INTEGRATING MUSIC AND MATHEMATICS FOR CONNECTING ACROSS MULTIPLE CONSTRUCTS OF FRACTIONAL UNDERSTANDING: AN RME TASK DESIGN JOURNEY

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ABSTRACT

Two South African curricular aims: appreciating creativity in mathematics and developing conceptual understanding, motivated this study. Negative views towards mathematics and challenges in teaching and learning fractions at primary school level are reported in literature, with the part-whole construct of fractions often the sole teaching focus.

Despite challenges in curriculum integration (high demands on teachers and diluting disciplines), benefits, such as motivation and creative thinking, are noted. I recognised music-mathematics integration as an opportune context for designing tasks to support learners in moving flexibly between the fraction as ratio, fraction as measure and part-whole constructs. Guided by Realistic Mathematics Education principles, I embarked on a participatory dual-design experiment in task design, grappling within three micro-Communities of Practice (micro-CoPs) and across two planes: the Design-Theorising Plane and the Grounded-Practice Plane.

In the Design-Theorising Plane, I worked with my two doctoral supervisors, grappling with design obstacles and finding resolutions. COVID-19 restrictions shifted our meetings to online platforms, allowing documentation and analysis of the task design process through recording functions. In the Grounded-Practice Plane, I worked within two separate micro-CoPs, both at independent schools (eight and two participating teachers respectively). Data on the teachers' interrogation and implementation of the designed tasks were obtained via formal and informal interviews. Their reflections informed ongoing adaptations to the task design. Data were analysed in a matrix I designed and via NVivo coding.

Findings include both the *product* of the task design journey (eight music-mathematics lessons, resources, and representations) and the *process* (ten groupings of Obstacle-Resolution Cycles). Three key questions (relating to music-mathematics fidelity; to task simplification for

implementation; and to appropriate music-mathematics representation) were used in addressing each Obstacle-Resolution Cycle. Designing tasks to teach the part-whole construct of fractions was relatively straightforward, but designing tasks to teach the fraction as ratio and fraction as measure constructs was more challenging. These constructs could not be conflated by superimposing the music and mathematical linear representations. Aligning them, however, allowed for moving flexibly between the constructs. The teachers reported that the integrated music-mathematics tasks and supporting resources enhanced their learners' fractional problem-solving abilities, simultaneously promoting more positive learner dispositions towards mathematics.

STATEMENT OF ORIGINAL AUTHORSHIP

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signed:

Tarryn Shirley Lovemore

Date: 14-02-2023

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LIST OF ACRONYMS AND ABBREVIATIONS

C2005	Curriculum 2005
CAPS	Curriculum and Assessment Policy Statement
CoP	Community of Practice
DBE	Department of Basic Education
NCS	The National Curriculum Statement
RME	Realistic Mathematics Education
RNCS	The Revised National Curriculum Statement
SANCP	South African Numeracy Chair Project
TIMSS	Trends In International Mathematics and Science Study

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I am appreciative to have received this endless support and I thank God for the many blessings.

I close these acknowledgements by listing the publications and presentations I authored/co-authored since becoming a member of the SANCP team.

Lovemore, T. (2020). *Enriching my teaching around the inverse order relationship in unit fractions at the grade 5 level through the inclusion of musical activities: An action research case study*. (Publication No. 38079) [Master's thesis, Rhodes University]. http://hdl.handle.net/10962/142429

Lovemore, T. (2021). Using music to teach fractions. In M. Qhobela & L.G. Mohafa (Eds.), *Electronic Book of Research Papers in Mathematics, Science and Technology Education: Short Papers, Snapshots, Posters and Symposia*, 154-157. University of Lesotho: SAARMSTE.

Lovemore, T., & Robertson, S-A. (2021). Primary school mathematics teachers' exploration of integration strategies within a community of practice. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Proceedings of the 43rd Annual Conference of the Mathematics Education Research Group of Australasia*, 434. Singapore: MERGA.

Lovemore, T., Robertson, S-A., & Graven, M. (2021). Enriching the teaching of fractions through integrating mathematics and music. *South African Journal of Childhood Education 11*(1), a899. https://doi.org/10.4102/sajce.v11i1.899

Lovemore, T., Robertson, S-A., & Graven, M. (2021). Using music note values to support the teaching of fractions. In M. Inprasitha, N. Changsri, & N. Boonsena, N. (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, 1, 218. Khon Kaen, Thailand: PME.

Lovemore, T., Robertson, S-A., & Graven, M. (2022). Aligning mathematical and musical linear representations to support fractional reasoning. In N. Fitzallen, C. Murphy & V. Hatisaru (Eds.), *Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia*, 354-361. Launceston: MERGA.

Lovemore, T., Robertson, S-A., & Graven, M. (2022). Developing fractional reasoning through body percussion. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.). *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education*, 4, 254. Alicante, Spain: PME.

Lovemore, T., Robertson, S-A., & Graven, M. (2022). Initiating a community of practice amongst primary school mathematics teachers: Trials and triumphs. [Paper presentation]. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of European Research in Mathematics Education (CERME12).* (pp. 3418-3425). ERME / Free University of Bozen-Bolzano.

Lovemore, T., Robertson, S-A., & Graven, M. (2022). Task design grapplings in integrating music and fraction representations. In K. R. Langenhoven & C. H. Stevenson-Milln (Eds.), *Book of Proceedings of the 30th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education*, 49-61. Western Cape: SAARMSTE.

CHAPTER 1: INTRODUCTION TO THE STUDY

1.1. Introduction

"Music is an unconscious exercise in mathematics in which the mind is unaware that it is dealing with numbers" – Wilhelm Leibniz (1646–1716)

The focus of this study is the integration of music into the teaching of fractions to young learners[1]. Such integration holds the potential to support both specific and general mathematics curriculum aims. At the specific level it may contribute to deepening aspects of learners' conceptual understanding; at the general level it may help to demonstrate that mathematics is a beautiful, elegant and creative part of human activity in cultural and social settings. Kilpatrick et al.'s (2001, p. 1), description of mathematics as "one of humanity's great achievements … of great sophistication and beauty" further supports this aim.

This is a qualitative design-research study. I believe that careful design of mathematics tasks which integrate other subject areas can support learners in developing greater confidence to solve problems creatively and encourage participation in mathematics. Tasks such as these can also support learners in developing deep conceptual understanding of the subject matter, that of mathematics and, in this instance, music. This study builds on my Master of Education (MEd) study, in which I explored the use of musical note values to teach fractions at the Grade 5 level (Lovemore, 2020). My focus has now moved on to how the use of music beats and notes might support and deepen learners' fractional reasoning across multiple constructs of fractions with the goal to achieve deep conceptual understanding of fractions. I use the term 'fractional reasoning' to indicate an active process of sense-making that subsequently leads learners to achieving 'fractional understanding' where they are able to flexibly use fractions. Considering the MEd as a preliminary iteration (one in which the part-whole construct of fractions predominated), I embarked on a new task design journey. I use the term journey and its many related terms (travellers, navigation etc.) to capture the process I followed in designing music-mathematics integrated tasks, sharing them with teachers, reflecting and adjusting the tasks to arrive at the resources, of which some samples and highlights are shared in Appendix 8.

Together with my fellow travellers (my two supervisors and ten intermediate phase teachers[2]) I navigated my way through multiple further iterations. This thesis shares the task design

[1] In South African usage, the term 'learner' refers to children in primary and secondary school, whereas 'student' refers to those enrolled in tertiary education.

[2] In the South African context, Grades 1 to 3 (7 to 9 years old) are known as foundation phase, Grade 4 to 6 (10 to 12 years old), as intermediate phase, Grade 7 to 9 (13 to 15 years old) as senior phase, and FET (further education and training) is Grades 10 to 12 (16 to 18 years old).

journey's itinerary, including the various layovers and delays encountered along the way, and culminates in arriving at a final destination: the research findings. These findings can, I believe, make a valuable contribution to the literature and practice of teaching fractions at the primary school level.

1.2 The Context of Mathematics Education in South Africa

The topic of 'education' elicits many emotions in South Africa. I start with a description of the ideal, based on the country's curriculum aims and then discuss some realities about the state of our country's mathematics education. This discussion provides insight into the national context of mathematics education, thus leading into the rationale for the current study.

In an attempt to redress inequalities from the apartheid era, the current South African curriculum, known as Curriculum and Assessment Policy Statement (CAPS), describes general aims of the curriculum as a whole. These include, among others:

- equipping learners… with the knowledge, skills and values necessary for self-fulfillment, and meaningful participation in society as citizens of a free country …
- [producing] learners that are able to identify and solve problems and make decisions using critical and creative thinking (South Africa. Department of Education [DBE], 2011a, pp. 4-5).

These general aims are applicable to all school subjects and lay the foundation on which individual subjects are developed. The CAPS mathematics curriculum (across all grades) is driven by specific aims and skills, including:

- developing a critical awareness of how mathematical relationships are used in social, environmental, cultural; and economic relations;
- developing confidence and competence to deal with any mathematics situation without being hindered by a fear of mathematics
- developing a spirit of curiosity and love for mathematics;
- developing an appreciation for the beauty and elegance of mathematics;
- developing recognition that mathematics is a creative part of human activity;
- developing deep conceptual understanding in order to make sense of mathematics; and
- developing acquisition of specific knowledge and skills necessary for:
    - The application of mathematics to physical, social and mathematical problems,
    - The study of related subject matter (e.g. other subjects)
    - Further study in mathematics;

- learn[ing] to pose and solve problems; and
- build[ing] an awareness of the important role that mathematics plays in real life situations including the personal development of the learner (South Africa. DBE, 2011a, pp. 8-9).

This study is further guided by the assumption that curriculum integration is a way to support teachers and learners in achieving these curriculum aims. The National Curriculum Statement Grade R – 12 aims for learners and teachers to recognise relationships within and across subjects (South Africa. DBE, 2011a). Such curriculum integration across subjects and integration with real-life situations has been acknowledged as a "driving principle" (Adler et al., 2000, p. 2) of the curriculum (South Africa. Department of Education [DoE], 2003; South Africa. DBE, 2011a; Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017; Jojo, 2019[3]).

Initial post-apartheid curricula (Curriculum 2005 and the Revised National Curriculum Statement) made the principle of integration an explicit and distinctive element in promoting meaningful learning through finding opportunities to link topics across the curriculum (Graven, 2002; South Africa. DoE, 2002; South Africa. DoE, 2003). The current CAPS curriculum documents also highlight the importance of identifying relationships across and within subjects and making real-life connections, but make no explicit mention of integration. Furthermore, no guidance is provided to teachers on how to implement making these connections. In their analysis of the 2021 mathematics matriculation examination results, the Department of Basic Education (DBE), does, however, note that learners find questions that integrate topics within mathematics challenging (South Africa. DBE, 2022, p. 197). This points to an implicit recognition of the desirability of teaching, not only assessing, mathematics in an integrated manner, integrating various concepts within mathematics and other subjects with mathematics. Further, curriculum documents do, in fact, highlight that the subjects of mathematics and music are both considered creative human activities, that both should have relevance to the real-life of the learners, and that both should be taught in ways that encourage creative problem-solving (South Africa. DBE, 2011a; South Africa. DBE, 2011b). I recognised this as an opportunity to

[3] A point to note with regards to the formatting of this thesis is that I opted to use the APA 7th Edition referencing conventions, however, I have made the deliberate decision to list multiple authors on the same theme chronologically, rather than alphabetically. The reason for this decision is to show the progression of the literature in chronological context, which I believe, for the purpose of this thesis, is more meaningful than an alphabetical list.

explore the design of music-mathematics integrated tasks which could support teachers and learners in achieving such curriculum aims. I use my findings from this study to suggest some ways in which integrating musical activities into mathematics lessons can provide opportunities for teachers to create real-life problem contexts for learners to engage in meaningful problem-solving to help deepen their fractional understanding. This, as I discuss in Chapter Two, fully aligns with the theoretical framing of the study, Realistic Mathematics Education (RME).

