Numerical investigation of flow to pumped wells in heterogeneous layered aquifers using radial flow models.

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ABSTRACT

Pumping tests are conducted to examine the performance characteristics of a well and the aquifer flow mechanisms. Numerical models that are able to incorporate all flow mechanisms are important for this purpose. Because of the convergent nature of groundwater flow approaching a pumped well, a three-dimensional R-θ-Z numerical model that simulates the groundwater flow behavior in heterogeneous aquifers is developed using the object-oriented programming approach. This programming method greatly simplifies the development of the model.

The model is applied in two dimensions to investigate the effect of changing the abstraction rate on the time-drawdown response in a heterogeneous aquifer. It is applied in three dimensions to investigate the effects of horizontal and vertical high permeability zones on groundwater flow. Under these circumstances, analytical approaches fail to produce a solution and the use of numerical models is crucial to understanding the groundwater flow behaviour.

Finally, the model is used to interpret the results of a pumping test conducted by Thames Water Utilities Limited (TWUL) in a UK Chalk aquifer. The aim is to investigate the flow processes and to study the impact of a nearby quarry on groundwater movement. Pumping test results show that the influence of the quarry is negligible despite its size and proximity to the pumped wells.

INTRODUCTION

Pumping tests are controlled field experiments that aim to determine the physical characteristics of an aquifer. They can also be used to validate a conceptual model based on the geological setting of the aquifer. The variation in aquifer properties from one layer to another or from one location to another is common in nature. In sedimentary rocks for example, vertical variations in properties may occur if the origin of the deposited material, or the post-depositional environment changes. Such changes may create confining materials or conversely high permeability layers within less permeable strata. In addition, well-compacted aquifers may develop special features such as faults or fractures when subject to different stresses. These features may provide an easy path for water movement and they may have the ability to store a large quantity of water by accretion. The interpretation of the results of pumping tests conducted in such aquifers needs tools that are capable of accounting for these special features.

Many analytical and numerical solutions have been developed for the interpretation of pumping test results. A numerical model has advantages over the analytical approach by allowing the inclusion of special features such as irregular boundaries, partially penetrating wells, changing aquifer properties, recharge, rivers, etc. Solutions that integrate both analytical and numerical techniques have also been developed to represent some of these features, for example Hemker (1999).

This paper describes the use of a layered R-θ, object oriented numerical model to investigate the effect of aquifer heterogeneity, caused by the occurrence of fractures. Fractures in both vertical and horizontal directions are discussed. Finally, the numerical model is used to interpret the results of a pumping test conducted by TWUL in a Chalk aquifer. The unconfined conditions in the aquifer together with the circumstances under which the test is conducted make it impossible to interpret the field results with conventional solutions.

MODEL DEVELOPMENT

The model applies the finite difference method to solve the basic equation describing flow through a porous medium under confined and unconfined conditions. The axi-symmetric flow equation to a well in a confined aquifer, in terms of the drawdown s and using the transformation function $a = \ln r$ that changes the distance in the radial direction from an arithmetic to a logarithmic value, is given by: $k_r / r^2 \left(\partial^2 s / \partial r^2\right) + k_r / r^2 \left(\partial^2 s / \partial \theta^2\right) + k_z \left(\partial^2 s / \partial z^2\right) = -S_s \left(\partial s / \partial t\right) + N$. Where k_r, k_θ and k_z are the hydraulic conductivities in the radial, circumferential and vertical directions respectively. S_s and N are the elastic storage and an external source term respectively. The principle of the finite difference technique is to replace the continuous problem domain with a discretised domain by laying a mesh consisting of an array of nodes over the area under study. In the model developed here, the intervals between the nodes in the radial direction increase in a logarithmic pattern. The vertical dimension is represented using layers in which one numerical layer represents one physical layer. The vertical flow between the nodes of different layers is determined by considering mass conservation at the face that separates the layers. Finally, the circumferential dimension is included by evenly distributing a number of vertical sections extending from the well face to the outer boundary.

