MATHEMATICAL AND STATISTICAL LITERACY:

an analysis based on PISA results

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ABSTRACT

In this paper we discuss the distinction between statistical literacy and mathematical literacy. The starting point of this discussion is the fact that within the Programme for International Student Assessment (PISA) statistical survey items are elaborated only from the mathematical literacy point of view. We first present theoretical elements on the differences between mathematics and statistics, between mathematical and statistical literacy and we elaborate on the growing interest in statistical literacy as a specific competence. Second we present results of an empirical analysis based on the PISA 2003 data. The analysis showed an extremely high correlation between mathematical and statistical literacy. In the conclusion we emphasise the necessity to reveal the notion of statistical literacy within the PISA results.

Keywords: mathematics – statistics – statistics education – mathematical literacy – statistical literacy.

RESUMO

Neste artigo discute-se a distinção entre letramento estatístico e letramento matemático. O ponto de partida dessa discussão é o fato de que no âmbito do Programa para Avaliação Internacional de Estudantes (PISA) itens da pesquisa relacionados à Estatística são elaborados apenas a partir do ponto de vista do letramento matemático. Em primeiro lugar apresentamos elementos teóricos sobre as diferenças entre matemática e estatística, entre letramento matemático e estatístico, tecendo considerações sobre o crescente interesse em letramento estatístico como uma competência específica. Segundo apresentamos resultados de uma análise empírica com base nos dados do PISA 2003. A análise mostrou uma correlação muito alta entre

matemática e estatística. Como conclusão destacamos a necessidade de revelar a noção de letramento estatístico dentro dos resultados do PISA.

Palavras-chave: matemática – estatística – educação estatística - letramento matemático – letramento estatístico.

Introduction

The question whether mathematical and statistical literacy are the same thing has not a straight answer. Looking at the historical, the theoretical and the political development of statistics and later on of statistical literacy, we can observe a diversity of perspectives and a shifted meaning of the notion of statistical literacy. We will argue that over the last decades, statistics has become recognized as distinct discipline. Statistics is no longer seen as a subfield of mathematics (MOORE, 2004; WATSON, 2006). It applies mathematical tools, although this is not unique to statistics since other disciplines also do this, e.g. architecture, designing, art. At the same time, there are core ideas, such as variation, data, uncertainty, which are not mathematical in nature. Kline (1985, p. 501) states that mathematics has to do with certainty, while statistics is a way to handle uncertainty. Whereas the former predicts what must happen in an individual case, the latter can tell us what happens to large groups but does not provide definite predictions about any one given case.

We have to consider that within the field of mathematics education there was – and is – an impressive growth on the conception of mathematics education and even on the notion of mathematics itself. Critical voices on the absolutist view on mathematics have given rise to new conceptions of mathematics and its implications for the field of education. The absolutist view on mathematics is defined by Ernest (1991, p. 7) as "it consists of certain and unchallengeable truths. According to this view, mathematical knowledge is made up of absolute truths, and represents the unique realm of certain knowledge […]". Besides the absolutist view of mathematics, a naturalist theory of mathematical knowledge is gaining popularity in the field. A number of alternate schools in the philosophy of mathematics have arisen, e.g. Humanistic mathematics, constructivists, intuitionists and the research program on ethnomathematics. Humanistic mathematics, for example, brings in an element of fallibility. Constructivists would stress the fact that mathematics is a product of the human cognition. Intuitionists

envisage that mathematics is built up from an empirically neutral mental basis (VAN KERKHOVE, 2007, p. 184). Bishop (1988, p. 18) argues that mathematics can be seen as *essentially a symbolical technology* based on skills or environmental activities of a cultural nature. Another perspective on mathematics is, as we see it, rightly labeled *ethnomathematics*. It refers to the study of mathematical practices, ideas and activities as embedded in their cultural context. Even the so-called academic Western mathematics is developed within a particular context, the same as other mathematical practices are. Proponents of the naturalistic approach to mathematics claim that mathematical knowledge is rooted in the cultural context of the knower (PINXTEN; FRANÇOIS, 2007, p. 214). Mathematics has no longer the status of an absolute certain discipline. In this sense, it comes closer to the identity of statistics.

