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Using a hedging network to minimize portfolio risk

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ABSTRACT

This paper develops a useful tool based on hedging networks that allows portfolio managers to allocate capital so as to build portfolios with low risk. We apply a popular measure from the network literature, the Katz centrality measure, to summarize how a security relates to other securities in the network (hedging relations) and to itself (unhedgeable component). We generate empirical evidence that picking stocks with the lowest value of the Katz centrality measure leads to portfolios with a low variance. We show that these portfolios achieve lower variance than other classical portfolio strategies, both in-sample and out-of-sample.

1. Introduction

In general terms, a network can be defined as a set of nodes and links that connects pairs of nodes. In finance literature, several papers (Pozzi, 2013; Diebold and Yılmaz, 2014; Peralta and Zareei, 2016; Výrost et al., 2019) use network theory to capture the complex structure of financial markets. These papers usually represent the financial market as a network where securities are nodes and the links among them are correlations. In this paper, we use hedging network, where each node is a security, and each link specifies the hedging relation between securities. A hedging relation can be defined as the contribution of a security j in hedging security i after eliminating the influence of other securities in the investment opportunity set. Basically, a security's return depends on its hedging relations with all other securities in the network, but also on a component (the unhedgeable component) not related to other securities' returns. Correlations can be noisy because they do not show the direct impact of securities on each other. Stated simply, two securities can be correlated because both are correlated with a third security. Thus, we use hedging relations that do not measure the effect of other securities on the relation between any two securities.

Based on hedging networks, this paper provides a useful tool for portfolio managers to allocate money and achieve portfolios with lower variance. We apply a popular measure in the network literature, the Katz centrality measure (Katz, 1953), to summarize how a security is related to other securities in the network (hedging relations) and to itself (unhedgeable component). Although financial markets are complex, and different network structures may exist, we provide empirical evidence that picking stocks with the lowest value in this centrality measure will allow us to achieve portfolios with the lowest risk. We show that these portfolios attain lower variance, both in-sample and out-of-sample, than other classical portfolio strategies (such as investing in securities with low correlation, minimum-variance with and without covariance-shrinkage estimator, market model, and an equally weighted portfolio which has been defended by some authors, such as DeMiguel et al., 2009 and Hsu et al., 2018). These results could be used by active portfolio managers to select stocks from a large dataset, but also by passive portfolio managers looking for diversification without holding a large number of securities.

This paper contributes to both the financial and networking literature. We contribute to the financial literature by proposing a new

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technique to achieve a better diversification strategy with fewer securities that works well out-of-sample. We also contribute to the new and growing literature on networks, proposing a new type of network (hedging network) that may now be tested in many other financial problems.

2. Hedging network

A network is defined as a pair of sets$\phi = \{V, E\}$, where V corresponds to the set of nodes and E the set of links connecting pairs of nodes. If there is a link from node i to node j, $(i, j) \in E$. The adjacency matrix, $A_{n \times n} = [A_{ij}]_{n \times n}$, summarizes the information in set E. The network is undirected if $A = A^T$. Moreover, if $A_{ij} \in \{0, 1\}$, the network ϕ is said to be an un-weighted network and when $A_{ij} \in \mathbb{R}$, the network is a weighted network.

Suppose we aim to hedge stock i with all the other available stocks in the market. Assuming $r_{i,t}$ as the return of stock i at time t, Stevens (1998) provides the following equation as "hedge regression":

$$r_{i,t} = \alpha_i + \sum_{k=1,\ k\neq i}^{N} \beta_{ik} r_{k,t} + \varepsilon_{i,t} \tag{1}$$

Where $\varepsilon_{i,t}$ is the part of $r_{i,t}$ that is not being hedged by other stocks. The variance of $\varepsilon_{i,t}$, named as the unhedgeable component of $r_{i,t}$, is denoted by υ_i. β_{ik}, the hedging relation, refers to the contribution of stock k in hedging stock i beyond the contribution of all other stocks (Goto and Xu, 2015).

