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GeoGebra Discovery at EGMO 2022[1]

GeoGebra Discovery na EGMO 2022

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ABSTRACT

This study will show the ability (or inability) of GeoGebra Discovery to deal with Euclidean geometry problems proposed at the recent European Girls' Mathematical Olympiad – EGMO (Hungary, April 6-12, 2022). After a brief introduction to the context of this Olympiad and to the program GeoGebra Discovery, the problems will be described and an attempt will be made to solve them with GeoGebra Discovery, finally pointing out the relationship between the difficulties encountered by the team members and by GeoGebra, which can contribute to the establishment of criteria on the interest (and complexity) of the automatically obtained results.

Keywords: *GeoGebra, Automatic Proof, Euclidean Geometry, Olympics, EGMO.*

RESUMO

Este estudo mostrará a capacidade (ou incapacidade) da GeoGebra Discovery de lidar com problemas de Geometria Euclidiana propostos na recente Olimpíada Europeia de Matemática de Meninas - EGMO (Hungria, 6 a 12 de abril de 2022). Após uma breve introdução ao contexto desta Olimpíada e ao programa GeoGebra Discovery, os problemas serão descritos e será feita uma tentativa de resolvê-los com a GeoGebra Discovery, finalmente apontando a relação entre as dificuldades encontradas pelos membros da equipe e pela GeoGebra, que podem contribuir para o estabelecimento de critérios sobre o interesse (e complexidade) dos resultados obtidos automaticamente.

Palavras-chave: *GeoGebra, Prova Automática, Geometria Euclidiana, Olimpíadas, EGMO.*

1. Introduction

GeoGebra is a Dynamic Geometry software that provides automated reasoning tools through the Relation, Prove, ProveDetails, Discover, Compare,

[1] Partially based on the English version of the communication "GeoGebra Discovery en la EGMO 2022" presented at the XX Jornadas para el Aprendizaje y Enseñanza de las Matemáticas (JAEM 2022), Valencia, Spain. https://20.jaem.es

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LocusEquation commands (see Section 3 for more details). It is able to provide mathematically rigorous information about the truth or falsity of a given geometric statement, and it can even create new geometric statement on its own. But all these features come without any explanation: it is a kind of oracle delivering very sharp answers.

Despite this severe limitation (lack of arguments to justify the obtained result), the performance of GeoGebra Discovery can have a relevant impact in education, as suggested by Hanna and Yan (2021), who devote one specific section to GeoGebra's automated proving tools, concluding that:

> *It is perhaps too early for empirical studies of classroom experience using the enhancements to GeoGebra… The key is to make a start, beginning with exploratory studies of the potential of these new tools at both the secondary and post-secondary levels.* (Hanna and Yan, pp 7-8, 2021)

Indeed, we consider that the mere existence and large availability (bearing in mind that GeoGebra is free, available on computers, laptops, tablets, smartphones…) of such technological oracle already implies an important impact on the classroom, making urgent the "…beginning with exploratory studies of the potential of these new tools" (Hanna and Yan, pp 7-8, 2021).

In this context, the goal of our contribution here is to start calling the attention of the community of educators about the need to address the similitudes and differences concerning the notion of "difficult" geometric problem, both for humans and for GeoGebra automated reasoning tools, by dealing with problems that have already being classified by humans as highly difficult, namely, those formulated in mathematical contests such as Olympiads (see Sections, 2, 4, 5 and 6).

There are some precedents in this regard, considering the notion of "difficulty" of a geometric statement, but not in relation to automated proving. For example, we could consider, as a measure of difficulty, the number of steps required to build a geometric construction describing the statement (higher number of steps=more complicated construction). This particular approach to measure the complexity of constructions has been studied since long, and has been adapted, more recently, to the Dynamic Geometry context, by Quaresma and collaborators, such as (Quaresma et.al. 2020, Santos et. al. 2019). A related notion, that of "interestingness" of theorems, is also subject to current research (see Gao et. al. 2019).

On the other hand, a detailed proposal of complexity measures for geometry problems addressed to talented students (eg. in the Mathematics Olympiad context), appears in the Master Thesis of Bak (2020), that aims to automatize the formulation and ranking of problems, but does not regard its automated solving. Or, regarding talented students in general, we could recall the recent work of Espinoza et. al (2022), providing a categorization of the difficulty of mathematics problems (not just geometric) posed for such students.

