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RESEARCH ARTICLE

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Life cycle, financial frictions and informal labor markets: the case of Chile

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ABSTRACT

We study the implications of economic policies on household's decisions. We focus on Chile in 2019. Using a life-cycle search model and survey data, we found that an equivalent change in labor tax rates and non-contributory pensions (NCP) have opposite effects on labor markets, specifically on informality and unemployment duration. NCP offers a milder trade-off as it produces a second-order increase in informality. However, due to the presence of informal labor markets and financial frictions, non-retired agents increase their current consumption only after a tax cut. That is, a positive wealth shock can reduce consumption. When we consider the impact on welfare, as households are assumed to value only consumption, cutting taxes seems to be preferred. We characterize labor market and consumption-savings decisions. We found two effects operating in opposite directions: substitution and wealth. The latter prevails suggesting that the life cycle aspects of the labor market are critical.

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1. Introduction

In late 2019 Chile was hit by a series of violent demonstrations triggered by a minor increase in the cost of public transport. The contrast between the mild cause and the drastic consequence suggests that those riots were generated by other, deeper, factors. After surveying the Chilean population, the literature (see for instance Zúñiga, 2019) noticed the main factors behind the protests: i) pensions were considered insufficient and ii) the cost of living was said to be too high.

Among the most direct instruments to tackle those problems we found labor taxes and non-contributory pensions. However, the aggregate impact of these instruments is far from being clear, especially in a model with incomplete markets and informal labor markets. Measuring these effects will allow policymakers to properly assess the underlying trade-offs and shed some light on the interaction between employment decisions,

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savings, and contributions to social security, which is probably one of the key concerns in the design of pension systems and labor market regulations in Emerging Markets. From a purely fiscal perspective, reforms in any of those two institutions may have complex effects on decisions to be formally employed, which also has important effects on the sustainability of the pension system. We focus on Chile mainly for two reasons: first, given the lower level of informality in this country (see Gasparini & Tornarolli, 2009), these policy instruments seem relatively effective when compared to other Latam countries. Second, as Chile has a full capitalization system, the life cycle aspects are critical to measure the labor market impact of any reform.

Using a search model with life-cycle features and data from Chile, included in the Harmonized Longitudinal Social Protection Survey (LSPS), this paper finds that tax rates and non-contributary pensions have differentiated effects on labor markets. We compute equivalent policies in terms of present value. We found that a tax cut generates a reduction in informality (henceforth IR) but rises unemployment duration (UD). A hike in non-contributary pensions causes the opposite effects *but offers a milder trade-off as it produces a second-order effect on informality*. However, due to the presence of informal labor markets and financial frictions, non-retired agents increase their current consumption only after a tax cut. *Thus, when we take into account the impact on welfare, as by assumption households only obtain utility from consumption, cutting taxes seems to be preferred.*

From a qualitative perspective, we can sharply characterize labor market decisions. We found two effects operating simultaneously: substitution and wealth. Due to the presence of risk averse agents and incomplete capital markets, the latter is responsible for the tradeoff between IR and UD. The wealth effect is associated with the discounted value of income when retired, which has a different quantitative impact depending on the state of the agent. Moreover, this effect implies that after any proposed policy shock the value of being unemployed changes in the same direction as the value of being employed, in any market, but is the least sensible among them all. Thus, the ratio of accepted offers moves in opposite direction with respect to the duration unemployment, generating the observed behavior.

The above paragraph suggests that the life cycle aspects of the labor market are critical to understand policy trade-offs. Moreover, the wealth effect, in the presence of liquidity constraints, can also account for the decrease in current consumption after a hike in non-contributary pensions. As agents move to informal labor markets because the value of a formal job offer relatively decreases after the shock, there is a reduction in current income that cannot be compensated with debt even though the agent is wealthier. Thus, expenditures and savings go down after the shock; a fact that makes taxes preferred to transfers. *That is, in this framework, a positive wealth shock can reduce consumption*.

From a methodological point of view, the paper estimates some of the parameters of the model using a simulation-based structural econometric method and calibrates the rest of them. Proposals for *future* reforms that have never been implemented in Chile cannot be properly addressed using reduced-form-based techniques (i.e., RCT) since exogeneous variability in each data set is not available. In this sense, the results in this paper suggest that the observed rigidity in the informality rate in Latin America (see for instance Gasparini & Tornarolli, 2009) is a consequence of demand side problems as employees respond elastically to several policy stimulus. Moreover, the model is used to obtain the effects of specific policy reforms through simulation methods. In this way, we provide a clear way to understand the possible mechanisms behind policy changes, complementing what may be found later through RCT's methods.

Finally, this paper develops a search model with life-cycle elements to quantitatively analyze the consequences of policy reforms using the Chilean data from the Longitudinal Social Protection Survey (LSPS) harmonized database. We model the life cycle of a worker. The first two-thirds of her life she is active in the job market and the last one she is retired. During the period of active life, the model captures the presence of two types of labor markets: formal, characterized by the presence of contributions to social security, and informal. Thus, we provide key statistics to identify reservation wages in a non-stationary search model with risk averse agents and incomplete markets. To compute the parameters of the model, this paper combines a simulated-method-of-moment approach with the more traditional calibration technique. We use three moments from the database: the average unemployment duration and the coefficient of variation of consumption for both active workers and retired agents. We use these moments to estimate two parameters, the probability of: i) losing a job for an employed worker, ii) receiving a new offer for an unemployed agent. The probability of being fired is slightly above (but within range) the values in the search literature. The rest of the parameters are calibrated, most of which use official information from the Chilean pension regulator or pinned down using an ad-hoc value as benchmark depending on the limitations found in the data set.

1.1. Relation with the literature

This paper contributes to two brands of literature. The first is that on job-search models with life-cycle elements. Examples of this literature are Low, Meghir, and Pistaferri (2010), Piguillem, Ruffo, and Trachter (2012), Claudio and Ruffo (2015) and especially Cirelli, Espino, and Sanchez (2017). In those papers, wages are assumed to follow a deterministic path or otherwise directly being a constant. This paper keeps the random nature of wage offers more traditional in search models, among other reasons, to analyze more in depth the role of changes in reservation wages in the mechanisms behind policy reforms. To our knowledge, that mechanism has not been exploited in those models. Of course, given the assumption of risk-averse workers, computing the reservation wage is far from being trivial given its dependence on asset holdings[1] and the life-cycle assumption. This is also one of the assumptions imposing constraints on the possibility of estimating parameters using simulation-based methods given the well-known curse-of-dimensionality problems.

The paper also provides a contribution to the search literature that considers informal labor markets. The latter include the work by Bosch and Pretel (2012, 2015), Meghir, Narita, and Robin (2015) and Flórez (2017). The papers by Bosch and Esteban-Pretel and Meghir et al. (2015) include the demand side of labor markets through a matching model, although they all abstract from savings and retirement considerations that are central in this paper. Flórez (2017) does consider savings in a search model with an informal sector,

[1]This problem goes back to the well-known analysis by Danforth (1979) and subsequent work.

although it ignores retirement and life-cycle elements. In this regard, this paper is a good complement of those four articles providing a focus on the interaction between formal versus informal labor-markets and retirement savings, an issue not addressed before in that literature.

The rest of the paper is as follows. Section 2 presents the search model. Section 3 presents the data sources and the estimation-calibration methods. Section 4 presents the result of the estimation and calibration of parameters. Section 5 presents the results of the policy exercises. Finally, section 6 concludes.

2. A life-cycle search model with non-segmented labor markets

This section introduces the search model. The first subsection presents the setup, while the following subsection presents some features that arise from the solution of this model.

2.1. The set-up

Consider a finite horizon economy with discrete time periods. The model considers an agent[2] whose age is counted in semesters (half years). Let $t = 0, 1, \ldots T$ denote the time-periods in which that agent may work, depending on the job opportunities encountered. Let $t = T + 1, \ldots, T + \mathcal{T}$ be the semesters of retirement for the agent until her death at age $T + \mathcal{T}$. This model assumes that all these terminal dates are known with certainty at the very beginning of her life. This assumption allows higher feasibility for the estimation procedure. Problems associated with the structural estimation of life cycle models are well known (see for instance, Pakes (1994)). As active life decisions may affect the terminal date of the model, we are faced with a functional operator with unknown qualitative properties. Note that while time is finite, reservation wages belong to a potentially big function space (i.e., the space of compact and continuous functions) due to their dependence on wealth. More to the point, the convergence of this operator is not guaranteed for an arbitrary initial condition. Thus, the numerical solutions, if they can be found, are sensible to the numerical procedure. To circumvent this problem, we derived a closed form solution for the retirement age decisions which depends critically on the non-stochastic nature of the terminal date of the model (see the appendix for details).

The agent's preferences can then be represented by an expected utility function depending on each period consumption, c_t. The Bernoulli (per-semester) utility function with respect to consumption is $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. The model assumes a discount factor per month equal to a constant β, which lies in the open segment (0,1).