Denoting the inverse covariance matrix by $\boldsymbol{\Sigma}^{-1} = \boldsymbol{\Psi} = [\psi_{ij}]_{N \times N}$, Stevens (1998) propose the following relation between $\boldsymbol{\Psi}$ and the hedge regression in Equation (1):

$$\psi_{ij} = \begin{cases} -\dfrac{\beta_{ij}}{\upsilon_i} & if\ i \neq j \\ \dfrac{1}{\upsilon_i} & if\ i = j \end{cases} \tag{2}$$

Subsequently, we deduct the hedging network from the inverse covariance matrix. The hedging network is formally defined as follows:

Definition. . The stock hedging network is $\Phi^W = \{N, \boldsymbol{\beta}\}$ where N is the set of stocks and $\boldsymbol{\beta} = \{(i, j) \in NxN\text{: } \beta_{ij} \neq 0\}$ the set of links. The intensity of an edge between node i and j is equal to $-\ \psi lon_i \psi_{ij}$ and the self-edge of node i in the network is equal to $\frac{1}{\psi_{ii}}$. The Φ^W is a weighted network whose adjacency matrix is given by $\boldsymbol{\beta}$. Therefore, a weighted link exists between stock i to stock j if β_{ij} is different from zero.

Assuming n risky assets with the covariance matrix, Σ. The vector of optimal weights, w, that minimizes the portfolio variance is as follows:

$$w^{minv} = \frac{\boldsymbol{\Psi} 1}{1' \boldsymbol{\Psi} 1} \tag{3}$$

And the variance of minimum-variance portfolio is:

$$\sigma_p^2 = w' \Sigma w = \frac{1}{1' \boldsymbol{\Psi} 1} = \frac{1}{\sum_{i=1}^{N} \frac{1}{\upsilon_i} - \sum_{i=1,\ i\neq j}^{N} \sum_{j=1}^{N} \frac{\beta_{i,j}}{\upsilon_i}} \tag{4}$$

From the above, we can observe that higher negative hedging relations ($\beta_{i,j}$) and lower unhedgeable components (υ_i) attain a lower portfolio variance and hence, a higher diversification benefit in a network structure.

3. Diversification using a centrality measure

In this section, we propose a measure to summarize the information in the hedging network for the purpose of building a portfolio with the lowest risk level. First, we evaluate the convenience of using this measure to achieve well-diversified portfolios in an in-sample analysis, and second, we evaluate and compare this to other conventional diversification strategies out-of-sample.

We use a popular measure in the network literature, the Katz centrality (Katz, 1953), that measures the relative influence of each node in a given network by taking into account the adjacent nodes (immediate neighbors), as well as non-immediate neighboring nodes that are connected indirectly through adjacent nodes. In our case, we calculate the Katz centrality measure (x_i) for an asset, using the information in hedging relations and the unhedgeable components:

$$x_i = \alpha \sum_j \beta_{ij} x_j + \boldsymbol{\nu}_i \tag{5}$$

Where β_{ij} is the hedging relation between asset i and j, and $\boldsymbol{\nu}_i$ is the unhedgeable component from asset i. α is between 0 and 1, and it governs the balance between the hedging relations ($\boldsymbol{\beta}$) and the unhedgeable component (decreasing the relevance of the hedging relations matrix as the value of α is reduced).

Table 1
Datasets description

Dataset Description	Source	Dates	Abbreviation	N
Monthly returns for the largest companies on the 31st of December 2019 with data for at least 20 years.	CRSP	From 1/2000 to 12/2019	20_CRSP	139
Monthly returns for the largest companies on the 31st of December 2019 with data for at least 30 years.	CRSP	From 1/1990 to 12/2019	30_CRSP	94
100 book-to-market and size portfolios	Ken French's Web site	From 01/1965 to 12/2019	BM-SIZE	100
100 size and investment portfolios	Ken French's Web site	From 01/1965 to 12/2019	SIZE-INVEST	100
100 size and operating profitability portfolios	Ken French's Web site	From 01/1965 to 12/2019	SIZE-PROFITAB	100

As we can see from (5), this centrality measure will be lower for stocks with negative hedging relations (or low positive hedging relations) and lower unhedgeable component, which are precisely the stocks that reduce the portfolio variance according to (4). Therefore, if a portfolio manager is choosing the assets with the lowest centrality to create a portfolio, she is investing in those assets that are reducing the portfolio variance.

4. Empirical evidence

In this section, we test whether a strategy based on the Katz centrality measure (selecting stocks with the lowest centrality value) actually achieves lower portfolio risk than other conventional portfolio strategies for different portfolio size. We first perform an in-sample analysis, then implement an out-of-sample test.

We employ five different datasets to generate robust results. We use two datasets from individual common stocks (20_CRSP and 30_CRSP) and three datasets from Fama-French portfolios (BM-SIZE, SIZE-INVEST, and SIZE-PROFITAB). For the individual stock datasets, we choose the 200 stocks with the highest capitalization in the American market (NYSE, NASDAQ and AMEX)[1] on the 31st of December 2019, and we include all stocks with less than 5 missing observations in 20 years (this is name as 20_CRSP) or in 30 years (30_CRSP). These datasets contain 139 and 94 common stocks, respectively. The other dataset are portfolios of stocks containing monthly excess returns (from the Ken French's Web site) based on different characteristics (book-to-market and size, size and investment, or size and operational profitability of the companies). The datasets[2] are described in detail in Table 1.

