Kantian Review (2023), **00**, 1–23
doi:10.1017/S1369415423000109

ARTICLE

The Unicity, Infinity and Unity of Space

Christian Onof

Imperial College London, London, UK and Birkbeck College, London, UK
Emails: christian.onof@bbk.ac.uk; c.onof@imperial.ac.uk

Abstract

The article proposes an interpretation of Kant's notions of form of, and formal intuition of space to explain and justify the claim that representing space as object requires a synthesis. This involves identifying the transcendental conditions of the analytic unity of consciousness of this formal intuition and distinguishing between it and its content. On this reading which builds upon recent proposals, footnote B160–1n. involves no revision of the Transcendental Aesthetic: space is essentially characterized by non-conceptual features. The article also addresses worries about the infinite magnitude and the unicity of space, by considering the characteristics and requirements of geometric constructions.

Keywords: space; unity of space; unicity of space; infinity of space; form of intuition; formal intuition; B160–1; Transcendental Deduction; Transcendental Aesthetic; geometric construction

1. Introduction

The problem of understanding Kant's notion of the unity of space and what it requires has recently attracted much attention in Kant scholarship. This problem stems from Kant's apparent revision, in the B version of the Transcendental Deduction (B-TD) of the claims he makes in the Transcendental Aesthctic (TAe).[1]

The TAe, in particular presents space as an intuitive representation characterized by its *unicity*: all spatial representations are part of a single space (B39/A25); space is also characterized as an *infinity*, i.e. an 'infinite given magnitude' (B39/A25). These features are presented in the Metaphysical Exposition, while the Transcendental Exposition (B40–1) presents the transcendental function of this representation, namely that it grounds the synthetic *a priori* truths of geometry.

The B-TD argues, in a famous footnote (B160–1n.) that the *unity* of space of the TAe presupposes a synthesis, and thereby introduces a distinction between a form of space providing the manifold, and the formal intuition of space which is the unified representation of space. In so doing, Kant explicitly revisits the TAe by explaining that, while this unity had been ascribed to sensibility in the TAe, it is now clear that it 'requires a synthesis' (B161n.). From the first part of the B-TD (B136–7), this entails that it stands under the unity of apperception.

Two opposite interpretative strands have emerged in relation to the problem of reconciling the TAe and the B-TD. The non-conceptualist approach traces its roots back to Heidegger's notorious interpretation of Kant. This is opposed to a conceptualist strand that similarly traces its roots to the early part of the twentieth century, in Marburg neo-Kantianism (Hermann Cohen and Paul Natorp), and ultimately to Hegel. The non-conceptualist argues that a unified representation of space is generated entirely independently of the understanding while the conceptualist has it that the unified representation of space is a product of the understanding and its categories.

In the early twenty-first century, while these positions were represented e.g. by Robert Hanna (2005) and Colin McLear (2015) on the one hand, and Eric Dufour (2003) on the other, broad versions of non-conceptualism and conceptualism came to define reference interpretations for the scholarship that followed. Henry Allison (1983) presented a broad non-conceptualist ('sensibilist') account of the representation of space as an intuition with properties that are independent of the understanding, while according a key role to the subject's grasp of this representation of space as presented in the B-TD. Béatrice Longuenesse (1998) and Michael Friedman (2012) sought to define a broad conceptualist ('non-sensibilist') account by having the representation of space arise from a pre-categorial synthesis.

So far, however, no semblance of a consensus has been achieved as to how to reconcile the TAe and B-TD on space. The concerns can be seen to cluster around two problems. These problems arise from the compatibility of, first, the notions of *unity and unicity* and, second, the notions of *unity and infinity*; I denote these problems PUU and PUI respectively:[2]

> PUU: The claim that the unity of space presupposes a synthesis is apparently incompatible with the claim that it is a unicity because:
>
> (a) the whole-part order of precedence characterizing the unicity claim apparently conflicts with the part-whole order of the product of synthesis (e.g. Williams 2018; Rosefeldt 2022);
>
> (b) the unity of a synthesis is not constrained to refer to a unique particular. Indeed, the unity of the consciousness unifying a manifold is a concept (A103); this implies that when the same synthesis is carried out again, its product falls under the same concept, but need not be the same particular. It has thus been argued that independent geometric representations are not in the same space (Guyer 2018).
>
> PUI: The claim that the unity of space presupposes a synthesis is seemingly incompatible with the claim that it is an infinite given magnitude because such a magnitude cannot be synthesized by finite beings (Guyer 2018).

The requirement that the synthesis at stake here must 'precede all concepts' (B160–1n.) defines an *additional constraint* that any proposal must take into account.

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

Below, I examine the two main strategies adopted by the various attempts, during the last decade, to address problem PUU(a); because of space constraints, this review focusses upon recent broad conceptualist and non-conceptualist proposals. This leads to developing an interpretation of the notions of form of, and formal intuition of space, with the help of which an investigation of PUU(a) will conclude that it is through the figurative synthesis that the formal intuition of space is produced in which space is intuited as horizonal object of a spatial figure. This defines a broad conceptualist interpretation of the unity of space as product of a synthesis. While the genesis of this representation of unitary space defines a part-whole order of priority, its content which is given by the form of intuition is non-conceptual and characterized by the reverse whole-part order. I then show that this interpretation of footnote B160–1n. and a proper understanding of geometric construction dispel concerns (PUI) about Kant's claim that space is an infinite given magnitude, and about the unicity of space required by different such constructions (PUU(b)).

2. Strategies to solve PUU(a)

Two strategies can be discerned to address PUU(a). The first has focused upon the nature of the synthesis in question and has been chosen by authors sympathetic to the broad conceptualist side of the debate. The second strategy has involved questioning in what sense the unity of space presupposes a synthesis. This strategy has typically been adopted, either alone or together with the first, by authors keen to preserve what is non-conceptual about space in our understanding of the formal intuition of space.

2.1 The first strategy

Jessica Williams' proposal takes its cue from an objection she levels against the broad conceptualist approach presented above. She argues that it fails to conform to the constraint that the synthesis for the representation of space as an object should precede all concepts. She thus argues (Williams 2018: 73–4) that Friedman's constructive procedure, which involves drawing a line, implies a role for a category of quantity; and that Longuenesse relies upon the figurative synthesis which accords with the categories (B152; Williams 2018: 74).

Rather than seek a pre-categorial synthesis, Williams (2018: 75) focuses upon 'a global aspect' of the original synthetic unity of consciousness (OSUA) to ensure that it allows for space's whole-part structure. She argues that this synthesis is 'the act of consciousness through which the subject grasps her own sensible nature as unified by her capacity to determine what is sensibly given' (p.: 75). This original synthesis defines the 'global aspect' of any particular 'local' synthesis of a given manifold (p. 77). But, unlike the latter, the first operates at a higher level than the categories (B131). Since this synthesis involves no composition, it addresses the worry raised in PUU(a).

Williams' proposal relies chiefly upon the nature of the Original Synthetic Unity of Apperception (OSUA) which 'precedes all concepts' (B131). Williams is right to say that this has a global aspect in that it is an *a priori* condition for all objective knowledge (2018: 75). But when Kant introduces the OSUA, he starts by indicating that

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

'I think must be able to accompany all my representations' (B131): there is no distinction of local and global here. When he further explains that this requires that 'I unite them in a self-consciousness, or at least can unite them therein' (B134), the actual uniting could be taken to refer to a local synthesis; but the mere possibility of uniting is certainly sufficient for the global aspect of the OSUA. There is no need for any other synthesis here: when I unify a 'local' manifold through the identity of the 'I think' that accompanies its parts, I am thereby aware that this is the same 'I' as in other possible such 'local' syntheses, and this is indeed necessary for the unity of my experience.

In other words, the TUA is the unity of the consciousness of the 'I think' that must be able to accompany *all* the subject's representations (B132). The consciousness of this unity requires *a* synthesis (B133), i.e. the OSUA is a condition of the analytical unity of apperception. But this synthesis can be any 'local' synthesis: through it, the identity of the 'I think' across all possible representations is manifested. A global aspect is therefore built in here: no higher synthesis is required.[3]

While I do not endorse Williams' proposal therefore, I agree with what motivates it, i.e. the belief that a broad conceptualist proposal must address non-conceptualist concerns about interpretations that reduce space to the product of a categorial synthesis.