For all periods $t = 0, 1, \ldots T$ the agent can be either unemployed or else employed in a job. The assumption is that there are two types of employed agents, who, when unemployed, may find one of two possible types of job opportunities with certain probabilities. The two types of potential jobs are a formal and an informal employment. Characteristics of each type of jobs are presented below. For unemployed agents, search

[2]Just as a convention we use the term "she" to refer to the agent in the subsequent paragraphs.

frictions imply that obtaining a new job opportunity of either type occurs with a probability α, while with the remaining probability the agent remains unemployed. After receiving an offer, the probability that the latter is a formal type is equal to p, while with the remaining probability the offer received is of an informal type. The formal job is characterized by a certain gross wage w^f, subject to tax rate equal to τ. The amount paid by the worker is her contribution to a pension fund, as it is the case in countries like Chile. The informal job is characterized by a tax-free wage w^I.

Unlike other structural models that include the labor demand side (whose information is not available in the (harmonized) LSPS database), where the formality decision comes from an implicit bargaining process with employers,[3] here wage realizations are assumed as exogenous with finite support. Let $\{w^I_{(1)}, w^I_{(2)}, \ldots, w^I_{(K)}\}$ be the support of w^I and let $\{w^F_{(1)}, w^F_{(2)}, \ldots, w^F_{(K)}\}$ be the one for the gross formal wage w^f. The probability of each possible wage realization is denoted as π_w. Each type of employed agent keeps her job the following period with respective probabilities δ^f and δ^I and lost with the respective remaining probabilities. When losing the job, the agent remains at least the period when fired as unemployed.

In every period, the agent (either before or after retirement) has available a savings technology, which can be empirically interpreted as a bank account or cash. Such savings technology yields a constant (real) gross per-period return equal to R.[4] For future reference, we denote as a_t the level of savings that the agent decides in date $t-1$ for the next date t. On the other hand, when an agent is formally employed, all her contributions are recognized as a pension payment after retirement. The yield on these contributions is known at the time of the contribution.[5]

Finally, after retirement, social security pays an amount additional to the contribution-yield transfer, which is viewed as the non-contributory branch of the retirement payments. These non-contributory pension payments are assumed to be lump sum.

2.2. Bellman equations

Under the assumptions presented above it is possible to write the Bellman equations for the agents before and after retirement.

- Retired agents

Suppose that retired agents know that contributions yield a stream of transfers T_t in date t, with t larger than T. Thus, the appendix shows that the Bellman equation for retired people is simply:

[3]See Bosch and Pretel (2012, 2015) as examples of these types of matching models with endogenous informality decisions through bargaining.
[4]This is clearly a convenient assumption for computation-estimation purposes.
[5]This assumption is clearly made for computational and estimation purposes. The life-cycle intrinsic feature of this model makes policy functions non-stationary in a recursive sense. This introduces several computing complexities that threaten the feasibility of using any type of structural estimation methods. This is the main reason for excluding possibly more realistic assumptions regarding knowledge on the yield on retirement fund contributions.

$$V_t(Ra_t,\mathbf{T}_t,\mathbf{T}_{t+1},\ldots)=\max_{a_{t+1}}\left\{\frac{\left(Ra_t+\left(T_t+T_t^{NC}\right)-a_{t+1}\right)^{1-\sigma}}{1-\sigma}+\beta\Phi_t(R,\beta,\mathcal{T})\frac{\left(R^{\mathcal{T}-t}a_{t+1}+\sum_{s=t}^{\mathcal{T}}R^{\mathcal{T}-s}\left(T_s+T_s^{NC}\right)\right)^{1-\sigma}}{1-\sigma}\right\} \quad (3)$$

where:

$$\Phi_{\mathcal{T}}(R,\beta,\mathcal{T})=\frac{1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{\left[R+(\beta R)^{\frac{1}{\sigma}}\right]^{1-\sigma}},$$

$$\Phi_t(R,\beta,\mathcal{T})=\frac{1+\left(\beta R^{(\mathcal{T}-t)(1-\sigma)}\right)^{\frac{1}{\sigma}}\Phi_{t+1}(R,\beta,\mathcal{T})}{\left[R^{\mathcal{T}-t+(\beta R^{\mathcal{T}-t-1})^{\frac{1}{\sigma}}\Phi_{t+1}(R,\beta,T)}\right]}$$

Here the variable T_s refers to the income received from pension funds, while T_s^{NC} refers to the non-contributory side of social-security payments. The value function corresponding to the very first period of retirement is equal to:

$$V_T\left(Ra_T,\sum_{s=0}^{\mathcal{T}}R^{\mathcal{T}-s}T_s\right)=\frac{\left[Ra_T+\sum_{s=0}^{\mathcal{T}}R^{\mathcal{T}-s}\left(T_s+T_s^{NC}\right)\right]^{1-\sigma}}{1-\sigma}\Phi_T(R,\beta,\mathcal{T}) \quad (4)$$

The just-retired agent knows the total value of the accumulated retirement fund, $\sum_{s=0}^{\mathcal{T}}R^{\mathcal{T}-s}T_s\equiv Y$, with certainty at the retirement date T.

Equation (4) depends on the accumulated wealth during the active part of the life cycle, which is denoted Y. This wealth will be used to finance consumption and savings decisions after retirement. The model assumes that, at least for countries where most formal workers contribute to a funded system, that the amount received as a transfer from social security is just the pro-rata fraction of all those accumulated funds:

$$T_t=\frac{Y}{\mathcal{T}-T},\ t=T+1,\ldots,\mathcal{T}$$

For countries with pay-as-you-go systems, such as Uruguay, the amount of contributed payments is tied to a reference value, which usually depends on the number of periods with positive contributions to the system. This fact implies that the agent must incorporate at least two additional state variables to the problem: i) when she is active, the numbers of years with contributions to social security and the total amount of tax collection in every period of her retirement. Both state variables allow her to estimate the value of the transfers to receive from social security. Note that she must also make a forecast of the number of years with contributions at the retirement age. ii) When she is retired, the budget constraint of the government or, equivalently, the total amount of contributions of active workers, who of course belong to younger cohorts. Considering the numerical implications of these types of retirement systems, we restrict attention to fully founded capitalization as observed in Chile.

The value function for the period before retirement is presented next, together with the Bellman equations for the active life.

■ The agent's problem(s) before the retirement

We first start presenting the pre-retirement perception of post-retirement yields of the contributed side of pension payments. In the case of a country with fully funded system, the non-retired agent perceives Y as follows:

$$Y = \sum_{s=0}^{T} \rho^s \theta_s \tau_s w_s^f 1_s \tag{5}$$

where 1_s is an indicator function taking on the value of 1 in every period s when the agent is formally working at the formal wage w_s^f. In equation (5), the value of wealth at retirement consists of the capitalized value, at rate ρ, of all contributions. Note that it is assumed that there is a pension funds that keeps a fraction $1 - \theta_s$ every period in which the agent contributes. As we are working in a partial equilibrium framework, the resources collected by the pension fund are wasted. In the empirical section below, θ_s is assumed to be arbitrarily closed to 1 for all s, which implies that the presence of transaction cost related with the capitalization system does not affect household's decisions significantly. Thus, $\theta_s \tau_s w_s^f 1_s$ can be interpreted as the actual net contribution of the worker to the fully founded capitalization system in period s.

This information allows writing a Bellman equation for active workers, one for each employment status. If in a given date t, the agent is unemployed then:

$$V_t^U(Ra_t, Y_t) = \max_{a_{t+1} \in A_U} \left\{ \frac{(Ra_t + b - a_{t+1})^{1-\gamma}}{1-\sigma} + \alpha\beta \left\{ p \left[\sum_{w_{t+1}^f} \max\left\{ V_{t+1}^f (Ra_{t+1} + (1- \tau_{t+1}) w_{t+1}^f; Y_{t+1} \right); V_{t+1}^U (Ra_{t+1}, Y_t) \right\} \pi_{w_{t+1}}^f \right] + (1-p) \left[\Sigma_{w_{t+1}^I} \max\left\{ V_{t+1}^I (Ra_{t+1} + w_{t+1}^I; Y_t); V_{t+1}^U (Ra_{t+1}, Y_t) \right\} \pi_{w_{t+1}}^I \right] \right\} + \beta(1-\alpha) V_{t+1}^U (Ra_{t+1}, Y_t) \right\} \tag{6}$$

In equation (6) the variable V_{t+1}^f denotes the value of becoming formally employed in the next period, while V_{t+1}^I denotes the value of becoming informally employed. Then, according to this equation an unemployed worker receives an offer with probability α and remains unemployed with probability $1 - \alpha$. Conditional on receiving an offer, she gets to choose between a formal/informal offer and unemployment with probability $p/1 - p$.

The term Y_t denotes the value of wealth, accumulated up to period t, which is available for the household in the fully funded pension system. Note that we are expressing wealth in terms of consumption goods available at the date of retirement T, according to equation (5), as we are using the compound rate ρ^{T-t}. Thus, the law of motion for Y_t is:

$$Y_{t+1} = Y_t + 1_t^f \rho^{T-t} \theta_t \tau_t w_t^f \tag{7}$$

Note that we have restricted the choice set for assets, which depends on the employment status: $a_{t+1} \in A_U$, where U stands for "unemployed". Moreover, A_U is compact and all its elements are positive, which implies that households can accumulate assets but not issue debt. Although we allow A_U to be different with respect to A_F (formally employed) or A_I

(informally employed), we will always assume the absence of debt contracts. Moreover, as the database only provides information about real state wealth, we must assume the same initial condition a_0 for all types of households.

