



This is a postprint/accepted version of the following published document:

López Morales, Manuel José; Dinis, Rui; García Armada, Ana. Near-Optimal Detection of CE-OFDM Signals with High Power Efficiency via GAMP-based Receivers. In: 2022 IEEE Globecom Workshops (GC Wkshps) Proceedings, Rio de Janeiro, Brazil, 4-8 December 2022. IEEE, Jan. 2023,, Pp.7-12

DOI: https://doi.org/10.1109/GCWkshps56602.2022.10008696

© 2022 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Near-Optimal Detection of CE-OFDM Signals with High Power Efficiency via GAMP-based Receivers

Manuel Jose Lopez Morales Universidad Carlos III de Madrid Madrid, Spain mjlopez@tsc.uc3m.es Rui Dinis DEEC-FCT Nova University of Lisbon Lisbon, Portugal rdinis@fct.unl.pt Ana Garcia Armada Universidad Carlos III de Madrid Madrid, Spain agarcia@tsc.uc3m.es

Abstract—A quasi-optimum receiver based on the generalized approximate message passing (GAMP) concept is proposed for constant envelope orthogonal frequency division multiplexing (CE-OFDM) signals. Large modulation index results in large power efficiency for CE-OFDM, but the phase modulator introduces nonlinear distortion effects, precluding good performance for a simple phase detector. Our simulation results show that the GAMP receiver can achieve quasi-optimum performance and it can outperform the linear OFDM and CE-OFDM with phase detectors, for both additive white Gaussian noise (AWGN) and frequency selective channels.

Index Terms—OFDM, phase modulation, GAMP, nonlinearity.

I. INTRODUCTION

rthogonal frequency division multiplexing (OFDM) has been widely adopted in communications systems due to its robustness against multipath propagation, large spectral efficiency and ease of implementation. Since OFDM suffers from high peak-to-average-power (PAPR) [1], its use with highly efficient nonlinear (NL) power amplifiers remains difficult. To overcome these difficulties, constant envelope OFDM (CE-OFDM) was proposed [2] and it was shown to outperform the conventional linear OFDM [3]. In millimeter wave and THz scenarios, which are of interest for the future 6G wireless communications technology, the power constraints are really severe so it is interesting to utilize energy efficient modulation techniques such as the CE-OFDM, which is an interesting alternative to classical OFDM. For low modulation index, the phase modulation introduces a DC component that degrades the performance, while for large modulation index, which is of interest since it results in a larger energy efficiency [3], we have phase excursions over $\pm \pi$ that lead to nonlinear distortions, which degrade the performance of the phase detector used with CE-OFDM [2]. State-of-the-art receivers for CE-OFDM are proposed in [4], [5], based on three types: brute force search (prohibitively large complexity), phase demodulator and

978-1-6654-5975- 4/22 © 2022 IEEE

linear receivers (both valid only for low modulation indexes). However, nonlinear distortion effects can lead to performance improvements, as shown in [6].

Several techniques have been applied to approach the optimum detection performance but they are limited by performance or complexity [6]. A gaussian message passing for massive MIMO-NOMA is proposed in [7]. One of the most recent receivers for nonlinear OFDM is based on the generalized approximate message passing (GAMP) algorithm [8]–[10]. A GAMP receiver for classical complex OFDM nonlinearities is proposed in [8]. A fast GAMP (fGAMP), which approximates some statistics to a normal distribution, thus degrading the performance, is proposed for classical OFDM nonlinearities [10]. However, the design of low complexity quasi-optimum receivers for CE-OFDM is still an open issue.