A majority of South Africa's school mathematics learners achieve poorly on national and international assessments. Reporting on the latest (2021) school leaving Grade 12 (matriculation) mathematics results, the Department of Basic Education (DBE) noted that "the 2021 examination revealed a slight improvement of candidates' understanding of basic concepts across some topics in the curriculum" (South Africa. DBE, 2022, p. 184), with only 37,6% of those writing the mathematics examination achieving above 40% and 57,6% of learners achieving at or above 30%. This "slight improvement" (South Africa. DBE, 2022, p. 184), is seen after three years of decline. In 2018 only 58% of learners achieved at the over 30% level (South Africa. DBE, 2019, p. 132), dropping, in 2019, to only 54,6% of learners achieving above the 30% mark (South Africa. DBE, 2020).

The Trends in International Mathematics and Science Study (TIMSS) has provided data on South African Grade 8 and 9 learners for 24 years (1995 – 2019). Reddy and her team show that while there has been some improvement in achievement, South Africa still ranks in the three lowest performing countries out of all countries who participated in the study (Reddy et al., 2022a, p. 14). The results from TIMSS 2019, showed that only 41% of South African Grade 9 learners demonstrated "basic subject knowledge and skills" (Reddy et al., 2022a, p. xiii). Grade 5 learners were included in TIMSS from the 2015 assessment period, with only 39% of the learners showing proficiency in basic mathematics, and a slight decrease to 37% of learners in the 2019 study (Reddy et al., 2022b). This means that 63% of Grade 5 learners "have not acquired basic mathematical knowledge" (Reddy et al., 2022b, p. 17). The TIMSS reports further highlight the inequalities amongst provinces. In the TIMSS 2015, for example, the Eastern Cape Province, in which the current study took place, ranked lowest, indicating that 74% of learners could not do basic mathematics (Reddy et al., 2016). While TIMSS 2019 showed that the Eastern Cape performance improved slightly, the province's average score was significantly lower than the national average (Reddy et al., 2022b, p. 19). Specifically looking at fractions, TIMSS 2019 showed that South African Grade 5 learners performed the lowest in questions relating to fractions and decimals, and measurement, with only 30% of the fraction items being answered correctly. These statistics indicating that a majority of South African learners are unable to do basic mathematics, are alarming, perhaps most particularly in light of claims such as those made by, for example, Kilpatrick et al. (2001, p. 1), that without basic mathematics, citizens cannot fully participate in the everyday life of their society. This is a point Reddy et al. (2022a) too make, namely, that mathematical skills and knowledge are vital if people are to actively participate in an increasingly "knowledge and technology-based economy" (p. 2). Based on the statistics provided in the preceding paragraph, more than half of South Africa's learners are ill-prepared to participate effectively in their local, national or global environments. This flies directly in the face of the general CAPS aims of equipping learners to meaningfully participate as citizens in a free society (South Africa. DBE, 2011a). It is a small wonder, therefore, that South African mathematics teachers are reported to experience low morale (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017). Spaull (2019) claims that many teachers are faced with the challenge of having insufficient pedagogical and content knowledge. Angie Motshegka, the Minister of Basic Education, is quoted as acknowledging that mathematics teaching is "often poor quality", teachers are unable to answer questions or demonstrate proficiencies in the curriculum they are responsible for teaching, and there is particularly a "major gap at lower grade levels" (Motshegka, 2013, as cited in Jojo, 2019, p. 5).

Investigating the issue of mathematics teacher proficiency in South African primary grades, Venkat and Graven (2017) have argued for increased support for teachers to help in achieving the general and specific curriculum aims with a specific focus on building teachers' morale and sense of agency. They further believe that gaps in primary mathematics teachers are partly due to narrow views of what mathematics is, exacerbated by a limited understanding of curriculum progression (Venkat & Graven, 2017). Research into teachers' and learners' attitudes, dispositions and beliefs about mathematics indicates narrow and disconnected views (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017; Jojo, 2019). Tasik and Edo (2019, p. 295) warn of the dangers of the prevalent view that "learning mathematics means memorising an abstract formula that does not make sense". Graven (2012), similarly, noted that learners often viewed mathematics simply as 'sums', 'working with numbers' and doing 'tests'. Once more, learners often have negative experiences in mathematics, resulting in feelings of "helplessness and hopelessness in accessing mathematics" (Graven, 2015, p. 2). Spangenberg and Myburgh (2017) suggest that teachers' beliefs about mathematics subsequently permeate learners' beliefs, affecting their participation in mathematics. Kilpatrick et al. (2001) identified a 'productive disposition' as a crucial element in learners' development of mathematical proficiency. It is, therefore, not surprising that Jojo (2019) calls for efforts to change South African mathematics teachers' attitudes and beliefs, pointing to how important it is that learners come to hold positive beliefs about, and confidence in, their ability to do mathematics in ways that make sense to them.

1.3 Rationale for The Study

The central motivation behind this study is to contribute towards supporting mathematics teachers and learners in meeting the general and specific curriculum aims of developing learners into creative problem finders and problem solvers within their communities (DBE, 2011a). By exploring ways for integrating music into teaching fractions, I hope not only to help young learners develop a positive, broader view of mathematics as a creative human activity, paving their way towards recognising its beauty and elegance, but also to help deepen their conceptual understanding of different fractional concepts and constructs.

Edelson and Johnson (2003), in the United States, argued that integration of mathematics and music will enable teachers to "help children achieve national and state learning standards in mathematics as well as the creative arts" (p. 65). The South African curriculum for Creative Arts, (including music, drama, dance and visual arts) emphasises that the arts, effectively applied, can benefit literacy and improve retention rates (DBE, 2011b, p. 9). While integration remains an important principle in the curriculum, as mentioned earlier, this needs to move beyond mere rhetoric. I posit that teachers should be supported with practical ways of integrating subjects, rather than simply paying lip service to the curriculum aims. For this reason I embarked on a task design journey, along with my fellow travelers (my two doctoral supervisors and the study participants), to explore ways in which intermediate phase (Grade 4 to 6) teachers can integrate music into their fraction teaching.

The context of music lends itself well to explore various mathematics concepts, such as counting, pattern, sequencing, spatial reasoning, one-to-one correspondence and fractional understanding (Geist et al., 2012). It was not, however, these synergies alone that prompted my focus on integrating music and fractions at the primary school (intermediate phase) level. Fractions are regarded as "a mathematically and culturally indispensable concept" (Cortina et al., 2019, p. 20), and developing sound fractional knowledge is a fundamental part of mathematical reasoning (Tzur 2016; Dole 2010; Cortina et al., 2014; Hilton et al., 2016). It is, however, notoriously challenging to teach and learn fractions; the multiple, interrelated constructs of fractions being one of the things that makes it so challenging (Siemon et al., 2015). I saw calls, such as those made by Tzur (2016), Cortina et al. (2014) and Cortina et al. (2019) to move towards activities that actively encourage deepened conceptual understanding, trialling new strategies, new instructional sequences and new resources to support meaningful teaching of fractions as a golden opportunity to explore ways in which the integration of musical elements may benefit the teaching and learning of fractions.

Some research has been done on the use of musical note values to teach fractions at primary level (see for example, Courey et al., 2012; Azaryahu et al., 2019; Azaryahu & Adi-Japha, 2020). My own MEd study too was an action research case study aimed at enriching my teaching of fractions at the Grade 5 level through the use of music note values (Lovemore, 2020). There is, however, a gap. Most of the literature appears to focus primarily on the part-whole construct of fractions. This fuelled my decision to explore ways in which tasks could enable flexible movement across and between multiple constructs of fractions (the part-whole construct as well as fraction as measure and fraction as ratio). I elaborate further on this gap in the next chapter (Literature Review).

With the key motivation of achieving curriculum aims through curriculum integration, the known challenges around fraction teaching, and the synergies between mathematics and music, I turned to Realistic Mathematics Education (RME) to guide me on my task design journey. This decision came after reading a wide range of literature (Treffers, 1986; Freudenthal, 1991; Streefland, 1991; Gravemeijer, 1994; van den Heuvel-Panhuizen, 2003; Hough & Gough, 2007; Cobb et al., 2008; Tasik & Edo, 2019; van den Heuvel-Panhuizen, 2020). I selected RME as the theoretical framing for the study because its key principles resonated with the general and specific aims outlined in South Africa's mathematics curriculum. Freudenthal (1991), for example, argued that mathematics is a human activity, and that teachers and task designers should encourage meaningful participation from learners by providing them with realistic problem scenarios requiring them to engage in informal ways of reasoning, which are subsequently developed by the teacher into formal, abstract mathematical concepts and representations. Again, music seemed an opportune realistic context from which to develop such meaningful tasks. I therefore initiated a trio of Communities of Practice (CoPs) (Lave & Wenger, 1991), the first being a kind of micro-CoP comprising myself and my two supervisors; the second and third being myself working in collaboration with two separate groups of practising intermediate phase teachers at two well-resourced independent schools in the Eastern

Cape Province of South Africa. My work with these teachers cohered with Gravemeijer and van Eerde's (2009) notion of a 'dual-design' experiment. Together with the teachers I embarked on two cycles of joint trialling, reflecting on and adjusting a series of mathematics-music tasks.

The choice of trialling the tasks with teachers working in two well-resourced independent schools was deliberate. Mathematics assessment statistics show that independent schools generally perform better than the majority of their state school counterparts. Disparities in achievement exist across schools of diverse socio-economic status. Such disparities are widely attributed to unequal access to resources and infrastructure. As Reddy et al. (2016; 2022b) reported, independent schools and more affluent fee-paying state schools achieve above South Africa's national average. The TIMSS 2015 results indicated that 60% of the learners in independent schools achieved scores demonstrating basic knowledge in mathematics (Reddy et al., 2016). From an ethical perspective (upon which I elaborate in Chapter 3), it seemed appropriate to not burden a state school with engaging in the trialling of a new method of teaching fractions while having to contend with other high demands and limited resources. Within the DBE and within the curriculum there is a complete push for curriculum coverage which teachers need to stipulate week-by-week. In this context, teachers have very little freedom to trial and innovate the kinds of interventions that I was looking at in this study. Given, further, that independent schools generally are able to manage their curriculum coverage arrangements with greater flexibility, I identified such schools as ideal spaces in which to trial – with teachers as co-researchers – various music-mathematics integrated tasks. Furthermore, there was willingness and encouragement from the principals for teachers to engage in the study. It should, however, be noted that the task design is intended for a wide range of schools, including the schools in the South African Numeracy Chair Project. These materials are intended, after the doctoral study, for implementation and usage in a wide range of South African schools. Work towards packaging these resources for use, with supplementary resources, such as a storybook, is already underway for use in schools, including township and rural schools, in a way in which it can fit in to the curriculum.