The understanding that statistics is not just mathematics has given rise to a new conception of statistical literacy and to a new field of study which is called statistics education. This field of study has emerged as an important discipline –with its own conferences and journals– that supports the teaching and learning of statistics. Statistics education is an emerging field that grew out of two main disciplines –statistics and mathematics education– and it is currently establishing itself as a unique field of study (GARFIELD; BEN-ZVI, 2008). While it is closely related to mathematics education, they are not identical. In this paper we want to elaborate on the way in which statistical education and statistical literacy have become a separate subfield. Our research is based on the recognition that statistics is not a purely subfield of mathematics. At the same time we have to recognize that there is a strong relationship between mathematical and statistical literacy. This quasi paradoxical fact generates the question "What can both disciplines learn from each other?" (FRANÇOIS; BRACKE, 2006). In the following section, we first elaborate theoretically on the notion of statistical literacy and its historical growth. Afterwards, we present an empirical analysis of the relationship between statistical and mathematical literacy based on PISA 2003 data. Finally, in the discussion, we compare the theoretical and empirical analyses.

The notion of statistical literacy

The term 'statistical literacy' can be used to describe the knowledge which people need in order to understand and make decisions based on the analysis of

statistics. Haack (1979) states that in order to interpret statistics people need to consider and to scrutinize certain aspects which include the source, the type of data, definition and measurement problems, and certain considerations concerning the survey sample. As with most authors who began to develop the concept of statistical literacy, Haack emphasizes elements which are basically related to the technical dimension of statistics knowledge. This perspective of statistical literacy seems to be based on accepted academic uses of statistics.

Different authors introduce wider perspectives of statistical literacy related to the kinds of statistical skills which are needed by people in everyday life (e.g., Evans 1992). Wallman (1993, p. 1) states in her Presidential Address to the American Statistical Association that "statistical literacy is the ability to understand and critically evaluate statistical results that permeate our daily lives–coupled with the ability to appreciate the contributions that statistical thinking can make in public and private, professional and personal decisions." If we compare this wider perspective on statistical literacy, given by Wallman (1993), with the description of *mathematical literacy* given by the Programme for International Student Assessment (PISA) 2003 establishment, we can see a high correspondence. "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (Organisation for Economic Co-operation and Development [OECD], 2004: 37). In line with this definition, Gal (2002; 2004) emphasizes the need for statistical literacy for all citizens who interpret statistics in various everyday situations. Furthermore, Gal suggests that when people read statistics in the media they have to make inferences, quite often in the presence of irrelevant or distracting information, and they may also have to apply mathematical operations to data contained in graphs. Figure 1 illustrates Gal's perspective of statistical literacy.

Figure 1. A statistical literacy model (Adapted from Gal, 2002).
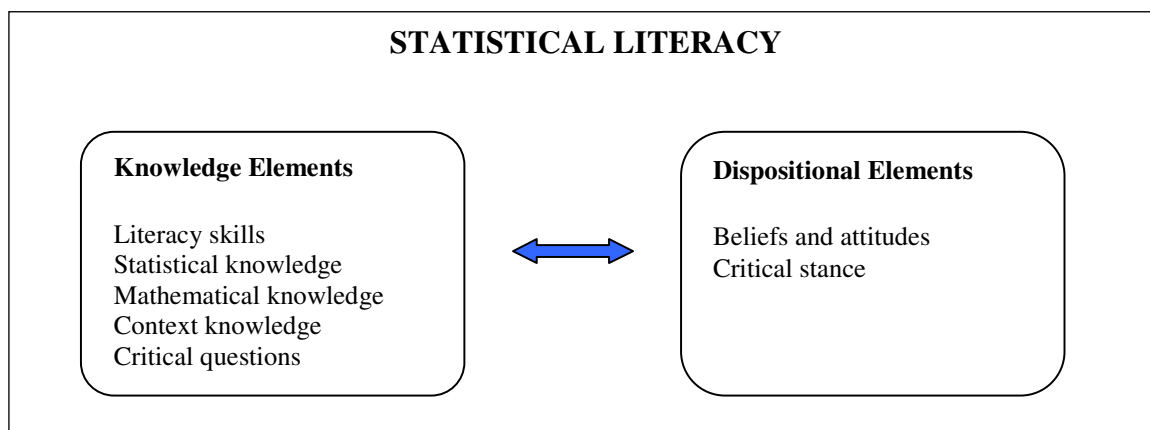
The statistical literacy model (Figure 1) represents two ranges of elements which when combined can enable readers to understand statistical messages. On one side of the diagram there are knowledge elements which involve cognitive components of statistical literacy (e.g., rational understanding of the data such as knowing how to decode and make calculations about it). On the other side are dispositional elements which comprise a range of 'non-cognitive' aspects (e.g., a person who interprets a graph can have knowledge, experiences and beliefs which might differentiate his/her interpretation of the graph). According to Gal, statistical literacy is based on the interaction of the components which comprise each range of elements. Gal's statistical literacy model underlines the fact that the academic or formal schooling background is not the only determinant of the use of statistical skills, as has been discussed in other studies (e.g., MONTEIRO, 2005). To develop statistical literacy, it may be necessary to work with learners in ways that go beyond instructional methods currently in use. To implement all knowledge bases supporting statistical literacy, topics and skills that are normally not stressed at school may have to be addressed (GAL, 2004, p. 73).

