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ORIGINAL RESEARCH

Mathematical models applied to the prediction of doping in male athletes

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Abstract

The compartmental model is a mathematical model (usually described by a set of differential equations) that describes how individuals from different compartments (or groups) that represent a population, interacts. The model is suitable especially for epidemic model, modeling spread of disease but also in simulation of interaction among social groups. The compartmental model has few assumptions to be feasible: "the infection/contamination rate" can be a function of many parameters (seasonality, epidemic waves, dependence of social distancing, policy of awareness, policy, and so one). The main assumption is that the population is homogeneous but, in reality, the heterogeneity of population (spatial localization, seasonal, demography) plays an important role in accuracy of models. Our approach was based on another method that has been used in the last years, the inclusion of a temporal function including heterogeneity in the influence that conduct to doping similar to rate of infection from epidemic models. In this paper, a new model is proposed for quantitative analysis of doping in a particular selected sport. Almost all the models in doping use the biological markers and effect of doping in declared by athletes involved in use of banned substances in a quantitative analysis over a group of high-performance athletes. The proposed compartmental model SEDRS (Susceptible-Exposed-Doped-Recovered-Susceptible) includes the heterogeneity shaped by awareness, due to social interaction that transmit the anti-doping policy. These measures are patterned by social interaction, especially during competitions and training, and this approach is included in system of integrodifferential equations. A heterogeneous (SEDRS) model is numerically solved and the solutions show how the social factor can contribute to decay of doping phenomenon of male athletes and the quantifiable influence in a healthier atmosphere in sport. The scope of the paper is the prediction of doping cases based on SEDRS model.

Keywords

Anti-doping policy; Compartmental models; Probability distribution; Heterogenous models

1. Introduction

The Romanian Doping Control Laboratory (RCDL) is the only laboratory in Eastern Europe accredited by the World Anti-Doping Agency [1]. In 2000, with the Olympic Games in Sydney, Romanian sport received an important blow: four of the tricolour athletes participating in the Olympics in Australia were suspended for doping. More than 100 Romanian athletes failed the anti-doping test between 2019 and 2020. Most of them are handball players or bodybuilders, but none are professional tennis players. During this period, the most athletes who resorted to doping were found in handball and bodybuilding [2].

It is not only a national problem, but a global one, which has become a public health problem in recent years, affecting all sports disciplines. We can ask why athletes resort to such practices, what amount of a prohibited substance must be found in a person's body to be suspended from the activity, how an anti-doping test was falsified in the past, on what criteria are those tested and how act in situations where minors are involved. A moment of reference is represented by the "World Conference on Doping", Lausanne, 02–04 February 1999, when for the first time in the history of sports, the IOC (International Olympic Committee), the international sports federations, the National Olympic Committees, governmental and non-governmental non-governmental groups, and the media that through the final decision (Lausanne Declaration) gave a new turn to the anti-doping campaign [3].

Depending on what type of advantage is desired by male athletes, of course the substances used also differ. We will start by saying that the reason for using a doping substance differs greatly from a person who practices athletics sport as a hobby, to an athlete who practices performance sports. Most of the time, those who do not do use the doping to improve their athletics performance, dope to look better (usually, they try to increase muscle mass as much as possible and counterpart, they also try to decrease adipose layer as little much as possible). For performance athletes, things are usually different.

In the 1970s and 1980s, the East German government decided to dope its athletes with various banned substances, mainly steroids, believing that success in sports would demonstrate the superiority of communism [4]. The athletes had no choice and had to obey this direction, even if they could feel how their bodies were changing because of the steroids. Competitors began to suspect potential doping because East German athletes dominated many competitions too categorically. In total, more than 9000 athletes received banned performance-enhancing substances, and many of them were left with lifelong health problems.

