



## History of Mathematics in High School: a proposal for didactic material based on (and despite) the Common Core

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**Abstract:** This article examines the notorious contrast between the importance given in and outside the classroom to the use of History of Mathematics in teaching Mathematics. Based on that, we propose a framework of didactic material that encourages the use of historical elements in the fulfillment of the Brazilian Common Core for High School (BNCC-EM). The material consists of a historical recount, related to certain skill, and mathematical tasks inspired by it. We also present one example supported by three seminal contributions from the 17th century, due to Michael van Langren, Christiaan Huygens, and Edmond Halley. Throughout recount and tasks, we emphasize the use of History of Mathematics for the study of mathematical procedures and concepts, about which the BNCC-EM itself, contrary to previous milestones in Education, says nothing.

**Keywords:** History of Mathematics. Mathematics Education. Curriculum. High School Education.


## Historia de las Matemáticas en la Enseñanza Secundaria: una propuesta de material didáctico a partir (y a pesar) del Currículo Básico


**Resumen:** Este artículo reflexiona acerca del notorio desajuste entre la atención prestada a la Historia de las Matemáticas dentro y fuera del aula. A partir de esta reflexión, se propone aquí un formato de material didáctico que incentive el uso de elementos históricos en el cumplimiento del Currículo Básico brasileño para la escuela secundaria (BNCC-EM). La propuesta consta de un informe histórico, vinculado a una determinada habilidad, y tareas matemáticas inspiradas en él. Para ilustrar el formato, presentamos un ejemplo respaldado por tres contribuciones del siglo XVII debidas a Michael van Langren, Christiaan Huygens y Edmond Halley. Tanto en la construcción del informe como en la elaboración de tareas, se buscó enfatizar el uso de la Historia de las Matemáticas para el estudio de conceptos y procedimientos matemáticos, sobre qué la BNCC-EM misma, contrariamente a hitos anteriores en Educación, guarda silencio.

**Palabras clave:** Historia de las Matemáticas. Educación Matemática. Currículum. Enseñanza Secundaria.

## História da Matemática no Ensino Médio: uma proposta de material didático a partir (e apesar) da BNCC-EM

**Resumo:** Este artigo reflete sobre o notório descompasso entre a atenção dada à História da Matemática dentro da sala de aula e fora dela. A partir dessa reflexão, propõe-se aqui um formato de material didático que incentive o emprego de elementos

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históricos no cumprimento da Base Nacional Curricular Comum do Ensino Médio (BNCC-EM). A proposta consiste em um relato histórico, vinculado a uma determinada habilidade, e tarefas matemáticas nele inspiradas. Para ilustrar o formato, apresentamos um exemplo apoiado em três contribuições do século XVII, devidas a Michael van Langren, Christiaan Huygens e Edmond Halley. Tanto na construção do relato como na elaboração de tarefas, busca-se enfatizar o uso da História da Matemática para o estudo de conceitos e procedimentos matemáticos, sobre o que a própria BNCC-EM silencia, ao contrário de marcos anteriores da Educação.

**Palavras-chave:** História da Matemática. Educação Matemática. Currículo. Ensino Médio.

## 1 Introduction

Literature broadly supports the adoption of History of Mathematics (HM) in mathematics teaching (FREUDENTHAL, 1981; FAUVEL, 1991; ERNEST, 1998; D'AMBRÓSIO, 1999; FAUVEL & VAN MAANEN, 2002; LARA, 2016; FURINGHETTI, 2020). In the classroom, however, the enthusiasm is not the same. Among the reasons given for teachers' resistance to accepting this approach is the lack of preparation and access to appropriate teaching material (SIU, 2006; BRITTO & BAYER, 2007; SANTOS, 2017). Moreover, the implementation of the National Common Core Curriculum for High School (BNCC-EM) worsened the mismatch, which echoed strongly in the publishing market since the new legal framework, unlike earlier documents, does not mention the historical perspective in mathematics and its technologies (BRASIL, 2018a).

This study examines arguments for and against this historical perspective and in view of the favorable arguments, it aims to propose a format of didactic material that encourages high school teachers to use the HM in compliance with the Common Core Curriculum. This study is exploratory, based on a bibliographic and documentary survey, the result of a master's thesis written by the first author and guided by the second. The work is organized as follows: at first, we approach the attention given to the history of mathematics (HM) in official documents of education; then, we focus on the distance between the potentiality attributed to the HM and its real approach in everyday life, in the classroom; based on these reflections, we present a proposal for a teaching material linked to a specific skill dictated by the BNCC-EM, which is composed of a historical account, aimed at teachers, and mathematical tasks, aimed at students; then we present an example of this material, and finally we discuss our efforts in light of the issues raised.

## 2 The HM in the official documents of education: the silence of the BNCC

Opening the way for broad reform of education, the Federal Constitution of 1988 determined that the Union must legislate on “guidelines and bases of national education” and decided that “minimum content must be established [...] to ensure common basic training”. (BRASIL, 1988, articles 22 and 210).

However, this Common Base was only defined more than twenty years later. In between, the teaching programs were guided by the National Curricular Parameters (PCN). The 1999 High School PCNs expressly encourage the mathematics historical perspective. Among 18 skills to be developed, it mentions “[...] relating stages in the history of mathematics with the evolution of humanity” (BRASIL, 1999, p. 46). It argues that “[...] the history of science and mathematics [...] is relevant for learning that transcends the social relationship, as it also illustrates the development and evolution of the concepts to be learned” (BRASIL, 1999, p. 54).

The document, known as PCN+, detailing educational guidelines that complement the PCNs, also gives space to the historical approach to mathematics. Within the scope of the competencies related to “research and understanding”, the student is expected: “[...] to understand the construction of mathematical knowledge as a historical process, in close relationship with the social, political, and economic conditions of a given time” (BRASIL, 2002a, p. 117).

Finally established in 2018, the BNCC-EM reorganized skills and abilities to be developed in the area of mathematics and its technologies as general basic training. One might expect HM to have its explicit endorsement here. But that did not happen. None of the five competencies or 43 skills dictated mentions the historical perspective of Mathematics. The third version of the document, at the end of the presentation of competencies, still shows a vague incentive to the historical perspective:

This perception of the unity of mathematics, besides the diversity of its practices, also serves to show that the development of the subject is the result of human experience throughout history. Thus, it is not a perfect building that emerged ready-made from the minds of a few privileged beings to be studied for pure intellectual delight [...].