Tobias Rosefeldt opts for an approach[4] that, like Williams', draws upon another type of synthesis that is meant to do the same work as her original synthesis in providing answers to PUU (and PUI). This synthesis supposedly does not give priority to the parts over the whole and is therefore tailor-made to address problem PUU.

The textual evidence for this *decomposing synthesis* is clear. It is found in the discussion of types of regresses that Kant presents as preliminary material for the resolution of the mathematical Antinomies (A512/B540ff.). Through the decomposing synthesis (A513/B541) it is 'possible to go back to infinity in the series of its inner conditions' (A514/B542) in the Second Antinomy (A523–4/B551–2). As Kant explains, when a whole of matter is given in empirical intuition, so are its *possible* parts. The decomposing synthesis actualizes these parts.

Rosefeldt's proposal involves applying this synthesis to space as it is given. He argues persuasively that the representation of space is essentially obtained through a determination of space which lets the background horizon appear because it is not possible to represent space otherwise than through a determination of a part of it. So the representation of what, in Husserlian terms, is a horizon (Rosefeldt 2022: 8) is interpreted as product of a (decomposing) synthesis (pp. 5–6).[5]

Rosefeldt's is an ingenious proposal, but it faces an objection to the way the decomposing synthesis is applied here. Kant clearly states a condition for the decomposing synthesis which, while fulfilled by an empirical intuition of outer sense, cannot be in the case of space. Namely, what the intuition of outer sense refers to should be 'given with certain boundaries' (A513/B541). Why this condition? If boundaries were not specified, it would be impossible to say that the whole of a material object is given.

One might think that since space is given as a whole (as infinite magnitude), the problem could be obviated. In terms of the dialectic this would mean that the applicability of the decomposing synthesis to space would rely upon its being clarified how space could be given as an infinite magnitude to our finite cognitive capacities (problem PUI). But Rosefeldt's answer to this question *presupposes* the applicability of the decomposing synthesis to space.

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

Although this need not define a *vicious* circle, there is a deeper reason for thinking that the decomposing synthesis does not apply to space. This lies in Kant's stipulating, in relation to this synthesis, that 'the condition of this whole is its part' (A513/B541). This would exclude this synthesis's application to space, which precisely is not conditioned by its parts. Why does Kant stipulate this condition? Because the synthesis of decomposition, like all syntheses, fundamentally relies upon a part-whole order of priority, now expressed in terms of parts as *conditions* for the whole. This Kant expresses by stating that the antinomial regress defined by iterations of the decomposing synthesis goes 'back to infinity in the series of its inner conditions' (A514/B542). So the decomposing synthesis gathers parts together which, *insofar as they are conditions of the whole*, thereby determine the whole through a synthesis of these parts (*ad infinitum*). Like all syntheses, the decomposing synthesis therefore has a part-whole order of priority: the 'decomposing' characteristic of the synthesis lies merely in its actualizing the parts that were only a potentiality in the representation of the whole.[6] This synthesis cannot, therefore, play the role Rosefeldt identifies, of generating the parts of space from the whole of space.

However, as I shall argue below, Rosefeldt's proposal seems exactly on the right track insofar as it identifies the representation of the whole of space as involving that of some of its parts.

2.2 The second strategy

Two proposals have explored the second strategy of identifying a different sense in which the unity of space requires a synthesis so as to formulate interpretations that are broadly non-conceptualist; and two other proposals have combined this strategy with the first, by also questioning the default assumption that the synthesis at stake in the formal intuition of space is the figurative synthesis.

Andrew Roche (2018) distinguishes the unity of space as a 'brute given' at one time from the unity that experience requires, which is that of space *over time*. In so doing, he is addressing a standard concern with a 'brute given' understanding of space. The issue was flagged, among others, by Williams (2018: 84) and Raysmith (2021: 10): if the manifold in intuition is already given in a unified intuition that is independent of the categories, this contradicts Kant's claim that the way appearances are given is such that their unity is that of syntheses that accord with the categories (cf. A84/B116ff.).

By having the unity over time of the representation of space conform to the categories, Roche seems to avoid this problem: the whole-part order of priority of the unity of space at one time is thus brought under a part-whole unity which produces the formal intuition of space understood as the intuition of 'one space … throughout time' (Roche 2018: 51). Since the synthesis in question structures the manifold 'at a level not as high as [that] of concept application' (p. 48), this therefore provides a solution to PUU(a).

But the proposal makes the big assumption that Kant's claim in the TAe that space is singular (B39/A25) only concerns a single moment in time. There is however no such restriction in Kant's affirming that 'if one speaks of many spaces, one understands by that only parts of one and the same unique space' (Roche 2018: 48).

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

Roche's interpretation overlooks this and introduces a requirement of synthesis for the *unicity* of space over time which sits uncomfortably with a broadly non-conceptualist reading.

Onof and Schulting (2015) distinguish between the unicity of space as defining a notion of unity of space *tout court*, and the fact that space requires the figurative synthesis *to be taken as* a unity, thus drawing upon Allison's (1983) interpretation. This proposal spells out the nature of the non-conceptual content of the representation of space and explains how its whole-part order of priority does not clash with the requirement of a synthesis. However, the claim that the unicity of space defines the unity at stake in footnote B160–1n. leads to the worry (e.g. Williams 2018: 82; see above) that an intuition of space which is apparently a brute given potentially threatens the success of the second half of the B-TD. Indeed, the latter relies upon the unity of intuitions necessarily being that of apperception. In this article, I present a revision of this proposal that addresses this worry.

James Messina (2014) proposes an interesting way of driving a wedge between the non-conceptualist 'Brute Given' and the conceptualist's 'Synthesis Reading' by endorsing the claim that the unity of space is synthetic but denying that the formal intuition of space is produced by a synthesis. Rather, Messina (2014: 23–4) claims that the OSUA characterizing our discursive cognition requires the unity of the manifold of space as a necessary condition and that this synthetic unity is guaranteed by the OSUA, whereby the latter is understood in a different sense. By drawing upon an analogy with God's representation of the world as a 'synthetic universal' (*CPJ*, 5: 407), Messina proposes that this sense defines the synthetic unity of the formal intuition as 'a whole that makes possible the figurative synthesis of the manifold contained within it' (pp. 23–4).

Although the interpretation he gives of the outline of the B-TD does not contain any particularly contentious claim, Messina does not spell out his understanding of its structure. It is understood by many commentators (see Onof 2022: 443) that, in the first part of the B-TD, Kant's claim that judgement relies upon the manifold being given in 'one (*Einer*) empirical intuition' (B143) defines a condition for its being brought to consciousness that is addressed in the second part of the TD. That is, the unity of an intuition is achieved *through* the figurative synthesis as Messina agrees (2014: 11).

However Messina's interpretation (2014: 24) leads him to the conclusion that '[o]n pain of regress, this figurative synthesis could not generate the synthetic unity of our formal intuition' of space because he understands the OSUA itself as *requiring* a unity of space. But taking this as a condition upon the *unity* of the act of apperception goes against Kant's clear statement that 'through this alone [i.e. the OSUA] is the unity of the [sensible] intuition possible', which must surely also be valid of the formal intuition of space.

What is required for the second part of the B-TD is rather that space is a *unicity*, i.e. providing the 'one [*Einer*] intuition' (B143) required for the OSUA. The figurative synthesis can then determine space through geometric constructions.[7] Such geometric constructions provide us with unified intuitions which, I shall argue in this article, also define formal intuitions of space, i.e. 'intuitions themselves (which contain a manifold), and thus with the determination of the unity of this manifold in them' (B160).

While the interpretation of the B-TD cannot be further addressed here, the above concerns are reflected in the problematic nature of Messina's (2014: 23) claim that there can be a *synthetic unity* that does not involve a synthesis. Messina argues that 'what makes this representation [of space] synthetic is both that it is a whole (in Kant's strict sense) *and* that it makes possible a combination of the manifold contained within it'. But, aside from this questionable understanding of 'synthetic', the claim that our discursive cognitions, as opposed to God's intellectual intuition, can be conscious of space as such a synthetic whole without a synthesis has not been explained. It seems rather that it is precisely a feature of discursive cognition that it cannot cognize a whole without concepts being involved (explicitly or as schemata), i.e. without synthesis (e.g. Allison 1983: 65–6). It follows that an intuition of space as unity, i.e. a formal intuition, 'presupposes a synthesis' as, *pace* Messina (pp. 25–6), Kant states unambiguously (B160–1n.).