The doping is present in all the sport as in Major League Baseball, where 47 players have been suspended for using banned substances (including steroids, HGH (Human Growth Hormone), testosterone and amphetamines) since 2005. The usage of performance-enhancing substances in sport by athletes (that is, doping), is a prohibited practice with global tendencies [5–9]. In order to be a performance-enhancing substances, three basic criteria are taken into account:

- The substance has the potential to increase the men athlete's performance;
- The substance is a potential risk for the mens athlete's health;
- The substance is a potential risk for the mens athlete's health.

The presence of forbidden substance in the body of an athlete is identifiable by biomarkers, working on a standardized analyzing procedure and by using standardized tests [10, 11].

In the basic compartmental models SIR (Susceptible-Infectious-Recovered), it is assumed that the men populations is homogeneous, meaning that the individuals are considered to be identical and to have random contacts. Generally speaking, epidemic models contain much heterogeneity such as: population heterogeneity, spatial heterogeneity, temporal heterogeneity, which can be reflected in model parameters [12, 13]. The epidemics on spatial distributions are the subject of the papers that deals mainly with various types of networks: Random Regular Networks (Berenbrink P, Elsässer R, and Friedetzky T, 2008), Erdős-Rényi (Erdős P and Rényi A, 1959) model, Scale Free networks, Small World Networks (Barabási A-L and Albert R, 1999), Barabási-Albert model (Barabási A-L and Albert R, 1999), and Watts-Strogatz model (Watts DJ and Strogatz S, 1998).

In [14], a general approach over Scale Free (SF) networks is presented and a fraction of connectivity applied for SIR epidemic using SF is described in [15]. Stochastic epidemics and rumor processes are described in [16] using few random network topologies: homogeneous networks [17], ER (Erdős-Rényi) random graphs [18], Barabási-Albert (BA) scale-free networks [19], and random geometric graphs [20].

A proposal for small world networks (SWN) by using mean field, pair approximation and probabilistic Markov chain. The strategies are applied to COVID-19-inspired SEIR (susceptible-exposed-infected-removed) epidemic process with quarantine and isolation strategies [21].

A SIR structure of disease in network is proposed by focusing on random partner in the population, rather than a random individual driven to a SIR disease in a random network with heterogeneous degree [22]. The mass action susceptible for SIR is constructed for homogeneous population, that is the infection rate is the same for the entire individual [22]. The dynamic rate of infection using edge-based compartmental modelling is proposed in [23].

An interesting approach, based on parametric Bayesian methods for heterogeneity learning alogorithm in SIRS is proposed in [24]. The Markov chain Monte Carlo algorithm (MCMC) is used in calculation of posterior distribution for a SIRS model using a spatial environment. Parameter estimation and clustering information are applied to a hierarchical form for transfer rates in prediction of COVID-19 [24].

The heterogeneous mean-field theory is suitable for epidemic networks. In [25], the authors proposed to apply heterogeneous mean-field to solve the epidemic threshold for uncorrelated networks (λ_c = <k>) where <k> is the average degrees of network. The <k> measure is involved in transmission rate of infection in a simple or complicated formula [26, 27].

The incidence rate of a disease (number of people infected per time unit) has a major role in the dynamics of epidemic models. Usually, this rate is assumed to be bilinear with respect of and, but depending on how many compartments we have the model, the form can include other functions.

$$g(S,I) = \beta SI \; (bilinear) \tag{1}$$

$$g\left(S,I\right) = \frac{\beta SI}{1+a_1 I} \; (saturated \; I) \tag{2}$$

$$g\left(S,I\right) = \frac{\beta SI}{1+a_1 S} \; (saturated \; S) \tag{3}$$

$$g\left(S,I\right) = \frac{\beta SI}{1+k_1 I+k_2 S} \; [28, 29] \tag{4}$$

$$g\left(S,I\right) = \frac{\beta SI}{\left[\left(1+k_1 I\right)\left(1+k_2 S\right)\right]} \; [30] \tag{5}$$

$$g\left(S,I\right) = \frac{k(S/I)^m}{\left[1+\alpha(I/S)^h\right]} \; [27] \tag{6}$$