But none of these previous approaches attempt to connect simultaneously three items: “intrinsic difficulty of geometric statements”, “hardships of their solution by humans”, “hardships of their solution by dynamic geometry programs” (see Quaresma and Santos (2022) for a contribution that, in some sense, could be considered as a precedent, since it addresses items 2 and 3 for some very standard problems).

We think this triple consideration approach could lead to some conclusions of relevance, from both the technological and educational perspective. For example, improving the performance of GeoGebra, helping to establish internally some reasonable time-out for the running algorithms and launching some sign to the user announcing it might expect a delayed or no answer. It could also contribute to improve GeoGebra’s performance by selecting, in the output of some of the commands that automatically display properties of a given figure, to show only relevant results, avoiding obvious, trivial ones, etc. On the other hand, learning about the differences and coincidences of GeoGebra and human users could lead to adapting GeoGebra’s reasoning tools to the needs of students with high capacities or in the context of mathematical contests (e.g.: reconsidering (in a double sense) the proposal to such students of problems that GeoGebra is able to deal with immediately, or choosing, as a challenge, problems that GeoGebra is unable to solve). Finally, as machine algorithms are made by humans (at least in the current times!), learning about the difficulties of GeoGebra solving problems can help understanding human and intrisic difficulties of the considered statements.

Some little steps towards supporting these reflections appear already in Section 7 in this paper, where we present one conclusion pertinent in the educational context and another one closer to the involved technology. But, as already mentioned there, the reader should bear in mind that this paper aims to present a proposal, and a couple of examples. A proposal to be developed along the next years as part of my work for the doctoral dissertation in mathematics education.

2. EGMO (European Girls’ Mathematical Olympiad)

The EGMO (European Girls' Mathematical Olympiad, [EGMO], 2022), the European Girls' Mathematical Olympiad, is an international competition similar in style to the International Mathematical Olympiad (IMO). It is an exam that takes place over two days and in which each participating team can send up to four high school students. It was created in 2012, at the initiative of the United Kingdom, inspired by the Chinese Women's Mathematics Competition (which is now an international event), with the aim of opening more spaces for the participation of women in international mathematical competitions. Spain has participated uninterruptedly since 2016 (seven times). The Olympics Commission of the Royal Spanish Mathematical Society ([RSME], 2022), which manages the Spanish Mathematical Olympiad (OME), configures the Spanish team of the EGMO based on the results of the OME. See (Royal Spanish Mathematical Society: European Girls' Mathematical Olympiad [RSME-EGMO], 2022), for more details on the Spanish participation in this Olympiad over the years.

This year the eleventh edition of the European Women's Olympiad (EGMO) took place in the city of Eger (Hungary), from April 12 to 16, 2022, with the participation of 57 countries, 31 of them European, but also including -outside of the official competition-- countries from all continents, such as USA, Peru, Australia, Mongolia, Japan or Tunisia (European Girls' Mathematical Olympiad 2022, [EGMO2022], 2022). The Spanish team--four students between 16 and 17 years old--achieved the best position in the history of her participation, occupying position 22 by countries (not counting the invited countries, in that case it would be 38 of 57), with one participant achieving a bronze medal and two other participants achieving honorable mentions (Spanish Team: European Girls' Mathematical Olympiad 2022, [Spanish Team: EGMO2022], 2022).

3. GeoGebra Discovery

GeoGebra Discovery (Kovács, 2022) is an experimental version of GeoGebra. It contains some state-of-the-art features of GeoGebra that are under heavy development and thus not yet designed for everyday use, so they are not included in the official version of GeoGebra. It is planned that each feature, once deemed sufficiently stable, will also be added to the official version of GeoGebra.

As described in Recio et. al. (2020), GeoGebra Discovery offers the user the ability to perform a rich variety of geometric tasks:

• automatically test the truth or error of a given conjectural statement (Prove and ProveDetails commands),

• automatically discover how to modify a given figure so that an incorrect statement becomes true (also called automated discovery, LocusEquation command),

• automatically discover the relationship between some specific elements of the given figure (Relation command),

• automatically discover all the statements that are true and that involve an element in the figure selected by the user (command Discover),

• automatically discover "all" properties of a certain type (perpendicularity, parallelism, ...) involving all elements of a given figure (available as a web application at http://autgeo.online/ag/automated-geometer. html?offline=1).

In addition, some of these commands have been improved by including the possibility of dealing with geometric properties that are verified on real numbers (inequalities).