One of the challenges for mathematics learning in high school is exactly to provide students with the view that it is not a set of rules and techniques, but is part of our culture and our history (BRASIL, 2018b, p. 522).

But even the passage just quoted was suppressed from the final version. We

note that, in this sense, the BNCC-EM distances itself from the PCNs and from the BNCC for Elementary School, where it is evident that “[...] it is important to include the History of Mathematics as a resource that can arouse interest and represent a significant context for learning and teaching mathematics” (BRASIL, 2018a, p. 298).

Pinto (2018, p. 15) observes that this silence of the BNCC-EM extends to other emerging fields of mathematics, such as ethnomathematics: “A quick look at research in Mathematics Education allows us to affirm that those theoretical-methodological approaches constitute references for a teaching practice that respects the diversity and plurality of Brazilian public schools”. Although it is not the scope of this work, we add that HM did not find space among the skills in the area of mathematics related to the so-called formative itineraries – beyond the basic general education of which BNCC-EM takes care. Those itineraries propose “[...] educational situations and activities that students can choose according to their interest”, according to an ordinance that establishes references for its preparation (BRASIL, 2019a, p. 94).

### **3 HM in textbooks: “some interest”**

The resolution establishing the BNCC-EM determines its complete implementation in 2022. The 2021 edition of the National Book and Teaching Material Program (PNLD) marked the definitive realignment of curricula and pedagogical proposals (BRASIL, 2019b). This edition is the first of the program aimed at high school since the approval of the base. Like the BNCC-EM, its public notice also does not mention the historical perspective. By contrast, previous notices expressly recognized that mathematics is “produced and organized throughout history” as “[...] part of people's routine, activities of the other sciences, and technologies” (BRASIL, 2015, p. 57).

One wonders what impact BNCC-EM's silence on HM has on the publishing market. It is true that many authors adopt historical elements out of conviction - in this regard, we have the historical example of the “Curso Elementar” (REIS & REIS, 1892), which, in footnotes, presented historical data and biographical information about a series of mathematicians and thinkers. But it is equally true that the attention paid to HM in textbooks has gained importance in recent years in the wake of PCNs and government notices based on them. Let us look at two studies in this regard.

After analyzing five didactic collections and interviewing their authors, Gomes

(2008, p. 160) identified the concern with contextualization and the use of HM as an auxiliary element, noting that its use still does not take place “[...] in an organic, enlightening, meaningful, and problematizing way”. Gomes considers that although some authors recognize hardships in dealing with the HM, they feel “almost ‘compelled’” to do so, which he interprets as “[...] an indicator of how a public policy related to the circulation of textbooks in the country exerts an indirect power over authors”.

From the analysis of six high school collections approved in the 2015 edition of the PNLD, Pereira (2016, p. 91) points out “some interest” in using the history of mathematics, “[...] either by the requirement of such use in the books' assessment by that Program or by the author's own will”. He also remarks: of the references to HM analyzed, only about 13% have the function of didactic strategy, “[...] which [...] plays the role of providing the student with the development of some mathematical reasoning, leading them to understand the mathematical content or concept”.

#### **4 The HM in the classroom: encouragements and objections**

Besides textbooks, in recent decades, there has also been a growing supply of paradidactic titles, educational products, publicity works, academic works, and journals specialized in HM (VIANNA, 1998; LARA, 2016; BRACHO & MENDES, 2019). We draw particular attention to the rich repository of the free-access Brazilian Center for Reference in Research on the History of Mathematics (Centro Brasileiro de Referência em Pesquisa sobre História da Matemática - CREPHIMat) (BRACHO & MENDES, 2019).

Still, the teacher resents the lack of material. Britto and Bayer (2007) investigated the use of HM by teachers from public and private schools in 35 municipalities in Rio Grande de Sul. They found that teachers consider the resource important but are not used to using it. Among the justifications, the teachers feel unprepared and resent the lack of material. Santos (2017) raised the point of view of high school teachers who work in public schools in Itajubá, Minas Gerais. Among the difficulties, the researcher points out the lack of time, material, teacher training, students' interest, and mandatory topic in official documents.

It is idle to speculate that the teachers lack commitment. Rather, we must admit that the product of much research on HM is simply not reaching them (BRACHO &

MENDES, 2019). And we add that it is also possible that it does not meet their expectations when it reaches them. Scientific publications and products from graduate programs can be quite demanding reading. On the other hand, easily understandable publicity works are not concerned with establishing correspondence between the HM and the classroom.

In any case, the use of HM-centered teaching resources depends on the teacher's conviction that their use can, in fact, enrich their class. In the introduction, we cited several authors who have no doubt about this. In the classroom, however, the perception can be quite different. In Chart 1, we reproduce a synthesis of Fauvel's Fauvel (1991, p. 4) "good reasons" in favor of using the HM and the complaints that Siu (2006, p. 268-269), admitting to playing "the devil's advocate", attributes to teachers.

Chart 1: Reasons for and against adopting the HM in the classroom

Arguments in favor of the HM	Arguments against the HM
<p>It helps to increase motivation.</p> <p>It gives mathematics a human face.</p> <p>It helps organize curriculum topics.</p> <p>Showing students how the concepts developed makes it easier to understand them.</p> <p>It changes the perception that students have of mathematics.</p> <p>Comparing the ancient and the modern gives value to modern techniques.</p> <p>It helps students develop a multicultural approach.</p> <p>It opens up opportunities for investigations.</p> <p>Obstacles from the past help explain what students find difficult.</p> <p>Students find comfort in realizing that they are not the only ones struggling.</p> <p>It encourages faster students to study harder.</p> <p>It helps explain the role of Mathematics in society.</p> <p>It makes math less scary.</p> <p>It helps to sustain interest and excitement in Mathematics.</p> <p>It offers opportunities for interdisciplinary work.</p>	<p>"I don't have time for that in class!"</p> <p>"That's not Math!"</p> <p>"How do you ask a question about that on the test?"</p> <p>"That doesn't improve students' scores!"</p> <p>"Students don't like it!"</p> <p>"Students regard it as history and they hate history!"</p> <p>"Students find it as tedious as math!"</p> <p>"Students don't have the knowledge to appreciate it!"</p> <p>"Progress in mathematics is about making difficult problems a routine, so why look back?"</p> <p>"We are short of material!"</p> <p>"Lack of training!"</p> <p>"I'm not a historian. How can I be sure of the accuracy of the content?"</p> <p>"Telling what really happened can be pretty crooked and can confuse rather than clarify"</p> <p>"Reading original texts is very difficult"</p> <p>"Can't this encourage chauvinism and nationalism?"</p> <p>"Is there any empirical evidence that students learn better?"</p>

