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Visual approach to the imaging of magic mirrors (Makyohs)

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ABSTRACT

The imaging of Japanese magic mirrors (Makyohs) is discussed by comparing the visual images of the backside relief pattern under different illumination conditions and the projected Makyoh image, rather than considering the back relief pattern as the input. It is also hypothesised that for a large number of magic mirrors, the Makyoh imaging is beyond the Laplacian regime under optimum viewing conditions, which accentuates the bright regions in the image; these correspond to reflecting highlights or step edges in the visual image. Published magic mirror images support this hypothesis. Simulations of the visual and Makyoh images are also presented.

1. Introduction

Magic mirrors are ancient artefacts originating from the Far-East (China and Japan) (Saines and Tomilin, 1999). Such mirrors are flat or slightly convex round bronze mirrors with a backside relief pattern. If a light beam falls on the surface, the mirror projects an image that resembles the back pattern as if the mirror were transparent. From the 19th century till recent days, such mirrors have been extensively studied by European and Asian scholars. It was clarified that the image is formed by a small relief of the front (reflecting) surface that locally focus and defocus the reflected beam. It was also concluded that in Chinese and Japanese mirrors (the latter called Makyoh) this surface relief is produced by different means, causing also differing imaging properties. In Chinese mirrors, the deformations are induced thermally upon cool down after casting (Murray and Cahill, 1987), while in the Japanese mirrors, the front relief pattern is formed during fabrication by elastic and plastic mechanical transfer of the back pattern (Watanabe, 1965; Thompson, 1893). The Japanese mirror and its fabrication are more represented among optics scholars; the present paper deals also with this kind of mirror. The making of these mirrors involves first casting the mirror with the back relief, thinning by filing, then polishing the front surface while exerting a strong pressure on the mirror plate using a tool called ‘distorting rod’, going repeatedly over the whole surface (Saines and Tomilin, 1999; Thompson, 1893). During this process, the mirror locally bends, and the bending is easier where the mirror is thinner; that is, less material is removed from these regions. The thickness variations finally give the front surface relief, e.g. a back-face protrusion will induce a corresponding front depression focusing the reflected beam resulting in a bright patch in the image.

The recent interest towards Makyoh imaging is propelled also by its application as a powerful topographic surface defect inspection tool (Makyoh topography) in semiconductor technology (Hahn et al., 1990) and its relevance for the field of freeform optics (Wooyoun et al., 2021). A related concept is the magic window (Berry, 2017; Hufnagel et al., 2022), where a transparent sheet with slight thickness (or refractive index) variations causes an effect similar to the magic mirror for a light beam traversing the sheet.

Previous approaches to modelling the image formation (Berry, 2006; Gitin, 2009) were aimed at finding correspondence between the back surface height map and the intensity map of the projected Makyoh image. These studies have pointed out the edge-detection property (Berry, 2006) or attempted to determine the point spread function linking the back pattern elevation and the Makyoh image intensity (Gitin, 2009); both works were based on the Laplacian approximation of imaging and a convolution-based backside pattern transfer model. A somehow different approach (Gamo, 1984; Hibino et al., 1990) defines a mechano-optical modulation transfer function for periodic back relief on the analogy of the optical modulation transfer function. These approaches are fruitful when the fundamental mechanism of the image formation is of interest; also, they are appropriate for the Makyoh-topography application. In the present paper a new approach is introduced, which is more suitable for the interpretation of the ‘magic’ imaging properties of the ancient mirror: we look at the relation of the mechano-optical image formation mechanism and the visual perception of the back relief of the Makyoh. Additionally, a hypothesis is presented regarding the Laplacian approximation.

2. The image formation mechanism of Makyoh

The back relief of the Makyoh usually consists of low-raised patterns of various scenes, objects, animals, Chinese characters, images of Buddha or Jesus etc. (Saines and Tomilin, 1999; Thompson, 1893). These back relief heights are in the mm order, and the raised parts are either stepped with constant heights or are slightly convex. The global

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shape of the front face of the mirrors is either flat or slightly convex. Thompson publishes images of several mirrors, together with their projected images in Thompson (1893); two typical of them are reproduced in Fig. 1.

