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THE CHOPSTICK AUCTION

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Discussion paper

The Chopstick Auction*

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Abstract

The Chopstick Auction is an auction game in which the highest bidder receives two valuable objects, and the second highest bidder one worthless object. This game models the exposure problem, as the second highest bidder has to pay. We analyze the Chopstick Auction with incomplete information. For two risk neutral bidders, the Chopstick Auction has an efficient equilibrium and is revenue equivalent with the second-price sealed-bid auction in which three chopsticks are sold as one bundle. If bidders are loss averse, the Chopstick Auction is either inefficient, or raises less revenue than the second-price sealed-bid auction. In case of three bidders, the Chopstick Auction has no symmetric equilibrium. We use the Dutch DCS-1800 auction as illustration.

Keywords: Chopstick Auction; Exposure Problem; Loss Aversion; DCS-1800.

JEL classification: D44, L96

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1 Introduction

In February 1998, the Dutch government auctioned licenses for second generation mobile telecommunication. Two big lots and sixteen small lots were sold using a variant of the ascending multiple object auction format that had been used by the FCC to sell PCS licenses in the US.[1] The big lots (A and B) consisted of 75 DCS-1800 channels each, 15 small lots (1,...,15) consisted of 12 or 13 DCS-1800 channels each, and 1 small lot (16) contained 22 channels.[2] The Dutch government decided to split the spectrum in such small fractions in order to give bidders enough flexibility to get an optimal division of the channels. Incumbents bidders (KPN[3] and Libertel[4]) were not allowed to bid on lots A and B, and newcomers (Telfort,[5] Dutchtone,[6] TeleDanmark, Orange/Veba[7] and Airtouch[8]) were allowed to bid on all lots. A newcomer was believed to need one big lot or at least five small lots in order to operate a feasible network for mobile telecommunication. A set of less than five small lots would, if we neglect the possibility of resale, be of no value to a newcomer.[9]

[1] See, McMillan (1994), Cramton (1995, 1998), McAfee and McMillan (1996), and Milgrom (2000) for descriptions and discussions of these auctions.

[2] One channel is equivalent to 0.2 MHz.

[3] KPN used to be the state monopolist of telecommunication and mail in The Netherlands.

[4] Libertel was at that time a consortium of Vodaphone and the Dutch bank ING.

[5] Telfort was at that time a consortium of British Telecom and Dutch Railways.

[6] DutchTone was bidding under the name of Federa, a consortium of Deutsche Telekom, France Telecom and two Dutch banks. After the auction, Deutsche Telekom withdrew from the consortium.

[7] Orange and Veba are mobile telecom operators in the UK and Germany respectively.

[8] Airtouch is a US baby bell.

[9] Resale was in fact possible, but only to a party that did not actively participate in the auction, and then only with the approval of the Minister of Economic Affairs.

The lots were sold according to the following rules.[10] There is a sequence of rounds, in which bidders submit bids on lots. For each lot, the minimum bid in round 1 equals 0. For the following rounds, the minimum bid on each lot is equal to the current highest bid on this lot plus a small bid increment of at most 10% of the current highest bid. Each bidder is eligible to bid in round $t+1$ if either she submits at least one bid in round t, or she is overbid in round t on at least one of the lots she currently has the highest bid on. When eligible to bid, a bidder is allowed to submit bids on all lots.[11] The only exception to this rule is that a bidder is not allowed to be active on both lot A and lot B. At the beginning of round $t+1$, each bidder receives information about bidding activity in round t, and information that is relevant for the current round. The auction ends when in a certain round no bids are submitted. Each lot is allocated to the bidder with the current highest bid for a price equal to this bid.

The Dutch rules differ from the ones used for the US auctions in at least four ways. First, bidders are not allowed to withdraw their bid when an inefficient lock-in is imminent. Second, there is no activity rule which forces bidders to remain active on a given fraction of the number of channels they desire to obtain. Third, there is no common knowledge about who has the highest bid on which lots in a round. Fourth, the auction is asymmetric in the sense that incumbents are restricted on the lots they are allowed to bid on. In this paper, we will discuss the effect of the first difference on the outcome of the auction. For discussion of the effects of the other differences,

[10] See Van Damme (2000) for a more detailed description of the auction rules.

[11] In fact, in the Dutch DCS-1800 auction, a bidder was only allowed to bid on lots for which he paid a relatively small deposit before the start of the auction. All bidder had paid the deposit for all lots they were eligible to bid on.

see Van Damme (2000).

