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Computational analysis of conductive heat transfer in a rectangular slab of stable boundary using Monte Carlo method

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Abstract. Heat transfer is of immense importance in many engineering studies. Monte Carlo technique are broadly utilized in the operation research fields and atomic physics in which difficult problems above the existing tools of theoretical mathematics were resolved. The target of this study is to confirm the short duration in which heat transfer occurs in a quadrilateral slab where the temperature is provided throughout the borderline. An effort was made towards providing appropriate condition for the explanation of the heat transfer in a substance. The Shrinking Periphery Monte Carlo technique was utilized to obtain heat transfer in a helical pattern, upward and downward movement, which was compared with the standard Monte Carlo technique. The result of the study showed that the dimension of the quadrilateral slab determines the duration to calculate temperature dissemination in the system. The study revealed that the helical pattern is the shortest route in computing run time for temperature dispersal in a slab. The helical pattern is paramount in determining temperature distribution in a quadrilateral slab of stable state. The application of this technique to examine the conduction of heat in quadrilateral slabs produced good outcomes

Keywords: Monte Carlo Method; Rectangular slab; temperature distribution; heat transfer; Helical pattern; Upward movement.

1. Introduction

Numerous heat transfer procedures necessitate the skill of temperature distribution above the surface, where there is an application of heat transfer. It helps to study its use in engineering construction and design in air-conditioning buildings [1]. Temperature is the necessary parameters in the problems of heat conduction. The temperature at any point in a material is distinct by its arithmetical value because of the scalar amount, whereas heat drift is known by its significance and route [2]. Heat transfer is the transmission of heat across system borders exposed to temperature dissimilarity [3]. In thermodynamics, energy can be conveyed by the collaborations of a system with its environments [4]. Conduction is the interchange of energy by direct contact between particles of a substance comprising temperature discrepancies which occurred in gaseous, liquefied and solidification stages [5, 6]. Furthermore, areas with higher kinetic energy will transfer heat energy to regions with lower molecular energy through straight molecular impacts [7, 8]. Zhang et al. [9] stated that numerical simulation is a potent tool that utilized Monte Carlo method (MCM) to determine the laser transmission in bio-tissue. Pfefer et al. [10] established a voxels Monte-Carlo (VMC) method and validated its capacity in the model of light spreading in a multifaceted tissue slab with embedded blood vessels. Dare [11] developed a hybrid method for numerical modelling of heat conduction problems. However, the author observed that it takes longer run time to calculate the temperature



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distribution in comparison with the finite difference method. [12] utilized probability MCM to model heat conduction complications. However, the technique witnesses slow execution time but can regulate solution at a point more quickly.

Cho [13] studied the utilization of particulate transport Monte Carlo method to analysed heat conduction arbitrary geometries. But, the author noticed the limitation of the particulate transport Monte Carlo method. Ravichandran and Minnic [14] examined merely treated boundary conditions linked with heat conduction at the nanoscale and slight attention on numerical analysis. Dare and Ofi [1] simplified Monte Carlo method by developing an improved Monte Carlo method where the domain was characterized without extreme computational load. The simple MCM is to distinguish random numbers chosen in a manner that directly generate random procedure of the original problem and to provide the required process from the behaviour of these random numbers [15, 16]. A random walk is an essential factor to solve the Monte Carlo solution of differential equation [17, 18]. This research aims to determine the short duration in which heat transfer occurs in a quadrilateral slab where the temperature is provided throughout the borderline. [19, 20]

The shrinking technique is utilized in this study because of the small calculation period required for movement conclusion. The random walk for heat conduction problem which is a process whereby the temperature at a point in the conduction area is attained by distribution of numerous walkers from the point and permitting them to move randomly in the region until they hit a borderline where they are absorbed. The point temperature where they are dispersed is computed by increasing the temperature at each boundary point [21].

2. Monte Carlo Method

According to [12], Monte Carlo application to heat transfer was used to obtain temperature dissemination in a slab. He stated that probability methods could be applied to heat conduction problems in the bodies of random shapes. He applied a random walk to the MCM of differential equations. Heat conduction problems involving irregular shapes and difficult borderline can be resolved roughly by numerical analysis. Equation 1 is applied successively to every nodal point in the conduction region after the initial guess is made for the nodal values. A two-dimensional model flow was considered, and fluid thermally properties are invariant with temperature. The author observed that the time to travel each step varies randomly and uses a square grid of mesh size.

