Effect of imperfect competition on infrastructure charges

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Abstract

The text explores the optimal infrastructure charges of an unbundled activity where the infrastructure manager sells the use of the infrastructure to operators providing services to a downstream market made up of atomistic customers. This situation has been widely analysed under the assumption that the upstream market is competitive, but more rarely in the case of imperfect competition. Typical examples are the railways activity in Europe and air transport. Various market structures are considered, illustrated by situations encountered in the transport field: a single mode operated by a single operator, two operators competing within the same mode, and two modes competing in a Bertrand way. In each case, situations are analysed using analytic formulae with a simplified demand function and a simplified cost function, and performing simulations with sensible parameter values drawn from current average situations. The main result is that the analysed imperfections make a dramatic departure from the conventional Marginal Cost pricing doctrine. Conclusions are drawn regarding infrastructure charging policy.

Keywords: Imperfect competition; Transport infrastructure; Rail; High speed train; Marginal cost; pricing; Differentiation; Pricing behaviour; Market power; Lerner index.

1. Introduction

The liberalisation of public services has created a great interest in Infrastructure Charges (IC), especially in Europe with the reforms leading to the unbundling of infrastructure management and operations. In this framework, the general doctrine commonly addressed (Proost and alii (2004), Quinet (2005)) is the Short Run Marginal Cost Pricing (SRMC), where the IC is equal to the Short Term Infrastructure Marginal Cost.

Inside this framework, a growing interest arose about differentiation of Infrastructure Charges. Among the many situations explored by the research program "Different", a

particular one deserves interest and is the subject of this text: the imperfect competition in transport markets.

Strangely enough, while a lot of attention has been paid to the situations of perfect competition, little consideration has been given to situations of imperfect competition. What happens in that case? How should optimal infrastructure charges deviate from SRMC? How should they vary according to the degree of competition? What are the consequences of alternative IC levels on welfare, on the revenues of the operators and on the consumers' surplus? This contribution explores these questions, using as an example the case of the railways, in which an infrastructure manager (IM) sells the use of the infrastructure to operators. These operators act in an imperfectly competitive market and provide services to atomistic customers. The IM sets the IC that all rail operators have to pay.

The effects and consequences of alternative Infrastructure Charges (IC) can be assessed either through theoretical considerations based on economic analysis or through tests of real situations. In the framework of imperfect competition, the first approach gets rapidly limited due to the complexity of mathematical derivations. As a consequence, only a few very general and well-known results can be derived through such a method.

The second approach, the numerical test of real situations, allows to use the power of computer calculation and to test more varied and complicated situations. But it needs some numerical assumptions so as to simulate the behaviours of the actors of the game.

The text is organized as follows: section 2 presents the basics of optimal IC in a framework of imperfect competition. Section 3 develops the modelling principles. Section 4 presents the data used. Section 5 presents the simulations and their results, and section 6 concludes.

2. Optimal IC under imperfect competition

Transport markets and especially rail markets are characterized by imperfect competition[1]: for long distance passenger traffic, there is in general just one rail operator (RO), the competition is intermodal, with air transport, and it often happens on each relation that there is just one or a few air competitors. For medium and short distance passenger traffic, there are in general just one or very few competing rail operators, and the main competition comes from road transport; road transport is regarded as being operated under pure competition conditions between road hauliers, having no strategic behaviour, and in the case where one RO is competing only with road transport, everything looks as if the RO were a monopoly. On-track competition is more frequent in freight transport, but here again, the competitors are just a few on each single relation.

In such a situation, the classical doctrine of marginal social cost pricing does not apply. The rigorous formulae giving the charge should be derived from a general equilibrium model (GEM) taking into account the real features of the economy.

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[1] The following results are drawn from Quinet 2007 ("Effect of market structure on optimal pricing and cost recovery") and Meunier 2007 ("Sharing investment costs and negotiating railway infrastructure charges"), both communications to the Second International Conference on Funding Transportation Infrastructure Leuven, Belgium, September 20-21, 2007

Unfortunately the formulae are not easy to handle (see for instance Mayeres I., Proost S. (2001)).

Another less rigorous but more tractable procedure can be used, in the framework of partial analysis, and some strictly localised departures from the first best situation are allowed. This procedure is the one used in the well-known Ramsey formula, where a budget constraint of the operator is modelled, or where the distortion of taxes is captured through a cost of public funds. And the result is that in this case, at the social optimum, the Lerner index (percentage of increase of price - here, the IC - compared to the short run marginal cost) is inversely proportional to the price-elasticity of the demand[2]. But this result holds only in case of perfect competition in the downstream market. Our aim is to follow this way in order to explore the consequences on the optimal charge level in cases of imperfect competition between transport operators, where cost of public fund and, possibly, externalities, are introduced. The procedure is similar to the modelling framework exposed in Suter and alii (2004) on the Molino model; still, it is much less sophisticated and does not take into account the phases of investment funding. Using a simpler process, it allows putting more attention to the transport market and to its imperfections. Let us present the analytical results in two particular cases (the derivations of the formulae are given in Appendix 3).

The first case will be a profit maximizer monopoly. Let us derive the algebraic formula for the optimal IC, using the following symbols and assumptions:

- The demand function of the down-stream market is a linear one:
- $Q = f(p) = \alpha p + \beta$, $\alpha < 0$ and $\beta > 0$
- where Q is the traffic and p is the price paid by the users to the rail operator (RO)
- The operating cost of the RO is assumed to be constant and equal to: c' per unit of traffic
- The operating cost of the infrastructure manager (IM) is assumed to be constant per unit of traffic and equal to: b
- The IM sells the paths to the RO at a price: t per unit of traffic
- Then the cost per unit of traffic for the RO is constant and equal to: $c = c' + t$
- The RO generates an external cost of e per unit of traffic
- The Cost of Public Funds (or shadow variables of possible budget constraints) are $(\lambda - 1)$ for the IM and $(\lambda' - 1)$ for the RO

It is easy to show that the RO, aiming at maximising its profit: $Q(p-c'-t) = Q(p-c)$, chooses the price p such as:

$$p = (c/2) - (\beta/2\alpha) = \frac{\alpha c - \beta}{2\alpha}$$

i.e.:

$$p(t) = \frac{t}{2} + \frac{\alpha c' - \beta}{2\alpha}$$

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[2] In presence of externalities, this opinion is wrong as shown in Quinet, Touzery et Triebel (1982) and in Oum and Tretheway (1988), and as it will be recalled later

The Welfare is:

W(p)=SU(p)+λPRim(p)+λ'PRro(p)-eQ(p)

where PRim and PRro are the profits of the IM and the RO, SU being the final users' surplus.

The optimal *IC* is the value of *t* which maximises *W(p)*.