It is worth mentioning the interaction between the existence of pension funds and a liquidity constraint environment: as there is a compulsory (minimum) level of savings represented by the fraction of wages taken by the government, which is devoted to the pension system $(\theta_t \tau_t w_t^f)$, as this wealth will be available after retirement, the precautionary savings motive is especially strong during the active fraction of the life cycle. Thus, agents are expected to be less liquidity constrained, the higher the labor taxes. If we interpret taxes as affecting disposable income, it is natural to expect more debt after a tax hike, making liquidity constraints a major obstacle to consumption smoothing. However, if we interpret the results from a life cycle perspective, it is reasonable to think that the lower bound on asset will not bind frequently. This intuition is confirmed in simulations.

For the formally employed at period t the Bellman equation is simpler: she can get fired with probability $1-\delta$, staying at least 1 period without a job, and she keeps its job with probability δ. Notice that we are not allowing for on-the-job-search.

$$V_t^f\left(Ra_t + (1-\tau_t)w_t^f; Y_t\right) = \max_{a_{t+1}\in A_F}\left\{\frac{\left(Ra_t+(1-\tau_t)w_t^f-a_{t+1}\right)^{1-\sigma}}{1-\sigma} + \left[\beta\delta V_{t+1}^f(Ra_{t+1} + (1- \tau_{t+1})w_t^f; Y_{t+1}\right) + \beta(1-\delta)V_{t+1}^U(Ra_{t+1}, Y_t)\right]\right\} \tag{8}$$

Moreover, for the informally employed agent the Bellman equation is similar

$$V_t^I\left(Ra_t + w_t^I; Y_t\right) = \max_{a_{t+1}\in A_I}\left\{\frac{\left(Ra_t+w_t^I-a_{t+1}\right)^{1-\sigma}}{1-\sigma} + \left[\beta\delta V_{t+1}^I\left(Ra_{t+1} + w_t^I; Y_{t+1}\right) + \beta(1-\delta)V_{t+1}^U(Ra_{t+1}, Y_t)\right]\right\} \tag{9}$$

The terminal condition for any of these value functions at the month previous to retirement (period $T-1$) is common to all of them. Let y_{T-1}^e be the (net-of-tax) income in that period. Then the corresponding Bellman's equation for any employment state e is:

$$V_{T-1}^e\left(Ra_{T-1} + y_{T-1}^e; Y_{T-1}\right) = \max_{a_T}\left\{\frac{\left(Ra_{T-1}+y_{T-1}^e-a_T\right)^{1-\sigma}}{1-\sigma} + \beta\Phi_T(R,\beta,\mathcal{T})\frac{\left[Ra_T+\sum_{S=0}^{T}\rho^s\theta_s\tau_s w_S^f 1_s+\sum_{s=0}^{\mathcal{T}}R^{T-s}T_s^{NC}\right]^{1-\sigma}}{1-\sigma}\right\} \tag{10}$$

Note that equation (10) is critical for the structural estimation process: due to the closed-form nature of the Bellman equation after retirement and because equation (10) is uniform across states U, f, I, it is not necessary to iterate over a function space (which contains the Bellman equations) starting from the time of the death of the agent. We can take any level of accumulated wealth and iterate forward using the closed-form solution in equation (10) and the associated Bellman equations in all the period after retirement. The last expressions assumes that the agent correctly perceives that the amount received after retirement is proportional to the total capitalized value of contributions while working (in the formal labor market). Again, the paper makes this assumption to facilitate computation feasibility, which is a key requirement for estimation purposes. As it will be stated in the last section, possible future research may generalize this simple assumption.

3. Data and estimation procedure

This section contains a description of the Longitudinal Social-Protection Survey (LSPS) and some other official data sources used to take the model to the data. Moreover, we describe the methodology used to estimate, calibrate and simulate the model. Additional details can be found in the appendix.

3.1. Description of the data set

This paper uses the harmonized database from the Longitudinal Social-Protection Survey (LSPS). Such database is based on national surveys from five countries: Chile (for which there are two years included here, 2006 and 2009), Colombia (year 2012), Paraguay (2015), El Salvador (2013) and Uruguay (2013). Each observation corresponds to an interviewed person from a household in a given country for the year in which the survey was performed. For some of the variables, there is retrospective information, while for the Chilean case the presence of two waves allows for a more complete longitudinal dimension in the variables included. However, the variability of the data set is not significant across years as the two waves are too close to each other: 2006 and 2009. For instance, the critical statistics used to estimate the model have a difference of at most 4 percentage points.

The harmonized database includes information on social security characteristics such as whether the interviewed person receives a contributed pension or a non-contributory pension, or is retired, and if so, the amount of income received from retirement pensions. Another variable informs if the interviewed person, given that she or he is an active worker, contributes to the formal retirement pension system. This information is key to trace a link between the formal-informal dimension in the model and its empirical counterpart. Below is an explanation about how this link is done.

The database also includes labor-market variables (current and past employment status, unemployment duration), income-related variables, education variables and demographic information, including age and sex. Income information does not come at the individual-level precision. Rather, for each observation the income level reported is the mean value corresponding to the quintile at which the observation belongs. That is, the income reported for an interviewed person i that belongs, say, to the second quintile, is the mean-income of the second quintile. The same happens to the information on expenditure, with the difference that the database provides five different categories of those expenditures. Given that the model only considers a unique consumption good, the paper aggregates the five expenditures component by adding them up. Proceeding in this way, the resulting “aggregate” expenditure has a more disperse and wider support than each of the five expenditure components. Thus, this version uses this information on expenditures for the estimation-calibration stage of some of the parameters, to be described in the subsection below.

Across all countries included in the Survey, the paper mainly uses the Chilean data. There are three main reasons for this choice. The first is that it constitutes the Latin American country with the most mature privately funded pension system in the sub-continent. Although subject to several recent criticisms from different political actors, the Chilean system still now is highly valued by specialists.

Second, as explained before, the model in this paper is more suitable for a "fully funded system" like the one in Chile, at least concerning the contributions from formal jobs, rather than more mixed contributory-pension systems.

The third reason is that there is accurate information for variables not covered in the survey such as tax rates and other variables for Chile, while for other countries in the Survey the availability of the same variables is more doubtful. Such information directly comes from public sources,[6] and it is information necessary to complete the calibration of parameters in the model.

El Salvador also has a capitalization system. We use this country as a robustness check for the model mainly because the survey does not contain information about the duration of unemployment. We must complete the database using an external source. Considering that unemployment spells are critical to calibrate search models, we can only use this country to test if the qualitative properties of the model and the transmission channels working for Chile after several policy experiments are not sensitive to a different set of empirical moments. The rest of the countries does not have a privately funded pension system and, thus, are excluded.

3.2. Quantitative Methodology

This subsection describes the methodology followed in this paper, both for estimating – calibrating the model parameters and for policy exercises. It first starts with a brief review on the literature and then it specifies a rationale for the chosen method.

A large fraction of the empirical literature on search models uses ML-based methods.[7] That literature exploits both the risk-neutrality assumption of the standard model and longitudinal data on unemployment duration to construct likelihood functions that can be used for estimation. Yet, with risk-averse workers, this task becomes much more cumbersome, mainly because of the unavailability of exact closed-form solutions for the likelihood function in such cases. Papers like Lentz (2009) use a numerical approach to compute policy functions embedding the latter into more analytical likelihood functions.[8] Availability of higher frequency data may limit the application of ML methods in highly non-linear search models. The latter is particularly applicable to this paper, given the use of a Survey with low-frequency data.

In contrast, other pieces of the empirical literature focuses on simulation-based methods.[9] A subset of these articles estimating structural search models,[10] including some assuming risk-averse workers such as Lise (2013) and Haan and Prowse (2017), use

[6]See, e.g., https://www.spensiones.cl/portal/orientacion/580/w3-propertyvalue-6138.html.
[7]For a survey on that methodology see, e.g., Eckstein and Gerrard (2007).
[8]Launov and Wälde (2013) proceeds in a similar way but using a model with risk neutral workers estimated for the German case.
[9]For an early survey on simulation-based estimation methods see Stern (1997). For a more recent survey see Aguirregaribia and Mira (2010).
[10]For example, Meghir et al. (2015).

different variants of the simulated methods of moments (SMM) proposed by Fadden and Daniel (1989) and Pakes and Pollard (1989). The typical SMM procedure starts using simulated data. Then the method uses an algorithm to minimize a weighted distance between sample and simulated moments. We first select some variables and moments associated with them. The simulated data is constructed using the model for given values of the parameters.[11] As a second step, we compute the empirically meaningful set of parameters, as a subset of them are the control variables in the mentioned minimization problem.