When unconfined aquifers are involved, the free surface boundary has to be taken into consideration. An equation that represents the movement of this boundary can be found in the work of Neuman (1972) and Todsen (1976). This equation has a non-linear form and must be solved using explicit techniques. Rushton (1979) reduces the complexity of this equation by considering that the multiplication of the relevant hydraulic gradients yields a very small number that can be ignored. The non-linearity of the equation disappears and the moving water table equation becomes: $\partial \varphi / \partial t = 1 / S_y \left(k_z \, \partial s / \partial z\right)$, which can be rewritten as $k_z \, \partial s / \partial z = S_y \, \partial \varphi / \partial t$ and then $q_z = S_y \, \partial \varphi / \partial t$. Where φ is a function that represents the location of the water table and S_y is the specific yield. The free surface is then taken into account by adding q_z to the basic flow equation. The free surface is included in the model as an object that is constructed when unconfined aquifers are considered. This object changes its location based on the head values at its constituent nodes at the end of each time step. The conductance values between the water table and the aquifer nodes are then recalculated.

THEORETICAL INVESTIGATIONS

This section deals with groundwater behaviour in a fractured aquifer. The source wave, which represents the location from where water is released at any given time, is used to explain the response of the aquifer to abstraction under a given set of circumstances. This source wave, also investigated numerically by Brown (1997) and analytically by Hermance (1998), can be represented analytically by integrating the Theis solution to the radial flow equation. Discretisation of the aquifer and the use of a logarithmic scale for the radial axis produce a source wave or a pulse that has constant size at all times in a confined homogeneous aquifer.

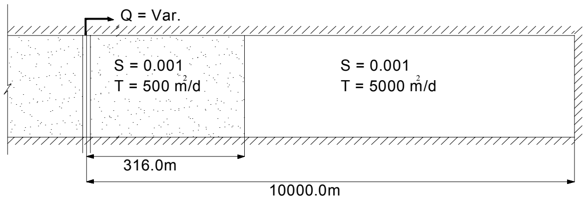

Figure 1. Aquifer with two transmissivity zones

To introduce this approach, the relationship between the location of this pulse and the shape of the time-drawdown curve is examined first. A confined aquifer composed of two different materials (Figure 1) is considered. The first material extends radially from the abstraction well to a distance of 316 m and has a transmissivity value of 500 m^2/d. The second material covers the remaining part of the aquifer and has a transmissivity of 5000 m^2/d. The well radius is set to a value equal to 0.001 m and the outer boundary is assumed to be impermeable and is 10 km away from the centre of the well. An abstraction rate of 1000 m^3/d is applied for one day and then increased to 4000 m^3/d for another 9 days. The resulting time-drawdown curve, shown by the solid line in Figure 2, can be split into five distinct parts. The first, stretching from time zero to 0.03 days, is characterised by a straight line that matches the Theis solution based on a transmissivity value of 500 m^2/d. The source wave at any time earlier than 0.03 days is completely within the first aquifer zone as shown by the pulse determined at time t = 0.00316 days and plotted in Figure 3. The second part falls between 0.03 days and 0.3 days and is characterised by a curve that reflects the fact that water is released from both aquifer zones between these times. This is shown by the deformed pulse plotted in Figure 3 and calculated at time t = 0.0316 days. Because both transmissivity values are involved, this part of the time-drawdown curve does not follow a straight line solution. The third part of the curve is a straight line that corresponds to a Theis solution with a transmissivity value of 5000 m^2/d. This can be explained by the fact that any source wave at a time later than 0.3 days falls completely within the second aquifer zone as shown by the pulse determined at time t = 1 day in Figure 3.

