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Discussion paper

Auctions for Extra Capacity in an Oligopolistic Market with Network Effects*

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Abstract

We study auctions in which firms can win a license for extra capacity in an oligopolistic market with network effects. When total market profit does not depend on the winner of the extra capacity, the largest firm wins the license in a first-price sealed-bid auction. Moreover, for each firm an auction mechanism exists that assigns the license to the firm, is dominant strategy implementable, and maximizes revenue. Both auction mechanisms are collusion proof. These results do not necessarily hold when total market profit is not fixed. We apply the developed theory to the market of petrol stations in the Netherlands.

1 Introduction

In February 1999, an MDW-study group advised the Dutch government that the market for petrol along the Dutch highways lacks serious competition.[1]

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[1]The report, which is in Dutch, is available on internet at http://www.ez.nl/publicaties/pdfs/11B88.pdf.

The market is characterized by high levels of market concentration with a Herfindahl-Hirschman Index of 3135, and a total market share of the largest four firms equal to 75%. Moreover, the margin on a liter petrol is higher than in surrounding countries as shown in Table 2. The lack of competition in the market worried the study group, and one of the suggestions to the Dutch government was to auction new licenses for petrol stations in order to let new firms enter the market, or give small firms the opportunity to grow.

The Netherlands	Belgium	Germany	France	United Kingdom
0.14	0.14	0.12	0.09	0.05

Table 2. Profit margins on petrol as a fraction of the price, measured in 1996. Source: Coopers & Lybrand (1996). "Investigation on the Price Structure of Euro 95 and Diesel Oil in The Netherlands, Belgium, Germany, France and Great Britain." (In Dutch).

The study group conjectured that a standard auction will not lead to an economically efficient outcome, because competition will be decreased as the largest firms in the market (Shell, Esso, BP and Texaco) will win all the new licenses. There are two reasons why this is likely. First, because of network effects, the willingness-to-pay for a license is higher for a large firm than for a small firm. With network effects we mean that a large firm is ceteris paribus able to gain more profit per outlet than a small firm. In the petrol market, the network effects are probably due to loyalty schemes or advertising, which are both more effective for large firms than for small. Second, a decrease in competition will lead to higher profits, so that there is an incentive for large firms to preempt the market, i.e., to buy licenses with the aim of preventing new competitors to enter the market. The economic literature suggests that in standard auctions, the chances for small firms to win capacity are small. For instance, Gilbert and Newbery (1982) show that monopoly persists when the incumbent monopolist and a potential newcomer compete to get a patent. In a related study, Jehiel and Moldovanu (2000a) find that when both incumbents and potential entrants bid for several licenses, all licenses will be sold to incumbents in case the number of incumbents exceeds the number of licenses, or all incumbents acquire a license if the number of licenses exceeds the number of incumbents.[2]

We performed an empirical analysis in order to test for the presence of network effects in the petrol market along the Dutch highways. We modelled the sales $F(n)$ per passing vehicle per petrol station of a firm with n petrol stations located at the Dutch highways with the following expression.

$$F(n) = \alpha + \beta n + \sum_i \gamma_i x_i + \eta.$$

The x_i's are the characteristics of the petrol station, such as location with respect to the nearest city, local competition, and facilities at the site. α, β, and the γ_i's are one-dimensional parameters, and η is a disturbance term for which the standard OLS assumptions are assumed to apply. Using data from petrol stations along the Dutch highway, we estimated β to be significantly larger than 0. We concluded that indeed network effects are present in the

[2]However, Krishna (1993, 1999) finds that newcomers may be able to beat incumbents when licenses are auctioned sequentially.

petrol market.[3,4]

The aim of this paper is to answer two questions for an environment with network effects. First, does the largest firm in the market win a license when the license is sold in the first-price sealed-bid auction? Second, is there a feasible auction mechanism which implements four targets in an environment with network effects, namely (1) the mechanism guarantees an economically efficient outcome, (2) it generates as much revenues as possible for the government, (3) it is not sensitive to collusion, and (4) it is easy to implement?

We will answer these questions in a complete information model, which is given in Section 2. We assume that there is a seller, who desires to sell a license to one firm out of a set of several firms, which compete in an oligopolistic market characterized by network effects. The firm which acquires the license imposes a negative externality on all its competitors by stealing part of their market share. In order to incorporate the network effects, the profit per outlet for a given firm is increasing in its total number of outlets. The size of the "pie" (total market profits) to be divided among the firms depends on which firm gets the license. We assume that the size of the pie is increasing in the size of the winning firm.

In this model, the four targets are formalized as follows. First of all, we do not explicitly calculate a measure for efficiency of the feasible auction

[3] The study was performed as part of the project of the Ministry of Finance, and can also be found in the report, which can be found on the internet at http://www.ez.nl/publicaties/pdfs/11B88.pdf.

[4] There are several other empirical studies on network effects. Gandal (1994) observes that network effects exist in the market for computer spreadsheet programs. In a study on the adoption by banks of automated teller machines, Saloner and Shepard (1995) find network effects in the bank sector.

mechanism. Instead, we assume that efficiency requires that the largest firm does not win the license. Secondly, the revenue target is straightforward: the feasible auction mechanism should maximize revenue over all feasible auction mechanisms. Thirdly, following Jehiel and Moldovanu (1996), we use the emptiness of the α-core as a measure for collusion proofness of the studied feasible auction mechanisms. Finally, a feasible auction mechanism is easy to implement if the bidding firms play a dominant strategy.