Given that several of these commands deal in a combinatorial way (that is, proposing all the combinations of three points found in the construction and checking if they are aligned, etc.) it is important (and difficult) to establish criteria on the interest of the results obtained: they are trivial, they are complex…etc.

4. Two problems of Euclidean geometry

The EGMO contest proposes the resolution of 6 problems, three on each of the two days (Problems: European Girls' Mathematical Olympiad 2022, [Problems: EGMO2022], 2022). This year, of the six problems, two (1 and 6) have been of Euclidean geometry, another of them is of a combinatorial nature and the other three have to do, roughly speaking, with numbers. Next, let us state problems 1 and 6:

Problem 1. Let ABC be an acute triangle with BC < AB and BC < AC. Consider the points P and Q on the segments AB and AC, respectively, such that P ≠ B, Q ≠ C and BQ = BC = CP. Let T be the circumcenter of triangle APQ, H be the orthocenter of triangle ABC, and S be the point of intersection of lines BQ and CP. Prove that the points T, H and S are on the same line.

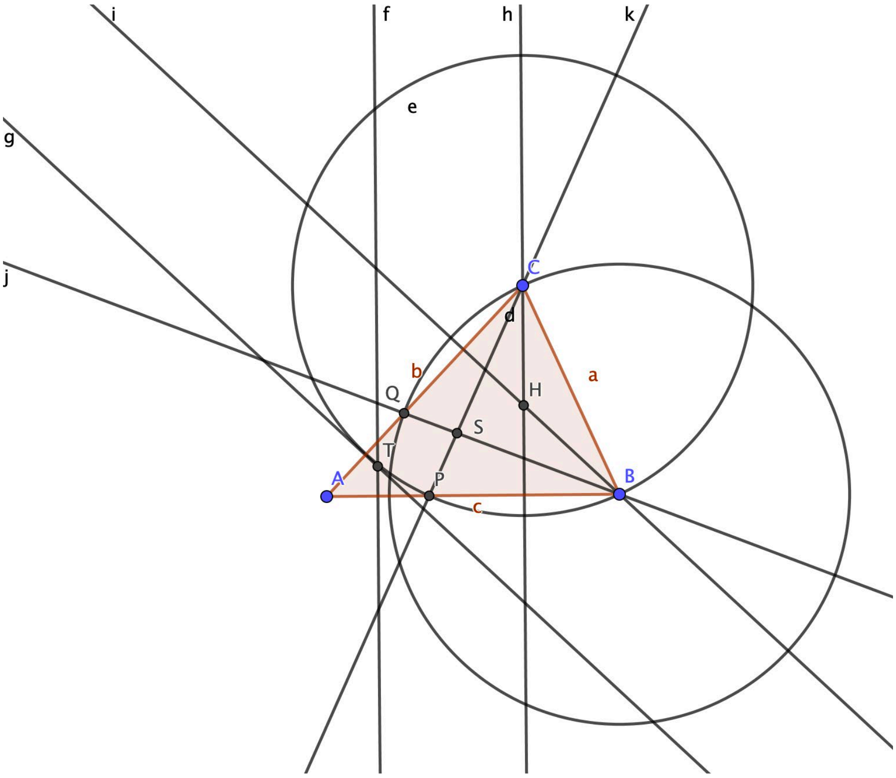

FIGURA 1: Problem 1. T has been constructed as the intersection of the perpendicular bisectors of AP and AQ, f and g.
SOURCE: The author

Problem 6. Let ABCD be a cyclic quadrilateral with circumcenter O. Let X be the point of intersection of the bisectors of the angles ∠DAB and ∠ABC; point Y that of the bisectors of the angles ∠ABC and ∠BCD; Z is the bisector of the angles ∠BCD and ∠CDA; and let W be the point of intersection of the bisectors of the angles ∠CDA and ∠DAB. Let P be the point of intersection of lines AC and BD. Points O, P, X, Y, Z, and W are assumed to be distinct.

Prove that O, X, Y, Z, and W lie on the same circle if and only if P, X, Y, Z, and W lie on the same circle.