Source: Fauvel (1992) and Siu (2006)

To some extent, many of the objections listed by Siu (2006) can be considered overcome. For example: “This is not math!”. Resistance seems to reflect the discipline's late and irregular inclusion in teacher education programs. In Brazil, for example, it was only in the 2000s that the “content of education science, history and philosophy of science and mathematics” was recommended as a common part of all undergraduate courses in the area, according to the National Curriculum Guidelines for the Mathematics, Research and Degree Courses (BRASIL, 2002b, p. 6). In this regard, in an analysis of curricular elements of classroom teaching courses in Mathematics in Ceará, Carmo and Queiroz (2020, p. 1) found that the HM discipline is heterogeneous and that its didactic use “is still very timid”.

Anyway, with or without formal recommendation, it must be made clear that HM has been systematically considered part of mathematics. In the index of *Mathematics Subject Classification*, which lists all areas and subareas of mathematics (ASSOCIATE EDITORS OF MATHEMATICAL REVIEWS AND ZBMATH, 2020), “History and biography” is the second of 98 items — the first is “General topics”. For many, HM is not only a part, but a fundamental part, of mathematics. Heiede (1992, p. 152) is emphatic: “If you teach mathematics, you also need to teach HM”. And challenges: “They didn't teach you logarithm if you haven't heard of (John) Napier”.

Other suspicions listed by Siu (2006) seem, at first sight, insurmountable. It would be imprudent, for example, to discredit the teacher who intuits that the student does not like HM. However, there are also many reports of teachers who are satisfied with its use in the classroom — such as those who write. We note from the outset that this type of objection directly clashes with the HM pioneers' intentions (HEPPEL, 1893; CAJORI, 1894) and that Fauvel (1991) lists as the primary reason for the didactic resource: HM helps to increase the motivation for learning.

Many teachers' frustrations may derive from extreme expectations for the teaching resource. One cannot expect that a historical reference is enough to motivate a classroom. “The most naive”, remark Miguel and Miorim (2019, sp), “end up attributing to history an almost magical power to change the student's attitude”. Fauvel (1991) warns that the adoption of HM is not easy — especially, we add, if the teacher does not have the appropriate training or teaching material. For Heiede (1992), HM should not be treated as a way to make Mathematics more fun. Taken as a panacea, the use of HM can, in fact, be quite disappointing.

There are also some open questions in the list of resistances pointed out by Siu (2006). For example, is there any empirical evidence that students learn better that way? Siu (2006, p. 275) bluntly answers the question: the evidence that the HM makes the student learn better is sparse and not always positive. The results, he says, seem to indicate more notable positive effects on affective rather than cognitive aspects. “In classes where HM is used, students like the subject more, but do not necessarily perform better on tests”, he sums up.

The question is hard, but perhaps misleading. How do you measure which students are learning best? And what is to be understood by the use of HM? The answers will necessarily be sparse, as they depend on different practices. We believe that it is possible to overcome this issue by invoking two reasons given by Fauvel (1991) in favor of the HM. The resource “it gives mathematics a human face” and “it helps explain the role of mathematics in society”. These are properties that are linked to the notion of integrated knowledge, defended by Freudenthal (1981). Higher grades may be a bonus, as the author writes, but they are no reason to adopt, or discard, HM in the classroom.

Among the suspicions compiled by Siu (2006), there are also delicate questions, such as: can HM encourage chauvinism and nationalism? It is true that, in the course of history, there have been rivalries between mathematicians from different countries, an example of which is the famous dispute over the paternity of Calculus. But the historical approach has precisely the power to dispel myths in this regard. A more serious risk, taking chauvinism in a broader sense, is reinforcing the stereotype of mathematics as an essentially male activity. Few women are mentioned in history books: Hypatia of Alexandria (c. 370-415 AD) and, more rarely, the Frenchwoman Marie-Sophie Germain (1776-1831). But this risk is not removed, it seems to us, by disguising the fact that the best-known landmarks and the main theorems taught to the student are actually attributed to men. Rather, it is an opportunity to discuss the place of women in society, yesterday and today. The trajectory of Sophie Germain, who, to be taken seriously, signed her correspondence as Monsieur Leblanc (BOYER & MERZBACH, 2011), is quite illustrative of the barriers that have already impaired women from participating in scientific activity.

## **5 Proposal of teaching material**

Given the encouragements and objections discussed, in this work, we propose



a format of textual didactic material linked to a specific skill dictated by the BNCC-EM, made of a historical account and mathematical tasks inspired by it.

We took the term report from the typology of textual genres by Rose (2012). In the teaching sphere, the author classifies them according to four purposes: to engage, inform, propose, and evaluate. Historical accounts are borderline types, linked to the functions of informing and engaging, which are precisely the objectives of this material. As we have seen, the teacher has his/her reasons for distrusting the use of the HM. Thus, before trying to motivate the students, we must give attention to the teacher's engagement: it is essential that he takes possession of the historical content to be convinced that the HM enriches classroom work.

According to Rose (2012), reports can be explanatory (*historical accounts*), when they establish causal relationships, or not (*historical recounts*). They can be biographical and autobiographical. In common, they develop according to phases marked in time from an initial stage of guidance. Thus, they are not confused with properly academic genres, such as essays and articles.