Messina's interpretation cannot be discussed in any further depth here. Rather, I aim to show that it is possible to avoid certain questionable interpretative choices in it with a reading of the unity of space as requiring a synthesis, without threatening the non-conceptual content of the representation of space.

While Messina interprets the unity of space's requiring a synthesis as actually a requirement of synthetic unity *without* a synthesis, Thomas Raysmith (2021) focuses upon a different sense in which the unity of space requires a synthesis. He thereby also deviates from the standard assumption that the synthesis in question is the figurative one. To avoid the problem flagged earlier of a brute given unity of space, i.e. that space is *actually* given in a manner which does not accord with the TUA, Raysmith (2021: 10–12) first proposes that the form of space, which he takes to be 'original space', is a 'unitary representation' that is a 'transcendental ground … for the possibilities of … spatial … representations', i.e. which 'giv[es] a manifold of merely *possible* representation[s]'.

Raysmith (2021: 14–15) then argues that what requires a synthesis is not the formal unity of space but our ability 'to think this unity', and therefore that since this must involve a 'concept of original space' a synthesis is indeed required. That is, the synthesis required by the unity of space is that of these features qua marks of this concept: it is through such a synthesis 'that we first think this unity [of space]' (p. 14). As textual evidence, he adduces Kant's stating that it is the synthesis 'through which all concepts of space and time first become possible' (B161n.).

This interpretation of the synthesis requirement is problematic. If, as Raysmith claims, this were 'the synthesis of the various conceptual marks that results in the formation of the concept of original space' (Raysmith 2021: 15), Kant would not have characterized it as 'first' making 'concepts of space' 'possible'. First, the plural 'concepts of space' does not refer to the concept of space in general in the TAe (B40), but geometric concepts: Kant makes it clear that this footnote is dealing with space as object, as is 'required in geometry' (B160n.). Second, the synthesis first makes such concepts possible: it does not therefore issue in a concept, i.e. it is not an intellectual synthesis. Rather, as I shall argue below, Kant's claim refers to the figurative synthesis as the 'first application' (B152) of the understanding to sensibility.

Further, the conceptualization which, as Raysmith correctly observes, is found in the TAe (B37), is only useful in philosophical reflection upon space. The conditions of possibility of such reflection are surely not what is at issue in the B-TD: in particular

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

footnote B160–1n. explicitly examines 'space as object (as is really required in geometry)'.

Raysmith could reply that Kant refers to the synthesis in question as one through which 'the understanding determines sensibility' (B161n.). But this would not justify its being interpreted as conceptual since the figurative synthesis is also 'an effect of the understanding on sensibility' (B152), but Raysmith does not regard *that* as conceptual.[8]

While Raysmith's understanding of the synthesis requirement does not provide a convincing solution to PUU(a), his proposal with respect to the form of space as ground for the possibilities of spatial representations, on the other hand, provides a new insight into how we should understand original space. I shall make a similar proposal in my interpretation below.

This brief survey of recent proposals cannot fully do justice to them. But while none has been found fully satisfactory, as we shall see below, the interpretation I propose will draw upon several ideas they have generated. The first part of this interpretation broadly belongs to the second strategy but with the different aim of showing that the footnote's claim that the unity of space requires a synthesis should not be taken as a revision of the TAe's presentation of what is the essentially non-conceptual content of the representation of space. Note that my proposal will not seek to deny what I take to be essential to interpreting B160–1n., i.e. that the claim that 'the formal intuition gives unity of the representation ... this unity ... presupposes a synthesis' (B160n.) cannot be interpreted in any other way than as the requirement that the unity of the (formal) intuition of space requires a synthesis.

3. The footnote and its compatibility with the TAe

Below, I present a proposal that builds upon several ideas reviewed above, to revise Onof and Schulting's (2015; hereafter OS2015) proposal, so as to address the worry that it presents the unity of space as that of a brute given (e.g. Messina 2014; Williams 2018; Guyer 2018), which would be problematic for the B-TD. In that proposal, space *tout court* characterized by its unicity and topological features is said to define the unity which 'precedes all concepts' (B161n.). The role of the 'synthesis' (B160n.) is then to enable this unity to be grasped as such, i.e. for the subject to be conscious of it. But this runs counter to Kant's claim that the unity of all syntheses is that of the categories (B160–1n.). To address this worry, I propose *not* to interpret the *unicity* of space of the TAe as defining a unity independent of the TUA (OS2015: 13–15). The alteration will have implications for how the notions of 'form of' and 'formal' intuition are understood and how they relate to the TAe. Below, I first focus upon what is needed to clarify these two notions, and thereafter tackle the three problems this article is addressing.

With this alteration, the proposal moves in principle from the broad non-conceptualist to the broad conceptualist camp; I however view it as essentially resulting from a clarification of the use of the concept of 'unity'. So it does not involve rejecting OS2015's claims about the non-conceptual content and origin of the representation of space. Rather, it is motivated by the conviction that the sense in which something like space, with a whole-part structure, could be called a unity prior to any synthesis, is not one accessible to our discursive cognition but only to a 'problematic'

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

(in Kant's sense) intuitive understanding (see the discussion of Messina's proposal above).[9]

3.1 Space in the Transcendental Aesthetic and Transcendental Deduction

A reader who has just finished the TAe would not be surprised at Kant's claiming in the B-TD that space has a unity that belongs to it: while she might comment that the word 'unity' was not mentioned explicitly in the TAe, it is clear that Kant is describing a structured *a priori* representation, so that it must be a unity in some sense. But how is this unity to be understood?

Kant gives us a clue by including in the *metaphysical* exposition (or in point (4) in the A edition) a statement that really pertains to the topic of the *transcendental* exposition (point (3) in the A edition).[10] Namely, he indicates that the pure intuition of space provides the ground for 'geometric principles' (B39/A25). Why does Kant include this statement in the metaphysical exposition? The answer lies in noting that it belongs to Kant's discussion of the unicity of space. Why does this unicity account for the pure intuition of space grounding geometric principles? Kant says that 'when one speaks of many spaces, one understands by that only parts of one and the same unique space' (B39/A25). This means that the intuition of space defines an *analytic unity of consciousness* (AUC): it is analytically true that the property of being in space is common to all parts of space, because they are all parts of the one unique space. To put it differently, space is a *totum analyticum* (Allison 1983: 43): the whole of space precedes its parts. This AUC of the intuition of space is the ground of all geometric principles, since all such principles are obtained by constructions in the same one space whose structure they characterize (B40; see further).

But once the reader has finished the first part of the B-TD (e.g. B131–8), these claims of the TAe seemingly clash with Kant's claiming that all cognition, and therefore in particular that of geometry, has the 'synthetic unity of consciousness ... [as its] objective condition' (B137–8). For the condition for geometric knowledge is now presented as lying in the understanding. The implication of this claim for space is presented in B160–1n.: the unity of '[s]pace, represented as object (as is really required in geometry) ... presupposes a synthesis'. This crystallizes the tension with the TAe, but it is therefore a tension concerning the broader issue of the grounding of geometry and thereby of all *a posteriori* knowledge of objects in space which depends upon the possibility of such geometric knowledge (A137/B176; B271/A224): all this depends upon clarifying the unity of space and its requirements.

Kant's shedding light upon these matters in B160–1n. has been taken by many to involve a 're-reading' of the claims of the TAe (e.g. Longuenesse 1998: 216). It is worth pointing out that, if that were the case, it would be strange that Kant, in rewriting the *CPR*, discusses this issue merely in a footnote. Rather, the issue raised in the footnote only arises because Kant's synthetic method in the *CPR* (cf. *Prolegomena*, 4: 274) prevented him from addressing it earlier: Kant defines the various transcendental conditions for the constitution of experience in a progressive manner and syntheses could only be discussed in the Transcendental Analytic.

