

## Tilburg University

### Announcing climate policy

Smulders, Sjak A.; Tsur, Y.; Zemel, A.

*Publication date:*  
2010

*Document Version*  
Early version, also known as pre-print

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*

Smulders, S. A., Tsur, Y., & Zemel, A. (2010). *Announcing climate policy: Can a green paradox arise without scarcity?* (CESifo Working Paper; No. 3307). CESifo Group Munich.

#### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

#### **Take down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Announcing Climate Policy: Can a Green Paradox  
Arise without Scarcity?

Sjak Smulders  
Yacov Tsur  
Amos Zemel

CESIFO WORKING PAPER NO. 3307  
CATEGORY 10: ENERGY AND CLIMATE ECONOMICS  
DECEMBER 2010

*An electronic version of the paper may be downloaded*

- *from the SSRN website:* [www.SSRN.com](http://www.SSRN.com)
- *from the RePEc website:* [www.RePEc.org](http://www.RePEc.org)
- *from the CESifo website:* [www.CESifo-group.org/wp](http://www.CESifo-group.org/wp)

# Announcing Climate Policy: Can a Green Paradox Arise without Scarcity?

## Abstract

Unintended consequences of a pre-announced climate policy have been studied in a variety of situations. We show that early announcement of a carbon tax gives rise to a “Green-Paradox,” in that it increases polluting emissions in the interim period (between announcement and actual implementation), irrespective of the scarcity of fossil fuels. The phenomenon holds both when the announced implementation date is taken as a credible threat and when households are skeptical about the (political) will or capability of the government to implement the policy as announced. The paradoxical outcome is driven by consumption-saving tradeoffs facing households who seek to smooth consumption over time.

JEL-Code: Q43.

Keywords: climate policy, carbon tax, green paradox, uncertainty.

*Sjak Smulders*  
*Department of Economics and CentER*  
*Tilburg University*  
*PO Box 90153*  
*5000 LE Tilburg*  
*The Netherlands*  
*J.A.Smulders@uvt.nl*

*Yacov Tsur*  
*Department of Agricultural Economics and*  
*Management*  
*The Hebrew University of Jerusalem*  
*P.O. Box 12 Rehovot*  
*76100, Israel*  
*tsur@agri.huji.ac.il*

*Amos Zemel*  
*Department of Solar Energy and Environmental Physics*  
*The Jacob Blaustein Institutes for Desert Research*  
*Ben Gurion University of the Negev*  
*Sede Boker Campus*  
*84990, Israel*  
*amos@bgu.ac.il*

June 15, 2010

“.....If you have to shoot, shoot, don’t talk!” – Tuco in *The Good, the Bad and the Ugly*.

# 1 Introduction

Climate policy making is complex because costs and benefits are uncertain, arise at different points of an extended time span, and fall on different countries and generations. Nevertheless, it is generally felt undesirable to postpone action until all complexities are sorted out. A way to deal with the complexities is to announce policies well ahead of their implementation, in order to give involved parties a chance to prepare their compliance with the regulation.

Two main problems are associated with such an “announce-in-advance” approach to policy making, namely the “leakage effect” and the “announcement effect”. The first arises when participation and coverage of the policy are incomplete. If not all of the polluting sectors or countries are subject to the policy, the response of unregulated sectors and countries could undo some of the pollution reductions in the regulated sectors and countries. The leakage effect may be *partial* (emission reduction in one country is partly offset by increased emission in other countries), it may be *negative*, i.e., with reduced emission also in the unregulated countries [see 1, 4], and it can be *positive*, i.e., give rise to an increased overall emission (the reduction of regulated countries is more than offset by increases in unregulated countries). The latter case is referred to as the *green paradox* [see, for example 7, 5].

The announcement effect corresponds to actions taken by affected parties immediately after the announcement and before the policy is implemented. Two types of announcement effects may occur. The favorable type occurs when polluters start abating pollution and accumulating credits, partly because it is less costly to spread abatement over time rather than concentrate efforts at a short period. The unfavorable type – vis-à-vis the intention to reduce emission – occurs when firms increase emission. They might have stocks of polluting inputs that they would like to quickly use before they are no longer allowed to do so. More subtly, but essentially through the same mechanism, the total stock of polluting resources might be inelastic in supply, because a nonrenewable resource

like oil, gas, or coal is involved. This is why an announcement of carbon taxation may induce resource owners to lower prices and induce users to burn more fossil fuels. This mechanism is studied in [3]. Its robustness vis-à-vis the presence of different types of backstop resources (cheap or expensive, clean or polluting) is considered in [9].

In this work we study unfavorable effects of announcing a carbon tax well in advance. In particular, we show how the announcement entails an increase in the use of fossil energy until the implementation date. Our approach deviates from the existing literature in three main ways. First, we abstract from scarcity of energy resources, i.e. there is no stock of energy resources that owners are eager to deplete. While the papers cited above all rely on the scarcity of the polluting input to generate the paradoxical announcement effect, we show that scarcity is not required. This situation seems relevant to abundant resources like coal for which the scarcity rent is likely to be very small.

Second, we account for the consumption-saving decisions made by households and for the effect of capital stock on energy demand. We thus pay attention also to the dynamics at the energy demand side, rather than focusing on the supply side. The build-up of capital is time-consuming so that investment behavior is forward looking. When the climate policy is announced, investment responds and the change in capacity affects energy demand before the climate policy is implemented. The existing literature typically assumes the resource stock to be the only predetermined stock variable [e.g. 6]. Abstracting from resource scarcity allows us to focus on other investment decisions without losing tractability.

Third, we allow for a competition between conventional (dirty) and alternative (clean) energy technologies. We show that the timing and technological opportunities of these are crucial in determining the paradoxical announcement effects. In doing so we extend the model of [8], which investigates the incentives to build up solar energy capacity, to the situation of pre-announced policies. The more general interpretation is that we deal with investment in productive capacity as well as equipment and knowledge capital for

abatement and alternative energy supply. We show that, depending on the relative cost of both types of investment, a green policy announcement may paradoxically result in increased emissions during the interim period.

The conventional view underlying the early announcement of a climate policy is that, if firms start investing in abatement capital or alternative energy supply in anticipation of the implementation of the policy, pollution falls because of the announcement. However, our model suggests that the paradoxical result of *increased* pollution is more likely. To show this, we assume in Section 3 that the policy takes the form of announcing that a carbon tax will be implemented at some future date. At the time of implementation, energy use falls so that output is lower. Since households want to smooth consumption, there is a rationale for accelerating investment early on, giving rise to a larger capital stock. The accumulation of capital raises the demand for energy before implementation since capital and energy are (imperfect) complements as factors of production. Thus, announcing a policy aimed at reducing the use of fossil energy well in advance gives rise to the opposite effect until the policy is actually realized. The result holds both when the regulation policy involves a mild tax rate which reduces fossil use but does not induce the use of alternative, clean (solar) energy as well as when the tax rate is high enough to trigger a transition to solar energy.

In section 4 we extend the results by considering uncertainty as yet another driver of paradoxical effects. We incorporate uncertainty into the model by assuming that the government announces the intention to levy the carbon tax, but the date of implementation depends on political conditions and is therefore uncertain. The distinction appears to be important as it affects the underlying mechanism that drives the paradox. In particular, the continuity of the consumption process plays a key role in deriving the early announcement effect when the implementation date is known with certainty. In contrast, under uncertain implementation date, the consumption path undergoes a discontinuous jump at the (random) implementation date. Nevertheless, we establish the “green paradox” also

under uncertainty, and show that it is driven by the same economic forces: anticipating that the tax will reduce energy use in the future induces households to enhance saving today in order to accumulate more capital that can substitute for the lower energy input. Prior to implementation of the tax policy, the increased capital stock is associated with increased energy input, hence the paradoxical outcome. Indeed, since uncertainty regarding implementation appears to be a common feature characterizing climate policies, the negative effect of the paradox may be significant.

Of course, saving (capital accumulation) comes at the expense of consumption, and the realization of the paradoxical effect depends on a condition relating the production elasticity of capital to the elasticity of marginal utility of consumption. This condition holds in any empirically relevant calibration and the paradoxical early announcement effect appears to be robust.

## 2 Competitive allocation

We build on the framework of Tsur and Zemel [8], who analyzed the penetration of solar technologies in a competitive economy. In subsection 2.1 we briefly describe the competitive (unregulated) economy and summarize the relevant properties characterizing it. In subsections 2.2 and 2.3 we extend the framework to better suit the present focus regarding early-announcement-green-paradox phenomena.