Noting that $\partial SU/\partial p=-Q$, and replacing the other terms by their expressions, it turns out that:

$$t-b=\frac{\frac{Q_b}{\alpha}(1-2\lambda+2\lambda')+e}{(2\lambda-\lambda'-\frac{1}{2})}$$

where Q_b is the traffic obtained when the IC is equal to the marginal infrastructure cost.

Let us present also the case of a duopoly, representing competition between air and rail. The demand functions are:

$$Q_r=\alpha p_r+\gamma p_m+q_r \qquad \text{for rail traffic}$$
$$Q_m=\gamma p_r+\beta p_m+q_m \qquad \text{for air traffic}$$
$$\alpha<0 \quad \beta<0 \quad \gamma>0$$

The profit of the RO is:

$$PR_r=Q_r(p_r-c_r)=-\alpha(p_r-c_r)^2=-\frac{1}{\alpha}Q_r{}^2$$

with the similar relation for the competitor *m*. The welfare is:

$$W=SU_r+SU_m+\lambda PR_{IM}+\lambda'PR_r+\lambda''PR_m-e_rQ_r-e_mQ_m$$

We assume for simplicity that both operators are purely private: *λ'=λ''=1*
and that rail externalities are negligible when compared to air externalities: $e_r \approx 0$.

Maximisation then leads to:

$$t-b=\frac{Q^r{}_b(\frac{\partial p_r}{\partial c_r}+\lambda-2)+Q^m{}_b\frac{\partial p_m}{\partial c_r}+e_m\frac{\alpha\beta\gamma}{\gamma^2-4\alpha\beta}}{\alpha\left[(\frac{\partial p_r}{\partial c_r}-1)(\frac{\partial p_r}{\partial c_r}+2(\lambda-1))\right]+\beta(\frac{\partial p_m}{\partial c_r})^2}$$

These developments confirm that the optimal IC under imperfect competition is quite different from the classical SRMC pricing principle. But, even in the simple cases analysed using linear demand functions, the algebraic formulae are complex and not

easy to interpret. This point is an argument for using numerical simulations in order to explore the properties of the IC in situations of imperfect competition.

3. The modelling framework of the simulations

Simulations could be made on a large scale, for instance at the country level. The overall model would use as entries the cost and demand functions for each route of each operator, the ICs to be tested on each route, as well as the structure of the competition (if any) between the operators; the outputs would be the prices and the traffics for each mode and various other outputs such as the profits of the firms or the welfare.

In practice, the implementation of this model is hampered by the lack of data: we have no good knowledge of the cost functions of the operators at the level of each route; the type of competition between the operators is not known precisely. The lack of data prevents us from achieving econometric calculations and induces us to use more simple and crude methods, restricting the ambitions of the modelling framework.

The method implemented here can be entitled "sensible simulation", and presents the following features:

- It involves a simple network: one or a few origin-destinations, one or two modes serving these relations
- The agents are: the final consumers, the transport operators (one or two rail operators, zero or one operator using another mode) and the infrastructure manager. The rail operator(s) pay an IC to the infrastructure manager. The IC has no fixed part tariff nor quantity rebates, it just uses a fixed unit price
- The demand functions are either linear or logit
- Cost functions are linear
- The parameters of the cost and demand functions are not calibrated on a specific real situation, they are set up in order to reproduce typical situations that are determined in relation to the common knowledge of the specialists of the field.
- Other parameters may be introduced such as cost of public funds or externalities
- Operators are supposed to adopt a continuum of possible behaviours between two extreme ones: the marginal cost pricing corresponding to the behaviour of an operator aiming at maximizing the welfare and, at the other end of the spectrum, the profit maximizing behaviour. The operator's utility function is assumed to be some kind of linear average between these extreme utility functions. Alternatively, this type of utility function can be interpreted as the result of more or less tight price regulation from the transport regulator.
- A variety of competition situations are represented, including:
  - For rail: monopoly, duopoly
  - For the competing mode: perfect competition, monopoly
  - For the type of duopoly competition: Bertrand competition
  - In case of oligopoly, the services provided by the operators are deemed to be imperfect substitutes.

It turns out (see Appendix 1) that these various competition situations can be represented using a single formula for each operator, that generalizes the Ramsey-Boiteux formula:

(p-c)/p=-s/ε

where *p* is the price of the operator, *c* is its marginal operating cost, *ε* is the own price-elasticity of the operator and *s* is a parameter representative of the behaviour of the operator or of the strength of the price regulation as seen above. Values of *s* are varying from 0 (case of perfect competition or own-market welfare maximisation behaviour, or extremely tight price regulation) to 1 (case of profit maximizing monopoly, Bertrand competition with profit maximizing operators, or no price regulation). Parameter *s* may be interpreted as a measure of the market power effectively exerted by the operator.

The simulation process is the following one:

- A set of sensible and reasonable estimates of some parameters is fixed, aiming at representing current typical situations: prices and traffic levels, costs of the operators and of the infrastructure manager, price-elasticities (a single elasticity in the case of a monopoly, 4 elasticities in the case of a duopoly).
- From this data set, the parameters of the demand function and the parameter s are deduced,
- After this calibration phase, the optimization phase aims at finding the IC that maximises the welfare, taking into consideration possible costs of public funds and external costs.

Appendix 2 details the corresponding calculations. They have been achieved through Mathematica and Excel softwares.

4. The data

The most difficult data to obtain are data on costs, since much of them are covered by secrecy. Prices are also difficult to gather due to the increasing use of yield management, that leads to high discrimination of the demand and to differentiation and multiplication of prices. The data base is shown in the following table:

Table 1: Main Data Set.

	Link	*A*	*B*	*C*	*D*	*E*	*F*
Market structure		Monopoly	Monopoly	Monopoly	Duopoly	Duopoly	Duopoly
Operator's prices	p1	43	43	52	48	59	50
	p2				62	102	85
Operator's costs	c1	16	13	16	19	24	16
	c2				55	80	65
Elasticities	E11	-0,9	-1	-0,9	-1,5	-1,2	-1,5
	E22				-1,7	-1,5	-1,5
	E12				0,8	2,3	2,5
	E21				1,5	1,5	1,5

	Link	A	B	C	D	E	F
Traffics	Q1	0,31	0,26	0,33	0,31	0,24	0,17
	Q2				0,39	0,45	0,51
Infrastructure charges used as references	RIC	12,60	12,60	5,80	16,15	20,56	10,28
Infrastructure Cost	b	2,06	2,06	2,13	3,44	5,52	5,21
Environmental Costs	e1	1	2,25	1,75	3,5	4,5	4
	e2	4,4	9,9	7,7	15,4	19,8	17,6

Situations A to C are "monopoly-like" situations of competition between high speed train and motorways for diverse travel distances, so as to represent more or less tough competition conditions and market shares for rail. Situations D to F are situations where high speed train is competing with air transport, again for diverse travel distances so as to represent a range of competition situations and a variety of relative competitive advantages for rail.