For modelling the image formation, we consider a globally flat mirror. The back-to-front pattern transfer can be described by the convolution (smoothing) with a Gaussian (Berry, 2006; Gitin, 2009):

$$h_f(x,y) = -\frac{A}{2\pi d^2}\, exp\left(-\frac{x^2+y^2}{2d^2}\right)^{*} h_b(x,y) \tag{1}$$

where h_f and h_b are the front and back relief respectively, both measured outward the mirror plate. A and d are the usual Gaussian parameters; A is dimensionless. The negative sign expresses the fact that a back protrusion induces a locally concave front surface region, as described in the Introduction. The value of d is in the order of the mirror thickness (~mm), while A is around ~ 1/1000 as the front surface undulations are in the μm order (Gamo, 1984). In the one-dimensional representation, the prefactor becomes $A/\sqrt{2\pi}d$. The mechanical pattern transfer thus has two major effects: it attenuates the relief heights and smooths the abrupt features. The front surface will therefore be mostly composed of smoothed steps and depressions of various widths, corresponding to backside steps and protrusions. A narrow back depression would not induce any front deformation, since no significant bending of the plate would occur during polishing.

Following Berry's seminal paper (Berry, 2006), the Laplacian representation became popular in interpreting magic mirror imaging. According to this model, the image intensity produced by the magic mirror relative to that of a perfectly flat mirror can be approximated as

$$I(x,y) = 1 + 2L\nabla^2 h_f(x,y) \tag{2}$$

where L is the mirror-to-screen distance; that is, the intensity variations follow the Laplacian of the front surface relief. This formula is valid only if L is much smaller than the local curvature radii of the surface, therefore it predicts a weak image contrast and also a similar share of dark and bright areas. Berry also notes (Berry, 2006) that a global convex curvature of the mirror extends the Laplacian regime of the range of screen distance (the global curvature can be taken into account by introducing an effective L value in (2), see (Riesz, 2019). However, a large number of real magic mirror images (Thompson, 1893; Dember, 1931; Molisch, 1927) and descriptions (e.g. (Swinson, 1994) show a character that the Laplacian description would not predict: accentuated, bright areas (mostly narrow lines or spots) and only rare dark areas on a gray background. This characterises mostly the Japanese mirrors: e.g., in his book (Thompson, 1893), Thompson shows images of six Japanese mirrors from his collection and all exhibit this feature (see again Fig. 1).

Indeed, for a well visible image in the presence of some background illumination, a pattern with outstanding bright patches is favourable (Swinson, 1994). These Makyohs therefore seem to operate in the imaging regime where stronger focusing occurs by the concave front surface areas. This occurs at higher L values. The focusing property upon varying L has also been observed by several authors (Saines and Tomilin, 1999; Swinson, 1994), also for magic windows (Hufnagel et al., 2022). To describe this imaging region, (2) is no longer adequate. In fact, (2) can be regarded as a limiting case, a small-signal linear approximation (Riesz, 2006) of the more general formula (Riesz, 2000)

$$I(f_x,f_y) = \frac{1}{1 - 2L\nabla^2 h_f(x,y)}, \tag{3}$$

which was derived as an expression of the intensity change of the reflected light pencil along its propagation direction. The argument of the intensity becomes $(f_x, f_y) = (x, y) - 2L\nabla h_f(x, y)$; this expresses the mapping of the surface points by the surface gradients, that is, (f_x, f_y) are the coordinates on the screen where the ray reflected from the mirror's point (x, y) falls. (The full formula includes also a Gaussian curvature term in the denominator (Riesz, 2000) but we neglect it in our simple analysis; also, in one dimension this term naturally drops. Note also that although Berry's approach included the foundations of the complete description, only the Laplacian approximation was developed in (Berry, 2006); later it was expanded for magic windows (Berry, 2017).