### 2.1 The economy and energy tradeoffs

The economy consists of a final good sector, an energy sector and households that own capital and labor. The final good is produced by capital, energy and labor inputs via a constant-returns-to-scale technology, represented by the per-capita production function  $y(\cdot, \cdot)$  of per-capita capital  $k$  and per-capita energy  $x$ , satisfying, for  $k > 0$  and  $x > 0$ , the

standard properties:

$$\begin{aligned}
y(0, x) = 0; \quad y(k, 0) = 0; \quad y_k(k, x) > 0; \quad y_x(k, x) > 0; \quad y_k(0, x) = \infty; \\
y_{kk}(k, x) < 0; \quad y_{kx}(k, x) > 0; \quad y_{xx}(k, x) < 0; \\
y_{kk}(k, x)y_{xx}(k, x) - y_{kx}^2(k, x) > 0; \quad y_k(k, x)k + y_x(k, x)x < y(k, x).
\end{aligned} \tag{2.1}$$

In (2.1), the subscripts  $k$  and  $x$  signify partial derivatives with respect to these inputs.

### 2.1.1 Energy sector

The energy sector consists of fossil energy firms and solar energy firms.<sup>1</sup> Fossil energy is generated by burning fossil fuels at a constant marginal cost  $\zeta$ . Solar energy is generated by capital input, denoted  $s$ , designated for that purpose (photovoltaic panels, solar thermal collectors, wind turbines etc.), such that each unit of  $s$  generates  $b$  power units, where  $b > 0$  is a technological conversion parameter. Once the solar infrastructure  $s$  has been installed, there is no additional variable cost associated with solar energy generation. The cost of solar energy is therefore solely due to the capital cost. The two sources of energy are perfect substitutes, thus

$$x = x^f + bs, \tag{2.2}$$

where  $x^f$  stands for the fossil energy supply rate.

Forward-looking solar energy firms determine their investment policy based on current and future prices of energy and capital in order to maximize the present-value of their profit stream [see details in 8]. Their investment policy determines the stock of solar energy capital according to

$$\dot{s}(t) = \iota(t) - \delta s(t), \tag{2.3}$$

where  $\iota(t)$  is the investment rate in solar capital and  $\delta > 0$  is a depreciation rate.

---

<sup>1</sup>Solar energy is used generically to represent alternative, non-polluting energy technologies.



### 2.1.2 Final good firms

So long as fossil energy is used, the competitive energy firms drive the energy price to the marginal cost of fossil energy supply  $\zeta$ . At this price, final good firms demand energy up to the level at which the value of its marginal product equals the energy price:

$$y_x(k(t), x(t)) = \zeta. \quad (2.4)$$

The demand for capital is set at the level where the value of its marginal product equals the price of capital – the interest rate. At each point of time  $t$ , the equilibrium interest rate equates the capital demand and capital supply, where the latter is determined by households' saving decisions.

### 2.1.3 Households

At time  $t$ , the representative household's wealth equals  $k(t) + s(t)$ , yielding the income  $y(k(t), x^f(t) + bs(t)) - \zeta x^f(t)$ . This income is used for final good consumption,  $c(t)$ , and saving (gross wealth increase),  $\dot{k}(t) + \delta k(t) + \dot{s}(t) + \delta s(t)$ . Thus, the household budget constraint at time  $t$  is

$$y(k(t), x^f(t) + bs(t)) - \zeta x^f(t) = c(t) + \dot{k}(t) + \delta k(t) + \dot{s}(t) + \delta s(t),$$

which, noting (2.3), can be expressed as

$$\dot{k}(t) = y(k(t), x^f(t) + bs(t)) - \zeta x^f(t) - \iota(t) - \delta k(t) - c(t). \quad (2.5)$$

The household derives felicity from consumption according to a strictly concave and increasing instantaneous utility function  $u(\cdot)$  and seeks the consumption-saving policy that maximizes

$$\int_0^{\infty} u(c(t)) e^{-\rho t} dt \quad (2.6)$$

subject to the budget constraint (2.5), taking parametrically the final good firms decisions (2.4) and the solar firms investment policy, where  $\rho$  is the pure (utility) rate of discount.

### 2.1.4 Equilibrium

In a perfect-foresight, competitive equilibrium, the expectations of all participants are fulfilled and no party has an incentive to alter decisions. The competitive (equilibrium) allocation is characterized in Tsur and Zemel [8]. We summarize the salient properties of the competitive allocation, making use of the following notation:

$$\zeta^c = (\rho + \delta)/b \quad (2.7)$$

is the critical fossil energy price which just equals the imputed cost of the solar capital required to produce one power unit at a capital price (interest rate)  $\rho + \delta$ ;

$$X(k, \zeta) = \arg \max_x \{y(k, x) - \zeta x\} \quad (2.8)$$

is the energy input demanded by final good firms with capital stock  $k$  when the energy price is  $\zeta$ , i.e., the energy input  $x$  that satisfies (2.4). Under assumption (2.1),

$$X_k(k, \zeta) = \frac{-y_{kx}(k, X)}{y_{xx}(k, X)} > 0. \quad (2.9)$$

Thus, in a competitive allocation, energy input increases with the stock  $k$ . This intuitive property underlies the “paradoxical” outcome derived below.

It is verified in Tsur and Zemel [8] that investing simultaneously in both final good capital  $k$  and solar capital  $s$  can be optimal only when the values of their marginal products are equal, i.e., when

$$y_k(k, x) = by_x(k, x). \quad (2.10)$$

The  $by_x$  term on the right hand side represents the marginal return to solar capital: an additional unit of solar capital allows for the saving of  $b$  units of fossil energy with the marginal product  $y_x$ . The marginal product of capital  $y_k$  on the left hand side represents the opportunity cost of solar investment.

It is also verified (*op. cit.*) that in a steady state, where  $\dot{k} = \dot{s} = 0$ , the condition

$$y_k(\hat{k}, \hat{x}) = \rho + \delta \quad (2.11)$$

must hold. Steady state levels are indicated by a hat “ $\hat{\cdot}$ ” over a variable. A steady state occurs when increasing capital further no longer contributes to the households intertemporal utility. Equation (2.11) states that this happens when the net gains from investment  $(y_k - \delta)$  equal the time cost of investment  $(\rho)$ . The steady state properties of the competitive allocation are summarized in:

**Proposition 1.** *The competitive allocation processes converge in the long run to a steady state, based either on fossil energy alone or on solar energy alone, according to:*

- (i) *If  $\zeta \leq \zeta^c$ , then no investment in solar energy ever takes place ( $s(t) = 0$  for all  $t \geq 0$ ), and the capital-energy steady state levels  $(\hat{k}, \hat{x})$  are determined by conditions (2.4) and (2.11), with  $\hat{x}^f = \hat{x}$ .*
- (ii) *If  $\zeta > \zeta^c$ , then the steady state is based solely on solar energy with  $(\hat{k}, \hat{x})$  determined by conditions (2.10) and (2.11),  $\hat{x}^f = 0$  and  $\hat{s} = \hat{x}/b$ .*

This result is intuitive: if the energy price is low relative to the discount rate ( $\zeta < (\rho + \delta)/b \equiv \zeta^c$ ), the relatively high time cost of capital prevents the accumulation of sufficient capital to render solar energy competitive relative to the fossil resource. In contrast, if  $\zeta > (\rho + \delta)/b \equiv \zeta^c$ , the low discount rate generates abundance of capital in the steady state and, in turn, low opportunity cost of solar investment. This, together with the high fossil price, generates high returns to replacing fossil energy.

Economies satisfying condition (i) are called *fossil-based* economies, while those satisfying condition (ii) are called *solar-based*. These labels describe long term behavior. When the initial capital stock  $k_0$  is small, energy is first derived exclusively from fossil sources at the rate  $X(k, \zeta)$  and investment in solar capital is delayed. For fossil-based economies, this policy holds at all times. In solar-based economies (where  $\zeta > \zeta^c$ ), investment in solar capital begins as soon as  $k$  reaches the threshold level  $k_m(\zeta)$ , defined by

$$y_k(k_m(\zeta), X(k_m(\zeta), \zeta)) = b\zeta. \quad (2.12)$$

Noting (2.4) and (2.10),  $k_m(\zeta)$  marks the lowest  $k$  level at which investment in solar capital is worthwhile.