5. The simulations

The modelled cases are, according to the available data, the following ones:

- A rail operator monopoly on a single origin-destination (O-D) link
- A rail operator competing with an operator from another mode, both in situation of monopoly within their mode on a single O-D link
- A rail duopoly on a single O-D link, with two hypotheses for the infrastructure charges: either a single infrastructure charge, or a differentiated infrastructure charge (the two competitors do not have the same IC then).

Simulations provide several results. Some of them are confirmation of already well-known results. Other ones pertain to the sensitivity of the results to calibration parameters such as the shape of the demand function or costs and prices. A last series gives indications about how interesting it would be to introduce some differentiation.
In the following sub-sections, we selected some simulations so as to illustrate the specific points that came out from each simulation theme. The following tables will show only one or a few simulation situations taken from situations A to F, since the other ones would not give much more additional information, and so as to keep tables relatively simple and easy to read. We did not precise for each table the whole set of parameter values that were used, since they were too numerous, but the key parameters that change from table to table are highlighted.

5.1. Consequences of Marginal Cost pricing in some cases of imperfect competition

First, as clearly shown by the theoretical formulae given above, as long as there is no tax distortion, i.e. the CPF (Cost of Public Funds) parameter is 1, the optimal IC are

low, and may be lower than the marginal infrastructure costs, in case of monopoly at least. Optimal ICs are even in some cases negative[3], which means that the rail service should be subsidized. This result is classical: in order to avoid the monopolistic distortion of prices vis-à-vis costs and to induce the monopoly to fix its price at the level of the marginal cost, it is necessary to decrease the prices of its inputs, and the single input on which the IM can act is the IC. This point is exemplified for instance in the case of monopoly, as shown by table 2:

Table 2: Comparison of optimal IC and marginal infrastructure cost in the case of a monopoly.

Link	Costs of Public Funds		Optimal Infrastructure Charge	Marginal Infrastructure Cost
	IM	RO	0IC	b
C	1	1	-34,2	2,1
	1,3	1	-5,7	2,1
	1,5	1	4,3	2,1
A	1	1	-18,9	2,1
	1,3	1	5,9	2,1
	1,3	1,3	-1,5	2,1

This table shows also that this result highly depends on the value of the CPF. The optimal IC level increases with the CPF of the IM. Additional simulations indicate that in the monopoly case under review (the *s* parameter being equal to 1 and without any externality), optimal IC is close to the marginal infrastructure cost for values of CPF around 1,4 for the IM and 1,0 for the operator; and that in a large number of cases tested, values of CPF in the range [1,5; 1,8] rise the optimal IC close to the (observed) reference level of IC.

The same results appear in the case of a duopoly (for example, a duopoly between rail and air transport for passengers), as shown in table 3. Here, the near coincidence between optimal IC and marginal cost is observed for slightly lower values of CPF than in the monopoly case; this result is in line with the expectation: when competition gets tougher, the optimal IC becomes higher than the marginal infrastructure cost.

Table 3: Comparison of optimal IC and marginal infrastructure cost in the case of a duopoly.

Link	Costs of Public Funds		Optimal	Marginal
	IM	RO	IC	infra cost
D	1	1	-22,1	3,4
	1,3	1	2,0	3,4
	1,3	1,3	-4,1	3,4

Taking into account the external costs increases the IC if the mode is less environment friendly than its competitor, and decreases it in the reverse situation which is usually the case for rail vis-à-vis air or road transport. Table 4 shows examples of these effects.

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[3] In the case of a profit maximiser monopoly, when CPF of the IM and of the operator are equal, optimal IC increase with these CPF and become equal to the marginal cost of infrastructure when CPF are infinite.

We see that, in the range of values considered in our simulations, external costs tend to have observable but lower impacts on prices and optimal charges, as compared to the impact of CPF (in this table, CPF of the IM is 1,5 while CPF of the Rail Operator is 1,0)

Table 4: Effects of external costs.

Link	External Costs		Rail Price	Optimal IC
	e1	e2		
B	0	0	32,9	0,4
B	2,25	9,9	30,9	-2,0
E	0	0	64,4	20,4
E	4,5	19,8	59,1	14,0

Another striking fact is the change in welfare induced by changes in the IC. It is clear from table 5 and figure 1 that the changes in welfare are small and that the effect of a sub-optimal IC bears mainly on the revenues of the IM and the operator's revenues and consumer surplus.

Table 5: Consequences of a sub-optimal IC (CPF of the IM=1,3; CPF of the RO=1,0).

Link	Comment	IC	Welfare	Revenues	
				IM	operator 1
D		16,1	45,1	5,6	5,9
D		9,0	46,3	2,9	7,6
D	In this simulation the IC is the marginal cost of infrastructure	3,4	46,5	0,1	8,9
D	In this simulation the IC is the optimal one	2,0	46,7	-0,9	9,7

So as to give an idea of the relative orders of magnitude obtained in our simulations, a loss of IM revenue through a reduction of IC level often benefits to the rail operator for one third and to the consumers for two thirds.

The relative share of the effect of external costs within the change in welfare depends highly, of course, on the unit level of externality gain or loss. This relative share depends also highly on the relative number of clients that rail takes from the competing mode within the total rail traffic increase obtained when the IC gets lower.

For sensible estimates of the unit level, this relative share of external costs' effects varies widely from a few per cent (the far more frequent case in our simulations) to the great majority of welfare gains. Figure 1 shows the relative orders of magnitude as taken from one of our simulations.
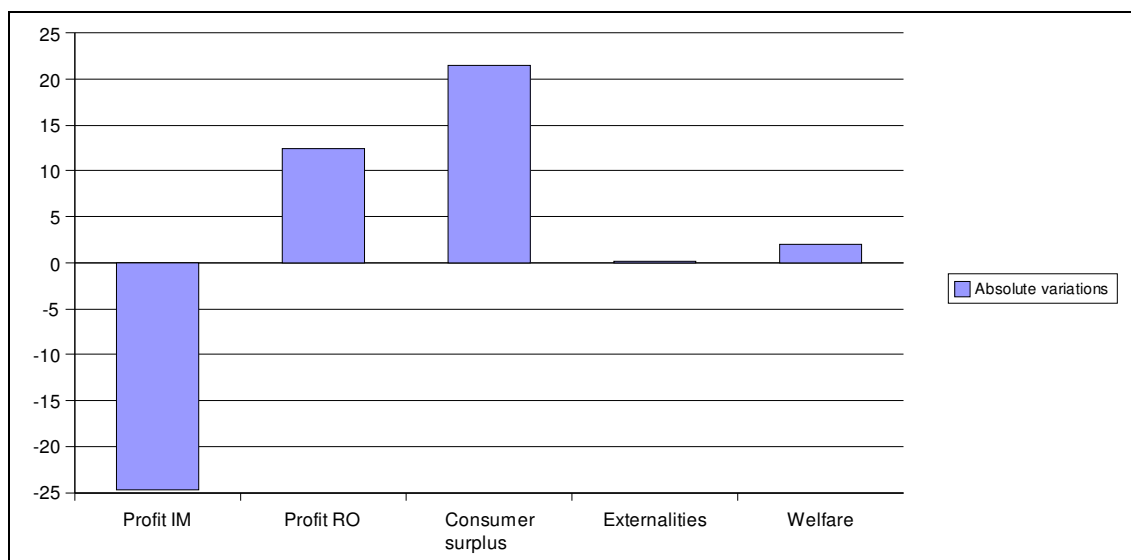
Figure 1: variations in profits of infrastructure manager, profits of rail operator, consumer surplus, externalities and welfare when the IC is set at its optimal level (link A, CPF(IM)=1,3 CPF(RO)=1).