1.4 Problem Statement

As indicated above, the general and specific aims of the South African mathematics curriculum are not being adequately met. Teachers are expected to highlight links within and across subjects with little support for integration strategies. Fraction teaching and learning is particularly challenging, and I saw integration with music as a possible strategy in teaching for deep conceptual understanding of fractions. Furthermore, as noted above, literature focuses largely on the part-whole construct of fractions, whereas, teachers are expected to provide contexts for learners to experience and move flexibly between the multiple and interrelated constructs of fractions. This study strove to explore ways to support the teaching and learning of multiple constructs of fractions, through the design of integrated mathematics and music lessons.

1.5 The South African Numeracy Chair Project

My study has been undertaken with the support of the National Research Foundation's South African Numeracy Chair Project (SANCP). This is chaired by Professor Mellony Graven. The Chair runs several initiatives, including after-school 'maths clubs' for learners and various teacher development programmes. Such Chair activities provide researchers and teachers with opportunities to trial and develop confidence in novel mathematics teaching strategies (see Stott et al., 2017). A particular focus of the Chair's teacher development programmes is on building a "supportive teacher community" and creating "research-informed resources" (SANCP, 2021). Resonating with the aims of my study, the aims of the Chair include improving the quality of primary school mathematics teaching and learner performance, encouraging research of practical and sustainable interventions for improving numeracy in primary grades and in so doing helping to address the mathematics education crisis (SANCP, 2021). As part of the SANCP team, Stevenson-Milln (2018) trialled ways of merging music and mathematics by designing and carrying out an action research study where she worked with two Grade R[4] teachers and their learners. As noted previously, my own MEd study similarly explored ways of integrating mathematics and music at the Grade 5 level, and SANCP's ongoing support provided me with an opportune space for a continuation of this research at the doctoral level.

1.6 Research Objectives

My study's aim has been to design and to trial integrated mathematics and music tasks for deepening intermediate phase learners' fractional reasoning across the multiple, interrelated concepts of fractions. The design of the tasks has been guided by the theoretical underpinnings of realistic mathematics education.

[4] In the South African context, Grade R (6 years old) refers to Reception Year (i.e., the year before Grade 1).

The research objectives which guided me in attempting to achieve the research aim were to:

- trial ways to integrate music and mathematics for developing deep conceptual understanding of fractions.
- design tasks which integrate musical and mathematical representations and concepts for connecting across multiple constructs of fractional understanding.
- explore the integration of music and mathematics for meeting general curriculum aims of appreciating mathematics as a creative and elegant human activity.
- understand/describe teachers' experiences of integrating music and mathematics for fractional understanding.

The overarching research question driving the current study is:

In what ways can music and mathematics be integrated via realistic mathematics education task design principles so as to facilitate connections across multiple constructs of fractional understanding?

The sub-questions are:

1) In what ways can musical and mathematical representations and concepts be integrated for developing conceptual understanding of fractions?
2) What realistic/meaningful tasks can integrate music and mathematics for connecting across multiple constructs of fractions?
3) How can integrated music and mathematics tasks support teachers in achieving the general and specific curriculum aims?
4) What are teachers' experiences of integrating music and mathematics for fractional understanding?

1.7 Significance

The significance of this study is multifaceted. Firstly, the study highlights ways in which curriculum integration can practically be implemented at the intermediate phase level. The specific example of mathematics and music can be used to demonstrate links across the curriculum that develop recognition of the beauty and elegance of mathematics, and show mathematics as a creative human activity, all of which may help in promoting positive views and beliefs of mathematics, rather than the disconnected, negative views that many teachers and learners are reported to hold (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017; Jojo, 2019). The tasks designed, together with their accompanying learning support materials can be used to support teachers in deepening fractional understanding of their learners, working flexibly between the different constructs of fractions. Ideally, they could also be further trialled and adapted to suit particular classroom contexts.

Developing understanding, rather than rote memorisation of procedures, will almost certainly help towards addressing South African (and international) learners' low achievement levels in mathematics.

The study provides a kind of route map of a realistic mathematics task design quest in search of appropriate ways for exploiting the integration potential of music and fractions. The journey has been challenging, and has involved significant grappling to find optimal ways of ensuring both mathematical and musical fidelity, and, as such, makes a methodological contribution towards future integration and task design projects that both researchers and teachers may find useful.

1.8 Organisation of the Thesis

The remainder of the thesis is divided into five Chapters. A discussion and analysis of the relevant literature follows. It includes discussion of the theoretical framework, RME, which guided the study. Chapter 3 then outlines the methodological decisions and procedures. Thereafter, Chapters 4 and 5 present the analysis and discussion of the findings. In the concluding chapter I explore some of the implications of these findings and suggest possibilities for future research.

A point I wish to highlight is that the order in which I refer to mathematics and music is not used consistently. This was a deliberate decision as I intend showing that both subject areas are considered equally important, in line with Bresler's (1995) co-equal style of integration.

With RME theory as my compass, I now share the details of my task design journey towards integrating music and mathematics in ways that support moving flexibly between multiple constructs of fractions at the primary school level.

CHAPTER 2: THEORETICAL FRAMING AND LITERATURE REVIEW

2.1 Introduction
2.2 Realistic Mathematics Education (RME)
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2.1 Introduction

Amongst the general aims contained in the South African mathematics curriculum are that learners develop a love for mathematics, recognising it as a beautiful and elegant human activity, and that they develop confidence and competence in conceptual understanding (South Africa. DBE, 2011a). In South Africa the achievement levels are shown to be particularly low by national and international assessments, with narrow and often disconnected views of mathematics contributing to a lowering of teacher morale (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017; Spaull, 2019; South Africa. DBE, 2020; Reddy et al., 2022a, 2022b). The teaching and learning of fractions is particularly notorious for being a challenging part of the curriculum (Streefland, 1991; Siemon, 2003; Courey et al., 2012; Cortina et al., 2015; Azaryahu et al., 2019; Getenet & Callingham, 2021). I see the connection between mathematics and music as one possible way of designing intervention tasks that would allow me to capitalise on the synergies between these two subjects. This could support teachers towards meeting the above-mentioned general curriculum aims of helping learners see the beauty and creativity of mathematics, while simultaneously meeting the specific aim of deepening learners' conceptual understanding of fractions.

My 'route map' for this chapter is that I begin with a discussion of the key literature on Realistic Mathematics Education (RME). RME provides the main theoretical framing of my study. The design of music-mathematics integrated tasks is the focus of my study, and I saw RME principles as being especially key to ensuring effective task design. My aim was to design tasks that began with a real and engaging experience for learners (musical activities) upon which teachers could then build in strengthening their learners' fractional reasoning and understanding, while simultaneously meeting the broader curriculum aim of helping them appreciate the beauty and creativity of mathematics.

My literature review route then takes me into discussion on curriculum integration, in general terms; its waxing and waning across time; and the benefits and challenges experienced by teachers and researchers in the design and implementation of integration strategies. I briefly discuss literature on integration with mathematics as well as with the arts. I then more specifically refer to literature on integration of music and mathematics. I outline the curriculum expectations of fractions in intermediate phase and discuss literature on the teaching and learning of fractions, with a specific focus on the multiple constructs of fractions. I also outline the music curriculum expectations and share some background information on musical notation and the synergies I see this as having with fractional reasoning and understanding.

I then briefly explore some literature on CoPs – despite my intention of working within a Community of Practice (CoP) having encountered some initial turbulence. I end this leg of my thesis journey with a discussion of some key literature dealing with task design that, together with the RME literature, influenced me in my own design of the various music-mathematics tasks developed and trialled in the course of my study.

2.2 Realistic Mathematics Education (RME)

Freudenthal, the founder of RME, writes in his *Revisiting Mathematics Education: China Lectures*, that long ago mathematics meant a plural of four arts, "the sum of arithmetic, geometry, astronomy and music" (1991, p. 1). Freudenthal's view of mathematics as a human activity, a sort of "mental art" (1991, p. 2), and the resulting RME principles, unsurprisingly, resonates with my study goals. This is precisely why I have chosen it as my broad theoretical framing for the study.

With the general and specific curriculum aims of helping learners recognise mathematics as a creative human activity and of developing deep conceptual fractional understanding (South Africa. DBE, 2011a) as the driving motivation for this study, I found RME to be an appropriate lens through which to approach the integration of music and mathematics. This was further supported by Julie and Gierdien's (2020) evaluation of RME as an ideal framing for South Africa's democratic curriculum changes aimed at fostering learner-centered, active and engaged classrooms. Moreover, considering reports on the narrow and disconnected views of many South African mathematics teachers and learners (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017), I heeded Tasik and Edo's (2019) proposition that RME can be a "solution" towards improving motivation and attitudes in mathematics learning; along with van den Heuvel-Panhuizen's (2020, p. 9) argument that RME fosters a more "positive attitude towards mathematics".

Freudenthal believed that learners should be given opportunities to "reinvent mathematics by organising or mathematizing real world situations" (Cobb et al., 2008). Therefore, RME emphasises the need to teach and learn mathematics within a context that allows learners opportunities to apply mathematics thinking in solving real-life problems and which promotes the use of contexts where learners can learn mathematical concepts through natural activities (Gravemeijer, 1994; Hough & Gough, 2007; Tasik & Edo, 2019; van den Heuvel-Panhuizen, 2020). This approach is viewed as infinitely more beneficial than simply rote learning number facts, methods and formulae (Freudenthal, 1991). It provides learners with opportunities to have concrete, hands-on experiences with mathematics, to employ informal strategies, which can then subsequently be linked to more formal representations of "mathematical sameness" (van den Heuvel-Panhuizen, 2003). The approach further suggests making use of a context in which resources and activities plan for engagement of learners of all ages and abilities (Freudenthal, 1991; Streefland, 1991; van den Heuvel-Panhuizen, 2003; Cobb et al., 2008).

Cobb et al. (2008) describe in detail, three central tenets of RME as a design theory. It was these tenets that guided my design of the integrated music-mathematics tasks:

- Firstly, starting points in a teaching and learning sequence should be "experientially real" (Cobb et al., 2008, p. 108) for learners. This enables them to actively engage in the mathematical activity from the beginning through the use of informal modelling and ways of speaking. Freudenthal (1991) refers to the beginning of a teaching and learning sequence being "common sense" (p. 23). He also stresses the importance of using situations as a starting point, from which to experience mathematics, in such a way that the situations encourage learners to recognise the need to further engage in the mathematical activity. The task design of the lesson and activity sequences in my study provided learners with the opportunity to actively engage in a kinaesthetic activity (jumping across a river and beating rhythms on a drum or clapping), from which a music-mathematics teaching and learning sequence emerged.