In Section 3, we discuss the outcomes of the model in the pure network effects case, i.e., when the size of the pie does not depend on which firm wins the license. With three or more firms, the largest firm wins the license in the first-price sealed-bid auction in any Nash equilibrium, so that the outcome of the auction is not efficient. However, we find that for each firm i, there exist a feasible auction mechanism in which firm i wins the license, and which implements (almost) revenue maximization in strictly dominant strategies. This feasible auction mechanism is a take-it-or-leave-it mechanism, in which the seller, when a firm chooses not to participate, assigns the license to the firm that imposes the worst threat on it (in terms of lost market share). Finally, as the α-core is empty, the take-it-or-leave-it mechanism is collusion proof, so that the four targets are reached.

In Section 4, we study the market when the size of the pie varies with the identity of the winner of the license. There is an equilibrium in which the largest firm wins the license. However, we construct an example in which it is not the largest firm who wins the license in the first-price sealed-bid auction. But, in equilibrium, the second highest bidder submits a bid that exceeds its willingness-to-pay against the winner, and we will show that this equilibrium can be excluded by iterated deletion of weakly dominated strategies. We conjecture that only Nash equilibria exist in which the largest firm wins survive iterated deletion of weakly dominated strategies, which implies that the outcome of the first-price sealed-bid auction is not efficient. Moreover, we show that any feasible auction mechanism that (almost) maximizes revenue, must necessarily assign the license to the largest firm, so that in our model all four targets cannot be reached at once, as there is a conflict between efficiency and revenue maximization. Also, we find that the α-core need not be empty, so that there need not exist a collusion-proof auction mechanism.

2 The model

A seller owns a license for an outlet in an oligopolistic market with n incumbent firms, labeled $1, ..., n$. Let

$$N \equiv \{1, ..., n\}$$

denote the set of firms. We will use i, j, k and l to represent typical firms in N. If the market situation is such that firm j has m_j outlets in the market, $j = 1, ..., n$, then firm i's profit is given by

$$\Pi_i(\mathbf{m}) \equiv S_i(\mathbf{m}) * P(\mathbf{m})$$

with $\mathbf{m} \equiv (m_1, ..., m_n)$ the vector of number of outlets, $P(\mathbf{m})$ total market profits, and $S_i(\mathbf{m})$ firm i's profit share.

We assume that firm i's profit share is given by

$$S_i(\mathbf{m}) \equiv \frac{f(m_i)}{\sum_{l=1}^{n} f(m_l)}, \tag{1}$$

where f has the following properties.

$$f(0) = 0, \tag{2}$$

$$f(m_i) > f(m_j) \quad \text{if } m_i > m_j, \text{ and} \tag{3}$$

$$f(m_i + 1) - f(m_i) > \ f(m_j + 1) - f(m_j) \quad \text{if } m_i > m_j \tag{4}$$

Equation (2) indicates that a firm with no outlets in the market makes no profit, (3) indicates that profit for a firm is increasing in the number of its outlets, and (4) is a convexity condition on f.

Equations (1)-(4) are sufficient to establish network effects in the market in the sense that the profit per outlet for a firm is increasing in the total number of outlets the firm has. Proposition 1 shows that (1)-(4) imply that profit per outlet is increasing in the number of outlets.

Proposition 1 *Suppose that (1)-(4) are satisfied. If* $m_i > m_j$*, then* $\Pi_i(\mathbf{m})/m_i > \Pi_j(\mathbf{m})/m_j$.

Proof. Let $m_i > m_j$. Then the result follows immediately with the following observation.

$$\begin{aligned}
\frac{f(m_i)}{m_i} &= \frac{1}{m_i}\sum_{h=1}^{m_i}[f(h) - f(h-1)] \\
&> \frac{1}{m_i}\sum_{h=1}^{m_j}[f(h) - f(h-1)] + \frac{m_i - m_j}{m_i}[f(m_j) - f(m_j - 1)] \\
&> \frac{1}{m_i}\sum_{h=1}^{m_j}[f(h) - f(h-1)] + \frac{m_i - m_j}{m_i m_j}\sum_{h=1}^{m_j}[f(h) - f(h-1)] \\
&= \frac{1}{m_j}\sum_{h=1}^{m_j}[f(h) - f(h-1)] \\
&= \frac{f(m_j)}{m_j}.
\end{aligned}$$

■

The seller plans to sell the license using an auction. Let $\bar{m}_j$ denote the number of outlets firm j has in the market before the license is sold. We assume $\bar{m}_1 > \bar{m}_2 > ... > \bar{m}_n$.[5] Let $\bar{\mathbf{m}} \equiv (\bar{m}_1, ..., \bar{m}_n)$ and $\mathbf{e}_i$ be the vector with the ith entry equal to 1, and the other entries equal to zero. We make the following assumption on P.

$$P(\bar{\mathbf{m}} + \mathbf{e}_i) \geq P(\bar{\mathbf{m}} + \mathbf{e}_j) \text{ if } \bar{m}_i > \bar{m}_j.$$

In words: the larger the firm that wins the license the larger total market profits. Define

$$U_i(j) \equiv \Pi_i(\bar{\mathbf{m}} + \mathbf{e}_j) - \Pi_i(\bar{\mathbf{m}})$$

as the utility of firm i when firm j wins the license. The willingness-to-pay for a firm i depends on which firm is considered by firm i as its opponent. We will say that firm i is willing to pay a specific amount "against" firm j, where the willingness-to-pay is given by the difference in utility for firm i when it gets the license, and when firm j gets the license, i.e., $U_i(i) - U_i(j)$.