As a conclusion of this first set of simulations, it appears that, depending on the circumstances (level of CPF, externalities, market structure), marginal social cost pricing can be either a good approximation or lead to non negligible welfare losses; in any case it leads to important changes in the distribution of welfare. Circumstances under which non negligible welfare losses may occur are variable; if we dare to give some hints from our simulations, this situation seems to be more likely to occur when CPF(IM) is low and rail has a strong market power.

5.2. Effects of differentiation of the IC

Infrastructure Charges can be differentiated in many ways. Simulations have been designed to explore some of them. A first set of simulations relates to individual differentiation criteria: operator's marginal costs, elasticities and marginal infrastructure costs; then, the question of averaging the IC level over several links is treated: does it make sense, what is the loss in welfare, what are the impacts on the operators' profits? Finally, the case of competing rail operators is treated. We will now address these points.

5.2.1. How much should IC be different when operator's costs are different?

Table 6 below shows the impact of differences on the operators' marginal costs: the effect of an increase of operator's marginal cost is to decrease the optimal tariff. The decrease seems to be similar in situation of duopoly than in situation of monopoly, but it could well be lower in other cases than those simulated since, in a duopoly, the competitor exerts an effect which limits the market power of the operator. In any case it appears that the positive but rather low effect on welfare implies important effects because of the distributive effects between the agents: infrastructure manager, operators, and consumers. This point, illustrated further down for the issue of IC averaging (see figure 2), is a general conclusion of all the simulations.

Table 6: Effect of differences on operator's marginal cost.

	Market Structure	Operators' Costs	Prices		Optimal IC
			P1	P2	
C	Monopoly	16,2	50,3		4,3
		19,5	52,9		4,0
E	Duopoly	24,0	64,4	90,3	20,4
		28,8	67,9	90,8	19,9

5.2.2. How much should IC be different when demand elasticities are different?

The following table shows that the optimal tariff is rather sensitive to the demand characteristics.

Table 7: Effect of differences in elasticities in the case of monopoly.

	Elasticities	Prices	Optimal
	e11	p1	IC
B	-1,0	46,4	-9,2
B	-1,5	36,9	-5,9

Note: the values of p1 and of the Optimal IC differ from the values given in table 4 because in table 4 the s1 parameter is 0,4 while it is 1 in table 7.

In the case of a duopoly with logit demand function, assessing the effect of elasticity is a bit difficult technically as elasticities depend on the value of the parameter «h» of the demand function that represents the weight given to the price: the higher h, the higher the elasticities, everything else being equal. The test has been to increase h by 15%; the results are shown in the following table:

Table 8: Effect of differences in elasticities in the case of duopoly.

Link	h	p1	p2	Optimal IC
F	0,042	52,99	73,57	7,32
F	0,047	50,70	72,80	7,71

The optimal IC is sensitive to the elasticities: the higher the elasticities, the higher the IC. This point is understandable: when elasticities are high, the market power of the operators is lower and the IC can be increased without reducing too much the consumers' surplus.
From these results two conclusions can be drawn:

-   First, it is important to have a good knowledge of elasticities, since the optimal tariff is highly varying with them. Unfortunately these elasticities are known with a large uncertainty, and efforts should be made to improve our knowledge in this field.
-   Second, it may be wise to differentiate the infrastructure tariffs according to the characteristics of the demand.

5.2.3. Differentiation according to the infrastructure costs

Table 9 shows the effect of differences in infrastructure costs. It relates to situations where the same link bears several traffics, for instance freight traffic and passenger traffic, or passenger trains with different number of carriages or different types of carriages (for instance double and simple deck), which damages to the track are different. Wrong prices signals come from an abusive assimilation of different ICs, but the impact seems to be rather minor when compared with the impact of other elements such as cost of public funds or elasticity level.

Table 9: Effects of changes in infrastructure costs.

	Infrastructure Costs operator 1	s Parameters		Prices		Optimal IC	Traffic mode1	Traffic mode 2
		s1	s2	p1	p2			
A	2,06	0,29		40,25		10,18	0,18	0,29
A	1,03	0,29		39,11		9,03	0,19	0,29
D	3,44	0,42	0,18	42,44	61,18	9,14	0,30	0,30
D	1,72	0,42	0,18	41,13	61,14	7,59	0,31	0,30

It appears in our simulations that an increase in infrastructure costs leads to an increase in OIC of the same order of magnitude.

5.2.4. Averaging of IC between links

Table 10 shows that averaging the optimal ICs over two or three links does not induce a large loss in welfare if the differentiated ICs are not too far. But if they are far from each other, the loss may be important and the effect can be to exclude profitable services from the market. This point is a caveat for the temptation to use a unique IC over a too large set of links whenever the characteristics are different in terms of both costs and demand.

Table 10: Effect of IC averaging (in this table, the first group of rows relates to fully differentiated tariffs; the second group of rows relates to a uniform tariff per km; the additional welfare lines show simply the sum of welfare values for the 3 market cases).

	s1	s2	p1	p2	Optimal IC	Length of the link in km	Q1	Q2	Welfare
A	1,00	-	51,68	-	-10,11	200	0,14	-	27,82
E	1,00	1,00	87,41	128,80	13,45	900	0,36	0,35	-15,76
F	1,00	1,00	60,27	101,83	8,33	700	0,31	0,33	-6,83
									5,23
A	1,00	-	62,92	-	2,56	200	0,11	-	27,49
E	1,00	1,00	86,14	128,56	11,52	900	0,37	0,35	-15,78
F	1,00	1,00	60,71	101,90	8,96	700	0,31	0,34	-6,84
									4,88