As an illustration, we consider the imaging of an abrupt back surface step in one dimension. On the front face, a smoothed step will appear described by the error function (convolution by Gaussian) as

$$h_f(x) = \frac{s}{2}\,\mathrm{erf}\left(\frac{x}{\sqrt{2}\,d}\right) \tag{4}$$

where s is the height of the *front* surface step. The second derivative (that is, the Laplacian in one dimension) will then be the first derivative of a Gaussian, so the intensity according to (3) will be

$$I(f_x) = \frac{1}{1 + 2Lx\frac{s}{d^3\sqrt{2\pi}}\exp\left(-\frac{x^2}{2d^2}\right)}, \tag{5}$$

while f_x is given by

$$f_x = x - 2L\frac{s}{d\sqrt{2\pi}}\exp\left(-\frac{x^2}{2d^2}\right) \tag{6}$$

Fig. 2 shows the imaging of this backside step in one dimension as a function of L with realistic parameters (s = 1 μm, d = 1 mm). Approximations according to (2) and (3) but neglecting the $x \rightarrow f_x$ mapping are shown as well for comparison. The Laplacian approximation is valid

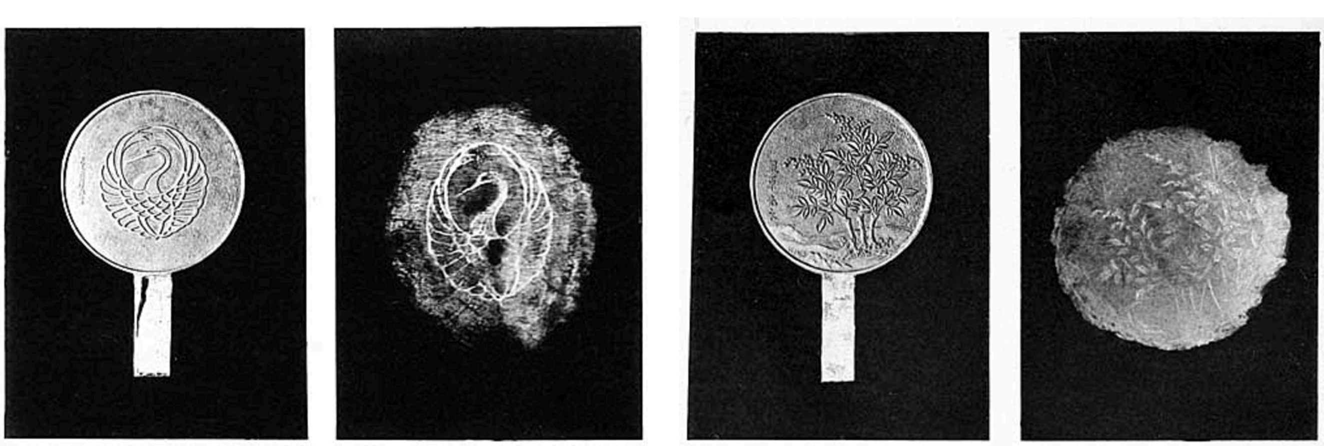

Fig. 1. Back and projected images of two original Japanese Makyoh mirrors: (a) with stepped back surface, (b) with convex back patterns (from (Thompson, 1893); public domain).