In contrast to the outcome of the American auctions,[12] the outcome of the Dutch DCS-1800 auction was probably not efficient. From Table 3, it can be seen that identical objects[13] were sold for substantial different prices.[14] In other words, the Law of One Price was not satisfied, which should have been the case in a perfect market, and which indicates inefficiency. Another indicator of inefficiency was the fact that there was resale of channels after the auction. Ben[15] was authorized by the Dutch authorities to acquire the licences bought by TeleDanmark and Orange/Veba in the auction.

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[12] See Cramton (1998).

[13] In fact, there were small differences. Some of the frequencies could not be used for regions near the Belgium and/or the German border, and the A and B lots also included some GSM frequencies. Van Damme (2000) argues that the effects of these heterogoneities on the value per frequency are not large enough to explain the large price differences.

[14] The incumbents, KPN and Libertel, seem to have profited from the auction design. They paid a lower price for their frequencies than two of their upcoming competitors, even though they were limited in the sense that they were not allowed to bid for the large lots.

[15] Ben was at the time a joint venture of Belgacom (70%) and TeleDanmark (30%).

Lot	**C**	**Winner**	**P**	**P/C**
A	75	Dutchtone	600	8
B	75	Telfort	545	7.3
1	13	Libertel	40.4	3.1
2	12	KPN	40.2	3.4
3	13	Orange/Veba	38	2.9
4	12	Telfort	40.5	3.4
5	13	KPN	43	3.3
6	12	TeleDanmark	41.1	3.4
7	13	KPN	40.4	3.1
8	12	KPN	39.1	3.3
9	13	Orange/Veba	46.5	3.6
10	12	TeleDanmark	41.25	3.4
11	13	KPN	42.98	3.3
12	12	TeleDanmark	39.9	3.3
13	13	KPN	39.9	3.1
14	12	KPN	40.5	3.4
15	13	Libertel	45.5	3.5
16	22	TeleDanmark	71.5	3.3

Table 3. Summary of the outcome of the DCS-1800 auction. **P** stands for the final price of the lot in millions of Dutch guilders, **C** for the number of channels the lot consisted of, and **P/C** for the price (in millions of Dutch guilders) paid per channel.

We conjecture, following Van Damme (2000), that the auction format that was used in the Netherlands leads to lower bids and to a less efficient auction outcome than the American auction format, as the Dutch auction format confronts bidders with the *exposure problem*, whereas the American format does not, as bidders are allowed to withdraw their bid. As a consequence of the fact that a bid could not be withdrawn in the Dutch DCS-1800 auction, the money a bidder spent on the small lots on which she had the highest bid should be regarded as sunk costs. Bidders then rather play the war of attrition than a standard auction game. Being aware of the possibility of losing money on the small lots, bidders were very active on the large ones, and bid hardly on the small ones. Such bidding behavior probably leads to inefficient outcomes and a low revenue. In the literature on multiple object auctions, this problem is referred to as the exposure problem, as bidders are *exposed* to the risk of paying more for an object than what it is worth to them.[16]

Motivated by the Dutch DCS-1800 auction, and with the aim to get a better understanding of the exposure problem, we will study a stylized model of a multiple object auction in which the exposure problem is present. The model is defined in Section 2. A seller simultaneously sells three chopsticks in an auction, which we will call the Chopstick Auction (CSA). In CSA, the price, which is the same for each object, is raised continuously. Bidders have the opportunity to step out at each price, until one bidder is left. The outcome of CSA is such that the second highest bidder gets one chopstick for the auction price p, and the highest bidder gets two chopsticks for a price of $2p$. We assume that bidders' marginal values are zero on the first chopstick, positive on second, and zero on the third. Bidders are incompletely informed about the other bidders' marginal value for the second chopstick. As the second highest bidder wins a worthless chopstick for a positive price, bidders face the exposure problem in CSA.

We will investigate whether an auction with an exposure problem is less efficient and/or yield less revenue than an auction in which the exposure

[16]See Bykowsky et al. (1998), and Milgrom (2000).

problem is not present. In order to do so, in Sections 3 and 4, we will compare CSA with the second-price sealed-bid auction, in which the three chopsticks are sold as one bundle (SPSB). From standard auction theory, we learn that SPSB has an efficient equilibrium (in dominant strategies), in which each bidder submits a bid equal to her value for the chopsticks. In the case of two risk neutral bidders with identical value distribution functions, we show that CSA has a unique symmetric Bayesian-Nash equilibrium. CSA is efficient, and revenue equivalent with SPSB. However, in the case of loss averse bidders, SPSB has either a more efficient outcome or a higher expected auction revenue than CSA. With three bidders, under general assumptions on the value distributions and the utility functions of the bidders, we derive an impossibility result: CSA has no symmetric equilibrium. We conjecture that this result implies that CSA has no efficient equilibrium, so that the seller, when aiming at efficiency, is better off by replacing CSA with SPSB.