Figure 2 shows that the positive (but often rather low) effect on welfare of diverse infrastructure pricing strategies may imply important effects because of the distributive effects between the agents: infrastructure manager, operators, and consumers.
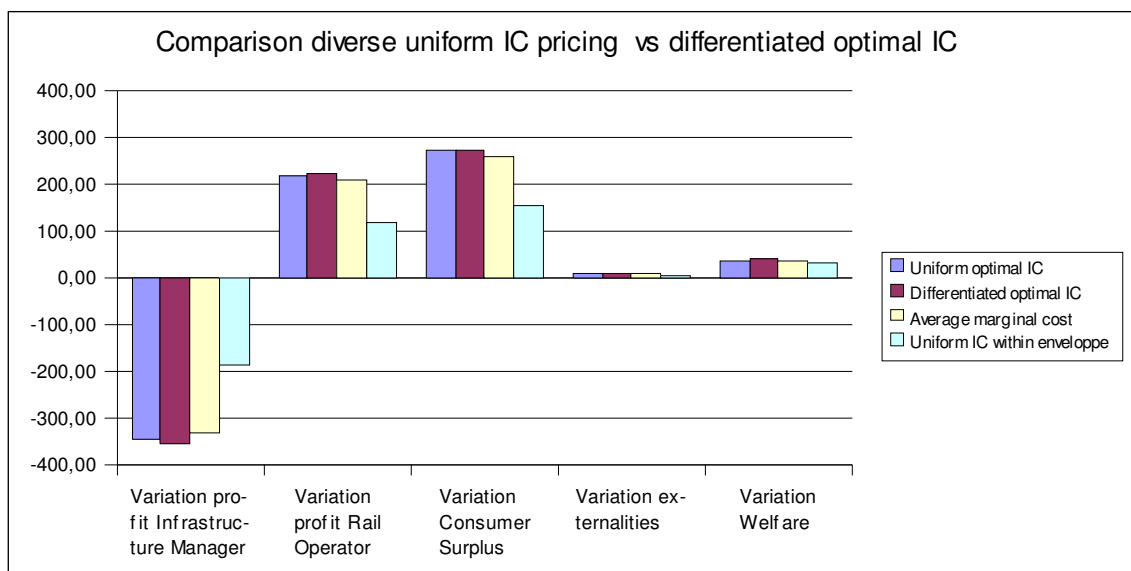

Figure 2: relative differences of diverse alternative IC pricing strategies as compared to actual (different) infrastructure charges on a set of 3 markets (A, B, C). Variation values are added over the 3 markets.

The 4 infrastructure pricing strategies presented in figure 2 are:

- uniform optimal IC: optimal IC level under the constraint that all markets A,B, C have to pay the same IC
- differentiated optimal IC: the uniformity constraint is suppressed, A, B and C pay different ICs
- average marginal cost: all markets A,B, C pay the same IC, equal to average marginal cost over the 3 markets
- uniform IC within envelope of reference ICs (RICs): this is the optimal IC within the interval [Min(RIC(A),RIC(B),RIC(C); Max(RIC(A),RIC(B),RIC(C)].

Simulations made for sub-markets that were all over-charged as compared to optimal levels, as well as for sub-markets that were all under-charged, showed the same results: low impact of differentiation on welfare, but possibly high impact on revenue distribution. Still, the impact of IC differentiation on welfare may become more important when the envelope of actual (reference) ICs does intersect the envelope of optimal ICs. This can be the case when demand or supply parameters are broadly dispersed; for instance, when both freight and passenger markets are considered.

5.2.5. Does it make sense to differentiate the IC of two competing rail operators?

The situation here is a duopoly on rail: both operators run rail services, and they are competing in a Bertrand mode. Their market shares and quality characteristics are different. Is it good to differentiate their tariffs? The evidence obtained from our data set is that, generally, differentiation between rail operators induces a very small extra welfare, as shown in the following table 11, where the last line displays the values

obtained with uniform optimal IC and the line before displays the results for differentiated optimal ICs :

Table 11: Effect of tariff differentiation in a situation of competing rail operators.

CPF IM	CPF operator	s1	s2	p1	p2	Optimal IC operator 1	Optimal IC operator 2	Q1	Q2	W
1,5	1,0	0,35	0,12	66,7	55,7	30,2	33,8	0,4	0,4	-10,8
1,5	1,0	0,35	0,12	68,2	54,2	32,1	32,1	0,4	0,4	-10,9

But in some cases when one of the operators does not bring much welfare (either because of its bad quality of service or of its cost inefficiency), a differentiated IC allows to exclude it from the market, while a uniform IC allows this inefficient operator to remain in the market, at the price of a loss of welfare. Still, even though being somewhat inefficient, an operator may play a strategic role for keeping an incentive for the main operator to behave reasonably.

5.3. Impacts of changes in the market structure and in the operators' behaviour

Imperfect competition is often changing. New entrants can appear then disappear, and the market structure then comes from monopoly to duopoly and vice-versa. Do these changes have an important effect on IC?

First, let us consider a possible misunderstanding of the market structure: while the true market structure is duopoly with air, the IM does not take this point into account and assumes that the market structure is a monopoly. It estimates the market structure (i.e. the parameter s1 and the demand function). In that case large mistakes result from ignoring the competition, as shown in the following table that explores the consequences of such mistakes.

Table 12: Effect of changes in market structure: monopoly versus duopoly.

	Market Structure	s Parameters		Prices at optimal IC level		Comment	Welfare
		s1	*s2*	*p1*	IC		
D	Monopoly	1,0		50	-0,3	Optimal IC	-11
D	Duopoly	0,4	0,2	45,5	12,8	Optimal IC	-3,7
D	Duopoly	0,4	0,2	34,6	-0,3	IC fixed at the level of Optimal IC if the market structure were monopoly	-5,0

Indeed, taking into account the regulating effect of the competitor increases optimal IC levels, therefore reducing the optimal charging policy's negative impact on IM revenues. It is interesting to assess the loss of welfare incurred by charging as if the market structure were a monopoly whereas, actually, it is a duopoly. In the case of table 12, this loss of welfare would be equal to 1,3, corresponding to a 5% increase in the marginal cost for rail.

Another example of the effect of a change in the market structure is expressed through changes in the value of the parameter s. As shown in the table 13, the behaviour of the rail's competitor (the value of s2) does not impact too much the optimal solution, while

large changes from the initial value s1 lead to important differences between the calculated IC and the optimal one.

Table 13 Impact of the values of the behaviour parameters s1 and s2

Link	s Parameters		Prices		Optimal IC
	s1	s2	p1	p2	
C	0,52	1,00	50,3	97,7	4,3
	1,00	1,00	59,2	97,7	-12,6
	0,00	1,00	39,0	97,7	23,0
F	0,84	0,17	53,0	73,6	7,3
	0,84	1,00	69,6	354,7	10,3
	1,00	1,00	65,9	100,8	11,6
	0,00	1,00	50,5	97,7	34,3

In our simulations, we observed that the parameter s allowing for a competitor's pricing behaviour is frequently strictly less than unity, and takes values often very low, for instance between 0 and 0,5. The point is not surprising as far as the historical rail operator is concerned; it had clear welfare goals not so long ago, and may still be impregnated with such objectives. It is more surprising for the air competitors, as they are clearly in the private sphere, and their aims should be purely profits. This result could indicate that the behaviours of the operators are much inspired by welfare concerns or, more realistically, that strategic considerations (may they come from demand considerations or competition concerns) do lead them to exert less market power than what would be expected at first sight. The price regulation exerted by the State, although acknowledged as mild, may also contribute to lower this parameter. But this situation may change in the future, and not acknowledging these changes would lead to large mistakes.