We now describe how the solar-based economy evolves over time. To that end, it is expedient to introduce the wealth variable

$$w = k + s \tag{2.13}$$

and its particular threshold level

$$w_{ms}(\zeta) = k_m(\zeta) + X(k_m(\zeta), \zeta)/b. \tag{2.14}$$

The characterization in [8] of the solar-based processes is summarized in:

**Proposition 2.** *The competitive processes of solar-based economies (with  $\zeta > \zeta^c$ ) evolve along the following three consecutive phases:*

**Fossil Phase:** *While  $w(t) < k_m(\zeta)$ , only fossil energy is used at the rate  $x^f(t) = X(k(t), \zeta)$  and no investment in solar energy is undertaken ( $s(t) = 0$ ).*

**Mixed Phase:** *While  $k_m(\zeta) \leq w(t) < w_{ms}(\zeta)$ , both fossil and solar energy are used simultaneously with  $k(t)$  fixed at  $k_m(\zeta)$ ,  $w(t) = k_m(\zeta) + s(t)$  and solar energy gradually replacing fossil energy:  $x^f(t) = X(k_m(\zeta), \zeta) - bs(t)$ .*

**Solar Phase:** *While  $w(t) \geq w_{ms}(\zeta)$ , no fossil energy is used ( $x^f(t) = 0$ ) and both types of capital grow simultaneously towards the steady state.*

During the final, solar phase both  $k$  and  $s$  grow simultaneously and the arbitrage condition (2.10) implies that solar capital equals the function  $S(w)$  defined, implicitly, by the relation

$$y_k(w - S(w), bS(w)) = by_x(w - S(w), bS(w)). \tag{2.15}$$

From (2.15) we deduce, using assumption (2.1), that

$$S'(w) = \frac{by_{kx} - y_{kk}}{2by_{kx} - y_{kk} - b^2y_{xx}} \in (0, 1) \tag{2.16}$$

hence both solar capital,  $S(w)$ , and the final good capital,  $k = w - S(w)$ , increase as wealth  $w$  grows from  $w_{ms}(\zeta)$  to the steady state value  $\hat{w} = \hat{k} + \hat{x}/b$ , where  $\hat{k}$  and  $\hat{x}$  are defined in Proposition 1(ii). During the two earlier phases, one of the assets remains fixed while wealth  $w$  increases:  $s(t) = 0$  during the fossil phase and  $k(t) = k_m(\zeta)$  during the mixed phase.

The intuition behind proposition 2 is similar to that behind the previous proposition. Starting with a low initial capital stock, the return to investing in  $k$  is larger than the return to replacing fossil by solar. Once a sufficiently high level ( $k_m$ ) of capital has been accumulated, the opportunity cost of solar investment has fallen enough to make solar investment profitable. Simultaneous investment in  $k$  and  $s$  cannot apply immediately since once  $k$  exceeds  $k_m$ , the return to further investment in capital,  $y_k$ , falls below the return to solar investment, which is fixed by the fossil energy price as long as fossil is still needed to satisfy energy demand ( $by_x = b\zeta$ ). Only after solar capital  $s$  has expanded enough to fully replace fossil fuels ( $bs = X(k_m(\zeta), \zeta)$ ), investment in both assets becomes attractive. Then, abundant solar energy supply reduces the marginal productivity of energy below the fossil fuel price. This reduces the returns to further investment in solar, renders investment in  $k$  profitable, and allows for simultaneous investment until the return has fallen low enough to induce households stop investing further.

## 2.2 The GDP function

Further insight can be gained by introducing the maximized value added (or GDP) function:

$$G(w, \zeta) = \max_{x,s} \{y(w - s, x) - (x - bs)\zeta\} \quad \text{s.t.} \quad s \geq 0, \quad x \geq bs. \quad (2.17)$$

It is verified that at each point of time, given total wealth  $w$ , the values of  $x$  and  $s$  (and  $k = w - s$ ) obtained from the solution of (2.17) are consistent with the competitive processes characterized in Propositions 1 and 2.<sup>2</sup> We can thus use the Propositions to derive an explicit GDP expression for each of the three phases.

---

<sup>2</sup>A third constraint:  $k > 0$ , or equivalently  $s < w$ , is guaranteed by (2.1).

During the fossil phase, while  $s(t) = 0$ ,  $w(t) = k(t) < k_m(\zeta)$  and  $x^f(t) = X(w(t), \zeta)$ , the GDP function becomes

$$G(w, \zeta) = y(w, X(w, \zeta)) - \zeta X(w, \zeta),$$

which increases with  $w$  at a diminishing rate. To see this, use (2.4)-(2.9) and assumption (2.1) to obtain  $G_w(w, \zeta) = y_k(w, X(w, \zeta)) > 0$  and  $G_{ww}(w, \zeta) = [y_{kk}y_{xx} - y_{kx}^2]/y_{xx} < 0$ . Moreover, the function decreases with  $\zeta$ :

$$G_\zeta(w, \zeta) = -X(w, \zeta) < 0.$$

During the mixed phase, while  $k_m(\zeta) < w(t) < w_{ms}(\zeta)$ ,  $k(t)$  is fixed at  $k_m(\zeta)$ ,  $s(t) = w(t) - k_m(\zeta)$  and  $x^f(t) = X(k_m(\zeta), \zeta) - bs(t)$ , and the GDP function is linear in wealth  $w$ . To see this, let

$$y_m(\zeta) \equiv y(k_m(\zeta), X(k_m(\zeta), \zeta))$$

and use (2.14) to write

$$G(w, \zeta) = y_m(\zeta) - b\zeta[X(k_m(\zeta), \zeta)/b + k_m(\zeta) - w] = y_m(\zeta) + b\zeta[w - w_{ms}(\zeta)].$$

Thus,  $G_w(w, \zeta) = b\zeta > 0$  and  $G_{ww}(w, \zeta) = 0$ .

During the solar phase, while  $w(t) > w_{ms}(\zeta)$  and  $x^f(t) = 0$ , the GDP function, expressed in terms of  $S(w)$  of (2.15), assumes the form

$$G(w, \zeta) = y(w - S(w), bS(w)),$$

which is independent of  $\zeta$ , since this price is irrelevant for production and investment decisions when fossil energy is no longer used. Using (2.10), we find that  $G_w(w, \zeta) = y_k(w - S(w), bS(w)) > 0$ , while (2.16) and assumption (2.1) imply

$$G_{ww}(w, \zeta) = y_{kk}[1 - S'(w)] + by_{kx}S'(w) = b^2 \frac{y_{kx}^2 - y_{kk}y_{xx}}{2by_{kx} - y_{kk} - b^2y_{xx}} < 0.$$

Thus, the GDP function increases with  $w$  at a diminishing marginal rate.

We summarize the above discussion in:

**Lemma 2.1.** *The GDP function can be expressed as*

$$G(w, \zeta) = \begin{cases} y(w, X(w, \zeta)) - \zeta X(w, \zeta), & w < k_m(\zeta) \\ y_m(\zeta) + b\zeta[w - w_{ms}(\zeta)], & k_m(\zeta) \leq w < w_{ms}(\zeta) \\ y(w - S(w), bS(w)), & w \geq w_{ms}(\zeta) \end{cases} \quad (2.18)$$

and satisfies: (i)  $G(\cdot, \zeta)$  and  $G_w(\cdot, \zeta)$  are positive and continuous.

(ii)  $G_{ww}(\cdot, \zeta) < 0$  for  $w < k_m(\zeta)$  or  $w > w_{ms}(\zeta)$ , and  $G_{ww}(\cdot, \zeta) = 0$  for  $k_m(\zeta) < w < w_{ms}(\zeta)$ . (iii)  $G(w, \cdot)$  is continuous and non-increasing.

The GDP function, thus, resembles a neoclassical production function, albeit with a constant marginal product of (total) capital during the mixed phase. The latter property arises because investment during the mixed phase amounts to installing solar capacity in order to replace fossil fuels, which have a fixed marginal product  $y_x = \zeta$ . Thus, the return to investment equals the savings in fossil fuel cost which are constant per unit of investment.

The GDP function  $G$  and the return to capital  $G'$  are depicted in Figure 1 as functions of total wealth  $w$  along the three phases.<sup>3</sup> Note that the linear branch of the mixed phase joins smoothly the curves corresponding to the fossil and solar phases. Note further, in the lower panel, that when  $\rho + \delta$  exceeds  $b\zeta$ , it equals the return to capital  $G'$  at some state  $w < k_m$ , i.e. along the fossil phase, hence the steady state must be fossil based (cf. equation (2.23) below). In contrast, when  $\rho + \delta < b\zeta$ , the steady state condition holds in the solar phase, giving rise to a solar based economy.

## 2.3 Dynamic characterization via the GDP function

Adding (2.3) and (2.5) gives

$$\dot{w}(t) = G(w(t), \zeta) - \delta w(t) - c(t). \quad (2.19)$$

Let

$$V(w|\zeta) = \max_{\{c(t)\}} \int_0^\infty u(c(t))e^{-\rho t} dt \quad (2.20)$$

---

<sup>3</sup>For brevity of exposition, we suppress the  $\zeta$  argument from all functions when no confusion arises. Thus,  $G'(w)$  stands for  $G_w(w, \zeta)$ .