That there is an explanatory dearth in the TAe that the TD must address is clear. When Kant says that 'all geometric principles ... are derived from intuition' (B39/A25), the question 'how' arises. A clue lies in Kant's indicating that 'the general

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

concept of spaces in general, rests merely on limitations' (B39/A25), but how these limitations fulfil the required transcendental function of grounding geometry requires clarification. This is the question that Kant addresses directly at B160–1n. by explaining that a synthesis is required for the unity of space.

I therefore understand B160–1n. as clarifying the tension by focusing upon space itself as geometric object, and simultaneously defusing it. That is, this footnote *complements* the TAe: while the TAe ascribed the unity to space, the footnote clarifies that, although this unity is in space, sensibility does not produce it. This unity is that of the consciousness, in having the pure intuition of space, of this unique space; it involves a synthesis that could not have been discussed in the TAe. But of course, the tension will only be properly defused once it has been clarified *why* a synthesis is required, a question related to problem PUU(a) which I examine further.

3.2 The formal intuition of space and the spatial manifold

This clarification will help make sense of Kant's distinction between form of and formal intuition which, in OS2015 (p. 30) could be understood as characterizing two types of intuition, namely, the manifold in intuition prior to any synthesis, and the synthesized manifold. To avoid any contradictory claims about how concepts of space and geometry are grounded, when Kant explains that 'an *a priori* intuition … grounds all concepts of [space]' (B39), I agree with Messina (2014: 19) and Raysmith (2021: 8) that we must interpret the intuition in question to refer to the *formal intuition* of space Kant introduces there, i.e. the intuition which 'gives unity of the representation' (B160n.).[11] This claim is further substantiated by noting that the distinction between form of intuition and formal intuition (B160–1n.) differentiates that which gives the manifold of an intuition from the resulting unified intuition.[12] While this sheds light upon the notion of formal intuition, that of form of intuition is less clear. To clarify these issues, a short excursus into the way Kant understands intuitions will be useful.

Conceptualists typically take it that Kant operates with only one notion of intuition, namely, a representation referring to a determinate object as a result of its manifold having been brought under the Transcendental Unity of Apperception (TUA). The opening sentence of the TAe justifies such a claim: 'intuition' is that 'at which all thought as a means is directed as an end' (A19/B33).

However, the same opening passage of the TAe refers to an 'empirical intuition' as an 'intuition which is related to the object through sensation' and which has an 'undetermined object' (B34/A20). So in fact, there are two types of intuition (see Gardner 1999) in Kant's epistemology, and quite understandably so. There is an empirical intuition which is related through sensation to an undetermined object. As Kant explains in the B-TD, I then 'make the empirical intuition … into perception through apprehension of its manifold'[13] (B162), i.e. through a synthesis whose unity, Kant has just shown in the B-TD, is that of apperception (TUA). This yields an intuition of an object that is determined in some respect: this is the intuition that is the 'end' of the cognitive process of the apprehension of an object.

Both types of intuition have in common that they are representations which are essentially distinct from concepts (A19/B33) in how they relate to their object so that their origin and content are non-conceptual. But that there are two distinct types of

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

intuition is essential to understanding Kant's epistemology, and is further confirmed by the dichotomy between raw appearances (B34) and empirical objects as unified appearances (A248).

But is the distinction between empirical intuition and (unified) intuition reflected in a corresponding distinction for space? In other words, to what extent is it useful toward making sense of the distinction between the notions of form of intuition and formal intuition of space, as well as their relation? The temptation is to map the dichotomy empirical intuition/(unified) intuition onto that of form of intuition/formal intuition of space, since Kant distinguishes the form of intuition which gives the manifold from the formal intuition in which this manifold is unified. But the reason one should resist this transposition to space of the distinction in question is very simply that empirical intuition is defined as 'that which is related to the object through sensation' (B34/A20): it has no equivalent in pure intuition. That is, there is no intuition of a manifold of space prior to its being unified as formal intuition of space: I cannot have a conscious representation of space without any determination (geometric or sensible) in it, as Rosefeldt points out (2022: 13–14).

If this transposition does not work because there is no manifold of sensations for space, let us try to shed light upon the form of intuition by first clarifying the nature of the manifold of space. According to B160–1n., the form of intuition 'gives the manifold'. But the manifold of space is described in the TAe as 'rest[ing] … on limitations' of the 'essentially single' space. This is *prima facie* puzzling because the limitations are what is brought about through an act of synthesis, and indeed, Kant includes in the same sentence the claim that the general concept of spaces also rests upon such limitations. The only way to make sense of this claim for the manifold is to take it that, prior to any such limitations, the manifold is not intuited. That is, since there is no equivalent for space of an empirical intuition of the manifold prior to synthesis, it is through the synthesis that the manifold that is synthesized is first intuited.

The manifold is therefore best described as non-existent prior to synthesis and as first *actualized* in a formal intuition of space. Raysmith (2021) is therefore exactly right to view the manifold as a set of possibilities. He however understands this manifold as the collection of all possible spatial representations. This would entail that the determination of this manifold, in e.g. constructing a geometric figure, would amount to picking one of these possibilities and representing it as actual. What is missing in this process is the *combinatory* role of the synthesis of such possibilities as parts of the manifold.

In answering the question of what space is, Kant offers that it is 'relations that only attach to the form of intuition alone' (B37–8/A23).[14] This suggests that the manifold is rather the set of possibilities of a certain type of relations, spatial relations. All relations between points are constructible from the *combination* of *basic spatial relations* which are either arcs of circles or straight lines between two different points ('On Kästner's Treatises', 20: 410–11; see Kant 2014). I therefore propose to interpret the manifold as the collection of all possible such basic spatial relations. And indeed, in the Axioms of Intuition, Kant describes how spatial relations are produced in the simplest case of drawing a line. This involves 'successively generating all its parts from one point' (B203/A162). There is therefore a combination of basic spatial relations in the synthesis: even if I draw a straight line between A and B, I am relating points along the line through basic straight line relations keeping the direction

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

fixed.[15] This also provides grounds for viewing the synthesis that is at stake in the formal intuition of space as the figurative synthesis.[16] I shall assume this for now and fully substantiate this claim later.

3.3 The form of intuition and non-conceptual content of the representation of space

While the manifold is a manifold of possibilities, the *form of space* is what gives it. However, this notion is not identifiable with a deflationary understanding of the form of space as mere potential for spatial representations (e.g. Longuenesse 1998: 21). I understand this notion of giving rather as identifying the transcendental condition for all such possibilities. It is not an intuition because it is what enables spatial intuitions; spatial intuitions, as explained above, involve determinations of space so that it is only as determined in some way that space can be intuited (see also Blomme 2012: 149–50).

So, since the form of intuition is what gives the manifold, and the latter is a set of possibilities for basic spatial relations between two points, the form of space is what gives these epistemic possibilities and, in that sense, is an *epistemic capacity* characterizing sensibility.[17] The role of the synthesis will then be to actualize certain parts of the manifold of possibilities, i.e. relations between points in space, as they are bound together in the act of drawing.

Before continuing, it will be pointed out that the form of sensibility in the TAe is also described as a pure intuition (B34/A20).[18] Does this conflict with my interpretation so far? This however is not the form of sensibility in isolation, but insofar as it contributes to a determinate representation: Kant takes the example of the representation of a body at B35/A20–1. As such, it is indeed as pure intuition of space, characterized by 'extension and form', that it contributes to the determinate intuition of the body. In support of this reading, consider Kant's 'conclusions' about space (A26/B42):

> [I]t can be understood how the form of all appearances can be given in the mind prior to all actual perceptions, thus *a priori*, and how as a pure intuition, in which all objects must be determined, it can contain principles of their relations prior to all experience.

When considering the form of all appearances (of outer sense) 'as a pure intuition', it contains 'principles' of the (spatial) relations of objects which are determined in it, i.e. geometric principles. The intuition in question is therefore the formal intuition of space.