In an auction, each firm i simultaneously and independently submits a bid $b_i \in B_i$, where B_i is the set of bids for firm i. In particular, we will study the first-price sealed-bid auction and a take-it-or-leave-it mechanism. In case the auction is the first-price sealed-bid auction, we assume the sets of potential bids B_i to have the form

$$B_i = \{0, \epsilon, 2\epsilon, ...\}$$

[5] This is with some loss of generality. The nongeneric cases can be treated in a completely analogous way, but the case differentiation is more tedious.

with ϵ the smallest money unit,[6] where ϵ is very small relative to all the other parameters. In the take-it-or-leave-it mechanism, B_i has the form

$$B_i = \{\text{“participate”},\ \text{“not participate”}\},$$

which indicates that each firm can either participate or not.

An auction has the following outcome functions

$$\widehat{p} : B_1 \times ... \times B_n \rightarrow [0, 1]^n$$

with

$$\sum_j \widehat{p}_j(b_1, ..., b_n) \leq 1,$$

and

$$\widehat{x} : B_1 \times ... \times B_n \rightarrow \Re^n.$$

If $\mathbf{b} = (b_1, ..., b_n)$, then $\widehat{p}_i(\mathbf{b})$ is interpreted as the probability that firm i gets the license, and $\widehat{x}_i(\mathbf{b})$ is the expected payment of firm i to the seller. For simplicity, we assume that the $\widehat{x}_i$'s are multiples of ϵ. When firm i chooses "not participate" in the take-it-or-leave-it mechanism, $\widehat{p}_i(\mathbf{b}) = \widehat{x}_i(\mathbf{b}) = 0$. We refer to this assumption as the "no-dumping assumption". The firms are risk neutral and have additively separable utility function for money and the allocation of the object, so that if $\mathbf{b}$ is submitted, firm i's utility is given by

$$\sum_{l=1}^{n} \widehat{p}_l(\mathbf{b}) U_i(l) - \widehat{x}_i(\mathbf{b}).$$

A *strategy* for a firm i is the choice of a bid (or a randomization over several bids) from the set B_i. A *feasible auction mechanism* is an auction including

[6] Following Jehiel and Moldovanu (1996), we make this assumption in order to avoid problems related to the existence of a Nash equilibrium in pure strategies.

strategies, which form a Nash equilibrium of the auction. An *optimal auction* is a feasible auction mechanism that gives the seller the highest expected revenue. An *almost optimal auction* is a feasible auction mechanism that gives the seller the highest expected revenue minus at most $n\epsilon$. We say that an (almost) optimal auction is *dominant strategy implementable* if each firm plays a strictly dominant strategy. Dominant strategy implementation implies that the auction game is easy to play by a firm, as its optimal bid does not depend on the strategies of the other firms.

Following Jehiel and Moldovanu (1996), we use the concept of α-*core* from cooperative game theory to define collusion proofness of the studied feasible auction mechanisms. The α-core is the core of the α-game, which is a TU game in which the player set consists of the seller and the n firms. The characteristic function is for each coalition defined as the maximal utility the coalition is able to obtain under the assumption that the complement takes the worst action against the coalition.

Formally, let player 0 denote the seller, and let $v : 2^{\{0\}\cup N} \rightarrow \Re$ be the characteristic function of the α-game. Let $S \subseteq \{0\} \cup N$. We distinguish two situations, namely $0 \in S$, and $0 \notin S$. In the case $0 \in S$, the complement has no options available, and the best thing the coalition S can do is transfer the license to the firm which maximizes total utility of the firms in S, so that $v(S) \equiv \max_{i \in S} \sum_{j \in S\setminus\{0\}} U_j(i)$. In the case that $0 \notin S$, the worst action the complement can take is assign the license to firm $i \notin S$ that imposes the "worst threat" on the firms in S. Hence, $v(S) \equiv \min_{i \notin S} \sum_{j \in S} U_j(i)$. Then

$x \equiv (x_0, x_1, ..., x_n) \in \Re^{n+1}$ is an element of the α-core if and only if

$$\sum_{j \in S} x_j \geq v(S)$$

for all $S \subseteq \{0\} \cup N$, and

$$\sum_{j \in \{0\} \cup N} x_j = v(\{0\} \cup N).$$

Each feasible auction mechanism is called *collusion-proof* if the α-core is empty. We use the emptiness of the α-core as a measure for collusion-proofness as it indicates that no cooperative agreement is stable against a deviation from a coalition. An implicit assumption that we will make throughout the paper is that the seller has complete commitment power, in the sense that he is able commit to any feasible auction mechanism he desires. We have to make this assumption, as the emptiness of the α-core suggests that such commitment is not stable. The strength of the α-core lies in the fact that it is the least sharp core concept, so that if the α-core is empty, other cores are empty as well (Jehiel and Moldovanu, 1996).