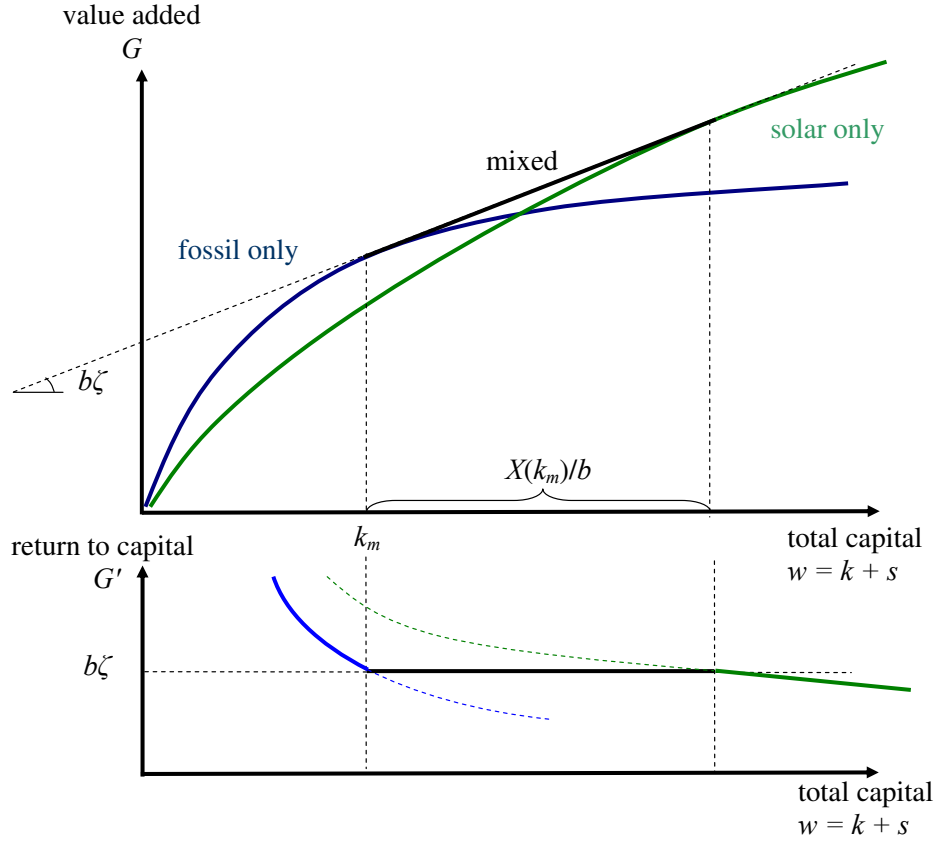


Figure 1: GDP and return to capital as functions of total capital  $w$ .

subject to (2.19), given  $w(0) = w < k_m(\zeta)$  and a constant fossil price  $\zeta$ . Using (2.18) it can be shown that:

**Lemma 2.2.** *The consumption and wealth processes,  $c(\cdot)$  and  $w(\cdot)$ , corresponding to (2.20) equal the competitive consumption and total capital processes,  $c(\cdot)$  and  $k(\cdot) + s(\cdot)$ , characterized in Propositions 1-2.*

The proof entails comparing the solution of (2.20) with the competitive processes characterized in the Propositions and is omitted. In fact, the necessary conditions for (2.20) specify the dynamics of the consumption process as

$$\dot{c}(t) = c(t)\sigma(c(t))[G_w(w(t), \zeta) - (\rho + \delta)], \quad (2.21)$$

where

$$\sigma(c) = -u'(c)/[u''(c)c] \quad (2.22)$$



is the intertemporal elasticity of substitution. Consumption grows in proportion to the difference between the net return to investment  $G_w - \delta$  and the time preference rate  $\rho$ . The boundary conditions include the initial capital  $w_0$  and the steady state levels  $\hat{w}(\zeta)$  and  $\hat{c}(\zeta)$ , satisfying  $\dot{c} = \dot{w} = 0$ , which are obtained from:

$$G_w(\hat{w}(\zeta), \zeta) = \rho + \delta \quad (2.23)$$

and

$$\hat{c}(\zeta) = G(\hat{w}(\zeta), \zeta) - \delta\hat{w}(\zeta) = \hat{w}(\zeta) \left[ \frac{\rho + \delta}{v(\hat{w}(\zeta), \zeta)} - \delta \right]. \quad (2.24)$$

In (2.24),

$$v(w, \zeta) \equiv \frac{G_w(w, \zeta)w}{G(w, \zeta)} < 1 \quad (2.25)$$

is the capital elasticity of the GDP function, or “capital share” for short.<sup>4</sup>

Since the system of differential equations (2.19) and (2.21) is autonomous,  $c$  can be expressed as a function of  $w$ . Taking the time derivative of  $c(w)$  and using (2.19), (2.21) and (2.25) we obtain

$$\begin{aligned} c'(w) &= \sigma(c(w))c(w) \frac{G'(w) - \rho - \delta}{G(w) - \delta w - c(w)} \\ &= \sigma(c(w)) \frac{c(w)}{w} \frac{v(w)G(w) - (\rho + \delta)w}{G(w) - \delta w - c(w)}. \end{aligned} \quad (2.26)$$

Equation (2.26), together with the steady state condition  $c(\hat{w}) = \hat{c}$ , defines the competitive consumption  $c(w)$  for every capital stock  $w$ . This, in turn, allows to determine the time trajectories of  $w$  and  $c$  as follows: First use (2.19) to find  $w(t)$ , implicitly, from

$$t = \int_{w_0}^{w(t)} \frac{dw}{G(w, \zeta) - \delta w - c(w)}, \quad t > 0.$$

The time trajectory of consumption is then given by  $c(w(t))$ .

### 3 Early announcement paradox

The discussion so far ignores the externalities associated with fossil energy due, e.g. to polluting emissions. A common policy addressing such externalities entails taxing

---

<sup>4</sup> $G(w, \zeta) - G_w(w, \zeta)w$  vanishes at  $w = 0$  and is non-decreasing in  $w$  hence is positive at  $w > 0$  which implies the inequality at (2.25).

emissions. In the present setting, such a policy is equivalent to increasing the fossil energy price  $\zeta$ . We call such price increase “carbon tax” and denote it by  $\tau$ . If the tax is imposed abruptly, the economy will respond promptly by switching from the competitive allocation corresponding to the initial (low) price  $\zeta^l$  to the allocation associated with the higher price  $\zeta^h = \zeta^l + \tau$ . Imposing such a policy by surprise entails discontinuities in the consumption-saving processes and is unlikely to score high in public opinion polls. Policymakers, then, opt to announce the tax policy well ahead of its actual implementation in order to allow gradual adjustments to the forthcoming changes.

We show that the early announcement gives rise to an effect akin to the green paradox, whereby the use of fossil energy will actually increase, rather than decrease, during the intermediate period between the announcement of the tax policy and its actual implementation. We assume that initially (without the carbon tax) the economy is fossil-based, i.e.,  $\zeta^l < \zeta^c$  (otherwise, no intervention is needed) and show that the result holds both when the after-tax economy remains fossil-based and when it switches to the solar-based type. In the latter case, we assume that the economy remains at the fossil phase until the tax implementation date  $T$ .

We use the superscripts  $l$  and  $h$  to denote competitive allocation processes under  $\zeta = \zeta^l$  and  $\zeta = \zeta^l + \tau \equiv \zeta^h$ , respectively. The allocation processes corresponding to the early announcement policy, with  $\zeta = \zeta^l$  until some known time  $T$  and  $\zeta^h = \zeta^l + \tau$  thereafter, are indicated by the superscript  $r$ . The latter are the result of

$$V^r(w_0|\zeta^l, \zeta^h) = \max_{\{c(t)\}} \left\{ \int_0^T u(c(t))e^{-\rho t} dt + e^{-\rho T} V(w_T|\zeta^h) \right\} \quad (3.1)$$

subject to (2.19) with  $\zeta = \zeta^l$ , given  $w(0) = w_0 < k_m(\zeta^h)$ . In (3.1),  $w_T = w(T)$  is free and serves as the initial wealth for the post-tax problem  $V(w_T|\zeta^h)$ , defined in (2.20) with  $\zeta = \zeta^h$ . The regulated value  $V^r(w_0|\zeta^l, \zeta^h)$  depends on both fossil prices, while the post-tax value  $V(w_T|\zeta^h)$  in (3.1) depends only on the higher price.