From these findings, we conclude that the Dutch government could have improved the DCS-1800 auction by designing an auction in which bidders do not suffer from the exposure problem. There are at least three ways for auction designers to prevent auctions from suffering from the exposure problem. The first is that the auction designer offers large packages of objects rather than small ones. Specifically, the Dutch government may have obtained a better auction outcome in the DCS-1800 auction by not splitting up the spectrum in such small lots, despite the fact that in that case, the bidders are not given the opportunity to "let the market decide" on the division of the spectrum.[17] In the model, we have shown that the seller is weakly better

[17] "Letting the market decide" has another important drawback, namely that the realized market structure may be very concentrated, which is a concern when thinking about

off if he auctions the chopsticks as one bundle rather than as three different objects.

A second way to get rid of the exposure problem is a withdrawal rule. Such a rule gives bidders the opportunity to withdraw their bid when an inefficient lock-in is imminent. In the FCC auctions in the US, and also in the DCS-1800 auction and the UMTS auction in Germany,[18] a withdrawal rule was implemented. After the auction, bidders were allowed to withdraw their bid on certain licenses. A withdrawing bidder had to pay a penalty in case the final price of the license was lower than her bid. It is questionable if such a withdrawal rule completely solves the exposure problem, as bidders still face a considerable risk of having to pay the penalty. In our model, the driving force behind the results is that the losing bidder has to pay money, so that these results remain valid with the introduction of a withdrawal rule with penalty.

A third way to get around the exposure problem is to allow for combinatorial bids. In our model, rational bidders will only submit strictly positive bids on bundles of two or three chopsticks. If the payments rules are such that the winning bidder pays the highest bid of its opponents on a bundle of two, the auction reduces to the second-price sealed-bid auction, which we have shown to be (weakly) better than the Chopstick Auction. However, allowing for combinatorial bids may lead to other problems. Bykowsky et al. (1998) identify the *threshold problem* in such auctions, which states that small bidders may have to solve complicated coordination problems in order to be able

consumer surplus (Jehiel and Moldovanu, 2000; Klemperer, 2001).

[18] See Jehiel and Moldovanu (2000) for a theoretical analysis of the German UMTS auction.

to overbid large bidders. Another problem is that in the case of a large number of objects, determining the winning combination may be computationally intractable. In fact, Rothkopf et al. (1998) show the winner determination problem to be NP-hard. Also, bidding in the case of combinatorial bids is complicated for the bidders. For instance, in the Dutch DCS-1800 auction, a bidder has the possibility to submit bids on all $2^{18} - 1 \approx 250,000$ possible combinations of licenses!

Several papers of Robert Rosenthal and co-authors are closely related to ours. Krishna and Rosenthal (1996), and Rosenthal and Wang (1996) analyze multiple object auctions with two types of bidders, namely "local" bidders who are interested in only one object, and "global" bidders who try to acquire several. The global bidders, in competition with the local ones, face the exposure problem when attempting to realize synergies between the objects. The equilibrium outcome of the auction is typically not efficient. Szentes and Rosenthal (2001a, 2001b) construct equilibria in the first-price sealed-bid, the second-price sealed-bid, and the all-pay version of CSA with complete information. The value of a bundle of chopsticks is the same for each bidder. In equilibrium, these auctions are efficient. The most important different between Rosenthal's studies and ours is that all mentioned papers consider one shot auctions, whereas CSA is an ascending auction, as are the PCS auctions in the US and the DCS-1800 auction in the Netherlands.

Some other papers in the literature on multiple object auctions are related to ours as well. Bykowsky et al. (1998) gives an illustrative example in which in equilibrium the auction outcome is either inefficient, or at least one of the bidders ends up paying more for the purchased items than they are worth to her. Ausubel and Cramton (1998) stress the importance of efficiency of the auction outcome in terms of revenues for the seller in auctions of perfectly divisible objects. Ausubel and Cramton (1999) show that efficiency of the auction outcome is necessary for revenue maximization when the auction is followed by a perfect resale market and when the seller cannot commit to not selling some objects. Milgrom (2000) constructs an example in which, in contrast to our results, the seller realizes a less efficient outcome when using larger packages (but gets a higher revenue). Klemperer (2001) lists issues that are of practical importance in the design of multiple object auctions. The results derived in this paper indicate that the warning "avoid the exposure problem" should be added to this list.