A non-monotone incidence rates can be used to describe the effect of social environment along with the effect of increasing of awareness of negative effects of doping (in time):

$$g\left(S,I\right)=\ \beta e^{-ml}SI\ with\ m>0\ [31,32] \tag{7}$$

$$g\left(S,I\right)=\frac{\beta SI}{1+aS+bI^{2}}with\ a,b>0\ [33] \tag{8}$$

$$g\left(S,I\right)=\frac{\beta SI}{1+\omega_{1}I+\omega_{2}I^{2}}with\ \omega_{1},\ \omega_{2}>0\ [34] \tag{9}$$

In the next section a method of calculation for heterogenous rate of infection using Poisson, Gamma and negative binomial distribution inspired by [24, 35–39] is presented.

The scope of the proposed model is the prediction of doping cases based on SEDRS compartmental model in heterogenous environment. Also, by manipulating the function parameters *via* g(S, I) function, it is possible to construct an optimal strategy to reduces the number of doping cases in a population.

2. Materials and methods

In order to model the doping for a quantitative analysis, we define a compartmental model inspired by SIRS epidemic model, a SEDRS (S-Susceptible, E-Exposed, S-Doped, R-Recovery) model. The compartmental model for homogenous men population is presented in Fig. 1, where $g\left(S,\ D\right)=\beta\cdot D/N$ is the "infectious" rate, that is the doping rate, β is the rate of male athletes that occasionally tested a drug, γ is rate of male athletes that enter in a program of recovery from performance-enhancing substances consume, α is the rate of male athletes that fails in recovery program, ϕ is the rate of male athletes that completed the recovery and return to sport but they continue to take banned substances, Λ is recruitment rate for new male athletes, and μ is the rate of deceased male athlete's as a result of performance-enhancing substances consume abuse.

The system of ordinary differential equations corresponding to compartmental model from Fig. 1. is:

$$\begin{cases} \frac{dS}{dt}=\Lambda N-g\left(t\right)S+\phi R \\ \frac{dE}{dt}=g\left(t\right)S-\alpha E \\ \frac{dD}{dt}=\alpha E-\left(\mu+\gamma\right)D \\ \frac{dR}{dt}=\gamma D-\phi R \\ N=S+E+D+R \end{cases} \tag{10}$$

The compartment D correspond to I-Infectious compartment in epidemic SEIRS model. In some cases, there are taken into time dependence formula for g(t), useful to control the waves of epidemics [40] or heterogeneous environment [40–44].

The model can have new men's athletes or no flux of new athletes, modeled by parameter Λ. In this last case, the model will have $\Lambda = 0$, and the letter Λ can be omitted. In our case we selected the realistic case $\Lambda > 0$. The rate of transmission of "doping phenomenon" is in $\beta \times$ D/N, that is the most common approach inspired from rate of infection from deterministic compartmental models (SEDR in this case). The heterogeneity in transmission can be modeled by the number of contacts among individuals that conduct to influence the start to use performance-enhancing substances by individuals. A more realistic approach would be that the spread of doping among high-performance athletes is influenced by decisions of their coaches, trainers and doctors.

They are important factors that influence the athlete's behavior regarding doping. Direct interactions between athletes and their mutual influence in changing behavior towards doping should not be neglected either. This has also been enhanced promoted by the constant emergence of new performance-enhancing substances, that significantly increase performance and have not yet been included on the banned substances list, that are banned only during competitions or are still under study to clearly establish the level of influence they have on athletes. As an example, we can mention the use of anabolic steroids in the 1960's.