The  $l$  and  $h$  processes, corresponding to  $V(w|\zeta^l)$  and  $V(w|\zeta^h)$ , were characterized in

the previous section. The early-announcement allocation processes, corresponding to  $V^r(w|\zeta^l, \zeta^h)$ , are characterized below for the Cobb-Douglas technology specification

$$y(k, x) = \phi k^\alpha x^\gamma \quad (3.2)$$

with  $\alpha + \gamma < 1$  and  $\phi > 0$ . We maintain the general form of the utility function  $u(\cdot)$  but assume that

$$v(w, \zeta)\sigma(c) < 1 \quad (3.3)$$

over the entire feasible ranges of capital and consumption. With the inter-temporal elasticity of substitution typically falling short of unity [see, e.g., 2] assumption (3.3), which plays a major role in the analysis below, holds under any empirically relevant calibration. The early announcement paradox is a direct consequence of the announcement-induced decline in consumption (see Figure 2 and the appendix for a proof):

**Lemma 3.1.** *Suppose that the final good technology (3.2) and assumption (3.3) hold. Then,  $c^r(w) < c^l(w)$  prior to the implementation date  $T$ .*

Note that the result is purely an announcement effect: The two consumption processes proceed under the same (low) fossil energy price and the same production technology (both are based entirely on fossil energy). The difference between  $c^l(w)$  and  $c^r(w)$  stems from the fact that the former is determined under the expectation that fossil energy will remain cheap, whereas the latter is carried out under the expectation that the price of fossil energy will increase at time  $T$ . Moreover, the comparison in Lemma 3.1 applies also when the after-tax price is high enough to change the economy into a solar-based type. (In this case, however, the comparison is restricted to the fossil phase of the regulated process.) Indeed, it is not a particular post-tax behavior that drives the effect but rather the anticipated reduction in fossil energy use.

The Cobb-Douglas specification (3.2) is not essential and is used here merely to simplify the analysis. In contrast, the elasticity condition (3.3) is essential and embodies the tradeoffs underlying the effect: the loss in utility due to reduced consumption in the

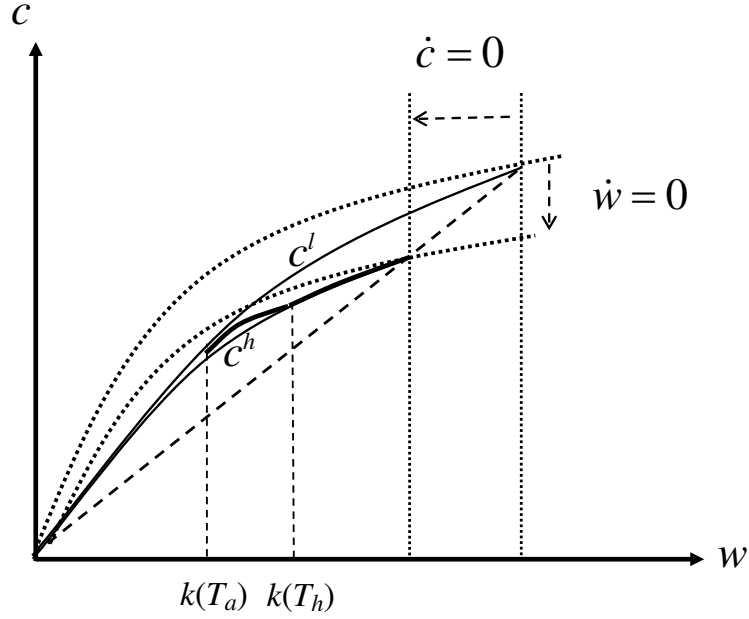


Figure 2: Consumption-capital curves under low and high fossil prices. The two vertical and two curved dotted lines depict the steady-state loci  $\dot{c} = 0$  and  $\dot{w} = 0$  under the two prices, with arrows indicating the transition from low to high price. The consumption curves  $c^l(\cdot)$  and  $c^h(\cdot)$  are indicated by the thin solid lines starting at the origin and ending at the steady states, defined by the intersection of the corresponding steady state loci and the diagonal dashed line  $c = \hat{R}w$  of (B.1). The regulated curve  $c^r(\cdot)$  (thick solid line) leaves  $c^l(\cdot)$  at the announcement time  $T_a$  to join  $c^h(\cdot)$  continuously at the tax implementation time  $T_h$ .

interim period on the one hand, and the increased post-implementation productivity due to capital accumulation on the other.

The consumption relation of Lemma 3.1 entails the opposite relation for the corresponding capital processes. To see this, note, using (2.19), that the time trajectories of  $w^j$ ,  $j = l, r$ , are given, implicitly, by

$$t = \int_{w_0}^{w^j(t)} \frac{dw}{G(w, \zeta^l) - \delta w - c^j(w)}, \quad j = l, r. \quad (3.4)$$

Thus, the relation  $c^l(w) > c^r(w)$  implies that, prior to implementation,

$$w^r(t) > w^l(t).$$

Moreover, during the the fossil phase  $w = k$  and the larger capital implies, noting

(2.9), larger energy use

$$X(w^r(t), \zeta^l) > X(w^l(t), \zeta^l). \quad (3.5)$$

With energy being derived solely from fossil sources during the interim period, the early announcement actually increases the use of fossil energy, compared to the case in which no announcement is made. We summarize this property in:

**Proposition 3.** *Suppose that the final good technology (3.2) and assumption (3.3) hold. Then, announcing that a carbon tax will be implemented  $T$  years from now increases the use of fossil energy in the interim period, prior to implementation, compared to the situation in which no announcement is made.*

The Proposition manifests the paradoxical outcome in our setting. Anticipating the future increase in the price of fossil energy, households respond by reducing consumption and increasing saving in order to smooth consumption at the time of transition to the carbon tax regime. Higher saving rates imply larger capital stocks and enhanced energy demand, in contrast to the original purpose of the announced policy.

## 4 Uncertain implementation date

In the previous section we assumed that households know the carbon tax implementation date precisely and adjust their behavior so as to smooth consumption at the transition time, giving rise to the paradoxical outcome. Here we show that the paradox persists also when agents are skeptical about the government (political) capability or willingness to implement the policy as announced, and take the implementation date  $T$  to be uncertain. We confine attention to the case in which the economy is fossil-based both before and after the carbon tax is imposed.

Suppose that the implementation of the carbon tax  $\tau$ , under which the price of fossil energy increases from  $\zeta^l$  to  $\zeta^h = \zeta^l + \tau$ , is considered to take place at some unknown future date  $T$ . The realization of  $T$  may depend on the successful ratification and

implementation of some international treaty, or on other developments in the global arena, and is taken as exogenous to the economy under consideration. Thus, from the vantage point of the economy, the hazard rate  $\pi$  corresponding to the random time  $T$  is constant. The payoff, conditional on  $T$ , is

$$\int_0^T u(c(t))e^{-\rho t} dt + e^{-\rho T} V(w_T | \zeta^h),$$

where  $V(w|\zeta)$  is defined in (2.20).

A constant hazard  $\pi$  implies that  $T$  is exponentially distributed and the expected payoff is

$$E_T \left\{ \int_0^T u(c(t))e^{-\rho t} dt + e^{-\rho T} V(w_T | \zeta^h) \right\} = \int_0^\infty [u(c(t)) + \pi V(w(t) | \zeta^h)] e^{-(\rho+\pi)t} dt.$$

The household problem with uncertain carbon tax implementation date becomes

$$V^\pi(w_0 | \zeta^l, \zeta^h) = \max_{\{c(t)\}} \int_0^\infty [u(c(t)) + \pi V(w(t) | \zeta^h)] e^{-(\rho+\pi)t} dt \quad (4.1)$$

subject to (2.19) with  $\zeta = \zeta^l$ , given  $w(0) = w_0$ . We compare the unregulated emission path corresponding to  $V(w_0 | \zeta^l)$ , under which no carbon tax is contemplated, with that corresponding to  $V^\pi(w_0 | \zeta^l, \zeta^h)$ , under which a carbon tax  $\tau = \zeta^h - \zeta^l$  will be imposed at an uncertain time  $T$ . The allocation processes corresponding to  $V^\pi(w_0 | \zeta^l, \zeta^h)$  are identified by the superscript  $\pi$ .

The capital process  $w^\pi(\cdot)$  follows (2.19) with  $\zeta = \zeta^l$  (the prevailing low price until the tax is imposed) while equation (2.21) becomes

$$\dot{c}^\pi(t) = \sigma(c^\pi(t))c^\pi(t) [G_w(w^\pi(t), \zeta^l) - (\rho + \delta) + P(w^\pi(t))], \quad (4.2)$$

where<sup>5</sup>

$$P(w) \equiv \pi \left( \frac{u'(c^h(w))}{u'(c^\pi(w))} - 1 \right). \quad (4.3)$$

---

<sup>5</sup>In deriving  $P(w)$ , use has been made of the property  $dV(w|\zeta^h)/dw = u'(c^h(w))$ , obtained by noting that the current-value costate (shadow price of capital) corresponding to  $V(w|\zeta^h)$  equals  $u'(c^h(w))$ .

