PAPER

Superposition Signal Input Decoding for Lattice Reduction-Aided MIMO Receivers

Satoshi DENNO[†a)], ***Senior Member***, **Koki KASHIHARA**[†], ***Nonmember***, ***and*** **Yafei HOU**[†], ***Senior Member***

SUMMARY This paper proposes a novel approach to low complexity soft input decoding for lattice reduction-aided MIMO receivers. The proposed approach feeds a soft input decoder with soft signals made from hard decision signals generated by using a lattice reduction-aided linear detector. The soft signal is a weighted-sum of some candidate vectors that are near by the hard decision signal coming out from the lattice reduction-aided linear detector. This paper proposes a technique to adjust the weight adapt to the channel for the higher transmission performance. Furthermore, we propose to introduce a coefficient that is used for the weights in order to enhance the transmission performance. The transmission performance is evaluated in a 4×4 MIMO channel. When a linear MMSE filter or a serial interference canceller is used as the linear detector, the proposed technique achieves about 1.0 dB better transmission performance at the BER of 10^{-5} than the decoder fed with the hard decision signals. In addition, the low computational complexity of the proposed technique is quantitatively evaluated.

key words: *soft decision, weighted sum, lattice reduction, linear detectors, low computational complexity*

1. Introduction

Transmission speed of wireless communications has been raised to several Gbps to comply with the demand that higher quality services should be provided in wireless networks. The fifth generation cellular system provides users with several Gbps wireless communication, for instance. Many techniques have been introduced for achieving such high speed wireless communications, such as orthogonal frequency division multiplexing (OFDM), adaptive modulation and coding (AMC), error correction coding, and multi-antenna techniques. Among those techniques, multi-input-multi-output (MIMO) spatial multiplexing, one of multi-antenna techniques, has played a main role in increasing the transmission speed of wireless communication systems. The MIMO spatial multiplexing is regarded as a key technique to achieve higher speed wireless transmissions even in the sixth generation cellular system. Although many techniques have been proposed for MIMO spatial multiplexing, linear detectors have been widely applied in those systems because of their low computational complexity. Because linear detectors can not achieve the optimum performance that the maximum likelihood detection attains, many techniques have been considered for the performance improvement. One of them is the lattice reduction that makes linear detectors achieve higher diversity gains with small additional computational complexity [1]–[4]. However, the lattice reduction has linear detector output signals transformed to hard decision signals. On the other hand, channel coding is one of the techniques to improve the transmission performance. Especially, soft input decoding attains better transmission performance than hard input decoding. The log likelihood ratio (LLR) is usually applied as soft signals to soft input decoders [5]. Although channel decoders attain the optimum performance with the input of the LLR, because high computational cost is necessary to calculate the LLR, MAX-log approximation has been widely applied. The LLR calculation still requires high computational complexity even if the MAX-log approximation is used, because the calculation is implemented with the brute force search. Some techniques have been proposed to reduce the computational cost of the LLR calculation with assistance of lattice reduction-aided linear receivers [6]–[11]. They shrink the vector space searched by the brute force search, which reduces the computational cost. This approach has been extended to iterative receivers [12]. Though those techniques achieve superior transmission performance with less computational complexity than the original technique, the transmission performance is easily degraded as the vector space is a little bit too shrunk.

This paper proposes a novel approach to soft input decoding, which is completely different from the conventional approach described above. The proposed approach generates some vectors near by a linear detector output signal vector, which are summed with appropriated weights. The weighted sum is fed to the channel decoder as a soft input signal. We propose a technique to adjust the weights adapt to the channels, which makes the noise power in the soft input signal equal to that in the linear detector output signal. While most channel decoders achieve the optimum decoding performance with input signals that are distributed with the Gaussian distribution, actually, the soft input signals are not Gaussian distributed, which degrades the performance even though the weights are adjusted. To mitigate performance degradation, we introduce a coefficient that is used to modify the adjusted weights for the transmission performance enhancement.

Next section describes a system model, and the proposed technique is introduced in Sect. 3. The performance of the proposed technique is confirmed in Sect. 4, and Sect. 5 remarks conclusions.

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[†]The authors are with Graduate School of Natural Science and Technology, Okayama University, Okayama-shi, 700-8530 Japan.
a) E-mail: denno@okayama-u.ac.jp
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Throughout the paper, $\mathrm{E}[\zeta]$, j, and c^* represent the ensemble average of a variable ζ, the imaginary unit, and complex conjugate of a complex number c. $\Re[c]$ and $\Im[c]$ represent a real part and an imaginary part of a complex number c, respectively. Superscript T and H indicate transpose and Hermitian transpose of a matrix or a vector, respectively. In addition, $\mathbf{A}^{-1}$, $\mathbf{A}^{-\mathrm{H}}$, $\mathbf{A}_m$ and $\mathbf{A}_{(k,n)}$ indicate an inverse matrix of a matrix $\mathbf{A}$, that of a matrix $\mathbf{A}^{\mathrm{H}}$, an mth column vector and a (k, n) element of a matrix $\mathbf{A}$, respectively.

2. System Model

We assume a wireless link where a transmitter with N_{T} antennas sends spatially multiplexed signal streams for a receiver with N_{R} antennas without any precoding. The number of the signal streams is the same to that of the transmit antennas. The information bit stream is encoded with a convolutional code, the encoded bit stream is provided to modulators via an interleaver. The modulator output signals are transmitted from the antennas. The transmitted signals are received at the receiver. We apply linear detectors to the receiver, such as MMSE filters and serial interference cancellers (SICs). Because the performance of linear receivers is inferior to that of the optimum receiver, i.e., the maximum likelihood estimation (MLD), we apply the lattice reduction to those detectors for performance improvement. Let $\mathbf{X} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ denote a transmission signal vector, if we use the LLL algorithm to implement the lattice reduction [13], a received signal vector $\mathbf{Y} \in \mathbb{C}^{N_{\mathrm{R}}\times 1}$ can be written with a unimodular matrix $\mathbf{T} \in \mathbb{C}^{N_{\mathrm{T}}\times N_{\mathrm{T}}}$ as,

$$\begin{aligned}\mathbf{Y} &= \hat{\mathbf{H}}\mathbf{X} + \mathbf{N} = \hat{\mathbf{H}}\mathbf{T}\,\mathbf{T}^{-1}\mathbf{X} + \mathbf{N}\\ &= \mathbf{H}\mathbf{Z} + \mathbf{N}\end{aligned} \tag{1}$$