In this paper, we consider CE-OFDM schemes with strong non-linear distortion caused by large modulation indices and present powerful receivers based on the GAMP and fast GAMP algorithms which are able to approach the optimal performance. Extending the GAMP and fGAMP [8], [10] to CE-OFDM signals is not straightforward, and it is addressed in this manuscript. Furthermore, the damped GAMP, which is not always considered in the literature (e.g. [10]), is implemented to avoid the instabilities of the GAMP and is adapted to suit the particularities of CE-OFDM signals. We effectively implement the damped GAMP for CE-OFDM signals and we show its robustness against both AWGN and multipath channels, even when imperfect channel estimation occurs, which is also not considered in the literature. Our performance results show that the proposed receiver can work well for CE-OFDM with large modulation indexes and can outperform the linear OFDM for the same spectral efficiency when channel coding is applied. We also analyze the complexity of the proposed receivers and compare them with the ones in the literature.

II. CE-OFDM SIGNALS AND OPTIMUM DETECTION

A. Signal Model

Each CE-OFDM symbol is composed of N complex data symbols that form the block $\mathbf{S} = [S_0, S_1, \dots, S_{N-1}]^T$. To guarantee a real-valued input to the phase modulator, the data

This work received funding from the European Union (EU) Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie ETN TeamUp5G, grant agreement No. 813391, the Spanish National Project IRENE-EARTH (PID2020-115323RB-C33) (MINECO/AEI/FEDER, UE) and Portuguese FCT Instituto de Telecomunicações project UIDB/50008/2020.

symbols are constrained to have Hermitian symmetry, which means that $S_m = 0$ for $m \in \{0, N/2\}$ and $S_{N-m} = S_m^*$, otherwise. The data symbols are selected from a Kary quadrature amplitude modulation (K-QAM) constellation. The constellation is normalized to have a unit power. An oversampled version of the CE-OFDM signal is obtained by adding N(M - 1) idle subcarriers to the data block, i.e., with $S_m = 0$ for $m = N, \ldots, NM - 1$, with M denoting the oversampling factor. The frequency domain vector **S** is transformed into the time domain via inverse discrete Fourier transform (IDFT) operation $\mathbf{s} = \mathbf{FS} = [s_0, s_1, \cdots, s_{NM-1}]^T$, where **F** is an NM by NM matrix with elements

$$\mathbf{F}_{n,m} = \frac{1}{\sqrt{NM}} e^{\frac{j2\pi nm}{NM}}, \quad n,m = 0,\dots,NM-1.$$
 (1)

A cyclic prefix (CP) of sufficient length is appended to the time-domain vector. The *n*-th sample of s is approximately Gaussian with zero mean and variance $\mathbb{E}[|s_n|^2] = \sigma^2 = (N-2)/(NM)^2$, according to the central limit theorem (CLT). These time-domain samples are submitted to a phase modulator, giving the transmitted signal samples in time [3] with f() denoting the phase modulation and b the modulation index

$$y_n = f(s_n) = \exp\left(j2\pi b s_n/\sigma\right).$$
 (2)

Later, the signal is transmitted via a multipath channel with impulse response $\{h_n\}$. The received signal is represented as

$$r_n = y_n \cdot h_n + w_n = \exp\left(j2\pi b s_n/\sigma\right) \cdot h_n + w_n, \quad (3)$$

where w_n represents the *n*-th AWGN sample with zero mean and variance $\mathbb{E}\{|w_n|^2\} = \sigma_w^2 = N_0$, with N_0 the one-sided noise power spectral density. A long enough cyclic prefix was added to (2), so r_n is represented in the frequency domain as

$$R_m = H_m Y_m + W_m, \tag{4}$$

where R_m , H_m , Y_m and W_m , $m = 0, 1, \dots, NM - 1$ are the DFTs of the signals r_m , h_m , y_m and w_m , respectively.

Typically, the detection of CE-OFDM in a multipath environment is made via a frequency-domain equalizer (FDE) and a phase detector, which can get a "linear" version of the phase, provided that there is no phase ambiguity [2]. Note that CE-OFDM retains the robustness against multipath fading and ISI, and high spectral efficiency of classical OFDM [2].