2 The model

Consider a situation with n bidders, $n \in \{2,3\}$, labeled $1, ..., n$, who wish to eat Chinese food. However, none of the bidders has anything to eat with. Suppose that a seller sells 3 chopsticks in the Chopstick Auction[19] (CSA) which has the following rules. The price starts at zero, and is continuously raised. Bidders have the opportunity to quit the auction at any price they desire. The seller informs all remaining bidders when one of the bidders quit. The auction ends when one bidder is left, who wins two chopsticks, and pays two times the price at which the second highest bidder quits. The second

[19] The credit for the name of this auction game goes to Mary Lucking-Reiley. Thanks to Balasz Szentes and Robert Rosenthal for pointing this out to me.

highest bidder wins one chopstick and pays the price at which she quits.[20] We will compare CSA with the second-price sealed bid auction in which the three chopsticks are sold as one bundle (SPSB).

The value $V_i(s)$ bidder i attaches to owning s chopsticks is given by

$$V_i(s) = \begin{cases} v_i & s = 2, 3 \\ 0 & s = 0, 1, \end{cases} \tag{1}$$

where v_i is a private signal of bidder i. In words, a bidder attaches a value of v_i to winning two chopsticks, and no value to winning only one chopstick or to winning a third one. We assume that v_i is drawn independently from the other signals from the interval $[\underline{v}, \overline{v}]$, with $0 \leq \underline{v} < \overline{v}$, according to a strictly increasing, continuous probability distribution $F_i(.)$ with density $f_i(.) \equiv F_i'(.)$. Sometimes we will take the simplifying assumption that bidders draw their signal from the same distribution.

Each bidder is an expected utility maximizer. The utility for bidder i who buys a set of chopsticks which gives her value V_i for a price of p_i is given by $U_i(V_i - p_i)$. For every i, U_i is assumed to be a continuous function which is strictly increasing, with $U_i(0) = 0$. For the sake of tractability, we assume in Section 3 that the bidders are either risk neutral (i.e., $U_i(x) = x$ for all x) or loss averse, which will be defined later. In Section 4, we use general utility

[20] In this auction, ties are broken as follows. In case of two (remaining) bidders, when a tie takes place at a price of p, a fair coin is tossed. If tails comes up, the bidder with the lowest label wins two chopsticks for a price of $2p$, and the other bidder wins one chopstick for a price of p. If heads come up, the outcome is reversed. When the auction is played by three bidders, in the first stage, either two or three bidders may decide to step out at the same price of p. In either case, the game ends immediately. When two bidders step out, then the third bidder gets two chopsticks for a price of $2p$. With 50-50 probability, one of the other bidders is awarded one chopstick for a price of p. When all three bidders decide to step out at p, the bidders' labels are randomly ordened in such a way that each ordering is equally likely. The first bidder then gets two chopsticks for a price of $2p$, and the second one for a price of p. The third neither gets nor pays anything.

functions.

In CSA, there is one winner, the bidder who wins both chopsticks, and one "real" loser, which is the bidder who buys one worthless chopstick for a positive price. Table 4 shows the effect of the quitting order on the utility levels of the bidders in the case of three bidders, when the price of a chopstick is equal to p. From Table 4, it becomes clear that CSA can also be seen as an English auction, in which the winner pays the bid of the second highest bidder, and the second highest bidder pays half of her own bid.

Bidder	**Quits**	**# Chopsticks won**	**Payment**	**Utility**
i	First	0	0	$U_i(0) = 0$
j	Second	1	p	$U_j(-p)$
k	Third	2	$2p$	$U_k(v_k - 2p)$

Table 4. Possible outcomes of CSA.

We assume that the seller aims at reaching the following two goals.

Efficiency: Generate an efficient outcome, i.e., the bidder with the highest signal obtains two chopsticks;

Revenue: Given that *Efficiency* is fulfilled, maximize expected auction revenue.

3 Two bidders

Consider CSA with two bidders. The game ends immediately when one of the bidders indicates to quit. Therefore, the strategy of a bidder is a bid in the interval $[0, \infty)$ for each realization of her signal.

3.1 Risk neutral bidders

In order to keep the model tractable, we restrict our attention to identical distributions, i.e., both bidders draw their signal from the same distribution $F \equiv F_1 = F_2$.

Proposition 1 gives equilibrium bidding in CSA when both bidders are risk neutral. By a standard argument, this bid function must be strictly increasing and continuous. Let $U(v, w)$ be the utility for a bidder with signal v who behaves as if she has signal w, whereas the other bidders play according to the equilibrium bid function. A necessary equilibrium condition is that

$$\frac{\partial U(v, w)}{\partial w} = 0$$

at $w = v$. From this condition, a differential equation is derived, from which the equilibrium bid function is uniquely determined.

Proposition 1 *Let $n = 2$. Suppose both bidders are risk neutral, and draw their signals from the same distribution function F. Let $B(v)$, the bid of a bidder with signal v, be given by*

$$B(v) = (1 - F(v)) \int_{\underline{v}}^{v} \frac{f(x)x}{(1 - F(x))^2} dx.$$

Then B is the unique symmetric Bayesian Nash equilibrium of CSA. The outcome of the auction is efficient.