3 Constant total market profits

Suppose that total market profit is constant, i.e., total market profit does not depend on the distribution of the outlets over the firms. Without further loss of generally, we assume

$$P \equiv 1.$$

Before we establish equilibrium bidding in the first-price sealed-bid auction, we derive two useful lemmas. Lemma 2 indicates that in the case of three or more firms, each firm gets more utility when a small competitor wins than when a large competitor wins. Lemma 3 shows that firm 1 is always willing to pay more against firm $i \neq 1$ (as its willingness-to-pay is given by $U_1(1) - U_1(i)$), than firm i is willing to pay against firm 1 (as its willingness-to-pay is given by $U_i(i) - U_i(1)$).

Lemma 2 *Let $n \geq 3$. For all $i, j, k \in N$, $i \neq j \neq k \neq i$, with $\bar{m}_j < \bar{m}_k$,*

$$U_i(j) > U_i(k).$$

Proof. Let $i, j, k \in N$, $i \neq j \neq k \neq i$, with $\bar{m}_j < \bar{m}_k$. By (4),

$$f(\bar{m}_k) + f(\bar{m}_j + 1) < f(\bar{m}_k + 1) + f(\bar{m}_j)$$

so that

$$\frac{f(\bar{m}_i)}{f(\bar{m}_k) + f(\bar{m}_j + 1) + \sum_{l \neq k,j} f(\bar{m}_l)} > \frac{f(\bar{m}_i)}{f(\bar{m}_k + 1) + f(\bar{m}_j) + \sum_{l \neq k,j} f(\bar{m}_l)}$$

which implies

$$\Pi_i(\bar{\mathbf{m}} + \mathbf{e}_j) - \Pi_i(\bar{\mathbf{m}}) > \Pi_i(\bar{\mathbf{m}} + \mathbf{e}_k) - \Pi_i(\bar{\mathbf{m}})$$

which is by definition equivalent to

$$U_i(j) > U_i(k).$$

■

Lemma 3 *Let $n \geq 3$. For all $i \in N \backslash \{1\}$,*

$$U_i(i) - U_i(1) < U_1(1) - U_1(i).$$

Proof. Let $i \in N \backslash \{1\}$. By (4),

$$f(\bar{m}_1) + f(\bar{m}_i + 1) < f(\bar{m}_1 + 1) + f(\bar{m}_i)$$

or, equivalently, with (2) and (3)

$$\frac{1}{1 + \sum_{l \neq 1,i} f(\bar{m}_l)/[f(\bar{m}_1) + f(\bar{m}_i + 1)]} < \frac{1}{1 + \sum_{l \neq 1,i} f(\bar{m}_l)/[f(\bar{m}_1 + 1) + f(\bar{m}_i)]}.$$

With some manipulation we obtain

$$\Pi_i(\bar{\mathbf{m}} + \mathbf{e}_i) - \Pi_i(\bar{\mathbf{m}} + \mathbf{e}_1) < \Pi_1(\bar{\mathbf{m}} + \mathbf{e}_1) - \Pi_1(\bar{\mathbf{m}} + \mathbf{e}_i)$$

which is equivalent to

$$U_i(i) - U_i(1) < U_1(1) - U_1(i).$$

■

Proposition 4 shows that when the seller sells the license using the first-price sealed-bid auction, in case of three or more firms, firm 1 wins the license in *any* Nash equilibrium. In the case of two firms, there is an equilibrium in which each firm wins with probability $\frac{1}{2}$. The first part of the proposition follows intuitively from Lemma 3, as in the case that firm 1 and any firm $i \neq 1$ are in direct competition, firm 1 is prepared to pay more for the license than firm i.

Proposition 4 *Suppose $P \equiv 1$. Let $n \geq 3$. Then, in any Nash equilibrium of the first-price sealed-bid auction, firm 1 wins the license. If $n = 2$, then there is a Nash equilibrium in which each firm wins with probability $\frac{1}{2}$.*

Proof. Let $n \geq 3$. Let $(p_1, ..., p_n)$ denote a Nash equilibrium. We prove the proposition by contradiction. In order not to perform a tedious case differentiation, we suppose the following holds for some $i \in N\backslash\{1\}$, some $j \in N\backslash\{i\}$ and all $l \in N\backslash\{i, j\}$ (other cases proceed in an analogous way).

$$p_i > p_j > p_l.$$

If these strategies are played, another firm than firm 1 wins the auction. These bids constitute a Nash equilibrium if

$$U_i(i) - p_i \geq U_i(j) \tag{5}$$

and

$$U_k(i) \geq U_k(k) - p_i \text{ for every } k \in N\backslash\{i\} \tag{6}$$

are satisfied. Condition (5) indicates that firm i has no incentive to submit a bid strictly lower than p_j. Condition (6) indicates that none of the firms other than i is willing to overbid the bid of i.

The contradiction is established, as (5) and (6) imply

$$\begin{aligned} p_i &\leq U_i(i) - U_i(j) \\ &\leq U_i(i) - U_i(1) \\ &< U_1(1) - U_1(i) \\ &\leq p_i \end{aligned}$$

where the second inequality follows from Lemma 2, and the third inequality from Lemma 3. A similar argument establishes that there is no equilibrium involving mixed strategies, in which a firm other than firm 1 wins the license with strictly positive probability.