The direction in which the particle will move is like a game of chance by starting to compute temperature at the centre of the square grid (X, Y). The particle moves from one point to another among the nodal point of the grids. The walk is ended whenever the particle arrives a mesh point on the boundary and the temperature at that point is scored. He represented the terminus temperature of the first walk as Tw (1). A second particle is removed from X, Y, and it moves among the grid point until it reaches the boundary where it is scored as Tw (2). He released the particle to determine the temperatures in the third and fourth position.

The Monte Carlo solution for T(X, Y) = $\frac{1}{N} \sum_{j=1}^N Tw(j)$

Where Nth = total number of the particle released from X, Y

Tw (j) = Tw (1) + Tw (2) + Tw (3) + Tw (4)

Tw (j) = end point temperatures

Monte Carlo Methods follow a particular pattern.

- i. Define a domain of potential inputs.
- ii. Generates data inputs randomly from the enclosure and perform a determined computation on them.
- iii. Aggregate the solutions of the distinct computations into the concluding result.

Assume the section to be rectangle in shape with border temperature being constant, a Fortran programming utilizing Monte Carlo procedures to calculate for the internal temperatures are developed [22]. The temperature was specified all along the periphery of the rectangular slab. Placing nodes within the slab that are spaced uniformly, with uniformly spaced nodes, the values of ΔX and ΔY is the same. Equation 1 was utilized to calculate temperature dispersal in a shrinking boundary

method. Figure 1 shows the discretized rectangular slab. Figure 2 shows the graph of temperature distribution in a helical pattern of a rectangular slab.

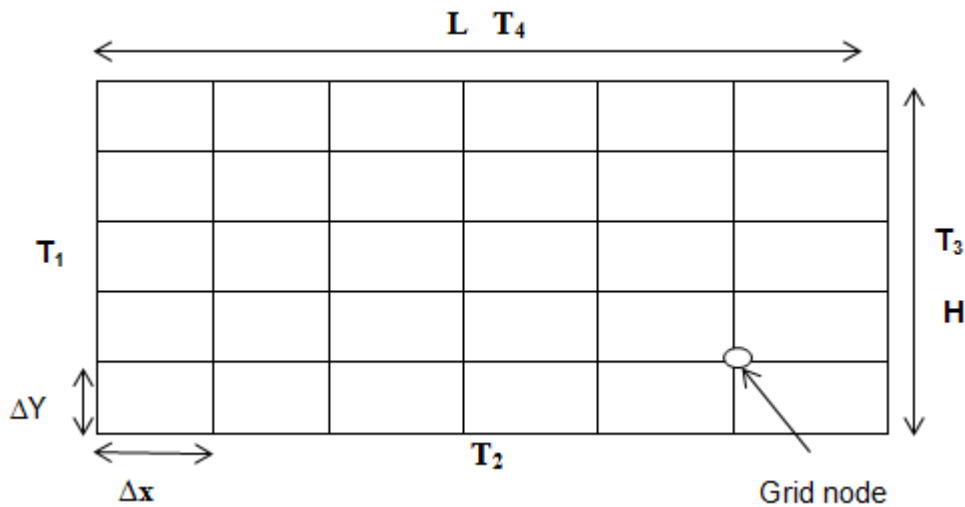


Figure 1. Discretized Rectangular slab

T1, T2, T3, T4 = Temperature distribution at the boundary

L = Length of the rectangle

H = Height of the rectangle

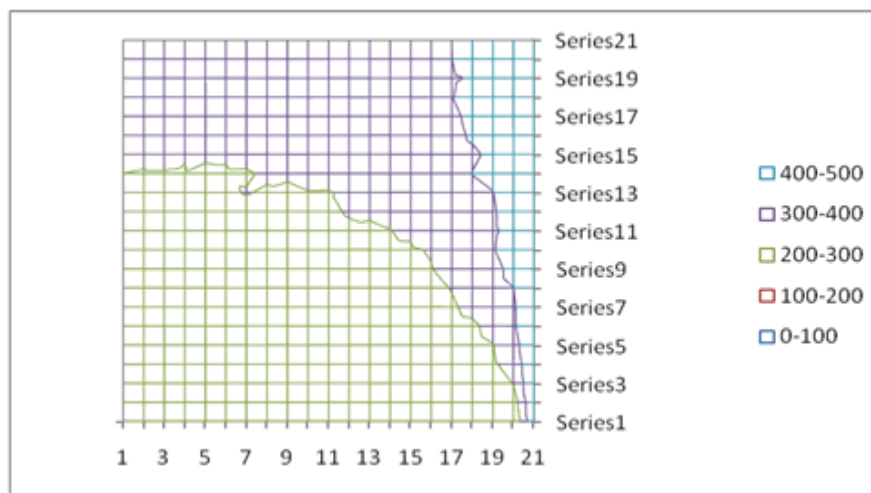


Figure 2. Surface Graph Chart of Helical Pattern

$$T(X,Y)=P_x +,T(x+1,y)+P_x-,T(x-1,y)+P_y+,T(x,y+1)+P_y,T(x,y-1) \tag{1}$$

If the moving particle is instantly the point X, Y. then it has an equivalent possibility of moving to any of the points (x+1, y), (x-1, y), (x, y+1), (x, y-1).