Assuming this interpretation is correct and that the synthesis required for the formal intuition of space is the figurative synthesis, let us look at how the manifold is combined in the figurative synthesis involved in drawing geometric figures (B151–2).[19] Insofar as this brings about a pure intuition, we can turn to the TAe where Kant indicates that 'extension and form' 'belong to the pure intuition' (B35/A21). *Extension* defines the possibility of joining points in space, while *form* defines that of doing it in different ways.[20] To link points, one can therefore (see Kant 2014 (Kant's comments on Kästner), 20: 410–11) draw a straight line or construct a curved line (using a 'mental compass' to draw arcs of circle of any radius). This is confirmed a

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

little further on when Kant indicates that, for objects in outer sense, '[i]n space their shape, magnitude, and relation to one another is determined, or determinable' (A22/B37). Extension is determinable as magnitude (e.g. how long the line is), while form is determinable as shape (e.g. the curvature of the line).[21]

From this understanding of the figurative synthesis, it is then possible to infer as transcendental conditions the characteristics of the possibilities given by the form of space which, as explained above, is an epistemic capacity functioning as transcendental condition of spatial representations:

1. Insofar as this synthesis produces a spatial figure, the basic spatial relations constituting the manifold must connect points belonging to the same space: space must be a *unicity*; here, note that it could be argued that a different occurrence of a figurative synthesis does not require the same space: I return to this issue in the final part of the article.
2. These points must be mutually external, so I can connect them through synthesis: this requires that a *whole be given prior* to locating points in it, i.e. *prior to its parts*.
3. In carrying out these operations (extension, joining), no knowledge of the particularities of a region of space must be required: all regions of space must have *identical* properties.
4. This space must not have any limitations insofar as I must be able indefinitely to extend the line I am drawing, so the form of space must be characterized as *unlimited*.[22]
5. It must be possible to join any two points in space through other points in space, so the form of space must be *gap free.*

These are therefore five characteristics of the *non-conceptual content* of space as manifold given by the form of space, and therefore of the content of its formal intuition (see OS2015).[23] That is why they are identified in the metaphysical exposition: these are, respectively, the *singularity and isotropy*, the *topological mereology,* the *homogeneity*, the *infinite magnitude* and the *continuity* of space.[24]

Note that, while they are defined as marks of the concept of space (B37), this in no way conflicts with the non-conceptual nature of the features of space that they identify:[25] the concept of space contains all the determinations of space as the result of a philosophical exposition. This is the only way in which non-conceptual features can be identified (see OS2015: 34–5; Blomme 2012: 147).

Having shed light upon Kant's notion of form of intuition, I now turn to problems PUU(a), PUI and PUU(b).

4. The solution to problem PUU(a)

4.1 A useful distinction

I want to draw attention to the distinction between space as (1) endowed with a *whole-part* structure, and space considered as (2) representation (formal intuition) *whose unity presupposes a synthesis of such parts* through which reference to an object is achieved. Claim (1) refers to space as *content* of an intuitive representation, i.e. that of which I am conscious in having a representation of space, as Williams (2018: 80)

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

explains. Claim (2) is about the *representation* of 'space considered as object' (B160–1n.). This important distinction enables us to differentiate between the properties of space as *content* of a representation (formal intuition) and space as this *representation*.

This is in fact equivalent to Messina's (2014: 16) distinction between the representation of space and the object it refers to. To clarify the relation between these distinctions, I need to explain in what way a *representation* can be distinguished both from its *content* and the *object* it defines. Take the example of a ball: I distinguish between my intuitive *representation* (unified intuition) of a ball, which characterizes my inner state which has a certain duration in my consciousness (I am representing a ball); its *content*, which is three dimensional and spherical and has various other space-filling characteristics I am conscious of in having this intuition; and the empirical *object*, whose properties include these spatial characteristics of the content and others such as mass, electric charge, etc. In the context of transcendental idealism, however, Kant often says that an object just is a representation (A385). But if this idealism is not misinterpreted as a form of Berkeleyan idealism, objects cannot be ontologically identified with representations. Rather, Kant's statements should be taken as indicating that there is nothing more to being an object than the possibility of representations (intuitive and conceptual) of it (Schulting 2017: 6). This amounts to identifying the object with the content of such possible representations: the ball is therefore nothing more than the possibility of intuitions of it (as described above) and concepts of it as having mass, electric charge, etc. It is in this sense that Kant's claim in the TAe that space is an intuition (A24/B39) should be understood: this status of space (and time) is the cornerstone of the transcendentally ideal conception of empirical objects as contents of possible representations.

Pace Messina (2014: 16), however, my point is precisely that content and representation are *not* required to have the same structure: indeed, why would they be? The claim that space precedes its parts is, *pace* Raysmith (2021: 7–8), a characterization of space as the content of what I am conscious of in having a formal intuition of space. This content is provided by the form of space: it is the manifold of space, i.e. space containing all the possibilities of basic spatial relations between two points, and has a whole-part order of priority.[26] This order need not, however, be a feature of the formal intuition itself. Indeed, the claim of whole-part mereological priority does not imply an ability to *represent space* as a whole *before* (and therefore independently of) representing any of its parts.

The tension with the claim that the formal intuition of space requires a synthesis thus disappears if we take OS2015's space *tout court* to refer to the content of the (formal) intuition of space rather than to any kind of (intuitive) representation of a brute given. Problem PUU does not disappear for that, however. For it may still seem puzzling that a content with a whole-part priority should be represented by a representation, the formal intuition of space, that is the result of a synthesis which composes parts into a whole. The puzzle is, namely, that this representation must somehow be able to represent space as having the whole-part order of priority. This puzzle can be seen as motivating, in particular, Williams' and Rosefeldt's attempts to identify a synthesis that could do justice to the whole-part order of the content of the representation it brings about.

4.2 The requirements of the AUC and the formal intuition of space

To address this, it is useful first to consider that a synthetic unity is needed to represent an AUC, as Kant explains in another important footnote of the B-TD: 'only by means of an antecedently conceived possible synthetic unity can I represent to myself the analytical unity' (B133n.; see A25/B40).

What does this mean? Simply that, in the case of concepts, I cannot think the analytic unity of, for example, 'red' without representing it as common feature of a manifold of representations that include 'red' together with any other possible representations, i.e. 'red *x*', 'red *y*', . . . (B133n.). And to think of it as such a common feature requires that this manifold be thought as a synthetic unity, i.e. as referring to the possibility of synthesizing such a manifold.

These claims are of course relevant to the conditions of possibility of thinking of the *concept* of space analysed by Kant in the TAe's Metaphysical Exposition (B37). However, I claim that they also apply to the *intuition* of space which B160–1n. is concerned with, since space is an AUC (see Dufour 2003). This enables the identification of conditions of this analytic unity which, we shall see, are the grounds for Kant's claim in B160–1n. that the unity of space presupposes a synthesis.

To show this, note both the commonality and difference between the cases of concepts and intuitions. What is common to both is that, just as a concept is an analytic unity of that which is common to all representations containing this conceptual determination together with other possible representations, space is what is common to all representations of spatial regions with other possible space-filling properties. Therefore, just as the AUC of a concept depends upon a synthetic unity of its marks together with other possible determinations, the analytic unity of the consciousness of a (formal) intuition of space depends upon a synthetic unity of the parts of space combined with possible space-filling properties. This introduces a dependence upon a *possible* synthesis.

What distinguishes the two cases is expressed (i) by Kant's stating that a common concept contains an (infinite) set of possible representations 'under itself' while the intuition of space contains an (infinite) set of possible representations 'in itself' (A25/B39–40); and (ii) by the different defining features of intuitions and concepts: an intuition is a presentation of an object as *immediately given* (whether determined or not). As Parsons (1983: 112) shows, (ii) goes beyond Hintikka's negative characterization of the presentation of the object as not given through conceptual marks: it involves an object being 'directly present to the mind, as in perception'. In the case of the (unified) intuition of an empirical object, it presents the object *as* actual: it might *be* actual (perception) or produced by the imagination as 'the faculty for representing an object even without its presence in intuition' (B151), therefore *as if* it were actual.

