2				
1. Computation_3	.39	3.17**	.17	.17***
2. Verbal WM_2	-.10	-.69	.16	-.01
Visuo-spatial WM_2	.15	1.22		
3. Verbal STM_2	.04	.29	.14	-.02
Visuo-spatial STM_2	.02	12		
4. Comparison_2	-.01	-.03	.14	.00
Approximate addition_2	.16	1.27		
5. Verbal intelligence_2	.21	1.80 [a]	.15	.01
Performance intelligence_2	-.03	-.25		

Note. *$p≤.05$; **$p≤.01$; ***$p≤.001$; [a] $p=.07$; [b] $p=.08$.

Shifting to models entailing Problem solving_3 as outcome variable, and whose data are reported in Table 3.9, Grade 1 precursors explained the 34% of its variance. Unique significant predictions were given by WM (Visuo-spatial WM, *β*=.23, *p*<.05), ANS (Comparison, *β*=.37, *p*<.001), and Intelligence (Verbal intelligence, *β*=.26, *p*<.01). When entering also Computation_3 in the first step, the final model accounted for the 39% of Problem solving_3 variance. Besides Computation_3 (*β*=.28, *p*<.05), other significant predictors were ANS (Comparison, *β*=.32, *p*<.01) and Intelligence (Verbal intelligence, *β*=.21, *p*<.05). Visuo-spatial WM effect approached significance (*β*=.17, *p*=.08). With regard to Grade 2 precursors, they explained the 4% of the outcome variable variance. Predictions only approaching significance could be highlighted (Visuo-spatial WM, *β*=.24, *p*=.07; Approximate addition, *β*=.24, *p*=.07; Verbal intelligence, *β*=.22, *p*=.08). When entering Computation_3 firstly, Problem solving_3 explained variance reached the 15%, but the only contribution approaching significance was that of Verbal intelligence (*β*=.21, *p*=.07), beside the significant involvement of Computation_3 (*β*=.39, *p*<.01).

By summing up the main findings emerging on Grade 3 math achievement, it is possible to point out the substantial predictive role of Grade 1 precursors, with main contributions given by Visuo-spatial WM, Comparison and Verbal intelligence. Predictability of Grade 2 precursors instead decreased in comparison to the previous grade and main precursors were represented by Visuo-spatial WM, Approximate addition and Verbal intelligence. Afterwards are represented the global hierarchical models achieved when controlling for overall math achievement of previous grades, thus inspecting the possible additional contribution of the inspected precursors.

3.4.4 Global longitudinal models

Table 3.10 reports regression parameters of longitudinal models achieved by taking into account also previous math achievement. Regarding Grade 1 precursors, the prediction on Global math_3 was computed by inserting in the first two steps ENT and Global math_2 scores. Results showed that, besides these two measures (respectively, *β*=.36, *p*<.01, *β*=.23, *p*<.05), unique additional effects were provided by WM (Verbal WM, *β*=.20, *p*<.05; Visuo-spatial WM, *β*=.24, *p*<.01) and Intelligence (Verbal intelligence, *β*=.23, *p*<.05). While the final model accounted for the 54% of Global math_3 variance, Grade 1 precursors alone were responsible of the 9%. With regard to Grade 2 precursors, the only additional unique prediction was given by WM (Verbal WM, *β*=.26, *p*<.05), apart from that of previous achievement in Global math_2 (*β*=.32, *p*<.05) entered in the first step. The final model accounted for the 24% of global variance, of which the 6% was explained by Grade 2 precursors alone.

Table 3.10

Hierarchical Regression Models for Global math_3 Inclusive of Previous Achievement

Variable	β	t	Adjusted R^2	R^2 change
Grade 1				
1. ENT_1	.36	3.17**	.41	.41***
2. Global math_2	.23	2.40*	.45	.04*
3. Verbal WM_1	.20	2.16*	.50	.05**
Visuo-spatial WM_1	.24	2.69**		
4. Verbal STM_1	-.12	-1.25	.50	.00
Visuo-spatial STM_1	.10	1.24		
5. Comparison_1	.13	1.27	.50	.00
Approximate addition_1	.03	.31		
6. Verbal intelligence_1	.23	2.58*	.54	.04*
Performance intelligence_1	-.07	-.65		
Grade 2				
1. Global math_2	.32	2.55*	.18	.18***
2. Verbal WM_2	.26	2.03*	.23	.05*
Visuo-spatial WM_2	.20	1.63		
3. Verbal STM_2	-.10	-.81	.22	-.01
Visuo-spatial STM_2	.03	.25		
4. Comparison_2	-.08	-.69	.22	.00
Approximate addition_2	.21	1.57		
5. Verbal intelligence_2	.12	.97	.24	.02
Performance intelligence_2	-.22	-1.76		

Note. **p*≤.05; ***p*≤.01; ****p*≤.001.

3.5 Discussion

The general aim of the current study was to investigate to which extent math cognitive precursors assessed at a specific time point are suitable in the prediction of future math performance. In particular we explored the predictive power of both general and specific precursors and, within the latter, attention was dedicated to the basic, approximate skills pertaining to ANS. In order to achieve this purpose, 78 of the first-grade children tested in Study 1 were followed longitudinally till third grade. Precursors were also tested at the beginning of Grade 2 (beside the previous testing in Grade 1), and their prediction power was explored in relation to Grades 2 and 3 math skills. These were considered on their whole, but separate analyses were also conducted in relation to the specific competences of computation and word problem solving. In fact, while the former is widely explored, less attention is commonly dedicated to problem solving, especially for what concerns specific precursors and ANS in particular.

The hypotheses were that the assessed precursors could be suitable in predicting math achievement also in subsequent grades. Working memory (WM) was expected to be the main precursor, hypothesized to retain a significant role even when controlling for prior achievement. Actually, the main findings put on evidence the strongest predictive power of overall WM and principally of the visuo-spatial component. Significant precursors across grades were also observed to be ANS and intelligence, despite the additional contribution (beyond that given by influencing the level of the other precursors) provided by the latter seemed to progressively decrease. In general, the role of short-term memory (STM) was subjected to that of WM, with the exception of Computation_3 that resulted to be predicted also by Visuo-spatial STM assessed in Grade 1. Moreover, the significant role of WM lasted even when controlling for achievement in previous grades. In this case, it was the only precursor to last as a unique additional predictor (except for Grade 1 intelligence), corroborating findings positing its leading role as a math learning precursor.

By discussing the overall outcomes in relation to literature findings, it can be highlighted that our results are globally in agreement with studies reporting the fundamental role of WM in math performance across grades (e.g., Fuchs et al., 2010; Passolunghi et al., 2008) and opposing those postulating WM recruitment to be no longer significant in following grades, for instance in Grade 3 (Meyer et al., 2010) or in Grade 2 when taking into account prior achievement (De Smedt et al., 2009). Regarding STM, its involvement seemed to be subjected to that of WM, even if some studies found significant contributions either in computation or problem solving, both beyond and instead of WM (e.g., Meyer et al., 2010).

The other intriguing finding concerned specific precursors and in particular the approximate component of number sense. ANS was in fact proven to be crucial in the prediction of math learning also in following grades, therefore opposing findings from studies which indicated its role to be non-significant (Iuculano et al., 2008; Rousselle & Noël, 2007) or to be relevant only in very early learning (Bonny & Lourenco, 2012; Desoete et al., 2012). Our outcomes are however in line, among all, with those of Inglis et al. (2011) who reported the ANS fundamental role also in older children.

Nevertheless, some notable differences could be observed in relation to specific math skills and in dependence to the different testing times. By comparing computation and problem solving abilities across grades, the first observation to be highlighted is that, in Grade 2, problem solving seemed to require no further abilities than computation, with the exception of Grade 2 Approximate addition, which gave an additional unique contribution. On the other hand, in relation Grade 3 performance, ANS (no relevant for computation), Verbal intelligence and marginally Visuo-spatial WM, all assessed in Grade 1, provided an additional independent quote of variance to problem solving with respect to computation. Contrary to Grade 2, in Grade 3 intelligence appeared to be fundamental only for skills requiring greater reasoning abilities, i.e., problem solving. Computation_3 resulted to be predicted only by memory measures. This can be explained by the fact that computation at this stage mainly entails application of procedures and storage and retrieval of facts in and from memory (see Delaney et al., 1998). These processes probably don't require particular intellectual capacities nor ANS skills, which are both more likely to be recruited when having to solve exercises less automatically and procedurally.

Another important outcome that clearly comes out is the decreased predictive power of Grade 1 precursors in Grade 2 (but not Grade 3) math performance, and that of Grade 2 precursors in relation to Grade 3 (but not Grade 2) math. In fact, beside the similarities partly illustrated, precursors assessed in the two different school years appeared to give a different contribution especially when analyzed in their components. In relation to math achievement in Grade 2, only WM assessed in Grade 2 appeared to contribute, whereas that measured the grade before resulted to be non-predictive. Regarding intelligence, main relevance had the performance component assessed in Grade 1, but the verbal one tested in Grade 2. With respect to Grade 3 math, it was Comparison of Grade 1 but Approximate addition of Grade 2 to have a major predictive role. Further, Visuo-spatial WM seemed to be more crucial in Grade 2 and the verbal component to become a significant predictor only in Grade 3.

On their whole, these findings suggest a consistent developmental evolution of cognitive skills and abilities to occur in first grades of primary school. Regarding memory, our results appeared to be in line with studies reporting the significant involvement of verbal WM to emerge starting from the age of 7, while before it was the visuo-spatial component to be mainly recruited (McKenzie et al., 2003; Meyer et al., 2010). This could depend both on the cognitive development of children (who begin to rely consistently on verbal resources), but also to the taught math skills which can require progressively increasing verbal manipulation and reasoning. This findings is supported also by the greater role across grades of the verbal component of intelligence at the expenses of the performance, fluid aspect, despite many studies reported the latter to be that more crucially involved in the learning processes, especially of mathematics (e.g., Passolunghi et al., 2008). In this perspective, performance, language-independent intelligence can be more crucial earlier, when children don’t master language perfectly but also when math competences don’t require a huge verbal manipulation.

Regarding ANS, Comparison assessed in Grade 1 was probably more predictive than concurrent Approximate addition being it more suitable in exploring ANS in so young children (as suggested in Study 1). In Grade 2, Approximate addition can be understood more easily than in the previous grade and be also more similar to math tasks children are used to solve during classroom activities.

3.5.1 Neurophysiological evidences

Results of this study can find substantiation from neuroimaging and electrophysiological studies which explored which are the brain regions to be recruited during the execution of math-related tasks. It is widely accepted that the parietal lobe is a cortical area highly specialized for mathematics. Within this area, bilateral intraparietal sulcus (IPS) has been shown to activate during the execution of number sense tasks tapping ANS (Dehaene, Piazza, Pinel, & Cohen, 2003). Importantly, the approximate system seems to activate even when adults perform exact symbolic tasks (Dehaene et al., 2003; Piazza, Pinel, Le Bihan, & Dehaene, 2007), putting forward its crucial role also in carrying out formal arithmetic. Furthermore, IPS activation during non-symbolic magnitude comparison tasks has been demonstrated to differentiate children with versus without MLD (Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007). However, some authors (Szűcs, Devine, Soltész, Nobes, & Gabriel, 2013) reported an IPS dysfunction in disabled children associated to the execution of working memory tasks, suggesting that IPS could be not necessarily specific to ANS performance. The debate concerning ANS recruitment remains therefore open.

Anyway, in line with our findings, imaging studies conducted on children performing math tasks reported a stronger activation in pre-frontal regions, associated to working memory and executive functions, than in the parietal ones (Kaufmann et al., 2006). This suggests the fundamental recruitment in children of cognitive resources necessary to supply for a non-complete specialization and development of brain areas specific to math. Progressively, as children grow up, a shift from frontal to parietal areas during the execution of math tasks is observed, thus suggesting the increasing specialization of this brain region. On the whole, these outcomes are likely to support the leading involvement of working memory in math learning at least until children reach cognitive maturation.

3.5.2 Concluding remarks

The current study has been developed with the chief purpose of checking the potential that cognitive abilities tested early in time have in predicting math performance in following grades of primary school. To the best of our knowledge, there were no studies investigating longitudinally and extensively both general and specific math precursors, further with particular interest to the approximate component of number sense. We were therefore aimed at proving if all these cognitive abilities could confirm their effective role as math precursors across grades. Hence, the fact of proving their significant involvement across time could support the precocious intervention on these precursors with proper training interventions in order to prevent future math difficulties.

The outcomes emerging from this study confirmed that math learning is a complex process that entails a broad spectrum of cognitive abilities. The greater role of one of these abilities over the others closely depends to the math skill that is considered but also to the stage of children's development. It is therefore suggested, beyond possible generalizations, to understand which cognitive precursors are more essential in a particular grade in order to know which of them can be monitored when encountering students with a suspected disorder in this field.

Nonetheless, the study is characterized by some limitations, first of all the number of participants. In fact, it hadn't been possible to follow longitudinally a larger sample of children, even if this was recommended. With a larger sample size it would be possible to carry out path analyses to better put on evidence the interrelation between math precursors and math skills tested at each grade. As a consequence, the achieved results should be corroborated by replicating the study on a larger sample.

The other critical point concerns tested abilities and related tasks. In this study the administered tasks were indeed equivalent but inevitably different between the two assessment times because they have been selected, among those available, in reference to children's age. Actually, batteries are developed either for preschoolers or for children by Grade 3, and therefore the first grades of primary school are normally not filled. The administration of the same tasks in all grades would however provide the possibility to make closer comparisons.

The following remark regards the reduced predictive power of overall precursors in particular assessment times, for instance of Grade 1 precursors in relation to Grade 2 math performance. This suggests the necessity to widen the spectrum of tested abilities in reference to particular stages of development. For instance, it would be useful to test also executive functions other than working memory, and inhibition first of all (see Blair & Razza, 2007; Passolunghi et al., 1999). Especially in relation to problem solving, also language capacities should be investigated. Future evaluations should include also symbolic, exact number sense skills, probably more relevant in upper grades than the approximate, non-symbolic ones. In this way, it could be also possible to check if ANS could maintain a significant role even when controlling for them.

CHAPTER 4

Study 3

Math performance in second graders: How are cognitive and non-cognitive factors linked up?