For the construction of the report, we started with the sentences that define a specific skill of the Common Core, paying attention to the terms used and the notions they imply. To choose the historical elements that best fit the text of the legal framework, we initially took into account the following criteria: historical relevance, availability of sources, and adequacy to the teaching stage. Regarding relevance, the emphasis given in reference works was considered. As for the availability of sources, we gave preference to documents that can be accessed via the internet, free of charge. As for the adequacy to the teaching stage, we focused on those that address mathematical properties of current use in the classroom. In a second step, we confronted our initial choices with the reflections made in the previous item: then, we sought to select the historical elements that not only fit the text of the BNCC-EM, but also correspond to incentives given for the use of HM, in particular, those of giving “a human face to mathematics” and “explain the role of mathematics in society”, as Fauvel (1991) intended.

For the writing of the reports, we sought a narrative tone as friendly as possible, as far as scientific rigor allows. It is a tone befitting the dual function of engaging and informing. It is certainly not a simple task, but the effort is necessary, not only to captivate the teacher, but to allow the gathered information to eventually be conveyed

to the student without the embarrassment of the jargon. It is the effort made by some of the pioneers of the HM, especially the Frenchman Jean Étienne Montucla (1725-1799), author of *Histoire des Mathématiques* (MONTUCLA, 1758), the first classical history of mathematics (VOGEL, 1974), and still a good reading (STRUICK, 1987).

A task is taken as the “organizing element of the learner's activity”, according to Ponte (2014, p. 14). “The activity [...] essentially concerns the student and refers to what he does in a given context. On the other hand, the task represents only the objective of each of the actions in which the activity unfolds”, explains the author. Thus, the tasks proposed here are not confused with didactic activities, as much as the reports are not confused with lesson plans or didactic sequences. They seek to demonstrate that the HM can be used to bring anecdotes and curiosities and investigate mathematical concepts and establish procedures.

Ponte (2014) distinguishes tasks in terms of structure, according to the degree of indeterminacy of the question; and as for the challenge, according to the perceived difficulty. Closed, reduced-challenge tasks are classified as exercises. High-challenge closed tasks are problems. Open-ended, reduced-challenge tasks are known as explorations. And high-challenge open tasks are investigations. We will seek here to explore the different types of tasks, taking into account the diversity of pedagogical practice.

Taken as a whole, reports and tasks retain the characteristics of educational products, “tools developed by the professionals in training that contain organized knowledge aiming to make pedagogical practice viable” (FREIRE, ROCHA, & GUERRINI, 2017, p. 380). However, we prefer to treat them with a broader and more common expression in the school environment, “didactic material”, to emphasize that this is not presented as a finished product like other resources of the type. Rather, it is expected to serve as a starting point for the teacher when, according to his/her inclinations, he/she decides to adopt HM in his/her classes. In this regard, we recall Tezza (2002, p. 8), for whom “unfinishedness” is the most outstanding quality of teaching material: “We must always be suspicious of definitive textbooks; a good material is perhaps rather a suggestion of material”. We hope this is our contribution.

## 6 Example of teaching material

This example draws on the pioneering contributions of Michael Florent van

Langren (1598-1675), Christiaan Huygens (1629-1695), and Edmond Halley (1656-1742) to the development of graphic representation techniques in the-17th century Europe. We propose to associate it with the BNCC-EM skill identified by the EM13MAT102 code, which can be developed through all high school grades and is described as follows:

Analyze tables, graphs, and samples of statistical research presented in reports published by different media, identifying, when applicable, inadequacies that may lead to interpretation errors, such as inappropriate scales and samples (BRASIL, 2018a).

From a historical point of view, the content is linked to the Great Navigations, the dispute between the Netherlands and Spain for political hegemony and the Scientific Revolution.

## 6.1 Historical report

### **Guidelines: Graphics in the 17th century**

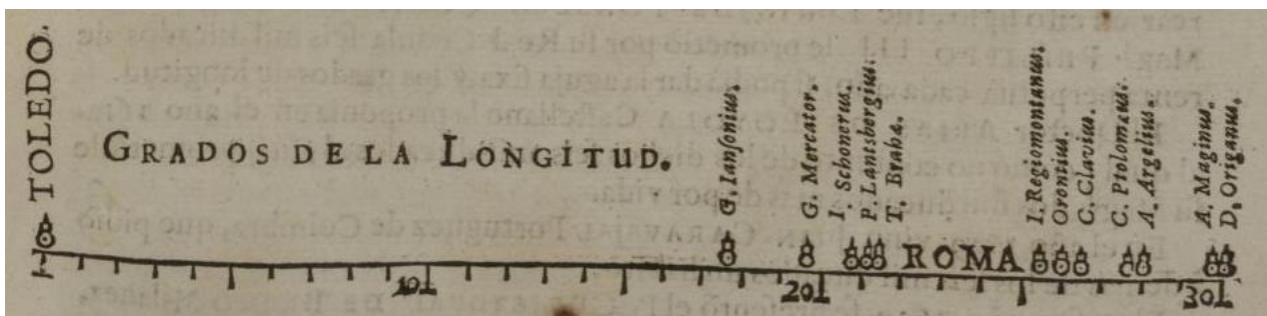
Graphics/charts are everywhere these days: in the press, in advertising, in the classroom, in government, and business reports. They can be taken as a “universal language” (FUNKHOUSER, 1937, p. 270) that facilitates to understand and analyze information. It has not always been like that. Like other languages, this one was also developed and develops in the course of history. Although there are prehistoric registers of maps of the sky and Earth, the dissemination of diagrams depended on circumstances given only in the 17th century. In the theoretical field, concepts of analytical geometry and probability were established. In practice, measurement instruments were improved, and systematic surveys of statistical data emerged. In short, “some data of real interest, some theory that would give them meaning and some ideas of visual representation” emerged (FRIENDLY, 2008, p. 21). Let us look in detail at three outstanding “ideas of visual representation” of the 17th century.