For $n = 2$, it is readily checked that a Nash equilibrium is established when both firms submit a bid equal to

$$\frac{f(\bar{m}_1 + 1)}{f(\bar{m}_1 + 1) + f(\bar{m}_2)} - \frac{f(\bar{m}_1)}{f(\bar{m}_1) + f(\bar{m}_2 + 1)}.$$

■

Consider for each $i \in N$, take-it-or-leave-it mechanism M^i, which has the following rules.[7] The firms simultaneously decide whether to participate or not. Let $T \subseteq N$ denote the set of firms who decide to participate, and let

$$w(j) \in \arg\min_{k \neq j} U_j(k)$$

denote the firm that imposes the "worst threat" on firm j. For each $T \subseteq N$, we define the winner of the license and the payments made to/by the seller.

(1) If $|T| \leq n - 2$, then the seller keeps the license, and all firms who participate receive ϵ.

(2) If $T = N \backslash \{k\}$, then the winner is $w(k)$. Each firm $j \in T$ is required to pay $U_j(w(k)) - \epsilon$ (where a negative amount indicates that firm j receives money from the seller).

(3) If $T = N$, then the seller gives the license to firm i. Each firm j is required to pay $U_j(i) - U_j(w(j)) - \epsilon$.

Proposition 5 shows that implementing a feasible auction mechanism which results in another firm than firm 1 to win the license does not necessarily lead to a loss in revenue compared to the optimal auction in which

[7] This take-it-or-leave-it mechanism, in a somewhat different form, was introduced by Jehiel et al. (1996) in a situation with negative externalities.

firm 1 wins. The intuition behind Proposition 5 is the following. First of all, each take-it-or-leave-it mechanism M^i is defined such that the seller transfers/asks money to/from each participating firm such that the firm's utility is ϵ higher compared to the situation in which it does not participate, so that for each firm participation is a strictly dominant strategy. Secondly, if all firms participate, each firm j pays $U_j(i) - U_j(w(j)) - \epsilon$ to the seller. The sum of the utilities of all firms from the allocation of the license to firm i is zero, as market profit remains constant. Therefore, $\sum_j U_j(i) = 0$ for all i. As $U_j(w(j))$ does not depend on i, total payments to the seller do not depend on i as well. Finally, $U_j(i) - U_j(w(j))$, is the maximal willingness to pay for firm j given that firm i wins the license, so that $\sum_j \{U_j(i) - U_j(w(j)) - \epsilon\}$ is the highest possible revenue from any mechanism minus $n\epsilon$.

Proposition 5 *Suppose $P \equiv 1$. Then for each $i \in N$, M^i is a feasible auction mechanism that (1) assigns firm i the license, (2) is dominant strategy implementable, and (3) is almost optimal.*

Proof. We start by showing that for each firm, participation in M^i is a strictly dominant strategy. Consider firm j, and assume that the firms in $N\backslash\{j\}$ play a pure strategy profile such that the set of participating firms is a set $T' \subseteq N\backslash\{j\}$. There are three possible cases.

(1) $T' = N\backslash\{j\}$. If firm j does not participate, its utility is $U_j(w(j))$. If j participates, then its utility equals

$$U_j(i) - [U_j(i) - U_j(w(j)) - \epsilon] = U_j(w(j)) + \epsilon.$$

(2) $T' = N\backslash\{j,k\}$. If firm j does not participate, its utility is 0. If j participates, then the license is allocated to $w(k)$, and j is required to pay

$U_j(w(k)) - \epsilon$. Its utility equals

$$U_j(w(k)) - [U_j(w(k)) - \epsilon] = \epsilon.$$

(3) $|T'| < n - 2$. If firm j does not participate, its utility is 0. If j participates, then its utility equals ϵ.

Participation is strictly better for firm j than nonparticipation in each of the three possible cases. Therefore, participation is a strictly dominant strategy.

The seller's revenue of M^i when all firms play their dominant strategy is given by

$$\begin{aligned} R(M^i) &= \sum_{j=1}^{n}[U_j(i) - U_j(w(j)) - \epsilon] \\ &= -\sum_{j=1}^{n} U_j(w(j)) - n\epsilon \\ &\equiv R. \end{aligned}$$

The first equation follows as

$$\begin{aligned} \sum_{j=1}^{n} U_j(i) &= \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_i) - \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}}) \\ &= \sum_{j=1}^{n} S_j(\bar{\mathbf{m}} + \mathbf{e}_i) * P(\bar{\mathbf{m}} + \mathbf{e}_i) - \sum_{j=1}^{n} S_j(\bar{\mathbf{m}}) * P(\bar{\mathbf{m}}) \\ &= \sum_{j=1}^{n} S_j(\bar{\mathbf{m}} + \mathbf{e}_i) - \sum_{j=1}^{n} S_j(\bar{\mathbf{m}}) \\ &= 0 \end{aligned}$$

by definition of the S_j's and the assumption $P \equiv 1$.

The highest expected revenue the seller can obtain given that firm i wins the license, is given by

$$\sum_{j=1}^{n}[U_j(i) - U_j(w(j))] = -\sum_{j=1}^{n} U_j(w(j)) = R + n\epsilon.$$

This expression follows immediately when taking into account that for each firm j the maximal willingness-to-pay for having firm i win the license is $U_j(i) - U_j(w(j))$. Therefore, the expected revenue from M^i is at most $n\epsilon$ lower than from any other feasible auction mechanism. ■

The first-price sealed-bid auction and the take-it-or-leave-it mechanisms M^i are collusion proof. This follows immediately from the following proposition, which shows that the α-core is empty. We prove this proposition by contradiction. Each coalition consisting of the seller and one the firms obtains strictly positive utility when the seller transfers the license to the firm. Moreover, the coalition consisting of all firms, but without the seller, can obtain at most zero utility, as the license cannot be transferred, and the seller has no "threat" available, because by the no-dumping assumption, he cannot transfer the license to one of the firms. These two properties together imply that the grand coalition should get a strictly positive payment. However, the grand coalition gets at most zero utility, as total market profits are assumed to be independent of whom owns the license, so that a contradiction is established.