6. Conclusions

This text explores the consequences of IC differentiation in a situation of imperfect competition, in the framework of a partial equilibrium model. In such situations of imperfect competition, the short run marginal social cost doctrine should be adapted. It appears that the optimal IC depends highly on the market structure and on the cost of public funds, and that in the simulations performed it rejoins the marginal cost only for costs of public funds of roughly 0,5 (λ values of 1,5), a rather high value compared to the current estimates.

Simulations of optimal tariffs have been made for various situations:

-   market situations: monopoly, duopoly with another operator running a substitute service on another mode (air transport), duopoly with another rail operator.
-   operator's behaviours: profit maximizing, welfare maximizing or intermediate behaviour
-   various costs of public funds.

Various possible IC differentiation situations have been explored as well as the consequences of the reverse procedure: averaging over several services that differ only in one of these characteristics. The estimated impacts have been observed as regards optimal tariffs, prices of the operators, traffics and welfare.

From this simulation exercise, several conclusions can be drawn:

- Market structure has an important impact on the optimal IC; so the ICs of two services similar in everything except the market in which they are run should differ. Generally speaking, IC levels for monopoly should be lower than for a duopoly. In many cases marginal cost pricing leads to non negligible welfare losses, and in any case it provides very different welfare distribution than the optimal pricing
- The possibility for IC differentiation between two sub-markets can stem from differences in costs or demand or market structure (or a combination of these features) of these services
- In any case, IC differentiation brings small welfare changes when the two - or more- sub-market situations are close to each other; in such cases, the tiny improvement in welfare may lead to huge consequences on the distribution of welfare between the agents: the operators, the infrastructure manager and the consumers
- As far as costs are concerned, differentiation between two operators whose operating costs are different seems to bring minor welfare gains, which could possibly be more important when these operators are monopolies than when they bear competition with another mode. In the case of two links having different infrastructure costs - or two operators whose damages to the track are different- differentiation may have observable welfare consequences and is to be recommended, especially when the operator is a monopoly; in the case of a duopoly, the market power of the operator is limited by the operator of the other mode and differentiation, though desirable, may be less important
- In some cases, averaging the tariffs of several services may have important effects if these services have very different characteristics of costs and demand; in particular, it may happen that the average tariff excludes some profitable services, ending up in a large loss of welfare
- In case of a duopoly on track – meaning that two rail operators compete on the same track - welfare does not seem to be highly sensitive to an averaging of tariffs
- Last but not least, making non differentiated IC come closer to optimal IC levels could be much more worth than trying to differentiate finely around the initial IC levels, if those levels are far from the optimal ones.

Besides data requirements, the research field of imperfect competition in rail markets seems to be quite important if we want to explore more these important issues and to have a better understanding of what the final indirect impacts of infrastructure charging are, once interactions between competitors and demand converge to equilibrium. Trying to open and explore this "black box" of interactions is highly desirable, since the very basic usual representations such as perfect competition assumptions are clearly far from being fulfilled.

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Appendices

Appendix 1: Modelling operators' behaviour

The purpose of this annex is to show that various competition situations and behaviour of the operators can be expressed by a formula akin to the Ramsey formula:

$$\frac{p-c}{p} = -\frac{s}{\varepsilon}$$

Where:
p is the price of the operator
c is its marginal cost (assumed to be constant)
ε is the own price-elasticity of the operator
s is a parameter taking values between 0 and 1.

Profit maximiser monopoly

In that case, the previous formula holds with the value 1 for the parameter s; it is the classical formula giving the price of a monopoly.

Price-taker monopoly (or perfect competition)

In that case, the operator sets its price equal to its marginal cost, and the parameter s is 0.

Public monopoly subject to budget constraint

Then the price is defined by the previous relation which boils down to the Ramsey formula, with $s = \dfrac{\lambda - 1}{\lambda}$ using our conventions for the CPF parameter. In that case, the parameter s depends on the tightness of the budget constraint: 0 if the budget constraint is not binding, 1 if the budget constraint is very high (so high that the operator has to behave like a monopoly to meet this constraint).

Mixed behaviour monopoly

Historical operators were very much like public firms aiming at welfare maximizing. Their behaviour is changing more or less quickly. It may happen that their behaviour is not yet profit maximising; another interpretation may be that the operator is subject to a more or less tight regulation. In that case, it is sensible to assume that their objective function is a combination of profit and welfare, this combination being characterized by a parameter s such that:

OF(p) = s*q*(p - c) + (1 - s)*[SU(p)+q*(p - c)]

Maximizing this objective function with respect to p leads to the same result as in the previous cases:

$$\frac{p - c}{p} = -\frac{s}{\varepsilon}$$

Mixed behaviour, operators acting in a Bertrand duopoly market

If the two operators provide (partially) substitute goods, each of them maximizes its objective function with respect to its price, which leads to the following relations:

$$\underset{p_i}{Max}[OF_i(p_1, p_2)]$$

for i=1,2

where:

$$OF_i(p_1, p_2) = s_i * q_i * (p_i - c_i) + (1 - s_i) * \left[SU(p_1, p_2) + q_i * (p_i - c_i)\right]$$

This leads to the twin relations:

$$\frac{p_i - c_i}{p_i} = -\frac{s_i}{\varepsilon_{ii}}$$

The numerical simulations used in the text are based on this type of Bertrand competition: the operators use the prices as an optimisation tool.

Appendix 2: Detail of the simulation process

The simulation process has two phases: first, calibration of the parameters of the model; second, optimisation of the IC of the IM. This process is presented in the case of a rail operator in competition with an air operator, the market structure being a Cournot duopoly; the extension to other market structures is straightforward.

First step: behaviour calibration

The first phase starts from sensible values of current observable variables. They are, in the chosen case:
The rail traffic: *qm*
The infrastructure cost: *bi* and charge: *ti*
The cost of the rail operator: *cr* (including the infrastructure charge *pi*) ; its price *pr*
The cost, price and traffic of the air operator: *cm, p*m, and *qm*
The four demand own and cross elasticities.