Although the amplification process of CE-OFDM signals can be distortionless, the phase of these signals with high modulation index results in a nonlinear distortion due to phase excursions beyond $\pm \pi$. They create an in-band distortion and an out-of-band (OOB) radiation. Whilst the latter may be mitigated through filtering, the former behaves as an additional noise that degrades the performance [11]. While a Bussgang receiver can reduce the in-band nonlinear distortion, it does not take advantage of the inherent information [3].

B. Optimum Detection

The asymptotic optimum performance of a given modulation is conditioned by the pairwise error probability (PEP) between its symbols. In the presence of nonlinearities, the squared Euclidean distance between two OFDM data sequences differing in one bit is $D_{\rm NL}^2 = \sum_{m=0}^{NM-1} \left| Y_m^{(2)} - Y_m^{(1)} \right|^2 = 4GE_b$, where $G = D_{\rm NL}^2/D^2 = D_{\rm NL}^2/4E_b$, which represents the optimum asymptotic gain [3] and can be computed as

$$G = \sigma^2 \frac{\int_{-\infty}^{\infty} |f'(s)|^2 p(s) ds}{\int_{-\infty}^{\infty} |f(s)|^2 p(s) ds},$$
(5)

where p(s) is the Gaussian probability density function (PDF) associated to CE-OFDM signals and f'(s) represents the first derivative of f(s). The asymptotic gain depends only on the modulation index, i.e. $G = (2\pi b)^2$ [3]. Thus, the optimum performance of the CE-OFDM is given by

$$P_b \approx Q\left(\sqrt{G\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{(2\pi b)^2 \frac{2E_b}{N_0}}\right),\tag{6}$$

where E_b is the average power times the bit duration.

A receiver based on the phase modulator is optimal only for small modulation indexes (i.e. $2\pi b < 0.7$). For greater values, the phase modulator suffers from phase ambiguities, which degrades the performance with respect to the optimum, as shown in [3]. An optimum receiver based on the maximum like-lihood detection makes a brute force search of all the possible transmitted sequences in reception [4], and has an unfeasible complexity of $K^{N/2-1}$. Therefore, this letter presents a quasi-optimum detector based on the GAMP algorithm, described in the next section, which can outperform the phase modulator and even the linear OFDM, while avoiding the unbearable complexity of the brute force search algorithm.

III. SUB-OPTIMUM RECEIVER FOR CE-OFDM SIGNALS

A. Belief Propagation Receiver

The formulation and the derivations below are based on the AWGN channel, and the case of the multipath channel will be extended in Sec. III-D. The received signal (3) in vector form is $\mathbf{r} = [r_0, r_1, \dots, r_{NM-1}]^T$, which is simplified to

$$\mathbf{r} = f(\mathbf{s}) + \boldsymbol{w} = \mathbf{h} \otimes f(\mathbf{FS}) + \boldsymbol{w}. \tag{7}$$

Eq. (7) is equivalent to a general problem statement for the GAMP algorithm [12], which belongs to a class of Gaussian approximations of loopy belief propagation for dense graphs. The GAMP sum-product variant approximates the minimum mean-squared error (MMSE) estimates of y and S. The GAMP algorithm algorithm [12] is administrated in the Gaussian of the signal structure particularities of CE-OFDM. They differ in the expectation and variance in the output nonlinear step and the symbols placing. Besides, the integral defined in (12) has a complex input and

Algorithm 1: Basic GAMP decoder with damping (β)