The increasing attention paid to statistical literacy also raises certain discussion points. Carvalho (2001) emphasizes that several authors (e.g., WALLMAN, 1993) view statistical literacy as a panacea to solve the problem of the lack of statistical knowledge in several sectors of the society. However, Carvalho highlights the need to discuss issues related to development and transferability of statistical literacy. One important issue related to the role of statistical literacy is associated with the development of active and critical citizens who can read and interpret statistics making connections to

different areas and reading the world and its complexity. Therefore, statistical literacy should enable people to do more than just reading the data but should allow them to criticize and propose alternative interpretations to a given set of data. School systems have a crucial role in developing statistical literacy which enables students to understand why and how statistics is a way of describing the world (FRANKENSTEIN, 1998; MOREIRA, 2002). Garfield and Ben-Zvi (2008) distinguish between statistical literacy, statistical reasoning, and statistical thinking where statistical literacy provides the foundation for reasoning and thinking. They prefer the definition for statistical literacy as "a key ability expected of citizens in information-laden societies, and [it] is often touted as an expected outcome of schooling and as a necessary component of adults' numeracy and literacy. Statistical literacy involves understanding and using the basic language and tools of statistics: knowing what basic statistical terms mean, understanding the use of simple statistical symbols, and recognizing and being able to interpret different representations of data" (GARFIELD; BEN-ZVI, 2008, p. 34). This basic knowledge makes it possible to reason with statistical ideas and to make sense of statistical information. At this stage, students must be able to connect one concept to another and to combine ideas about data and chance, what is called statistical reasoning. The final stage of statistical thinking includes a deep understanding of the theories underlying statistical processes and methods. It also includes the critical competence of understanding the constraints and limitations of statistics and statistical inferences. That is why this stage of statistical thinking is called "the normative use of statistical models" by Garfield and Ben-Zvi (2008).

Despite the diversity of perspectives and the emergent issues related to statistical literacy, mathematics and statistics educators started to agree that statistical literacy is a specific area besides mathematics literacy. Statistics education and statistical literacy have become a separate and unique field with its own associations, its own journals and international conferences.

Relation between mathematical and statistical literacy in PISA 2003

In this section, we would like to grasp the relationship between mathematical literacy and statistical literacy based on an analysis of empirical data from PISA 2003 survey. On the one hand there is the broad, complex and shifted meaning of statistical literacy as explained in the theoretical section which we could call 'real' statistical literacy. On the other hand there is the way in which the PISA 2003 survey is putting the concept of statistical literacy in operation. It could be that measuring 'real' statistical literacy is beyond the PISA 2003 survey and moreover that 'real' statistical literacy is beyond the ability typically seen in 15-year olds (BROERS, 2006).

Broers (2006) argues that the propensity to think statistically cannot easily be transferred in a classroom situation. Pupils are taught a rather technical oriented curriculum e.g. how to summarize data graphically and numerically, and how various models may be used to infer probabilistic statements on target populations, while statistical literacy is a complex competence.

It can be the fact that statistical literacy (and mathematical literacy) is not that easy to learn but they are key abilities expected of citizens in information-laden societies, and it is an expected outcome of schooling (GARFIELD; BEN-ZVI, 2008). The only thing we can proof is the fact that elements of probability theory and statistics take part in the (mathematics)curriculum and that PISA tries to measure those elements. In the following section we will explain how the elements of probability theory and statistics are included within the PISA 2003 survey.