$\hat{\mathbf{H}} \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{T}}}$ and $\mathbf{N} \in \mathbb{C}^{N_{\mathrm{R}}\times 1}$ in (1) denote a channel matrix between the transmitter and the receiver and an additive white Gaussian noise vector. In addition, $\mathbf{H} \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{T}}}$ and $\mathbf{Z} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ represent a transformed channel matrix and a transformed transmission signal vector defined as $\mathbf{H} = \hat{\mathbf{H}}\mathbf{T}$ and $\mathbf{Z} = \mathbf{T}^{-1}\mathbf{X}$, respectively. In this paper, the unimodular matrix is calculated based on the LLL algorithm as follows.

$$\begin{pmatrix}\hat{\mathbf{H}}\\ \frac{\sqrt{2}\sigma}{\sigma_d}\mathbf{I}_{N_{\mathrm{T}}}\end{pmatrix}\mathbf{T} = \mathbf{Q}\mathbf{R} \tag{2}$$

In (2), $\sigma \in \mathbb{R}$, $\sigma_d \in \mathbb{R}$, $\mathbf{I}_{N_{\mathrm{T}}} \in \mathbb{C}^{N_{\mathrm{T}}\times N_{\mathrm{T}}}$, $\mathbf{Q} \in \mathbb{C}^{(N_{\mathrm{R}}+N_{\mathrm{T}})\times N_{\mathrm{T}}}$, and $\mathbf{R} \in \mathbb{C}^{N_{\mathrm{T}}\times N_{\mathrm{T}}}$ represent a standard deviation of the AWGN, amplitude of the modulation signals, the N_{T} dimensional identity matrix, an orthogonal matrix, and an upper triangular matrix. The orthogonal matrix satisfies the following, $\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{I}_{N_{\mathrm{T}}}$. The linear detectors estimate the transformed vector $\mathbf{Z}$ from the received signal vector, which is provided to a slicer. Let $\hat{\mathbf{Z}} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ denote an estimated signal vector by the linear detectors, the slicer outputs a hard decision signal vector $\bar{\mathbf{Z}}_0 \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$, which is defined as $\bar{\mathbf{Z}}_0 = \lfloor\hat{\mathbf{Z}}\rceil$ where $\lfloor\bullet\rceil$ indicates the floor function that searches possible nearest integer to the input. The transmission signal vector is estimated as $\bar{\mathbf{X}} = \mathbf{T}\bar{\mathbf{Z}}_0$

As is described above, when the LLL algorithm is used for the performance improvement, the slicer has to be applied to detector output signals. The output signals from the slicer are regarded as hard decision signals. We propose a low complexity technique that converts the hard decision signals into the soft signals for achieving a higher coding gain.

3. Superposition of Hard Decision Vectors as Soft Signals

When the AWGN is added in the system shown in (1), the AWGN is also included in the output signal vector from the linear detector. If the AWGN is big, the output signals are less reliable. When we believe that the hard decision signal vector $\bar{\mathbf{Z}}_0 \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ is the same to the transmission signal transformed with the unimodular matrix, we can define a reliability of the hard decision vector $\bar{\mathbf{Z}}_0$ in the following equation.

$$P(0) = \exp\left(-\frac{|\hat{\mathbf{Z}} - \bar{\mathbf{Z}}_0|^2}{2\sigma_{\mathrm{p}}^2}\right) = \prod_{n=1}^{N_{\mathrm{T}}}\exp\left(-\frac{|\hat{z}(n) - \bar{z}_0(n)|^2}{2\sigma_{\mathrm{p}}^2}\right) \tag{3}$$

In (3), $P(0) \in \mathbb{R}$, $\hat{z}(n) \in \mathbb{C}$, and $\bar{z}_i(n) \in \mathbb{C}$ represent the reliability of the vector $\bar{\mathbf{Z}}_0$, an nth entries of the vector $\hat{\mathbf{Z}}$ and $\bar{\mathbf{Z}}_0$, which are defined as $\hat{\mathbf{Z}} = (\hat{z}(1)\cdots\hat{z}(N_{\mathrm{T}}))^{\mathrm{T}}$ and $\bar{\mathbf{Z}}_0 = (\bar{z}_0(1)\cdots\bar{z}_0(N_{\mathrm{T}}))^{\mathrm{T}}$. In addition, $\sigma_{\mathrm{p}}^2 \in \mathbb{R}$ denotes an equivalent variance, which is defined in the following section.

If the noise is bigger than half of the minimum Euclidean distance in the modulation signal constellation, the linear detector output signal will not be correct and one of the other signals will be correct. Since $\bar{\mathbf{Z}}_0$ is the hard decision signal vector provided by the slicers, the other vector can be defined as,

$$\bar{\mathbf{Z}}_m = \bar{\mathbf{Z}}_0 + \Delta\mathbf{Z}_m. \tag{4}$$

$\bar{\mathbf{Z}}_m \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ and $\Delta\mathbf{Z}_m \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ in (4) denote an mth candidate vector and an mth difference vector which is defined below. Since the modulation signals are designed to locate on the integer lattice, the element of those vectors are Gaussian integers. Let $b_m(l) \in \mathbb{N}$ be defined as $0 \le b_m(l) \le 2N_{\mathrm{T}} - 1$ where l represents an index of the non-zero term in the mth difference vector $\Delta\mathbf{Z}_m$[†], a variable $\zeta(m) \in \mathbb{N}$ is defined as follows.

$$\zeta(m) = \sum_{l=0}^{N_{\mathrm{L}}-1} b_m(l)\,(2N_{\mathrm{T}})^l \tag{5}$$

$$\text{subject to}\begin{cases} b_m(l_1) \ne b_m(l_2) & l_1 \ne l_2\\ b_m(l_1) < b_m(l_2) & l_1 < l_2\end{cases}$$

N_{L} represents the number of the non-zero terms in the real and the imaginary parts of the vector $\Delta\mathbf{Z}_m$. $b_m(l_1)$ expresses a location of the l_1th non-zero element in the mth difference

[†]The index l ranges as $0 \le l \le N_{\mathrm{L}} - 1$ where N_{L} is defined afterwards.

vector $\Delta\mathbf{Z}_m$. In addition, when we define a set that contains all the indexes $\zeta(m)$, the m_1th biggest element in the set is denoted by $\zeta(m_1)$. For example, if the real part and the imaginary part of the nth entry in the vector $\Delta\mathbf{Z}_m$ are only non-zero values, we will regard that 2 non-zero terms are included in the vector $\Delta\mathbf{Z}_m$. In other word, while the index m denotes the index that specifies the candidate vector, $\zeta(m)$ defines the vector explicitly with $b_m(l)$ as follows.