4.1 Theoretical background

With regard to learning and academic achievement, two main and almost independent research lines can be identified: one is focused on the evaluation of children's cognitive abilities and the other on behavioral and emotional-motivational aspects. Considering the latter, studies were normally conducted on youths starting from pre-adolescence, whereas less attention has been dedicated to younger children.

Among this kind of constructs investigated in relation to academic proficiency, and specifically to that of math, great attention has been devoted to self-efficacy, with quite robust results about its key involvement (e.g., Bandura, 1986; Pajares, 1996). More confused are findings concerning other, more general self-perceptions, for instance self-concept and self-esteem. Major concerns regard their possible involvement in the academic setting and the directionality of this involvement (are these factors influencing achievement or vice versa?), but also their role along development.

While these factors are supposed to be positively related to achievement, others have been proven to be negatively associated. In particular, many studies attempted at investigating the involvement of anxiety and depressive symptoms (e.g., Blechman et al., 1986; Fisher et al., 1996), despite also in this case the direction of the effects is not so clear. Major debate subsists about the role of a subtype of anxiety that is specific to mathematics and therefore defined as math anxiety. Actually, it is not known when, along with development, this can become (negatively) associated to math performance, since some studies failed in finding such association in young children (e.g., Krinzinger et al., 2009).

Finally, as hinted before, the relationship among these factors and cognitive abilities is less explored. An important exception is represented by anxiety (both general and specific to math), studied also in relation to its detrimental impact on working memory (Ashcraft & Kirk, 2001; Eysenck & Calvo, 1992) but not on other cognitive resources.

4.2 The current study: Aims and hypotheses

Starting from these considerations, the present study, that will be indicated as Study 3, was mainly aimed at investigating which constructs, non-cognitive in nature, can hold a significant involvement in math proficiency in very young students. In this case, the inspection of the cognitive abilities was faded in the background, since in the two previous studies it has been extensively carried out with results founding positive substantiation from literature. As a consequence, the main focus of the current study was the evaluation of self-perceptions and affective constructs. Secondarily, we wanted also to explore the relationship of the same constructs with cognitive math precursors in the determination of math proficiency.

More in detail, the main goal of the current study has been the early investigation of the mentioned constructs in order to explore when the same can become relevant in terms of math academic performance. With this purpose, the assessment has been conducted on a large sample of children attending second grade. Self-perceptions, i.e., self-efficacy, self-concept and self-esteem have been selected as factors supposed to be "protective" of a good academic attainment. On the other hand, general anxiety, math anxiety and depression were thought to be "vulnerability" factors, therefore likely to be related to low attainment.

Considering the constructs tightly related to math, i.e., self-efficacy and math anxiety, the purpose was to explore the possible direct relation between self-efficacy and math performance and also the direction of this link in young students. Another tricky point to be solved regarded the possible involvement of math anxiety, since some studies in young children reported its significant involvement (Young et al., 2012), whereas others failed in finding such association (Krinzinger et al., 2009; Thomas & Dowker, 2000). In relation to the other assessed variables, i.e., general anxiety, depression, self-concept and self-esteem, we wanted to explore if they could have either a direct or mediated role on math achievement and also if this one could in turn affect them.

A secondary aim was that of investigating the possible relation of all these variables with the cognitive precursors that have been proven, in the previous studies, to be crucial for math learning (intelligence, memory and ANS).

The hypotheses were that all these variables could significantly affect, even indirectly, math performance. Specifically, we attended self-efficacy and general anxiety to have a direct impact and to be also influenced in turn by math achievement. That is, we expected children with a lower performance to become also, as a consequence of repetitive failure, less self-efficacious and more anxious. Concerning math anxiety, the hypothesis was that its relationship with math achievement could begin to establish starting from second grade, at least for some children. All the other investigated aspects were supposed to be related significantly but in a lower extent to math performance. More in detail, on the basis of some literature findings (see Marsh, 1996), we expected self-concept, and specifically math self-concept, to have an important impact and to be closely associated to self-efficacy. The involvement of self-esteem and depression was supposed to be present but more likely to be indirect.

Regarding the relationship between cognitive and non-cognitive factors, the assumption was that a significant link could be found, at least between memory and both general and math anxiety.

Nonetheless, we expected anxiety to significantly impact also ANS scores, despite no one study, to our knowledge, inspected the effects of this construct on approximate skills pertaining to number sense. In fact, it was hypothesized that anxiety could act by impairing the recruitment of cognitive abilities in general, therefore also the ability of discriminating magnitudes.

Two important clarifications are finally needed. Firstly, it has to be specified that anxiety and depression aren't here treated as clinical conditions, but only the presence of some of the symptoms characterizing the two states has been evaluated. Secondly, in this investigation the outcome was represented by math *performance*, rather than math *learning*. In other words, it is difficult to assess if these factors act only at the moment of math testing or are so pervasive and detrimental that are able to impair also the acquisition and application of new notions during everyday classroom activities. Anyway, it can be supposed that the ongoing interfacing of these factors can have an impact also in the long term thus affecting ultimately, if strongly established, also learning processes.

4.3 Method

4.3.1 Participants

203 children attending Grade 2 were initially recruited for the study. Children attended ten different classrooms across six primary schools in Northeastern Italy. From the initial sample, 3 children were excluded because diagnosed for neurological diseases, and 2 because they were repeating the year. The resulting sample was represented by 198 children (105 males; mean age: 7 years, 7 months).

4.3.2 Procedure

The study took place in the second term of Grade 2. Formal consent was obtained from the school headmaster, class math teachers and parents before proceeding. The assessment of non-cognitive constructs and cognitive math precursors was conducted in two different but consecutive moments. For all children cognitive constructs were tested in a first session, then the non-cognitive measures were collected. Math skills were tested last.

Non-cognitive constructs were investigated by means of questionnaires administered to the whole class in three different sessions across two consecutive weeks. Each session lasted approximately 30 min. Testing began after having collectively given the instructions and made sure that children had understood them. It was also checked if they could appreciate the difference between the terminology used to classify responses (e.g., the difference between *Sometimes* and *Often*). Response modality included no more than four options in Likert-like scales.

Cognitive precursor abilities were instead collected individually and in two different sessions lasting approximately 30 min. Testing took place in a quiet room outside the classroom and in the range of two consecutive weeks for each child. All the administered tasks were preceded by related task instructions.

4.3.3 Measures

4.3.3.1 Non-cognitive measures

4.3.3.1.1 Self-perceptions

Self-efficacy. This scale has been developed following the indications of Bandura (2006) with the aim of specifically assessing the presence of self-efficacy in the mathematical domain. The questionnaire is made up by 12 statements, 8 of them tapping math self-efficacy (e.g., "I can perform even the most difficult additions.") and the others inserted as distracters ("I am good at writing.").

Children could give a dichotomous response: *Yes, true*, if sentences described them well, or *No, not true*, if not. Responses reflecting the presence of self-efficacy are scored 1 point, 0 points otherwise. The maximum score is 8 points for math self-efficacy (12 points for the overall scale).

Self-Description Questionnaire for Preschoolers (SDQP, Marsh, Ellis, & Craven, 2002; Italian ed. under adaptation by Corsano & Camodeca). This questionnaire has been developed to assess self-concept in preschool children, but is appropriate for children till second grade. Questions investigate aspects of the Self very relevant at this age and are structured in 6 subscales: Physical ability (e. g., "Can you run fast?"), Appearance (e. g., "Do you have a nice looking face?"), Peer relations (e. g., "Do you have a lot of friends?"), Parent relations (e. g., "Do your parents always listen to you?"), Verbal (e. g., "Are you good at reading?"), and Math (e. g., "Do you know lots of numbers?"). Each subscale is made up of 6 questions, with the exception of 8 questions for the Parent relations subscale, giving a total of 38 items.

The questionnaire is structured as a Likert-type scale, with 4 possible response choices: *Not true*, *Rarely true*, *Quite true*, and *True*. To each response can be assigned from 0 (*Not true*) to 3 (*True*) points, with a maximum score of 114 points.

Culture-Free Self-Esteem Inventory for Children (CFSEI, Battle, 1992; Italian edition revisited in Tressoldi & Vio, 1996). The scale assesses self-esteem in children aged 6 to 18. Sentences explore self-esteem in different domains and are organized in 4 subscales: General (e. g., "I am as clever as the majority of people."), Interpersonal relationships (e. g., "I have a lot of friends."), Emotional state (e. g., "I feel happy as the majority of people."), and Behavior (e. g., "I normally succeed in doing important things."). The total number of statements is 40, i.e., 10 for each subscale.

For each item, children have two possible choices: *Yes, true*, if they describe them well, or *No, not true*, if not. Responses reflecting the presence of good self-esteem are scored 1 point, 0 points otherwise. The maximum score is 40 points.

4.3.3.1.2 Negative affect

Depression and Anxiety in Youth Scale (DAYS, Newcomer, Barenbaum, & Bryant, 1994; Italian ed., 1995). This questionnaire has been developed in order to assess clinical symptoms of anxiety and depression in infancy and adolescence (6 to 19 years of age) basing on DSM-III-R. For the purposes of the current study, two scales were selected: the self-rating scale (to be completed by children) and the scale to be filled in by teachers (the scale for parents has not been administered).

The original self-rating scale (hereinafter Anxiety-Self and Depression-Self) includes 22 items, 11 exploring anxiety and 11 depression symptoms. The scale we used has been partly modified to be more suitable to younger children and entailed the removal or reformulation of some of the items. The final administered scale is made up by 16 items, 8 for anxiety (e. g., “I easily get scared.”) and 8 for depression (e.g., “I happen to cry.”).

Response modality includes a 4-points Likert-like scale indicating the frequency with which the anxiety or depression symptom is experienced: *Never*, *Sometimes*, *Often*, and *Always*. Scoring ranges from 0 points (corresponding to *Never*) to 3 (*Always*). The maximum score is 24 point for the anxiety subscale and 24 points for the depression subscale (48 points for the overall scale).

The scale for teachers (hereinafter Anxiety-Other and Depression-Other) has been maintained in the original version. The questionnaire includes 20 statements, 7 for anxiety (e. g., “He/she easily changes his/her mood.”), and 13 for depression (e. g., “He/she is often sad.”) with a dichotomous response modality: for each child, math teachers have to indicate if that sentence was true or false. 1 point is assigned to each response indicating the presence of an anxiety or depression symptom, 0 points otherwise. The maximum score is 7 points for the anxiety subscale and 13 points for the depression subscale points (20 points for the overall scale).

Scale for Early Math Anxiety (SEMA, translated and adapted from Wu & Menon, 2012). This scale has been developed to investigate early math anxiety in second and third grades, by reviewing the largely known MARS-E scale (Suinn, Taylor, & Edwards, 1988) suitable for children at upper grades of primary school. The questionnaire is structured in two parts with 10 sentences each. In the first part, children are asked to imagine to have to solve the illustrated math problems (e. g., “What time will it be in 20 minutes?”) and indicate which level of nervousness they would feel if they actually had to solve them. In the second part, 10 sentences describing common situations happening during math lessons are listed (e.g., “Your teacher gives you a bunch of subtraction problems to work on.”). Students have to imagine these situations and indicate the level of nervousness they would experience if these really took place.

A 4-point Likert-like response modality is provided (instead of the 5-point scale of the original version): *Not nervous at all*, *A little nervous*, *Somewhat nervous*, and *Very nervous*. Each response is scored 0 (*Not nervous at all*) to 3 (*Very nervous*), giving a maximum of 60 points.

4.3.3.2 Cognitive measures

In order to assess cognitive precursors, in the current study were used the same tasks of Grade 2 assessment in Study 2. In turn, some of these tasks were also administered in Study 1. More detailed information can be therefore achieved by referring to the pertinent Method sections. Tasks are listed below:

Intelligence: *Vocabulary* and *Block design* (from WISC-III, Wechsler, 1991; Italian edition, 2006), respectively for the evaluation of verbal and performance intelligence. Vocabulary entails children to give exhaustive definitions of selected words; Block design requires the reproduction of bi-dimensional configurations by means of solid cubes. Maximum age-reference score is 19 points for both tasks.

Short-term memory (STM): *Forward word recall* (from Passolunghi & De Beni, 2001) and *Forward digit recall* (from WISC-III, Wechsler, 1991; Italian edition, 2006) for the verbal component; *Path recall* (adapted from Lanfranchi et al., 2004) for the visuo-spatial component. These tasks require the reproduction of increasing number (respectively of words, digits and moves on a grid) in the same experimenter's presentation order. Maximum score is 10, 16 and 8 points respectively.

Working memory (WM): *Backward word recall* (from Passolunghi & De Beni, 2001) and *Backward digit recall* (from WISC-III, Wechsler, 1991; Italian edition, 2006) for the verbal component; *Path dual task* (adapted from Lanfranchi et al., 2004) for the visuo-spatial component. The first two tasks entail the repetition of increasing number of words and digits, respectively, in the inverted order to that used by the experimenter. The last task involves the selective recall of the first step of a path of increasing length traced on a grid and the simultaneous performance of a secondary task. Maximum score is 6, 14 and 8 points respectively.

Approximate Number System (ANS): *Magnitude comparison of intermixed quantities* or *Comparison-Intermixed* (adapted from Iuculano et al., 2008), *Magnitude comparison of separate quantities* or *Comparison-Separate* (adapted from Piazza et al., 2010), and *Approximate addition* (adapted from Iuculano, Tang, Hall, & Butterworth, 2008). The goal in the first two tasks is the identification, without counting, of the most numerous array of dots simultaneously presented. The goal is similar for the last task, but in this case dots are presented in succession in three different arrays, of which the first two are supposed to be summed and then compared to the third. Maximum score is 40 points for both comparison tasks and 24 points for Approximate addition.

4.3.3.3 Math skills

Also considering math skills, the administered tests were the same provided for Grade 2 students in Study 2.

Math tests were the same used in Study 2 (Grade 2 assessment). They consisted in:

Computation: 16 arithmetic operations to be performed in 10 min. Maximum score is 16 points (1 for each correct response).

Word problem solving (from Giovanardi Rossi & Malaguti, 1994): 5 word problems to be solved in 15 min. The overall number of expressions to be set out and solved is 6. Maximum score is 9 points (1.5 for each correct result).