If the demand function is linear, the parameters to be estimated are the 5 parameters a, b, c, kr, km such that:

Qr=a*pr+c*pm+kr
Qm=c*pr+b*pm+km

The behaviour parameters of the operators: *s1* and *s2*
s1 is for instance such that the rail operator's behaviour is to maximize:

$$s1*qr*(pr-cr)+(1-s1)*\left[\int pr\,dqr-cr*qr\right]$$

The calibration phase aims at giving good estimates of the demand function's parameters and of the behavioural parameters (here: the seven parameters *a, b, c, kr, km, s1*, s*2*) that reproduce the four elasticities, the two prices, and the two traffics.

This is obtained by minimizing the sum of the squares of the relative differences between, on the one hand, a set of the calculated values that conform perfectly to the model used and, on the other hand, the observed values of each parameter.

The procedure is the following one: let X*i be the data observed and collected, and Yj the demand function and behavioural parameters. To each set of value of Yj corresponds a set of calculated values of Xi: *Xi(Y1, Y2, …Yj)*. The optimal set of Yj minimizes the sum:

Σi [(X*i-Xi(Y1,Y2, ..Yj))²/ X*i ²]

Second step: Optimisation

Once the parameters of the demand function and of the operator's behaviour are estimated, they are used in a maximisation process which aims at maximizing the total welfare. We assume that the infrastructure charge has to be determined by the IM in order to maximize welfare, with possibly a cost of public funds (or a budget constraint).

The second step consists in finding the infrastructure charge that maximizes the welfare:

$SU+\lambda*PRim+\lambda'*PRr+\lambda''*PRm-e_r*Qr-e_m*Qm$

Where: *SU* is the consumer's surplus, λ, λ' and λ'' are costs of public funds (or dual variables of budget constraints), e_r and e_m are environmental costs.

Appendix 3: Derivation of optimal infrastructure charges for linear demand - monopoly and duopoly

We will present here the derivations that give the expressions for optimal infrastructure charges, in the simple case where the value for parameter *s* is 1 (pure profit maximisation).

1. Monopoly

The first case will be a profit maximizer monopoly. Let us derive the algebraic formula for the optimal IC, using the following symbols and assumptions:

- The demand function of the down-stream market is :
  $$Q = f(p) = \alpha p + \beta \quad , \quad \alpha < 0 \quad and \quad \beta > 0$$
  where *Q* is the traffic and *p* is the price paid by the users to the rail operator (RO)
- The operating cost of the RO is assumed to be constant and equal to : *c'*
- The operating cost of the infrastructure manager (IM) is assumed to be constant per unit of traffic and equal to: *b*
- The IM sells the paths to the RO at a price *t* per unit of traffic
- Then the cost per unit of traffic for the RO is constant and equal to: $c = c' + t$
- The RO generates an external cost of *e* per unit of traffic
- The Cost of Public Funds (or shadow variables of possible budget constraints) are *(λ-1)* for the IM and *(λ'-1)* for the RO

It is easy to show that the RO, aiming at maximising its profit: *Q(p-c'-t)=Q(p-c)*, chooses the price p such as:

$$p = (c/2) - (\beta/2\alpha) = \frac{\alpha c - \beta}{2\alpha}$$

ie

$$p(t) = \frac{t}{2} + \frac{\alpha c' - \beta}{2\alpha} \qquad \text{(E0)}$$

$$\frac{\partial p}{\partial t} = \frac{1}{2} \quad ; \quad \frac{\partial Q}{\partial t} = \frac{\alpha}{2} \qquad \text{(E1)}$$

$$Q(t) = \frac{\alpha t}{2} + \frac{\alpha c' + \beta}{2} \qquad \text{(E2)}$$

$$Q(t) = -\alpha(p(t) - (c' + t)) \qquad \text{(E3)}$$

The welfare is:

$$W(p) = SU(p) + \lambda PRim(p) + \lambda' PRro(p) - eQ(p)$$

where *PRim* and *PRro* are the profits of the IM and the RO.
The optimal *IC* is the value of *t* which maximises W.

Noting that $\partial SU / \partial p = -Q$, using (E1) and replacing the other terms by their expressions, it turns out that since:

$$W(t) = SU(p(t)) + \lambda \big[(t-b)Q(t) \big] + \lambda' \big[(p - (c'+t))Q(t) \big] - eQ(t)$$

$$\frac{\partial W}{\partial t} = \frac{-Q}{2} + \lambda \left[(t-b)\frac{\alpha}{2} + Q \right] + \lambda' \left[(p - (c'+t))\frac{\alpha}{2} + (\frac{1}{2} - 1)Q \right] - e\frac{\alpha}{2}$$

and using (E3):

$$\frac{\partial W}{\partial t} = \frac{-Q}{2} + \lambda \left[(t-b)\frac{\alpha}{2} + Q \right] + \lambda' \left[(-\frac{Q}{\alpha})\frac{\alpha}{2} - \frac{1}{2}Q \right] - e\frac{\alpha}{2}$$

$$\frac{\partial W}{\partial t} = Q \left[-\frac{1}{2} + \lambda - \lambda' \right] + \lambda \left[(t-b)\frac{\alpha}{2} \right] - e\frac{\alpha}{2}$$

But, if Q_b is the traffic in the case of an infrastructure charge equal to the marginal infrastructure cost, (E2) gives :

$$Q(t) = \frac{\alpha t}{2} + \frac{\alpha c' + \beta}{2\alpha} = \frac{\alpha(t-b)}{2} + (\frac{\alpha b}{2} + \frac{\alpha c' + \beta}{2\alpha})$$

$$Q(t) = \frac{\alpha(t-b)}{2} + Q_b$$

Therefore :

$$\frac{\partial W}{\partial t} = \left[(t-b)\frac{\alpha}{2}\right]\left[(-\frac{1}{2}+\lambda-\lambda')+\lambda\right] + Q_b(-\frac{1}{2}+\lambda-\lambda') - e\frac{\alpha}{2}$$

$$\frac{\partial W}{\partial t} = [(t-b)]\left[\frac{\alpha}{2}(2\lambda-\lambda'-\frac{1}{2})\right] + Q_b(-\frac{1}{2}+\lambda-\lambda') - e\frac{\alpha}{2}$$

since α<0, *W* is concave in *t* as long as :

$$\left[2\lambda-\lambda'-\frac{1}{2}\right] > 0$$

and *W* gets its maximum for IC value *t* such as:

$$t-b = \frac{\dfrac{Q_b}{\alpha}(1-2\lambda+2\lambda')+e}{2\lambda-\lambda'-\dfrac{1}{2}}$$

2. Duopoly

Let us present also the case of a duopoly, representing competition between air and rail. The demand functions are:

$$Q_r = \alpha p_r + \gamma p_m + q_r \qquad \text{for rail traffic}$$

$$Q_m = \gamma p_r + \beta p_m + q_m \qquad \text{for air traffic}$$

$$\alpha < 0 \quad \beta < 0 \quad \gamma > 0$$

The profit of the RO is (through profit maximisation):

$$PR_r = Q_r(p_r - c_r) = -\alpha(p_r - c_r)^2 = -\frac{1}{\alpha}Q_r{}^2$$

with a similar relation for the competitor *m*, and the equivalent formulations:

$$Q_r = -\alpha(p_r - c_r) \quad and \quad Q_m = -\beta(p_m - c_m) \qquad \text{(E4)}$$

These expressions also give us:

$$\frac{\partial Q_r}{\partial c_r} = -\alpha(\frac{\partial p_r}{\partial c_r}-1) \quad and \quad \frac{\partial Q_m}{\partial c_r} = -\beta\frac{\partial p_m}{\partial c_r} \qquad \text{(E5)}$$

We assume for simplicity that both operators are purely private: $\lambda'=\lambda''=1$
and that rail externalities are negligible when compared to air externalities: $e_r \approx 0$.