Let $a_m(i) \in \mathbb{N}$ and $c_{l,m}(i) \in \mathbb{N}$ $i = 0 \cdots, N_\mathrm{T} - 1$, $l = 0, \cdots, N_\mathrm{L} - 1$ indicate integers taking 0 or 1, the nth element of the vector $\Delta\mathbf{Z}_m$ is defined as follows.

$$\begin{aligned} c_{l,m}(i) &= \delta(b_m(l) - i) \\ a_m(i) &= \sum_{l=0}^{N_\mathrm{L}-1} c_{l,m}(i) \\ n &= \mathrm{int}\left(\frac{i}{2}\right) + 1 \\ \Delta z_m(n) &= (r_{2n-1} a_m(2n-1) + \mathrm{j} \cdot r_{2n-2} a_m(2n-2))\,\Lambda \end{aligned} \tag{6}$$

In (6), $\Delta z_m(n) \in \mathbb{C}$, $\mathrm{int}(\alpha)$, and $\delta()$ denote the nth element of the vector $\Delta\mathbf{Z}_m$, a function to output integer part of the input $\alpha \in \mathbb{R}$, and the Kronecker's delta function[†]. In addition, $\Lambda \in \mathbb{N}$ and $r_n \in \mathbb{N}$ represent a Euclidean distance between the possible modulation signals, and a random coefficient, i.e., $r_n = \pm 1$. As is shown in (6), $a_m(i)$ expresses a location of a non-zero term in the difference vector $\Delta\mathbf{Z}_m$. In other words, If $a_m(2n-1)$ is 1, the real part of the nth element in the difference vector $\Delta\mathbf{Z}_m$ is set to the non-zero term. In this paper, we assume that the candidate signal vectors coming from the detectors are only neighbor to the transmission signal vector in spite of the AWGN variance. In a word, the distance Λ is set to the minimum Euclidean distance between the possible adjacent modulation signals.

This paper propose a soft signal vector that is a weighted sum of the candidate signal vectors with the weight of the reliability as,

$$\underline{\mathbf{Z}} = \sum_{m=0}^{N_\mathrm{S}-1} P(m)\,\bar{\mathbf{Z}}_m \tag{7}$$

In (7), $N_\mathrm{S} \in \mathbb{N}$, $\underline{\mathbf{Z}} \in \mathbb{C}^{N_\mathrm{T}\times 1}$, and $P(m)$ denote the number of the candidate vectors, the soft signal vector, and the reliability defined in (3) where $\bar{Z}_m$ and $P(m)$ replace $\bar{Z}_0$ and $P(0)$, respectively. The soft signal vector can be uniquely obtained when the equivalent noise variance σ_p^2 is given. We propose the equivalent noise valiance estimation technique in the following section.

3.1 Equivalent Noise Variance

If we believe that the linear detector output hard decision signal vector $\bar{\mathbf{Z}}_0$ is correct, the noise power in the soft signal vector $\underline{\mathbf{Z}}$ can be defined as follows.

$$\begin{aligned} e_\mathrm{Z}\left(\sigma_\mathrm{p}^2\right) &= \mathrm{E}\left[\left|\underline{\mathbf{Z}} - \gamma\left(\sigma_\mathrm{p}^2\right)\bar{\mathbf{Z}}_0\right|^2\right] = \mathrm{E}\left[\left|\sum_{m=1}^{N_\mathrm{S}-1} P(m)\left(\bar{\mathbf{Z}}_m - \bar{\mathbf{Z}}_0\right)\right|^2\right] \\ &\simeq \sum_{m=1}^{N_\mathrm{S}-1} \mathrm{E}[P(m)^2]\,|\Delta\mathbf{Z}_m|^2 \end{aligned} \tag{8}$$

In the above equation, $e_\mathrm{Z}\left(\sigma_\mathrm{p}^2\right) \in \mathbb{R}$ and $\gamma\left(\sigma_\mathrm{p}^2\right) \in \mathbb{R}$ represent the noise power in the soft signal vector and the average amplitude of the hard decision signals $\bar{\mathbf{Z}}_0$, which is defined as,

$$\gamma\left(\sigma_\mathrm{p}^2\right) = \left(\sum_{m=0}^{N_\mathrm{S}-1} P(m)\right) \tag{9}$$

In the derivation of (8), we use the assumption that the difference vectors $\Delta\mathbf{Z}_m$ $m = 0, \cdots, N_\mathrm{S}-1$ are uncorrelated with each other as is defined in (6). As is described above, the linear detector output signal $\hat{z}(m)$ consists of the transmission signals and the AWGN. Therefore, the ensemble average of square of the reliability $P(m)$ can be calculated in (10). In (10), $G^{-1}(m)$ denotes a amplitude of the AWGN in the mth element of the detector output vector $\hat{\mathbf{Z}}$.

As the number of the candidate vectors N_S increases, the complexity of the proposed technique becomes higher. As is shown in (6), the number of the candidates vector index m gets higher as the number of the non-zero terms N_L increases. For low computational complexity, the number of the non-zero terms N_L in the vector is restricted to be less than 3 in this paper, i.e., $N_\mathrm{L} \le 2$. Because the real part and the imaginary part are dealt as the independent terms, in other words, two terms are at most not zero in all the terms, $\Re[\Delta z_m(1)]$, $\Im[\Delta z_m(1)]$, $\Re[\Delta z_m(2)]$, $\cdots$, $\Im[\Delta z_m(N_\mathrm{T})]$. If the restriction is applied, the noise power $e_\mathrm{Z}(\sigma_\mathrm{p}^2)$ can be derived theoretically as shown in (11).