$P_{x+} = P_{x-} = P_{y+} = P_{y-}$ = the probabilities of the points.

Figure 3 shows the flow chart for the Monte Carlo. Multidimensional conduction problem consists of finding the temperature distribution in the conducting material [22]. This involves the solution of the differential equation. The general conduction equation is

$$dQ_s = dQ_c + dQ_g \tag{2}$$

$$\left[\frac{1}{K} \left(\frac{\partial T}{\partial t} \right) \right] \approx \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K}. \quad (3)$$

dQc = the total heat into the volume element.

dQg = total energy produced within the volume element.

dQs = stored energy as a result of temperature rise.

The duration of program when performed in a quadrilateral slab using shrinking boundary Monte Carlo technique was performed firstly. It was followed by program run for the computation of temperature using the borderline quadrilateral slab helical pattern. Then, the determinations of the calculation program run from the upward direction. Finally, the downward movement was performed. The quadrilateral comprises of twenty-one nodal points. It consists of 2D quadrilateral borderline with continuous thermal conductivity. Table 1 shows the temperature distribution in a rectangular slab using MCM.

Steps used in the computation of temperature distribution are as follows:

- Chose a point within the rectangular domain
- Define the lowest distance of the particle point to boundary
- Is it the minimum distance to the boundary then
- Chose a random number (F) to determine angle Displacement (Θ), $\Theta=2\pi F$
- Use the value of d & Θ to move to the next point within rectangular domain
- Is the distance between nodes calculation before and new points less than new points to the boundary
- Score and record the node temperature
- Is number of walks step selected sufficient
- Calculate the average temperature of the boundary nodes taken
- End of program

3. Result and Discussion

The computational run results for the numerous model of the Monte Carlo method are made to know the duration for the temperature dispersal to occur are obtained. The results are listed as found. The duration of a helical program run for the simulation of heat conduction in a system using shrinking boundary Monte Carlo Method is performed first. Secondly, the program run time for the computation of temperature is performed using the upward movement. Finally, determination of the simulation from the downward motion in the enclosed rectangular slab.

Table 1 shows the helical simulation temperature results for 6 x 5 grid. Table 2 shows the upward Simulation temperature results for 6 x 5 grid. Table 3 shows the downward simulation temperature results (K) for 6 x 5 grid

Table 1. Helical Simulation temperature results (K) for 6 x 5 grid Run Time = 3.55sec

300.0	300.0	300.0	300.0	300.0	300.0
200.0	261.8	278.4	284.4	289.6	400.0
200.0	228.7	249.6	263.6	275.1	400.0
200.0	238.6	258.3	272.0	281.2	400.0
500.0	500.0	500.0	500.0	500.0	500.0

Table 2. Upward Simulation temperature results (K) for 6 x 5 grid Run Time = 3.76sec

300.0	300.0	300.0	300.0	300.0	300.0
200.0	261.8	278.4	284.4	289.6	400.0
200.0	228.7	249.6	263.6	275.1	400.0
200.0	238.6	258.3	272.0	281.2	400.0
500.0	500.0	500.0	500.0	500.0	500.0

Table 3. Downward simulation temperature results (K) for 6 x 5 grid Run Time = 3.79sec

300.0	300.0	300.0	300.0	300.0	300.0
200.0	261.8	278.4	284.4	289.6	400.0
200.0	228.7	249.6	263.6	275.1	400.0
200.0	238.6	258.3	272.0	281.2	400.0
500.0	500.0	500.0	500.0	500.0	500.0

Table 4. Helical Sampled Solution Points and Deviation Test Results

Sample identity	Coordinate	Sample value	Relation value	Deviation	Percentage deviation
1	2, 4	215.10	219.00	3.9	1.8
2	4, 4	260.30	263.00	2.7	1.0
3	6, 8	248.10	248.80	0.7	0.3
4	8, 8	275.20	276.70	1.5	0.5
5	10, 12	278.70	277.70	-0.1	0.0
6	12, 15	289.90	291.80	1.9	0.7
7	14, 15	296.90	303.20	6.3	2.1
8	16, 20	302.00	301.20	-0.8	0.3
9	18, 18	315.30	315.60	0.30	0.1
10	20, 20	314.90	335.40	20.5	6.1