Input: r Output: \hat{S} Parameters: t_{max} , F, $g_{in}(\cdot)$, $g_{out}(\cdot)$, σ_w^2 , β 1) Initialization: 1 t = 1, $\hat{S}(1) = \tilde{S}(1) = \hat{x}(1) = \mathbf{0}_{NM,1}$, $\mu^S(1) = \mathbf{1}_{NM,1}$ 2) Output linear step: 2 $\mu_n^q(t) = \frac{1}{NM} \sum_{m=0}^{NM-1} \mu_m^S(t)$, $\forall n$ 3 $\hat{q}_n(t) = \sum_{m=0}^{NM-1} F_{n,m} \hat{S}_m(t) - \mu_n^q(t) \hat{x}_n(t-1)$, $\forall n$ 3) Output non-linear step: 4 $\hat{x}_n(t) = (1 - \beta) \hat{x}_n(t-1) - \beta g_{out}(\hat{q}_n(t), \mu_n^q(t), r_n)$, $\forall n$ 5 $\mu_n^x(t) = (1 - \beta) \tilde{S}_m(t-1) - \beta \frac{\partial g_{out}(\hat{q}_n(t), \mu_n^q(t), r_n)}{\partial \hat{q}}$, $\forall n$ 4) Input linear step: 6 $\tilde{S}_m(t) = (1 - \beta) \tilde{S}_m(t-1) + \beta \hat{S}_m(t)$, $\forall m$ $\mu_m^v(t) = \left(\frac{1}{NM} \sum_{n=0}^{NM-1} \mu_n^x(t)\right)^{-1}$, $\forall m$ 7 $\hat{v}_m(t) = \tilde{S}_m(t) + \mu_m^v(t) \sum_{n=0}^{NM-1} F_{n,m}^* \hat{x}_n$, $\forall m$

- 5) Input non-linear step:
- **8** $\hat{S}_m(t+1) = a_{in}(\hat{v}_m(t), u_{-}^v(t)), \forall m$



Fig. 1. GAMP decoder architecture.

a real output and different integral limits and domain, which differs from the implementation in [8]. Also, (18) and (19) have complex values as input from the OAM constellation but result in a real output and the scalar nonlinear functions $g_{in}(\cdot)$, $g_{in}'(\cdot),\,g_{out}(\cdot)$ and $g_{out}'(\cdot),$ depend on the employed modulation format and the shape of the nonlinearity f(z). Each iteration of the GAMP consists of four steps. The first one (output linear) produces estimates of an intermediate vector $\{\hat{q}_n\}$ with corresponding variances $\{\mu_n^q\}$. The second one (output nonlinear) produces estimates of intermediate vector $\{\hat{x}_n\}$ with corresponding variances $\{\mu_n^x\}$. The third one (input linear) produces estimates of intermediate vector $\{\hat{v}_n\}$ with corresponding variances $\{\mu_n^v\}$. And the last one (input nonlinear) produces estimates of vector $\{\hat{S}_n\}$ with corresponding variances $\{\mu_n^S\}$. Since \mathbf{F} is the Fourier transform matrix, the input and output linear steps can be efficiently implemented using fast Fourier transform. A general architecture of the GAMP decoder is illustrated in Fig. 1. In the following sections, we drop the element indexes n and m and the iteration number t notation for clarity.

B. Output nonlinear step

The output function $g_{out}(\hat{p}, \mu^p, r)$ of the sum-product loopy belief propagation algorithm is given by [13]

$$g_{out}(\hat{q}, \mu^q, r) := \frac{\hat{s}_0 - \hat{q}}{\mu^q}, \quad \hat{s}_0 := \mathbf{E}[s|\hat{q}, r, \mu^q]$$
(8)

and the negative derivative of $g_{out}(\cdot)$ is given by [13]

$$-\frac{\partial}{\partial \hat{q}}g_{out}(\hat{q},r,\mu^q) = \frac{1}{\mu^q} \left(1 - \frac{\operatorname{var}[s|\hat{q},r,\mu^q]}{\mu^q}\right), \quad (9)$$

where the expectation and variance over the distribution are

$$p(s|\hat{q}, r, \mu^q) \propto \exp\left(-\frac{||r - f(s)||^2}{2\gamma\sigma_w^2} - \frac{||\hat{q} - s||^2}{2\mu^q}\right).$$
 (10)