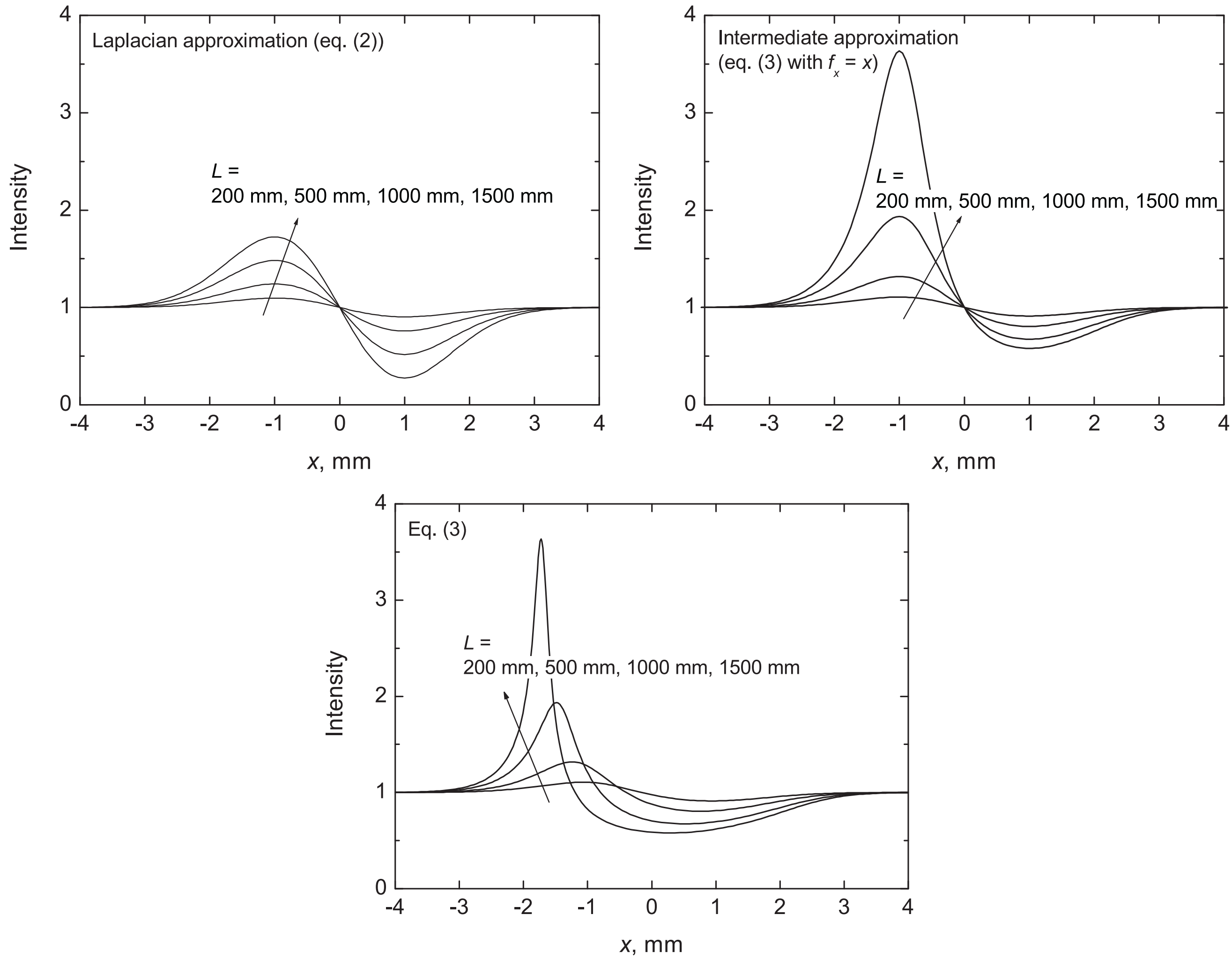

Fig. 2. Makyoh image intensity profiles produced by a one-dimensional abrupt back surface step at various screen distances, calculated using different models: (a) Laplacian approximation, (b) intermediate approximation (Eq. (3) with $f_x = x$), (c) Eq. (3); x is the transverse coordinate across the step; the step is at $x = 0$.

below about 20% contrast only. The gradient-related mapping has a large effect, making the intensity peaks narrower, the dark areas smeared and the peak position shifted. In Berry (2017), Berry also discussed the various approximations and found similar effects for quasi-periodic surface profiles of a magic window. In Makyoh topography, when the observation occurs at a higher L setting resulting in higher contrast, this effect is also present and must be taken into account.

3. Visual image of the back relief

The exact quantitative modelling of the visual imaging of the backside is difficult as the observed image depends not only on the back surface relief but on the material's reflection properties and the illumination conditions as well. We can however assume that the mirror's back surface (a non-machined cast metal surface) has diffuse reflection properties with a specular component. The illumination is usually scattered ambient light (room or temple), which may, however, have some directional component. The observation direction of the viewer person is assumed perpendicular to the mirror.

A simple one-dimensional model for the visual imaging now can be formulated as follows. If the surface reflection is perfectly diffuse, the reflection is governed by Lambert's law, specular reflection is not observed; the intensity in a given observation direction will be proportional to cos(α) where α is angle of the surface normal to the observation direction, independent of the illumination's directional distribution. The specular reflection component for the illumination's directional component can be modelled as a reflected intensity proportional to $\cos^n(2\alpha - ß)$, where ß is the direction of illumination and the n ($\gg 1$) exponent makes the reflected lobe narrower than Lambertian. If the cosine arguments are outside $\pm\pi/2$, the reflection is zero for the directional component (occluding effect). Self-shadowing effects are neglected.