Channel decoders achieves the optimum performance with soft input signals in the AWGN channel. We can expect that the best decoding performance can be achieved by making the soft signals have the similar characteristics of received signals in the AWGN channel. When linear filters are employed in MIMO channels, however, the signal power to noise power ratio (SNR) of some detector output signal streams could become much lower than that of the other signal streams, which degrades the overall transmission performance. The lattice reduction can almost equalize the SNR of all the signal streams, i.e., all the element of the vector $\hat{\mathbf{Z}}$. The proposed decoding makes the soft signal vectors $\underline{\mathbf{Z}}$ have the SNR performance of the estimated signal vector $\hat{\mathbf{Z}}$.

For the aim, the proposed technique makes the noise power $e_\mathrm{Z}\left(\sigma_\mathrm{p}^2\right)$ equal to the noise power in the estimated signal vector $\hat{\mathbf{Z}}$ by adjusting the equivalent variance σ_p^2.

$$\bar{\sigma}_\mathrm{p}^2 = \arg\min_{\sigma_\mathrm{p}^2}\left[\left|J(\mathbf{H}) - e_\mathrm{Z}\left(\sigma_\mathrm{p}^2\right)\right|^2\right] \tag{12}$$

$J(\mathbf{H}) \in \mathbb{R}$ in (12) indicates the noise power in the estimated signal vector $\hat{\mathbf{Z}}$ with a channel matrix $\mathbf{H}$. Although

[†]The Kronecker's delta function $\delta(n)$ is defined as,
$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}.$$

$$\begin{aligned}
\mathrm{E}[P(i)^2] &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\prod_{n=1}^{N_\mathrm{T}}\exp\left(-\frac{|\hat{z}(n)-\bar{z}_i(n)|^2}{\sigma_\mathrm{p}^2}\right)P(t_1,\cdots,t_{2N_\mathrm{T}})\,\mathrm{d}t_1\cdots\mathrm{d}t_{2N_\mathrm{T}} \\
&= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\prod_{n=1}^{N_\mathrm{T}}\exp\left(-\frac{\left|\frac{t_{2n-1}}{G(n)}-\Re\left[\Delta z_i(n)\right]\right|^2+\left|\frac{t_{2n}}{G(n)}-\Im\left[\Delta z_i(n)\right]\right|^2}{\sigma_\mathrm{p}^2}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{2N_\mathrm{T}}\prod_{n=1}^{2N_\mathrm{T}}\exp\left(-\frac{t_n{}^2}{2\sigma^2}\right)\mathrm{d}t_1\cdots\mathrm{d}t_{2N_\mathrm{T}} \\
&= \prod_{n=1}^{N_\mathrm{T}}\exp\left[-\frac{\left(\left|\Re\left[\Delta z_i(n)\right]\right|^2+\left|\Im\left[\Delta z_i(n)\right]\right|^2\right)G^2(n)}{2\sigma^2+\sigma_\mathrm{p}^2}\right]\left(\frac{\sigma_\mathrm{p}^2G^2(n)}{2\sigma^2+\sigma_\mathrm{p}^2G^2(n)}\right) \qquad (10)
\end{aligned}$$

$$\begin{aligned}
\sum_{m=1}^{N_\mathrm{S}-1}E[P(m)^2]\,|\Delta\mathbf{Z}_m|^2 &= \left\{3\sum_{n=1}^{N_\mathrm{T}}\Lambda^2\exp\left[-\frac{\Lambda^2G^2(n)}{2\sigma^2+\sigma_\mathrm{p}^2G^2(n)}\right]\right. \\
&\left.+4\sum_{n_1<n_2)}^{N_\mathrm{T}}\Lambda^2\exp\left[-\frac{\Lambda^2G^2(n_1)}{2\sigma^2+\sigma_\mathrm{p}^2G^2(n_1)}-\frac{\Lambda^2G^2(n_2)}{2\sigma^2+\sigma_\mathrm{p}^2G^2(n_2)}\right]\right\}\prod_{n=1}^{N_\mathrm{T}}\left(\frac{\sigma_\mathrm{p}^2G^2(n)}{2\sigma^2+\sigma_\mathrm{p}^2G^2(n)}\right) \qquad (11)
\end{aligned}$$

the minimization should be achieved by the stochastic gradient method for accelerating the minimization, we apply dichotomizing search algorithm for the minimization, because the differential of the error $e_\mathrm{Z}\left(\sigma_\mathrm{p}^2\right)$ with respect to σ_p^2 is a little bit complex. Because the power $e_\mathrm{Z}\left(\sigma_\mathrm{p}^2\right)$ increases monotonically as the equivalent power variance σ_p^2 becomes higher, the optimum σ_p^2 can be found easily without being trapped by local minima.

3.1.1 Soft Decoding with LR-MMSE

While the proposed soft decoding can be applied to any types of lattice reduction-aided linear detectors, we show a configuration of the proposed soft decoding with a lattice reduction-aided minimum means square (MMSE) filter (LR-MMSE). As is well known, an MMSE weight matrix $\mathbf{W}\in\mathbb{C}^{N_\mathrm{R}\times N_\mathrm{T}}$, an estimated signal vector $\hat{\mathbf{Z}}$ from the linear detector, and a hard decision signal vector $\bar{\mathbf{Z}}_0$ are obtained as follows.

$$\begin{aligned}
\mathbf{W} &= \mathbf{H}\left(\mathbf{R}^\mathrm{H}\mathbf{R}\right)^{-1} \\
\hat{\mathbf{Z}} &= \mathbf{W}^\mathrm{H}\mathbf{Y} \\
\bar{\mathbf{Z}}_0 &= \lfloor\hat{\mathbf{Z}}\rceil \qquad (13)
\end{aligned}$$

The noise power in the estimated signal vector is shown in the following.

$$\begin{aligned}
J(\mathbf{H}) &= \mathrm{E}\left[\left|\mathbf{Z}-\mathbf{W}^\mathrm{H}\mathbf{Y}\right|^2\right] \\
&= \sigma_\mathrm{d}^2\,\mathrm{tr}\left[\mathbf{T}^{-1}\mathbf{T}^{-\mathrm{H}}(\mathbf{I}_{N_\mathrm{T}}-\mathbf{W}^\mathrm{H}\mathbf{H})\right] \qquad (14)
\end{aligned}$$

The term $G(n)$ has to be set as,

$$G(n) = \frac{\sigma^2}{\sigma_\mathrm{d}^2\left(\mathbf{T}^{-1}\mathbf{T}^{-\mathrm{H}}\left(\mathbf{I}_{N_\mathrm{T}}-\mathbf{W}^\mathrm{H}\mathbf{H}\right)\right)_{n,n}}. \qquad (15)$$