For the CE-OFDM nonlinear model (2), we can define

$$\mathbb{E}[s|\hat{q}, r, \mu^{q}] = \frac{I_{1}}{I_{0}} \quad \text{and} \quad \operatorname{var}[s|\hat{q}, r, \mu^{q}] = \frac{I_{2}}{I_{0}} - \left|\frac{I_{1}}{I_{0}}\right|^{2}, \quad (11)$$

where the integrals I_u , u = 0, 1, 2 are given by

$$I_u = \int_{-\infty}^{\infty} s^u e^{-\frac{||r-f(s)||^2}{2\gamma \sigma_w^2} - \frac{||\hat{q}-s||^2}{2\mu^q}} ds.$$
 (12)

Eq. (12) is obtained using numerical integration since it cannot be expressed in closed-form using elementary functions. Nevertheless, $E[s|\hat{q}, r, \mu^q]$ and $var[s|\hat{q}, r, \mu^q]$ can be approximated following [10] for the fGAMP, as

$$\mathbf{E}[s|\hat{q}, r, \mu^{q}] = \frac{\mu^{q}r + \sigma_{w}^{2}\hat{q}}{\mu^{q} + \sigma_{w}^{2}}, \quad \mathbf{var}[s|\hat{q}, r, \mu^{q}] = \frac{\sigma_{w}^{2}\mu^{q}}{\mu^{q} + \sigma_{w}^{2}}$$
(13)

when $|\hat{q}| \leq 1$, and when $|\hat{q}| > 1$ we have

$$\mathbf{E}[s|\hat{q}, r, \mu^{q}] = \frac{\mu^{q}r + t^{2}\sigma_{w}^{2}\hat{q}}{\mu^{q} + t^{2}\sigma_{w}^{2}}, \quad \operatorname{var}[s|\hat{q}, r, \mu^{q}] = \frac{t^{2}\sigma_{w}^{2}\mu^{q}}{\mu^{q} + t^{2}\sigma_{w}^{2}}.$$
(14)

C. Input nonlinear step

The sum-product variant of the loopy belief propagation algorithm for the input nonlinear step $g_{in}(\cdot)$, $g'_{in}(\cdot)$ is [13]

$$g_{in}(\hat{v},\mu^v) := \mathbb{E}[S|\hat{v},\mu^v] \tag{15}$$

$$\mu^{v} \frac{\partial}{\partial \hat{v}} g_{in}(\hat{v}, \mu^{v}) := \operatorname{var}[S|\hat{v}, \mu^{v}].$$
(16)

In the GAMP algorithm, v is interpreted as a Gaussian noise corrupted version of S with noise variance μ^v . Therefore, for K-QAM modulation, the input nonlinear step is expressed as

$$\mathbf{E}[S|\hat{v}, \mu^{v}] = \sum_{i=1}^{K} d_{i} P(d_{i}|\hat{v}, \mu^{v}), \qquad (17)$$

$$\operatorname{var}[S|\hat{v}, \mu^{v}] = \sum_{i=1}^{K} |d_{i} - \operatorname{E}[S|\hat{v}, \mu^{v}]|^{2} P(d_{i}|\hat{v}, \mu^{v}), \quad (18)$$

where d_i , $i = 1, 2, \dots, K$ is the set of *K*-QAM constellation points, e.g. $\mathbf{d} = \frac{1}{\sqrt{2}}[1+j, 1-j, -1+j, -1-j]$ for 4-QAM constellation, and conditional probabilities

$$P(d_i|\hat{v}, \mu^v) = \frac{\exp\left(-\frac{|d_i - \hat{v}|^2}{2\mu^v}\right)}{\sum_{l=1}^K \exp\left(-\frac{|d_l - \hat{v}|^2}{2\mu^v}\right)}.$$
 (19)

For OOB subcarriers, $\mathrm{E}[S|\hat{v},\mu^v]=0$ and $\mathrm{var}[S|\hat{v},\mu^v]=0.$