Proposition 6 *Suppose $P \equiv 1$. Then the α-core is empty.*

Proof. The characteristic function has the following properties. The best a coalition of the seller and one of the firms can do is transfer the license from the seller to the firm. Therefore, for all $i = 1, ..., n$,

$$v(\{0, i\}) = U_i(i).$$

The coalition of all firms yields zero utility, as the worst threat of the seller against this coalition is to keep the license (in fact, by the no-dumping assumption, the seller has no other options), so that

$$v(N) = 0.$$

Moreover, the grand coalition gets zero utility, as total market profit is 1 regardless who (the seller or one of the firms) owns the license. Therefore

$$v(\{0\} \cup N) = 0.$$

We prove the proposition by contradiction. Suppose that the α-core is not empty. Let $x \equiv (x_0, x_1, ..., x_n)$ be an element of the α-core. The following inequalities must hold.

$$x_0 + x_i \geq v(\{0, i\}) = U_i(i),\ i \in N, \tag{7}$$

$$\sum_{i=1}^{n} x_i \geq v(N) = 0, \tag{8}$$

and

$$x_0 + \sum_{i=1}^{n} x_i = v(\{0\} \cup N) = 0. \tag{9}$$

Equation (7) implies, by adding over all $i = 1, ..., n$,

$$nx_0 + \sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} U_i(i).$$

Then, using (8), we get

$$nx_0 + n\sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} U_i(i)$$

or, equivalently

$$x_0 + \sum_{i=1}^{n} x_i \geq \frac{1}{n}\sum_{i=1}^{n} U_i(i). \tag{10}$$

It is easily checked that (3) implies that $U_i(i) > 0$ for all $i \in N$. But then (10) contradicts (9). Therefore, the α-core must be empty. ■

4 General total market profits

Assume that the size of the pie may vary with the identity of the winner of the license. We assume

$$P(\bar{\mathbf{m}} + \mathbf{e}_1) > P(\bar{\mathbf{m}} + \mathbf{e}_j) \text{ for all } j \neq 1, \tag{11}$$

so that when firm 1 wins the license, total market profits are maximized.[8]

It seems very likely that firm 1 wins the first-price sealed-bid auction, given the fact that firm 1 wins the license in any Nash equilibrium in the case that total market profits are constant. Indeed, according to Proposition 7, there is at least one Nash equilibrium in which firm 1 wins the license.

Proposition 7 *Suppose (11) holds true. There is a Nash equilibrium of the first-price sealed-bid auction in which firm 1 wins the license.*

[8]This assumption is with some loss of generality, as the non-generic case $P(\bar{\mathbf{m}} + \mathbf{e}_1) = P(\bar{\mathbf{m}} + \mathbf{e}_2)$ is excluded.

Proof. Let p_i be the bid for firm $i \in N$. It is readily checked that the following bids constitute a Nash equilibrium in which firm 1 wins the license.

$$\begin{aligned} p_1 &= U_2(2) - U_2(1), \\ p_2 &= p_1 - \epsilon, \text{ and} \\ p_k &= 0,\ k \in N \backslash \{1, 2\}. \end{aligned}$$

■

In the case of two firms, firm 1 always wins the license. The reason is that the willingness to pay for firm 1 against firm 2 is strictly larger than the willingness to pay for firm 2 against firm 1.

Proposition 8 *Suppose (11) holds true, and $n = 2$. Then, in any Nash equilibrium of the first-price sealed-bid auction, firm 1 wins the license.*

Proof. The proof is by contradiction. Let p_1 and p_2 be the bids of firm 1 and firm 2 in a Nash equilibrium, with $p_2 \geq p_1$. This is an equilibrium only if

$$U_1(2) \geq U_1(1) - p_2,$$

and

$$U_2(2) - p_2 \geq U_2(1).$$

Note that the first (second) condition refers to firm 1 (2) having no incentive to deviate to a bid above p_2 (below p_1). These conditions together imply that an equilibrium in which firm 2 wins only exists if

$$U_1(1) - U_1(2) \leq U_2(2) - U_2(1). \quad (12)$$

However, by assumption,

$$P(\bar{m}+e_1) > P(\bar{m}+e_2)$$

so that

$$P(\bar{m}+e_1)(S_1(\bar{m}+e_1)+S_2(\bar{m}+e_1)) > P(\bar{m}+e_2)(S_1(\bar{m}+e_2)+S_2(\bar{m}+e_2))$$

as $S_1+S_2=1$ by definition. Then, with some straightforward manipulation,

$$U_1(1)-U_1(2) > U_2(2)-U_2(1),$$

which contradicts (12). ■

However, in Example 9, we find an equilibrium of the first-price sealed-bid auction, in which it is not the largest firm that wins the license.