Proof. The following observations imply that a symmetric equilibrium bid function must be strictly increasing. First, a higher-value type of a bidder cannot exit before a lower-value type of the same bidder would exit. (Suppose the lower type is indifferent between two different strategies, giving her two different probabilities of being the ultimate winner of two chopsticks. The higher type then strictly prefers the strategy with the higher probability to win. Therefore, she will never quit earlier than the lower type.) Furthermore, there is no range in which the bid function is flat. (Suppose there is the bid function is flat at a price level of p. Then each bidder being in the range of signals that bid p exits the auction with positive probability at p. But if this is the case, then each bidder strictly prefers staying just a bit longer.)

Let $\tilde{B}$ be a symmetric and strictly increasing equilibrium bid function. If the other bidder bids according to $\tilde{B}$, the expected utility of a bidder with signal v who bids as if she has signal w is given by

$$U(v,w) = -(1-F(w))\tilde{B}(w) + \int_{\underline{v}}^{w} f(x)(v - 2\tilde{B}(x))dx.$$

The first (second) term of the RHS refers to the case that the bidder makes the second highest (highest) bid.

The FOC of the equilibrium is

$$\frac{\partial U(v,w)}{\partial w} = -(1-F(w))\tilde{B}'(w) - f(w)\tilde{B}(w) + vf(w) = 0 \qquad (2)$$

at $w = v$. Rearranging terms we find

$$\frac{(1-F(v))\tilde{B}'(v) + f(v)\tilde{B}(v)}{(1-F(v))^2} = \frac{f(v)v}{(1-F(v))^2},$$

which is equivalent to

$$\frac{\tilde{B}(v)}{1-F(v)} = \int_{\underline{v}}^{v} \frac{f(x)x}{(1-F(x))^2}dx + C,$$

for some C. C must be zero (C must be at least zero, otherwise a bidder with signal $\underline{v}$ submits a negative bid; if C is larger than zero, a bidder with signal $\underline{v}$ submits a strictly positive bid. As $\tilde{B}$ is (by assumption) strictly increasing, this bidder submits the lowest bid with probability one, and has to buy one chopstick for a positive price. Clearly, she is strictly better off by bidding zero.) Also the SOC is fulfilled as $sign(\frac{\partial U(v,w)}{\partial w}) = sign(v - w)$. It is readily checked that B is a solution.

What remains to be checked it that B is strictly increasing. From (2), B is strictly increasing if and only if $B(v) < v$ for (almost) all v. This is true, as

$$\begin{aligned} B(v) &= (1 - F(v)) \left[\int_{\underline{v}}^{v} \frac{f(x)x}{(1 - F(x))^2} dx \right] \\ &= v - \underline{v} - (1 - F(v)) \int_{\underline{v}}^{v} \frac{1}{(1 - F(x))} dx \\ &< v. \end{aligned}$$

As B is strictly increasing, CSA is efficient.

The uniqueness of the equilibrium follows with the Revenue Equivalence Theorem which states that the expected payment made by any bidder given her signal is unique by the efficiency of the outcome and the utility of the lowest type. As the equilibrium bid function is strictly increasing, and the utility of the lowest type is always zero in an efficient equilibrium, B is the unique equilibrium bid function. ■

Using CSA, the seller reaches both his goals *Efficiency* and *Revenue*. By the Revenue Equivalence Theorem (Myerson, 1981), CSA yields the same expected revenue as any other efficient auction mechanism in which the bidder with the lowest signal obtains zero expected utility. This follows from the fact that CSA is an auction of a single object, namely a set of two chopsticks, which is allocated efficiently according to Proposition 1. Therefore, there is no efficient auction that can improve the revenues for the seller in comparison with CSA, so that the seller reaches both *Efficiency* and *Revenue*. More specifically, the seller is indifferent between using CSA and SPSB to sell the three chopsticks.

Corollary 2 *Let $n = 2$. Suppose both bidders are risk neutral, and draw their signals from the same distribution function. When the seller uses either CSA or SPSB, then his goals Efficiency and Revenue are fulfilled.*