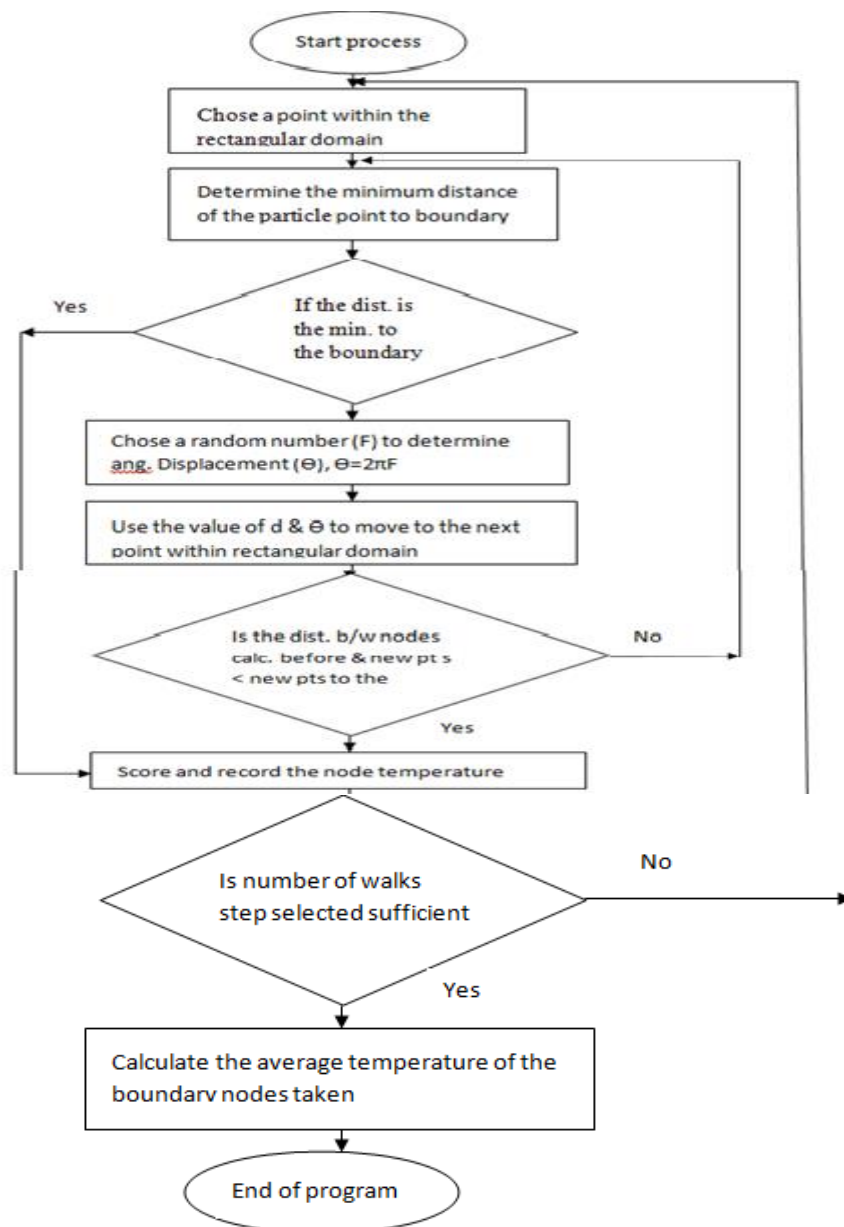


Figure 3. Flow chart for Monte Carlo technique

Table 4 shows the helical tested result points and deviation results. The peak deviation is valued at 20.5, while the percentage deviation is 6.1. Also, sample points with lesser values have reduced variations, while higher values have large deviations. Generally, the results can be compared with the finite difference method [17].

4. Conclusion

To conclude, this work has proved that Shrinking boundary MCM can be used to model steady-state heat conduction in a rectangular slab. It shows clearly that 6 x 5 grids for complete characterization of the temperature of the domain can be realized. The calculation time for the various motions was made to know the shortest time for the temperature dispersal to occur. It was observed that the mean of the program run for different dimension of the slab shows that helical is shortest. The utilization of the shrinking technique to determine heat conduction in a quadrilateral slabs produced positive outcomes. The acquired results are similar to values found by finite difference technique.

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