Comparing (4.2) with (2.21), we see that the uncertainty in  $T$ , represented by the hazard  $\pi > 0$ , affects the  $c^\pi(w)$  process via the  $P(w)$  term, the sign of which depends on the relative magnitudes of  $c^h(w)$  and  $c^\pi(w)$ .

Equation (2.26) becomes

$$\frac{dc^\pi(w)}{dw} = \sigma(c^\pi(w))c^\pi(w) \frac{G_w(w, \zeta^l) - (\rho + \delta) + P(w)}{G(w, \zeta^l) - \delta w - c^\pi(w)}, \quad (4.4)$$

from which  $c^\pi(w)$  is obtained, using the boundary condition  $c^\pi(\hat{w}^\pi) = \hat{c}^\pi$ , where the steady state values  $\hat{w}^\pi$  and  $\hat{c}^\pi$  are defined by

$$G(\hat{w}^\pi, \zeta^l) - \delta\hat{w}^\pi - \hat{c}^\pi = 0 \quad (4.5)$$

and

$$G_w(\hat{w}^\pi, \zeta^l) - (\rho + \delta) + P(\hat{w}^\pi) = 0. \quad (4.6)$$

The following relation between  $c^j(w)$ ,  $j = l, h, \pi$ , is established in the appendix:

**Lemma 4.1.** *Suppose that the final good technology (3.2) and assumption (3.3) hold.*

*Then, the relations*

$$c^h(w) < c^\pi(w) < c^l(w) \quad \forall w \in (0, \hat{w}^\pi] \quad (4.7)$$

*hold prior to the realization of the implementation date  $T$ .*

Uncertainty, then, reduces consumption but not by as much as would be implied by a prompt implementation of the carbon tax.

We can use equation (3.4) again, with  $j = l, \pi$ , to obtain

$$w^l(t) < w^\pi(t),$$

which implies that

$$X(w^\pi(t), \zeta^l) > X(w^l(t), \zeta^l)$$

at each point of time prior to the realization of  $T$ . This establishes the early announcement paradox in the case of uncertain  $T$ . We summarize this result in

**Proposition 4.** *Suppose that the final good technology (3.2) and assumption (3.3) hold. Suppose further that households are skeptical about the government capability to implement its policy as announced and take  $T$  to be uncertain. Then, the early announcement of a future carbon tax policy increases fossil energy use in the interim period (prior to the tax implementation), compared to the situation in which no tax is contemplated (and no announcement made).*

While households are unable in this case to ensure a completely smooth consumption process at the uncertain implementation date, they can prepare for the carbon tax by increasing savings (i.e., accumulating a larger capital stock as compared to the situation where no carbon tax is anticipated) in order to use their larger wealth as a buffer to moderate discontinuous change in consumption when the carbon tax is eventually realized. In the meantime, alas, the larger capital enhances the demand for fossil energy and the polluting emissions that come along with it.

## 5 Concluding comments

It is tempting to announce the intention to levy an emission tax at some later date in order to grant households and firms a “grace period” during which they can adjust to the forthcoming regime. This good intention turns out to be counterproductive, as it increases emissions during the interim period before the tax is implemented. This paradoxical outcome arises disregarding the scarcity of fossil fuels. Indeed, setting aside scarcity allows tracing the paradoxical outcome to delicate saving-consumption tradeoffs in household decisions, dominated by a condition relating the elasticities of (marginal utility of) consumption and of capital.

The paradoxical outcome stems from the households’ desire to smooth consumption at the transition to the after-tax regime. Imposing the tax renders essential fossil energy inputs more expensive, which in turn decreases production and consumption. Anticipating these future effects, households reduce consumption and increase saving well before the



tax is implemented, giving rise to a faster buildup of capital in order to mitigate the fall in energy use at the time the carbon tax is imposed. In the meantime, the larger capital entails a higher rate of fossil energy use. Actually, when the tax implementation time  $T$  is known with certainty, we find that the consumption process is continuous at  $T$ . This “consumption smoothing motive” is the driving force underlying the early announcement paradoxical outcome.

The paradoxical effect persists also when the carbon tax implementation time  $T$  is uncertain. While the economic forces at work are similar to those driving the early announcement effect under known  $T$ , the two mechanisms operate differently. Under uncertain  $T$ , households cannot predict the tax implementation date at which they should smooth the consumption process. In fact, consumption will undergo a discontinuous jump at this date and the adopted processes are tuned so as to minimize the expected utility loss associated with the jump. The solution involves delicate tradeoffs but the paradoxical effect of increased fossil energy use persists at all times until the tax policy is realized.

## Appendix

### A Preliminaries

Under the Cobb-Douglas specification (3.2), the GDP function (2.18) becomes

$$G(w, \zeta) = \begin{cases} A(\zeta)w^\beta & \text{if } 0 < w \leq k_m(\zeta) \text{ (fossil phase)} \\ b\zeta(w + \bar{w}(\zeta)) & \text{if } k_m(\zeta) < w \leq w_{ms}(\zeta) \text{ (mixed phase)} \\ Qw^\nu & \text{if } w_{ms}(\zeta) < w \text{ (solar phase)} \end{cases} \quad (\text{A.1})$$

where

$$\beta \equiv \alpha/(1 - \gamma) < 1, \quad (\text{A.2})$$

$$\nu \equiv \alpha + \gamma < 1, \quad (\text{A.3})$$

$$A(\zeta) \equiv \phi(1 - \gamma) (\phi\gamma/\zeta)^{\gamma/(1-\gamma)}, \quad (\text{A.4})$$

$$Q \equiv \phi(b\gamma/\alpha)^\gamma (\alpha/\nu)^\nu, \quad (\text{A.5})$$

$$k_m(\zeta) \equiv \left[ \frac{\beta A(\zeta)}{b\zeta} \right]^{1/(1-\beta)} \quad (\text{A.6})$$

$$w_{ms}(\zeta) \equiv k_m(\zeta)\nu/\alpha \quad (\text{A.7})$$

and

$$\bar{w}(\zeta) \equiv k_m(\zeta)(1-\beta)/\beta = k_m(\zeta)(1-\nu)/\alpha. \quad (\text{A.8})$$

The capital share, defined in (2.25), becomes

$$v(w, \zeta) = \begin{cases} \beta & \text{if } 0 < w \leq k_m(\zeta) \\ w/(w + \bar{w}(\zeta)) & \text{if } k_m(\zeta) < w \leq w_{ms}(\zeta) \\ \nu & \text{if } w_{ms}(\zeta) < w \end{cases} \quad (\text{A.9})$$

Thus,  $v(w, \zeta)$  equals the constants  $\beta$  or  $\nu$  during the fossil or solar phases, respectively, with a smooth interpolation along the intermediate mixed phase.

## B Proof of Lemma 3.1

The solution of (2.26), under the appropriate boundary condition, is denoted  $c(w)$  and referred to as the  $c(w)$ -curve. Adding a superscript, as in  $c^j(w)$ , signifies the solution under  $\zeta = \zeta^j$ . Consider first the case  $\zeta^l + \tau = \zeta^h < \zeta^c$ , i.e., the economy is fossil-based both before and after the carbon tax. In this case, the economy remains in the fossil phase forever, with  $v(w, \zeta) = \beta$  all the way to the steady state  $\hat{w}(\zeta)$  given by (2.23). According to (2.24),  $\hat{c}(\zeta) = \hat{R}\hat{w}(\zeta)$ , where

$$\hat{R} = (\rho + \delta)/\beta - \delta \quad (\text{B.1})$$

is independent of  $\zeta$ . Omitting the  $\zeta$  argument when no confusion arises, the following characterization holds:

**Lemma B.1.**  $c(w) > \hat{R}w$  for  $w \in (0, \hat{w})$  and  $c(w) < \hat{R}w$  for  $w > \hat{w}$ .

*Proof.* Since  $c(\hat{w}) = \hat{R}\hat{w}$ , equation (2.26) cannot be used directly to determine  $c'(\hat{w})$  as both numerator and denominator vanish. However,  $c'(\hat{w})$  can be obtained by applying l'Hôpital's rule, yielding the quadratic equation

$$\Theta(c') \equiv c'^2 - \rho c' - \hat{R}\sigma(\hat{c})(\rho + \delta)(1 - \beta) = 0. \quad (\text{B.2})$$