3.2 Loss averse bidders

What is the effect on the outcome of CSA when bidders are loss averse rather than risk neutral? We model loss aversion in the following simplified way. Bidder i is called θ_i-loss averse if her utility function $U_i(.)$ is given by

$$\begin{aligned} U_i(x_i) &= x_i \text{ for all } x_i \geq 0 \\ U_i(x_i) &= \theta_i x_i \text{ for all } x_i < 0, \end{aligned}$$

where $\theta_i > 1$ is the loss aversion parameter for bidder i. The interpretation of θ_i-loss aversion is the following. If a θ_i-loss averse bidder loses x_i units, then she perceives this as if she were a risk neutral bidder losing $\theta_i x_i$ units. More specifically, the realized utility u_i from CSA for bidder i having signal v_i, who buys $s \in \{1, 2\}$ chopsticks in CSA at a price of p per chopstick, is given by

$$u_i(v_i, s, p) = \begin{cases} v_i - 2p & \text{if } s = 2 \text{ and } v_i \geq 2p, \\ \theta_i(v_i - 2p) & \text{if } s = 2 \text{ and } v_i < 2p, \\ -\theta_i p & \text{if } s = 1. \end{cases}$$

Proposition 3 establishes that the seller strictly prefers SPSB over CSA. As SPSB has en efficient outcome, this auction fulfills *Efficiency.* There are two possibilities that have to be checked. If is CSA not efficient, then the Proposition is immediately established, as the targets of efficiency and revenue maximization are lexicographically ordened. If CSA is efficient, then it remains to be checked that SPSB yields strictly more revenue than CSA. Using the Envelope Theorem, we show that the expected utility for each bidder i given her signal v_i is higher than in SPSB, which implies that expected payments in CSA are lower than in SPSB.

Proposition 3 *Suppose that each bidder i is θ_i-loss averse. The seller who aims at fulfilling the criteria Efficiency and Revenue is strictly better off replacing CSA with SPSB.*

Proof. As SPSB has an equilibrium in weakly dominated strategies, in which each bidders bids her value for the bundle of three chopsticks, the outcome of SPSB is efficient, so that *Efficiency* is fulfilled. Myerson (1981) shows that for this auction, the interim utility for bidder i having signal v_i is given by

$$U_i^{SPSB}(v_i) = \int_{\underline{v}}^{v_i} P_i(x)dx,$$

with $P_i(x)$ the probability that x is the highest signal.

Let (p_i, δ_i) denote the outcome of CSA for bidder i, where p_i is her payment, $\delta_i = 1$ if she wins two chopsticks and $\delta_i = 0$ if she wins 0 or 1 chopstick.

Let $d_i(p_i, v_i, \delta_i)$ be the difference between the realized *value* in CSA (i.e., $\delta_i v_i$) and the realized *utility* level for bidder i having signal v_i and loss aversion parameter θ_i if the auction outcome is (p_i, δ_i). Hence,

$$d_i(p_i, v_i, \delta_i) = \begin{cases} \theta_i p_i & \text{if } \delta_i = 0, \\ p_i & \text{if } \delta_i = 1 \text{ and } p_i \leq v_i, \\ \theta_i p_i - (\theta_i - 1)v_i & \text{if } \delta_i = 1 \text{ and } p_i > v_i. \end{cases}$$

Call $d_i(p_i, v_i, \delta_i)$ the *subjective costs for bidder* i. Observe that $d_i(p_i, v_i, \delta_i) \geq p_i$ (the subjective costs are higher than the actual payments), and that $d_i(p_i, v_i, \delta_i)$ is (weakly) decreasing in v_i. Let $D_i(w, v_i)$ denote the expected value of $d_i(p_i, v_i, \delta_i)$ for bidder i with signal v_i, who bids as if she has signal w, while all the other bidders play according to their Bayesian-Nash equilibrium strategy.

Suppose that for CSA, *Efficiency* holds (otherwise SPSB is already better). Then the equilibrium probability for a bidder with signal v_i to win in the auction is given by $P_i(v_i)$. Given the equilibrium strategies of the other bidders, a bidder optimally announces her true signal v_i, maximizing

$$U_i(w, v_i) \equiv P_i(w)v_i - D_i(w, v_i)$$

with respect to w. Let

$$U_i^{CSA}(v_i) \equiv U_i(v_i, v_i).$$

By the Envelope Theorem,

$$\frac{d\tilde{U}_i(v_i)}{dv_i} = P_i(v_i) - D_i^2(v_i, v_i), \tag{3}$$

where $D_i^2(v_i, v_i)$ denotes the derivative of $D_i(v_i, v_i)$ with respect to its second argument. By definition, $D_i^2(v_i, v_i) \leq 0$. Integrating (3), and by individual rationality, we have

$$U_i^{CSA}(v_i) = \int_{\underline{v}}^{v_i} \{P_i(x) - D_i^2(x,x)\}dx + \tilde{U}_i(\underline{v}) \geq \int_{\underline{v}}^{v_i} P_i(x)dx = U_i^{SPSB}(v_i)$$

so that the interim utility of bidder i in CSA is (weakly) higher than in SPSB. This implies that the expected subjective costs in the CSA of bidder i are (weakly) lower than the expected payments in SPSB. Efficiency implies that there is always a bidder who buys one chopstick for a strictly positive price, so that the expected *payments* to the seller are strictly lower than the expected *subjective costs*. Therefore, the expected revenue of CSA is strictly lower than the expected revenue of SPSB. ■