3.1.2 Soft Decoding with LR-SIC

We show a configuration of the proposed soft decoding applied with a lattice reduction-aided serial interference canceller (LR-SIC). An SIC transforms the received signal vector with an orthogonal matrix $\mathbf{Q}$ defined in (2). The transmission signals are detected serially from the transformed received signal $\hat{\mathbf{Y}}\in\mathbb{C}^{N_\mathrm{T}\times 1}$, which is written in the following.

$$\begin{aligned}
\hat{\mathbf{Y}} &= \mathbf{Q}^\mathrm{H}\mathbf{Y} = \mathbf{R}\mathbf{Z}+\mathbf{Q}^\mathrm{H}\mathbf{N} \\
\hat{z}(n) &= \frac{\hat{y}(n)-\sum_{i=n+1}^{N_\mathrm{T}}R(n,i)\bar{z}_0(i)}{R(n,n)} \\
\bar{z}_0(n) &= \lfloor\hat{z}(n)\rceil \quad n = N_\mathrm{T},\cdots,1 \qquad (16)
\end{aligned}$$

In (16), $\hat{y}(n)\in\mathbb{C}$ and $R(n,i)\in\mathbb{C}$ denote the nth entry of the vector $\hat{\mathbf{Y}}$ and the (n,i) entry of the matrix $\mathbf{R}$. If we neglect the error propagation in the SIC, the power of the AWGN included in output signals from the SIC can be obtained as,

$$\begin{aligned}
J(\mathbf{H}) &= \mathrm{E}\left[\left|\mathbf{Z}-\hat{\mathbf{Z}}\right|^2\right] \\
&= \sum_{n=1}^{N_\mathrm{T}}\frac{2\sigma^2}{|R(n,n)|^2} \qquad (17)
\end{aligned}$$

Hence, the term $G(n)$ has to be set as,

$$G(n) = |R(n,n)| \qquad (18)$$

3.2 Coefficient for Equivalent Variance

As is described above, soft input decoders achieve the optimum decoding performance when the soft input signal is distributed with the Gaussian distribution. Actually, as is shown below, the distribution of the soft input signals is a little bit different from the Gaussian distribution in the proposed soft decoding. The distribution of the soft input signals is wider than the Gaussian distribution even though the variance of the Gaussian distribution is exactly the same to the noise power. To mitigate the performance degradation due to the non-Gaussian distribution, we introduce a coefficient κ (≤ 1) which is multiplied with the equivalent variance σ_{p}^2 to get the distribution narrower. The optimum coefficient κ is searched through computer simulation.

4. Simulation

The proposed soft decoding is evaluated by computer simulation in a MIMO channel where a transmitter with 4 antennas sends spatially multiplexed signal streams to a receiver with 4 antennas, i.e., $N_{\mathrm{T}} = N_{\mathrm{R}} = 4$. Quaternary phase shift keying (QPSK) and a half rate convolutional code with constraint length of 3 are employed. Independent identically distributed (i. i. d.) channel is applied, and every channel is modeled with Rayleigh fading based on Jakes' model [14]. As is explained, the number of the non-zero terms in the vector $\Delta\mathbf{Z}_m$ is at most 2, i.e., $N_{\mathrm{L}} \leq 2$. When the number of the non-zero terms N_{L} is 1, i.e., $N_{\mathrm{L}} = 1$, the number of the candidates vectors is $8\,(= {}_8\mathrm{C}_1)$. If the number of the non-zero terms is increased to 2, the number of the candidates vectors becomes $28\,(= {}_8\mathrm{C}_2)$. Hence, if the number of the non-zero terms is less than 3, $N_{\mathrm{L}} \leq 2$, the number of the candidate vectors except for the filter output vector $\bar{\mathbf{Z}}_0$ is 36, i.e., $N_{\mathrm{S}} = 36$. When the number of the non-zero terms is 1, $N_{\mathrm{L}} = 1$, the number of the candidate vectors except for the filter output vector $\bar{\mathbf{Z}}_0$ is 8, i.e., $N_{\mathrm{S}} = 8$. Simulation parameters are listed in Table 1.

Table 1 Parameters in computer simulation.

Channel model	Rayleigh fading
Modulation	QPSK / Single carrier
Number of antennas ($N_{\mathrm{T}}, N_{\mathrm{R}}$)	$(4, 4)$
Detector	MMSE & SIC
Lattice reduction	LLL ($\delta = 0.75$)
Block interleaver size	96×68
Number of candidate vectors N_{S}	8, 36
Forward error collection	Convolutional code ($K = 3, R = 1/2$)
Decoder	Soft input Viterbi algorithm

4.1 BER Performance

Figure 1 shows the BER performance of the proposed soft decoding. In the figure, "MMSE" and "LR-MMSE" indicate the performance of the soft decoding with the soft signal vector from the MMSE and that of the hard decoding with the hard decision signal vector from the LR-MMSE, respectively. The soft decoding with the MMSE are much inferior to the hard decoding with the LR-MMSE. In a word, the lack of the lattice reduction causes severe performance degradation, which exceeds the performance gain given by the soft decoding. The performance gain of the proposed soft decoding with $N_{\mathrm{S}} = 8$ and that with $N_{\mathrm{S}} = 36$ are about 0.7 dB and 1.0 dB at the BER of 10^{-5}, respectively.

Figure 2 shows the BER performance of the proposed soft decoding where the LR-SIC is applied as a linear detector. In the figure, "SIC" and "LR-SIC" indicate the performance of the soft decoding with the soft signals $\hat{z}(m)$ from the SIC defined in (16) and that of the hard decoding with the hard decision signals $\bar{z}_0(m)$ from the LR-SIC, respectively. The lack of the lattice reduction also deteriorates the performance even when the SIC is deployed. The BER performance of the proposed soft decoding with $N_{\mathrm{S}} = 8$ is almost the same to that with $N_{\mathrm{S}} = 36$. The proposed soft decoding achieves a gain of about 1.0 dB at the BER of 10^{-5}.