Since  $\Theta(0) < 0$  and  $\Theta(\hat{R}) = \hat{R}(\rho + \delta)(1 - \beta)[1 - \beta\sigma(\hat{c})]/\beta > 0$ , the positive root  $c'(\hat{w})$  of (B.2) is smaller than  $\hat{R}$ . Just below  $\hat{w}$ , then,  $c(w) > \hat{R}w$ . Suppose that  $c(w)$  and  $\hat{R}w$  cross at some state  $0 < \tilde{w} < \hat{w}$  where  $c(\tilde{w}) = \hat{R}\tilde{w}$ . Then  $c'(\tilde{w}) \geq \hat{R}$ . However, at  $\tilde{w}$  we can use (B.1), (2.26) and assumption (3.3) to obtain

$$c'(\tilde{w}) = \sigma(\hat{R}\tilde{w})\hat{R} \frac{\beta G(\tilde{w}) - (\rho + \delta)\tilde{w}}{G(\tilde{w}) - (\delta + \hat{R})\tilde{w}} = \beta\sigma(\hat{R}\tilde{w})\hat{R} < \hat{R}, \quad (\text{B.3})$$

hence the crossing cannot occur below  $\hat{w}$ . It can be shown in a similar manner that just above  $\hat{w}$ ,  $c(w) < \hat{R}w$  and a crossing cannot occur above  $\hat{w}$ .  $\square$

Note the crucial role of assumption (3.3) in establishing inequality (B.3). It shows how the diminishing marginal utility competes with the diminishing product of capital in shaping the form of the consumption–capital curve.

Next we compare  $c^l(w)$  and  $c^h(w)$  – the consumption curves under  $\zeta = \zeta^l$  and  $\zeta = \zeta^h$ , respectively. With  $\zeta^l < \zeta^h < \zeta^c$ , the two curves remain in the fossil phase forever. Observe that  $\hat{R}$  is independent of  $\zeta$  and the steady-states corresponding to both fuel prices lie on the straight line  $c = \hat{R}w$ . However, (2.23) and (A.1) imply that  $\hat{w}^l > \hat{w}^h$  and therefore  $\hat{c}^l$  is proportionately larger than  $\hat{c}^h$ .

According to Lemma B.1,  $c^l(\hat{w}^h) > \hat{R}\hat{w}^h = c^h(\hat{w}^h)$ , hence the low-price consumption curve lies above its high-price counterpart at  $w = \hat{w}^h$ . We establish now that this property holds for all capital stocks.

**Lemma B.2.** *Under assumption (3.3),  $c^l(w) > c^h(w)$  for all  $w > 0$ .*

*Proof.* The result holds for  $w = \hat{w}^h$ . Suppose that the two curves cross at some point  $(\tilde{w}, \tilde{c})$  with  $0 < \tilde{w} < \hat{w}^h$ . It follows that  $c^l(\tilde{w}) \geq c^h(\tilde{w})$ . Using (2.26) we find

$$\frac{\beta G(\tilde{w}, \zeta^l) - (\rho + \delta)\tilde{w}}{G(\tilde{w}, \zeta^l) - \delta\tilde{w} - \tilde{c}} \geq \frac{\beta G(\tilde{w}, \zeta^h) - (\rho + \delta)\tilde{w}}{G(\tilde{w}, \zeta^h) - \delta\tilde{w} - \tilde{c}}. \quad (\text{B.4})$$

All terms of (B.4) are positive, because both  $w$  and  $c$  increase below their corresponding steady states. Thus,

$$(\rho + \delta)\tilde{w}G(\tilde{w}, \zeta^h) + \beta(\delta\tilde{w} + \tilde{c})G(\tilde{w}, \zeta^l) \leq (\rho + \delta)\tilde{w}G(\tilde{w}, \zeta^l) + \beta(\delta\tilde{w} + \tilde{c})G(\tilde{w}, \zeta^h).$$

or

$$\beta(\delta\tilde{w} + \tilde{c})[G(\tilde{w}, \zeta^l) - G(\tilde{w}, \zeta^h)] \leq (\rho + \delta)\tilde{w}[G(\tilde{w}, \zeta^l) - G(\tilde{w}, \zeta^h)].$$

Along the fossil phase  $G(\tilde{w}, \zeta^l) > G(\tilde{w}, \zeta^h)$ , yielding

$$\beta(\delta\tilde{w} + \tilde{c}) \leq (\rho + \delta)\tilde{w}$$

or, using (B.1),

$$\tilde{c} \leq \hat{R}\tilde{w},$$

violating Lemma B.1. It follows that the  $c^l(w)$ -curve and the  $c^h(w)$ -curve do not intersect in the interval  $(0, \hat{w}^h]$ .

At  $w > \hat{w}^h$  the inequality (B.4) and the signs of its terms are reversed, but a crossing of the consumption curves can be ruled out via the same considerations, recalling that the curves lie below the straight line  $c = \hat{R}w$  when the total stock  $w$  exceeds their respective steady states.  $\square$

Let  $\lambda^r(t)$  and  $\lambda^h(t)$  represent the current-value costate variables (shadow prices) associated with  $V^r(w_0|\zeta^l, \zeta^h)$  and  $V(w_T|\zeta^h)$ , respectively. Under the early announcement regime, we see, noting (3.1), that the current-value shadow price of capital  $w$  equals  $\lambda^r(t)$  for  $0 \leq t \leq T$  and  $\lambda^h(t - T)$  for  $t > T$ .

The transversality condition corresponding to the free  $w_T$  in (3.1) is

$$\lambda^r(T) = \frac{dV(w_T|\zeta^h)}{dw}. \quad (\text{B.5})$$

However,  $dV(w_T|\zeta^h)/dw = \lambda^h(0)$ , hence the early-announcement shadow price of capital is continuous at  $T$  (though the emission rate jumps from  $X(w_T, \zeta^l)$  to  $X(w_T, \zeta^h)$  on that date). Moreover, since  $w^{j'}(c) = \lambda^j$ ,  $j = r, h$ , the consumption process is also continuous at  $T$ , implying  $c^r(w_T) = c^h(w_T)$ . Invoking Lemma B.2, thus, gives

$$c^r(w_T) = c^h(w_T) < c^l(w_T). \quad (\text{B.6})$$

Although  $c^r(\cdot)$  and  $c^l(\cdot)$  proceed under the same price  $\zeta^l$  during the interim period  $t \in [0, T]$ , each of them is carried out anticipating a different price  $\zeta$  from time  $T$  onwards

( $\zeta^h$  for the former and  $\zeta^l$  for the latter). This difference shows up in the transversality conditions: (B.5) for the early-announcement problem (3.1) and  $c(\hat{w}^l) = \hat{R}\hat{w}^l$  for the unregulated problem (2.20) with  $\zeta = \zeta^l$ . As a result,  $c^r(\cdot)$  and  $c^l(\cdot)$  behave differently already during the interim period, giving rise to (B.6).

Now, (B.6) implies that  $c^r(w) < c^l(w)$  for all  $w < w_T$ . To see this, note that both  $c^r(w)$  and  $c^l(w)$  satisfy the same first order differential equation (2.26) with  $\zeta = \zeta^l$ . Thus, if the curves meet at any state  $w$  they must coincide over the whole  $w$  domain in which they apply. We have thus established Lemma 3.1 when the post-tax economy remains fossil-based ( $\zeta^h < \zeta^c$ ).

Next we examine the case  $\zeta^l < \zeta^c < \zeta^h$ , in which the carbon tax induces a transition from a fossil-based to a solar-based type. In this case, Proposition 2 implies that the transition process does not follow the fossil phase all the way to  $\hat{w}^h$ . Rather, the economy switches to the mixed phase as soon as its capital stock reaches  $k_m(\zeta^h)$ , then to the solar phase when total wealth  $w$  reaches the state  $w_{ms}(\zeta^h)$  on the way to the steady state at  $\hat{w}^h > w_{ms}(\zeta^h)$ . Nevertheless, Lemma B.2 remains valid during the fossil-phase also in this case. We outline the proof below, omitting the tedious derivations.