3.1. WHY PISA DATA?

Currently, two international surveys evaluate student performances: PISA and Trends in International Mathematics and Science Study (formerly known as Third International Mathematics and Science Study – TIMSS) funded by the International Association for the Evaluation of Educational Achievement (IEA). These two evaluation systems develop their studies using different approaches. Hutchison & Schagen (2007) set up an analysis to describe the differences between TIMSS and PISA. They argue that PISA has stolen a march on TIMSS by introducing the life skills aspect to evaluation. To Hutchison & Schagen, this innovation demonstrates the importance of ongoing methodological development as part of any such study. In their

analysis they are differing TIMSS and PISA in four main ways. Firstly, TIMSS focuses on the curriculum-related tasks, whereas PISA is literacy based. Secondly, PISA items are aimed at life skills while TIMSS items are more knowledge oriented. Thirdly, TIMSS focuses on the extent to which students have mastered mathematics and science as they appear in school curricula, whilst PISA aims to capture the ability to use mathematical and scientific knowledge and skills to meet real-life challenges (HUTCHISON; SCHAGEN, 2007). Fourthly, TIMSS is explicitly organized around two frameworks, a curriculum framework (which envisages three layers: intended, implemented and attained curriculum) and an assessment framework. PISA focuses on skills for future life rather than on the grasp of the school curriculum.

PISA aims to assess reading literacy, mathematical literacy, scientific literacy and problem solving. The prime aim of the OECD/PISA assessment is to look "at young people's ability to use their knowledge and skills in order to meet real-life challenges rather than how well they had mastered a specific school curriculum" (OECD, 2005, p. 9). Whist PISA data emphasizes literacy and problem solving, TIMSS data is based on students achievement on the curriculum-related tasks. Since the focus of our research question is on mathematical literacy we want to elaborate PISA data which are explicitly focused on mathematical literacy. Besides the focus on mathematical literacy, PISA data has a larger proportion of mathematical items focusing on the domain of data and uncertainty. While 39% of PISA 2003 items are classified under the mathematical content domain 'data and uncertainty', for TIMSS, the corresponding score is only 13%. TIMSS items are mainly classified under the mathematical content domain 'algebra and number' (HUTCHISON; SCHAGEN, 2007, p. 238).

Why PISA 2003 data?

PISA conducts its survey on a three-year cycle, starting in 2000. In each round, one domain is taken as the main subject. For our purposes, the empirical data from PISA 2003 survey is most interesting because its main focus was on mathematical literacy, occupying about two-thirds of the testing, with the remaining testing time being divided between the other two minor domains, reading and scientific literacy. The main focus in PISA 2000 and PISA 2006 was respectively on reading and scientific literacy. Within

the PISA 2003 survey, four subfields of mathematics are defined. Each subfield is defined by its corresponding subscale:

1) space and shape subscale, related to spatial and geometric phenomena and relationships,

2) change and relationship subscale, related to mathematical manifestations of change, functional relationship and dependency among variables,

3) quantity subscale, related to numeric phenomena and quantitative relationship and patterns, and

4) uncertainty subscale, related to probabilistic and statistical phenomena and relationships (OECD 2004).

These four different subscales represent mathematical literacy in general. Besides these, two other scales are available, namely the reading and science scales. The relationship between the different subscales of mathematical literacy is not discussed on standard PISA 2003 reports (e.g., OECD, 2004). In this paper we discuss the correlations between the different subscales presented in the technical report on PISA 2003 data (OECD, 2005, p. 109).

Our discussion is motivated by our interest in analysing the interrelations of the different subscales of mathematical literacy, as well as the interrelations with the reading and science scales for the PISA 2003 data. In order to investigate the distinctiveness of statistical literacy, we are focussing on the uncertainty subscale because it is most clearly tied to statistical literacy. The uncertainty scale is related to probabilistic and statistical phenomena and relationships. Therefore, we especially want to explore the interrelationship between the *uncertainty* subscale and the other mathematical literacy subscales. Also the interrelationship between the other three mathematical literacy subscales (space and shape, change and relationship and quantity) has been investigated. Finally we want to mention the interrelationship with the reading and science scales. They are not the main topic of investigation but they can bring in some argument for what we will call *general literacy* behind the data.