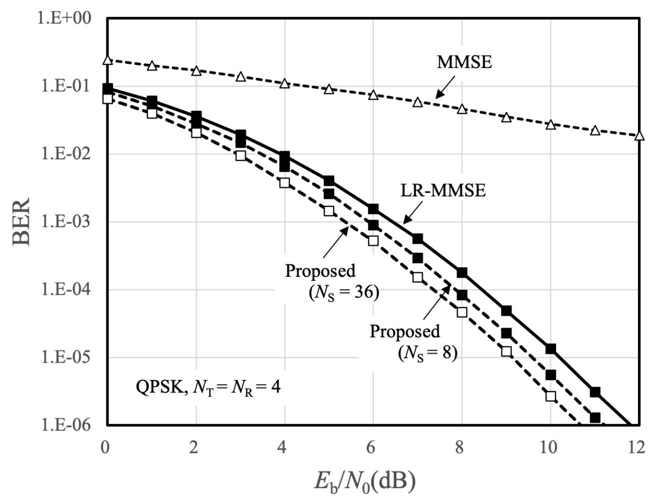

Fig. 1 BER performance of proposed soft decoding with MMSE.

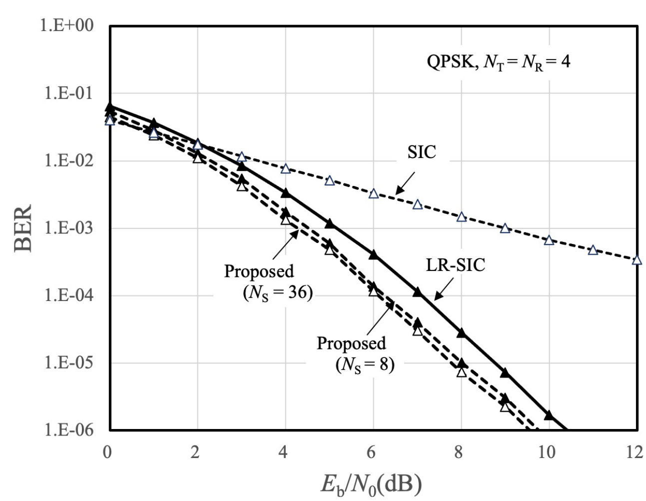

Fig. 2 BER performance of proposed soft decoding with SIC.

4.2 Distribution of Detector Output Signals

Figure 3 shows a cumulative distribution function (CDF) of the soft signals multiplied with the transmit signals, i.e., $\Re\left[x^*(m)\left(\mathbf{T}^{-1}\right)_m \underline{\mathbf{Z}}\right]$ and $\Im\left[x^*(m)\left(\mathbf{T}^{-1}\right)_m \underline{\mathbf{Z}}\right]$, which are regarded as the real part and the imaginary part of the received signal when the modulation signal $x(m) = 1 + \mathrm{j}$ is sent. This means that the negative values of those signals are regarded

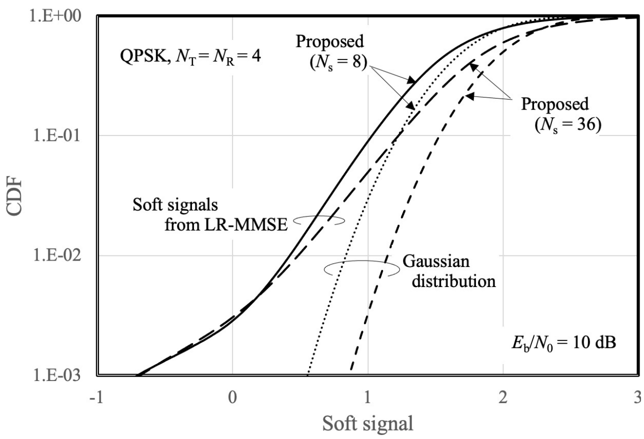

Fig. 3 CDF of MMSE detected signals.

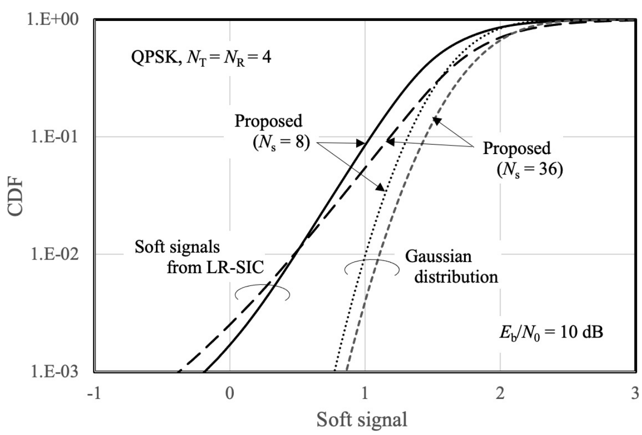

Fig. 4 CDF of SIC detected signals.

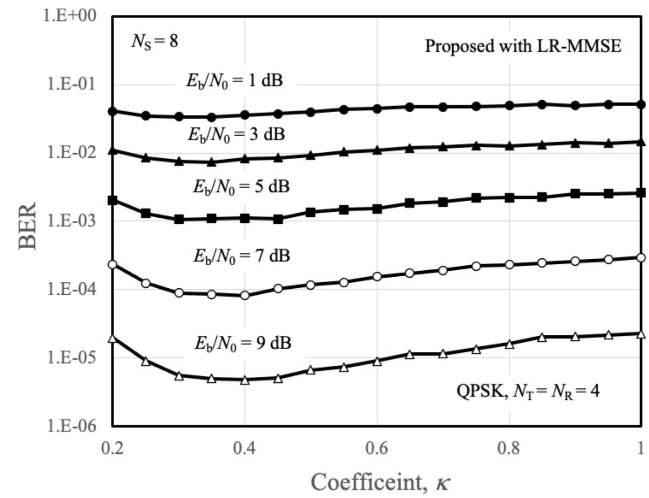

Fig. 5 BER v.s. Coefficient κ.