- The second tenet Cobb et al. (2008, p. 109) discuss is that the starting point of the teaching and learning sequence should be "justifiable in terms of the potential end points of the learning sequence". This means that the scenario allows for informal reasoning, speaking, and representations which have the potential to support a progressive movement towards the intended formal ways of representing mathematics (Cobb et al., 2008). In RME terms, this is referred to as horizontal mathematising: solving and organising real experiences using mathematical tools (van den Heuvel-Panhuizen, 2003). Treffers (1986) described horizontal mathematisation as moving from the physical model to the mathematical model. The lesson activities for my study have been designed with the end-goal being deep interconnected conceptual understanding of fractions, with the focus on moving flexibly between multiple constructs of fractions (fraction as measure, fraction as ratio, and the part-whole construct).

- The third tenet Cobb et al. identify is that mathematical activities should be designed in such a way as to support the "process of vertical mathematization" (2008, p. 109). The aim is that learners are guided through their informal modelling (such as drawings, diagrams and tables) towards a more formal mathematical notation. The end point should be developing mathematical "mental objects" and "mental operations" (Freudenthal, 1991, p. 19). In contrast to horizontal mathematisation, vertical mathematisation is working within the formal, abstract mathematical symbols, discovering mathematical relationships between concepts and forming generalisations (Treffers, 1986; van den Heuvel-Panhuizen, 2003). The design of teaching and learning activities in my study creates opportunities for learners to first draw their own model of the problem scenario. They are then guided by their teacher to represent it in musical notation and to clap the rhythms. The end point is for learners to be able to represent and work with formal mathematical modelling of fractions, on, for example, a number line.

Van den Heuvel-Panhuizen (2003) discussed the role that models play in bridging the gap between the horizontal and vertical mathematisation (i.e., connecting the real scenario to the formal, abstract mathematics). Models are representations of the problem scenario. They intentionally reflect the relevant mathematical concepts. Models can be visual images, diagrams, or informal sketches that support the progression to the advanced formal representations (van den Heuvel-Panhuizen, 2003).

I need to highlight van den Heuvel-Panhuizen's explanation of *real* in RME. She explains that the term 'realistic' often causes confusion. As RME stems from a Dutch background, the Dutch term 'zich realiseren' should be considered rather, as this actually means 'to imagine'. Van den Heuvel-Panhuizen (2003) therefore highlights that, while the realistic starting point need not necessarily be a real-life one, it should be experientially-real for the learners. This could involve real-world situations, but also imaginary, fantasy situations, which have meaning to learners insofar as such situations provide suitable contexts from which they may develop their mathematical reasoning. An example which Cortina et al. (2014) used to teach fractions from an RME perspective is a story about how ancient Maya measured with a wooden stick, which does not cover all lengths exactly, thus the need for using fractions in measurement arose.

A further reason for selecting RME as the theoretical framing of my study is the support and guidance this approach provides to teachers (in terms of stages of preparation, planning, presentation and reflection) for developing strategies and activities that promote class discussion and reasoning within various contexts, thus promoting also the progression from informal to formal representations and application (Hough & Gough, 2007; Edo & Tasik, 2019).

Important to consider are the criticisms of RME noted in literature. Two commonly expressed criticisms are firstly, the possibility of teachers feeling under pressure to have their learners demonstrate formal procedures and abstract representations which many believe is accomplished more quickly with more direct teaching through rote learning procedures; and secondly, concerns that RME allows too much freedom for learners in coming to their own solutions to solve a problem (van den Heuvel-Panhuizen, 2020). Van den Heuvel-Panhuizen (2020) refutes this second criticism, emphasising the vital role of the teacher in guiding the learners in selecting an appropriate solution strategy for solving contextual problems. With an awareness of these challenges and critiques, I retain my belief in RME providing an appropriate framework within which to conduct my study.

2.3 Curriculum Integration

Description of curriculum integration

Despite Fraser's description of curriculum integration as a way to create coherence within and across subjects, she notes that curriculum integration is "one of the most confused topics in education" (2013, p. 18). Confusion notwithstanding, similarities are recognisable across various authors' definitions of curriculum integration. Drake and Burns (2004, p. 7), for example, write that integrating the curriculum is about "making connections". Civil (2007) describes curriculum integration as "a philosophy of teaching in which content is drawn from several subject areas to focus on a particular topic or theme" (2007, p. 11). More recently, Kneen et al. (2020, p. 262) write of connections "across subject boundaries" thus blurring distinctions between traditional subjects and allowing for holistic learning opportunities. Similarly, Pluim et al. (2020) refer to an integrated curriculum as one where knowledge is "packaged in more open relations to each other, with less insulation between contents" (p. 716) such that teaching different subjects happens concurrently. The language used in these definitions all carry similarities with Adler et al.'s (2000) description of curriculum integration as collapsing and blurring boundaries around school subjects.

Curriculum integration, as Fraser (2013) notes, also draws on the prior knowledge and real-life experience of learners, something which resonates with the first of the key tenets of RME outlined in the preceding section. While a variety of synonyms are used to refer to curriculum integration (such as interdisciplinarity, cross-curricular, integrated teaching, among others), for my study, I stick to the term 'curriculum integration' and, drawing on the complementary views contained in the literature, I define it as **an intended connection of various school subjects, moving flexibly between concepts within a subject and linking learning to the real life of the learners**.

Integration is by no means a novel concept in education research either globally or within South Africa's education development (Bresler, 1995; Adler et al., 2000; Drake & Burns, 2004; Naidoo, 2010; Kneen et al., 2020). Internationally, curriculum integration approaches are reported as having been attempted from as early as the 1940s in the United States, Britain and New Zealand (McKinnon et al., 1991; Drake & Burns, 2004; McPhail, 2018). Indeed, Dowden (2011) traces the concept of curriculum integration as far back as "innovations developed by pioneering teachers" from the 1920s, while Fraser (2013) believes that curriculum integration roots are to be found in the early 1900s, such as in the work of John Dewey. Dewey (1902, 1938) believed that learners should experience the curriculum in an integrated manner, to reflect the interdisciplinary knowledge and skills used for everyday problem-solving.

Arguments for and against curriculum integration

Many attempts to integrate subjects and topics are reported as having been unsuccessful (Fraser, 2013; McPhail, 2018). Fraser (2013) distinguishes curriculum integration from the popular theme teaching of the 1960s and 1970s, which explored how subjects could contribute to a thematic unit; curriculum integration, on the other hand, she explains as teacher scaffolding and facilitating inquiry into a central issue by drawing on different subjects which can contribute to the issue, rather than forcing a way to combine all subjects in inauthentic ways. McPhail (2018, p. 69) cautions, however, that, if not carefully planned and designed, this sort of thematic or integrated approach may lead only to superficial learning.

It is during this 1970s period that Bernstein's theorising on curricular structuring began attracting attention. Bernstein (1971), highlighting the trend of educational institutions of allocating specific, differentiated time periods for content within a curriculum, described curriculum as "the principle by which units of time and their contents are brought into a special relationship with each other" (p. 86). He further explained that different subjects were allocated different amounts of time, and were considered either compulsory or optional (Bernstein, 1971) thus giving the impression that certain content or subjects were more important than others.

Bernstein (1971) distinguished between two broad types of curriculum, based on how strong the boundaries were between different content: ‘collection’ type and ‘integration’ type. He described the former as having strict boundaries separating topics into units or subjects, with clear content and set of achievement and assessment criteria; the latter as moving away from the insulation of subjects towards “some relational idea, which blurs the boundaries between subjects” (p. 93), allowing for relationships between topics to be explored within and across different contexts.

Kneen et al. (2020) mention how curriculum integration has been viewed as an approach to educational transformation worldwide. In the South African context, curriculum integration first became a topic for discussion in the early 1990s, a time of democratic transition in the country. Along the lines suggested by Bernstein’s (1971) ‘collection’ versus ‘integration’ distinction, an integrated curriculum was, as Adler et al. (2000, p. 2) note, seen to offer “a response to traditional curriculum practices” which transmitted disjointed, abstract knowledge. Integration was seen to provide a mechanism for achieving the country’s curriculum reform goal of making classrooms more grounded and learner-centered spaces.

Naidoo (2010) described how the first post-apartheid curriculum, known as Curriculum 2005 (C2005)[5], attempted to implement curriculum integration through combining certain disciplines or subjects into ‘Learning Areas’ (for example, combining History and Geography into Social Sciences). Flaws identified in C2005 led to the implementation in 2002 of the Revised National Curriculum Statement (RNCS). This was intended to optimise the balance between curriculum integration and conceptual progression. Naidoo (2010), reporting on Grade 9 teachers’ perceptions of this form/strategy of curriculum integration, noted that, while some teachers were open to the idea of integration, more than two-thirds of the teachers in her study were resistant to it. As was the case in several other countries, South Africa’s education community found that challenges arising from the attempts at integrated curriculum led to tensions around whether to abandon such attempts or to continue trialling other sorts of integration strategies. While benefits of integration strategies were recognised, concerns about the quality of the content of the disciplines remained (Naidoo, 2010; Dowden, 2011; Fraser, 2013; McPhail, 2018).

[5] In the South African context, Curriculum 2005 (C2005) was implemented in 1998 with the goal of redressing the inequalities of the education system under the apartheid regime. C2005 was, however, revised in the year 2000 and the new curriculum became known as the Revised National Curriculum Statement (NCS).

Despite the challenges, McPhail (2018, p. 56) notes a "re-emergence" of curriculum integration within the context of some teaching and learning areas for Twenty-first Century skills, such as critical and creative thinking, collaboration and communication. Kneen et al. (2020) similarly recognised movements favouring the use of curriculum integration as a strategy to help prepare learners to participate in a Twenty-first Century society. Barnes (2015) too argued for careful attention being paid to planning, sequencing, progression and assessment of curriculum integration for Twenty-first Century teaching and learning. With such a turbulent history, spanning over one hundred years, it is curious that, as McPhail (2018) observed, attempts at curriculum integration continue in education practice and research, a movement which Pluim et al. (2020) describe as "persistent" despite the critique (p. 719). It is, in my view, a worthwhile persistence. Ongoing reflection on the benefits of curriculum integration, taking due cognisance of the challenges experienced, and seeking research-based solutions for them, as is the motive behind my own study, may help to show our community how the challenges of curriculum integration might be overcome and its benefits harnessed.

Different degrees of integration

There are different degrees of integration, or, as Fraser (2013, p. 20) expresses it, there is a "continuum of curriculum integration". Much depends on the ways in which the different components are connected. I use this section to briefly discuss some such variations.

Bernstein (1971) classified the integrated approach according to whether it was teacher-based or teachers-based, and whether the integration was within a subject or across different subjects. Teacher-based integration is where a teacher spends an extended amount of time with the same group or class of learners during which time, s/he may either teach distinct subjects to the group, or choose to "blur the boundaries" between subjects (Bernstein, 1971, p. 93). Teachers-based integration, by contrast, involves teachers of different subjects collaborating in order to demonstrate relationships between their subjects (Bernstein, 1971). As shown in Figure 2.1, below, from these teacher/s-based types of integration, two further variations emerge: integration within a subject or integration across multiple subjects (Bernstein, 1971). Of the two integration pathways illustrated in Figure 2.1, my study, in general, followed the second, teachers-based pathway. My research participants and I worked together in CoPs in fine-tuning the overall design and implementation of the integrated music and mathematics lessons. We integrated both *within* a subject (mathematics: multiple constructs of fractions) and *across* subjects (mathematics and music). I have highlighted (in blue) this route on Figure 2.1. I do recognise, however, that the degrees of integration in my study were not always as 'neat' or 'straight-forward' as this highlighting implies. It could be argued that some of the integrated lessons (taught by just a single teacher) represented a teacher- rather than teachers-based degree of integration.