#### **Phase 1: An accurate picture of “huge mistakes”**

Son and grandson of cartographers, Michael Florent van Langren published in 1644, in Flanders, then part of the Spanish Netherlands, a treatise in which he claimed to have solved the so-called longitude problem. Estimating latitude through the mere observation of the Sun and stars was long known, but the calculation of longitudes, which is particularly relevant for ocean crossings, still challenged navigators. To

enhance his strategy, Van Langren decided to highlight the “enormous errors” (VAN LANGREN, 1644, p. 3) of estimates attributed to “eminent astronomers and geographers”, such as Tycho Brahe (1546-1601) and Nicolaus Mercator (1620-1687), for the difference in longitude between Rome and the historic city of Toledo, official residence of Spanish monarchs. To highlight the discrepancy, he displayed the estimates along a graduated line (Figure 1).

Figure 1: Estimates of the longitude difference between Toledo and Rome.

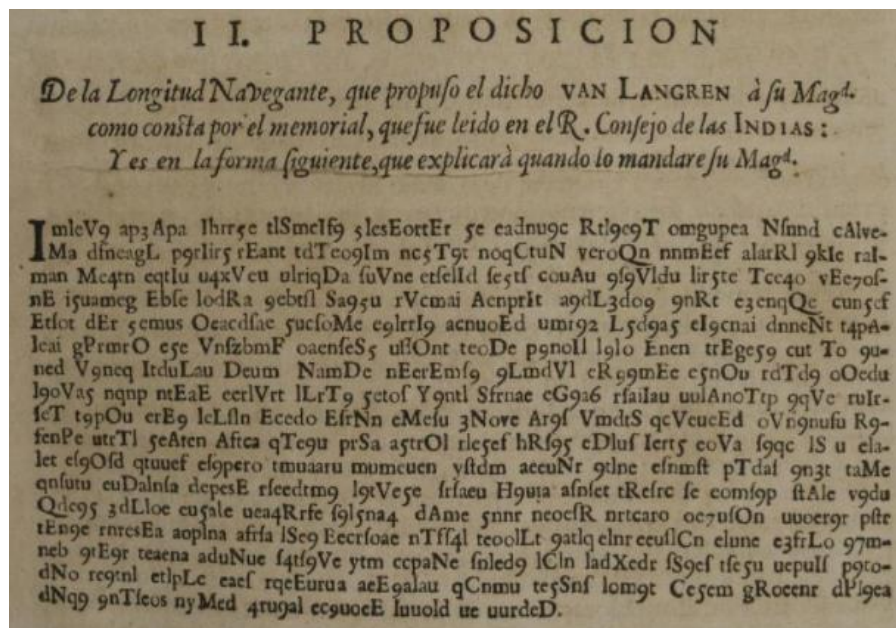


Source: “La Verdadera longitud por mar y tierra” [The true longitud by sea and land] (VAN LANGREN, 1644, p. 3)

The Van Langren diagram is considered to be the oldest graphic representation of a statistical nature (FRIENDLY et al., 2010). Besides pioneering, one can salute him for his excellence. The solution found makes it possible to promptly identify the range of values and also suppose a measure of central tendency – where, skillfully, “Roma” is spelled. Friendly et al. (2010, p. 5) note that Van Langren could have tabulated the data, but “only a graph speaks directly to the eye”. By the way, all values overestimate the difference between the cities: around  $16^{\circ}31'$ .

Van Langren's treatise was also famous for presenting his main result in ciphered form (Figure 2). Van Langren was aware of the value of solving the longitude problem, for which several crowns came to offer prizes, and he undertook to reveal the secret if King Philip IV so ordered.

Figure 2: Van Langren's encrypted strategy.



Source: Van Langren (1644, p. 8)

The secret remained undeciphered for centuries. In 2021, the Belgian press reported that the code had finally been cracked, and the alleged solution began to circulate on blogs and forums dedicated to cryptography. In any case, there is no evidence that Van Langren's strategy was enough to solve the problem, which would only be definitively overcome in the following century, with the invention of marine chronometers.

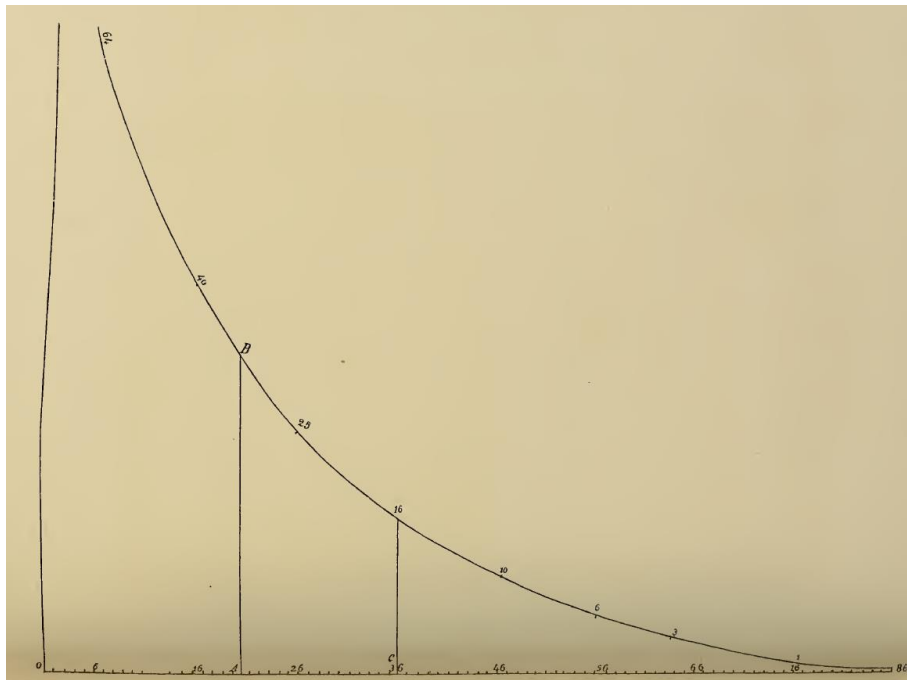
### Phase 2: The science of the modern state

We should note that Van Langren's work only has a statistical nature when considering the modern sense of the term. The word comes from the Latin "status", state, and was originally defined quite differently, as this passage from the 18th century attests: "[...] the science that is called statistics teaches us the political arrangement of all modern states of the known world" (VON BIELFELD, 1770, p. 269). The definition, therefore, did not apply to measures of longitude, but to data relating to the functioning of the states.