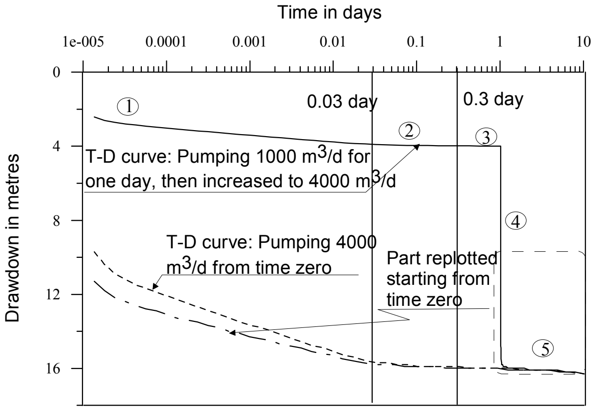

Figure 2. Time-drawdown curves

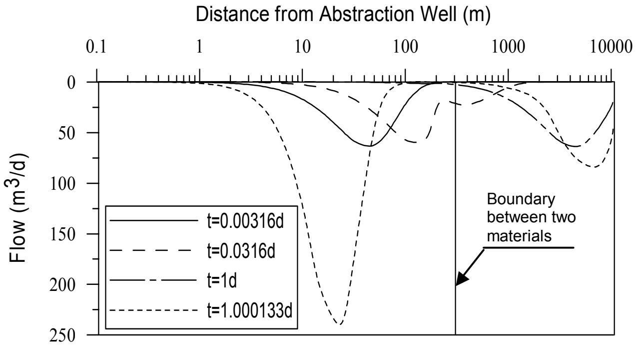

Figure 3. Source wave propagation

The fourth part of the curve corresponds to the start of the new abstraction rate. This part of the curve is re-plotted, taking the time at which the new rate begins as zero (Figure 2). It also contains three parts. These parts, however, do not match those of a curve resulting from a single pumping rate of 4000 m^3/d starting from time zero also shown in Figure 2. This is because, prior to the increase in abstraction rate, a source wave has already been developed by abstracting at 1000 m^3/d and this wave has reached the high permeability zone. This is illustrated by the presence of two pulses at time t = 1.000133 days in Figure 3. The increased abstraction rate leads to the development of an additional pulse in the low permeability zone and water is released from both zones concurrently. At later times, when the newly developed pulse reaches the high permeability zone and merges with the pulse already there, the time drawdown curves coincide and corresponds to a Theis-like solution with T = 5000 m^2/d. Part 5 of the curve arises under this condition. It can be concluded that, when a step-drawdown pumping test is conducted, care is needed when interpreting how the variation of aquifer properties influences the test results.

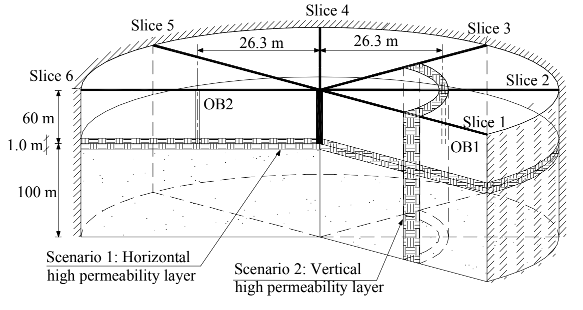

Figure 4. Three-dimensional aquifer

The presence of fractures in an anisotropic aquifer is investigated by considering a 160 m thick unconfined aquifer system illustrated in Figure 4. This system has horizontal and vertical hydraulic conductivities equal to 50 m/d and 5 m/d respectively and a specific storage and a specific yield equal to 0.00001 m^{-1} and 0.01 respectively. Two scenarios are considered. In Scenario 1 a thin, high permeability layer is assumed to stretch horizontally at a depth equal to 60 m below the initial position of the water table. This layer is 1 m thick and has horizontal and vertical hydraulic conductivities equal to 2000 m/d and 10 m/d respectively. In Scenario 2, a vertical high permeability zone is introduced at a distance equal to 26.3 m from the well in the form of an arc enclosing an angle of 45 degrees. The model is divided into eight segments in plan, each of which is bounded by a vertical section termed a slice. The high permeability zone therefore extends between slices 1 and 3. The well at the centre of the system is 1 m in diameter and is pumped at a rate 7000 m^3/d.