MAT-2, *Number* module (from Amoretti et al., 2007): 11 exercises, exploring different aspects of numerical competences, to be solved in 20 min. Maximum score is 11 points (1 for each correct exercise).

4.4 Results

4.4.1 Data Analysis and preliminary results

Main statistical analyses were conducted by means of the *PAW Statistics 21* statistical package. AMOS 21 package was used to perform path analyses. Descriptive and correlation analyses included raw scores achieved in each test (with the exception of the intelligence indices for which the age-referenced scales scores were reported). Path analyses were instead conducted by using standardized z-scores. This technique is an extension of multiple regression since it allows the assessment of the relationship between source variables, called *exogenous variables*, and the outcomes, defined *endogenous variables*.

Descriptive statistics with reliability measures and correlation results are reported in Table 4.1 and Table 4.2, respectively. From the rapid inspection of the correlation matrix, some significant relationships can be reported. It can be noted that Self-efficacy is positively correlated with all math measures (r=.24, p<.01 with Computation; r=.20, p<.001 with Problem solving), with the only exception of MAT-2. Significant correlations with all math outcomes except Problem solving can be highlighted for Self-esteem (r=.15, p<.05 with Computation; r=.21, p<.01 with MAT-2). Significant negative relationships with all math measures can be also noticed for Anxiety-Other (from r=-.20, p=.01 to r=-.30, p<.001) and Depression-Other (from r=-.17, p<.05 to r=.27, p<.001). Other significant and interesting relationships can be evidenced between cognitive precursor tasks and Anxiety-Other and Depression-Other, but also with Self-efficacy and Self-esteem. For more detailed information it is possible to inspect Table 4.2.

Table 4.1

Descriptive Statistics and Reliability Measures

	Task	Min	Max	Mean	SD	Skewness	Kurtosis	Reliability
Math measures	Computation	.00	16.00	11.03	4.30	-.85	-.21	.84
	Problem solving	.00	9.00	5.59	2.40	-.27	-.83	.69
	MAT-2	.00	11.00	6.66	2.43	-.52	-.24	.74
Cognitive factors	Vocabulary	1.00	19.00	9.93	3.45	-.19	.10	.74 [a]
	Block design	2.00	18.00	11.06	3.04	-.17	-.29	.80 [a]
	Forward word recall	.00	8.00	3.75	1.02	.06	2.61	.88
	Forward digit recall	2.00	14.00	7.91	2.03	.20	.06	.87 [a]
	Path recall	2.00	8.00	6.43	1.30	-.03	-.45	.70
	Backward word recall	.00	5.00	1.86	1.32	.20	-.95	.86
	Backward digit recall	.00	10.00	3.86	1.37	.50	1.72	.85 [a]
	Path dual task	.00	8.00	5.04	1.62	-1.41	2.26	.81
	Comparison-Intermixed	16.00	37.00	24.42	5.90	-1.17	2.17	.72
	Comparison-Separate	14.00	36.00	26.39	3.85	-.32	.26	.62
	Approx addition	7.00	23.00	17.22	2.81	-.77	.55	.65
Non-cognitive factors	Self-efficacy	1.00	6.00	5.27	1.09	-1.62	2.14	.66
	Self-concept	32.00	114.00	94.61	14.00	-1.73	2.36	.89
	Self-esteem	9.00	40.00	27.27	5.73	-.50	1.23	.79
	Anxiety-Self	.00	20.00	6.89	3.85	.42	-.21	.65
	Anxiety-Other	.00	7.00	1.47	1.52	1.03	.61	.65
	Math anxiety	.00	57.00	14.82	10.57	.80	.74	.87
	Depress-Self	.00	20.00	6.85	3.88	1.06	.38	.65
	Depress-Other	1.00	9.00	2.30	2.27	1.03	.51	.71

Note. Min= minimum; Max= maximum; SD= standard deviation.

[a] Reliability values taken from the manual (standardized tests). The remaining were computed on the sample.

Table 4.2

Correlation Matrix between All Variables

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	Computation	1.00																					
2	Problem solving	.54***	1.00																				
3	MAT-2	.65***	.57***	1.00																			
4	Vocabulary	.26***	.23***	.26***	1.00																		
5	Block design	.16	.16*	.29***	.34***	1.00																	
6	Forward word	.20**	.22**	.23**	.20**	.23***	1.00																
7	Forward digit	.13	.07	.08	.05	.10	.40***	1.00															
8	Path recall	.18*	.17*	.21**	.05	.27***	.23***	.25***	1.00														
9	Backward word	.15*	.14*	.24***	.19**	.14*	.44***	.15*	.18**	1.00													
10	Backward digit	.13	.18*	.28***	.17*	.14*	.37***	.35***	.16*	.45***	1.00												
11	Path dual task	.13	.19**	.13	.06	.06	.17*	-.06	.12	.18**	.18*	1.00											
12	Comp-Intermixed	.27**	.18*	.22**	.18*	.18*	.09	.18*	.12	.12	.06	.06	1.00										
13	Comp-Separate	.05	.10	.11	.09	.16*	.04	.14*	.05	.08	.14*	.13	.23***	1.00									
14	Approx addition	.15*	.23**	.18**	.02	.12	-.01	.04	.07	.09	.04	-.02	.02	.25***	1.00								
15	Self-efficacy	.24**	.20**	.13	.23**	.05	.15*	-.01	.14*	.09	-.01	.03	.06	.08	.11	1.00							
16	Self-concept	-.01	<.01	.06	.06	-.06	.04	<.01	-.01	-.01	<.01	<.01	.14*	.11	-.03	.22**	1.00						
17	Self-esteem	.15*	.06	.21**	.24***	.14*	.17*	.14*	<.01	.02	.14*	.04	.18*	.16*	.06	.29***	.34***	1.00					
18	Anxiety-Self	.10	<.01	-.06	.03	.14*	-.01	.03	<.01	-.03	.03	.06	.11	.05	-.06	-.19**	-.30**	-.23***	1.00				
19	Anxiety-Other	-.20**	-.20**	-.30***	-..05	-.04	-.27***	-.26***	-.19*	-.17*	-.26***	-.01	-.11	-.10	-.23**	-.31***	-.15	-.28***	.63***	1.00			
20	Math anxiety	-.04	-.09	-.12	-.22**	.03	-.01	.02	.04	.03	.02	.01	-.03	-.10	-.10	-.38***	-.26**	-.25***	.14	.24**	1.00		
21	Depress-Self	<.01	.02	-.11	.04	.05	-.11	-.04	-.05	-.04	-.04	-.03	-.04	-.03	-.04	-.19**	-.32***	-.31***	.17*	.54***	.24***	1.00	
22	Depress-Other	-.17*	-.17*	-.27***	-.15	-.17*	-.15	-.17*	-.18*	-.15	-.18*	<.01	-.08	-.10	-.18*	-.10	-.18*	-.13	-.23**	.08	.08	.26***	1.00

Note. *$p≤.05$; **$p≤.01$; ***$p≤.001$.

Shifting to path analyses, different path models have been drawn on the bases of theoretical assumptions. For models estimation, the Maximum Likelihood approach (Jöreskog & Sörbom, 1996) was selected, while performing an additional bootstrapping (Bollen-Stine) procedure.

Scores achieved in the administered questionnaires were treated as individual variables. In fact, particularly strong correlations were observed neither between close measures (e.g., Self-concept and Self-esteem). Regarding the DAYS questionnaire, however, scores corresponding to each subscales (anxiety and depression) were separately computed and converted in the correspondent z-scores. The same procedure was carried out for both forms, the one filled in by children (indicated as Self scales) and that by teachers (Other scales). Teachers' judgments were initially collected in order to compare their responses with those provided by children. Anyway, since the two profiles emerged to be not strongly correlated, both scores were inserted into path analyses.

Finally, in relation to the cognitive measures, unique composite scores were computed for each of the investigated constructs (intelligence, STM, WW and ANS) by calculating the mean value from the z-scores achieved in each task assessing them. Therefore, in this case there wasn't a distinction among the different components of the assessed cognitive precursors, since the purpose wasn't their detailed evaluation but rather their possible general relationships with the other assessed constructs. Considering the math variable, analyses were conducted by using z-scores achieved in both Computation and Problem solving, but also the composite score obtained from the mean of the three administered tests (Computation, Problem solving and MAT-2) and indicated as Global math. This measure has been used instead of MAT-2 since it was more complete and informative.

Alternative solutions were tested. Some relationships were very likely to be bidirectional, in particular considering the relation between anxiety and depression. Anyway, since literature results converge towards anxiety as preceding and predicting depression (e.g., Chorpita & Daleiden, 2002), this directionality was respected also in the present analyses. Other bidirectional links were those between Self-concept and Self-esteem and between these two constructs and depression. Considering the former relation, on the basis of theoretical assumptions we selected to define Self-concept as preceding Self-esteem (thus self-esteem is expected to be built on the concept that individuals have about themselves). In relation to the remaining relationships, a strongest model from the statistical viewpoint was achieved when considering depression to influence self-perceptions. For this reason, we maintained this directionality in all the subsequently-defined models.

4.4.2 Path model results

Path models were firstly defined by inserting Global math as outcome variable. In a second step, the specific abilities of Computation and Problem solving were taken into account. Models 1.1 and 1.2 (reported respectively in Figures 4.1.1 and 4.1.2 and related parameters in Tables 4.3.1 and 4.3.2) were the final selected models for Global math. They respectively accounted for the 22% and the 19% of the explained variance of this outcome variable. Both models were characterized by good fit indices (CMIN=56.55, df=55, p=.42, CFI=.996, NFI=.890, TLI=.994, RMSEA=.012, for Model 1.1; CMIN=58.42, df=55, p=.35, CFI=.992, NFI=.887, TLI=.987, RMSEA=.018, for Model 1.2). Anyway, GoF indices slightly favored Model 1.1 (AIC=154.55, BCC=162.05 vs. AIC= 156.42, BCC=163.92 of Model 1.2). We however chose to report both models since also Model 1.2 was highly acceptable from both the theoretical and statistical viewpoint.

The two models essentially differed in the crucial relationship standing between Global math, Self-efficacy and Anxiety-Other. Actually, Model 1.1 posited Self-efficacy to negatively impact on Anxiety-Other (β=-.23, p<.01), which directly influenced Global math (β=-.21, p<.01). In Model 1.2, relationships were in the reverse direction: Global math was observed to negatively affect Anxiety-Other (β=-.19, p=.01), which directly and negatively influenced Self-efficacy (β=-.23, p<.01).

Figure 4.1.1 Path Model 1.1

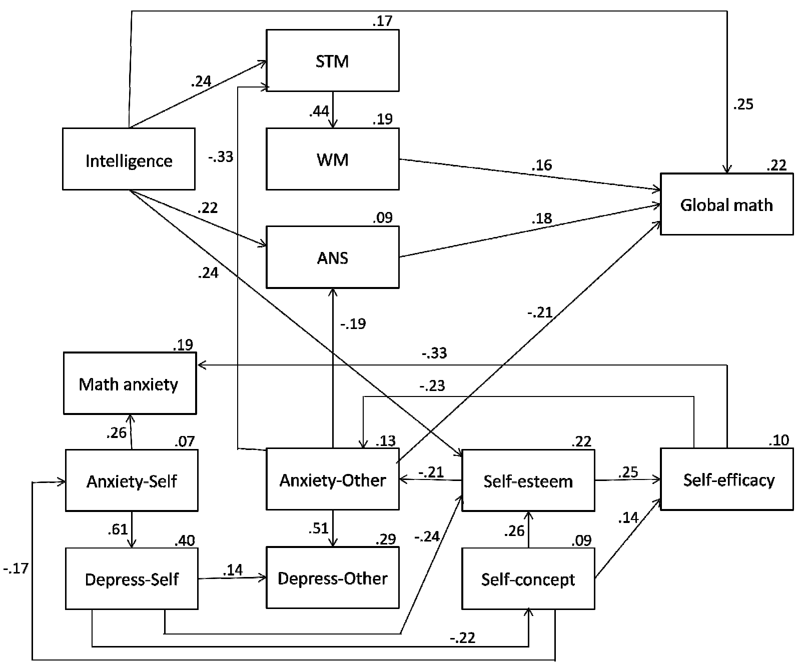

Figure 4.1.1. Standardized solution for Model 1.1 including Global math. Model fit indices: CMIN=56.55, df=55, p=.42, CFI=.996, NFI=.890, TLI=.994, RMSEA=.012.

Table 4.3.1

Standardized Parameters of Path Model 1.1

Endogenous variable	Exogenous variable	Direct effect	Indirect effect	R^2
STM	Intelligence	.24***		.17
	Anxiety-other	-.33***		
WM	STM	.44***		.19
ANS	Intelligence	.22***		.09
	Anxiety-Other	-.19**		
Self-efficacy	Self-concept	.14*		.10
	Self-esteem	.25***		
Self-concept	Depress-Self	-.22**		.09
Self-esteem	Intelligence	.24***		.22
	Self-concept	.26***		
	Depress-Self	-.24***		
Anxiety-Self	Self-concept	-.17*		.07
Anxiety-Other	Self-efficacy	-.23**		.13
	Self-esteem	-.21**		
Math anxiety	Self-efficacy	-.33***		.19
	Anxiety-Self	.26***		
Depress-Self	Anxiety-Self	.61***		.40
Depress-Other	Anxiety-Other	.51***		.29
	Depress-Self	.14*		
Global math	Intelligence	.25***	.07	.22
	STM		.07	
	WM	.16**		
	ANS	.18**		
	Self-efficacy		.06	
	Self-concept		.03	
	Self-esteem		.07	
	Anxiety-Self		-.01	
	Anxiety-Other	-.21**	-.06	
	Math anxiety			
	Depress-Self		-.02	
	Depress-Other			

Note. Direct and indirect effects are expressed as *β* values. Indirect effects are reported only for Global math.
**p*≤.05; ***p*≤.01; ****p*≤.001.