The great hallmark of the science of the "modern state" is the *Natural and Political Observations Made Upon the Bills of Mortality* (1662), by the Englishman John Graunt (1620-1674). In 1603, the King of England, James I, had ordered the weekly survey of baptisms and burials, which served to monitor the bubonic plague. Graunt pored over thousands of weeks' worth of data and synthesized it into tables. He noted, for example, the regularity of deaths due to certain diseases, such as tuberculosis, and the erratic frequency of deaths due to the plague, which would be overcome in the

following decade (MORABIA, 2013).

Figure 3: The lifeline modeled by Huygens.



Source: Huygens (1895, p. 531)

From the mortality tables organized by Graunt, the Dutchman Lodewijk Huygens (1631-1699) calculated what we now know as life expectancy and communicated the result by letter to his older and more famous brother, the astronomer and mathematician Christiaan Huygens (1629-1695). The latter, in a reply dated November 1669, sketched a continuous curve to represent the number of survivors out of a total of 100 births for each age (HUYGENS, 1895), reproduced in Figure 3.

Christiaan's graph is based on the following coordinate system: ages 0 to 86 are represented on the horizontal axis; on the vertical axis, the number of survivors at those ages out of a total of 100 births is recorded. In modern notation, we could define it as the representation of a certain real variable function  $f: \mathbb{R} \rightarrow \mathbb{R}$ , such that  $f(x)$  is the percentage of people who have reached the age  $x$ .

Christiaan advocates that the median of the number of years a person has left should be taken as their life expectancy. And shows how to infer it from the plotted curve. In modern notation: given the age  $x_1$ , just search graph  $x_2$  such that  $f(x_2) = \frac{f(x_1)}{2}$ . For example, using Christiaan's original markings: out of 100 born, only 32 reach the age of 20 (points A and B in Figure 3). Of these, only half will reach 36 (point C). So, a 20-year-old can expect to live another 16 years.

This exchange of letters is at the origin of life expectancy studies. Here we have a clear example of the use of a graph, not only for the presentation of results but for statistical inference. For Boyer (1947, p. 148), Huygens' lifeline would be the first example of a graph produced from statistical data. Friendly et al. (2010), giving precedence to Van Langren, consider Christiaan's sketch the first statistical graph of a continuous distribution.

### Phase 3: A “graph of success”

A few years after the correspondence of the Huygens brothers, the Englishman Edmond Halley (1656-1742) published what can be considered the first “great success” in the application of the coordinate system to scientific investigation (BENIGER & ROBYN, 1978, p. 2). His article, dedicated to the relationship between atmospheric pressure and altitude, was published in the oldest journal dedicated exclusively to scientific production, the English *Philosophical Transactions of the Royal Society*, created in 1665.

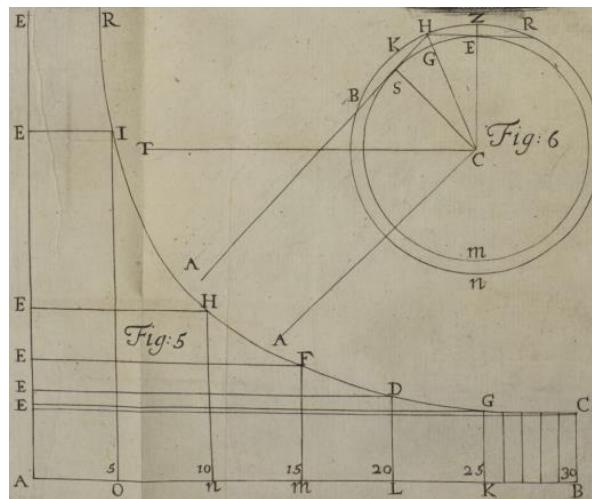
The atmospheric pressure was already known to decrease as altitude increased. The Italian Evangelista Torricelli (1608-1647), who invented the barometer in 1643, explained in a letter the relationship between magnitudes through the following analogy: “We live submerged at the bottom of an ocean of elementary air, which, by undoubted experiments, we know to have weight” (TORRICELLI, 1823, p. 33). And the further down that ocean, the greater the weight.

In his article, Halley starts from the observation that the “expansion of the air” and the “weight of the atmosphere” are inversely proportional (“reciprocal”), as much as volume and pressure, as Robert Boyle (1627-1691) had proposed in 1662. Taking the height of the mercury column in the barometer as a measure of the “weight of the atmosphere”, Halley ponders: it is “evident” that “with the help of the Hyperbola Curve and its Asymptotes”, the “expansions of the air” can be known from of any measure of the barometer (HALLEY, 1687, p. 105). The curve in question, the first bivariate graph of experimental data (FRIENDLY & DENIS, 2005), is presented at the end of the journal (Figure 4).

In the graph, the abscissas  $AB$ ,  $AK$ ,  $AL\dots$  are the measurements of pressure, i.e., the height of the column of mercury in the barometer, in inches. The ordinates, all represented by the letter  $E$ , are valid for the “expansion of air”. Assuming that “expansion of air” and “weight of the atmosphere” are inversely proportional, we have

that their product is constant. In modern notation, we have  $x \cdot y = k \in \mathbb{R}$ , which, in fact, is the equation of a hyperbola. Thus, for all points on the curve, the rectangle defined by abscissa and ordinate has the same area:  $A_{ABCE} = A_{AKGE} = A_{ALDE} = k$ , as Halley notes.

Figure 4: Halley's "Hyperbola Curve", identified as Fig. 5.



Source: Halley (1687, p. i)

It is not, however, a graph of pressure by altitude, as one might imagine. But Halley (1687, p. 105) shows how to take this last step: altitude corresponds to the sum of all “expansions of air”, taken “infinitely small” from sea level, where the column of mercury measures 30 inches (the abscissa  $AB$ , in the graph). Hence why Halley treats all ordered by  $E$ : he is not interested in their values, but in the areas under the curve. The modern reader can already intuit what this sum of “expansions of air” taken “infinitely small” is: the definite integral of a function  $f: \mathbb{R} \rightarrow \mathbb{R}$ , such that  $f(x) = \frac{k}{x}$ , where  $x \neq 0$  is the pressure and  $f(x)$  is the expansion of air, in the interval of  $x$  at 30, which is the barometer measurement at sea level:

$$\int_x^{30} \frac{k}{x} dx = k \cdot (\ln(30) - \ln(x)) = k \cdot \ln\left(\frac{30}{x}\right).$$

Halley did not have the weapons of integral calculus at hand, then in its dawn. On the other hand, he was aware of the work of Grégoire de Saint-Vincent (1584-1667), among others that studied the squaring of the hyperbola and recognized the proportionality relationship between the areas under the curve and the difference between logarithms (FRISINGER, 1974). That is all it took. Let us see how this notion fits into integral calculus.