This paper uses a mix of the SMM to compute a subset of all the parameters of the model and calibration procedures for the rest of them. The type of data available prevents the use of ML-based methods. On the other hand, the task of estimating parameters of a life-cycle model, which is highly non-linear, also imposes constraints on the total number of parameters to be estimated through SMM; forcing to calibrate the rest of the parameters. The reason for this limitation is that the distinctive features of the model, namely non-stationarity, and risk aversion, cause serious computation-feasibility concerns. These facts, at the same time, prevent us from estimating all parameters through SMM.

Computation in this context is important for two reasons. First, as mentioned above, a feasible numerical strategy is the first step towards the estimation of empirically meaningful parameters. Second, one of the main purposes of working with structural search models is to perform counterfactual policy exercises. Even if we do not have exogeneous variability in the date set, the structural estimation of deep parameters remains a feasible choice. Thus, the literature on empirically oriented labor search model emphasizes this role as the main value-added of those models relative to more reduced-form approaches.

There is a well-known literature on numerical dynamic programming applied to discrete choice models[12] that highlights important computational problems generated by search models, mostly related with the sensitivity of reservation wages with respect to endogenous variables. In order to circumvent these issues, the strategy here is to use the non-stationarity of the value function inherited from the finite horizon dynamic programming problem to capture in full the flexibility of reservation wages, and its dependence on wealth and policy variables. Thus, we are isolating the SMM from this type of (time) variability, a fact which allows to estimate some of the (time invariant) deep parameters.

The methodology permits us to obtain the optimal policy-functions (where the optimality is at the individual level) computed over a finite grid of points corresponding to the state variables: wages (or, else, the unemployment-benefit parameter), employment status, age and assets. Given the life-cycle nature of the model, there is a large degree of heterogeneity coming from the age profile. This may introduce a typical curse-of-dimensionality problem that appears in these dynamic models.

[11]Recently, Eisenhauer, Heckman, and Mosso (2015) compare both ML and SMM type of estimation methods in occupational choice models thorough Monte Carlo simulations. Although SMM performed reasonably well in several dimensions, that paper shows that ML methods tend to present the same or better properties more generally.

[12]For surveys on numerical dynamic programming and computational issues see, e.g., Rust (1997) and Judd (1998). Some papers propose several algorithms to overcome such problem. See, e.g., Keane and Wolpin (1994).

In this paper however, the structure of the data set implies a natural solution to these problems as the support in the observed distribution of wages is very coarse. In particular, given how income information comes from the LSPS, this support has only five elements. Each of them is linked to a particular quintile of the income distribution within the Survey. More to the point, the asset grid is also coarse enough to avoid this dimensionality problem.

We take the model to the data and try to replicate the observed behavior in 2 out of 5 possible economies reported in the survey: Chile 2006 and El Salvador 2013. There is a clear rationale behind this choice: Chile and El Salvador are the only two countries with a fully funded private pension system. However, we use the last country only as a robustness check because the available data set for El Salvador does not report variables which are critical to search models, especially related with the duration of unemployment. To complete the information provided in the survey, we use an external source to be described later.

As regards the value of estimated parameters, this paper follows state-of-the-art quantitative models with labor-search and risk-averse workers.[13] A well-known contribution is Lentz (2009). This paper estimates a search model with risk-averse workers, based on Lentz and Tranaes (2005), applied to Dutch weekly data. Lentz (2009) uses Dutch data on unemployment spells to eventually estimate a hazard rate for active workers. In this paper, for the case of Chile, the Survey provides information about months being unemployed during a whole year for each worker, which allows us to recover the duration of unemployment. Considering that the unemployment spell is a natural outcome of search models and because we have survey data for this variable, we use it to estimate a subset of the parameters in the model.

We estimate but not observe, the probability of being fired, $1 - \delta$ and the probability of getting a job offer, α. The LSPS database provides information on whether during the year before the interview took place the person was employed, and it reports if in the year of the interview she was unemployed. Yet, there is no information neither about when a worker lost her job between semesters within a given year nor receive a formal/informal job offer. As can be seen in the Bellman equations of employed and unemployed workers, these parameters are critical to determine the predictions of the model. Thus, we estimate them using the SMM.

For the identification and estimation of those two parameters, then, this paper uses an indirect approach, based on the information about income and expenditures, together with the duration of unemployment.[14] The paper computes the coefficient of variation of consumption (expenditures) for active workers and for retired people, according to the information in the Survey. Then, we use those two coefficients together with the average observed duration of unemployment to identify and estimate the probability of finding

[13]A more complete survey on identification in models of labor markets see French and Taber (2011).

[14]Regarding the wage probability distributions, several papers in the search literature such as, e.g., Low et al. (2010) estimate a wage equation by running auxiliary regressions using panel data with monthly or weekly frequency. Lentz (2009) however explains that such strategy may explain too little of the wage variation. Given the type of wage (income) data in the LSPS, the probabilities of realizations are directly calibrated from the relative frequency for each of the five realizations in the Survey.

Table 1. Moments used in the estimation procedure
Computed using the LSPS for Chile 2006.

Moment	Value
Average Duration of Unemployment in Semesters	1,5
Variation coeff of consumption, active workers	0,15
Variation coeff of consumption, retired	0,43

Source: own calculation from LSPS database

a new job for an unemployed worker and the probability of being fired. The estimation is done using the simulated method of moments. The paper uses an overidentified version of the method.

At first glance it may seem odd the use of the coefficient of variations on consumption to identify and estimate parameters related to job creation or job destruction. Yet, there is a rationale behind this choice. Suppose two different values for this coefficient. Given a value for the variance of wages, and given the borrowing constraint assumed in the model, what lies behind the agent facing higher coefficient of variation of consumption is that she must be more liquidity constrained relative to the agent with lower consumption variation. This implies that the former accumulated a lower amount of assets and then her reservation wage is lower than for the latter. Ceteris paribus the rest of the conditions in the model, the agent that hits more frequently the liquidity constraint should generate a shorter duration of unemployment than the other agent. If the observed duration does not decrease in the data set, then one possible way to make all these features consistent is that the probability of receiving a new job offer for an unemployed worker, α, should be lower and/or the firing probability, $1-\delta$, should be higher.

For Chile, the rest of the parameters are calibrated directly from the public webpages with official information on contribution rates, unemployment benefits, contributed and non-contributed pension payments (for retired agents). Further, the fraction of total tax payments devoted to the contributed pension system and the return on contributions to future pensions is also identified from public information about the return on pension funds. For El Salvador, the same statistics are not available, except for the case of tax rates. Thus, we use the parameters calibrated for the Chilean economy. Considering that we are using El Salvador as a robustness check for Chile, this seems appropriate.

Finally, this paper does not intend to identify and calibrate (or estimate) preference parameters (risk aversion and discount factor). The reason is that the Survey does not provide information on asset accumulation, together with the fact that the expenditures information is already used to identify and estimate the two labor-market related parameters referred in the paragraphs above. Thus, the values for those two preference-related parameters are set to standard values in the related literature (both from labor search and from macro papers). To verify the sensibility of the results to this assumption, we simulate the model for several values of these two parameters.

4. Results I: structural estimation and calibration

This section reports results concerning the estimation and calibration of deep parameters. Table 1, presented below, shows the three moments that are used to estimate the probability of receiving a job offer, α, and the probability of being fired, $1-\delta$ for Chile 2006.

The table includes the average duration of unemployment, computed as the number of months that a worker remains unemployed in the last 12 months,[15] the coefficient of variation of aggregate consumption for active workers and retired agents. As stated in the last section, for other parameters we use information from official Chilean sources.

Table 2 below presents the values of the parameters that are estimated and calibrated. Estimations are obtained using the SMM and the moments reported in Table 1. The calibrated values for wages are obtained directly from the LSPS data. The return on pension funds and the unemployment compensation are obtained from the Chilean social-security authority website. The tax rate on wages is obtained from the OECD. The interest rate is obtained from official statistics.

The estimated value of the firing probability is 0.068. This value is within the order of magnitudes relative to other estimates in the literature.[16] In fact, it is often slightly above those other estimates with few exceptions. Clearly, a straight interpretation of these differences in estimated values is that at least for the Chilean case (and presumably for other LAC countries) jobs in the "averaged" labor market (between formal and informal) may be slightly less secure than their counterparts in more developed countries, as ex-ante expected. Of course, this is due to a more de-regulated institutional framework.

On the other hand, the probability of finding a new job opportunity (of any kind) is estimated in 0.535. Comparing this with other results in the literature, this value is within the estimated bounds.[17] Of course, the estimated value of this parameter is affected by the assumed values for the probability of the offer being formal, p. This paper assumes a benchmark value of 0.5. Although this looks too simple, it assumes a value that facilitates the simulation-based method used in this paper. Finally, at the bottom of Table 2 appears the standard error of the joint estimation of those two parameters, equal to 0.0893.

The following eight rows of Table 2 report the calibrated values of three parameters related to formal workers: the tax rate on formal wages (τ), the fraction of tax payments included for future retirement payments (θ), and the (implicit or explicit) return on pension funds (ρ). As stated above, the web-page from the Superintendecia de Pensiones[18] provides information on several of those parameters such as the rate applied from labor income as the contribution to the pension fund. The return on funds comes also directly from the report of the above-mentioned website. The value of the fraction of total tax payments going to retirement funds is set to 90% to take into account fees and other deductions.