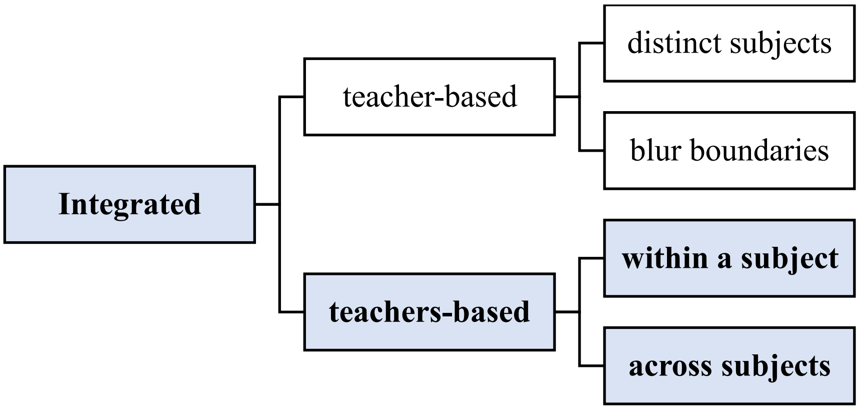


Figure 2.1: Bernstein's classification of an integrated approach (adapted) (1971, p. 91)

Drake and Burns (2004) identified three categories of curriculum integration: multidisciplinary, interdisciplinary and transdisciplinary, each of which depends upon the starting point for the integration:

- Multidisciplinary integration uses the discipline as a starting point and shows how different subjects might relate to one another around a common theme.
- Interdisciplinary integration has common skills and concepts as the starting point, from which the disciplines can then be identified. This approach places less emphasis on the subject areas and more emphasis on the skills and concepts.
- Transdisciplinary integration makes use of a real-life context as the starting point. From here, learners' questions guide the process of developing skills as learning progresses. Project-based learning would be an example of transdisciplinary integration.

Kneen et al. (2020), drawing on Bernstein's work in distinguishing the levels of curriculum integration, found that primary school teachers were better placed to use what Drake and Burns (2004) label the transdisciplinary approach. Primary school teachers tend to be generalists rather than subject-specialists, and, as such, tend to find it easier to adopt integrated strategies where a particular theme might guide their choice of content. Primary school teachers also tend to have more flexibility with curriculum approaches, especially where they spend the majority of a day with a single class of learners. Kneen et al. (2020) found, on the other hand, that secondary school teachers (known in South Africa as Senior Phase or Further Education and Training teachers) tended to opt for a more multidisciplinary approach, where the subject discipline took priority, and themes were worked around the subject.

Writing of arts integration, Bresler (1995), having observed integration practices at three schools over a three-year period, identified the following four styles. She based her style categorisation on how the art form was viewed in relation to whatever other curriculum area/s (‘academic’ or ‘core’ subjects) it was teamed with.

- The subservient style:
  This is the most prevalent style, as teachers do not need expertise in the art form. In this style of integration, the art form serves the academic subject. Teachers, nonetheless, use this style as it is a time-efficient way of meeting academic and art curriculum expectations, providing multiple representations of content in the academic subject (Bresler, 1995). It risks the neglect of the art form, which Kneen et al. (2020) also identified as a concern. An example of this style of arts integration includes singing songs related to topic themes. In such instances, the art element does not place especially high cognitive demands on learners.

- The co-equal, cognitive style:
  This style acknowledges the art form as being *equal* to the academic subject. Also referred to by Bresler (1995) simply as the co-equal style, this approach is the least prevalent and most challenging. She attributes this to the need for knowledge and skill in the art form, thus requiring the teacher to have an extensive background in the relevant art form. Such co-equal integration, Bresler found, engaged learners in higher-order cognitive skills, drawing as they did on specific art skills and knowledge. It also encouraged more active participation, critical reflection and appreciation for the arts. An example Bresler (1995) gives of this is teaching the concept of ‘lines’ across music (melodic lines), literature (prose lines), dance (movement lines) and visual arts (use of lines to show perspective).

- The affective style:
  In this approach teachers use art to enhance the mood or classroom atmosphere by playing background music, for example, to help learners recognise their emotions, to change the pace of the lesson, or even to quieten a noisy class (Bresler, 1995). Another element of this style is to encourage creativity. This, Bresler (1995) noted, occurred

mainly in early grades of children between the ages of five and eight, where they would be expected to engage in open-ended tasks relating to song, dance, visual images or play, to express themselves in projects. Bresler (1995) noted the usefulness of this style in supporting learners who found it difficult to express themselves in the usual ways, such as, those with language barriers or special needs.

- The social integration style:
  Complementing the academic curriculum, this style is used when schools use arts to enhance social and community relationships, such as performing concerts for parents, playing music or performing dances at social events, and celebrating cultures on holidays (Bresler, 1995). Sometimes these are organised by the specialist art teachers and sometimes by the classroom teachers.

While the styles Bresler identifies here are independent, they can be combined and different teachers use different combinations in various ways to suit their pedagogical needs. In my study, I combined the co-equal, cognitive style (maintaining fidelity in the mathematics and the music) and the affective style (using music as a context to experience mathematics, recognising it as a creative human activity).

Focusing specifically on mathematics teaching and learning in South Africa, Adler et al. (2000) identified three main degrees of mathematics curriculum integration: integration across mathematical concepts; integration of everyday life into mathematics; and integration between mathematics and other subjects.

To some extent, an RME-informed integrated task design could be seen to link to the transdisciplinary degree of Drake and Burns (2004). A problem scenario serves as a context for an experientially-real starting point (Cobb et al., 2008). This said, RME also holds firm to the discipline of mathematics, hence my study’s use of multiple integration strategies, three in particular:

- firstly within mathematics (Bernstein, 1971; Adler et al., 2000), with the aim of moving flexibly between multiple constructs of fractions;
- secondly, across the subjects of mathematics and music (Bernstein, 1971; Adler, 2000), or as Drake and Burns (2004) may refer to it as a multidisciplinary approach;
- and thirdly, integration with real-life contexts (Bernstein, 1971; Adler, 2000; Drake and Burns, 2004).

Figure 2.2, overleaf, illustrates how my study further blurred the boundaries in its use of a variety of integration strategies. Behr (1983) (as discussed in Section 2.7) links the part-whole construct of fractions to the other four constructs. This is why I have chosen to overlap the fraction constructs in the diagram. And, because musical note values are linked to the beats per bar, musical beat and note values also overlap.

Some benefits and challenges in curriculum integration

Along with educational transformation and an approach to foster Twenty-first Century skills, studies have shown various benefits of curriculum integration. McPhail (2018) suggests that it is partly *because* of teachers' acknowledgement of the potential benefits of an integrated approach to the curriculum, that curriculum integration continues to be explored in schools and in research.

Studies have reported that integrated lessons can increase learner attention and interest, as well as encourage engagement and participation (Naidoo, 2010; McPhail, 2018; Pluim et al., 2020; Tytler et al., 2021). Fraser (2013) states that learners are actively involved in such integrated lessons, as it provides them the opportunity to be part of the decision-making process. Through integrating curriculum with the everyday life of learners, it becomes more relevant to them, thus resulting in enthusiasm and higher levels of motivation (Adler et al., 2000; Naidoo, 2010; Barnes, 2015; McPhail, 2018).

Naidoo (2010) recounted how some South African teachers appreciated the opportunities which curriculum integration opened to engage with community-based projects. A further advantage, which Pluim et al. (2020) highlight, is that integrated curriculum with real-world context will have economic benefit for a society, as learners are exposed to everyday expectations and have to apply their learning to solve realistic problems. This will better prepare them to fully participate in society in future. Pluim et al. (2020) further note that curriculum integration can meet diverse needs in the classroom, as context and culture can be considered, thus allowing for a more democratic handling of a lesson's design and content.

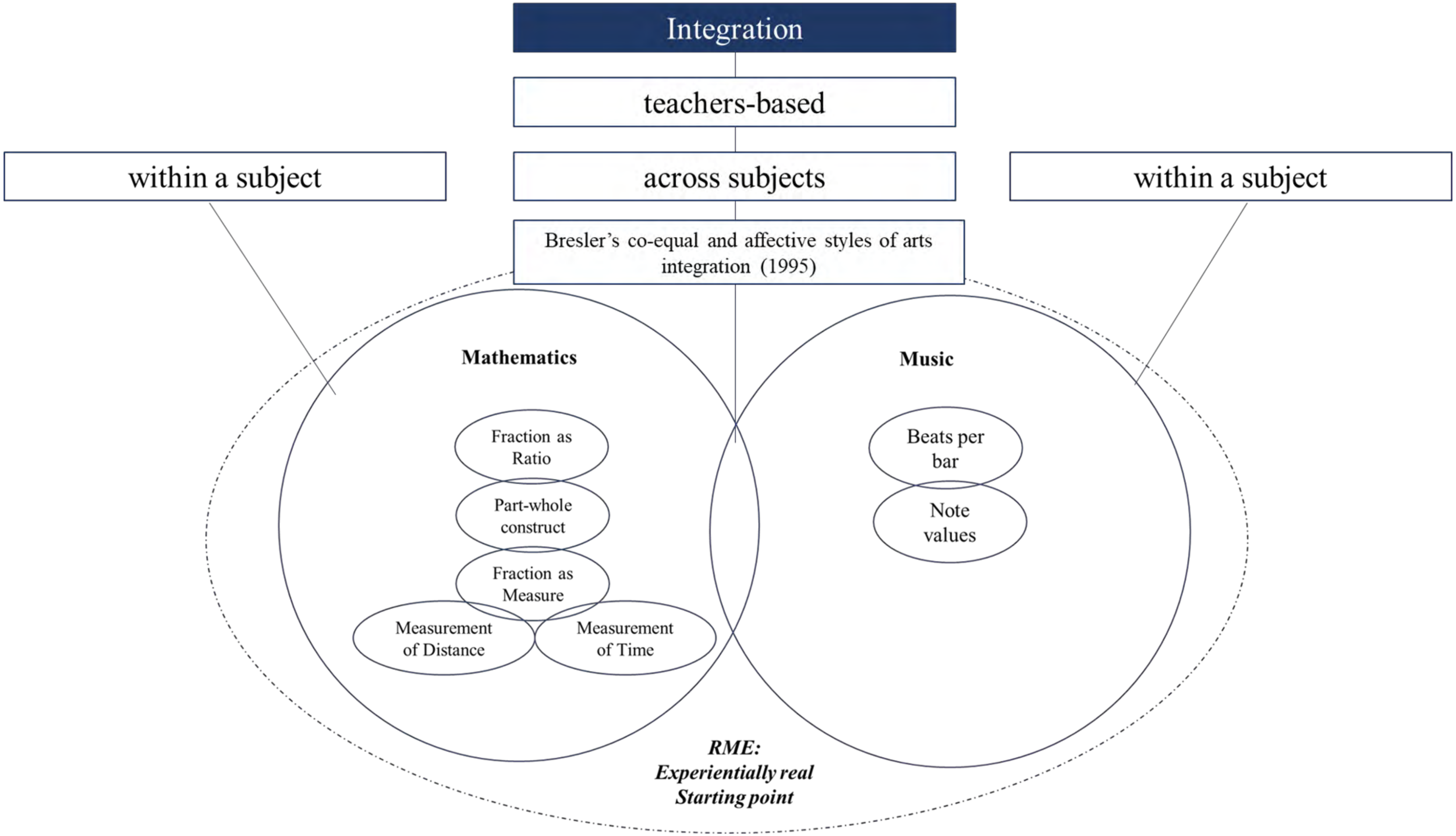


Figure 2.2: Visual representation of integration for my study

McPhail (2018) reports an eight-year longitudinal study conducted in the USA during the 1930s, which showed that curriculum integration resulted in a "slight improvement in academic scores" (p. 66). This may derive in part from the point made by Pluim et al. (2020) that curriculum integration allows learners to see a topic from multiple perspectives, engage in problem-solving and organising content around big ideas, all of which help develop deeper understanding of the topic.