By taking as reference Model 1.1, it is possible to highlight some other constructs to influence Global math, apart from the cognitive variables acting directly (Intelligence: *β*=.25, *p*<.001; WM: *β*=.16, *p*=.01; ANS: *β*=.18, *p*<.01). An indirect and notable influence was that of Self-efficacy (*β*=.06) and of Self-esteem (*β*=.07) directly projecting towards it. Other indirect effects were those of Self-concept (*β*=.03), Anxiety-Self (*β*=-.01), and Depression-Self (*β*=-.02), in addition to that of Anxiety-Other (*β*=-.06). Interestingly, Math anxiety did correlate neither directly nor indirectly with

Global math and was only directly predicted by Anxiety-Self (*β*=.26, *p*<.001) and Self-efficacy (*β*=-.33, *p*<.001).

Significant and direct associations can be observed also between cognitive and non-cognitive variables. Firstly, Anxiety-Other negatively predicted STM (*β*=-.33, *p*<.001, and thus WM indirectly, *β*=-.14), but also ANS (*β*=-.19, *p*<.01). Another significant direct impact was that of Intelligence on Self-esteem (*β*=.24, *p*<.001).

The inspection of the contribution of the non-cognitive variables alone was possible only for Model 1.1 (in Model 1.2, only cognitive variables provided direct effects). By removing the non-cognitive precursors, the remaining constructs were suitable in explaining the 9% of overall global math variability.

Figure 4.1.2 Path Model 1.2

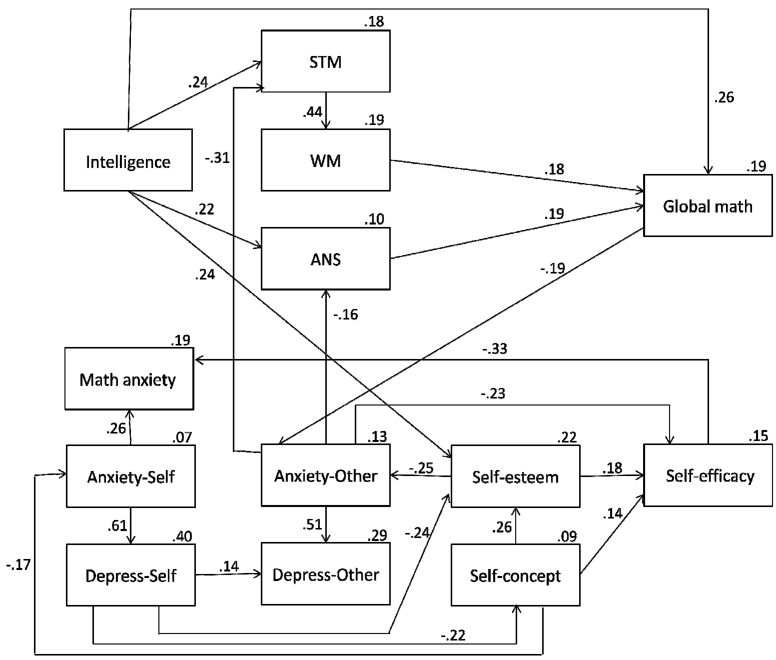

Figure 4.1.2. Standardized solution for Model 1.2 including Global math. Model fit indices: CMIN=58.42, df=55, *p*=.35, CFI=.992, NFI=.887, TLI=.987, RMSEA=.018.

Table 4.3.2

Standardized Parameters of Path Model 1.2

Endogenous variable	Exogenous variable	Direct effect	Indirect effect	R^2
STM	Intelligence	.24***		.18
	Anxiety-other	-.31***		
WM	STM	.44***		.19
ANS	Intelligence	.22***		.10
	Anxiety-other	-.16*		
Self-efficacy	Self-concept	.14*		.15
	Self-esteem	.18**		
	Anxiety-Other	-.23**		
Self-concept	Depress-Self	-.22**		.09
Self-esteem	Intelligence	.24***		.22
	Self-concept	.26***		
	Depress-Self	-.24***		
Anxiety-Self	Self-concept	-.17*		.07
Anxiety-Other	Self-esteem	-.26***		.13
	Global math	-.19**		
Math anxiety	Self-efficacy	-.33***		.19
	Anxiety-Self	.25***		
Depress-Self	Anxiety-Self	.61***		.40
Depress-Other	Anxiety-Other	.51***		.29
	Depress-Self	.14*		
Global math	Intelligence	.26***	.07	.19
	STM		.08	
	WM	.18**		
	ANS	.19**	<.01	
	Self-efficacy		<.01	
	Self-concept		<.01	
	Self-esteem		<.01	
	Anxiety-Self		-.01	
	Anxiety-Other		-.06	
	Math anxiety			
	Depress-Self		-.01	
	Depress-Other			

Note. Direct and indirect effects are expressed as *β* values. Indirect effects are reported only for Global math.
$^*p \leq .05$; $^{**}p \leq .01$; $^{***}p \leq .001$.

A second aim was to explore if the prediction of the investigated constructs could differ in relation to specific math abilities, i.e., Computation and Problem solving. For this reason, other new models were tested. As proven also in Study 2, problem solving abilities are strongly predicted by computation, therefore this link has been held also in the present investigation. The final path model, termed Model 2, was shown to explain the 14% of Computation and the 32% of Problems solving skills variance. Detailed relationships and path coefficients can be observed in Figure 4.2 and Table

4.4. Also for this model, fit indices were particularly good: CMIN=65.08, df=65, *p*=.47, CFI=1.000, NFI=.887, TLI=1.000, RMSEA=.002.

The important difference to be highlighted with respect to the previous models stands in the relationship between Self-efficacy, Anxiety-Other and math outcomes. In this case, Self-efficacy was shown to negatively affect Anxiety-Other (*β*=-.23, *p*=.01); this one negatively predicted Computation (*β*=-.19, *p*=.01), projecting towards Problem solving (*β*=.49, *p*<.001), and this last variable, in turn, was observed to impact Self-efficacy (*β*=.15, *p*<.05). Relationships between the other variables remained unchanged, with adjustments only in strength (see Table 4.4).

The removal of cognitive variables in order to prove the effect of the non-cognitive constructs alone proved that these were suitable in explaining the 6% of Computation and the 29% of Problems solving skills variability. Also this new solution was good in terms of statistical fit (CMIN=65.08, df=65, *p*=.47, CFI=1.000, NFI=.887, TLI=1.000, RMSEA=.002). Relationships among variables were the same of the previous complete model, with slight changes only in strength, especially between Self-efficacy and Anxiety-Other (*β*=-.22, *p*=.01), Anxiety-Other and Computation (*β*=-.22, *p*=.01), and Computation and Problem solving (*β*=.54, *p*<.001). By removing also Computation in the prediction of Problem solving, the remaining variables explained the 4% of its variance (fit indices: CMIN=21.02, df=21, *p*=.46, CFI=1.000, NFI=.937, TLI=1.000, RMSEA=.002). In this case, the influence of Anxiety-Other resulted to be direct (*β*=-.16 , *p*<.05).

Figure 4.2 Path Model 2

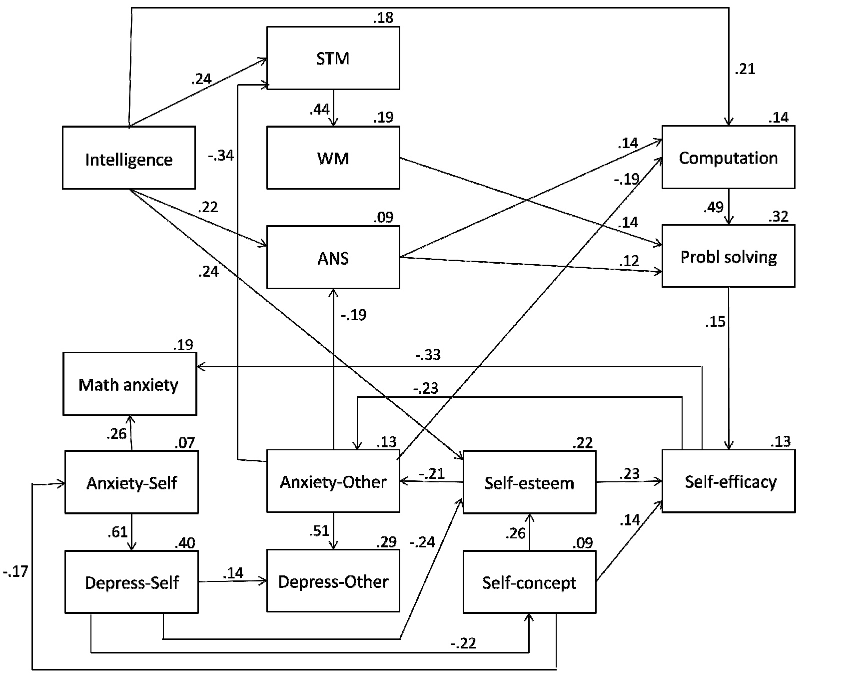

Figure 4.2. Standardized solution for Model 2 including Computation and Problem solving. Model fit indices: CMIN=65.08, df=65, *p*=.47, CFI=1.000, NFI=.887, TLI=1.000, RMSEA=.002.

Table 4.4

Standardized Parameters of Path Model 2

Endogenous variable	Exogenous variable	Direct effect	Indirect effect	R^2
STM	Intelligence	.24***		.18
	Anxiety-Other	-.34***		
WM	STM	.44***		.19
ANS	Intelligence	.22***		.09
	Anxiety-Other	-.19*		
Self-efficacy	Self-concept	.14*		.13
	Self-esteem	.23***		
	Problem solving	.15		
Self-concept	Depress-Self	-.22**		.09
Self-esteem	Intelligence	.24***		.22
	Self-concept	.26***		
	Depress-Self	-.24***		
Anxiety-Self	Self-concept	-.17*		.07
Anxiety-Other	Self-efficacy	-.23**		.13
	Self-esteem	-.21**		
Math anxiety	Self-efficacy	-.33***		.19
	Anxiety-Self	.26***		
Depress-Self	Anxiety-Self	.61***		.40
Depress-Other	Anxiety-Other	.51***		.29
	Depress-Self	.14*		
Computation	Intelligence	.21**	.05	.14
	STM		<.01	
	WM			
	ANS	.14*	<.01	
	Self-efficacy		.05	
	Self-concept		.02	
	Self-esteem		.06	
	Anxiety-Self		-.01	
	Anxiety-Other	-.19**	-.03	
	Math anxiety			
	Depress-Self		-.02	
	Depress-Other			
Problem solving	Intelligence		.12	.32
	STM		.06	
	WM	.14*	<.01	
	ANS	.12*	.07	
	Self-efficacy		.02	
	Self-concept		.03	
	Self-esteem		.04	
	Anxiety-Self		-.01	
	Anxiety-Other		.15	
	Math anxiety			
	Depress-Self		-.01	
	Depress-Other			
	Computation	.49***	<.01	

Note. Direct and indirect effects are expressed as *β* values. Indirect effects are reported only for Computation and Problem Solving.
*$p\leq.05$; **$p\leq.01$; ***$p\leq.001$.

4.4.4 More detailed analyses about self-perceptions

For the reason that self-concept and self-esteem are defined as multidimensional constructs not properly definable by a unique global factor, further analyses were conducted. In this way, we wanted to explore the possibly different contributions in math achievement of their different subcomponents.

Preliminary considerations can be drawn by inspecting the correlation matrix in Table 4.5 which reports the correlation between math tasks, self-efficacy and different sub-components of both self-concept and self-esteem. It can be noticed that no one self-concept subcomponent was significantly related to any math measure, with the exception of Self-concept-Math which was mildly associated to MAT-2 (r=.14, p<.05). Anyway, some subcomponents were more tightly related than others to Self-efficacy (r=.21, p<.01 for Self-concept-Appearance, r=.19, p<.01 for Self-concept-Parents, and r=.21, p<.01 for Self-concept-Math).

Considering self-esteem, it can be noticed that two subcomponents were more closely related to math performance, especially with MAT-2 (r=.20 , p<. 01 for Self-esteem-Relations; r=.24, p<.001 for Self-esteem-Behavior). These two subcomponents were also those mainly correlated with Self-efficacy (r=.29 , p<.001 for Self-esteem-Relations; r=.36 , p<.001 for Self-esteem-Behavior).

Starting from these preliminary findings, alternative path models were defined by substituting global measures of Self-concept and Self-esteem with those including the more salient subcomponents scores. These further models didn't provide improvements in overall model fit or outcome measure explained variance, rather determine a loss of significant relationships with scores associated to anxiety and depression.

Table 4.5

Correlation Matrix of Self-perceptions Subscales and Math Measures

		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Computation	1.00													
2	Problem solving	.54***	1.00												
3	MAT-2	.65***	.60***	1.00											
4	Self-efficacy	.24***	.20*	.13	1.00										
5	Self-concept-Activity	.04	<.01	.05	.14*	1.00									
6	Self-concept-Appearance	-.02	-.04	.13	.21**	.40***	1.00								
7	Self-concept-Peer	-.05	-.02	<.01	.16*	.40***	.47***	1.00							
8	Self-concept-Parents	-.04	-.02	-.04	.19**	.35***	.46***	.41***	1.00						
9	Self-concept-Verbal	.02	.05	.03	.03	.35***	.32***	.25***	.54***	1.00					
10	Self-concept-Math	.04	.09	.14*	.21**	.45***	.44***	.36***	.52***	.50***	1.00				
11	Self-esteem-General	.03	.05	.09	.04	.16*	.25***	.15*	.21**	.17*	.17*	1.00			
12	Self-esteem-Relations	.15*	.08	.24***	.29***	.18*	.35***	.39***	.13	.14*	.19**	.40***	1.00		
13	Self-esteem-Emotion	.02	-.10	.06	.16*	.08	.13	.16*	.15*	.13	.16*	.25***	.37***	1.00	
14	Self-esteem-Behavior	.25***	13	.21**	.36***	.10	.18**	.02	-.04	.02	.17*	.36***	.39***	.32***	1.00

Note. *p≤.05; **p≤.01; ***p≤.001.

4.5 Discussion

The investigation of which factors other than cognitive precursors contribute to the determination of math proficiency represented the principal aim of the current study. In particular the attention was focused on young children, i.e., in second graders. Children of this age, in fact, possess sufficient reading and comprehension capacities but also begin to form reliable conceptions about themselves and their abilities, both general and specific to the academic framework. Previous research has indeed been typically focused on older students, whereas findings on young children are sparse and more confused. The goal was, therefore, to assess as soon as possible along the school career the aspects that interact with math achievement.