[15]The Survey reports statistics for a spell of 24 months. Sorted by age, a) for agents with 18 to 42 years old, the average unemployment spell is 15.4 month (2.6 semesters) which imply that an average of 7.7 month in the last 12 months (1.3 semesters), b) for agents with 43 to 65 years old, the average unemployment spell is 18.7 month (3.1 semesters) which imply that an average of 9.3 month in the last 12 months (1.6 semesters). The weighted average of these 2 groups of agents is the reported 1.5.

[16]For example, Daniele (2008) gets a range of estimates for a similar parameter between 0.0087 and 0.011.Low et al. (2010) calibrates those two probabilities in 0.028 and 0.049 respectively.

[17]For example, comparing this value with the estimates in Daniele (2008) the latter predicts a range of offer probabilities from the estimated parameters in that paper, where the upper bound for that range is 0.562 for workers receiving high-wage offers in the first week of unemployed, a similar value to that estimated in this paper. On the other hand, the value used in Low et al. (2010) is slightly higher than in this paper (0.76 for low-education workers, 0.82 for high-education workers).

[18]See https://www.spensiones.cl/portal/orientacion/580/w3-propertyvalue-6138.html.

Table 2. Parameters, Chile 2006 Estimation comes from matching moments that appear in Table 1. Calibration is done using official data sources.

Parameter	Description	Value	Method
$1-\delta$	Layoff probability	0,068	Estimated through SMM (*)
α	Probability of receiving a new offer	0,535	Estimated through SMM (*)
p	Probability of a formal offer	0,5	Assumed
π_w	Probability vector of each wage	[0.14 0.21 0.20 0.24 0.21]	Calibrated from LSPS
θ	Compulsory savings	0,9	Calibrated from public source
ρ	Maximum return on pension funds	0,045	Calibrated from public source
τ	Tax on wages	0,1	Calibrated from public source
$R-1$	Net interest rate	0,03	Calibrated from public source
b	Unemployment compensation	0,7	Calibrated from public source
β	Subjective discount factor	0,95	Reproduced from literature
σ	Relative risk aversion	3,0	Reproduced from literature

(*) For this joint estimation the standard error is 0.0893
Sources: own estimation and calibration based on harmonized LSPS data, OECD, Superintentendencia de Pensiones Central Bank (Chile).

The value of the real interest rate on savings is calibrated from the average real interest rate on deposits reported from the official Chilean statistics. Ideally, the relevant calibration should include possibly other returns from other investment options, depending on the qualitative information found in LSPS on the types of investment that the interviewed person reports to have. Yet, given that such information is only qualitative, it is not possible to compute weighted averages of rates of return from the LSPS database.

The next row reports the calibrated value of the unemployment benefit, equal to 0.70. It corresponds to its normalized value according to the official information from the social security Chilean web site .[19] In fact, the payments corresponding to such benefits are decreasing in the duration of unemployment, a feature that is not present in the paper. It is always possible to add this assumption to the model. However, this generalization implies a heavy cost in terms of the computability of the model given the non-linearity of the utility function and the non-stationarity of the dynamic programming problem.

Finally, the last two rows present the values of the main two preference parameters. As stated in the section above, this paper uses values for those parameters directly taken from those found in the international literature.

Table 3 presents some quantitative properties of the calibrated-estimated benchmark model. The table reports the results obtained after simulating the model for five endogenous variables. Moments are taken across semesters of active life. The second column reports the results using data while the third does so running simulations. Averages and standard deviations are taken within the agent's first 50 years of life.

The first three rows of Table 3 report statistical moments for the three variables used to estimate the model. The main purpose of these reported statistics is to provide a measure of the precision of the match of the model.

[19]This normalization was used to scale the reported level of wages. The raw data is available at current prices ranging from 260 to 1,906 pesos, which represent the average value of the extreme quintiles. These are monthly values and thus were adjusted to represent biannual figures. Thus, as all variables in the model are real, the interpretation is that the vector of wages has a base year which is equal to the first period modeled for each cohort.

Table 3. Properties of the benchmark model, Chile 2006. Selected moments computed using simulated and actual data.

Variable	Data	Model
Average duration of unemployment (in semesters)	1,15	1,76
Coefficient of variation of consumption – active	0,15	0,24
Coefficient of variation of consumption – retired	0,43	0,4
Frequency of accepted formal job offers	–	0,81
Frequency of accepted informal job offers	–	0,35

Source: Own computations based on simulated and LSPS data

As can be seen in the table, the difference between actual and simulated unemployment duration does not look large. Indeed, the model overestimates the observed figure in less than 18%. The model also overestimates the coefficient of variation for active workers' consumption by 7 percentage points. This seems a weakness of the estimation method. Yet, part of this problem is attributed to the fact that the intertemporal consumption problem of the retired agents is assumed to be non-stochastic. Indeed, the last row of Table 3 shows that the model predicts a coefficient of variation in consumption for retired people lower than observed, although the difference is not large (3 percentage points). The deterministic nature assumed during retirement may introduce some distortions in the estimation procedure. We claimed before that an increase taxes may generate more precautionary savings. However, given the level of taxes, as agents do not need to self-insure themselves against future income shocks, they save less and thus hit the liquidity constraint more frequently. The reason behind this assumption comes from the trade-off between estimation precision and dimensionality problems mentioned before. A stochastic retirement problem would force us to numerically solve the value functions during retirement, losing the closed forms presented in the appendix.

The last rows show the values of two relative frequencies generated by the model. In the benchmark case, the fourth row indicates that about 81% of the received formal job offers are accepted (and so 19% of those offers are rejected). Similarly, the fifth row indicates that only 35% of the informal job offers are accepted, while the remaining 65% are rejected. These figures reflect the relatively low informality rate observed in Chile compared to other Latin American countries (See Gasparini & Tornarolli, 2009) and will be used as benchmark for the counterfactual policy exercises in the next section.

5. Results II: policy-evaluation exercises

This section is divided into two parts. In the first subsection, we present the results for two policy experiments applied to the benchmark case, Chile 2006. In the next subsection, we include some robustness checks. As preference parameters are borrowed from the literature, we verify if the results are sensitive to changes in the discount factor and the coefficient of risk aversion. Moreover, we applied the model to the other country in the survey with a fully funded private pension system, El Salvador 2013.

5.1. Benchmark Case: Chile 2006

As stated in the introduction, structural models are useful to analyze counterfactual policy exercises. This paper considers two alternative reforms for the case of Chile that are especially relevant after the protests observed in 2019.

The first exercise assumes a permanent decrease in the tax rate on formal wages, from 10% to 7%. Although this specific reform is not present in the current policy debate, a cut in the tax rate can be seen as a possible reaction by a government that is facing the increasing demands against the functioning of the Chilean pension system. Note that the shock is assumed to be permanent. In a general equilibrium framework, this type of reforms must be followed by a reduction in Government expenditure to insure an intertemporal balanced budget. For the sake of simplicity, we assume that this last reform does not affect the labor market during the life cycle. As time is assumed to be finite, this assumption is mild.

The second policy exercise is a 10% increase in non-contributory pensions, a frequently discussed type of policy with redistributive goals. This change can be seen as a possible reaction after several suggestions made to the Chilean Government.[20] In both cases, the focus is set on labor markets and formality-informality acceptance rates. Table 4 below shows the results of these policy exercises.

The two reforms are equivalent in terms of present value under the parameter structure presented above: the net present value of a 10% increase in non-contributary pensions equals the 3% of the mean average in the informal sector. As the Government is assumed to cut taxes by 3 percentage points, the increase in current disposable income equals the present value of the hike in non-contributary pensions. Note however that in the presence of liquidity constraints and given the possible uncertain career of any agent in this economy, there are multiple equivalent reforms. For the sake of simplicity, we just calibrate the policy shock for an informal employed worker that will never hit the liquidity constraint.

Panel A of Table 4 shows the results regarding the drop in the contribution rate from 10% to 7%. The first row shows an increase in the average duration of unemployment. This increase is proportionally quite big (about 23% from the benchmark case). The mirror image of this increase in the unemployment duration appears in the following two rows: for both types of job offers received by unemployed workers, the frequency of acceptance drops. In the case of formal jobs, the drop in the acceptance rate is about 4%, while for the informal jobs the drop is much larger, about 32%.

Those results can be explained by a dominant mechanism: a drop in the contribution rate decreases the income flow after retirement, affecting the agent in any state. However, the strongest impact of this effect is on formal workers, followed by informal employees and then on the unemployed ones. Thus, the relative value of being unemployed increases (as the value function decreases less than in any other state), making workers more demanding when receiving job offers and consequently raising the duration of unemployment. This mechanism can be seen as a type of *wealth effect*, which counter-balances a more traditional effect, namely, *a substitution effect*. In this effect, a decrease in the tax rate generates an increase in the value functions of the agent in any state as there is

[20]For example, the recent OECD report on Chile (2018) suggests the increase of the "solidarity pillar" as a strategy to improve inclusiveness for the old in the Social Security system.