While a key critique of curriculum integration is that it involves time demands on teachers, McPhail (2018) and Pluim et al. (2020) highlight that there is practical benefit of integrating curriculum, by minimising the overlapping content across subjects. Through careful planning, teachers can avoid unnecessary repetition across segregated subjects, and rather cover the content once, in a meaningful and connected way. This will inevitably save time in the classroom. Furthermore, such integration also encourages collaboration amongst teachers, to plan, share expertise and resources and support one another across disciplines. In so doing, teachers can model collaboration for their learners (Bernstein, 1971; McPhail, 2018; Pluim, 2020). In contrast to these benefits, Naidoo (2010), reported on the concerns of South African Grade 9 teachers, who felt that combining subject disciplines into Learning Areas resulted in overlaps and unnecessary repetition. These teachers recognised the need for collaborative planning of integrated lessons, but they were not given enough time to do so.

While many benefits of curriculum integration are evident and discussed in literature, so too, many challenges have been experienced and documented throughout the literature. Fraser (2013) believes that one of the reasons why teachers and curriculum designers experience so many challenges can be ascribed to poor or incorrect understandings (or interpretations) of what curriculum integration is and what it entails. Of major concern across the literature is that the particular knowledge and skills within disciplines become diluted through blurring the boundaries of subjects (Drake & Burns, 2004; Naidoo, 2009; Fraser, 2013; Barnes, 2015; McPhail, 2018; Kneen, 2020; Pluim et al., 2020). Kneen et al. (2020) note that the challenge lies in upholding the "integrity" of the discipline knowledge (p. 259).

Literature establishes that curriculum integration is complex and difficult. Designing such integrated curricula, units, lessons, activities and resources proves to be a demanding process. Virtue et al. (2009), for example, share concerns that teachers may be confronted with many obstacles when moving towards an integrated approach. Adler et al. (2000) found that curriculum integration is not an easy or straightforward task. McPhail (2018) further claims that, even for experienced teachers, curriculum integration "remains a great challenge" (p. 71) and is "much harder than it looks" (p. 66). The main concerns about curriculum integration that surface in literature include demands on teachers and gaps in content knowledge and skills.

Demands are placed on teachers when they are expected to implement curriculum integration. Teachers' workload increases, as it takes careful and rigorous planning to design integrated lessons (Naidoo, 2010; McPhail, 2018). Decisions need to be made about what topics to combine, the degree of integration, how the integrated content will be delivered, and how much time or weighting should be allocated to each subject (McPhail, 2018). Teachers may not feel comfortable in teaching for integration, as they may be faced with concepts or subjects in which they are not experts (Adler et al., 2000; Pluim et al., 2020). They need adequate time and support to plan and prepare for implementing an integrated curriculum (Adler et al., 2000; Naidoo, 2010; McPhail, 2018; Pluim et al., 2020; Bicer et al., 2022).

Excessive teacher demands, and sometimes lack of expertise outside of their field, means that the potential exists for inappropriate balance between subjects and the dilution of the discipline knowledge (McPhail, 2018; Pluim, 2020). Naidoo (2010) notes that teachers felt their professional identities were tied to the subjects they studied and taught. They felt inadequate to deliver high quality lessons on subjects in which they lacked expertise and they found it difficult to motivate learners and show enthusiasm for subjects with which they were unfamiliar (Naidoo, 2010). Kneen et al. (2020) caution that the danger exists of weakening disciplinary knowledge through an integrated curriculum. McPhail (2018) similarly argues that using curriculum integration to promote engagement may result in stunted "cognitive advancement" (p. 66), which, as he notes, is necessary for progressing in learning. McPhail (2018) advises against "integration for integration's sake" (p. 66), arguing instead for meticulous planning for deep conceptual understanding of content from within each discipline.

Writing specifically of the South African context, Naidoo (2009; 2010), noted gaps in learners' discipline knowledge and understanding as a challenge when curriculum integration was implemented. Her research, based on the National Curriculum Statement (NCS), indicated that the teachers she surveyed, in the South African context, were not convinced of the links made between subjects in this curriculum revision, and that they were concerned about this negatively impacting the national Grade 12 school-leaving (matriculation) examinations. The teachers participating in her study expressed concern about the integrated Learning Areas resulting in poorer academic quality. Reasons given for this included:

- That it was challenging to structure the planning for conceptual development;
- That teachers felt the subject content was fragmented and 'watered-down';
- That it was difficult to assess an integrated curriculum;
- That teachers were concerned about how they would instill an appreciation for their individual subjects; and
- The textbooks and resources were of poor quality, often leaving out the "basics" (Naidoo, 2010).

In her 2010 article, Naidoo mentions the DBE's statement on their intentions for, yet again, revising the curriculum. She explained her interpretation of the changes being that curriculum planners sought to address teachers' concerns by placing greater emphasis on subject-based content knowledge, specifying exactly what and when content should be taught, and referring to 'subjects' instead of the 'Learning Areas'. This new curriculum became known as the Curriculum and Assessment Policy Statement (CAPS) (South Africa. DBE, 2011a), and was officially implemented in 2012 (and currently still stands). As noted in my introductory chapter, little explicit emphasis is placed on integration strategies in the CAPS documents, despite it being identified as one of the curriculum's general aims.

Hence, for instance, Naidoo (2009) noted a positive effect on learning if approximately 10% of the time was spent on integrated projects, after the knowledge had come from approximately 90% of subject discipline time. This allowed for developing deep conceptual knowledge of each subject, and then carefully selecting which content to meaningfully integrate across subjects. This form of integration showed educational gains and stronger emphasis on developing content knowledge and application of skills. Barnes (2015) similarly noted that curriculum integration and individual subject teaching "exists profitably side-by-side" (p. 262), as did McPhail (2018). He suggested that curriculum integration not be the dominant mode of teaching and learning, but that it rather supplement developing deep understanding in disciplines, his phrase "boundary maintenance before boundary crossing" speaking of a discipline focus first, followed by well-planned integrated activities for application (McPhail, 2018, p. 70).

Implementing an integrated curriculum need not necessitate abandoning subject disciplines entirely, but rather making space for both a discipline-based and an integrated approach could offer much value to teachers and learners (Presmeg, 2009; Bautista et al., 2016). In agreement with these authors' suggestions, Pluim et al. (2020, p. 731), promote a "Worldly Perspective" to curriculum, which they explain as a combination of integrated and disciplinary strategies, where they overlap in support of one another. Kneen et al. (2020), furthermore, describe the

Welsh response to developing an integrated curriculum, where the subject teachers' roles were considered crucial, and teachers were invited to be "Pioneer Practitioners" (p. 259) to plan the curriculum from a bottom-up approach. This, as Naidoo (2010) explained, is in sharp contrast to what happened in South Africa, where teachers were not involved in developing the integrated curriculum.

Despite the potential benefits of curriculum integration having been explored over a considerable time, challenges persist and little integration across subjects is evident in curriculum support texts provided to teachers in various subjects[6]. As has been indicated in this section, all the authors cited above recognise both benefits and challenges of curriculum integration and all agree that some form of a combination of curriculum integration and subject disciplines can be a possible way forward. This points to a need for further research into effective ways of integrating curriculum, and, in my next section, I turn specifically to literature on mathematics integration. After a brief overview of integration in the broader context of mathematics teaching and learning, I focus on literature relating to links between music and mathematics, and then, more specifically, on literature to do with integrating these two subjects to support the teaching and learning of fractional understanding.

2.4 Mathematics Integration

There has been much discussion around integration across Science, Technology, Engineering and Mathematics, commonly referred to as STEM (English, 2016; Kelley & Knowles, 2016; Holmlund et al., 2018; Gunawan & Shieh, 2020; Tytler et al., 2021). More recently, recognition for integrating the arts has resulted in the adjusted acronym STEAM education (MacDonald et al., 2020; Bautista, 2021). Of particular importance for mathematics education are the concerns expressed by both English (2016) and Tytler et al. (2021), that mathematics learning does not benefit as much as the other STEM subjects in integration projects, plus that obstacles exist around ensuring and maintaining integrity of the STEM disciplines. Such concern notwithstanding, research into integration opportunities for these subjects continues, in recognition, as Tytler et al. (2021) note, of the benefits for solving real-world problems, making learning relevant and meaningful.

South African researchers, Adler et al. (2000), explored the feasibility and desirability of integration within and across mathematics. Despite having been published 22 years ago, this study's findings still apply to the general aims of our current CAPS curriculum. Naidoo (2010)

[6] See weekly planners in curriculum documents across subjects (South Africa. DBE, 2011).

reported that teachers found mathematics difficult, if not impossible, to integrate. This is in line with Adler et al.'s (2000) study which found that integration across mathematics and other subject areas made it difficult to teach a *new* mathematical concept, but was more appropriate for *application* activities. When working with pre-service teachers Adler et al. (2000) found the attempted integrated lessons were mostly "textbook-style word problems" which superficially related a mathematics concept to another topic (p. 4). They concluded that it was more useful to have a mathematical concept as the starting point, from which to explore integration opportunities, rather than starting with another subject. They also worked with two in-service teachers. One of the teachers tried to integrate mathematics with everyday life contexts, and found that the demand this placed on the teacher was very high, as she felt her knowledge of the broad topics was not sufficient, and this resulted in poor sequencing of activities, imbalance of content, and the mathematics being reduced to learners copying from the teacher's board and the mathematics being overshadowed by the other subject. Adler et al. (2000) suggested that a possible way to combat this challenge was for teachers to collaborate across subjects so as to retain expertise across the topics, a solution which echoes Bernstein's (1971) model of teachers-based integration. The second in-service mathematics teacher took another approach. He taught a lesson which integrated mathematics concepts. This Adler et al. (2000) termed integration *within* mathematics. They expressed the view that this approach may offer the most success in terms of mathematics learning, as the integration stems from the mathematics itself, thus allowing for a clear focus on concepts. Adler et al. (2000) concluded that "there is a need to zoom in on the mathematics and out again repeatedly" (p. 9) in order to maintain a balance between the mathematics and the integrated concepts. This conclusion resonates with the suggestions made by Presmeg (2009) and Bautista et al. (2016) (discussed in Section 2.3 regarding making space for both a discipline-based and an integrated approach).