For this reason, it has been selected to evaluate some of the most relevant constructs. On one hand, self-perceptions in the form of self-efficacy, self-concept and self-esteem have been investigated as *protective* factors. On the other hand, *vulnerability* factors represented by anxiety (both general and specific for math) and depression have been inspected. General hypotheses were that all these constructs could be somehow significantly related to math performance even beyond cognitive precursors, with which they were however expected to be linked, especially for what concerned anxiety.

The expectations have been generally confirmed even if with some important exceptions. Firstly, it is important to point out that, in line with our hypothesis, the two factors directly and strongly associated to math performance were represented by general anxiety and self-efficacy. General anxiety was also observed to negatively influence the recruitment of cognitive precursors. Depression and self-esteem were shown to be related but in an indirect way. Notably, contrary to the expectations, math anxiety was not related to math performance and the contribution of self concept was not only indirect but also negligible.

Before discussing more in detail these findings, it is important to highlight one interesting and unexpected outcome. The instrument selected for the investigation of anxiety and depression provided also a rating scale to be filled in by teachers (who had to evaluate the presence or absence of anxiety or depressive symptoms in their students). It is interesting to note, however, that the correlations between self- and teacher-rating scales were quite low and that they presented different patterns of connections with the other assessed constructs. This was especially true for anxiety, where judgments given by teachers seemed to be more informative than those expressed by children.

Other studies using also different scales proved that the correlations between self- and observer-ratings are often only moderate (e.g., Albus, Maier, Shera, & Bech, 1990). The explanation in our study can be twofold: firstly, self-ratings can be more biased in so young children, who can be subjected to the particular mood of that day rather than consider their general feelings; secondly, responses given by teachers concerned a manifest behavior typically recordable in classroom settings (e.g., “He/she can’t sit for a long time on the chair”) and therefore possibly more strongly connected to academic outcomes.

In fact, it could be noted that anxiety as expressed by teachers was the only non-cognitive variable to directly predict math performance. Two almost equivalent models were possible and indicated anxiety to negatively influence global math proficiency but also to be influenced by the same. This entails a higher anxiety level to undermine math performance, but also a low math performance to determine a subsequent increase in anxiety.

Anxiety acted also indirectly by influencing, always negatively, memory performance. This finding corroborates literature results in accordance with the various models proposed about the negative influence of anxiety on the recruitment of cognitive resources (see Eysenck et al., 2007). Interestingly, anxiety was shown to negatively affect also the approximate number sense precursors represented by ANS. To the best of our knowledge, no one study explored before the influence of anxiety on approximate, non-symbolic number sense skills. Anyway, it could be supposed that also processes requiring discrimination abilities, even if immediate and spontaneous, can be impaired by an excessive arousal triggered by high anxiety.

Nevertheless, one important and crucial point regarded the involvement of math anxiety. Notably, this construct resulted to influence neither directly nor indirectly math proficiency. Furthermore, it was observed to be predicted by general anxiety (self-rating scale) despite not in a great extent. These findings suggest that, in younger students, math anxiety is well distinguishable from general anxiety (see Ashcraft & Krause, 2007), but has an impact neither to math performance nor to the related cognitive precursors. Hence, these results seemed to confirm those by Krinzinger et al. (2009) which proposed math anxiety at that age to be more related to personality aspects and general anxiety or be mediated by teachers or parents’ attitudes (Stevenson, Hofen, & Randel, 2000) and for this reason to be not significantly linked to math proficiency. Origins of math anxiety are very likely to further lie in the interaction with teachers, who thereby represent an additional source of variability (Ashcraft, 2002).

Anyway, math anxiety was shown to be negatively predicted by self-efficacy also directly and not only indirectly via general anxiety (for the association between self-efficacy and anxiety see Bandura, 1986). This suggests that a link with math performance, even if indirect, is going to establish. In fact, when inspecting the two key aspects of computation and problem solving separately, it could be shown that problem solving capacities, and also computation indirectly, positively affected self-efficacy beliefs. In this sense, it could be suggested that the achievement in specific math competences begins to be determinant for the shaping of self-efficacy and also of math anxiety indirectly. It is therefore likely for math anxiety to become in short time a significant negative predictor of math performance and possibly also of the related cognitive precursors.

4.5.1 Indirect influences

Apart from self-efficacy and general anxiety, all the other explored constructs (with the exception of math anxiety) were observed to be related to math performance only indirectly. Self-esteem, in particular, acted mainly throughout Self-efficacy and Anxiety-Other and was in turn negatively affected by both Depression-Self (see also below). Most of these relationships were observed to be bidirectional, even if statistical fit favored one solution over the others. For this reason it is quite difficult to determine if a variable mainly influences or it is influenced by another. Probably both possibilities are likely to occur.

More specifically, regarding self-esteem, many studies have reported its significant involvement in academic contexts (e.g., Hattie & Marsh, 1996). Actually, low self-esteem was observed to be linked to maladaptive learning strategies (Lloyd & Sullivan, 2003). In our study this construct seemed to be a background factor contributing to the determination of aspects more directly linked to math proficiency. Moreover, we have observed that, within self-esteem, the two subcomponents represented by relationships with others and by behavioral disposition seemed to be the most crucial. This can suggest the great impact that these two aspects have in the academic setting in young children. Finally, self-esteem has been also noticed to be predicted by high intelligence, thereby hinting how it can be boosted by success achieved in different contexts, success that is more easily attainable by children that are endowed with high intellectual capacities.

Closely related to self-esteem is self-concept, which anyway seemed to contribute to math performance only in the extent it determined self-esteem and self-efficacy levels. Contrary to the hypotheses and to some literature findings (e.g., Hattie & Marsh, 1996; Marsh, 1994), it didn’t hold a significant relation with math performance, either considering it as a whole or exploring more specific components such as math self-concept. As postulated by Marsh (2003), it could happen that only when children grow up their self-concepts become more strongly correlated with academic achievement. In fact, academic self-concept has been observed to exhibit high cross-situational variability in early grades, since young students have not yet accumulated sufficient experience on their scholastic capacities (Bong & Shaalvik, 2003). At this age, it could therefore happen that rather the appraisal about themselves represented by self-esteem could have a more influential impact.

Finally, with respect to depression, it appeared to hold a background role, mild but however present. It is also well known that depression is strongly related to learned helplessness (see Peterson, Maier, & Seligman, 1993), a negative disposition that arises as the consequence of repetitive failure and that can also undermine self-esteem, as highlighted before. In this sense, the contribution of depression is likely to become more salient in the context of a closer examination of children with learning difficulties than of the whole sample (e.g., Fristad et al., 1992; Wright-Strawderman & Watson, 1992).

4.5.2 Concluding remarks

This study was attempted at providing a comprehensive view of the factors that can contribute to the determination of math performance in very young students. The processes affecting academic performance were expected to be various and multifaceted, with a complex network of relationships between even different factors. Hence, a range of constructs has been assessed, by trying to select those expected to be the most relevant.

The investigated constructs were actually expected to act on math performance in a multiform way, for instance determining an avoidance behavior, i.e., escape from the evaluation (e.g., Ashcraft & Ridley, 2005; Bandura & Schunk, 1981), or an excessive mental arousal that impedes the recruitment of the necessary cognitive resources (e.g., Hopko et al., 1998). Despite the huge number of inspected variables, these couldn't anyway explain a very consistent quote of math performance variance. However, when removing the effects determined by the cognitive precursors, the remaining variables gave a still significant contribution.

To support the achieved findings, however, a greater sample size would be probably required, being consistent the number of variables that have been assessed. In fact, one issue has been represented by the lack of a strong correspondence between ratings given by children and those provided by teachers, factor that complicated the overall pattern of relationships and required the introduction of both scales in the model. Future investigation is therefore needed to delineate the extent of adequacy of these kinds of measurements and shed light on the link between self- and observer-ratings in young children.

Another close point to be discussed concerned that fact that cognitive precursors were assessed by means of an objective evaluation, while non-cognitive aspects by rating scales, whose responses by participants are less controllable. In this perspective, a more reliable index of these non-cognitive precursors could be achieved by evaluating manifest behavior. Regarding anxiety, physiological responses such as sweating, increased heat rate or pressure can be a more reliable index. In the same way, neuroimaging and neurophysiological investigations could inspect brain activations elicited by a certain emotional state.

To further complicate the picture, it has to be specified that the examined constructs are likely to act not necessarily in a linear way at each level of math performance. For instance, it has been proven that optimum levels of anxiety can determine a proper arousal state that enhances rather than depressing performance (e.g., O'Hanlon & Beatty, 1996). In the same way, the fact of possessing a proper amount of self-esteem can be more fruitful than an excessively high image about the Self (e.g., Dweck, 1999). Moreover, self-worth is expected to be related to a given domain in dependence of the emphasis and importance that the individuals attribute to that domain (e.g., Harter, 1983). In this perspective, if students are not interested in their math performance, the fact of achieving poorly can be not detrimental for their global self-image.

Finally, children can adopt coping strategies to successfully face with difficulties (e.g., Eisenberg, Fabes, & Guthrie, 1997) and this resource can alter the easily expected predictions. Children can for instance develop a sort of *mathematical resilience* (Johnston-Wilder & Lee, 2008), or generally an *academic resilience* (Martin, 2003), in front of failure and therefore have the helping-hand to overcome their school-related obstacles.

As a consequence, a more exhaustive model was likely to be achieved by inserting variables that are known to be affected by the constructs we measured and that probably act as mediators between the same and math performance. That is, variables such as motivation or persistence (e.g., Marsh et al., 1999) could probably more strongly predict math outcomes, because being more tightly and directly associated to them. Anyway, the purpose of this study was to explore constructs expressing how the child feels and behaves and results demonstrated that also in very young students variables apart from cognitive abilities have a role that cannot be neglected. Despite the picture is very complex and fragmented, these outcomes suggest how self-perceptions and affective aspects are linked with academic, math performance even if they act on the background. Acting on these aspects could provide important advantages from the academic viewpoint, but also the act of improving the latter can have beneficial effects on children's well-being.

CHAPTER 5

Study 4

Math anxiety and its effect on math performance in first grades of primary school.

5.1 Theoretical background

Math anxiety has been defined as a condition of tension that interferes with the execution of tasks involving numbers. This condition has been detected even in absence of general anxiety traits and has also been distinguished from more common test anxiety (see Hembree, 1990). Nevertheless, studies about math anxiety report findings sometimes contradictory, since some of them demonstrated this condition to precede a negative math performance while others interpreted it as a consequence of poor attainment (Ma & Xu, 2004). Anyway, math anxiety is more likely to be both the cause and the consequence of failure in math (e.g., Ashcraft, 2002), but its origins are still almost unknown.

In fact, research has been focused on prevalence on adults or young adults, for whom it has been demonstrated that this condition impairs the execution of complex but not easier math tasks (e.g., Ashcraft & Faust, 1994. Math anxiety is therefore hypothesized to interfere with the recruitment of the cognitive resources necessary to perform a math task requiring a certain cognitive load. Typically, it has been proven to have detrimental effects on computation skills involving carrying procedures (e.g., Ashcraft & Faust, 1994; Faust et al., 1996). Very few are the studies that, on the contrary, investigated math anxiety very early, i.e. in first grades of primary school. Some authors proved this condition to be experienced also by so young children (e.g., Young et al., 2012), but many of them lacked in finding an already-established association between it and math performance (e.g., Krinzinger et al., 2009; Thomas & Dowker, 2000). In Study 3, illustrated in the previous chapter, we have investigated in second graders math anxiety together with other constructs. Also in this case, a significant involvement of this condition in the determination of math proficiency was not detected. Math anxiety has been in fact observed to be present and already distinguishable from general anxiety, but only a trend about its negative association with math skills has been identified.

Moreover, only a few works have investigated math anxiety specifically in children affected by difficulties in math. Related findings generally indicate this subtype of anxiety to be present in these youths (e.g., Ma & Xu, 2004; Rubinsten & Tannock, 2010) and treatments aimed at reducing it were also proven to improve math performance. Nevertheless, also in this case, the target was represented by older students, whereas less research interest has been devoted to young children.

5.2 The current study: Aims and hypotheses

Since in Study 3 an association in second graders between math anxiety and math proficiency has not been found out, the first purpose of the current study, indicated as Study 4, has been the investigation of the same construct the following grade, i.e., in children attending third grade. The purpose was that of identifying, as soon as possible, when this condition can become relevant for math performance.

For this reason, math anxiety has been explored by means of an additional scale that allowed to control for general academic anxiety and also to distinguish anxiety related to the condition of learning mathematics from that arousing when being tested in it. Also general anxiety has been assessed in order to verify if potential associations have to be attributed to the presence of general anxiety traits or are specific to math. Even if the association was expected to be utmost unidirectional, i.e., from math anxiety to math performance, we wanted to explore also the reverse possibility. A related goal was to investigate if (and in case how) math anxiety can be differently experienced far from math testing or just before it. Hence, we administered the math anxiety questionnaire twice, in order to explore if this condition affects children in a wide extent or if it is limited to the examination context.

For the reason that the effects of math anxiety on math performance could not emerge by considering the whole sample, a further aim was to inspect its effects in relation to the level of students' math performance, thus by comparing groups characterized by low, average or high proficiency in the administered math tests.

Global hypotheses were that math anxiety could be present in third graders in a way that can interfere with the actual execution of math tasks. Moreover, it was hypothesized to act independently of general and academic anxiety. The interference between math anxiety and math performance was supposed to last also far from the examination, but the aspect of math anxiety related to math testing was expected to increase at the moment of math evaluation. The association between anxiety and performance in math, if not on the whole sample, was however hypothesized to emerge when comparing children characterized by a different level of math skills. Significantly higher levels of math anxiety were supposed to emerge in the group of low performing children, whereas a moderate anxiety degree was expected for the higher-performance groups.

5.3 Method

5.3.1 Participants

A sample of 82 third-graders was recruited for the study. Children attended four different classes across two different schools of Northeastern Italy. From the initial sample, 3 students had to be excluded because they were repeating the year. The final sample was therefore represented by 79 children (48 males, mean age: 8 years, 5 months).

5.3.2 Procedure

Data were collected before the end of first term of Grade 3. Before proceeding, formal consent was obtained from the school headmaster, math teachers and parents.