Proposition 3 is intuitive in the light of the Revenue Equivalence Theorem. In CSA, loss averse bidders bid the same as risk neutral bidders who pay $\theta_i b$ when their bid b is the second highest bid. By the Revenue Equivalence Theorem, in the case of efficiency, the expected payment of risk neutral bidders to the seller does not depend on θ_i. Loss averse bidders, however, only pay their bid b rather than $\theta_i b$ in the case they lose, so that they pay less than their risk neutral equivalents. Therefore, the seller is better off if he chooses an efficient auction in which the bidder cannot incur losses, such as SPSB.[21]

[21] In Proposition 3, CSA can be replaced by any auction in which the losing bidder has to pay, such as the all-pay auction. This is true, as the only property of the Chopstick Auction that is used in the proof is the fact that the losing bidder has to pay. Also, this result holds in the case of three or more bidders.

4 Three bidders

In the case of three bidders, CSA consists of two stages. In stage 1, each bidder decides at which point to leave the auction. At some point in time, one of the bidders leaves the auction, and the two remaining bidders enter stage 2. In stage 2, both remaining bidders have to make a decision about how long to stay, given the price at which the other bidder left.

A symmetric Bayesian Nash equilibrium is a Bayesian Nash equilibrium, in which bidders with the same value play the same strategy. Proposition 4 establishes that a symmetric Bayesian Nash equilibrium cannot exist. We prove this by contradiction. Suppose that a symmetric equilibrium exists. Then, by a standard argument, in both stages, a bidder with a low value must step out earlier than a bidder with a high value. Let bidders 2 and 3 step out according to the same strictly increasing bid function in stage 1. Then bidder 1 prefers not to bid according to this bid function. Intuitively, this can be seen as follows. Suppose that the price approaches the bid at which the other bidders would step out given that they have the same value as bidder 1. Bidder 1 knows that if one of the other bidders steps out earlier than her, then there is a high probability that she enters stage 2 having the lowest value. As also the bid function in the second stage is strictly increasing in value, with high probability, bidder 1 is the second highest bidder. In that case, she wins only one chopstick for a positive price, so that she makes a loss. Therefore, bidder 1 prefers to deviate to a lower bid, which is a contradiction to the assumption that the equilibrium is symmetric.

Proposition 4 *Let $n = 3$. Then CSA has no symmetric Bayesian Nash* *equilibrium.*

Proof. Suppose, in contrast, that a symmetric equilibrium exists. Then, for both stages, the equilibrium bid function must be strictly increasing. It must be weakly increasing by the same argument as used in the proof of Proposition 1. Also, no pooling can occur in equilibrium. (Suppose instead that there is some pooling at a price p. Then at least one of the two following situations occur. Either the bidder at the lower end of the interval of bidders who bid p makes a loss at p, so that she is better off by deviating to a lower price. Or the winner at the upper end of the interval gets a strictly positive expected utility, but then she can strictly improve by bidding slightly higher.)

However, in the first stage, bidder 1 prefers to deviate if both other bidders submit bids according to a strictly increasing bid function. Let B_1 denote the equilibrium bid function in the first stage. Suppose that the auction reaches a price $B_1(v_1 - \epsilon)$ before anybody quits. Then, bidder 1 gets zero utility when she quits at this point. In the event that one and only one of the other bidders has a signal in the interval $[v_1 - \epsilon, v_1]$, bidder 1 has the second highest signal, and she will win 1 (worthless) chopstick for a price of at least $B_1(v_1 - \epsilon)$. In the event that both bidders have a signal in the interval $[v_1 - \epsilon, v_1]$, bidder 1 wins both chopsticks. The first event happens with a probability which is of the order ϵ, and the second event with a probability of the order ϵ^2, so that bidder 1 strictly prefers not to bid $B_1(v_1)$, but to step out earlier. Therefore, a symmetric equilibrium does not exist. ■

In the case that all bidders are risk neutral, and draw their signals from the same distribution, Proposition 4 can also be derived with the Revenue

Equivalence Theorem. Suppose a symmetric equilibrium exists. It is shown in the proof of Proposition 4 that this equilibrium is necessarily efficient, implying that the expected payment by the bidders with the two highest signals in stage 2 is equal to the expected payment in SPSB, namely the expectation of the second-highest signal, $v_{(2)}$. When two bidders enter stage 2, they are already sure that they have to pay at least the price reached in stage 1, so that this payment can be considered as sunk costs. Stage 2, with the bidders who have the two highest signals, is also revenue equivalent with SPSB with these two bidders, so that the expected payment by the two highest bidders *above the sunk costs* is again given by the expectation of the second highest value, i.e., $v_{(2)}$. Hence, the costs the bidders commit themselves to in stage 1 should be equal to 0. This implies that an efficient equilibrium will be characterized by an immediate drop-out of the bidder with the lowest signal. Therefore, in stage 1, equilibrium bids should be equal to 0. But this cannot happen in equilibrium, as any bidder is better off by waiting a bit longer.