Example 9 *Consider the market with three incumbent firms and a potential entrant, with $\bar{m}_1=3$, $\bar{m}_2=2$, $\bar{m}_3=1$, and $\bar{m}_4=0$. The status quo total market profit equals 1, i.e.,*

$$P(\bar{\mathbf{m}})=1.$$

When the two largest firms acquire the license, total market profit remains at the initial level, so that

$$\begin{aligned} P(\bar{\mathbf{m}}+\mathbf{e}_1) &= 1, \text{ and} \\ P(\bar{\mathbf{m}}+\mathbf{e}_2) &= 1. \end{aligned}$$

When the smallest incumbent firm gets the license, the "pie" will shrink to $\frac{13}{15}$, i.e.,

$$P(\bar{\mathbf{m}}+\mathbf{e}_3)=\frac{13}{15}.$$

If the potential entrant obtains the license, no profit will be made in the market, so that

$$P(\bar{\mathbf{m}} + \mathbf{e}_4) = 0.$$

The other relevant parameters are given by

$$\begin{aligned} f(0) &= 0, \\ f(1) &= 7, \\ f(2) &= 15, \\ f(3) &= 24, \text{ and} \\ f(4) &= 34. \end{aligned}$$

Let $WTP(i,j)$ denote how much firm i is willing to pay for the license in order to prevent firm j from winning it. By definition, $WTP(i,j)$ can be written as

$$WTP(i,j) \equiv U_i(i) - U_i(j).$$

We find that

$$\begin{aligned} WTP(1,2) &= \frac{34}{34+15+7} - \frac{24}{24+24+7} \approx 0.17, \\ WTP(2,3) &= \frac{24}{24+24+7} - \frac{13}{15} * \frac{15}{24+15+15} \approx 0.20, \\ WTP(3,2) &= \frac{13}{15} * \frac{15}{24+15+15} - \frac{7}{24+24+7} \approx 0.11, \text{ and} \\ WTP(3,4) &= \frac{13}{15} * \frac{15}{24+15+15} \approx 0.24. \end{aligned}$$

Suppose that firm i bids p_i. Then it is readily verified that the following set *of bids establish a Nash equilibrium.*

$$\begin{aligned} p_1 &= p_4 = 0, \\ p_2 &= WTP(1,2) + \epsilon, \text{ and} \\ p_3 &= WTP(1,2). \end{aligned}$$

Note that the equilibrium which we derive in Example 9 can be excluded by iterated deletion of weakly dominated strategies. Firm 3 seems to play a rather foolish strategy in the sense that it is bidding much more that it would be willing to bid in order to prevent firm 2 from winning the license, but it does not play a dominated strategy. This follows from the fact that firm 3's willingness-to-pay against firm 4 is higher than its equilibrium bid. However, all firm 4's bids above zero are weakly dominated by a bid of zero, so that firm 3's strategy is deleted in the second iteration of deletion of weakly dominated strategies.

Example 9 and Propositions 4 and 8 together justify the following conjecture.

Conjecture 10 *Suppose (11) holds true. Then in any Nash equilibrium of the first-price sealed-bid auction that survives iterated deletion of weakly dominated strategies, the largest firm wins the license.*

Proposition 11 shows that, in contrast to the situation with constant total market profits, there is no (almost) optimal auction in which another firm than firm 1 wins the license. The proof follows the following lines. The maximal payment a firm j is willing to make given that firm i wins is given by $U_j(i) - U_j(w(j))$, so that the maximal revenue for the seller, under the restriction that firm i wins the license, equals $\sum_{j=1}^{n} [U_j(i) - U_j(w(j))]$. We show that the highest possible revenue given that firm 1 wins is strictly higher than the highest possible revenue given that any other firm wins. We finish the proof by showing that take-it-or-leave-it mechanism M^1, which is defined above and which assigns the license to firm 1, is almost revenue maximizing.

Proposition 11 *Suppose (11) holds true. Only for $i = 1$ there exists a feasible auction mechanism that (1) assigns firm i the license, (2) is an (almost) optimal auction. The maximal expected revenue equals*

$$P(\bar{\mathbf{m}} + \mathbf{e}_1) - \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)}). \tag{13}$$

where

$$w(j) \in \arg\min_{k \neq j} U_j(k)$$

denotes the firm that imposes the "worst threat" on firm j.

Proof. The highest expected revenue the seller can obtain given that firm $i \neq 1$ wins the license, is given by

$$
\begin{aligned}
\sum_{j=1}^{n}[U_j(i) - U_j(w(j))] &= \sum_{j=1}^{n}[\Pi_j(\bar{\mathbf{m}} + \mathbf{e}_i) - \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)})] \\
&= \sum_{j=1}^{n}[S_j(\bar{\mathbf{m}} + \mathbf{e}_i) * P(\bar{\mathbf{m}} + \mathbf{e}_i) - \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)})] \\
&= P(\bar{\mathbf{m}} + \mathbf{e}_i) - \sum_{j=1}^{n}\Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)}) \\
&< P(\bar{\mathbf{m}} + \mathbf{e}_1) - \sum_{j=1}^{n}\Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)}) \\
&= \sum_{j=1}^{n}[U_j(1) - U_j(w(j))].
\end{aligned}
$$

The last inequality follows by the assumptions on P. Therefore, the optimal auction assigns firm 1 the license. The only thing that is left to check is whether an auction exists which is almost optimal, and assigns firm 1 the license. Consider take-it-or-leave-it mechanism M^1, which is defined in the proof of Proposition 5. In this auction, each firm has a strictly dominant strategy to choose "participate", and if they do so, firm 1 is the winner of the license. The revenue of M^1 is equal to

$$\sum_{j=1}^{n}[U_j(1) - U_j(w(j))] - n\epsilon,$$

so that M^1 is an almost optimal auction in which firm 1 wins the license. ■

For generic P, the α-core need not be empty. This is established by Example 12, in which the α-core is not empty, and in which we show how an element in the α-core can be established by a take-it-or-leave-it offer from the seller to firm 1.