Thus, we quote Dr. H. Kay Dooley, a doctor of the US weightlifting team: "I don't think it's possible for a weightlifter to compete internationally without using anabolic steroids". Moreover, during the 1968 Olympic Games in Mexico City, male athletes and coaches did not debate the morality or appropriateness of doping; the only debate was on the efficacy of certain substances. The techniques used to improve performance and to circumvent its sound traces have been refined over time. Thus, after a lengthy investigation into the use of banned substances in Olympic sports, Bamberger and Yaeger concluded: "in several sports three distinct categories of top male athletes have emerged in the Olympics. The first is a small group of male athletes who do not use performance-enhancing substances" [29].

The first is a small group of male athletes who do not use performance-enhancing substances. The second is a large and growing group whose consumption of performance-enhancing substances remains undetected. These male athletes either take substances for which they are not being tested, use substances in amounts below the generous levels allowed by the International Olympic Committee, or use substances that, at the time of testing, mask the presence of performance-enhancing substances in their system at the time of testing.

The third group includes m athletes who use performance-enhancing substances to improve performance that are banned, and are actually caught. There now seems to be a consensus among different interest groups, including many athletes, doctors, coaches, administrators, organizers of sporting events, parents and spectators, who recognize that the use of these substances in most sports is a serious and growing problem. Given the wide variety that this parameter can take (the rate of transmission of the phenomenon of doping), the different types of actions prohibited by international conventions on doping in sport and the fact that data on the influence of each category of staff in the athlete's entourage has not yet been quantified, in the philosophy of this article we have gathered all these variables as social interaction.

The support personal of the men's athlete can be legally involved if they can be clearly honked to the performance-enhancing substances consumption case. This is another fact that favors the rate of transmission, resulting in an increase of the doping phenomenon is determined both by the number of

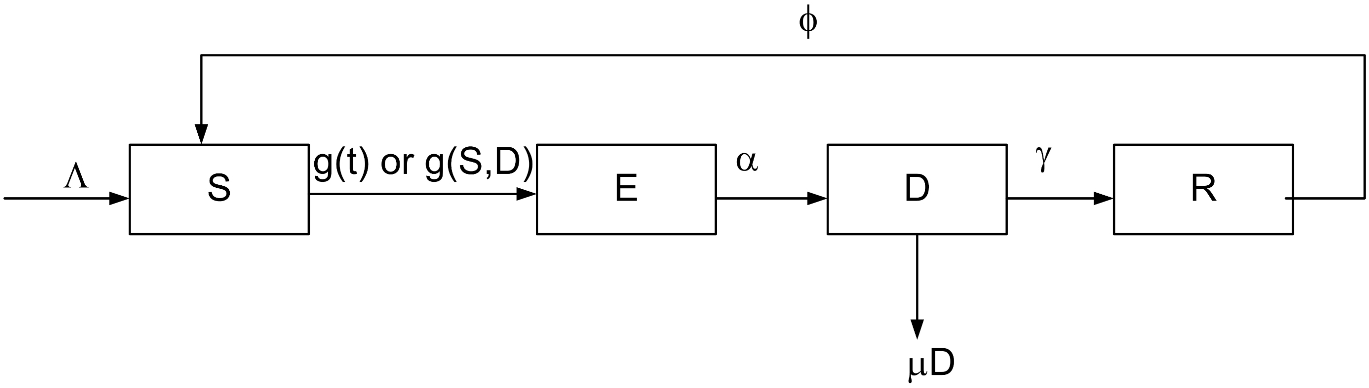

FIGURE 1. The proposed SEDRS model. S: Susceptible; E: Exposed; S: Doped; R: Recovery.

contacts between athletes and the number of support personnel that develop performance-enhancing substances consumption practices.

We follow the model of heterogeneity proposed in [36, 37] with respect of [24]. We consider the doping a form of incident ("accident" in [37]) the occurs for a number of people in S compartment.