2.5 Arts Integration

In Section 2.3 I briefly mentioned Bresler's work (1995) on aspects of arts integration, noting her description of it involving linking the various arts forms to what she termed the 'academic' or 'core' subjects such as mathematics, sciences and languages. Barnes (2015) notes that there are records showing that as far back as Plato's time (more than two millennia back), integrating skills of music, movement, drama, literature, philosophy and politics has commonly been practised (Barnes, 2015). Within formal schooling systems also, as An and Tillman (2014) note, teachers have long practised integrating arts into their curricula. They describe how the arts were incorporated by teachers in the 1950s to early 1960s as a means of encouraging creative thinking (An & Tillman, 2014). They, however, note that there was a decline in arts integration as the more rigid, discipline-specific focus on curriculum reverted to emphasising 'core' subjects such as mathematics and science, after the Sputnik era (An & Tillman, 2014). The 1970s, in part because of the influence of multiculturalist movements, saw a reignition of the notion of integrated curriculum, and the arts were once again included in the interdisciplinary curriculum with a view to making learning more relevant to a broader spectrum of learners (Bresler, 1995; An & Tillman, 2014). As noted earlier, however, Bresler (1995) found that classroom teachers were hesitant to integrate arts into their subjects, as they did not fully embrace the top-down curriculum demands placed on them with little or no support in carrying them out.

More recently the study by Kneen et al. (2020), involving both primary and secondary teachers, explored how arts could be integrated into the Welsh school curriculum. Their study emerged from growing concerns about the marginalisation of the arts in the curriculum. Despite reports, such as from Naidoo (2010), that arts teachers expressed a greater openness towards an integrated curriculum, Kneen et al. (2020) observed that arts integration was a lot more challenging than expected.

Barnes (2015) warns that "one-off arts projects" (p. 266) will not be successful in developing meaningful, relevant integrated learning opportunities. Ideas for how to make integration more meaningful and effective include An and Tillman's suggestion (2014) that teachers use arts integration to help learners cross borders between segregated subjects (comparable to Bernstein's (1971) notion of blurring boundaries between subjects). Eisner (1996), too, discourages the strict distinctions between purely academic subjects and the arts. Bresler (1995) refers to Broudy's work from 1972, calling for arts education to serve a holistic function of helping learners appreciate the arts as well as developing in their awareness and perception of creativity in the world around them, such as nature and clothing. This links well to South Africa's curriculum aims for Creative Arts (within the Life Skills subject area), which include:

- guiding learners to achieve their full physical, intellectual, personal, emotional and social potential;
- developing creative, expressive and innovative individuals;
- developing skills such as self-awareness, problem-solving, interpersonal relations, leadership, decision-making, and effective communication;
- providing learners with exposure to experiences and basic skills in dance, drama, music and visual arts including arts literacy and appreciation (DBE, 2011b, p. 10).

An and Tillman (2014) list examples, similar to Drake and Burns (2004), recounting how creative teachers in the USA use arts to deliver 'core' curriculum, for example using dancing to explore mathematical and scientific concepts. They also report that art forms such as music and theatre are related to increased success in reading and mathematics.

Various studies have shown benefits of integrating arts into other curriculum subjects, not unlike the literature on curriculum integration in general. Art integration promotes enjoyment, motivation, and increased learner engagement and confidence (An & Tillman, 2014; Kneen et al., 2020). Bautista et al. (2016) argued that arts integration can positively impact a learner's cognitive, emotional and social development holistically. An integrated art lesson can provide opportunities for open-ended questions which require critical, creative thinking and problem-solving, exploring concepts at complex levels (Bickley-Green, 1995; Bautista et al., 2016; Choutou, 2019). Bickley-Green (1995) refers to "instructional richness" (p. 15) that emanates from careful planning of mathematics and art integrated lessons. She even suggests, in contrast to Kneen et al. (2020), that arts integration can eventually result in "excellence in instruction" while facilitating deep conceptual learning of both the arts and the mathematics (Bickley-Green, 1995, p. 7). Authors appreciate the arts for providing multiple mediums for learners to explore concepts and to express their ideas (An & Tillman, 2014; Bautista et al., 2016). An and Tillman (2014) further recognised that arts integration has the potential to meet diverse needs of learners, through differentiation of more complex tasks and meeting the special needs of learners. Additionally, parental involvement, addressing societal issues, and linking to real-world experiences are possible with arts integration (Presmeg, 2009, An & Tillman, 2014; Choutou, 2019). Bickley-Green (1995) suggested that arts integration may benefit a society's economy, as it shows learners opportunities for application in careers, for example, engineering or design. This aligns with the South African CAPS general aims of "equipping learners… with the knowledge, skills and values necessary for self-fulfillment, and meaningful participation in society as citizens of a free country" (South Africa. DBE, 2011a, p. 4).

In contrast to Naidoo's (2010) findings in South Africa, arts integration, according to An and Tillman (2014) can decrease the fragmented nature of the discipline approach to the curriculum, and reduce redundancy, as topics are explored holistically. Like literature on curriculum integration, there too were challenges with arts integration strategies. Kneen et al. (2020) reported, in contrast to Bickley-Green (1995), teachers' concerns about subject development – integrating arts with other ('core') subjects could lead to the arts being neglected. There is a further challenge of ensuring that learners were given the opportunity to develop the full knowledge and skill set of the arts (Kneen et al., 2020). Bresler (1995) recognised that, despite the benefits of integrating the arts into the core subjects, the actual implementation of integration was challenging and received little attention in literature, which focused mostly on successful instances. Many studies report that teachers felt they lacked confidence and expertise in subjects other than their own specialist disciplines (Bautista et al., 2016; Choutou, 2019; Kneen et al., 2020). In Kneen et al.'s (2020) study, music appeared to be the art form in which teachers were least confident to teach. Other challenges identified include inflexible timetables, having to prepare learners for external assessments, teacher reluctance and scepticism (Bautista et al., 2016; Choutou, 2019; Kneen et al., 2020).

While some researchers, such as Bickley-Green (1995) and Bresler (1995), recognise arts integration as an opportunity for teachers across disciplines to collaborate, many describe the challenges in this. Lack of supporting resources and guidelines often leaves the onus on teachers to creatively plan and implement art integration in their lessons (Bautista et al., 2016; Choutou, 2019; Kneen et al., 2020).

Bresler (1993) also reported on non-specialist classroom teachers' integration of music, specifically. Bresler (1993), Whitaker (1996) and Giles and Frego (2004) all found that music was mainly incorporated into lessons through singing, listening to music, and movement, with the goals of teaching other subjects' content in a memorable way, taking a break from academic work or just for background music. These are examples of Bresler's (1995) subservient and affective styles of arts integration. Interestingly, Giles and Frego (2004), in their small-scale study with 18 teachers across three schools, found that these strategies of music integration were not dependent on learner age or grade. Bresler (1995), on the other hand, found the styles to be predominant with teachers of younger learners.

Bresler (1993), Whitaker (1996), and Giles and Frego (2004) also found that classroom teachers faced challenges when trying to integrate music: lack of time for adequate planning, lack of guidance or structures for support and minimal resources, as well as a pressure to achieve in academic subjects' assessments. This is a similar trend to what was noticed with general curriculum integration challenges. Whitaker (1996) expressed concern about some ways of integrating music leading to superficial views of its importance. Classroom teachers also were reluctant to teach musical concepts, such as in Bresler's co-equal style, as they felt apprehension and lacked confidence in their own musical skills and knowledge. Giles and Frego (2004) also reported that classroom teachers in their study felt that it was not their responsibility to teach music concepts and skills, but rather the music specialists'. They did, however, note a positive attitude amongst teachers towards music integration in their subjects. They, therefore, suggested that improving collaboration channels between classroom teachers and specialist music teachers, through providing time and space to collectively plan, would indeed give classroom teachers the tools needed to successfully integrate music in their teaching while upholding the integrity of all the subjects and the music (Giles & Fargo, 2004). Civil (2007) attributes enhanced teaching and learning of reading, sciences and mathematics to such collaboration and integration of music. As discussed in my Findings chapter, these were similar challenges I found myself navigating through the implementation of the music-mathematics integrated tasks.

2.6 Integration of Mathematics and Music

Mathematics-music connections have been noted since the times of such philosophers as Pythagoras, Plato and Aristotle (Papadopolous 2014; Amon, 2017). Although in Kneen et al.'s (2020) study, music was the art form teachers were the least confident about integrating, much research has been done on the benefits of integrating mathematics and music (Geist et al., 2012; Papadopolous, 2014; Civil, 2007; An et al., 2014, 2016; Holmes & Hallam, 2017; Samsudin et al., 2019; Harney, 2020).

Consistent with findings in some of the more generic investigations into integration discussed in Section 2.3, studies have shown that integrating music into mathematics lessons can attract learners' attention, increase motivation, confidence and participation in activities, and decrease anxiety (Edelson & Johnson, 2003; Civil, 2007; Geist et al., 2012; An et al., 2016; Samsudin et al., 2019; Lovemore, 2020). Edelson and Johnson (2003) found that the mathematics classroom environment was positively affected through implementing musical activities. Samsudin et al. (2019) found that incorporating music and movement into pre-school mathematics lessons enhanced children's emotional development, as they could express themselves and interact with their peers in a positive manner. In my Master of Education study (Lovemore, 2020), I similarly found that incorporating music activities into my Grade 5 mathematics lessons supported learner collaboration and communication in problem-solving. Samsudin et al.'s (2019) findings can be likened to those of Ariba and Luneta (2018), who found that Grade 1 (7 years of age) learners' attitudes and disposition to mathematical activities were improved through music integration. An and Tillman (2014) also noted a more positive disposition towards mathematics when music was integrated. These findings are hopeful in the South African context, as Graven and her colleagues report the negative disposition with narrow views towards mathematics (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017).

Researchers have found that music-mathematics integrated lessons have the potential to increase learner understanding of concepts (Greene, 1991; Samsudin et al., 2019; Harney, 2020). Harney (2020) believes that such integration allows teachers and learners to explore fundamental concepts that are found in both mathematics and music, which then leads to deeper conceptual understanding in both subjects. Civil (2007) and An and Tillman (2014) recognised that music-mathematics integration encouraged learners to engage in more complex problem-solving, through promoting discovery learning and identifying relationships.

In addition to the increase in learner creativity (Greene, 1991; An & Tillman, 2016; Harney, 2020), music integration also creates real-life scenarios across multiple contexts in which learners can see and use mathematics (Civil, 2007; An et al., 2016; Harney, 2020; Lovemore, 2020). Music further provides opportunities for learners to engage in mathematics through the use of multiple senses, including visual, auditory and kinesthetic representations of concepts (Samsudin et al., 2019; Lovemore et al., 2021a).

Geist et al. (2012), for example, found that mathematical learning can be enhanced through elements of music, such as rhythm, melody, beat and pattern. Giles and Frego (2004) also mention the use of rhythmic elements to teach mathematical concepts. Learners' spatial-temporal reasoning can be improved through mathematics and music (Civil, 2007; Holmes & Hallam, 2017). A quasi-experimental design study by Holmes and Hallam (2017) showed that spatial-temporal skills, considered to be higher-order mathematical skills, improved with music instruction, particularly rhythmical activities, in children aged four to seven years. Stevenson-Milln (2018) identified ways in which patterns in music could be used to support Grade R (5 to 6 years old) learning of patterns in numeracy.