Data analysis

In this paper we undertake a secondary analysis of the PISA 2003 data available at the OESO databank (OECD, 2005). The PISA 2003 results do not consider statistical literacy as a specific component to be evaluated. Therefore, that survey approaches statistical literacy only from the point of view of mathematical literacy. In order to

answer our research question, we analyze at two levels: 1) at the level of the countries, and 2) at the level of the individual students. At both levels, the relationship between variables is quantified with Pearson product-moment correlation coefficients.

At the level of the countries, we first analyze the interrelations of the four subscales of mathematical literacy: space and shape, change and relationship, quantity, and uncertainty. Second, we relate each scale to the general mathematics score (GMS) which refers to the original PISA 2003 mathematical literacy scale. Third, to exclude the correspondent subscale part from the general mathematics score, we create a new variable, the weighted mathematics score (WMS) that calculates –for each country– the weighted sum of three subscales, excluding the corresponding subscale.

At the level of the individual students, we concentrate on the correlations between the four mathematical subscales. These correlations are available in the Technical Report (OECD, 2005, p. 109).

At both levels (countries and individual students), we relate the four subscales of mathematical literacy to the reading and science scores.

PISA result scores use 'Raw Test scores' based on the techniques of modern item response theory (IRT). This makes it possible to construct a scale of mathematical performance, to associate each assessment item with a point score on this scale according to its estimated difficulty and to assign each student a point score on the same scale representing his or her estimated ability.

To facilitate the interpretation of the scores assigned to students, the scale was constructed to have an average score among OECD countries of 500 points and a standard deviation of 100, with about two-thirds of students across OECD countries scoring between 400 and 600 points (OECD, 2004, p. 45). In Table 1 we present the ranking for the first ten countries out of the 41 countries that participated at the PISA 2003 international comparative research (DE MEYER; PAULY; VAN DE POELE, 2004; OECD, 2004). Table 1 represents the four subscales of mathematical literacy, being space and shape, change and relationship, quantity, and uncertainty. Each subscale shows the ranking of the countries by their mean performance.

Table 1: Ranking of the First Ten Countries by Mean Performance on the Mathematics Subscales (Adapted from OECD, 2004)

Space & shape		Change & relationship		Quantity		Uncertainty	
Country	Mean	Country	Mean	Country	Mean	Country	Mean
Hong Kong	558	Netherlands	551	Finland	549	Hong Kong	558
Japan	553	Korea	548	Hong Kong	545	Netherlands	549
Korea	552	Finland	543	Korea	537	Finland	545
Switzerland	540	Hong Kong	540	Liechtenstein	534	Canada	542
Finland	539	Liechtenstein	540	Macao-China	533	Korea	538
Liechtenstein	538	Canada	537	Switzerland	533	New Zealand	532
Belgium	530	Japan	536	Belgium	530	Macao-China	532
Macao-China	528	Belgium	535	Netherlands	528	Australia	531
Czech Republic	527	New Zealand	526	Canada	528	Japan	528
Netherlands	526	Australia	525	Czech Republic	528	Iceland	528

Results

We first present the interrelations of the four subscales of mathematical literacy: space and shape, change and relationship, quantity, and uncertainty, at the level of the countries. We present the correlations of each mathematical subscale to the General Mathematics Score (GMS) and to the Weighted Mathematics Score (WMS). Then we present the correlations of the mathematical subscales with the reading and science scales. Table 2 will give the overview of the correlations.

Table 2: Correlations Between Mathematical Subscales at the Country Level (n=41)

Subscales	Space & shape	Change & relationship	Quantity	Uncertainty
Space & shape	1			
Change & relationship	.98	1		
Quantity	.98	.98	1	
Uncertainty	.94	.97	.95	1
GMS[a]	.99	.99	.99	.98
WMS[b]	.98	.99	.98	.96
Reading	.90	.94	.92	.96
Science	.96	.97	.95	.94

Note. [a] General Mathematics Score (GMS) refers to the original PISA 2003 mathematical literacy scale. [b] Weighted Mathematics Score (WMS) refers to the weighted sum of the three subscales, each time excluding the corresponding subscale in the column.

The extremely high correlations between the four subscales of mathematical literacy as presented in Table 2 are remarkable. All subscales are very highly correlated. These values suggest that there is no distinctiveness about the uncertainty subscale, which is most clearly related to statistical literacy since it is related to probabilistic and statistical phenomena and relationships. Although the correlation between the uncertainty subscale and the weighted mathematics score (WMS: excluding the corresponding variable) is the lowest of all mathematical subscales, it is still extremely high (r = .96).