The number of occurrences X_i can be modelled (like in other cases in real life's) by a Poisson distribution pmf (point mass function), and θ_i the number of effective contacts are depending of the i-th individual per unit time.

$$pmf\left(x,\ \theta\right)=\frac{\theta^{x}\cdot e^{\theta}}{x!} \tag{11}$$

Where $\theta > 0$. It is plausible that the mean occurrence rate θ is a variable that depends somewhat of the "awareness" or "proneness" of people to take performance-enhancing substances (PES), modeled by a gamma distribution.

$$f\left(\theta,\alpha,\beta\right)=\frac{\theta^{\alpha-1}\cdot e^{-\beta\cdot\theta}\cdot\beta^{\alpha}}{\alpha!} \tag{12}$$

The marginal probability of X is:

$$\Pr\left(X=x\right)=\int_{0}^{+\infty}p(x,\theta)\cdot f\left(\theta,\alpha,\beta\right)dx \tag{13}$$

This finally reduces to:

$$\Pr\left(X=x\right)=\frac{(x-\alpha-1)!}{(\alpha-1)!\cdot x}\cdot\left(\frac{\beta}{1+\beta}\right)^{\alpha}\left(1-\frac{\beta}{1+\beta}\right)^{x} \tag{14}$$

The number of occurrences has a negative binomial distribution. The probability that a susceptible individual to no take PDE, that is $X_i = 0$ is given by:

$$p=P\left(X_i=0\right)=\left(\frac{\beta}{1+\beta}\right)^{\alpha} \tag{15}$$

Let's denote m = β and k = α. The equation (15) becomes:

$$p=P\left(X_i=0\right)=\left(\frac{m}{1+m}\right)^{k} \tag{16}$$

The mean of negative binomial distribution is k/m. This can be considered the mean of the number of effective contacts of all susceptible individuals with doped individuals, that is k/m = $\beta \times$ D/N.

$$\frac{k}{m}=\frac{\beta D}{N}\rightarrow\frac{1}{m}=\frac{\beta D}{kN} \tag{17}$$

$$p=\left(1+\frac{\beta D}{kN}\right)^{-k} \tag{18}$$

The risk rate is risk = 1-e^{rate_of_infection}, and rate of change in the number of susceptible individuals becomes [39]:

$$g(t)=kln\left(1+\frac{\beta D}{kN}\right) \tag{19}$$

The system of equation (10) becomes for heterogenous model of partial differential equations consumption in (SEDRS) model for high performance sport:

$$\begin{cases}\frac{dS}{dt}=\Lambda N-k\ln\left(1+\frac{\beta D}{kN}\right)S+\phi R\\ \frac{dE}{dt}=k\ln\left(1+\frac{\beta D}{kN}\right)S+\alpha E\\ \frac{dD}{dt}=\alpha E-\left(\gamma+\mu\right)D\\ \frac{dR}{dt}=D-R\end{cases} \tag{20}$$

The g(t) in this case includes the effect of antidoping policy applied in sport by: information, awareness, training and prevention actions carried out both with high-performance athletes and with their team specialists and parents and sports medicine specialists. The influence of these actions on each category of staff leads to a mitigation of the doping phenomenon; in on the other hand contrary, the influence of coaches, fitness trainers and sports doctors can lead to increased consumption of banned substances.

3. Results

The simulations take into account an initial population: S_0 = 998, $E_0 = 2$, $D_0 = 0$, and $R_0 = 0$ (Figs. 2,3). It is interesting to observe the influence of two parameters, k and β, the shape of curve that describes the evolution in time of D(t) compartment. These two parameters are the components of g(t), in equation (20) (Figs. 4,5).