So, according to (i), the AUC of the intuition of space requires representing the intuition's possible parts *inside* it, but additionally according to (ii), as intuition, the formal intuition of space must be a representation of an *actual* object; and since space is not an object of perception, this intuition will be produced by the imagination. This therefore confirms that the *figurative synthesis* is involved here (which excludes other interpretative options examined earlier).[27]

The text also points to such an interpretation: the only synthesis that has been discussed in the pages leading to B160–1n. is this synthesis, and no other *a priori*

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

synthesis is discussed in sections 24 and 25 of the B-TD. Further, that the synthesis 'precedes all concepts' (B160n.) is ensured by the figurative synthesis being the 'first application' of the understanding, thereby grounding the possibility of the intellectual synthesis in which an object is determined conceptually (B152): pure sensible (geometric) concepts are involved, not as concepts, but rather as schemata (B176–7/A137–8).[28]

The problem now takes the following form: how is an actual representation of the whole of space, as required by the AUC, possible for our finite cognitive capacities? Setting aside any worry about how a finite subject's synthesis could produce a representation with an infinite magnitude (see below), we should first note that, since the manifold of space is a set of possibilities, the requirement is to make these possibilities manifest in an actual representation.

I will now show that it is sufficient that *one region of space* be represented as actual to fulfil this requirement. Indeed, a condition for selecting parts of the manifold to be synthesized into a figure is that the possibility of other parts that are not selected be recognized. This goes beyond the equivalent truth for concepts, namely, that if something is determined as a 'red tomato', this excludes its being a red nose, a red hood, a red flag, etc.: representing a red tomato does not bring up these other possibilities which lie *under* the concept 'red'. By contrast, determining points related by basic spatial relations as part of a figure[29] involves simultaneously determining others as what is left outside it. This is because these possibilities *are given together* by the form of space. This feature of the *content* 'space'[30] as whole prior to its parts accounts for why an intuitive *representation* combining some parts also manifests the others and therefore the whole.

In terms of categories, the points on the figure are selected under the category of reality while the others are determined under the category of negation. I take this to be faithful to the spirit of Williams' (2018: 78–9) claim that 'the subject's recognition of the possibility of accessing any part of boundless space' is key to the unity of space.

It will be objected that it is only the points immediately in the vicinity of the figure that are determined as outside it. But if this region of points delimiting the figure has a limit, the latter will also have to be determined by reference to what is outside, so the question of whether only the immediate vicinity is concerned resurfaces. This is where the *homogeneity* of space is crucial: the exclusion of the immediate 'outside' of the figure from inclusion in it is simultaneously that of the whole rest of space, since the rest of space is 'more of the same'.[31] This is why space as a whole is manifested as a *horizon* for the determination of a spatial figure (see OS2015: 48; Rosefeldt 2022: 8): a representation of a figure in space defines a representation which makes the whole of space manifest.[32]

Does this address PUU(a)? Insofar as representing one possible part of space as actual makes space as a whole manifest, the figurative synthesis is indeed a synthesis that, through combining some parts of the manifold of space (i.e. the content) into a figure, represents space qua whole as set of all possible spatial parts. But, it will be objected, what I have just described as a formal intuition of space is in fact an intuition of *a finite figure in space*, not of *space as containing* this finite figure. This, however, is just a matter of what object the subject attends to in producing the representation of the finite figure. In drawing a line, one can direct one's attention to the line that is produced as in geometry or, as Kant indicates, to something else, for example, to the

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

act through which the subject synthesizes the spatial manifold (B135; see Onof 2022: 450–2). In the first case, we have an intuition of a geometric object, in the second, one is conscious of one's self as manifested in its affection of inner sense.

The proposal here is that if, as another alternative, the subject directs her attention to the background space within which the figure is drawn, the intuition is thereby an intuition of space containing a figure. It is in this sense that, in a formal intuition, space is 'represented as an object' (B160–1n.): Kant is clear that space is *not* an object but a condition of objectivity (B65–6/A48–9), but it can be represented 'as an object' insofar as it is a *horizonal* object (OS2015: 47) defining the background to any constructed figure.[33] There will be different formal intuitions of space depending upon the figure that is drawn. Hence the concept of space of the TAe is a 'general concept of spaces in general' (A25/B39; see Blomme 2012: 147).

5. The solution to problems arising from the infinite magnitude of space

Problem PUI is the problem of our ability as finite cognizers to synthesize something of infinite magnitude. Since, in the proposed solution to problem PUU(a), the issue of the magnitude of space did not arise, the figurative synthesis's ability to produce a formal intuition of space by determining a figure in it addresses the worry raised by PUI: the infinite magnitude of space will not impede its function.

Problem PUI assumes of course that there is indeed a potential worry here arising from space being given as an infinite magnitude. It is noteworthy that Paul Guyer (2018) has recently questioned this claim about space. He argues that the judgement that space has an infinite magnitude must be grounded inferentially. Specifically, the claim of infinite magnitude (A25/B39) requires (i) the premise that any spatial region can only be represented as part of a larger space, and (ii) that the truth of premise (i) holds no matter how large the spatial region is (Guyer 2018: 184ff.).[34] He draws support (p. 186) for his claims from Kant's comments on Kästner's response to Eberhard (20: 412): there, Kant could be read as stating premise (i) and, by adding that it is true irrespective of the size of the region, implicitly to be making an inference.

If such an inference were required,[35] the claim that space is a given infinite magnitude would be a *regulative* claim of reason. But Guyer claims that, through subreption (analogously to the case of the Highest Being at A619–20/B647–8) it is taken by Kant interpreters to be *constitutive*.[36] If Guyer is right, one cannot assert that space is a *given* infinite magnitude: this claim would be what he calls a 'myth'.

5.1 The problem of space's infinite given magnitude

Guyer argues that, when I draw a figure in space, the infinity of space is not *given* but only *inferred* through the premise that the figure could have been of any size. But he is thereby assuming that the figure has a size. If it does, then there will indeed be an inference in which, having drawn a figure with a determinate magnitude, one then *further* observes that this magnitude is indifferent, and concludes to the infinity of space.

That there is a relative size of all the drawn parts of the figure is clear. But what are the grounds for claiming that there is an absolute size, by which I mean a size of the

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

figure itself, not simply relative to its parts? Such a magnitude requires a unit; but if a unit is chosen in drawing the figure, this still only determines the figure's size relatively to this unit which has been defined in drawing the figure.[37] What is missing and would be needed for an absolute size to be defined is a reference to something outside this particular intuition (and act of synthesis).

As a result, Guyer's premise can be rejected. But the issue he raises exhibits an important feature of geometric constructions (Guyer 2018: 195) which could nevertheless be thought to count against my claim. Namely, a single intuition *represents a set of possible intuitions* in geometric constructions (A713/B741). That is, in producing the intuition, 'we have taken account only of the action of constructing the concept, to which many determinations, e.g. those of the magnitude of the sides and the angles, are entirely indifferent' (A714/B742). Kant is here contrasting the magnitude that is represented by the construction, which is 'indifferent', and that of an actual physical constructed figure, which is determinate. Is this determinacy compatible with my above claim? I think not, because I take this claimed determinacy simply to be a determinacy with respect to a unit specified in the figure, and thereby to define only a relative magnitude.

That is, it would make no sense to ask how big my figure is with respect to the tree in front of me: this is because any such determination with respect to something outside the figure is excluded insofar as in the construction, 'we have taken account only of the action of constructing the concept' (A714/B742). This is precisely not what the landscape painter does in drawing a scaled down tree on his canvas. So determinacy of a geometric construction by the imagination must be defined internally to the figure.

To say that space as horizonal object contains a drawn figure whose magnitude is indeterminate is as much as saying that space is given as an infinite magnitude. This is how we should understand Kant's stating, in his response to Kästner: 'Now one cannot name a magnitude, in comparison with which each assignable [unit] of the same type is only equal to a part of it, anything other than infinite' (20: 419; Kant 2014: 309). It is certainly the case that, in reflecting upon this issue, an inference to the infinite magnitude is made, as Guyer detects in his reading of Kant's response to Kästner: but this is the subject of reflection making an inference, not the subject of experience for whom the figure has an indefinite absolute magnitude and for whom space is given as an infinite magnitude.[38]

That no inference is required can further be understood by identifying the categories that are involved in determining as an infinite magnitude the space given through the generation of its formal intuition. Consider first the case of the subject focusing upon the figure drawn in the figurative synthesis. I indicated above that the categories of *reality* (inside the figure) and *negation* (outside the figure) are involved. One can add that, in terms of quantity, the synthesis is under the category of *plurality*.