We generate portfolios of differing size (N stocks) based on the following strategies: Lowest Centrality, Highest Centrality, Lowest Correlation, and Naïve Diversification (1/N strategy). Therefore, for a portfolio with number of stocks N, a Lowest Centrality (Highest Centrality) strategy is generated by selecting the N stocks with the lowest (highest) centrality in the dataset and creating an equally weighted portfolio with these N selected stocks. Lowest Correlation strategies are based on the same procedure, selecting the N stocks with the lowest correlation coefficient in the dataset. We define Naïve Diversification as an equally weighted portfolio ($w_i = w_j = 1/N$), also called the 1/N strategy.[3] This is the simplest diversification strategy analyzed in DeMiguel et al. (2009) and Maillard et al. (2010), which find that this strategy can outperform more complex and sophisticated portfolio strategies, such as mean–variance.

Although we are interested in the strategy that selects stocks with the lowest centrality measure, we also introduce the strategy with the highest centrality, because, following our hypothesis, stocks with the highest centrality values should show the worst diversification benefits. In addition, we introduce the lowest correlation strategy to compare our strategy based on the Katz centrality measure to the classical theory based on return correlations. This allows us to demonstrate that negative hedging and negative correlation are not identical concepts.

First, we show in an in-sample analysis, the variance for each portfolio strategy (previously defined) holding from 2 to 40 stocks in the various datasets. From Fig. 1, we observe that the Lowest Centrality strategy (red line) is the best strategy, achieving a lower variance for any number of stocks in the majority of the dataset. This result confirms that the Katz centrality measure classifies stocks properly according to their contribution to diversification benefits. In addition, as discussed previously, the strategy based on holding stocks with the highest centrality values is the one with the worst performance in terms of diversification as we forecasted.

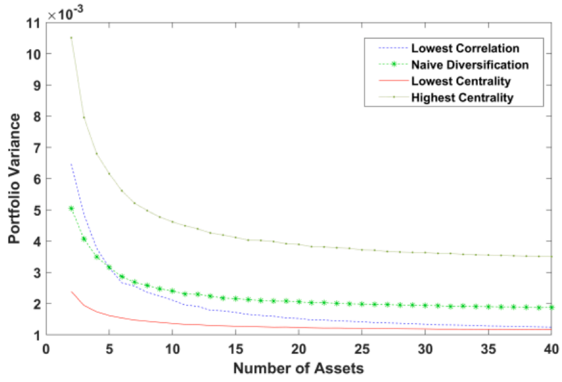
In addition, from Fig. 1 we can observe that the Lowest Correlation strategy is close to our strategy only if the number of stocks is high enough (around 30 stocks in 4 out of 5 dataset and 10 stocks in the SIZE-INVEST dataset). From this result, we may extract an interesting conclusion for portfolio management. For investors with reduced portfolio size (e.g., private investors), using the hedging networks proposed in this paper could help to achieve less risky portfolios.

Next, we examine the out-of-sample portfolio's variance of investing in a portfolio with a specific number of stocks (10, 20, 30, and 40) according to the strategies previously studied. In addition, we also compare the results with other standard portfolio strategies used

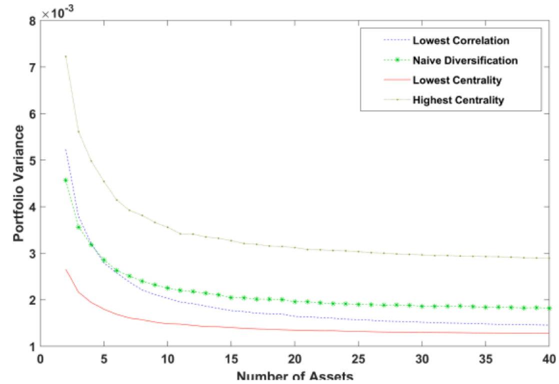
[1] We obtain both the returns and information about capitalization from the CRSP, and the specific tickets of the selected companies are available upon request.

[2] This dataset contains different types of assets (stocks and portfolios) and states of the market (bull and bear), being representative of different business cycles.