Table 4. Results of counterfactual policy exercise, Chile 2006.

Panel 4.A: Drop in contribution rate from 10% to 7%

This panel reports unemployment duration and the fraction of formal/informal offers accepted.
For the last two variables, there are two types of values reported: full model and no retirement.

Variable	Benchmark	After drop in τ
Average unemployment duration (semesters)	1,769	2,186
Fraction of formal offers accepted (full model)	0,813	0,783
Fraction of informal offers accepted (full model)	0,356	0,243
Fraction of formal offers accepted (no retirement)	0,633	0,795
Fraction of informal offers accepted (no retirement)	0,456	0,278

Panel 4.B: Increase in non-contributory pensions of 10%

This panel reports unemployment duration and the fraction of formal/informal offers accepted.

Variable	Benchmark	After increase in T^{NC}
Average unemployment duration (semesters)	1,769	1,351
Fraction of formal offers accepted (full model)	0,813	0,811
Fraction of informal offers accepted (full model)	0,356	0,369

Source: Own computations based on model

a hike in current or (near) future labor income. The impact effect is strongest in formal workers, which explains the mild reaction of acceptance rates in this sector: both substitution and wealth effect offset each other. Thus, all the action of the model is in the informal sector, where the acceptance rate lowers more than the increase in unemployment duration as the formal sector acts as a buffer.

The last two rows of panel A reflect a case in which the substitution effect is predominant. The table shows the results of a similar exercise on the fraction of formal and informal jobs accepted but modified in an ad-hoc way. This variation consists in dropping all semesters after retirement, keeping only the periods while actively working (or otherwise being unemployed), reducing the size of the wealth effect. In this case τ corresponds only to a proper labor income tax rate, not to a contribution to a retirement fund. Then, a drop in that parameter implies an increase in the propensity of accepting a new formal job and a drop in the propensity to accept an informal one; keeping the duration of unemployment relatively constant (which is not reported for expositional purpose). Moreover, this effect operates not only due to the standard mechanism present in traditional search models with risk-neutral workers but also reinforced by the effect on asset holdings. This gives rise to an *income* effect, which acts in the same direction as the wealth effect but as can be seen in simulations, is milder. That is, the hike in current income is smoothed to the next period, accumulating assets, which in turn increase the reservation wage and thus raising the unemployment duration in the near future. In the full model with retirement, wealth and income offset the substitution effect. That is, given the estimated and calibrated values of the parameters, this model shows that *the first and second effects are stronger than the last one.*

The difference between the two above-mentioned cases suggests that such a change depends crucially on the true planning horizon of workers.[21] Those results indeed suggest that the possible effects of changes in contribution rates are very different (essentially, the opposite) for workers with long-run horizons than for those with short-run horizons. Although the paper does not deal directly with behavioral issues like myopia, this rough exercise presents a first step towards understanding the role of

[21]This effect is also related to empirical literature linking financial literacy (as in Meier and Sprenger (2013) and information (as in Fuentes, Lafortune, Riutort, Tessada, and Villatoro (2017) to voluntary savings for retirement.

planning horizon when evaluating potential policy reforms in social-security contributions. Besides, to our knowledge at least, such result is novel in the search literature with life-cycle considerations.

The other relevant comment is related to the relationship between the propensity to accept formal/informal jobs. In the case of the full model, the drop in the contribution rate undoubtedly makes the unemployed worker even more reluctant to accept informal jobs than formal ones. This result is in line with the intuition that a drop in such rate makes labor formality less costly for workers. An even sharper result is obtained in the ad-hoc modified model, where the same policy generates a decrease in the number of informal jobs offers, which are accepted. Thus, *a drop in contribution rates generates the correct incentives on the labor-supply side, decreasing informality in labor markets, although making unemployment a phenomenon with higher duration*.

The second panel of Table 4 evaluates the effects of an increase in non-contributory pensions after retirement. Thus, the mechanism is driven purely by the wealth effect. Unemployment duration drops about 24% relative to the benchmark value. This drop in fact reflects an increase in the informality rate, measured by the increase in the fraction of new informal jobs by about 3%. On the other hand, the propensity to accept formal jobs drops very little (less than 1%). The last result is a bit surprising. Indeed, one would expect a more significant drop in the propensity to accept formal offers when increasing the solidarity pillar. Thus, what is at work here is a mechanism even more extreme to the one at stake in the contribution-rate full exercise, that is, a pure wealth effect. The value functions of the agents raise in all the possible states, but the value of being an informal worker increases the most (the other two moves in similar direction and magnitude).

Yet, from both the labor-market informality and the fiscal policies discussion, these results seem undesirable. Indeed, and from a more macro perspective, these results suggest that making non-contributory pension transfers more generous may imply lower incentives to contribute to the formal branch of the social security system and so it may ceteris paribus worsen the fiscal position of the government providing those transfers. However, if the orders of magnitude and the effects on the duration of unemployment are taking into account *an increase in non-contributory pensions are preferred to a cut in the labor tax rate*. In particular, the effects on unemployment duration are similar in magnitude in both policies (changing in a different direction, of course) but the changes in the informality rate are milder in the case of non-contributory pensions.

If we consider the effects on consumption, the above conclusion is reversed. Due to the presence of liquidity constraints, consumption only increases relative to the benchmark in the case of a tax cut. After a hike in non-contributory pensions, agents choose to increase the acceptance rate in the informal labor market, which has a lower average wage, and cannot front-load consumption as much as they want due to the increase in their wealth after retirement. This fact affects significantly the overall impact generated by non-contributory pensions. The table below shows the situation at hand.

-(+XX) implies a negative deviation of XX units with respect to benchmark

Table 5 present an ad-hoc criterion to evaluate the two reforms, based on deviations with respect to the benchmark simulations. It is assumed that the policymaker negatively values an increase in the informality rate (IR) and unemployment duration (UD) but positively values a hike in consumption (Cons.). Each of these deviations are equally

Table 5. Summary of simulated results.

Effects of a tax cut and a hike in non-contributary pensions.

Deviations with respect to benchmark

Policy	IR (I)	UD (II)	Partial Effect (I+ II)	Cons. (III)	Total Effect (I+ II+III)
Labor Tax	-(−31)	-(24)	+7	+(+10)	+17
NC Pensions	-(+3.5)	-(−24)	+21	+(−8)	+12.5

weighted. Thus, an increase in non-contributory (NC) pensions is preferred *if the Government cares only about the labor market impact of both reforms*, labeled Partial Effect. However, if authorities consider the overall impact of these measures, their Total Effect, this paper suggests that an increase of disposable income by means of a tax cut is preferred. Note that if policy makers are assumed to be benevolent, we arrive to the same conclusion as households only value consumption (i.e., the instantaneous utility function only depends on it). Thus, we provide evidence in favor of the relevance of financial frictions for the design of labor market reforms as they explain why taxes normatively dominates transfers.

5.2. Robustness Checks

We begin by fitting the model to El Salvador, which is the only country, besides Chile, with a private pension system. As was explained before, the closed-form nature of the retirement side of the model precludes us from dealing with a pay-as-you-go system as pensions will depend on observed wages. This fact affects severely the tractability of the model as it will involve adding a new class of households to the economy: those who do not get the sufficient number of formal working years. For those workers, non-contributory pensions are the only source of income when retired and, thus, the impact of this type of policies is radically different with respect to the one observed for private pension systems.

The first thing to be noted is that the survey does not include data on the duration of unemployment for this country, a critical parameter for the calibration of the model. Thus, we must borrow this figure from an external source.[22] More importantly, the spell of unemployment is substantially lower than the one in Chile, averaging around 0.4 semesters. As data sources are not comparable, taking into account these differences in the reported values, we take El Salvador as a robustness check for the qualitative results working at the model after a change in non-contributory pensions and labor taxes. Namely, the first policy must generate a reduction in duration and the second an increase in this parameter.

We have survey data for wages, its probability and consumption moments. For external parameters, we found data on taxes also from the OECD.[23] Tax rates are rather similar. For the rest of the parameters, as we could not find a reliable official source and considering the qualitative nature of the exercise, we use the ones reported for Chile. The Table below presents the values.

[22] https://webquery.ujmd.edu.sv/siab/bvirtual/Fulltext/ADOS0000660/C1.pdf
[23] http://dx.doi.org/10.1787/888,933,407,321

We now turn to the performance of the model applied to El Salvador. We are interested in the quality of the match compared to the one obtained for Chile.

If we measure the performance of the model in terms of the absolute deviation of the simulated moments with respect to actual data, applied to El Salvador, it has a fit which is slightly worse when compared to the one observed for Chile (0.83 vs 0.73 respectively). Note that there is a significant difference when it comes to the estimated values for the lay-off probability $1-\delta$ and the arrival rate α. For El Salvador both parameters are smaller (0.02 vs 0.07 for $1-\delta$ and 0.35 vs 0.54 for α, see Tables 2 and 7). This fact has important implications for the impact of economic policies. The table below illustrates the situation at hand.