4. Results and discussion

Now, consider three characteristic back relief features: a ridge with steep sidewalls, and two convex mounds with differing widths to demonstrate two characteristic behaviours as shown later. The mounds are modelled as Gaussians with d_b halfwidth for computational ease and transparency; the narrow one has halfwidth ~ d, and the wide one, larger than d (d is the convolution parameter with value around the mirror thickness, see Sec. 2). If s_b is the height of the back features, the front reliefs after carrying out the convolutions according to equation (1) will be:

$$h_f = -s_b A \frac{d_b}{\sqrt{d_b^2 + d^2}} exp\left(-\frac{x^2}{2\left(d_b^2 + d^2\right)}\right) \quad (7)$$

Taking $d_b = d$ for the narrow mound and we obtain:

$$h_f(narrow) = -\frac{s_b A}{\sqrt{2}} exp\left(-\frac{x^2}{4d_b^2}\right), \tag{8a}$$

and, neglecting the d-related terms for the wide one,

$$h_f(wide) = -s_b A exp\left(-\frac{x^2}{2d_b^2}\right) \tag{8b}$$

The ridge's relief is trivial to obtain based on (4) with the substitution $s = s_b A$.

Now we can plot the intensity profiles as calculated using (3) together with the profiles of the visual image of the back relief pattern (Fig. 3). The diffuse and the directional components of the latter are plotted separately, using the cosines without any proportionality constants. Again, $s_b = 1$ μm, $d = 1$ mm and $A = 1/1000$ was taken. The ridge width is 6 mm and $d_b = 3$ mm, that is, the narrow and the wide backside mounds have 1 mm and 3 mm halfwidths respectively; $ß = 90°$ (glancing incidence) and $n = 5$ was taken corresponding to about 50° scattering angle of the specular reflection. For simplicity, we assume the ridge walls perpendicular for the Makyoh image while sloping at $\alpha = 60°$ for the visual back image. $L = 900$ mm is chosen, just below the 1000 mm focal distance produced by the narrow mound.

Stressing the limitations of our simple models, we can conclude the following, based on the figure: If only diffuse illumination is present, the steeper parts of the back relief will appear darker, ridges therefore appear as bounded with dark lines for the spectator. Similarly, the sloping sides of the mounds are also darker. If directional lighting is present, the corresponding slopes facing toward the illumination source will appear highlighted, while the opposite sloping parts as dark. (These features are confirmed by everyday experience as well when looking at illuminated diffuse objects.) Narrow, high-aspect-ratio features can reflect a highlight upon a wide range of illumination angles (like the narrow mound in this paper), while shallow features may not produce reflection (or produce only weak one) in the observation direction (like the wider mound).

The obtained Makyoh images confirm the behaviour having been known already (Hahn et al., 1990; Berry, 2006; Gitin, 2009; Riesz, 2000): the edge-detection feature (Berry, 2006) is well visible for the ridge, however the bright peak stands out and is narrower than the black one, as discussed in Section 2. The mounds show the characteristic image of a bright spot surrounded by a dark ring, typical to that of pit defects in semiconductor wafers (Hahn et al., 1990), well documented in the Makyoh-topography literature.

For a ridge, or a single step bordering a figure or character, the Makyoh image is dominated by a bright line, this corresponds to a dark line for diffuse reflection or a bright one for directional illumination if the slope faces towards the light source. Further, the similarity between the mound's visual and Makyoh images is remarkable, however larger screen distances make the centre spot stand out. The directional lighting usually produces a highlight; this may correspond to a focused part of the Makyoh image. This effect works especially for narrow features. Fig. 4 demonstrates both effects for a modern Japanese mirror replica (manufactured following the traditional method) consisting of mainly narrow-line patterns; the similarity of the visual back and projected images is striking. The illumination of the back side is chiefly diffuse, with some lighting coming from the top, so the visual image of the ridges are two parallel dark lines, except the horizontally oriented ones which have the top edge highlighted. This corresponds to the Makyoh image of the ridges consisting of bright focused lines surrounded by faint dark lines.

Depending on the distance to the screen, the imaging can be in or outside the Laplacian regime. Choosing the distance of the screen (or wall), within some obvious limits, is the decision of the spectator of the mirror. It was hypothesised in the present paper that in order to obtain an image with well visible pattern and contrast, Makyoh mirrors are used in the non-Laplacian regime; this results in higher-intensity but narrower bright patches/lines but smaller-contrast and wider dark areas. Many published images (Thompson, 1893; Dember, 1931; Molisch, 1927; Swinson, 1994), as discussed in section 2, are in this regime, lending support to our hypothesis (see also Fig. 4). It is also reasonable to assume that the fabrication of these mirrors was somehow optimised for this kind of use.