Example 12 *There are two firms in the market. In the status quo situation, firm 1 has 2 outlets, i.e.,* $m_1 = 2$, *and firm 2 has 1 outlet, i.e.,* $m_2 = 1$. *Suppose that* f *is given by*

$$f(k) = k^2 \text{ for all } k \in \aleph.$$

The status quo total market profit is 1, i.e.,

$$P(2,1) = 1.$$

Suppose that if firm 1 wins the license, total market profit grows to 2, i.e.,

$$P(3,1) = 2.$$

and that total market profit remains unchanged if firm 2 wins the license, i.e.,

$$P(2,2) = 1.$$

With these parameters, we establish the following utilities.

$$\begin{aligned} U_1(1) &= 1, \\ U_2(1) &= 0, \\ U_1(2) &= -\frac{3}{10}, \text{ and} \\ U_2(2) &= \frac{4}{10}. \end{aligned}$$

The characteristic function of the related α*-game is then defined as follows*

$$\begin{aligned} v(0) = v(2) = v(1,2) &= 0 \\ v(0,1) = v(0,1,2) &= 1 \\ v(1) &= -\frac{3}{10} \\ v(0,2) &= \frac{4}{10}. \end{aligned}$$

The α-core is not empty. It is readily checked that the α-core is given by all vectors $(x_0, x_1, x_2) \in \Re^3$ with

$$\begin{aligned} x_0 + x_1 &= 1, \\ x_0 &\geq \frac{4}{10}, \\ x_1 &\geq 0, \text{ and} \\ x_2 &= 0. \end{aligned}$$

One element of the α-core is $(1 - \epsilon, \epsilon, 0)$. This outcome is for instance established with a take-it-or-leave-it offer from the seller to firm 1 to buy the license for a price equal to $1 - \epsilon$, where it is in firm 1's interest to accept the offer.

5 Concluding remarks

The analysis has confirmed the conjecture of the MDW-study group advising the Dutch government that a standard auction will lead to an increase of market concentration rather than a decrease the Dutch government aims at, so that the outcome of the auction is inefficient. The inefficiency is caused by the fact that the consumers do not participate in the auction (see also Jehiel and Moldovanu, 2000a). Of course, the issue of increasing market concentration due to auctions of licenses is not restricted to the petrol sector. For instance, participating in the UMTS auctions that took place in Europe in 2000 and 2001 seemed to be much more interesting for incumbents than for entrants (Jehiel and Moldovanu, 2000b; Klemperer, 2001; Van Damme, 2001).

The assumption of a constant pie may not be as unrealistic as it seems. In the Dutch petrol market, the firms developed an interesting way of coordinating on high petrol prices. When Shell, the market leader, decides to change its price, it announces this in the press. Within a few days, all the other firms follow Shell's example. For each firm, this coordination is very profitable as it leads to high prices, and there is no reason to believe that firms will deviate from this coordination in case the market structure changes somewhat. Therefore, independent of which firm wins the license, total market profits will not be affected.

Example 12 may cast some doubt on the usefulness of the α-core as a concept for strategy proofness of feasible auction mechanisms. More specifically, we would argue that an *empty* α-core does indicate collusion-proofness, but a *non-empty* α-core needs not imply that auctions are not collusion proof. In Example 12, none of the standard coalition agreements against the seller seem to work against the proposed take-it-or-leave-it offer of the seller, in which he is able to extract almost the entire surplus of firm 1: It is not in firm 1's interest to reject the take-it-or-leave-it offer; firm 2 is not willing to compensate firm 1 for rejecting the offer; firm 2 is not willing to bribe the seller not to sell the license to firm 1; it is not in the interest of the firms to form a cartel against the seller, in which they agree that firm 1 does not accept the offer; and so forth. In fact, the non-emptiness of the α-core indicates the stability of the agreement between the seller and firm 1. In contrast, such an agreement would not be stable in the case of an empty α-core. In that case we have to make strong assumptions on the commitment of the seller to the auction mechanism he chooses to transfer the license to one of the firms.

A similar reason that the α-core may not be an adequate concept for strategy proofness follows from the discussion in the paper of McAfee and McMillan (1992) on the formation of cartels in auctions. It is easily shown that the α-core is not empty in auction models in which no externalities are assumed. Still, in this setting, McAfee and McMillan assume the possibility of cartels of firms that cooperate against the seller. They rely on the assumption that there is an enforcement mechanism that forces the cartel members not to deviate from the cartel agreement. Deviators may be directly punished, or indirectly through grim-trigger strategies. As preventing collusive behavior is one of the major issues in auction design (Cramton and Schwartz, 1999; Klemperer, 2001), future research should lead to the construction of a better measure for the stability of collusion against the seller.

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