To examine the effects of the horizontal high permeability layer in Scenario 1, the results are compared to those produced by abstracting from an aquifer having the same configuration except for the high permeability layer. The rate of water released from all nodes forming the water table and the upper layer are plotted in Figure 5 at time t = 0.01 day. When the high permeability layer is included, the nodes releasing more than 500 m^3/d are located between 100 m and 1200 m away from the abstraction well but when the high permeability layer is removed, the nodes that release more than 500 m^3/d are located between 100 m and 900 m away from the well. The high permeability layer allows the nodes located further away from the abstraction well to contribute water at earlier times with the result that the distribution of storage release shown in Figure 5 changes markedly.

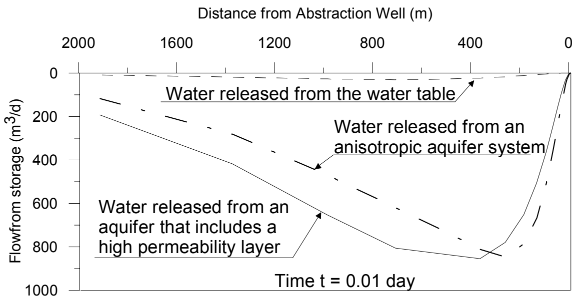

Figure 5. Source waves at time t = 0.01 d

In the second scenario, the vertical and the circumferential hydraulic conductivities of the high permeability zone are set to a large value of 1000 m/d, but the radial hydraulic conductivity remains unchanged. The variations with time of the groundwater head in the upper layer and at the water table are recorded at two observation wells OB1 and OB2. These wells are located at a distance equal to 26.3 m from the abstraction well but OB1 is in slice 2 and OB2 is in slice 6. The observations are therefore taken at the mid-point of the high permeability zone and at a location at the same radial distance but on the opposite side of the pumped well. The results are also compared to those produced from a similar aquifer system but one that excludes the high permeability zone.

The time-drawdown curves from both systems overlap at early times. However, as shown in Figure 6, lower drawdown values are recorded at later times when the high permeability zone is present. The drawdown values observed in the upper layer at OB1 in the high permeability case are much less than those observed at OB2. In addition, these values are very close to the water table drawdown at OB1. This behaviour is explained by the fact that when the source wave reaches the high permeability layer, at some time after pumping begins, flow is easily delivered from the water table to the abstraction well. The access to a large source of water, represented by the water table, explains the smaller drawdowns at later times when the vertical high permeability layer is included.

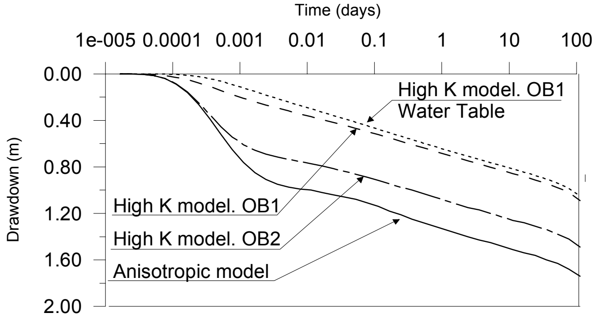

Figure 6. Time-drawdown curves. Scenario 2

FIELD INVESTIGATIONS

The numerical model is used to interpret the results of a pumping test conducted by TWUL in the Chalk aquifer in the southeast of the London Basin. The chalk is characterised in this area by fractures that have developed at a variety of depths and with several different orientations. In addition, quarrying activities have been carried out extensively in the area with one major operational quarry 600 m away from the pumped well.