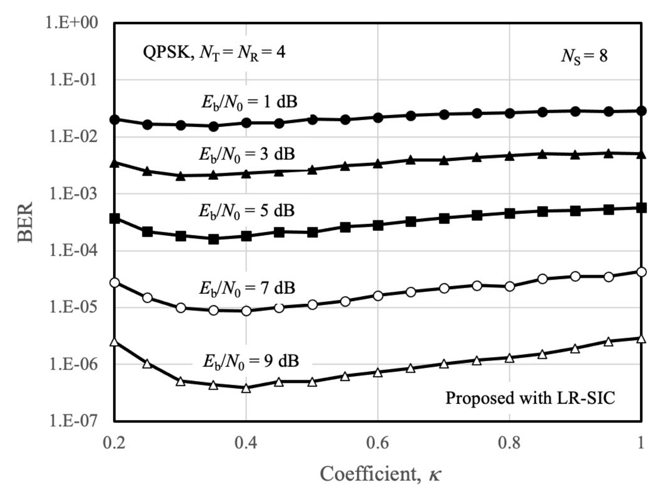

Fig. 6 BER v.s. Coefficient κ.

as erroneous signals. The LR-MMSE is applied to the proposed soft decoding in the figure. The E_b/N_0 is 10 dB. In the figure, the performance with $N_S = 8$ is compared with that with $N_S = 36$. In addition, the CDF of the Gaussian distributions with the variance of $e_Z\left(\sigma_p^2\right)$ and the mean of $\gamma\left(\sigma_p^2\right)$ are added as a reference. The distribution of the soft signal in the proposed soft decoding with $N_S = 8$ is almost the same to that with $N_S = 36$. Those distributions are quite different from the Gaussian distributions. As is expected, while the soft signal level at the CDF of about 0.5 almost agrees with that of the Gaussian distribution, negative soft signals occur with higher probability than those of the Gaussian distribution. Because the negative signals are regarded as the error signals as described above, the error signal defined as, $x(m) - \left(\mathbf{T}^{-1}\right)_m \underline{\mathbf{Z}}$, is distributed wider than the Gaussian signals. Figure 4 shows a CDF of the soft signals multiplied with the transmit signals, i.e., $\Re\left[x^*(m)\left(\mathbf{T}^{-1}\right)_m \underline{\mathbf{Z}}\right]$ and $\Im\left[x^*(m)\left(\mathbf{T}^{-1}\right)_m \underline{\mathbf{Z}}\right]$, when the LR-SIC is applied to the proposed soft decoding. In the figure, the distributions with $N_S = 8$ and $N_S = 36$ are compared. As is done in Fig. 3, the Gaussian distributions are also added. Similar as the distributions in Fig. 3, the distribution with $N_S = 8$ is almost the same to that with $N_S = 36$. However, the proposed decoding with the LR-MMSE generates the negative soft signals with higher probability than that with the LR-SIC, which means that the proposed soft decoding based on the LR-MMSE outputs the erroneous soft signals with bigger amplitude than that on the LR-SIC. The distribution of the soft signals are different from the Gaussian distribution in spite of the linear detectors used for the soft signal generation. This is the reason why we introduce the coefficient κ for the reliability $p(i)$ in the soft signal generation.

4.3 Performance with Respect to Coefficient κ

We analyze the performance with respect to the coefficient κ. Figure 5 and Fig. 6 show the BER performances v.s. the coefficient κ when the LR-MMSE and the LR-SIC are used in the proposed soft decoding, respectively. The number of the candidate vectors N_S is set to 8 for low complexity implementation, since the increase in the number of the vectors N_S achieves just a small performance gain as shown in

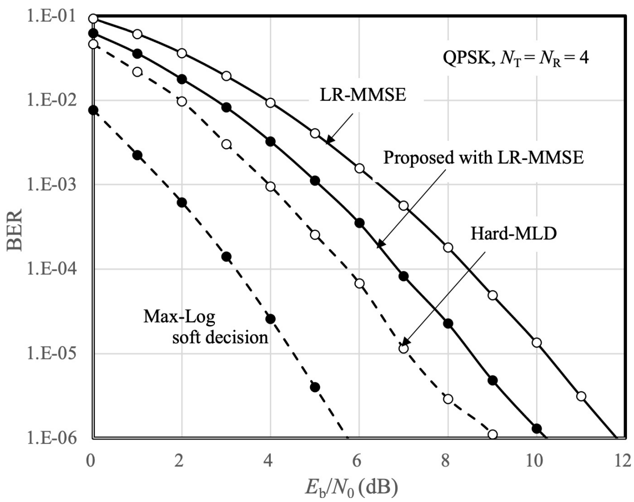

Fig. 7 MMSE based soft decoding compared with MLD based ones.

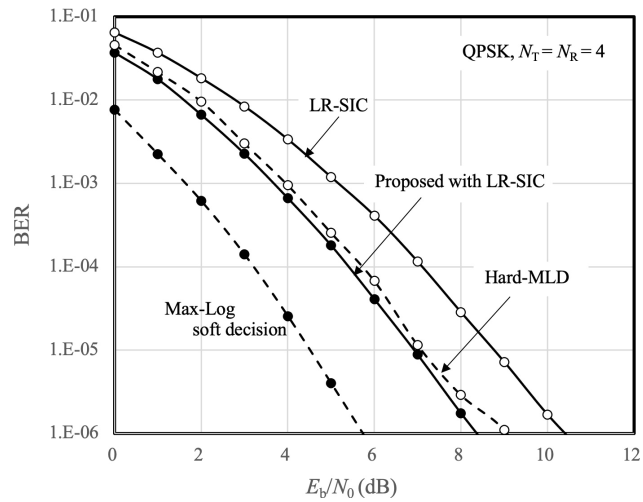

Fig. 8 SIC based soft decoding compared with MLD based ones.

Fig. 1 and Fig. 2. The BER performance is minimized at about $\kappa = 0.4$ in the both the soft decoding with the LR-MMSE and that with the LR-SIC, even if the optimum κ value slightly depends on the E_b/N_0. Therefore, we apply the coefficient κ value of 0.4 to the proposed soft decoding with not only the LR-SIC but also the LR-MMSE in the following performance evaluation.

4.4 Performance with Optimized Coefficient

Figure 7 shows the BER performance of the proposed soft decoding with the optimized coefficient κ of 0.4, where the LR-MMSE is applied to the proposed soft decoding. The number of vectors N_S is set to 8 for the reason described in the previous section. The performance of the conventional soft decoding with the Max-log MAP approximation is drawn in the figure as a reference. In addition, the performances of the hard decoding with the hard decision signals fed by the MLD and the LR-MMSE are added. The proposed soft decoding achieves a gain of about 1.5 dB at the BER of 10^{-5}. However, the performance of the soft decoding is about 4 dB inferior to that of the conventional soft decoding.