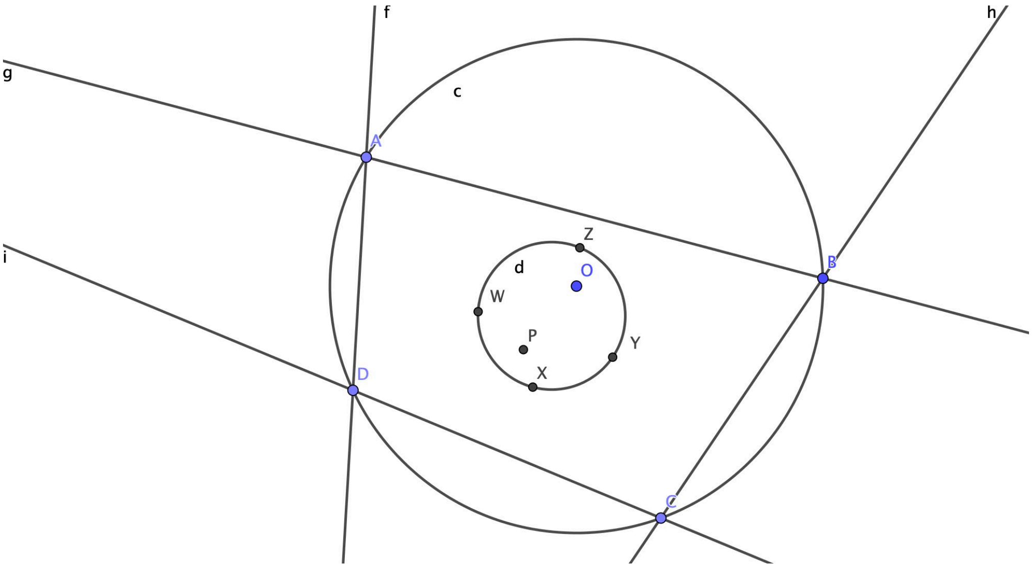

FIGURE 2: A circle c has been constructed with center at O and on it the four vertices ABCD of the quadrilateral. After constructing X, Y, Z, W, via the Bisector command, the circumference d passing through X,Y,Z is found (and which also happens to contain W). Next, some vertex of the quadrilateral is moved to study when O or P are in d.
SOURCE: The author

5. Solving through GeoGebra Discovery

Problem 1 is solved immediately with GeoGebra. As seen in Figure 3, if GeoGebra is asked to calculate Relation({T,S,H}), the answer (symbolically, mathematically correct, not approximate) is that the three points are aligned except in certain degenerate cases. In the same Figure you can also see the result of asking GeoGebra directly to show the alignment of these three points (and it answers "true").

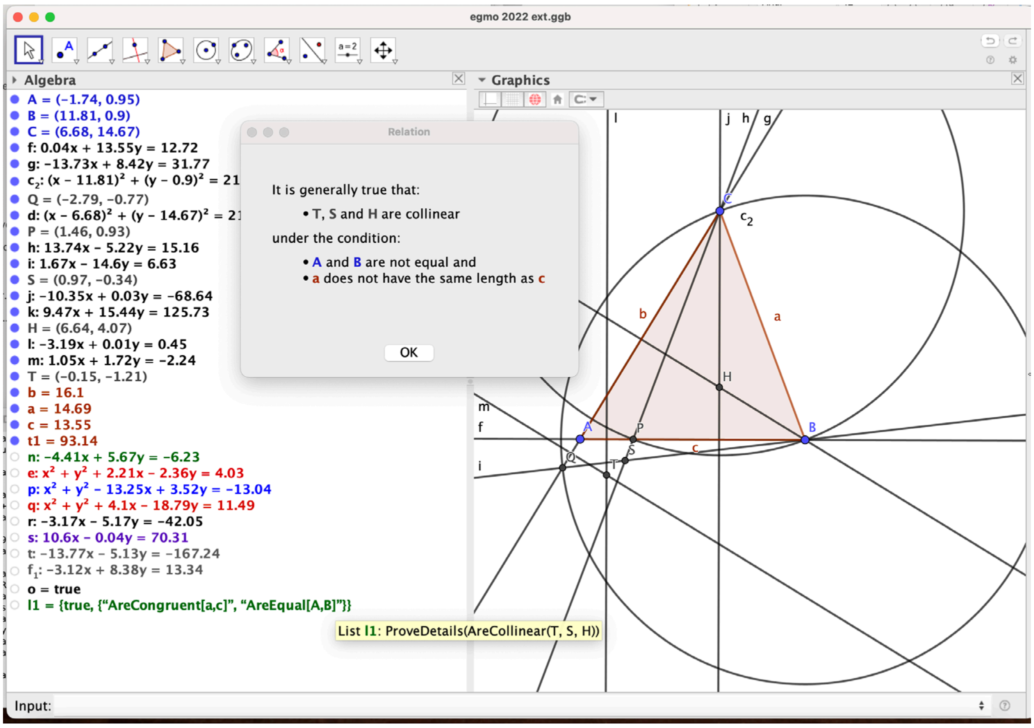

FIGURE 3: Solving Problem 1
SOURCE: The author

Note that it has not been necessary to use that the given triangle is acute with BC < AB and BC < AC. See Figure 3 for a solution where BC >AB.