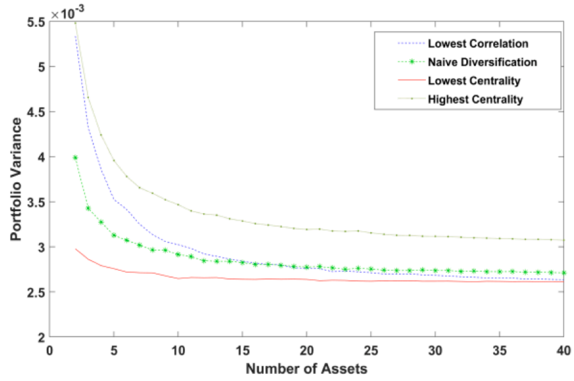
[3] For this strategy, our next results represent the average variance of considering 1,000 random, equally weighted portfolios selected from the N stocks in each dataset.



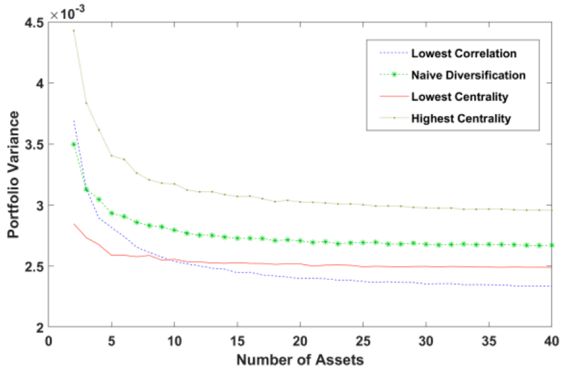
a) 20_CRSP



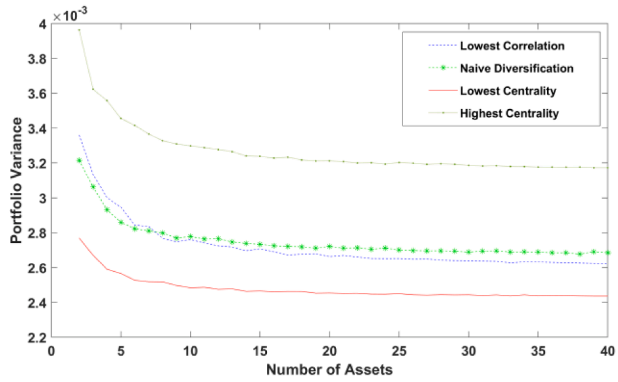
b) 30_CRSP



c) BM-SIZE



d) SIZE-INVEST



e) SIZE-PORFITAB

Fig. 1. This Fig. shows the portfolio variance decay in an in-sample analysis for a set of portfolio strategies holding from 2 to 40 stocks in five different datasets: (a) 20_CRSP, (b) 30_CRSP, (c) BM-SIZE, (d) SIZE-INVEST and (e) SIZE-PROFITAB.

in the literature in out-of-sample analysis: Market-Model, Minimum-Variance portfolio (sample covariance matrix), and Shrinkage Minimum-Variance portfolio (based on Shrinkage covariance estimation from Ledoit and Wolf (2003)). We employ the "rolling window" approach described in DeMiguel et al. (2009). Assuming, in total, T periods of stock returns, we set an M-period estimation window where the hedging networks (and the correlation coefficients or covariances) are estimated and the portfolios from each strategy are created. We maintain each portfolio for the next H-period observations, and compute the out-of-sample returns. We then roll the estimation window by including these new H observations and repeat the investment process. We re-estimate the network (and the other strategies) including these new H observations and removing the earliest H observations to the M-period estimation window.

Table 2
Annualized Portfolio Variance in an out-of-sample analysis (M=120, H=6)
In each row, the strategy achieving the lowest portfolio variance is in bold, and * denotes significance at the 10% level, ** denotes significance at the 5% level, and *** denotes significance at the 1% level for the test of equal variances between each strategy and the Lowest Centrality strategy.