Applying the relationship between areas and logarithms to the Halley curve, we



have:

$$\frac{A_{BKGC}}{\log AB - \log AK} = \frac{A_{BLDC}}{\log AB - \log AL} = \frac{A_{BMFC}}{\log AB - \log AM} = \frac{A_{BNHC}}{\log AB - \log AN} = r.$$

In general, the area under the hyperbola between  $x$  and  $AB$  can be given by:

$$A(x) = r \cdot (\log AB - \log x).$$

Taking  $AB$  per 30 inches (the pressure at sea level), we have:

$$A(x) = r \cdot (\log 30 - \log x) = r \cdot \frac{\log 30}{\log x}.$$

Passing the logarithm to the natural base, we get:

$$A(x) = r \cdot \frac{\log 30}{\log x} = r \cdot \frac{\ln\left(\frac{30}{x}\right)}{\ln(10)},$$

and by doing  $k = \frac{r}{\ln(10)}$ , we arrive at:

$$A(x) = k \cdot \ln\left(\frac{30}{x}\right) = \int_x^{30} \frac{k}{x} dx.$$

But how to determine the constant of proportionality  $k$ ? To do so, Halley takes the ratio between the densities of air and mercury at sea level to be 1:10,800. So, an inch of mercury corresponds to a column of air of 10,800 inches, or 900 feet. Here is the “expansion of air” at sea level, where the barometer measurement is 30 inches. Then,  $k = 30 \cdot 900$  feet, and so we can calculate the altitude of a place where the barometer measures  $x$  inches as follows:

$$A(x) = 30 \cdot 900 \cdot \ln\left(\frac{30}{x}\right) \text{ ou } A(x) = \frac{900}{0,0144765} \cdot \log\left(\frac{30}{x}\right).$$

And from this rule, Halley calculates the table of values given by Figure 5.

Today we know there are discrepancies in the values found by Halley, errors smaller than an inch of mercury (CREWE, 2003). The theme is an excellent invitation to an interdisciplinary approach. Based on Halley's reasoning, it is enough to isolate the pressure variable to arrive at the equation known as the barometric formula or Halley's Law:  $P(z) = P_0 \cdot e^{-\lambda z}$ , where  $P$  is the pressure,  $z$  is the altitude and  $\lambda$  a constant for air density, gravity, and atmospheric pressure at sea level.

Figure 5: Measurements of pressure, in inches of mercury; and altitude, in feet.

A Table shewing the Altitude, to given heights of the Mer- cury.		A Table shewing the heights of the Mercury, at given Al- titudes.	
Inch.	Feet.	Feet.	Inch.
30. — — — — —	0	0 — — — — —	30, 00.
29. — — — — —	915.	1000. — — — — —	28, 91.
28. — — — — —	1862.	2000. — — — — —	27, 86.
27. — — — — —	2844.	3000 — — — — —	26, 85.
26. — — — — —	3863.	4000 — — — — —	25, 87.
25. — — — — —	4922.	5000 feet — — — — —	24, 93.
20. — — — — —	10947.	1 mile — — — — —	24, 67.
15. — — — — —	18715.	2 — — — — —	20, 29.
10. — — — — —	29662.	3 — — — — —	16, 68.
5. — — — — —	48378.	4 — — — — —	13, 72.
1. — — — — —	91831.	5 — — — — —	11, 28.
0, 5. — — — — —	110547.	10 — — — — —	4, 24.
0, 25. — — — — —	129262.	15 — — — — —	1, 60.
0, 1. — — — — —	29 mil. or 154000.	20 — — — — —	0, 95.
0, 01. — — — — —	41 mil. 216169.	25 — — — — —	0, 23.
0, 001. — — — — —	53 mil. 278338.	30 — — — — —	0, 08.
		40 — — — — —	0, 012.

Source: Halley (1687, p. 106)

We see that Huygens and Halley do not explicit the algebraic expressions that they seek to represent graphically, so the discussion is almost entirely rhetorical. But what matters here, beyond the formulas, is to verify the importance of the graphic resource both for the description and for the interpretation of such diverse phenomena.

## 6.2 Mathematical tasks

Task 1 (Exploration). Collect guesses about some measure that we can, in a second moment, verify, such as the distance between two cities; the mass of an object; the duration of a song; the room temperature; the height of a building; etc.

- represent the data in the manner of the cartographer Michael Florent van Langren, i.e., mark the different estimates on a graduated line;
- determine mean, mode, and median of the data and check which of those values is more or less affected by outlier guesses;
- check the actual measurement and compare with the estimates taken.
- consider other ways of representing the results (bar graph, histogram, *boxplot*, table, etc.) and discuss possible advantages and disadvantages of the different strategies.

*Note to teacher.* This task can be done collectively, simply by gathering estimates from each student to a certain extent. It is possible that the actual measurement approaches the common sense of the room. It is also possible that the students' guesses are far from the real value. In this case, we can discuss the biases

that led students to overestimate or underestimate the measure. How familiar are they with the measured object? What did they take as a basis for comparison? As for the data representation strategy, the student should realize that Van Langren's solution is quite ingenious, even when compared to solutions generated by digital means. The teacher can draw students' attention to three attributes, beyond aesthetic judgment: clarity (the graph has a well-defined title, scale, and markings, without ambiguities), conciseness (the information has been reduced to the essentials, without redundancies) and organization (the graph allows you to readily estimate the range of measures and central trends). Its limitations can also be pointed out: it is clear that Van Langren's strategy is not useful for a very large collection of data. In this case, more efficient strategies can be discussed, such as histograms, supported by digital tools.