Consider now the focus being rather upon the background, i.e. space as a horizonal object: different categories will be involved. In terms of quantity, it must now be as a *totality* that it is determined. This still leaves us with the concern that, since the schema of quantity is number (A142/B182), this quantity would seem to have to be a finite number. But all depends upon the category of quality. As we saw above, the AUC of space as (formal) intuition requires that a figure be represented (figurative synthesis). Space is then represented as larger than this figure since it contains it, and,

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

because of the absence of scale, simultaneously as having a magnitude that is larger than any finite number, i.e. infinite (see also Raysmith 2021: 11; Rosefeldt 2022: 13).

I claim that the given infinite magnitude is thereby determined under the category of *limitation*. Indeed, this category corresponds to the form of the unity of judgement characterizing an *infinite judgement*, i.e. a judgement through which the 'infinite sphere of the possible is thereby limited only to the extent', here, that the magnitude is determined as not being any finite number (see A72-3/B97–8; see also Reflexion 3063, 16: 636). In this case, the judgement that the magnitude of space is infinite means that it 'is greater than any space that I can describe' (response to Kästner, 20: 418; see Guyer 2018: 186).[39]

6. The solution to problem PUU(b)

Problem PUU(b) questions how it is that the unity of the intuition of space always refers to the same space. The short answer is that space is characterized as a unicity by Kant so that all formal intuitions involve determinations of some parts of it. But what if this unicity of space is questioned?

When dealing with objects of outer sense, the claim that this oak tree is the same that I walked past yesterday finds its justification in the fact that it is a substance which has perdured since yesterday. In the case of a geometric figure my imagination produces, this is not true. As Guyer (2018: 190–1) points out, when I represent a triangle at some point in time, any figure I later represent is not related to the triangle in terms of spatial location for instance, i.e. as being near or far from the triangle I represented earlier. From this, however, he concludes *pace* OS2015 that it is not assumed that there is a single space that both triangles are drawn inside (p. 191).

While this argument has some immediate appeal, like the argument against the infinite magnitude of space, it relies upon a covert assumption which bears examination. Indeed, when inferring from the observation that I do not locate the triangle I represent now in relation to the one I represented yesterday, the conclusion that these triangles are in different spaces can only go through on the assumption that I am indeed *locating* the triangles in the space I represent them in. As Guyer seems to agree (2018: 191, n. 12), however, I do not locate a figure when I draw it in space: such locating would require at least one other reference point to be given prior to my construction. The proper inference to draw from this is that my figure is indeterminate as to its spatial (and indeed temporal) location. So when I draw another figure tomorrow, its location is equally indeterminate and that is the reason they are not located with respect to one another. It is not possible to infer from this that these figures are drawn in different spaces.

One might nevertheless worry that the space in which I draw my figure is some 'imaginative' space (and therefore a different one every time): how could it be the space in which empirical objects are located? But the difference between empirical objects and my figure is precisely determinacy (at least partial) versus indeterminacy of these aspects. It is in fact required that my geometric constructions should be in the same space in which I determine empirical manifolds of outer sense: only then can I determine these otherwise indeterminate constructions as applying to a particular manifold here and now. Only in this way can the figurative synthesis enable the generation of spatial locations for the objects of outer sense I determine: this is its role in the B-TD (Onof 2022: 456–7), as Kant explains when he says that the combination of 'everything that is to be determined in space' must agree with the 'combination' determined through the 'unity of the [figurative] synthesis' of the spatial manifold (B161).[40]

Further, the unicity of space is a requirement of geometric principles (see earlier), i.e. of geometry as science of space. If every triangle I drew were in a different space, geometry would require the relations between these spaces to be determined, which in turn requires a single embedding space (see also OS2015: 21). More broadly, the unicity of space follows from the fact that the space that is represented (in perception or geometry) is always the same set of all possible basic spatial relations between points as horizon for a particular actualized combination of such relations which is the spatial figure.[41]

7. Concluding summary

In this article, I have drawn upon several recent publications devoted to understanding the notion of the unity of space and the consistency between B160–1n. and the TAe, to propose an interpretation which revises Onof and Schulting 2015. I do not understand space as a brute given: a formal intuition of space has a unity which is that of apperception since it is produced by the figurative synthesis. The analytical unity of consciousness of this formal intuition, i.e. one's apperceptive consciousness of it, requires the representation through the figurative synthesis of a figure as combination of possible basic spatial relations between points. The set of such relations is given by the form of intuition which is an epistemic capacity characterizing sensibility. Insofar as these possibilities are prior to the representation, this content of the resulting intuition, i.e. the formal intuition of space as horizonal object, has a whole-part order of priority even though the synthesis producing it proceeds by combination of components of the manifold. Space, as set of all possible basic spatial relations, is unique and given as an infinite magnitude.

This account of the unicity, infinity and unity of space, has enabled problems PUU and PUI, raised by footnote B160n., to be addressed without needing to appeal to any other than the figurative synthesis; it has also clarified the cogency and grounds for the claims that space must be unique and given as an infinite magnitude.

Acknowledgements. This article was discussed in detail with Dennis Schulting for whose input I am grateful. I would also like to acknowledge Andrew Roche's perceptive comments which contributed to significantly improving the clarity of the argument, and Richard Aquila's careful reading of the final version which identified remaining unclarities.

Notes

1 The English citations from the *Critique of Pure Reason* are from Kant 1998, cited in standard A/B format. I use only one abbreviation throughout: 'OS2015' for Onof and Schulting 2015.

2 This loosely follows the two problems Tobias Rosefeldt (2022) flags, although he does not have a bi-partite unity-unicity problem.

3 Williams' proposal also faces a problem in its attempt to address PUU(b), as Rosefeldt (2022: 4) argues.

4 Although I classify his proposal as broad conceptualist, Rosefeldt's proposal is sympathetic to certain non-conceptualist views (Rosefeldt 2022: 15–17) so that it sits rather between the conceptualist and non-conceptualist camps.
5 Rosefeldt (2022: 12–13) also thereby provides an account of how to understand the infinite magnitude claim in addressing PUI.
6 Note that Rosefeldt acknowledges that 'decomposing synthesis also involves a composing synthesis' (2022: 7) in his example of drawing a line through a circle, but the composing synthesis on his account only concerns the drawing of the dividing line, not the circle, so has a limited role.
7 These, in turn, enable the determination of empirical manifolds by making the synthesis of apprehension possible (B161).
8 It is also unclear how, in the end, Raysmith's notion of formal intuition whose unity is that of original space avoids something like the problem of the Brute Given, insofar as empirical manifolds would be given with a form whose unity is alien to that of apperception.
9 I hinted at this in Onof (2022: 445) by using speech quotes in referring to a 'unity'.
10 This has puzzled some commentators who take it to be due to Kant's having poorly organized the content of the TAe (even in the B version!).
11 I differ however on the consequences of this identification.
12 The formal intuition is a representation of what Kant also calls metaphysical space (Kant 2014: 309; see Onof and Schulting 2014: 293).
13 The elements of this manifold correspond to sensations but are not identical to them insofar as they have the form of space. I thank the editor for getting me to reflect on this point.
14 Longuenesse (1998: 217ff.) puzzles over the apparent changing meaning of 'form of intuition'. While it seems reasonable to identify the form of sensibility in the TAe with the form of intuition, a problem arises when Kant claims that it is 'also called pure intuition' (B34/A20). But addressing this problem is not best dealt with by identifying form of and formal intuition. I examine this issue a little further.
15 It will be countered that there is a (non-countable) infinity of such points. However, my finite imaginative powers will link a finite number of points which are mutually connected with the basic relation 'straight line'.
16 The figurative synthesis in the B-TD takes over the function of the pure synthesis of apprehension of the A-TD (A99; see Onof 2022: 456–7). From Kant's presentation of the corresponding empirical synthesis, one can infer that generation/apprehension of a figure involves 'run[ning] through and taking together' the manifoldness of the possible basic spatial relations, thereby actualizing some of them.
17 As Henny Blomme (2012: 151) explains, a key difficulty in investigating the nature of space is that, in this case, form precedes matter.
18 My thanks go to a reviewer for getting me to comment upon this.
19 This is independent of whether this synthesis is the one at stake in B160–1n.
20 Note that this possibility depends not only upon the whole-part priority, but also upon its infinite magnitude insofar as there is no restriction upon how far extension proceeds (in any direction).
21 Relation is only relevant to the case of outer sense insofar as distinct empirical objects may be at stake.
22 *Pace* Guyer (2018: 11; see section 5), this space is needed to ground geometry: 'Thus, the geometer, as well as the metaphysician, represents the original space as infinite, in fact as infinitely given' (response to Kästner, 20: 419; Kant 2014: 309).
23 This non-conceptual content of space is essential to geometric axioms (A732/B761) and demonstrations (A734/B762) as Mathias Birrer (2017: 218–20) shows, and also as involved in the 'perceptual cognition of things' (Onof 2016: 220–2), which also accounts for animals' spatial cognition.
24 Why does Kant not spell out the fifth property in the metaphysical exposition? This is likely because, for Kant, this property, which is determined as 'continuity', can only be understood by reference to time, i.e. to the possibility of continuous motions. Note that the third property is not made explicit but implied by the validity of geometric principles anywhere in space.
25 *Pace* Friedman (1992: 60ff.), from the fact that continuity is definable in logical terms with the use of a combination of universal and existential quantifiers, it does not follow that the spatial property it identifies is reducible to such a definition (see OS2015: 36).
26 This characterisation of space is also a mark of the concept of space in the metaphysical exposition (B37–40).