Welfare maximisation then leads to:

$$0=\frac{\partial W}{\partial c_r}=-Q_r\frac{\partial p_r}{\partial c_r}-Q_m\frac{\partial p_m}{\partial c_r}+\lambda\left[(t-b)\frac{\partial Q_r}{\partial c_r}+Q_r\right]+\left[(p_r-c_r)\frac{\partial Q_r}{\partial c_r}+(\frac{\partial p_r}{\partial c_r}-1)Q_r\right]+\left[(p_m-c_m)\frac{\partial Q_m}{\partial c_r}+\frac{\partial p_m}{\partial c_r}Q_m\right]-e_m\frac{\partial Q_m}{\partial c_r}$$

and, using (E4) and (E5) so as to simplify the two following expressions:

$$-Q_r\frac{\partial p_r}{\partial c_r}+\left[(p_r-c_r)\frac{\partial Q_r}{\partial c_r}\right]=\alpha(p_r-c_r)(\frac{\partial p_r}{\partial c_r})+(p_r-c_r)(-\alpha(\frac{\partial p_r}{\partial c_r}-1))=\alpha(p_r-c_r)=-Q_r$$

$$-Q_m\frac{\partial p_m}{\partial c_r}+\left[(p_m-c_m)\frac{\partial Q_m}{\partial c_r}\right]=\beta(p_m-c_m)(\frac{\partial p_m}{\partial c_r})+(p_m-c_m)(-\beta(\frac{\partial p_m}{\partial c_r}))=0$$

Thus, we obtain for welfare maximisation:

$$0=Q_r(\frac{\partial p_r}{\partial c_r}-2+\lambda)+Q_m\frac{\partial p_m}{\partial c_r}+\lambda\left[(t-b)\frac{\partial Q_r}{\partial c_r}\right]-e_m\frac{\partial Q_m}{\partial c_r}$$

Introducing the quantity values that would be obtained if the infrastructure charge was set equal to b (from now on, the subscript b will be used for the value of the variable that is obtained for IC $=b$):

$$Q_r=\frac{\partial Q_r}{\partial c_r}(t-b)+Q^b{}_r$$

$$Q_m=\frac{\partial Q_m}{\partial c_r}(t-b)+Q^b{}_m$$

where the partial derivatives are constant, in the linear model.

We then obtain:

$$Q^b{}_r(\frac{\partial p_r}{\partial c_r}-2+\lambda)+Q^b{}_m\frac{\partial p_m}{\partial c_r}-e_m\frac{\partial Q_m}{\partial c_r}=-\frac{\partial Q_r}{\partial c_r}(t-b)(\frac{\partial p_r}{\partial c_r}-2+\lambda)-\frac{\partial Q_m}{\partial c_r}(t-b)\frac{\partial p_m}{\partial c_r}-\lambda\left[(t-b)\frac{\partial Q_r}{\partial c_r}\right]$$

or:

$$t-b=-\frac{Q^b{}_r(\frac{\partial p_r}{\partial c_r}-2+\lambda)+Q^b{}_m\frac{\partial p_m}{\partial c_r}-e_m\frac{\partial Q_m}{\partial c_r}}{\frac{\partial Q_r}{\partial c_r}(\frac{\partial p_r}{\partial c_r}-2+2\lambda)+\frac{\partial Q_m}{\partial c_r}\frac{\partial p_m}{\partial c_r}}$$

In the end, using again (E5):

$$t-b=\frac{Q^r{}_b(\frac{\partial p_r}{\partial c_r}+\lambda-2)+Q^m{}_b\frac{\partial p_m}{\partial c_r}+\beta e_m\frac{\partial p_m}{\partial c_r}}{\alpha\left[(\frac{\partial p_r}{\partial c_r}-1)(\frac{\partial p_r}{\partial c_r}+2(\lambda-1))\right]+\beta(\frac{\partial p_m}{\partial c_r})^2}$$

We can go further if we solve the two simple linear equations obtained by mixing (E5) with linear demand formulations, so as to obtain the exact expressions for prices, quantities, that are linear functions of c_r:

$$2\alpha\frac{\partial p_r}{\partial c_r}+\gamma\frac{\partial p_m}{\partial c_r}=\alpha c_r$$

$$\gamma\frac{\partial p_r}{\partial c_r}+2\beta\frac{\partial p_m}{\partial c_r}=0$$

Using the notation: $\Delta\equiv\gamma^2-4\alpha\beta$, the full set of prices and quantities obtained is the following:

$$p_r=p_r^b-\frac{2\alpha\beta(c_r-(c'+b))}{\Delta}\quad and\quad\frac{\partial p_r}{\partial c_r}=\frac{-2\alpha\beta}{\Delta}$$

$$p_r=p_r^b+\frac{\alpha\gamma(c_r-(c'+b))}{\Delta}\quad and\quad\frac{\partial p_m}{\partial c_r}=\frac{\alpha\gamma}{\Delta}$$

$$Q_r=Q^b{}_r-\alpha\frac{(2\alpha\beta-\gamma^2)(c_r-(c'+b))}{\Delta}\quad and\quad\frac{\partial Q_r}{\partial c_r}=-\alpha\frac{2\alpha\beta-\gamma^2}{\Delta}$$

$$Q_m=-\frac{\alpha\beta\gamma(c_r-(c'+b))}{\Delta}\quad and\quad\frac{\partial Q_m}{\partial c_r}=-\frac{\alpha\beta\gamma}{\Delta}$$

A fully explicit expression for optimal IC is then:

$$t-b=\frac{\Delta}{\alpha}\frac{Q^b{}_r(2\alpha\beta(3-2\lambda)+(\lambda-2)\gamma^2)+Q^b{}_m\alpha\gamma+\alpha\beta\varkappa_m}{2(\alpha\beta(3-4\lambda)+(\lambda-1)\gamma^2)(2\alpha\beta-\gamma^2)+\alpha\beta\gamma^2}$$