Questionnaires assessing anxiety and math tests were administered collectively to the whole class, after having provided related instruction and checked that children had understood them. General anxiety and math anxiety were tested in different sessions. The following week, also math skills were assessed by means of timed tests. Children were informed that they would have to complete math tests and before initiating they filled in one of the math anxiety questionnaires (MeMa) administered in one of the previous sessions. Only this questionnaire was administered twice, since it was more complete and informative than the other one (SEMA). In relation to math anxiety, the two assessment times will be indicated as Time 1 and Time 2.

5.3.3 Measures

Some questionnaires and tests were the same selected for Study 2 (Grade 3 assessment) for what concerned math tasks, and Study 3 for anxiety scales. More detailed description of the same can be therefore find out in the correspondent Method sections.

5.3.3.1 Anxiety

General anxiety has been investigated by the same questionnaire used in Study 3, i.e., *Depression and Anxiety in Youth Scale* (DAYS, Newcomer et al., 1994; Italian ed., 1995), by taking only the anxiety subscale. Also in this case, the self-rating scale (partly modified from the original version) and that to be filled in by teachers were both administered. Response modality includes a 4-point Likert-like scale indicating the frequency with which the anxiety symptom is experienced: *Never*, *Sometimes*, *Often*, and *Always*. Scoring ranges from 0 points (corresponding to *Never*) to 3 (*Always*). The maximum score is 24 point for the self-rating subscale. 7 points are the maximum score for the teacher-rating subscale (where dichotomous responses are provided).

Scale for Early Math Anxiety (SEMA, translated and adapted from Wu & Menon, 2012) is the same used in Study 3 and includes 20 sentences describing situations involving math. Students had to identify themselves with these situations and express the level of nervousness they would feel if they effectively experienced them. A 4-point Likert-like response modality is provided (instead of a 5-point scale of the original version): *Not nervous at all*, *A little nervous*, *Somewhat nervous*, and *Very nervous*. Each response is scored 0 (*Not nervous at all*) to 3 (*Very nervous*), giving a maximum of 60 points.

Math Anxiety MeMa Scale (adapted from Caponi, Cornoldi, Falco, Focchiatti, & Lucangeli, 2012) has been developed from the revision of the Italian version of MARS-E (Saccani & Cornoldi, 2005). This scale distinguishes from anxiety of learning mathematics, 16 items (e.g., "Look at the teacher explaining a complex arithmetic operation at the blackboard") and of being tested in the same subject, 8 items (e.g., "Perform a written math test"). Further, 6 items explore general academic anxiety, i.e., anxiety in other disciplines (e.g., "Have an oral test in geography"). Children had to express the degree of fear they would feel if the described situations were really happening. Since this questionnaire was developed for students from Grade 4, some of the items have been readapted.

The questionnaire is structured as a Likert-type scale, with 4 possible response choices indicating increasing fear level. To each response can be assigned from 0 (lowest fear level) to 3 (higher fear level) points, with a maximum score of 48 points for the learning subscale (MeMa-Learning), 24 points for the testing subscale (MeMa-Testing) and 18 for the academic subscale (MeMa-School).

5.3.3.2 Math skills

Math tests were the same used in Study 2 (Grade 3 assessment). They consisted in:

Computation: 16 arithmetic operations to be performed in 10 min. Maximum score is 16 points (1 for each correct response).

Word problem solving (adapted from Giovanardi Rossi & Malaguti, 1994): 6 word problems to be solved in 15 min. The overall number of expressions to be set out and solved is 8. Maximum score is 12 points (1.5 for each correct result).

MAT-2, *Number* module (adapted from Amoretti et al., 2007): 13 exercises, exploring different aspects of numerical competences, to be solved in 20 min. Maximum score is 13 points (1 for each correct exercise).

5.4 Results

5.4.1 Data analysis and preliminary results

Statistical analyses were conducted by means of the *PAW Statistics 21* statistical package. Preliminary statistical analyses included the raw scores achieved in each test (with the exception of the intelligence indices for which age-referenced scales scores were reported) and the following by using standardized z-scores. Considering math, a composite score was computed by averaging the z-scores achieved in the three tests (Computation, Problem solving and MAT-2) and it was termed Global math. As for the previous studies, analyses were performed by taking into account Global math (chosen instead of MAT-2 because considered more informative), but also Computation and Problem solving separately.

Descriptive and correlation results are reported in Tables 5.1 and 5.2, respectively. From the inspection of the correlation matrix, it is possible to notice the high correlation between MeMa scores assessed in Time 1 (MeMa_1) and scores achieved in SEMA (*r*=.59, *p*<.001 with MeMa-Learning_1; *r*=.60, *p*<.001 with MeMa-Testing_1). SEMA scores appeared to be correlated with math measures in a similar extent than MeMa_1 (significant association for both in relation to MAT-2 only: *r*=-.26, *p*<.05 with SEMA; *r*=-.21, *p*=05 with MeMa-Learning_1). Therefore, since MeMa_1 provided more information, it was taken as a unique math anxiety measure for subsequent analyses. Moreover, for the reason that the two MeMa subscales for learning and testing showed different correlation patters, they were treated separately instead of computing a composite score or summing them.

Another notable remark regarded the significant correlation between math measures and anxiety measured by teachers, defined Anxiety-Other (*r*=-.24, *p*=.05, to *r*=-.35, *p*<.01), but not with that expressed by children, or Anxiety-Self. As a consequence, despite math anxiety instead correlated with Anxiety-Self (*r*=.34, *p*<.01 with both MeMa-Learning_1 and MeMa-Testing_1), only Anxiety-Other has been taken into account in subsequent analyses.

Table 5.1
Descriptive Statistics and Reliability Measures

	Task	Min	Max	Mean	SD	Skewness	Kurtosis	Reliability
Math measures	Computation	.00	15.00	10.04	4.55	.18	1.12	.88
	Problem solving	.00	12.00	5.65	3.27	.30	-.89	.79
	MAT-2	1.00	11.00	6.14	2.29	.19	-.37	.80
Anxiety measures	Anxiety-Self	.00	20.00	7.79	4.69	.43	-.58	.84
	Anxiety-Other	.00	7.00	1.67	1.82	.94	.04	.72
	SEMA	.00	39.00	12.44	8.83	.95	.44	.87
	MeMa-Learning_1	.00	31.00	7.53	7.60	1.29	.97	.89
	MeMa-Testing_1	.00	23.00	8.30	6.28	.52	-.72	.89
	MeMa-School_1	.00	15.00	4.91	3.97	.78	-.16	.76
	MeMa-Learning_2	.00	34.00	6.23	7.74	1.82	3.01	.91
	MeMa-Testing_2	.00	24.00	6.30	5.49	1.30	1.42	.84
	MeMa-School_2	.00	17.00	4.23	3.96	1.17	1.08	.78

Note. Min= minimum; Max= maximum; SD= standard deviation.

5.4.2 Math anxiety at Time 1 vs. Time 2

The comparison between MeMa_1 and MeMa_2 has been carried out by considering the two subscales, i.e., Learning and Testing. Results showed that significant differences can be noticed neither for MeMa-Learning (*t*(78)=1.75, *p*=.08; MeMa-Learning_1, *M*=7.52, *SD*=7.56; MeMa-Learning_2, *M*=6.21, *SD*=7.81) nor for MeMa-School (*t*(78)=1.16, *p*=.25; MeMa-School_1: *M*=4.75, *SD*=4.09; MeMa-School_2: *M*=4.21, *SD*=3.40). Differences were instead significant for MeMa-Testing

($t(78)=3.15$, $p<.01$; MeMa-Testing_1: $M=8.43$, $SD=6.25$; MeMa-Testing_2: $M=6.28$, $SD=5.53$), but with lower values for the former than the latter.

Despite no significant differences could be observed for some of the subscales across Time 1 and Time 2 assessments, subsequent analyses have been carried out by considering all of them separately (also because characterized by different correlation patterns).

Table 5.2
Correlation Matrix between All Variables

		1	2	3	4	5	6	7	8	9	10	11	12
1	Computation	1.00											
2	Problem solving	.43***	1.00										
3	MAT-2	.38***	.67***	1.00									
4	Anxiety-Self	.06	-.01	-.10	1.00								
5	Anxiety-Other	-.35**	-.30**	-.24*	.18	1.00							
6	SEMA	-.16	-.15	-.26*	.29*	.01	1.00						
7	MeMa-Learning_1	-.14	-.16	-.21	.34**	-.01	.59***	1.00					
8	MeMa-Testing_1	-.08	-.02	-.08	.34**	-.08	.60***	.77***	1.00				
9	MeMa-School_1	-.06	-.16	-.17	.12	-.18	.51***	.70***	.67***	1.00			
10	MeMa-Learning_2	-.36**	-.25	-.28*	.23	.13	.36**	.74***	.47***	.41**	1.00		
11	MeMa-Testing_2	-.14	-.14	-.15	.19	-.24	.50***	.73***	.63***	.62**	.77**	1.00	
12	MeMa-School_2	-.23	-.18	-.14	.26*	-.15	.46***	.76***	.59***	.64***	.77**	.86***	1.00

Note. *$p≤.05$; **$p≤.01$; ***$p≤.001$.

5.4.3 Regression models

Regression models were conducted separately for MeMa_1 and MeMa_2. Models were firstly drawn by inserting in a single block the math anxiety measures collected at either Time 1 or Time 2 (i.e., MeMa-Learning_1, MeMa-Testing_1 and MeMa-Learning_2, MeMa-Testing_2) in order to inspect unique contributions. Subsequent analyses were carried out again by inserting in the first two steps Anxiety-Other (known also from Study 3 to directly and negatively predict math performance) and MeMa-School, in order to control for their effects and inspect the extent of their involvement.

Parameters for regression models defined with Global math as dependent variable are reported in Table 5.3. With regard to Time 1, the two MeMa math subscales accounted for the 4% of Global math variance, with the only significant prediction of MeMa-Learning_1 ($β=-.38$, $p<.05$). The insertion of Anxiety-Other and MeMa-School_1 lifted explained variance to 22%. Significant predictors were Anxiety-Other ($β=-.39$, $p<.01$), MeMa-School_1 ($β=-.42$, $p<.05$), and MeMa-Testing_1 ($β=.52$, $p<.05$). In relation to Time 2, the two MeMa math subscales could account for the 13% of Global math variance, but only MeMa-Learning_2 could give a significant prediction ($β=-.56$, $p<.01$). When adding to the model Anxiety-Other and MeMa-School_2, the explained variance raised up to

15%. Despite both Anxiety-Other and MeMa-School_2 added a significant quote of variance, in the final model only Anxiety-Other nearly gave a significant contribution (*β*=-.33, *p*=.07).

Table 5.3

Hierarchical Regression Models for Global math

Variable	*β*	*t*	Adjusted R^2	R^2 change
Time 1_Model 1				
1. MeMa-Learning_1	-.38	-2.12*	.04	.04
MeMa-Testing_1	.22	1.22		
Time 1_Model 2				
1. Anxiety-Other	-.39	-3.31**	.11	.11**
2. MeMa-School_1	-.42	-2.21*	.16	.05*
3. MeMa-Learning_1	-.31	-1.57	.22	.06*
MeMa-Testing_1	.52	2.50*		
Time 2_Model 1				
1. MeMa-Learning_2	-.56	-2.82**	.13	.13**
MeMa-Testing_2	.25	1.27		
Time 2_Model 2				
1. Anxiety-Other	-.33	-1.87 [a]	.09	.09*
2. MeMa-School_2	-.11	-.32	.17	.08*
3. MeMa-Learning_2	-.26	-.95	.15	-.02
MeMa-Testing_2	-.01	-.03		

Note. **p*≤.05; ***p*≤.01; ****p*≤.001; [a] *p* =.07.

Table 5.4 reports regression models achieved in relation to Computation. In the case of Time 1, variables negatively explained the 1% of Computation variance and no one of them was a significant predictor. When adding Anxiety-Other and MeMa-School_1, the model accounted for 9% of Computation variance, but the only significant contribution was that of Anxiety-Other (*β*=-.36, *p*<.01). In relation to Time 2, the two MeMa math subscales could explain the 13% of Computation variance, but only MeMa-Learning_2 gave a significant contribution (*β*=-.61, *p*<.01). Anyway, when inserting Anxiety-Other and MeMa-School_2 in the previous step, explained variance reduced to 11% and no one variable provided significant unique effect.

Parameters of models including Problem solving as dependent variable are listed in Table 5.5. Regarding Time 1, the two MeMa math subscales accounted for 3% of Problem solving variance. Only MeMa-Learning_1 effect was significant (*β*=-.36, *p*<.05). When adding also Anxiety-Other and MeMa-School_1, variables contributed to the 18% of Problem solving variance. Significant predictors were Anxiety-Other (*β*=-.34, *p*<.01), MeMa-School_1 (*β*=-.46, *p*<.05), and Mema-Testing_1 (*β*=.56, *p*<.01). In relation to Time 2, the two MeMa math subscales explained the 3% of Problem solving variance but no one variable had a significant predictive power. The addition of Anxiety-Other and MeMa-School_2 variables raised explained variance up to 6%. The only predictor to give a contribution approaching significance was Anxiety-Other (*β*=-.33, *p*=.08).

Table 5.4

Hierarchical Regression Models for Computation

Variable	β	t	Adjusted R^2	R^2 change
Time 1_Model 1				
5. MeMa-Learning_1	-.19	-1.06	-.01	-.01
MeMa-Testing_1	.07	.37		
Time 1_Model 2				
6. Anxiety-Other	-.36	-2.85**	.11	.11**
7. MeMa-School_1	-.18	-.88	.11	.00
8. MeMa-Learning_1	-.16	-.74	.09	-.02
MeMa-Testing_1	.22	.97		
Time 2_Model 1				
1. MeMa-Learning_2	-.61	-3.06**	.13	.13**
MeMa-Testing_2	.33	1.66		
Time 2_Model 2				
1. Anxiety-Other	-.20	-1.11	.06	.06
2. MeMa-School_2	-.22	-.64	.10	.04
3. MeMa-Learning_2	-.39	-1.44	.11	.01
MeMa-Testing_2	.34	.98		

Note. *$p \leq .05$; **$p \leq .01$; ***$p \leq .001$.