An asymmetric equilibrium of CSA is easily found, namely when one bidder decides to always stay in the auction, no matter what the other bidders do, and the other bidders step out immediately.[22] If these strategies are played, the auction outcome will be very inefficient, and the revenue will be zero. However, this type of equilibrium involves a dominated strategy, so that it is very unlikely to be played.

The impossibility result of Proposition 4 suggests that the seller is better off when he replaces CSA with SPSB. The nonexistence of symmetric equi-

[22] Also the second-price sealed-bid auction has such equilibria.

libria indicates that CSA probably has no efficient equilibrium either. This conjecture is based on the following consideration. Asymmetry implies that if three bidders have the same type, one of them steps out strictly earlier than the other two. Assuming continuous bid functions, this implies that the bidder who steps out first, also steps out earlier than the other two bidders when they have slightly lower values, so that the outcome is inefficient. This reasoning justifies the conjecture that SPSB is strictly better according to the seller's goals.

Conjecture 5 *Let $n = 3$. The seller who aims at fulfilling the criteria Efficiency and Revenue is strictly better off replacing CSA with SPSB.*

5 Concluding remarks

In this paper, we have studied the exposure problem in multiple object auctions. We have found in all the investigated settings that a seller who aims at efficiency and high auction revenues (weakly) prefers to sell the three chopsticks as one package in the second-price sealed-bid auction (in which the exposure problem is not present) over selling them using the Chopstick Auction (in which bidders face an exposure problem). We conclude that avoiding the exposure problem is an important issue in auction design.

The results for the Chopstick Auction can be straightforwardly generalized to allow for $L \geq 3$ objects being sold to $n \geq 3$ bidders who need $M \geq 2$ objects. Let $W \equiv \lfloor \frac{L}{M} \rfloor < n$ be the maximal number of "winners" in the auction. Assume there is a strictly positive number S of superfluous objects, i.e., $S \equiv L - M\lfloor \frac{L}{M} \rfloor > 0$. The outcome of the auction is such that the W highest bidders get the M objects they need, and $(n+1)$th highest bidder has to buy and pay for the S superfluous lots, which are of no value to her. For the case $W = n + 1$ results analogous to Propositions 1 and 3 can be derived using similar arguments. If $W > n+1$, i.e., if there is more than one bidder who does not win in the auction, then, analogous to Proposition 4, the auction has no symmetric equilibria.

Loss aversion, which we assumed for Proposition 3, seems to be a reasonable assumption for bidders in the Dutch DCS-1800 auction. In this auction, the bidders are "agents" trying to win valuable licenses for their "principals", the shareholders of the firms they represent. For the agents, leaving the auction with an expensive, but worthless, set of channels has more impact on the negative side (as they may lose their jobs), than has winning a valuable set on the positive side.

In the introduction of this paper, we have argued that in the presence of the exposure problem, bidders rather play the war of attrition than a standard auction game. In fact, Bulow and Klemperer (1999) found a result analogous to Proposition 4 for the *generalized war of attrition.* The generalized war of attrition is a game in which n bidders are bidding for $m(<n)$ prizes in a multiple object button auction. In this auction, bidders drop out while the price is rising, until m bidders are left. Those bidders win a prize, and pay the current price. Each bidder who drops out earlier, pays her bid plus c times the difference between the final price and her bid. In the limit $(c \to 0)$ of the unique efficient equilibrium, all but the $m+1$ bidders with the highest signals drop out immediately. However, this cannot be an equilib-

rium in the game with $c = 0$, as bidders have an incentive to deviate, and bid just above 0. Therefore, the generalized war of attrition has no symmetric equilibrium.

Several issues related to our model need further investigation. For instance, the effect of all remaining bidders being informed when one of the bidders quit is not well understood. More specifically, does the Chopstick Auction have symmetric equilibria if bidders would not observe each other leave the auction? Moreover, we have assumed that a bidder does not acquire any value when she wins only one chopstick. A question that may be interesting for further research is how the analysis would change if the marginal value of the first and the third chopstick are strictly positive. Finally, the impossibility result in the case of three bidders is not very informative about equilibrium bidding. A further study is needed to get a better understanding about how bidders behave in the Chopstick Auction in the case of three bidders.

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