Task 2 (Investigation). In Van Langren's graph, all values exaggerate the difference between Roma and Toledo. Discuss the following hypotheses:

- a) coincidence: ancient geographers could err on the side of plus or minus;
- b) bias in taking measurements: in the absence of precision instruments, human beings systematically tend to exaggerate long distances;
- c) bias in calculating longitude: the difference in degrees depends on the estimate of the Earth's radius, which was thought to be smaller.

*Note to teacher:* The hypothesis of coincidence is quite weak, which should be evident to the student: the probability that 12 estimates all fall above the true value by chance is very small. A possible bias in taking the measure involves the discussion of the methods available to each geographer, which goes far beyond the scope of the class. But generally speaking, students can debate their own perception of distance. Do you tend to exaggerate it? Do you tend to minimize it? Under what conditions? Finally, it is possible that the calculation of the longitude estimates collected by Van Langren embedded a systematic error. Tufte (1997) suggests that the discrepancy is the result of accounts that underestimate the radius of the Earth. But the discrepancy – almost 50% compared to the average of the data presented – seems too gross, even for the time. Thus, it is possible to assume a combination of errors, both in taking direct measurements and in the indirect calculation of longitude. Alternatively, one can even question the very fidelity of the data reproduced by Van Langren, whose sources are not known. In any case, it is worth noting that, contrary to what the cartographer supposed, a “true” measurement, free from error, is not allowed for the measurement

in question.

Task 3 (Exercise). In the graph made by astronomer and mathematician Christiaan Huygens, the curve represents the total number of survivors of a certain age, out of 100 born. For example, point C in the figure indicates that only 16 out of 100 people born reached 36. Analyzing the graph, answer:

- out of every 100 born, how many people reached the age of 50?
- out of every 100 born, half would be dead by what age?
- according to the Huygens graph, what would your life expectancy be?

*Note to teacher:* This exercise boils down to reading the graph. To solve item (a), for example, just take the ordinate of the point on the curve whose abscissa is 50 years: about 7 people. Beyond the mathematical result, this question can be used to discuss life expectancy in a “modern state” of the 17th century and the current indicators of human development. What factors increased life expectancy? What factors explain current inequalities?

Task 4 (Problem). Consider the real variable function  $f(x) = 27\,000 \cdot \ln\left(\frac{30}{x}\right)$ ,  $x \neq 0$ , defined from an article by Edmond Halley, such that  $f(x)$  is the altitude of a city, in feet, under atmospheric pressure  $x$ , in inches of mercury:

- suppose a city with altitude  $Y_1$  where the pressure measured in the barometer is  $X_1$ . What is the pressure  $X_2$  in another city whose altitude is half of  $Y_1$ ?
- suppose a city with altitude  $Y_1$  where the pressure is  $X_1$ . What is the altitude  $Y_2$  of another city where the barometer measure is twice that of  $X_1$ ?

*Note to teacher:* This problem explores the operative properties of logarithms and solving logarithmic equations. If necessary, the teacher can facilitate the task by assigning real values to the variables, in order to highlight the unknown to be determined. The answers may seem counter-intuitive to the students, and it is therefore worth emphasizing that the quantities in question are not proportional.

Task 5 (Investigation). Take the data from the table (Figure 4) produced by Halley and:

- sketch a graph of altitude by pressure;

- b) choose five cities, look up their altitudes and convert the values to feet, using the following approximation  $1 \text{ foot} = 0.3 \text{ meter}$ ;
- c) using the graph in item (a), find the atmospheric pressure of the five cities in inches of mercury. Convert to millimeters of mercury, using  $1 \text{ inch} = 25.4 \text{ mm}$ .

*Note to teacher:* This task does not offer major difficulties to the student, but it is an interesting opportunity to research the characteristics of the cities. The investigation can be well performed with a calculator and graphical tools, such as the Geogebra dynamic geometry software.

## 7 Final considerations

Throughout this research, we examined arguments for and against the use of HM in the classroom. In view of the apparent impasse, it was deemed convenient to propose not a finished product, but a format of didactic material designed to engage the teacher. It is an essentially simple format. The historical report, as we said, has the dual function of involving and informing the teacher; the tasks demonstrate the potential of the historical elements for the investigation of mathematical concepts and the establishment of procedures in the fulfillment of the common curriculum. To this end, the use of historical sources proves to be especially fruitful. Besides informing the difficulties and the context in which certain notions were developed, they also serve as inspiration for the development of exercises, problems, explorations, and investigations.

In the example given, we have seen how Van Langren, Huygens, and Halley use graphic representations to face concrete issues of their time. It richly illustrates, we think, the notion that the HM helps explain the role of mathematics in society, as Fauvel (1991) suggests. We do not expect, of course, that the students have the fluency and reading breath needed to go through 17th-century treatises. However, with the proper mediation of the teacher, they can identify its main elements, especially the excerpts that address the diagrams in question, available online.

The suggestion of mathematical tasks inspired by historical elements seeks to overcome the objection that the HM is not mathematics. It is true that many textbooks still reduce the HM to anecdotal history, to use the happy expression by Miguel and Miorim (2019, sp), but it is equally true that it can be taken as a strategy to clarify and

deepen mathematical concepts and procedures, as shown by Carvalho, Cavalari, and Cristóvão (2021). It was what we were looking for.

Of the questions raised, one seems to us, at this point, insurmountable: the question of motivation. We can admit that mathematics has good stories to tell. We can admit that good stories, including anecdotes, have the power to interest and even predispose the students in class. Thus, it seems to us, we can accept that the HM has motivating potential. This, however, is carried out or not according to the didactic circumstance. Enclosed in itself, a good story is not enough to increase the student's interest in the "cold logic" of mathematical ideas, as wanted by Cajori (1894, p. 3). Instead, in the reports and tasks proposed here, we sought to give attention to the difficulties that mark the development of mathematical knowledge to, precisely, remove the stigma of coldness that weighs on the subject matter. Here is one more reason to use the HM: to show that the apparent aridity of a mathematical result derives from the efforts of different people, from different civilizations, in the vibrant course of history.

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