Table 5.5

Hierarchical Regression Models for Problem solving

Variable	β	t	Adjusted R^2	R^2 change
Time 1_Model 1				
1. MeMa-Learning_1	-.36	-1.97*	.03	.03
MeMa-Testing_1	.25	1.41		
Time 1_Model 2				
1. Anxiety-Other	-.34	-2.79**	.07	.07*
2. MeMa-School_1	-.46	-2.36*	.11	.04 [b]
3. MeMa-Learning_1	-.26	-1.32	.18	.07*
MeMa-Testing_1	.56	2.63**		
Time 2_Model 1				
1. MeMa-Learning_2	-.33	-1.61	.03	.03
MeMa-Testing_2	.11	.54		
Time 2_Model 2				
1. Anxiety-Other	-.33	-1.78 [c]	.05	.05
2. MeMa-School_2	-.22	-.63	.11	.06 [a]
3. MeMa-Learning_2	-.02	-.07	.06	-.05
MeMa-Testing_2	-.07	-.18		

Note. *$p \leq .05$; **$p \leq .01$; ***$p \leq .001$; [a] $p = .06$; [b] $p = .07$; [c] $p = .08$.

We tested also the possible predictive power of math outcomes in the determination of math anxiety, in order to explore if the direction could be bidirectional. For this reason, Global math first of all, but also Computation and Problem solving were inserted as predictors of the different aspects of math anxiety. Global math effect (see Table 5.6) explained the 3% of MeMa-Learning_1 variance by approaching significance (β=-.21, p=.07). MeMa-Learning_2 was instead significantly predicted by Global math for the 12% of its variance (β=-.36, p<.01). This variable wasn't however suitable in the prediction of MeMa-Testing (R^2=-.01, p=.51, for MeMa-Testing_1; R^2=.01, p=.19, for MeMa-Testing_2). On the other hand, Computation and Problem solving (see Table 5.7) were not suitable in the prediction of MeMa-Learning_1 (R^2=.03, p=.30), MeMa-Testing_1 (R^2=-.02, p=.76) and MeMa-Testing_2 (R^2=-.01, p=.48). Nevertheless, they significantly predicted MeMa-Learning_2 (R^2=-.10, p<.05), by means of the significant contribution of Computation (β=-.30, p<.05).

Table 5.6

Hierarchical Regression Models for MeMa Subscales with Global math as Predictor Variable

Variable	β	t	Adjusted R^2	R^2 change
Time 1_Dep. variable: MeMa-Learning_1				
Global math	-.21	-1.85 [a]	.03	.03 [a]
Time 1_ Dep. variable: MeMa-Testing_1				
Global math	-.07	-.66	-.01	-.01
Time 2_ Dep. variable: MeMa-Learning_2				
Global math	-.36	-2.87**	.12	.12**
Time 2_ Dep. variable: MeMa-Testing_2				
Global math	-.18	-1.32	.01	.01

Note. *p≤.05; **p≤.01; ***p≤.001; [a] p =.07.

Table 5.7

Hierarchical Regression Models for MeMa Subscales with Computation and Problem solving as Predictor Variables

Variable	β	t	Adjusted R^2	R^2 change
Time 1_Dep. variable: MeMa-Learning_1				
Computation	-.09	-.72	.03	.03
Problem solving	-.12	-.94		
Time 1_ Dep. variable: MeMa-Testing_1				
Computation	-.09	-.71	-.02	-.02
Problem solving	.02	.13		
Time 2_ Dep. variable: MeMa-Learning_2				
Computation	-.30	-2.14*	.10	.10*
Problem solving	-.13	-.89		
Time 2_ Dep. variable: MeMa-Testing_2				
Computation	-.09	-.63	-.01	-.01
Problem solving	-.11	-.71		

Note. *p≤.05; **p≤.01; ***p≤.001.

5.4.4 Group differences

Having explored the predictive power of math anxiety on different math outcomes by considering students altogether, a consequent aim was the investigation of the same in relation to the different level of math skills. In this way, it could be possible to investigate if math anxiety can substantially differentiate children characterized by a different level of math competence. In order to identify groups differing by math level, the mean score achieved in Global math (which was also the outcome variable that resulted to be better predicted by math anxiety) was taken as reference. Hence, three groups were defined: low (at least 1 SD below the mean value, N=13), average (in the range of 1 SD below and above the mean value, N=48), and high (at least 1 SD above the mean value, N=18) math achievers.

The three groups were therefore compared by performing an analysis of variance. Results showed that significant differences could be observed only for MeMa-Learning_2 ($F(2, 78)=3.92$, $p<.05$, $\eta^2=.18$). No significant differences stood for MeMa-Testing_1 ($F(2, 78)=.70$, $p=.50$, $\eta^2=.04$), or MeMa-Testing_2 ($F(2, 78)=2.18$, $p=.13$, $\eta^2=.11$). Differences for MeMa-Learning_1 approached significance ($F(2, 78)=2.76$, $p=.08$, $\eta^2=.13$). Post-hoc analyses revealed group differences in MeMa-Learning_2 to subsist between low (M=1.03, SD=.41) and high (M=-.45, SD=.33) math achievers. Differences could be also evidenced in MeMa-Learning_1 between average (M=.39, SD=.23) and high (M=-.50, SD=.35) math achievers.

Finally, since the intriguing result emerging from regression analyses was the positive, instead of negative (as for the other math anxiety subscales) predictive power of MeMa-Testing, especially for Time 1, we performed an additional analysis by classifying children in three different groups on the basis of their level of MeMa-Testing_1 anxiety. The definition of the groups used the criterion of the mean value of anxiety for that subscale: low (at least 1 SD below the mean value, N=17), average (in the range of 1 SD below and above the mean value, N=44) and high (at least 1 SD above the mean value, N=18) math anxiety. Possible differences were inspected in relation to the different math skills. Results showed significant differences in relation to Global math ($F(2, 78)=4.82$, $p=.01$, $\eta^2=.12$), but neither for Computation ($F(2, 78)=2.38$, $p=.10$, $\eta^2=.08$) nor for Problem solving ($F(2, 78)=2.44$, $p=.10$, $\eta^2=.08$). Post-hoc analyses revealed group differences in Global math between low (M=-.31, SD=.21) and average (M=.27, SD=.13) math anxious and between the latter and high (M=-.31, SD=.17) math anxious children.

5.5 Discussion

The early identification of math anxiety and its possible association with math proficiency has represented the core investigation of the current study. In Study 3, in fact, a significant relationship between these two variables in second graders couldn't be observed, in agreement with other studies conducted on so young students (e.g., Krinzinger et al., 2009; Thomas & Dowker, 2000). The present research has however been carried out on third graders in order to inspect if the association could emerge in this grade. Since a trend had been however highlighted in Study 3, the hypotheses were that this link could be established in third grade, and that the same link could be bidirectional. In other words, we hypothesized a high level of math anxiety to impair math performance, but also low math performance to contribute to math anxiety raising up. Anyway, we expected main differences to emerge when comparing children with different levels of math achievement rather than when inspecting the whole sample.

In order to test these hypotheses, math anxiety was evaluated by means of two questionnaires; one of them was the same administered in Study 3 (SEMA) and the other (MeMa) was newly introduced. Since the correlation between the two measures was high and they presented similar correlation patterns, the analyses have been conducted by referring only to MeMa, which provided more information. The scale indeed allowed to independently inspect the aspects of anxiety related to learning mathematics and that associated to the condition of being tested in it. Also general academic anxiety could be assessed. In our analyses, the scores deriving from each subscale were separately treated.

The questionnaire has been proposed to children twice: far from (Time 1) and immediately before (Time 2) math testing. The administration didn't take place after testing in order to avoid biases deriving from success or failure in that single performance. Expectations were in favor of a greater math anxiety, at least for MeMa-Testing, at Time 2. Results went instead in the opposite direction: the only significant difference to emerge was in MeMa-Testing, but with values significantly lower for Time 2 than Time 1. The absence of a consistent increase in overall math anxiety at the moment of testing suggested this condition to be pervasive and long-lasting and therefore to have effects that go far beyond the testing setting (see also Lyons & Beilock, 2011). On the other hand, the decrease in math testing anxiety can indicate math anxiety, when experienced, to be more strongly associated to other aspects, for instance to that of learning this discipline. It can be in fact the act of struggling with concepts, rules and procedures in everyday lessons to be more overwhelming for so young students, whereas testing can be not yet seen as a stressful situation as happens in students attending upper grades.

These preliminary suggestions are in part corroborated by findings emerging from regression analyses. Actually, despite possible differences concerning overall math or computation and problem solving, the general outcome was represented by the stronger negative impact on math performance given by the level of MeMa-Learning, at least when the effects of general anxiety (that however represents the best predictor of math performance) and academic anxiety were not taken into account. This again indicates that, when present, math anxiety is more likely to arise from the difficulty in learning and applying math information. Anxiety in the context of testing could instead surface later in development, as hinted before.

Differences in anxiety associated to math testing came out neither when comparing children characterized by different levels of math capacities. Differences could be noticed only in relation to the aspect of learning mathematics and specifically when comparing low vs. high achievers (for MeMa-Learning_2), but also average vs. high (for MeMa-Learning_1). This finding puts forward the power of math anxiety to discriminate also the upper levels of math performance.

Furthermore, regression analyses reveled a general trend for a positive rather than negative impact of MeMa-Testing_1 (more than MeMa-Testing_2) on math performance. In order to elucidate this issue, we compared groups of students defined on the basis of the level of this specific subtype of anxiety. Results put on evidence that higher Global math scores were achieved by children with intermediate values of math testing anxiety. This result finds support from many relevant investigations demonstrating how moderate anxiety determines the proper level of arousal that is functional to an optimum performance (e.g., O'Hanlon & Beatty, 1996).

5.5.1 Relation between math anxiety and specific math competences

When taking into account global math performance or the individual aspects of computation and problem solving, slight differences could be noticed. Computation, in particular, was observed to be mainly predicted by general anxiety than by that specific for mathematics, indicating how this ability, which entailed the application of procedures with carrying steps, can be more affected by a general than specific condition of anxiety. In this perspective, the impact of general anxiety could be so strong that masks the effects of more specific math anxiety. Actually, as postulated by Eysenck and collaborators (e.g., Eysenck & Calvo, 1992; Eysenck et al., 2007), anxiety interferes with the recruitment of the cognitive resources, for instance working memory, that are essential for the execution of computation procedures (as also proven in Study 3). In this sense, the impact of general anxiety is likely to be stronger in relation to computation than other math skills since it determined, as proposed in our test, huge cognitive load and sustained attention.

The other research question was related to the possible bidirectional relation between math anxiety and performance. This investigation led to interesting results which are in line with those achieved by other authors (see Ashcraft et al., 2002). In fact, it could be stated that in third graders not only the relation is established in a way that high math anxiety impairs math attainment, but also that negative math outcomes can further contribute to increase math anxiety. In this case, both Computation and Global math were observed to predict the degree of math anxiety. The effect of Computation can become here significant for the chief role it assumes in the context of overall math learning in early grades of primary school. It is indeed the aspect to be more stressed by teachers and is frequently the object of math evaluation at school.

5.5.2 Concluding remarks

By summing up the main findings, this study suggested the interrelation between math anxiety and math performance to significantly strengthen approximately at the beginning of third grade. It has also been possible to observe that the link was characterized for being bidirectional already in so young student. In preceding grades of primary school, instead, children are likely to be aware about their feeling in a positive or negative way in relation to mathematics, but factors other than math anxiety appeared to be more salient in the determination of earlier math proficiency (see Krinzinger et al., 2009; Stevenson et al., 2000). Results of this work therefore allowed to shed light on the onset of the relation between math anxiety and math performance, while some previous studies couldn't find this association in early grades of primary school.

Nevertheless, in order to attain more robust evidences, these findings should be corroborated by replicating the study on a large sample of children. With a wider sample size, it would be also possible to better inspect math anxiety in students characterized by a low math achieving profile. Further, it would be also interesting to follow children longitudinally to inspect possible changes also in relation to the aspect of math testing anxiety. Hence, the level of math anxiety in early grades could be monitored to investigate if it can predict future math achievement; the same could be done for math performance, in order to assess with which extent it contributes in determining following math anxiety.

CHAPTER 6
General discussion

The present dissertation has been focused on the investigation of some of the most crucial factors that were thought to intervene in the process of learning mathematics in the first grades of primary school. Actually, only in recent years mathematical learning and related difficulties have begun to receive both research and clinical increasing attention. Nevertheless, two main investigation lines run almost parallel. The first and leading one is dedicated to identification and assessment of the cognitive abilities at the basis of math learning therefore defined as math precursors. Little attention instead receives the monitoring of constructs, typically non-cognitive in nature, which have a less direct, but however significant, impact on math achievement.

By joining these two principal research lines, the topic of this thesis has been represented by the combined evaluation of both kinds of factors in relation to math attainment of very young students. In this way, it has been possible to early identify which are the most relevant factors that, when deficient, are likely to characterize children with difficulties in math, therefore suggesting on which of them it is possible to intervene. The following paragraph illustrates the main findings emerged from the described studies. After that, limitations, future directions and implication of the same are discussed.

6.1 Summary and discussion of main findings

Studies 1 and 2 have been dedicated to the investigation of the cognitive abilities essential for math learning, with attention to both general and specific precursors. Study 3 investigated, on the other hand, non-cognitive factors involved in math proficiency and also their relationship with the above-mentioned cognitive abilities. Study 4 was specifically dedicated to the investigation of the subtype of anxiety characteristic of mathematics, since in the previous study the expected significant role didn't emerge. On the whole, studies focused on children from the entrance to primary school (early Grade 1) till Grade 3, thus trying to achieve a complete picture of early stages of math learning. Furthermore, a complete evaluation of math skills has been carried out, by analyzing them on their whole but also taking into account the specific competences regarding computation and word problem solving.