As can be seen in Table 8 and 9, comparing the results with those obtained for Chile, unemployment duration moves in the same direction after an increase in the non-contributory pensions but in the opposite one after a tax cut. That is, the unemployment spell lowers after both measures with respect to the benchmark. This fact can be attributed to the differences observed in the estimated parameters. The channels were explained before, but the order of magnitudes involved are different. An increase in transfers only involves the wealth effect. However, there is a tension between the substitution and the wealth/income effect after a change in ad-valorem taxes. As the lay-off probability is lower, the risk of taking a job is reduced. Thus, the strength of the substitution effect increases, implying a hike in the acceptance rate of the formal offers, which generates the reported values for the spell.

Now we turn to the robustness check for the results found for Chile. The table below confirms that changing the parameters borrowed from the literature, β and σ, do not improve the performance of the model. We report a 5% increase (+)/decrease (−) in each of the selected parameters with respect to the benchmark.

Table 6. Parameters, El Salvador 2013 Estimation comes from matching moments that appear in Table 1. Calibration is done using official data sources.

Parameter	Description	Value	Method
$1-\delta$	Layoff probability	0,02	Estimated through SMM
α	Probability of receiving a new offer	0,35	Estimated through SMM
p	Probability of a formal offer	0,5	Assumed
π_w	Probability vector of each wage	[0.25 0.16 0.18 0.20 0.19]	Calibrated from LSPS
θ	Compulsory savings	0,9	Calibrated from public source
ρ	Maximum return on pension funds	0,045	Calibrated from public source
τ	Tax on wages	0,09	Calibrated from public source
$R-1$	Net interest rate	0,03	Calibrated from public source
b	Unemployment compensation	0,7	Calibrated from public source
β	Subjective discount factor	0,95	Reproduced from literature
σ	Relative risk aversion	3,0	Reproduced from literature

Sources: own estimation and calibration based on harmonized LSPS data, OECD, Superintentendencia de Pensiones Central Bank (Chile).

Table 7. Properties of the benchmark model, El Salvador 2013.

Selected moments computed using simulated and actual data.

Variable	Data	Model
Average duration of unemployment (in semesters)	0.40	0.80
Coefficient of variation of consumption – active	0,11	0,54
Coefficient of variation of consumption – retired	0,32	0,34

Source: Own computations based on simulated and LSPS data

Table 8. Results of counterfactual policy exercise, El Salvador 2013.

Panel 4.A: Drop in contribution rate from 9% to 6%		
This panel reports unemployment duration.		
Variable	Benchmark	After drop in τ
Average unemployment duration (semesters)	0.80	0.62
Panel 4.B: Increase in non-contributory pensions of 10%		
This panel reports unemployment duration.		
Variable	Benchmark	After increase in T^{NC}
Average unemployment duration (semesters)	0.80	0.71

Source: Own computations based on model

Table 9. Robustness Check, Chile 2006.

Parameters/Coefficient	U. Duration	C.V. Active	C.V. Retired	Difference
Data	1.15	0.15	0.43	NA
Benchmark	1.76	0.24	0.40	0.73
$-\sigma$	0.93	0.68	0.39	0.78
$+\sigma$	0.98	0.69	0.61	0.89
$-\beta$	1.66	0.55	0.37	0.97
$+\beta$	1.42	0.57	0.39	0.73

As can be seen in Table 9 the parameter set selected to perform the policy exercise, available in Table 2, has the best relative performance, except when we increase β. In this case, the quality of the match is equivalent. There is something to be noted, however: there is no monotonic relationship between the predicted coefficient and the parameters related with preferences; a fact that suggests the presence of non-linearities in the model.

6. Conclusions

This paper presents a labor-search model with life-cycle elements, risk averse agents and imperfect capital markets with the major goal of quantifying counterfactual policy experiments. We found that from welfare perspective and given the necessity of a stimulus policy, a tax cut is preferred to an increase in non-contributory pensions. This result is robust to several parameter specifications and the channel working behind the model were observed in more than one country. These results depend critically on two of the main theoretical contributions of this paper: on one hand, the presence of risk averse agent and liquidity constraints. On the other, a life-cycle non-stationary search model. These features interact with each other to highlight the importance of an increase in current disposable income not only for the behavior of aggregate demand but also for important labor market statistics as the duration of unemployment.

There are two major directions for future research. The first one is of methodological order. Introducing life-cycle elements into search models always represents a challenge for estimation through SMM methods. The main challenge comes from the well-known curse-of-dimensionality issues, which in these models seem severe since the life-cycle elements introduce an additional dimension of heterogeneity. Undoubtedly this should constitute a focal point for future research to develop methods that allow improving the precision, extending the scope of the estimation (i.e., extending the number of parameters to be properly estimated) and also the possibility of estimating parameters with richer assumptions (e.g., dropping the assumption of deterministic lives after retirement, as assumed in this paper).

The second direction is related with the evaluation of economic policies and the use of these models to address it. The asymmetric results on formal job acceptance observed in Table 4 between the benchmark case and the model with agents not considering the retirement years represent a first step towards a more systematic line of research regarding the relevance of the planning horizons to the design of labor markets and social-security reforms. This is also particularly relevant when dealing with labor informality issues as can be seen in the response of consumption to future lump-sum transfers.

Other venues for future research include for example the introduction of the labor-demand side. In this regard, the random-matching models with bargaining (such as those by Bosch & Pretel, 2012, 2015) must be the starting point to introduce life-cycle issues. Yet, to our knowledge such extension has not been tried yet, possibly due to computational complexities, among others.

Other extensions include adapting search models with explicit intra-household analysis. Indeed, Adda, Dustmann, and Stevens (2017) and Haan and Prowse (2017) constitute recent contributions of models analyzing marriage, fertility and labor-search considerations that is explicitly estimated through simulation-based methods. No attempt to extend such analysis to Emerging Market countries is known at this time. Such extensions may imply richer and more subtle effects that a model without such considerations may generate.

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Appendix

Estimation and computation of the model

This appendix provides a rough description of the method used to compute and estimate the probabilities of being fired when employed and of finding a new job offer when unemployed. As suggested in other papers such as Low et al. (2010), a major challenge to the computation of the model presented in this paper is the effect of employment decisions and savings choices on the efficiency of the algorithm. Moreover, this difficulty is worsened by the endogeneity of reservation wages and the non-stationarity of the dynamic programming program. These problems forced us to limit the number of parameters estimated using simulation methods.

The algorithm is based on five codes,[24] each of which is allocated in a separate file. The first file is called "SLC". This file contains the active part of the life cycle. Agents take labor and saving decisions. Agents accumulate savings and make employment decisions according to equations (6), (8) and (9) in the main text. The second file is called "retNS". This file contains the consumption and saving decisions of the retired agents. As explained above, retired agents receive two types of transfers: contributive, associated with compulsory savings, and non-contributive transfers from the government. These decisions are made according to equation (3) in the main text. The third file is "CFNS" (NS means non-segmented labor market) puts together active and non-active decisions and compute the level of assets for the period previous to retirement (the last active period). This is done in equation (10). The file "EST" computes the "matched" parameters (δ,α) according to the observed values for the coefficient of variation of consumption and average duration of unemployment. Finally, the file "markovchain" simulates a finite state space Markov process.

The estimation procedure runs as follows. The first step is to run the EST file providing as an input the vector of the three moments used for estimation, using the syntactic expression ESTNS(0.15, 0.43, 1.5). As explained in the main text, the first and second inputs are the coefficient of variation of consumption for the active life and retired life respectively. The last number is the average duration of unemployment. The process for wages is in fact assumed to be iid (all rows of the transition matrix are equal). The values for the wages (formal and informal sector) are taken from the data as well as the probability associated with each possible value.

The EST file provides as output a first iteration for the estimated parameters, using the simulated method of moments, named as "alphastarNS" and "lambdastarNS" in the code. We repeat the procedure several times and compute the average across simulations. That is, the name "alphastarNSj" is the output of the j-th execution of the code "ESTNS". The reported valued must be an average of across the J simulations. These averages are the estimated calling (alphastarNS_Avg) and lay-off probability (lambdastarNS_Avg).

[24]All codes are written in MATLAB language.

The algorithm uses these average values "alphastarNS_Avg" and "lambdastarNS_Avg" to run the file "CFNS". This file provides a series of consumption, assets, and labor-decision statistics listed as an output of the code. The next step repeats the procedure several times and average the values across simulations for each time (i.e., $CANS_{i,t}$ is the consumption (C) of the agent during her active (A) life in a non-segmented (NS) labor market, in period "t", simulation "i". ($\frac{\sum_i CANS_{i,t}}{I}$) must be the reported value, where "I" is the number of simulations).

The files "CFNS", "retNS", "CFNS", "ESTNS" and "markovchain" are available under request.

Finally, in order to compute the model, we had to make an additional assumption. As suggested by Pakes and Pollard (1989), the backward induction nature of lyfe cycle models coulpled with the dependence of equation (10) on the expost value of savings Y_{T-1} make the problem intractable as the guess value for Y_{T-1} may not be realized ex-post after any possible simulation.