The analysis proceeds in two steps. First, we compare the consumption-capital relation  $c^c(w)$  obtained by solving (2.26) under  $\zeta = \zeta^c$  (the highest price for which the economy is still fossil based) with that corresponding to the slightly larger price  $\zeta^\epsilon = \zeta^c(1 + \epsilon)$  with  $\epsilon \ll 1$ , which barely suffices to turn the economy into a solar based type. While  $c^\epsilon(w)$ , corresponding to  $\zeta = \zeta^\epsilon$ , can be made arbitrarily close to  $c^c(w)$  during the fossil phase, i.e., for  $w \leq k_m(\zeta^\epsilon)$ , the  $c^\epsilon(\cdot)$  process includes also mixed and solar phases, where consumption changes appreciably. Integrating the equations of motion along the mixed phase, we find that  $c^c(w) > c^\epsilon(w)$  for all  $0 < w < k_m(\zeta^\epsilon)$  (i.e. during the fossil phase of  $c^\epsilon$ ). Since  $c^c(w) < c^l(w)$  for any  $\zeta^l < \zeta^c$ , it follows that  $c^l(w) > c^\epsilon(w)$  for any  $\zeta^l \leq \zeta^c$  and  $w \leq k_m(\zeta^\epsilon)$ .

In the second step we compare  $c^\epsilon(\cdot)$  with  $c^h(\cdot)$  for an arbitrary high price  $\zeta^h > \zeta^\epsilon$ .

Both processes evolve along the three phases, with transition states  $k_m(\zeta)$  and  $w_{ms}(\zeta)$  that depend on the fossil price. The comparison, thus, includes also regions where each process proceeds along a different phase. This situation complicates the analysis, but does not affect the final outcome, whereby  $c^h(w) < c^\epsilon(w)$  for  $w \leq k_m(\zeta^h)$ , i.e., in the region where both processes are in the fossil phase. However,  $c^\epsilon(w) < c^l(w)$  for any price  $\zeta^l \leq \zeta^c$ . Thus, while in its fossil phase,  $c^h(w) < c^l(w)$  for any price  $\zeta^l \leq \zeta^c$ .

With Lemma B.2 holding during the fossil phase also when the carbon tax switches the economy to a solar-based type, we can use the transversality condition requiring a continuous consumption process at time  $T$ , follow the arguments leading to Proposition 3 and establish the Paradox also for this case. Indeed, it is not the peculiar post-tax behavior that drives the paradoxical effect but rather the mundane desire of households to smooth consumption over time. To accomplish this, they build up larger capital prior to the tax implementation. Since capital and energy are complement inputs, the demand for energy increases as well.

## C Proof of Lemma 4.1

We consider the case in which the post-tax economy remains fossil-based. The  $c^\pi(w)$  process corresponds to the solution of (4.4) under the boundary condition  $c^\pi(\hat{w}^\pi) = \hat{c}^\pi$ , where  $\hat{w}^\pi$  and  $\hat{c}^\pi$  are defined by (4.5)-(4.6). We begin by comparing the steady state values  $\hat{w}^\pi$  and  $\hat{c}^\pi$  with their regulation-free counterparts. From (2.23) and (4.6) we obtain

$$G_w(\hat{w}^l, \zeta^l) - G_w(\hat{w}^\pi, \zeta^l) = P(\hat{w}^\pi). \quad (\text{C.1})$$

According to (4.3),  $P(\hat{w}^\pi)$  is small when  $\pi$  is small, hence  $\hat{w}^\pi$  is close to  $\hat{w}^l$  and (4.5) implies that

$$c^\pi(\hat{w}^\pi) = \hat{c}^\pi \approx \hat{c}^l = c^l(\hat{w}^l) \approx c^l(\hat{w}^\pi) > c^h(\hat{w}^\pi).$$

With  $u''(\cdot) < 0$ , it follows that  $u'(c^\pi(\hat{w}^\pi)) < u'(c^h(\hat{w}^\pi))$  and  $P(\hat{w}^\pi) > 0$ . Turning again to (C.1) and noting (cf. Lemma 2.1) that  $G_{ww} < 0$ , we find that  $\hat{w}^\pi > \hat{w}^l$  when the hazard

rate  $\pi$  is small. We show that this relation between the steady states extends to arbitrary positive values of  $\pi$ .

Consider the steady state  $\hat{w}^\pi$  as a function of  $\pi$  and assume that at some  $\pi$  value this function crosses the constant  $\hat{w}^l$  so that the left hand side of (C.1) vanishes. However, (4.5) holds for both  $c^\pi(\cdot)$  and  $c^l(\cdot)$  hence  $c^\pi(\hat{w}^\pi) = c^l(\hat{w}^\pi) > c^h(\hat{w}^\pi)$ . According to (4.3),  $P(\hat{w}^\pi) > 0$ , hence the right hand side of (C.1) is positive while the left hand side vanishes. Thus, the crossing cannot occur. We conclude, therefore that

$$\hat{w}^\pi > \hat{w}^l \quad \forall \pi > 0. \quad (\text{C.2})$$

Next, we compare  $c^\pi(\cdot)$  to  $c^l(\cdot)$ . Since  $\hat{w}^l$  represents the steady state for the  $w^l(\cdot)$  process, it follows that  $\dot{w}^l(t) = 0$  at this state. However, the steady state  $\hat{w}^\pi$  of  $w^\pi(\cdot)$  exceeds  $\hat{w}^l$ , hence  $\dot{w}^\pi(t) > 0$  when  $w^\pi(t) = \hat{w}^l$ . Thus, (2.19) implies  $c^l(\hat{w}^l) > c^\pi(\hat{w}^l)$ . We show that this relation cannot be reversed at other capital states. Suppose otherwise, that  $c^l(w^*) = c^\pi(w^*)$  (hence  $P(w^*) > 0$ ) at some capital state  $w^* < \hat{w}^l$  but  $c^l(w) > c^\pi(w) \forall w \in (w^*, \hat{w}^l]$ . It follows that  $c^{l'}(w^*) \geq c^{\pi'}(w^*)$ . However, we can write (4.4) as

$$\frac{dc^\pi(w^*)}{dw} = \frac{dc^l(w^*)}{dw} + \frac{\sigma(c^l(w^*))c^l(w^*)P(w^*)}{G(w^*, \zeta^l) - \delta w^* - c^l(w^*)} > \frac{dc^l(w^*)}{dw},$$

because the denominator of the second term is also positive at  $w^*$ . A crossing of the consumption curves, with  $c^{l'}(w^*) \leq c^{\pi'}(w^*)$ , can be ruled out also for  $w^* > \hat{w}^l$  using the same argument (since the denominator is negative above  $\hat{w}^l$ ). Thus,

$$c^\pi(w) < c^l(w) \quad \forall w > 0.$$

We wish to compare the uncertain consumption curve also to its high price counterpart,  $c^h(\cdot)$ . We use (C.2) to deduce from (C.1) that  $P(\hat{w}^\pi) > 0$ , hence  $c^h(\hat{w}^\pi) < c^\pi(\hat{w}^\pi)$ . To establish the same relation for smaller capital stocks, assume otherwise, that  $c^h(\tilde{w}) = c^\pi(\tilde{w})$  at some  $\tilde{w} < \hat{w}^\pi$ , where  $c^{h'}(\tilde{w}) \leq c^{\pi'}(\tilde{w})$  but  $P(\tilde{w}) = 0$  (hence  $c^{\pi'}(\tilde{w}) = c^{h'}(\tilde{w})$ ). This, however, implies (B.4) which can be ruled out by the same arguments used to establish Lemma B.2, completing the proof of Lemma 4.1.

## References

- [1] Copeland, B. and Taylor, M.: 2005, Free trade and global warming: A trade theory view of the kyoto protocol, *Journal of Environmental Economics and Management* **49**, 205–234.
- [2] Dasgupta, P.: 2007, Commentary: The Stern review’s economics of climate change, *National Institute Economic Review* **199**(1), 4–7.
- [3] Di Maria, C., Smulders, S. and van der Werf, E.: 2008, Absolute abundance and relative scarcity: Announced policy, resource extraction and carbon emissions, *Technical report*, Fondazione Eni Enrico Mattei Working Paper 2008.92.
- [4] Di Maria, C. and van der Werf, E.: 2008, Carbon leakage revisited: unilateral climate policy with directed technical change, *Environmental and Resource Economics* **39**, 55–74.
- [5] Eichner, T. and Pethig, R.: 2009, Carbon leakage, the green paradox and perfect future markets, *Technical report*, CESifo Working Paper 2542.
- [6] Hoel, M.: 2009, Bush meets Hotelling: Effects of improved renewable energy technology on greenhouse gas emissions, *Technical report*, CESifo Working Paper 2592.
- [7] Sinn, H.-W.: 2008, Public policies against global warming, *International Tax and Public Finance* **15**, 360–394.
- [8] Tsur, Y. and Zemel, A.: 2009, Market structure and the penetration of alternative energy technologies, *Technical report*, The Center for Agricultural Economic Research Discussion Paper 2.09.
- [9] van der Ploeg, F. and Withagen, C.: 2009, Is there really a green paradox? *Technical report*, OxCarre Research Paper 35.