D. Equalization for frequency-selective channels

Canonical approximate message passing algorithms assume that the underlying random variables are independent. In multipath channels this is not true, since statistical dependencies are introduced between the elements of $\{r_n\}$. We may solve this issue by using the hybrid-GAMP [14] but is out of the scope of the paper due to its complexity and we rely on combining the canonical GAMP with a MMSE frequency-domain equalizer (FDE) [15]. In the FDE equalizer, the vector \mathbf{r} is transformed into the frequency domain vector $\mathbf{R} = [R_0, R_1, \dots, R_{NM-1}]$ by means of DFT. The frequency-domain equalized signal $R_m^{(eq)}$ is given by

$$R_m^{(eq)} = R_m \frac{H_m^*}{|H_m|^2 + \sigma_w^2}, \quad m = 0, 1, \cdots, NM - 1 \quad (20)$$

where $\{H_m\}$ is the DFT of the channel impulse response h zero-padded to length NM. The output of the equalizer is used as an input to the conventional GAMP (Algorithm 1). A similar approach is often used in single carrier systems with cyclic prefix or in constant envelope OFDM systems [2].

E. Complexity analysis

The complexity of the proposed GAMP decoder is mainly dependent on the number of DFT/IDFT involved in each iteration, the integrals and the complex products present in the process. The complexity of the output linear step is conditioned by the IDFT (implemented via IFFT), which has a total of $4NM \log(NM)$ complex products. The output non-linear step is conditioned by the 3 integrals (Eqs. (12) and (11)), with each integral being solved numerically with a total of N_S samples, and each sample of the integral composed of 2 complex products, making a total of $6N_S$ complex products which are repeated for each subcarrier (total of NM). The input linear step is conditioned by the DFT (implemented via FFT), with $4NM \log(NM)$ complex products. Last, the input non-linear step is limited by the calculation of a total of 2Kcomplex products in (18) and (19), which are repeated NM times. The total number of complex products of one iteration of the GAMP is repeated a maximum of t_m times, so the total complexity is

$$N_{CP} = t_m N M (8 \log(NM) + 2K + 6N_S).$$
(21)

It is worth noting that the fGAMP is equivalent to the GAMP with $N_s = 0$. Comparatively, the complexity of other techniques such as the ones presented in [6] are proportional to

 K^{NM} (exponential with the number of subcarriers), demonstrating that the proposed technique is much less complex.

IV. PERFORMANCE RESULTS

In this section we provide the BER performance results for the receiver proposed in this paper for quasi-optimum detection of CE-OFDM signals. We consider both an ideal AWGN channel and a multipath channel with 16 symbol-spaced paths. constant power delay profile (PDP), uncorrelated Rayleigh fading and no Doppler effects. We consider N = 1024 subcarriers, with 4-QAM modulation and Gray mapping. An LDPC 1/2-rate code is applied to account for the spectral inefficiency of CE-OFDM and an output backoff (OBO) of 5dB is applied to account for the PAPR>1 of the linear OFDM. The selected OBO is the minimum in practice [16], so the gains for the GAMP are potentially larger. The different modulations account for the spectral efficiency differences between linear OFDM and CE-OFDM. The oversampling factor is M = 4 and we consider the nonlinearly model defined in (2), which is assumed to be known in the receiver. Contrarily to OFDM where the channel coding is critical, we can have good CE-OFDM performance without it, since the NL behaves as a kind of channel coding [8], due to the implicit diversity effects [6]. Additional gains would be achieved if a conventional code is employed with CE-OFDM, since a concatenated coding scheme would result from combining the conventional one and the inherent to the GAMP with NL. We assumed perfect synchronization and channel estimation at the receiver, unless otherwise stated by defining a non-perfectly estimated channel as $\hat{H}_m = \sqrt{1-e}H_m + H_e$, where $H_e \sim \mathcal{CN}(0, e)$. The integration required by (12) was performed numerically using the midpoint rule, with $N_s = 60$ samples. The maximum number of iterations of the GAMP algorithms was set to $t_m = 50$, although the algorithm can stop before that if there are no errors, and the damping factor was set to $\beta = 10^{-\delta \cdot E_b/(10N_0)}$, with $\delta = 1.5$ for the AWGN channel and $\delta = 0.7$ for the multipath channel. Please note that the parameters give a trade-off between complexity and performance, since the GAMP algorithm has several degrees of freedom. The optimization of the parameters (i.e. an adaptive damping factor) is still an open issue, and we could potentially reduce the complexity and increase the performance, but this is out of the scope of the paper. With our choice of parameters, our goal is to show that the GAMP based receiver can work.