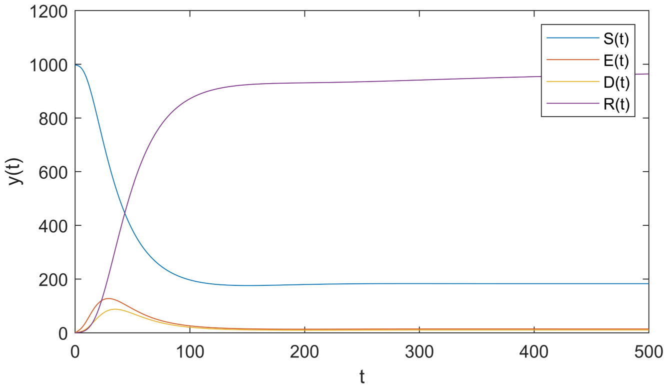

FIGURE 2. Simulation of the proposed SEDR model (Λ = 0.02, κ = 0.01, β = 2.0, α = 0.15, γ = 0.2, $\mu = 4 \times 10^{-5}$, ϕ = 0.002).

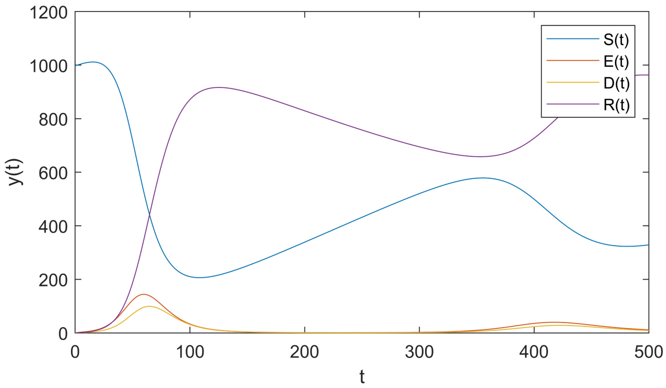

FIGURE 3. Oscillations in the proposed SEDR model (Λ = 0.02, κ = 0.1, β = 2.0, α = 0.15, γ = 0.2, $\mu = 4 \times 10^{-5}$, ϕ = 0.002).

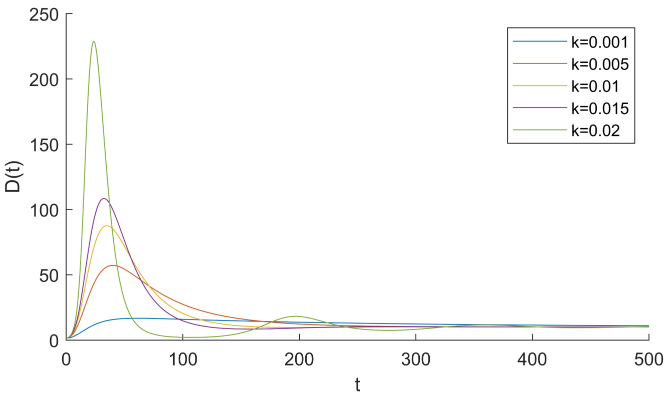

FIGURE 4. The proposed SEDRS model, simulation for κ variable and the rest of parameters are constants (Λ = 0.02, β = 2.0, α = 0.15, γ = 0.2, $\mu = 4 \times 10^{-5}$, ϕ = 0.002).

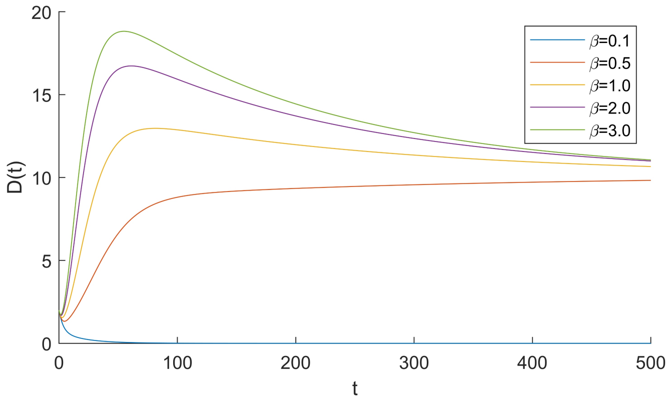

FIGURE 5. The proposed SEDRS model, β variable and the rest of parameters are constants (Λ = 0.02, κ = 0.01, α = 0.15, γ = 0.2, $\mu = 4 \times 10^{-5}$, ϕ = 0.002).