The dominance of the accentuated bright lines on a gray/dark background makes an impression of a 'negative' image (in modern terms) as opposed to e.g. a drawing in black ink on a white paper (see Fig. 1). The gradient effect makes the bright lines/patches narrower thus enhancing the visually perceived sharpness of the Makyoh image.

Finally, we have to make a remark regarding those magic mirrors that show different behaviour, namely, no outstanding bright patches. For example, Berry (Berry, 2006) observed dark bands/areas over a grey background which hints to opposite sign of the front relief than expected on the basis of the pattern transfer mechanism outlined in the Introduction. This indicates a front relief formation different from that of Japanese mirrors; it most likely characteristic to the Chinese mirrors. In fact, many types of magic mirrors exist with diverse and sometimes

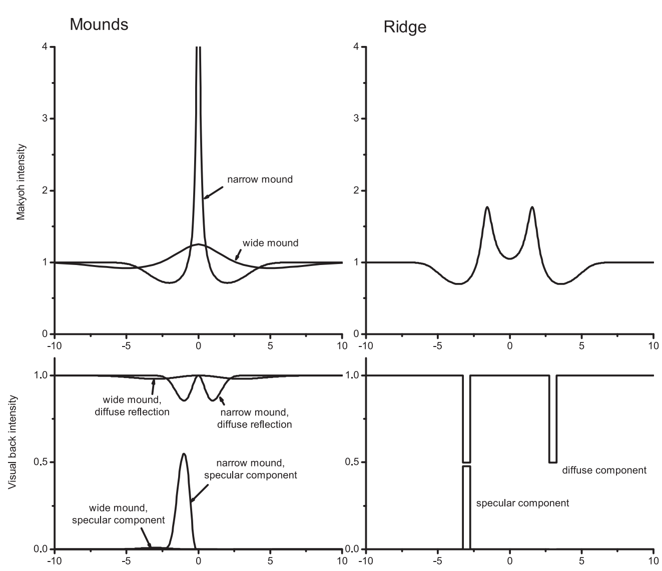

Fig. 3. Calculated one-dimensional intensity profiles of the reflected Makyoh image on the screen and the visual images of the back relief for a backside ridge as well as a narrow and a wide mound.

Fig. 4. Photo of the visual image and the projected Makyoh image of a modern Japanese mirror replica (with kind permission from kyotocrafts.store, © 2020, Kyoto Crafts Store).

obscure origin. The secret nature of manufacturing tradition and the sparse documentation further hampers any systematic study. Also, many present-day studies are made on modern replicas whose fabrication technology is different or unknown (e.g. Mak and Yip, 2001). It is also possible that some published images are in the Laplacian regime; these are likely produced by convex mirrors, as discussed in section 2. To sum up, it is unreasonable to expect that any model would describe *all* existing magic mirrors; we can state however that our post-Laplacian model adequately describes a certain class of mirrors.

5. Conclusions

Previous approaches to understand Makyoh imaging (Berry, 2006; Gitin, 2009; Gamo, 1984; Hibino et al., 1990) have attempted to find relations between the back pattern elevation and the Makyoh intensity, mostly based on the Laplacian approximation. Our approach, introduced in the present paper, compare the visual images of the back pattern and the projected image to find similarities between them; in addition, a post-Laplacian imaging was hypothesised.

Comparing the fundamental contrast (image) forming mechanisms, we can conclude the following: For the projected Makyoh image, intensity is determined by the second order properties of the reflecting front surface (local curvatures), while the intensities of the back visual image are by first-order properties (slope) of the back pattern; it may have also a directional bias and highlights. What is common in the two mechanisms is the sensitivity to the surface slope change, providing similar visual cues. The second-order mechanism produces alternating dark/bright patches, but the focusing effect (post-Laplacian regime) accentuates the bright and diminishes the dark one, making it more similar visually to the first-order mechanism producing usually a single patch for a slope change. An additional interesting property is that the Makyoh image in this imaging regime has a 'negative' visual character.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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