A. Optimal modulation index for minimum BER

Our first goal is to find the optimal modulation index b for the CE-OFDM (2). If an optimal decoder is used at the receiver side, the nonlinearly distorted OFDM signals can outperform their linear counterpart, especially for larger modulation index [6]. However, since the proposed GAMP receiver for CE-OFDM signals is not necessarily optimal, it is reasonable to assume there is an optimal modulation index. Fig. 2 shows the BER performance for different E_b/N_0 values versus the modulation index for an AWGN channel and for a multipath channel. The optimal modulation index depends on the E_b/N_0 , and is found in the range $2\pi b = 1.3 - 1.4$, values for which the phase detector performs poorly (see [3] Fig. 1).



Fig. 2. BER for different E_b/N_0 (ρ) ratios in dB and different modulation indexes for both an AWGN and multipath (MP) channel.

B. BER vs E_b/N_0 performance

The BER performance for CE-OFDM systems with the phase detector, the proposed GAMP and the maximum-likelihood [3] receiver is shown in Fig. 3. The GAMP approaches the optimum performance in low and high modulation indexes while the phase detector can only for low ones.



Fig. 3. BER vs. E_b/N_0 for low $(2\pi b = 0.5)$ and high $(2\pi b = 1.4)$ modulation indexes (b) for the phase detector (PD), the GAMP, the optimal [3] receivers for the AWGN channel.

Fig. 4 shows the BER performance of the CE-OFDM systems with the proposed GAMP-based receiver in an AWGN channel. Clearly, the BER performance of CE-OFDM can be significantly better than that of the linear OFDM. For $2\pi b = 1.4$, at BER= 10^{-6} , a gain of about 6 dB is obtained over the linear OFDM. With respect to the phase detector, the gain is infinite, since the BER does not go below $5 \cdot 10^{-2}$ for any E_b/N_0 value. It can be seen that the fGAMP [10] results in a large performance degradation and a small complexity reduction for the CE-OFDM (GAMP 2 million complex products per iteration and fGAMP 0.5 million).



Fig. 4. BER vs. E_b/N_0 for different modulation indexes (b) for the GAMP, phase detector (PD), fGAMP and the optimal [3] receivers for the AWGN channel, comparing with linear OFDM with 1/2-rate LDPC with OBO 5 dB.

Fig. 5 shows the BER performance of the CE-OFDM system in the multipath channel used in Fig. 2. The receiver combines a GAMP-based decoding and MMSE frequency-domain filtering. The CE-OFDM with MMSE FDE and GAMP receiver allows a large gain when compared to linear OFDM, e.g. 5dB at BER= 10^{-4} . The performance of OFDM schemes in frequencyselective channels is improved by the inherent diversity effects created by the nonlinear effects. Furthermore, there is a tradeoff between the complexity and the performance for the GAMP parameters. When reducing t_m , the performance and complexity decrease, and when δ is reduced, there are instabilities and the performance is potentially worse.

V. CONCLUSIONS

In this letter, we proposed a GAMP-based receiver for CE-OFDM signals. It can be used when the modulation index of the CE-OFDM signal is large, while the conventional phase detector fails due to the phase ambiguity. Our performance results show that the CE-OFDM with a GAMP-based receiver, for large modulation indexes, outperforms a conventional linear OFDM both for AWGN and multipath channels, provided that an MMSE-based FDE equalizer is used in the latter. This is something remarkable since these configurations are classically regarded as unfeasible.