The model can be developed more explicitly and other components can be included as could be another component as social interaction in competition or the weather conditions in competition (warm, humidity and altitude). This model can be made by dividing the compartment D in more parallel compartment and averaging the total effects.

The model described by (2) has applied to a data provided by [45] (Fig. 6). Even the data are in the form of short time series, an optimization based on Levenberg Marquardt curve fit was applied using the equation (20). The Fig. 7 shows the results for these two set of data for one step ahead prediction for year 2020.

4. Discussion

There are many factors that can be taken into account also. Social aspect can influence also the attitude of sportive. Not to be neglected is the influence of the spectators and the general publics who, for the beauty of the sports show, accepts or even encourages the consumption of doping substances by athletes. As with other types of drug abuse, doping in sports is primarily a demand-driven problem.

These examples from Figs. 2,3,4,5, show the flexibility of proposed model in order to fit to different shapes of D compartment: saturation, increasing to a peak and decreased, and having periodic peaks. The function g(t) used in first experiments has a nonlinear shape and the influence on dynamics of SEDRS model can cover a typical classical antidoping method of control: more control and new policy when the doping phenomenon escalation (the slope from D(t) curve increases abruptly).

The second set of data (Fig. 6) is a real one even the number of samples is small. The first simulation (Fig. 7A) offers goods approximation inside the interval but the prediction for last step is not very good. This occurs because it is no signs from previous samples that next will be and abruptly decrease for doping cases. The second simulation (Fig. 7B) offers good prediction and the tendency of decreasing slope is very well detected by SEDRS model.

The proposed model overcome the supervised learning model in this case of small dataset because of absence of the overfitting case. The model has limitations, basically

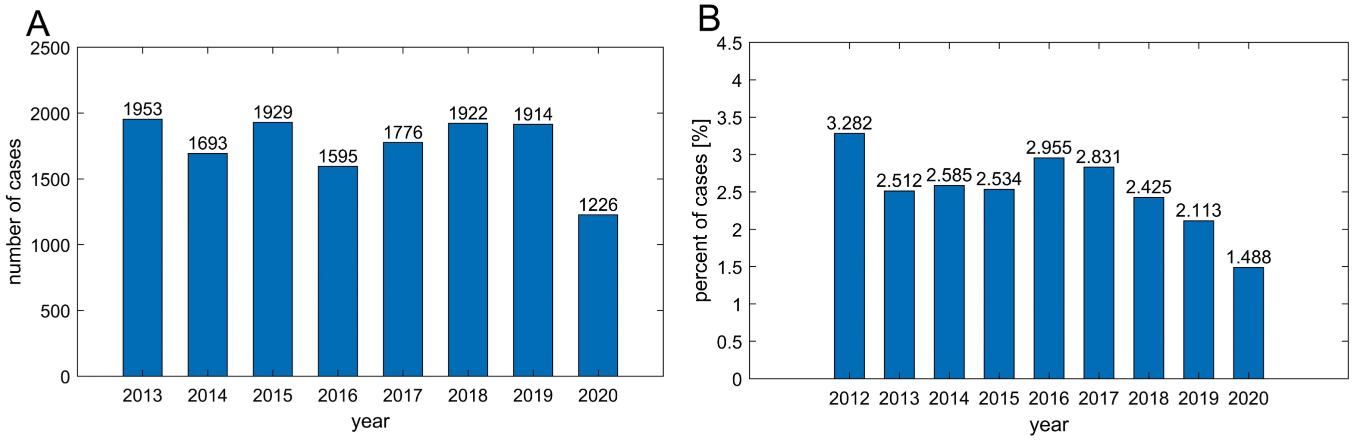

FIGURE 6. Dataset used in second simulation. (A) The Olympic doping cases between 2013–2020. (B) The percent of doped non-Olympic sportive between 2011–2021.