Thus, we assume that the last period of active life depends only on a_T, the asset level selected in period $T-1$, and not on the value of the program in the month before retirement, V^e_{T-1} in equation (10). That is, the value function for the last period of active life, $T-1$, *relevant for the period before that* is different with respecto to the value function used to compute optimal decisions *for the period inmmediately after* $T-1$. The former is equal simply to:

$$V^e_{T-1}\left(Ra_{T-1}+y^e_{T-1};Y_{T-1}\right)=\max_{a_T}\frac{\left(Ra_{T-1}+y^e_{T-1}-a_T\right)^{1-\sigma}}{1-\sigma}$$

The latter is equation (10). This assumption implies that our simulations underestimate the effect of retirement wealth on labor markets. However, the level of assets a_T is increasing in the value of the program in the first period of retirement as it is computed using equation (10). We solve the active part of the life cycle using a given value for a_T and then compute a_T as a solution of equation (10). This strategy implies that we have a different value for, say, unemployment duration for each possible level of assets in the grid and then we select the one associated with the maximal level of a_T according to equation (10).

Closed forms for the retired agents

Before-the-Last period

This is the period characterized as age $T+\mathcal{T}$ and month 11. **W**e solve the problem using months but computations are done for semesters (i.e., every year has 2 periods instead of 12). Set-up problem:

$$\max_{a_{T+\mathcal{T},12}}\frac{\left(Ra_{T+\mathcal{T},11}+\tau_{T+\mathcal{T},11}-a_{T+\mathcal{T},12}\right)^{1-\sigma}}{1-\sigma}+\beta\frac{\left(Ra_{T+\mathcal{T},12}+\tau_{T+\mathcal{T},12}\right)^{1-\sigma}}{1-\sigma} \tag{A1}$$

FOC

$$\left(Ra_{T+\mathcal{T},11}+\tau_{T+\mathcal{T},11}-a_{T+\mathcal{T},12}\right)^{-\sigma}=\beta R\left(Ra_{T+\mathcal{T},12}+\tau_{T+\mathcal{T},12}\right)^{-\sigma} \tag{A2}$$

or

$$\left(Ra_{T+\mathcal{T},12}+\tau_{T+\mathcal{T},12}\right)=(\beta R)^{\frac{1}{\sigma}}\left(Ra_{T+\mathcal{T},11}+\tau_{T+\mathcal{T},11}-a_{T+\mathcal{T},12}\right) \tag{A3}$$

so

$$a_{T+\mathcal{T},12}(*) = \frac{(\beta R)^{\frac{1}{\sigma}}\left(Ra_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},11}\right) - \tau_{T+\mathcal{T},12}}{R + (\beta R)^{\frac{1}{\sigma}}}$$

$$c_{T+\mathcal{T},11}(*) = \frac{R\left(Ra_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},11}\right) + \tau_{T+\mathcal{T},12}}{R + (\beta R)^{\frac{1}{\sigma}}}$$

$$c_{T+\mathcal{T},12}(*) = \frac{(\beta R)^{\frac{1}{\sigma}}\left[R\left(Ra_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},11}\right) + \tau_{T+\mathcal{T},12}\right]}{R + (\beta R)^{\frac{1}{\sigma}}}$$

so

$$U_{T+\mathcal{T},11} = \left[R\left(Ra_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},11}\right) + \tau_{T+\mathcal{T},12}\right]^{1-\sigma} \frac{\left[1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]}{\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{1-\sigma}}$$

Two months before death

The savings problem is given by:

$$\max_{a_{T+\mathcal{T},11}} \frac{\left(Ra_{T+\mathcal{T},10} + \tau_{T+\mathcal{T},10} - a_{T+\mathcal{T},11}\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]\left[R\left(Ra_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},11}\right) + \tau_{T+\mathcal{T},12}\right]^{1-\sigma}}{\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{1-\sigma}}$$

The FOC is

$$\begin{aligned} &\left(Ra_{T+\mathcal{T},10} + \tau_{T+\mathcal{T},10} - a_{T+\mathcal{T},11}\right)^{-\sigma} \\ = \quad & \beta R^2 \frac{\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]\left[R\left(Ra_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},11}\right) + \tau_{T+\mathcal{T},12}\right]^{-\sigma}}{\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{1-\sigma}} \end{aligned}$$

Equivalently this expression, after some algebra, becomes:

$$\begin{aligned} &\left[R\left(Ra_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},11}\right) + \tau_{T+\mathcal{T},12}\right] \\ = \quad & \beta^{\frac{1}{\sigma}} R^{\frac{2}{\sigma}} \frac{\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma}}}{\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}}} \left(Ra_{T+\mathcal{T},10} + \tau_{T+\mathcal{T},10} - a_{T+\mathcal{T},11}\right) \end{aligned}$$

Which in turn implies:

$$a_{T+\mathcal{T},11}(*) = \frac{\beta^{\frac{1}{\sigma}R^{\frac{2}{\sigma}}}\left[1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma}}\left[\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}}\right]^{-1}\left(Ra_{T+\mathcal{T},10} + \tau_{T+\mathcal{T},10}\right) - \left(R\tau_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},12}\right)}{R^2 + \left[\beta^{\frac{1}{\sigma}R^{\frac{2}{\sigma}}}\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma}}\right]\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{\frac{-(1-\sigma)}{\sigma}}}$$

$$c_{T+\mathcal{T},10}(*) = \frac{R^2\left(Ra_{T+T,10} + \tau_{T+T,10}\right) + \left(R\tau_{T+T,11} + \tau_{T+T,12}\right)}{R^2 + \left[\beta^{\frac{1}{\sigma}R^{\frac{2}{\sigma}}\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma}}}\right]\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{\frac{-(1-\sigma)}{\sigma}}}$$

$$c_{T+\mathcal{T},11}(*) = \frac{\beta^{\frac{1}{\sigma}R^{\frac{2}{\sigma}}}\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma}}\left[\left[R+(\beta R)^{\frac{1}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}}\right]^{-1}\left[R^2\left(Ra_{T+\mathcal{T},10} + \tau_{T+\mathcal{T},10}\right) + \left(R\tau_{T+\mathcal{T},11} + \tau_{T+\mathcal{T},12}\right)\right]}{R^2 + \left[\beta^{\frac{1}{\sigma}R^{\frac{2}{\sigma}}\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma}}}\right]\left[R+(\beta R)^{\frac{1}{\sigma}}\right]^{\frac{-(1-\sigma)}{\sigma}}}$$

Consequently the value function for the retired agent three months before she dies becomes:

$$U_{T+\mathcal{T},10} = \frac{\left[R^2\left(Ra_{T+T,10}+\tau_{T+T,10}\right)+\left(R\tau_{T+T,11}+\tau_{T+T,12}\right)\right]^{1-\sigma}}{\left[R^2+\left[\beta^{\frac{1}{2}R^{\frac{2}{\delta}}}\left[1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma}}\right]\left[R+(\beta R)^{\frac{1}{\sigma}}\right]^{\frac{-(1-\sigma)}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}}}\left[1 + \beta\frac{\beta^{\frac{1-\sigma}{\sigma}}R^{\frac{2(1-\sigma)}{\sigma}}\left[1+\beta(\beta R)^{\frac{1-e}{\sigma}}\right]^{\frac{1-\sigma}{\sigma}}}{\left[R+(\beta R)^{\frac{1}{\sigma}}\right]^{\frac{1-\sigma^2}{\sigma}}}\right]$$

Time of retirement(induction)

By induction, then by the first month of retirement the value is

$$U_{T,1} = \frac{\left[R^{12\mathcal{T}}a_{\mathcal{T}} + \sum_{s=1}^{12\mathcal{T}} R^{12\mathcal{T}-s}\tau_s\right]^{1-\sigma}}{1-\sigma}\Phi_{T,1}(\beta, R)$$

with

$$\Phi_{T,1}(\beta, R) \equiv \frac{\left[1 + (\beta R^{(12\mathcal{T}-1)(1-\sigma)})^{\frac{1}{\sigma}}\Phi_{T,2}(\beta, R)\right]}{\left[R^{12\mathcal{T}-1} + (\beta R^{(12\mathcal{T}-2)})^{\frac{1}{\sigma}}\Phi_{T,2}(\beta, R)\right]^{1-\sigma}}$$

More in general, for any year t such that $T \leq t \leq T + \mathcal{T}$ then

$$\Phi_{t,m}(\beta, R) \equiv \frac{\left[1 + (\beta R^{(12\mathcal{T}-(12-m)t)(1-\sigma)})^{\frac{1}{\sigma}}\Phi_{t,m+1}(\beta, R)\right]}{\left[R^{(12\mathcal{T}-(12-m)t)} + (\beta R^{(12\mathcal{T}-(12-m+1)t)})^{\frac{1}{\sigma}}\Phi_{t,m+1}(\beta, R)\right]^{1-\sigma}}$$

Where the last difference equation is modulo 12 and where

$$\Phi_{T+\mathcal{T},12}(\beta, R) = \frac{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{\left[R + (\beta R)^{\frac{1}{\sigma}}\right]^{1-\sigma}}$$