The PISA 2003 reading and science scales show –as expected– the lowest correlations with all mathematics subscales. Still, the correlations are very high.

In order to discuss at the level of the individual students, we present in Table 3 the correlations between the mean performance on the four mathematical scales for all participating students (n = 276 165). For this analysis, we use the available data from the Technical Report (OECD, 2005, p.190).

Table 3: Correlations Between Mathematical Subscales at the Student Level (n = 276165) (Adapted from OECD, 2005, p. 190)

Subscales	Space & shape	Change & relationship	Quantity	Uncertainty
Space & shape	1			
Change & relationship	.90	1		
Quantity	.90	.93	1	
Uncertainty	.89	.92	.90	1
Reading	.68	.74	.73	.74
Science	.74	.77	.76	.78

As is the case at the country level, with the student level, all subscales are very highly correlated. The magnitude of the correlation is similar (around .90) for all subscales, again indicating no specific behavior for the uncertainty subscale. Correlations between the mathematical literacy subscales and the PISA 2003 reading and science scales are lower in magnitude but also rather high for all subscales (ranging between .68 and .78).

Conclusions and further research

Within the PISA 2003 survey, statistical literacy is seen only from the point of view of mathematical literacy. Items related to probabilistic and statistical phenomena and relationships appear under the cover of a mathematical *uncertainty* subscale.

Looking at the differences and at the similarities between mathematical and statistical literacy at the theoretical level, there is an increasing tendency to agree that statistical literacy is delineated as a specific area. On the other hand, looking at the analysis based on the PISA 2003 data, we recognize an extremely high relationship between statistical and mathematical literacy at both the country and the student level. These data show that countries and students that have a high or low score on mathematical literacy subscales in general also have a high or low score on statistical literacy.

Comparing with the scores at the reading and science scales, we note the lowest correlations with all mathematics subscales which appear at the student level. Even so,

the correlations are very high. A first hypothesis for these high correlations is that in general there is one factor we could call *general literacy* behind the data.

Generally, educators have to take up the challenge to prepare students to be literate consumers, statistically and mathematically. Mathematical and statistical literacy are highly related. Mathematics teachers and statistics teachers are challenged in the same way. In this sense we suggest that a reciprocal influence between mathematics and statistics educators should be beneficial.

Although statistics is viewed as a unique discipline, collaboration among statisticians and mathematics educators is still desirable. There is the practical argument that statistical content is most often taught in mathematics curriculum and in departments of mathematics. In additional to that, we bring a new argument from our research that the mathematical subscales with the overarching idea of patterns in quantity, patterns in space and shape and patterns in change and relationship, are highly interrelated with the fourth overarching idea of uncertainty which includes the elements of probability theory and statistics. Therefore mutual influence and inspiration between mathematics education researchers and statistic education researchers as well as collaboration between mathematics and statistics educators seems advisable.

A second hypothesis for the high relation between statistical and mathematical literacy within PISA 2003 data is the emphasis on *application* within the PISA survey. In future research, comparing statistical and mathematical literacy in the PISA survey with The Trends in International Mathematics and Science Study (TIMSS), where *calculation* rather than *application* is more central, could give us further interesting insights.

While in literature, statistics has become recognized as distinct discipline, it has not in the international comparative PISA research which assesses how far 15-year-old students (near the end of compulsory education), have acquired some of the knowledge and skills which are essential for full participation in society. However, looking at the different sections of mathematical literacy, the section in PISA on uncertainty is very clearly tied to statistical literacy. Why should not theoretical developments of statistical literacy –as a distinct discipline– become attuned to contemporary international research on pupils' skills to reveal the notion of statistical literacy within the PISA research?

Therefore further research is required at the level of the students' scores. Since about 90% of performance variation occurs within countries, country averages give only part of the picture (OECD, 2004, p. 60). Furthermore, an analysis at a lower level is needed. Subscales other than u*ncertainty* also include aspects related to statistics. For instance, all items that demand combinatorics are included in the *Quantity* subscale and some graphs of in the items of *Change and Relationship* subscale have a statistical nature. Therefore we could say that further analysis based on the raw data of the PISA results should reveal the notion of statistical literacy within the interpretation of results and in PISA reports.

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