	Lowest Centrality	Naive Diversific.	Market model	Minimum-variance	Minimum-variance (Shrinkage)	Lowest Correlation	Highest Centrality
30_CRSP							
10	**0,0141**	0,0203***	0,0234***	0,1038***	0,072***	0,0195***	0,0851***
20	**0,0138**	0,0203***	0,0234***	0,1038***	0,072***	0,0187***	0,0523***
30	**0,0139**	0,0203***	0,0234***	0,1038***	0,072***	0,0187**	0,0408***
40	**0,0151**	0,0203***	0,0234***	0,1038***	0,072***	0,0185*	0,0339***
20_CRSP							
10	**0,0097**	0,0152***	0,0152***	0,0362***	0,0403***	0,0138**	0,0502***
20	**0,0091**	0,0152***	0,0152***	0,0362***	0,0403***	0,0115*	0,0360***
30	**0,0088**	0,0152***	0,0152***	0,0362***	0,0403***	0,0111**	0,0307***
40	**0,0091**	0,0152***	0,0152***	0,0362***	0,0403***	0,0107**	0,0283***
BM-SIZE							
10	**0,0230**	0,0307***	0,0235	0,0439***	0,0525***	0,0658***	0,0690***
20	**0,0233**	0,0307***	0,0235	0,0439***	0,0525***	0,0354***	0,0507***
30	0,0239	0,0307***	**0,0235**	0,0439***	0,0525***	0,0306***	0,0445***
40	0,0249	0,0307***	**0,0235***	0,0439***	0,0525***	0,0297***	0,0408***
SIZE-INVEST							
10	**0,0210**	0,0303***	0,0229**	0,0409***	0,0572***	0,0345***	0,0639***
20	**0,0214**	0,0303***	0,0229***	0,0409***	0,0572***	0,0278***	0,0510***
30	**0,0222**	0,0303***	0,0229	0,0409***	0,0572***	0,0271***	0,0452***
40	0,0230	0,0303***	**0,0229**	0,0409***	0,0572***	0,0273***	0,0421***
SIZE-PROFITAB							
10	**0,0205**	0,0302***	0,0226***	0,0489***	0,029***	0,0288***	0,0592***
20	**0,0221**	0,0302***	0,0226	0,0489***	0,029***	0,0291***	0,0481***
30	0,0229	0,0302***	**0,0226**	0,0489***	0,029***	0,0285***	0,0425***
40	0,0236	0,0302***	**0,0226**	0,0489***	0,029***	0,0283***	0,0397***

In our baseline results, we consider the estimation window (M) equal to 120 months, the holding-period (H) of 6 months and α value of 0.6. In addition, we repeat the analysis for other specifications (M=120, M=180 M=240; and H=1, H=6, H=12) and other α values (0.1, 0.2, 0.4, 0.6, 0.8, 0.9) and the conclusions remain the same.[4]

Table 2 shows the out-of-sample results (measured by annualized variance) for each strategy and portfolio size. Each column represents a different portfolio strategy (previously described). Across the rows, we show the results for portfolios of different size from 10 to 40 stocks and dataset. Bold Fig.s indicate the strategy with the lowest risk for each row, and we test for equality of variances for each specific strategy against the Lowest Centrality strategy[5]. Table 2 shows that our proposed strategy of holding the securities with the lowest centrality achieves lower portfolio risk than any other strategy in the majority of the datasets. In the case of individual common stock datasets (30_CRSP and 20_CRSP), the Lowest Centrality strategy achieves a lower variance than any other strategy, and it is statistically significant for any number of stocks in the portfolio. However, in the case of Fama-French portfolios, we observe that our strategy outperforms the rest of strategies only for a low number of assets (among 10 and 20), but it is not worse than the other strategies from an statistically perspective (as the variance is not statistically different from the Market Model in the case of 30 and 40 assets, in general).

We also observe that the Naïve Diversification strategy is generally better than the Minimum-Variance or Minimum-Variance (Shrinkage) strategy that is in some sense in concordance with DeMiguel et al. (2009) results. The Lowest Correlation strategy has a larger variance than the Lowest Centrality portfolio in almost every case demonstrating that they are different strategies. In addition, we observe that the Highest Centrality strategy achieve always worse results in terms of diversification, which confirms that our main hypothesis on the performance based on Katz centrality measure is working properly.

5. Conclusion

We analyze portfolio diversification based on network theory, and we propose a new type of networks based on hedging relations among stocks instead of correlations. We propose using the Katz centrality measure, which considers both variables (hedging relations and unhedgeable component) and allows portfolio managers to select the most suitable stocks to achieve a well-diversified portfolio. Next, we use empirical data to evaluate the convenience of using this centrality measure to achieve well-diversified portfolios in both in-sample and out-of-sample analysis. This strategy based on holding stocks with the lowest centrality provides lower portfolio variance than some other traditional strategies, such as selecting stocks by correlation coefficient, the minimum-variance portfolio or a naïve strategy (consisting of an equally weighted portfolio of every stock in the opportunity set). We observe this result for different datasets and both an in-sample and out-of-sample analysis. Another interesting result is that the number of securities needed in a portfolio to be well-diversified is lower using the proposed strategy than using other traditional portfolio strategies.

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[4] We do not report the results to save space. They are available upon request.
[5] We follow the bootstraping method in Ledoit and Wolf (2011).