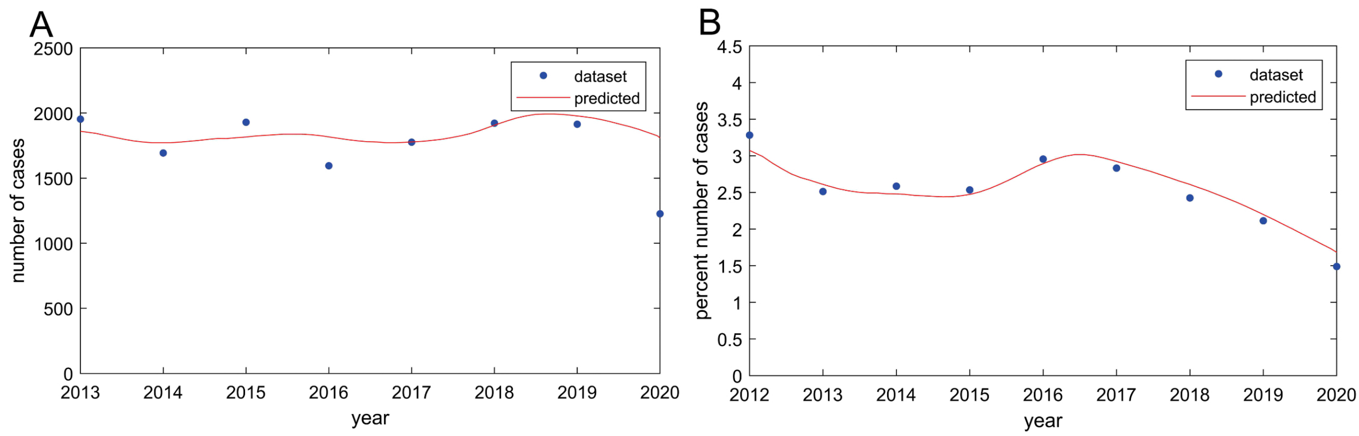

FIGURE 7. Simulation of the proposed SEDR model. (A) The Olympic doping cases between 2013–2020. (B) The percent of doped non-Olympic sportive between 2011–2021.

due to small dataset used. Also, there is a technical problem how to model a more strength method to limit the doping phenomenon in force, applied on short interval of time.

The social measures have their role, also. Those who rely on the results of sports competitions have taken a different position: bookmakers. In order to attract customers, it is necessary to quantify the correctness of each competition and the participation of each athlete. Thus, they became promoters of a clean sport.

5. Conclusions

In this paper a new model based on ODEs namely SEDRS has been proposes. The final form of this compartmental model was modified by heterogeneity using doping form as an incident modeled by Poisson distribution.

The results are in concordance with real data, even the experiments used small dataset. The approach will be extended to other dataset inasmuch they will be available from antidoping public reports. More research will be concentrated to a better approach related to D compartment, that is, two or more compartment depending on doping motivation could improve the precision of model over real data.

In the future development, an approach based on short-time exponential time depending sum of impulses (like in control of epidemic waves by vaccination and social measures as quarantine) will be investigated.

This approach will take into account the possibility that the system of ODEs to be stiff (small parameter approach) and the sensitivity of good choice for initial values of parameter in order to have a convergence to global minim and best fitting of model and experimental data.

AVAILABILITY OF DATA AND MATERIALS

The data from this article is taken from free website https://www.statista.com/statistics/1287434/share-non-olympic-doping-violations-worldwide/.

AUTHOR CONTRIBUTIONS

CGV and MT—designed the research study. DA and AG—performed the research. DA—provided help and advice on modeling and simulation. IMI—analyzed the data. MR, CM and IC—wrote the manuscript. All authors contributed to editorial changes in the manuscript. All authors read and approved the final manuscript.

ETHICS APPROVAL AND CONSENT TO PARTICIPATE

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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