6.1.1 Math cognitive precursors

Studies focused on the evaluation of the cognitive abilities at the basis of math learning are typically interested in two kinds of factors: *general*, which include intelligence, memory and other executive functions, and *specific*, which regard mathematics more tightly and include the capacities of manipulating magnitudes, quantities and numbers. Authors struggled in finding which of these abilities may rise the role of precursors of math learning. Only in very recent years, studies have begun to explore both kinds of factors in order to assess their relative contribution (see Cirino, 2011; Fuchs et al., 2011; Passolunghi & Lanfranchi, 2012; Xenidou-Dervou et al., 2013). Nonetheless, it is not yet elucidated the precise role of some of these abilities. Debate subsists, for instance, regarding the role of more active or passive memory processes (respectively, working memory and short-term memory). Within specific precursors, confused are findings concerning the role of the Approximate number system (ANS) which is a very basic, language-independent ability of making approximate judgments about magnitudes. Almost unexplored are also the relationships among all these precursors.

Keeping in mind these considerations, we aimed at elucidating which are the factors mainly involved in math learning very early, from the entrance to primary school till Grade 3. This is what has been carried out in Studies 1 and 2. In fact, the earlier the monitoring of the key math precursors, the greater the possibilities to overcome potential difficulties in this discipline (by developing proper intervention programs). More precisely, Study 1, illustrated in CHAPTER 2, dedicated to the investigation of working memory, short-term memory, intelligence and ANS just at the beginning of schooling. In order to test the relation between them and with early math skills, path analyses were carried out. Results showed that the investigated precursors largely predicted early math performance. The main precursors appeared to be intelligence, in both its verbal and performance aspects, both verbal and visuo-spatial working memory, and visuo-spatial short-term memory. Within verbal memory, a greater role was assumed by the capacity of recalling digits rather than general verbal information. This capacity was more closely associated to early math skills entailing number knowledge than to the more basic, number knowledge-independent ones. Within ANS, a relevant role was assumed by the ability of making magnitude comparison, but not by the more complex approximate addition.

Our findings are in line with studies highlighting the fundamental role of working memory (e.g., Bull & Scerif, 2001; Gathercole & Pickering, 2000a; Holmes & Adams, 2006; Passolunghi et al., 2007, 2008) and especially of the visuo-spatial capacities (e.g., Bull et al., 2008; Jarvis & Gathercole, 2003; Passolunghi & Mammarella, 2010, 2012). These are fundamental at different levels, from the visualization of numbers along a mental number line to the carrying out of arithmetic operations in column. The aspect of memory for digits suggested possible math difficulties to rely already at the level of retaining numerical information (e.g., Geary et al., 2004; Passolunghi & Cornoldi, 2008). Concerning ANS, and specifically magnitude comparison, many studies lacked in finding its significant involvement in math learning (e.g., Holloway & Ansari, 2009; Rousselle & Noël, 2007; Soltész et al., 2010). Other authors proposed a significant role, when found out, to closely depend on working memory capacities, necessary to perform these tasks because of some perceptual constraints with which they are set up (e.g., Gilmore et al., 2013). By taking into account magnitude comparison, in our study such association with working memory hasn't emerged, suggesting this ability to provide an independent unique contribution to early math achievement.

Once explored the interaction among the cognitive precursors and their impact on early math learning, the subsequent aim was the inspection of the longitudinal predictive role on math performance in the following grades, i.e., in Grades 2 and 3. It has indeed been proven that the assessment of math precursors and competence at the beginning of school is predictive of future math performance up to ten years later (Clarren, Martin, & Townes, 1993). This finding endorsed our choice of the early detection of cognitive math profiles. This investigation has represented the topic of Study 2, reported in CHAPTER 3. Here, math precursors were put in relation to global math skills, but also to the individual abilities of performing computation and solving word problems. Moreover, the same precursors were tested also in Grade 2 in order to inspect possible differences in the predictive power in dependence to their assessment time.

Findings from this longitudinal study demonstrated that both general and specific precursors held a significant involvement also in the prediction of math achievement in subsequent grades, with a main role held by working memory. Additional cognitive resources were needed, especially in Grade 3, for problem solving in relation to computation. In general, the principal precursor across grades was represented by working memory and in particular by the visuo-spatial component. The role of verbal working memory appeared to emerge in upper grades. The role of short-term memory was generally subjected to that of more active memory processes and also the direct involvement of intelligence seemed to partially decrease. Considering ANS, it maintained a predictive role even in following grades, therefore indicating that is crucial also for the acquisition of more formal and complex math skills (as suggested by Inglis et al., 2011). Anyway, when taking into account also prior achievement, almost the only predictive power to last was that of working memory, confirming its leading role as math precursor.

Another interesting result emerging from Study 2 concerned the different predictive ability of precursors when assessed either in Grade 1 or Grade 2. In fact, the components of the investigated precursors assumed a different role in dependence of when they had been evaluated. For instance, in relation to working memory assessed in Grade 1, it was possible to observe that both verbal and visuo-spatial components were highly predictive of math performance two years later, whereas, when tested in Grade 2, the only predictive component resulted to the visuo-spatial one. This suggested the consistent cognitive development children experience in the first grades of primary school and therefore implies the necessity to pay attention to aspects possibly differing in dependence to when they are tested.

6.1.2 Self-perceptions and affect

Once having achieved an overall framework around the cognitive abilities acting as primary precursors of math learning, the subsequent interest was that of exploring if also in very young students factors other than cognitive abilities can be involved in math attainment. Typically, these constructs are explored in old children (e.g., college students) and less attention is dedicated to pupils. For this reason, in Study 3, described in CHAPTER 4, we explored some of the constructs that have been proven, at least in adolescents, to be relevant in academic contexts. The target sample was represented by second graders, since these children possess sufficient comprehension skills and begin to develop adequate perceptions about their capacities and feelings. The examined constructs included, on one hand, perceptions about the Self: self-efficacy, that is specific to mathematics, and more general aspects such as self-concept and self-esteem. On the other hand, affective factors were assessed: anxiety, both general and specific for mathematics, and depression. A wealth of literature findings confirms the involvement of self-efficacy and anxiety, positive for the former (e.g., Pajares, 1996) and negative for the latter (e.g., Fisher et al., 1996), but the other constructs are frequently underestimated. A related aim consisted in the investigation of the possible interrelationship between cognitive and non-cognitive factors. From literature, it is for instance known that anxiety affects memory resources (e.g., Ashcraft & Kirk, 2001; Eysenck et al., 2007), but studies about its impact on ANS are lacking.

Path analyses results reveled that non-cognitive constructs could explain a significant quote of math performance variability. Specifically, self-efficacy and general anxiety were observed to be directly involved in the determination of math performance. More in detail, self-efficacy was observed to be influenced by math attainment, i.e., the better the math performance, the higher the self-efficacy level. Self-efficacy was also proven to influence indirectly math performance, since it negatively impacted on general anxiety, in turn observed to negatively affect math achievement. Anxiety effects were proven to be both direct and indirect. In fact, not only it impaired memory resources, as stated in other studies, but it was also observed to affect ANS. Other aspects to hold a significant, even if indirect role were depression and self-esteem, role that was positive for the former and negative for the latter. This finding seemed to contradict recent perspectives minimizing the effects of low self-esteem (Baumeister, Campbell, Krueger, & Vohs 2003). Nevertheless, one tricky result concerned the absence of a significant involvement of self-concept and particularly of math anxiety, which appeared to impact neither math performance nor the related cognitive precursors. Only a trend for its association with math performance could be noticed. This finding was in line with those achieved by other authors (Krinzinger et al., 2009; Thomas & Dowker, 2000) reporting math anxiety in so young students to be experienced but not in a way that interferes with math attainment.

For this reason, we wanted to inspect when math anxiety could strengthen its link with math performance. This has represented the purpose of Study 4, described in CHAPTER 5, where we specifically focused on math anxiety by testing it in the first term of Grade 3. Math anxiety assessment took place both far from math testing and just before it. We administered a questionnaire that allowed to distinguish anxiety related to learning mathematics from that associating to the math testing condition. Moreover, the evaluation of general academic anxiety was also possible.

Results demonstrated that math anxiety didn't increase at the moment of testing, indicating that it is felt even beyond the context of math evaluation and is therefore rooted in children that experience it. More in detail, in children of this age math anxiety appeared to be related to math proficiency but only in the aspect of learning the subject, indicating that is the subject itself (or how it is taught) to have a main detrimental effect. The link was bidirectional, indicating that math anxiety is likely to be both the cause and the consequence of a poor math performance. Finally, it has been observed that a better math performance is associated to a moderate level of math testing anxiety, thus confirming the fact that it provides an optimum level of arousal that is fundamental for a successful performance.

6.2 Limitations and future directions

This dissertation has focused on two fundamental aspects within academic achievement and therefore tried to integrate the assessment of both cognitive and non-cognitive constructs. In trying to perform this investigation, some difficulties have been inevitably encountered. Hence, some limitations of the studies have to be underlined.

We sought to perform research on wide samples of children in order to accomplish reliable results. Anyway, especially in the case of the longitudinal study on math precursors (Study 2), the sample size was relatively small and this limited the analyses that could be performed. Since a lot of variables were evaluated and the design was rather complex, the investigations should be replicated on a larger sample.

Always considering the evaluation of cognitive precursors, our choice has been that of concentrating on some fundamental aspects, especially on memory and the approximate component of number sense. In some cases it happened that these variables couldn't account for a very consistent quote of math achievement variance and results therefore directed towards the investigation of other possibly crucial abilities. For instance, executive functions such as inhibition could be explored, as well as the attentional processes. Important could be also the evaluation of the exact, symbolic component of number sense. In this way, it could be also checked if this component fully mediates the influence of ANS on math learning or if ANS can provide an additional unique contribution. Furthermore, aspects other than ANS accuracy could be monitored. Reaction times, for instance, could be in older children an index more significant than accuracy. Hence, also this parameter should be collected in future investigations.

Another suggestion for future studies entails the necessity to monitor which cognitive abilities are more crucial in the determination of both concurrent and following math learning at a certain stage of development and instruction. In performing this investigation, it would be also interesting to inspect if memory and ANS, proven to be almost dissociated in young children, develop in parallel or become interrelated at a certain point along development.

Also regarding the investigation of non-cognitive factors, a longitudinal monitoring of the same in relation to math achievement is suggested. Light should be shed on these constructs and also on different profiles emerging from either self- or observer-ratings in young children. In our study, indeed, different patterns of associations could be drawn for profiles emerging from these two different evaluations, making it difficult to understand which was a more reliable and pertinent outcome and thus to disentangle differential results.

Concerning math anxiety, further research is needed to elucidate the involvement of aspects related to both math learning and math testing and their evolution as children grow up. Since sample size was also in this case relatively small, specific analyses on low math performing children couldn't be conducted. As a consequence, it was not possible to deeply inspect the possible increasing relevance of the assessed factors in this specific subsample.

Actually, all these studies should be replicated on a sample represented by low-achieving and/or disabled students to explore if in this specific target both cognitive and non-cognitive constructs can assume possibly different relevance and interfacing.

6.3 Implications of the research

The studies reported in the present dissertation have been focused on samples of typically developing children. Despite the investigations haven't been carried out on children with learning difficulties or at risk for, the final aim consisted in providing the tools to which also teachers can resort for the early identification of math difficulties. There is indeed large consensus about the fact that a precocious identification and the consequent application of the proper intervention programs can recover deficits in math and be also beneficial for learning disabled children. It is fundamental to stress on the early detection and intervention since it is easier to intervene on malleable cognitive resources in developing young children.

By discussing results in Study 2, it has been pointed out that there are brain regions that are specific to math and related precursors. It has been proven that these cerebral regions underpinning math cognitive precursors have a reduced volume or decreased connections with the other areas in

MLD individuals (e.g., Molko et al., 2003). Nevertheless, brain plasticity is a magnificent phenomenon that can be promoted by proper training programs. This means that, even if our brain structures have suffered as a consequence of a certain event (e.g., pre-term birth, malnutrition, cerebral traumas and so forth), our capacities have however a certain potential to be strengthen. These possibilities are higher if the intervention is precocious. For this reason, many are the intervention programs that have been specifically developed for very young children.

With regard to math learning, trainings have been set up to potentiate precursor abilities, once proven their effective role in math learning. Many of these trainings have focused on memory and other executive functions (e.g., Holmes, Gathercole, & Dunning, 2009; Passolunghi & Costa, in press; Thorell, Lindqvist, Bergman, Bohlin, & Klingberg, 2009), others on number sense, with greater attention to the symbolic processes (e.g., Arnold, Fisher, Doctoroff, & Dobbs, 2002; Butterworth & Laurillard, 2010; Young-Loveridge, 2004). In relation to number sense, also the act of practicing with linear board games involving numbers has been demonstrated to improve math skills (e.g., Ramani & Siegler, 2008; Whyte & Bull, 2008). Nevertheless, our findings suggest that, at least in very young children, also approximate number abilities can be potentiated in order to strengthen early math capacities. Despite many authors subjected the role of approximate, non-symbolic skills to that of the more formal, symbolic ones, it is undeniable that also the former have a significant impact. Hence, it could be useful to develop trainings and games that are suitable for preschool children and that involve non-symbolic quantities.

Concerning factors other than cognitive abilities, findings from our studies put forward the necessity to intervene from different viewpoints. Actually, the way children feel and perceive themselves is tightly related to their academic performance. Children spend a lot of time at school and events happening in the academic context permeate many aspects of their life. For this reason, having proved the triggering of a vicious cycle between children's feelings and thoughts and their academic performance (where high anxiety and depression and low perceptions about themselves and own capacities contribute to a poor performance, but where these can be in part also the consequence of the same), it is fundamental for teachers to act in the aspect pertaining to them, i.e., by trying to promote a successful performance.

Mathematics is, among all the school subjects, the one that most strongly impacts students' self-esteem. Actually, a common way of thinking is that one person succeeds in math only if he/she is particularly clever and a failure is often interpreted as a sign of inadequate intelligence and capacities. Therefore, apart from the possibility to work on cognitive precursors also indirectly (throughout math exercises expressly developed to tap them, see Passolunghi & Bizzarro, 2005), math teachers could adopt simple strategies to promote academic proficiency. For instance, Hackworth (1992) proposed some activities directed at reducing math anxiety. Teachers were recommended to help and assist students in understanding the origins of their anxiety and in being involved as active learners in the acquisition of new skills. In this way, students will become more confident with the discipline and boost also their pride. From this point of view as well, it is fundamental to act as early as possible, since it has been proven that negative attitudes are going to establish across time and have widespread detrimental effects (e.g., Ma, 1999; Petersen et al., 1993).

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