The abstraction well was drilled at a large diameter (~ 4 m) to a depth of 32.8 m, and then a borehole, of 0.3 m diameter, was drilled from the base of the well to a depth of 62.3 m. An arched adit, 435 m long, was constructed at a depth of 30.5 m from the surface in an easterly direction. Fortunately, the adit does not de-water during the test and it therefore contributes to enhanced directional transmissivity rather than providing a large storage component. The pumping test consists of three abstraction phases with different rates and durations. During the first two abstraction phases there are short periods of recovery due to problems with the pump. These show as short term rises in the field data. They are in fact subject to detailed simulation in the model but the results are then combined with the appropriate phase.

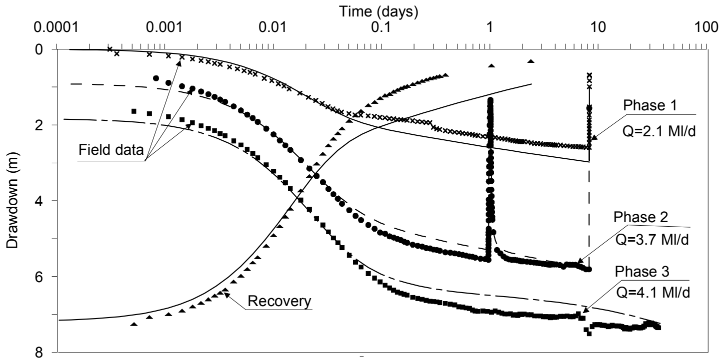

Figure 7. Pumping test results

The presence of fractures and the variation of the pumping rate have led to the theoretical investigations described in the previous section. However, a simplified conceptual model was able to produce numerical results that match the field data (Figure 7). This conceptual model consists of two layers; the first extends to the base of the well and the second represents the remainder of the Chalk. The upper layer has a transmissivity of 900 m^2/d, a vertical hydraulic conductivity of 0.5 m/d, a specific storage of $1x10^{-5}$ m^{-1} and a specific yield of 0.01. The lower layer has the same parameter values as the upper layer but with a transmissivity of 100 m^2/d. A large well diameter of 5 m, however, is needed to match the numerical and the field curves.

The possibility of simulating the groundwater processes in a fractured aquifer using a relatively simple conceptual model highlights two main points. First, the fractures in the area are not highly developed so they are not clearly reflected by the field data. Second, these fractures are still able to direct water from regions away from the quarry towards the pumped well and obscure the effects of the quarry. The acidisation process that is often used to develop new boreholes in the Chalk explains the need for introducing a larger well size. Finally, because of the small value of the vertical hydraulic conductivity, the inclusion of the adit as a high transmissivity conduit does not greatly affect the simulated results.

SUMMARY

When the abstraction rate is increased in an aquifer with variable properties a new source wave, equivalent to the increase in abstraction, develops in the area near the well and then propagates outwards and merges with the pulse already propagating. The time-drawdown curve in this case does not conform with the one resulting from the use of superposition theory which relies on an aquifer having homogeneous characteristics. In both horizontal and vertical directions, fractures help deliver water from the areas remote from the well and reduce the drawdown at the abstraction well. The vertical fractures studied here, however, do not necessarily represent a fault that may alter the structure of the aquifer and result in a low hydraulic conductivity. Finally, the field application shows that fractures can obscure the effects of major physical features.

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REFERENCES

Brown I.T., 1997. Numerical modelling of pumping tests in unconfined aquifers. Ph.D. Thesis, The University of Birmingham, England.
Hemker C.J., 1999. Transient well flow in vertically heterogeneous aquifers. J. Hydrology, 225, 1-18.
Hermance J.F., 1998. Mathematical primer on groundwater flow. US Imports and PHIPES.
Neuman, S.P., 1972. Theory of flow in unconfined aquifers considering delayed response of the water table. Water Resources Research, 8, 4, 1031-1045.
Rushton K.R., Redshaw S.C., 1979. Seepage and groundwater flow. John Wiley and Sons, Chichester.
Todsen M., 1971. On the solution of transient free-surface problems in porous media by finite-difference methods. J. Hydrology, 12, 177-210.