On the other hand, in Figure 4, one can see how GeoGebra Discovery automatically finds the statement about the collinearity of T, S, H, if asked to find theorems holding about point S.

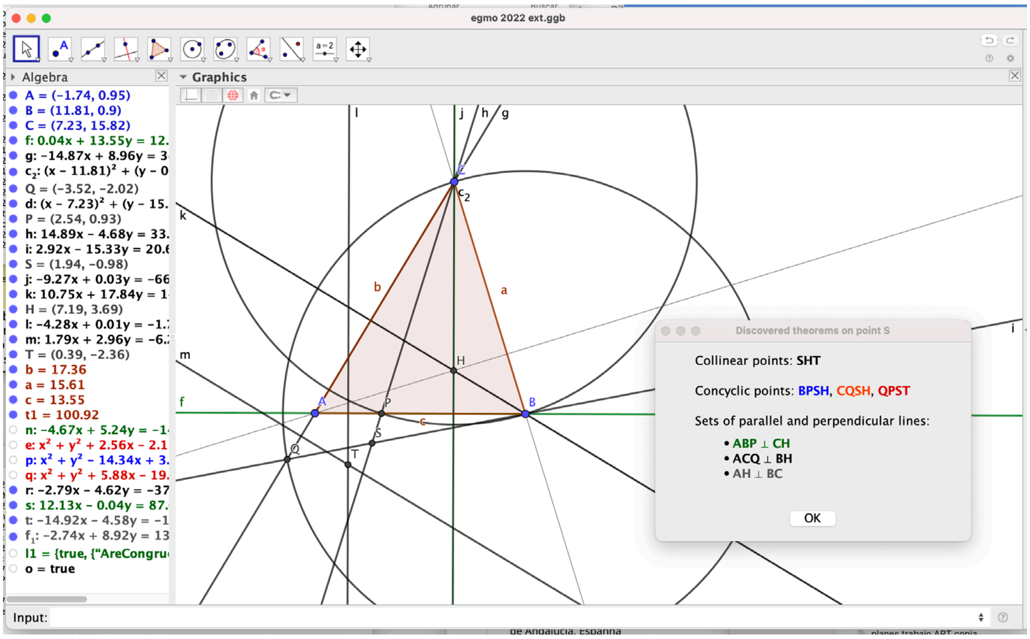

FIGURE 4: Alignment of T, S, H found simply by asking GeoGebra to Discover all theorems involving S.
SOURCE: The author

On the contrary, GeoGebra is not capable of solving Problem 6. We start, in the first place, trying to verify (with Relation or Prove, etc.) what seems to be true at first glance: that X, Y, Z, W are in the same circumference. But it does not work. Nor is it possible to find the locus of, say, vertex C so that O, X, Y, Z (and, if the above conjecture is true, W) are on the same circle. The idea is that, once we know what properties C must have for O, X, Y, Z to be co-cyclic, perhaps we can study whether such properties also imply that P is on the same circle.

But in all cases, GeoGebra is not able to finish the computations, perhaps because the Bisector command involves a construction where you must choose between the inner or outer bisector, and it involves handling signs, which makes the entire calculation process very complex.

6. Team score on problems 1 and 6.

The four participants of the Spanish team obtained the following scores (where 7 points is the highest):

Problem 1: C1=1, C2=7, C3=0, C4=1

Problem 6: C1=0, C2=1, C3=0, C4=0

Note that C1 and C3 obtained Mention and C2 Bronze Medal. It can also be seen in the list of results on the EGMO 2022 website that the total Spanish score (9 points obtained by 4 people, out of a total of 28 possible points in Problem 1; and 1 point in total for Problem 6) would place the Spanish team among the last ten, if only these two problems where considered. Other data accessible on that website shows that the mean score for Problem 1 in all the participants was slightly more than 4.6 points, compared to 1.05 for the mean score for Problem 6, the lowest after Problem 3 (0.78), a problem in which the Spanish team scored better than in Problem 6 and in Problem 2.