27 Prauss (2019: 407) argues that Kant's understanding of the synthesis generating a line as combinatory misses its 'original-dynamic' (*Ursprünglich-Dynamisch*) character, but he does not thereby examine the role of the manifold. It seems to me that his proposal as to how an *analytic* continuum (parts) of space is obtained from a synthetic continuum (whole) is rather relevant to understanding how the form of space gives the manifold. Prauss's proposal interpreted in this way would be compatible with my understanding of the manifold in terms of basic relations (although *pace* Prauss 2019: 399, n. 6, these would involve 'trinit[ies]' (*Dreiheit*) of two points separated by an extension (*Ausdehnung*) rather than a point separating two extensions).

28 There is no requirement that the synthesis be pre-categorial: as Williams (2018: 73–4) points out, pre-categoriality is apparently not fulfilled by what Longuenesse and Friedman propose, much as they endorse this requirement.

29 The figure's spatial relations are thereby determined but, as I show further, the magnitudes only relatively so.

30 In response to a reviewer's comment, this content is only a set of possibilities prior to the synthesis. It is this set that characterizes the form of sensibility within which empirical manifolds are received.

31 Note that the rest of space being infinite in magnitude is not a problem here: I return to this in section 5.

32 This addresses the worry Messina (2014: 20–1) identifies for (broad) conceptualist readings which would seem to have to deny that the figurative synthesis produces a determinate intuition if it is to be responsible for the unity of space. The formal intuition of space is always an intuition of space with a determination in it, in line with Rosefeldt's (2022: 13-4) claim that space cannot be represented as empty.

33 This brings about the same differentiation background/foreground or horizon/figure that Rosefeldt does through the decomposing synthesis he proposes, but without resorting to any other synthesis than the figurative synthesis.

34 Guyer does not endorse the claim that this larger one is unique, an issue I discuss below.

35 Rosefeldt's (2022: 11) appeal to Husserl's eidetic variation could be interpreted as a case of the use of reason to infer the infinity of space.

36 It is however unclear that interpreters are typically guilty of a hypostatization of space.

37 Note that angles are themselves determined trigonometrically through *relative relations* of lengths.

38 Thanks to a reviewer for suggesting this.

39 Although I endorse Blomme's (2012: 158–9) mapping of Kant's third and fourth characterizations of space onto determinations under the categories of quality and quantity (A24–5/B39–40), I differ on how to understand how these categories are applied.

40 This metaphysical space is distinct from the notion of absolute space of the *Metaphysical Foundations of Natural Science*, which is a determination of the former required by Newtonian science.

41 Roche (2018: 50) is therefore wrong to assume that the given unicity of space is only instantaneous.

References

Allison, H. E. (1983) *Kant's Transcendental Idealism: An Interpretation and Defense*. New Haven: Yale University Press.

Birrer, M. (2017) *Kant und die Heterogenität der Erkenntnisquellen*. Berlin: De Gruyter.

Blomme, H. (2012) 'The Completeness of Kant's Metaphysical Exposition of Space'. *Kant-Studien*, 103, 139–62.

Dufour, E. (2003) 'Remarques sur la note du paragraphe 26 de l'Analytique transcendantale: Les interprétations de Cohen et de Heidegger'. *Kant-Studien*, 94(1), 69–79.

Friedman, M. (1992) *Kant and the Exact Sciences*. Cambridge, MA: Harvard University Press.

—— (2012) 'Kant on Geometry and Spatial Intuition'. *Synthese*, 186, 231–55.

Gardner, S. (1999) *Kant and the Critique of Pure Reason*. Abingdon: Routledge.

Guyer, P. (2018) 'The Infinite Given Magnitude and Other Myths about Space and Time'. In O. Nachtomy and R. Winegar (eds), *Infinity in Early Modern Philosophy* (Berlin: Springer), 181–204.

Hanna, R. (2005) 'Kant and Nonconceptual Content'. *European Journal of Philosophy*, 13(2), 247–90.

https://doi.org/10.1017/S1369415423000109 Published online by Cambridge University Press

Kant, I. (1998) *Critique of Pure Reason*. Trans. and ed. P. Guyer and A. W. Wood (Cambridge Edition of the Works of Immanuel Kant). Cambridge: Cambridge University Press.

—— (2014) 'On Kästner's Treatises'. Trans. C. Onof and D. Schulting. *Kantian Review*, 19, 305–13.

Longuenesse, B. (1998) *Kant and the Capacity to Judge: Sensibility and Discursivity in the Transcendental Analytic of the Critique of Pure Reason*. Trans. C. T. Wolfe. Princeton: Princeton University Press.

McLear, C. (2015) 'Two Kinds of Unity in the Critique of Pure Reason'. *Journal of the History of Philosophy*, 53(1), 79–110.

Messina, J. (2014) 'Kant on the Unity of Space and the Synthetic Unity of Apperception'. *Kant-Studien*, 105(1), 5–40.

Onof, C. (2016) 'Is there Room for Non-Conceptual Content in Kant's Critical Philosophy?'. In D. Schulting (ed.), *Kantian Nonconceptualism* (London: Palgrave), 199–226.

—— (2022) 'The Transcendental Synthesis of the Imagination and the Structure of the B Deduction'. In G. Motta, D. Schulting and U. Thiel (eds), *Kant's Transcendental Deduction and the Theory of Apperception: New Interpretations* (Berlin: de Gruyter), 437–60.

Onof, C and D. Schulting (2014) 'Kant, Kästner and the Distinction between Metaphysical and Geometric Space'. *Kantian Review*, 19(2), 285–304.

—— (2015) 'Space as Form of Intuition and as Formal Intuition: On the Note to B160 in Kant's *Critique of Pure Reason*'. *Philosophical Review*, 124(1), 1–58.

Parsons, C. (1983) *Mathematics in Philosophy*. Ithaca, NY: Cornell University Press.

Prauss, G. (2019) 'Zur Begreifbarkeit der Ausdehnung von Zeit und Raum'. *Kant-Studien*, 110(3), 397–412.

Raysmith, T. (2021) 'Kant's Original Space and Time as Mere Grounds for Possibilities'. *Kantian Review*, 27(1), 23–42.

Roche, A. F. (2018) 'Kant's Transcendental Deduction and the Unity of Space and Time'. *Kantian Review*, 23(1), 41–64.

Rosefeldt, T. (2022) 'Kant on Decomposing Synthesis and the Intuition of Infinite Space'. *Philosophers' Imprint*, 22, 1–23.

Schulting, D. (2017) *Kant's Radical Subjectivism*. London: Palgrave/Macmillan.

Williams, J. J. (2018) 'Kant on the Original Synthesis of Understanding and Sensibility'. *British Journal for the History of Philosophy*, 28(1), 66–86.

Cite this article: Onof, C. The Unicity, Infinity and Unity of Space. *Kantian Review*. https://doi.org/10.1017/S1369415423000109