Fig. 5. BER vs. E_b/N_0 for $2\pi b = 1.4$ for the GAMP and fGAMP receiver for the multipath channel, with different GAMP parameters with $N_s = 60$. Linear OFDM with 1/2-rate LDPC with OBO 5 dB. A case with imperfect channel estimation (ISI) is shown for $e = \sigma_{w}^2$.

REFERENCES

- S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Communications*, vol. 12, no. 2, pp. 56–65, 2005.
- [2] S. C. Thompson, A. U. Ahmed, J. G. Proakis, J. R. Zeidler, and M. J. Geile, "Constant Envelope OFDM," *IEEE Trans. on Communications*, vol. 56, no. 8, pp. 1300–1312, 2008.
- [3] J. Guerreiro, R. Dinis, and P. Montezuma, "On the Detection of CE-OFDM Signals," *IEEE Communications Letters*, vol. 20, no. 11, pp. 2165–2168, 2016.
- [4] A. U. Ahmed, Reception and Performance Enhancement Techniques for Constant Envelope OFDM. University of California, San Diego, 2014.
- [5] A. U. Ahmed and J. R. Zeidler, "Novel Low-Complexity Receivers for Constant Envelope OFDM," *IEEE Trans. on Signal Processing*, vol. 63, no. 17, pp. 4572–4582, 2015.
- [6] J. Guerreiro, R. Dinis, and P. Montezuma, "Optimum and sub-optimum receivers for OFDM signals with strong nonlinear distortion effects," *IEEE Trans. on Communications*, vol. 61, no. 9, pp. 3830–3840, 2013.
- [7] L. Liu, C. Yuen, Y. L. Guan, Y. Li, and C. Huang, "Gaussian Message Passing for Overloaded Massive MIMO-NOMA," *IEEE Transactions on Wireless Communications*, vol. 18, no. 1, pp. 210–226, 2019.
- [8] S. V. Zhidkov and R. Dinis, "Belief Propagation Receivers for Near-Optimal Detection of Nonlinearly Distorted OFDM Signals," in *IEEE* 89th Vehic. Tech. Conference (VTC2019-Spring), 2019, pp. 1–6.
- [9] Y. Ma, N. Wu, J. Andrew Zhang, B. Li, and L. Hanzo, "Parametric Bilinear Iterative Generalized Approximate Message Passing Reception of FTN Multi-Carrier Signaling," *IEEE Trans. on Communications*, 2021.
- [10] C. Yang, X. Liu, Y. L. Guan, and R. Liu, "Fast GAMP Algorithm for Nonlinearly Distorted OFDM Signals," *IEEE Communications Letters*, vol. 25, no. 5, pp. 1682–1686, 2021.
- [11] T. Araújo and R. Dinis, Analytical evaluation of nonlinear distortion effects on multicarrier signals. CRC Press, 2015.
- [12] S. Rangan, P. Schniter, A. K. Fletcher, and S. Sarkar, "On the Convergence of Approximate Message Passing With Arbitrary Matrices," *IEEE Trans. on Information Theory*, vol. 65, no. 9, pp. 5339–5351, 2019.
- [13] S. Rangan, "Generalized approximate message passing for estimation with random linear mixing," in 2011 IEEE International Symposium on Information Theory Proceedings, 2011, pp. 2168–2172.
- [14] S. Rangan, A. K. Fletcher, V. K. Goyal, E. Byrne, and P. Schniter, "Hybrid Approximate Message Passing," *IEEE Trans. on Signal Processing*, vol. 65, no. 17, pp. 4577–4592, 2017.

- [15] D. Falconer, S. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Comm. Magazine*, vol. 40, no. 4, pp. 58–66, 2002.
- [16] H. Ochiai and H. Imai, "On the distribution of the peak-to-average power ratio in OFDM signals," *IEEE Transactions on Communications*, vol. 49, no. 2, pp. 282–289, 2001.