7. Final conclusions

Two conclusions: one, related to our curriculum; another, more computer related. In the first place, and without the intention of presenting a statistically valid study (which would require a much larger sample), it seems to emerge, from the observation of the behavior of the Spanish team in the involved Euclidean geometry problems, that the Spanish participants have a certain deficit (compared to most participating countries) of knowledge in this area, even considering only the simpler Problem 1 – where the score of the Spanish team is clearly below the average (2.25 vs. 4.6). The same goes for Problem 6, where our mean is 0.25. And without this implying underestimating the participants who, as we have pointed out, obtained two honorable mentions and a bronze medal. It is, clearly, a curricular issue.

The other conclusion, regarding the use of GeoGebra in the secondary class, points out, on the one hand, the immediacy and ease with which this program offers students rigorous certainty about the truth of certain properties; a feature which can hinder their interest in undertaking such tasks. And, on the other hand, the difficulty – which, properly treated, can become a source of inspiration and motivation – that seems inherent in other types of problems, such as Problem 6, due to its formulation ("prove that xxx if and only if yyy") and elements involved (angles, bisectors, which imply a framework with real numbers, much more complicated to process than the complex ones)…Perhaps a systematic study of the characteristics that are behind these difficulties could help to make automatic discovery algorithms in GeoGebra more fine-tuned to focus on obtaining interesting results from a perspective that, precisely, should be defined more precisely...

References

Bak, P. (2020): Automated generation of planar geometry Olympiad problems. *Master Thesis*. Pavol Josef Šafárik University in Košice.

Espinoza, J.; Lupiáñez, J.L.; Segovia, I. (2022): A Study of the Complexity of Problems Posed by Talented Students in Mathematics. *Mathematics*,*10*,1841. https:// doi.org/10.3390/math10111841

European Girls' Mathematical Olympiad. [EGMO] (April 26, 2022). https://www.egmo.org

European Girls' Mathematical Olympiad 2022. [EGMO2022] (April 26, 2022). https://www.egmo.org/egmos/egmo11/

Gao, H.; Li, J.; Cheng J.: (2019): Measuring Interestingness of Theorems in Automated Theorem Finding by Forward Reasoning Based on Strong Relevant Logic. In: 2019 *IEEE International Conference on Energy Internet (ICEI)*, pp. 356–361, doi:10.1109/ICEI.2019.00069.

Hanna, G.; Yan, X. (2021): Opening a discussion on teaching proof with automated theorem provers. *For the Learning of Mathematics* 41(3):42--46.

Kovács, Z.: GeoGebra Discovery. A GitHub project. (April 26, 2022). https://github.com/kovzol/geogebra-discovery.

Problems: European Girls' Mathematical Olympiad 2022, [Problems:EGMO2022], 2022). https://www.egmo.org/egmos/egmo11/solutions.pdf

Quaresma, P.; Santos, V.; Graziani, P.; Baeta, N. (2020): Taxonomy of geometric problems. *Journal of Symbolic Computation* **97**, 31–55.

Quaresma, P.; Santos, V. (2022): Four Geometry Problems to Introduce Automated Deduction in Secondary Schools. In: J. Marcos, W. Neuper and P. Quaresma (Eds.): *Theorem Proving Components for Educational Software 2021 (ThEdu'21):* EPTCS 354, 2022, pp. 27-42, http://eptcs.web.cse.unsw.edu.au/paper.cgi?ThEdu21.3

Real Sociedad Matemática Española. Comisión de Olimpiadas. [RSME], (April 26, 2022). https://www.rsme.es/la-sociedad/organizacion-interna/comisiones-comites-y-grupos/comision-de-olimpiadas/

Real Sociedad Matemática Española: European Girls' Mathematical Olympiad. [RSME-EGMO], (April 26, 2022). https://www.rsme.es/european-girls-mathematical-olympiad/

Recio, T.; Van Vaerenbergh, S.; Vélez, M. P. (2020). Herramientas de Razonamiento Automático en GeoGebra: qué son y para qué sirven. *Unión, Revista Iberoamericana de Educación Matemática*. Año XVI - Número 59. Agosto 2020, páginas 08-15. https://union.fespm.es/index.php/UNION/article/view/202

Santos, V.; Baeta, N.; Quaresma, P. (2019): Geometrography in Dynamic Geometry. *The International Journal for Technology in Mathematics Education*, vol. 26, no. 2, June 2019, pp. 89—96.

Spanish Team. European Girls' Mathematical Olympiad 2022. [Spanish Team: EGMO2022] (April 26, 2022). https://www.egmo.org/egmos/egmo11/countries/country45/