Figure 8 shows the BER performance of the proposed soft decoding with the optimized coefficient $\kappa = 0.4$, where the LR-SIC is applied to the proposed soft decoding. The number of vectors N_S is also set to 8 for low complexity implementation. The performances of the conventional soft decoding and that of the hard decoding with the MLD are drawn in the figure. The proposed soft decoding achieves about 2.0 dB better BER performance than the LR-SIC, and outperforms the hard decoding with the MLD. The soft decoding with the LR-SIC approximately attains 0.5 dB higher gain than that with the LR-MMSE. The performance gap between the proposed soft decoding and the conventional soft decoding is reduced to about 2.5 dB.

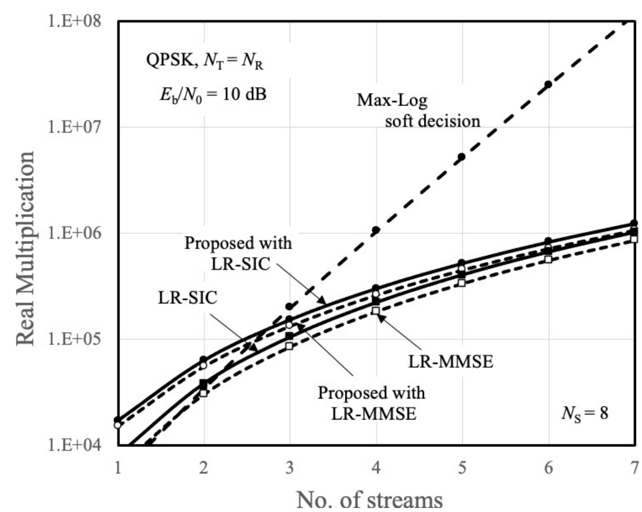

Fig. 9 Complexity of the proposed soft decoding.

4.5 Complexity Analysis

As is usually done, the computational complexity is evaluated in term of the number of multiplications in this section. The complexity of the proposed soft decoding is compared with that of the conventional soft decoding with the max-log MAP approximation in Fig. 9. The abscissa and the ordinate mean the number of the multiplexed signal streams and the number of the multiplications, respectively. The complexities of the LR-SIC and the LR-MMSE are added to show how the proposed decoding itself increases the complexity. This figure shows that proposed decoding needs small additional complexity, which is much smaller that the complexity of the linear detectors. When the number of the signal streams is less than 3, our proposed soft decoding requires more number of the multiplications than the conventional soft decoding. However, the proposed soft decoding has lower computational complexity than the conventional soft decoding as far as the number of the streams is more than 2. While the complexity of the conventional soft decoding grows exponentially as the number of streams increases, the complexity of the proposed soft decoding increases just parabolically. For instance, when the number of streams is 4, the complexity of the proposed soft decoding is about one tenth as much as the conventional soft decoding.

5. Conclusion

This paper has proposed a novel approach for the soft input decoding in MIMO systems. The proposed soft input decoding applies a linear filter assisted with the lattice reduction to get a hard output signal vector. Some hard decision vectors near by the hard output signal vectors are summed with weights, and the weighted sum are provided as a soft input signal to a soft input decoder. The soft decoding on the approach makes the soft input signals have the same variance to that of the detector output signals. We derive the variance of the soft signals theoretically and show how the variance is made to agree with the variance of the detector output signals. Although soft input decoders achieve the best performance if the soft input signals are distributed with the Gaussian distribution, the distribution of the soft signals is different from the Gaussian signals in the proposed soft decoding. We introduce a coefficient to adjust the distribution of the soft signals to maximize the transmission performance.

The transmission performance is evaluated in a 4×4 MIMO channel by computer simulation. The proposed soft decoding achieves a gain of about 1.0 dB at the BER of 10^{-5} when the linear filters such as the LR-MMSE and the LR-SIC are applied, as long as the number of the candidate hard decision vectors is less than 37. If the coefficient is introduced to the proposed soft decoding, the proposed soft decoding achieves a gain of about 2.0 dB. The proposed soft decoding with the LR-SIC outperforms the hard decoding with the MLD. The performance gap between the conventional soft decoding and the proposed soft decoding is reduced to about 2.5 dB, while the complexity of the proposed soft decoding is one tenth as much as that of the conventional soft decoding.

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Satoshi Denno received the M.E. and Ph.D. degrees from Kyoto University, Kyoto, Japan in 1988 and 2000, respectively. He joined NTT radio communications systems labs, Yokosuka, Japan, in 1988. In 1997, he was seconded to ATR adaptive communications research laboratories, Kyoto, Japan. From 2000 to 2002, he worked for NTT DoCoMo, Yokosuka, Japan. In 2002, he moved to DoCoMo communications laboratories Europe GmbH, Germany. From 2004 to 2011, he worked as an associate professor at Kyoto University. Since 2011, he is a full professor at graduate school of natural science and technology, Okayama University. From the beginning of his research career, he has been engaged in the research and development of digital mobile radio communications. In particular, he has considerable interests in channel equalization, array signal processing, Space time codes, spatial multiplexing, and multimode reception. He received the Excellent Paper Award and the communication societies Best Paper Award from the IEICE in 1995 and 2020, respectively.

Koki Kashihara received the B.E., and M.E. from Okayama University, Okayama Japan in 2020 and 2022, respectively. His research interests are in spatial signal processing techniques for wireless communications. He joind with Chugoku Electric Power Transmission & Distribution Co., Inc in 2022.

Yafei Hou received his Ph.D. degrees from Fudan University, China and Kochi University of Technology (KUT), Japan in 2007. He was a post-doctoral research fellow at Ryukoku University, Japan from August 2007 to September 2010. He was a research scientist at Wave Engineering Laboratories, ATR Institute International, Japan from October 2010 to March 2014. He was an Assistant Professor at the Graduate School of Information Science, Nara Institute of Science and Technology, Japan from April 2014 to March 2017. He became an Assistant Professor at the Graduate School of Natural Science and Technology, Okayama University, Japan from April 2017. He is a guest research scientist at Wave Engineering Laboratories, ATR Institute International, Japan from October 2016. His research interest are communication systems, wireless networks, and signal processing. He received IEICE (the Institute of Electronics, Information and Communication Engineers) Communications Society Best Paper Award in 2016 and Best Tutorial Paper Award in 2017. Dr. Hou is a senior member of IEEE and senior member of IEICE.