An et al. (2013) state that most mathematical concepts can be related to music in some way. Music and mathematics both have symbols which represent abstract ideas (Harney, 2020). Harney (2020) lists several mathematical concepts that can be linked to music, some of which include counting (for example, counting the beats in a bar or in a song), line, pattern, proportion, symbols, symmetry, shape, unit, ratio, relationships, part-whole concepts and measurement.

Samsudin et al. (2019) used music to help pre-school learners remember number names and enhance their understanding of time. An and Tillman (2014) carried out a study with pre- and in-service teachers, asking them to plan music-mathematics integrated lessons. They noticed four main types of music-themed activities that were designed to support mathematics learning:

(i) Listening and singing to introduce a topic, memorise content and entertain learners.

(ii) Composing and performing music, which was the most popular type of activity. In these activities learners could compose, decompose and recompose music using informal graphical notation to represent musical notes, such as colours, informal symbols and numbers. Learners could then play their songs using percussion instruments such as handbells and keyboards.

(iii) Musical notation activities that focused on teaching formal musical notation and linking it to numbers and fractions, as well as clapping and singing notes of varying lengths of time.

(iv) Designing and making musical instruments to practise concepts of measurement, shape and space.

In my study, all four of these activity types were incorporated in the task design. In relation to (i), however, body percussion, such as clapping, replaced the singing. For (ii) I designed tasks for learners to informally notate and perform body percussion, and in (iii) the formal musical notation of the music staff line was aligned with a number line. In terms of (iv), I designed and made the musical instruments from found objects (such as putting raw rice into an empty plastic bottle), which is also linked to the Grade 4 Natural Science and Technology curriculum (South Africa. DBE, 2011c).

In their study, An and Tillman (2014) found it interesting that most of the musical activities focused on the mathematical concepts related to numbers, operations and relationships. Elaborating on how measurement can be used in music-mathematics integrated lessons, Harney (2020) explains that the inverse order relationship of units of measures and iteration can be taught through music by demonstrating that one whole bar of music can be filled with one whole note, or two half notes or 16 sixteenth notes, therefore, the smaller the value of the note (measure of time), the more iterations will be needed to complete the bar. I too explored this inverse order relationship in my MEd study into ways of using music to support learners' developing deep fractional understandings (see Lovemore, 2020).

Fraction teaching and music

Several studies have been done on how fractions and music note values can be integrated. Fractions can be related to music note values[7], where mathematical relationships can be explored between notes of different time value (Harney, 2020; Lovemore, 2020; Lovemore et al., 2021a). Schmidt-Jones (2021, p. 27) explains that note values work “like fractions in arithmetic: two half notes or four quarter notes last the same amount of time as one whole note”. See Table 2.1 below for a graphic notation of the note values and fractional relationships, and Figure 2.3 from Schmidt-Jones. An and Tillman (2014) reported on how pre- and in-service teachers planned lessons to teach musical note values and relate this to fractions, with a focus on the part-whole relationship of fractions.

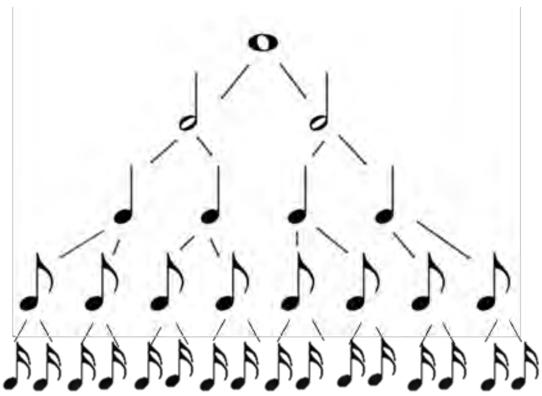
Note value symbol	**Musical notation naming conventions**		**Relative values of musical notes**
	British-English name	**American-English name**	
	Semibreve	Whole note	
	Minim	Half note	
	Crotchet	Quarter note	
	Quaver	Eighth note	
	Semi-quaver	Sixteenth note	

Table 2.1: Table representing musical note value symbols, naming conventions and relationships

[7]The duration of a given printed note relative to other notes in a composition, considered in relation to the tempo (speed) of the musical piece, also known as time value (Collins English Dictionary, 2022).

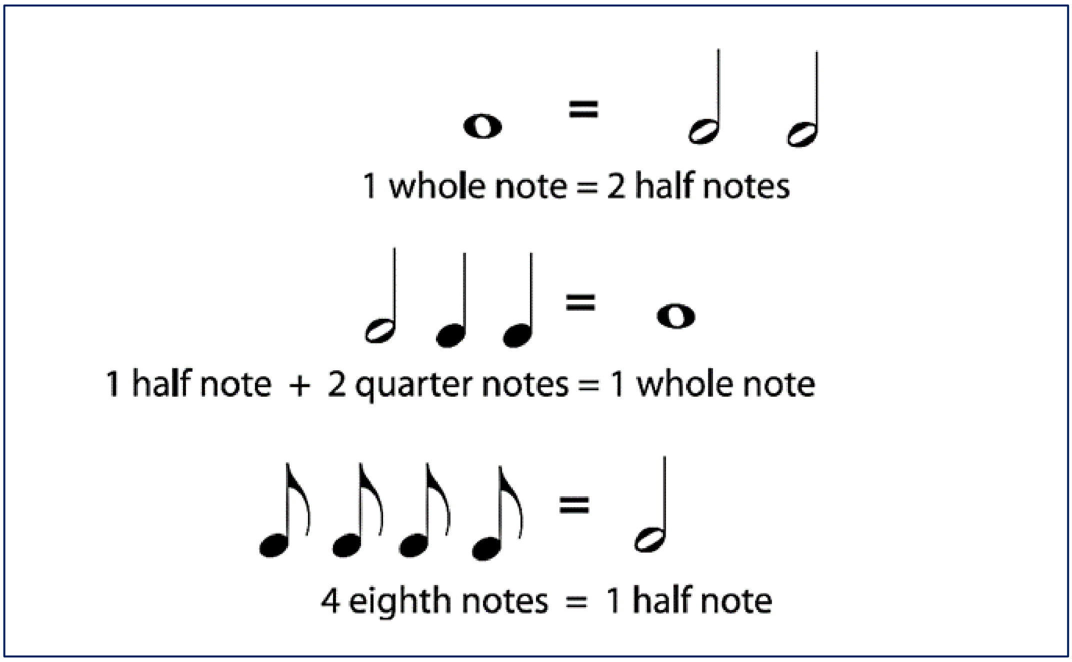


Figure 2.3: Schmidt-Jones' (2020, p. 28) demonstration of the relationship between different note values

Courey et al. (2012) designed an 'academic music intervention' programme in which a specialist music teacher and a researcher co-taught 12 lessons on "conceptual understanding of music notation, fraction symbols, fraction size, and equivalency" (p. 251) to grade 3 learners. They found that learners', particularly those with poor understanding of fractions, performed better in the post-test than the pre-test, thus showing significant gains. Inspired by Courey et al.'s study, Azaryahu et al. (2019) conducted a study aimed at comparing the 'academic music intervention' with their 'MusiMaths' programme (a holistic music-mathematics intervention). They found that both groups who received some form of musical intervention outperformed the control group that received no intervention, and the 'MusiMaths' group performed better than the 'academic music group' with application of fraction examples not used in the intervention. My Master of Education action research case study (Lovemore, 2020) explored integration of note values to enhance my own teaching of unit fractions, and the inverse order relationship, at the Grade 5 level. The Western notation and note-naming convention provided a context to link music and fractions. The paper plate activity, as shown in Figure 2.4 below, was a hands-on activity for learners to visually compare the relationship between note values, for example, two half notes make a whole note, also linking these note values to fraction representations ($\frac{1}{2}+\frac{1}{2}$ = 1 whole). I found the American-English note-naming convention useful in linking the note values to fractions (Lovemore, 2020; Lovemore et al., 2021a).

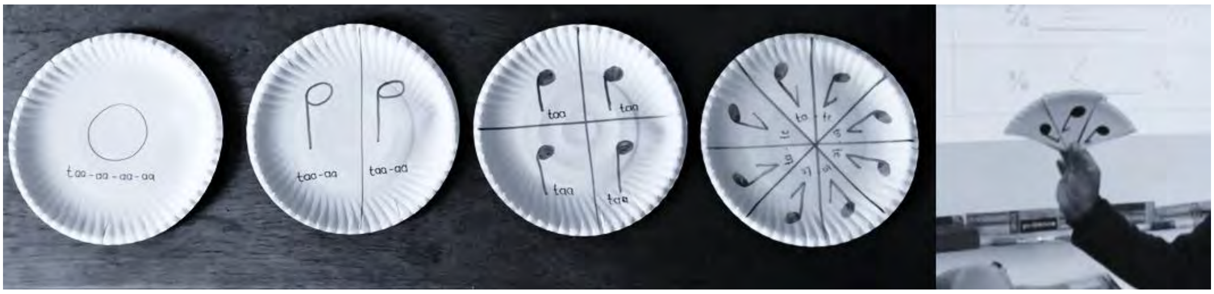


Figure 2.4: Paper plate activity for learners to visualise relationships between different note values (from Lovemore et al., 2021a)

A similarity between all these studies linking music note values and fractions is that they focus on the part-whole construct of fractions. The interventions also focused on using music which is in a 4/4 time-signature, meaning that the activities would only be applicable if the musical song had four beats per bar. The activities in these studies would only highlight fractions with denominators of two, four, eight, and sixteen, thus not including other denominators. Schmidt-Jones (2021) explains that third notes, sixth notes, tenth notes, etc. do not exist in music. Learners in the intermediate phase are, however, expected to work with fractions up to twelfths, including tenths and hundredths in Grade 6. This highlighting of the need to develop music-mathematics integrated activities which allow for working with multiple fractions represents a further opportunity for me in my own music-mathematics research endeavours. Courey et al. (2012) state that designing and implementing activities which move between fraction representations is “the most difficult aspects of fraction instruction” (p. 271). Finding such music-mathematics integrated lessons was a driving aim of the present study.

An et al. (2016) found that pre-service teachers experienced further challenges with actually implementing music-mathematics integrated lessons. Classroom management was challenging as learners did not always follow instructions during the incorporation of music into the lessons. Other challenges were difficulty in meeting learners’ diverse needs through the music-themed activities as well as lack of time to fully implement the integrated lesson for deep understanding. An et al. (2016) further state that there is limited research on how music can be used as a pedagogical approach to mathematics, and that further research is necessary on how to support teachers with this. This has resonance with Bresler’s (1995) findings that implementation of arts integration at the co-equal style, where both the art and the academic subject are considered important, is complex and the least prevalent. Bautista et al. (2016) similarly found that while most teachers are willing to trial curriculum integration strategies, they were implemented at a superficial level: attaining curriculum integration at a deep conceptual level is challenging. An and Tillman (2014) also found that pre-service teachers, while acknowledging the value in music-mathematics integration, struggled to use music activities beyond singing, thus neither achieving explicit music and mathematics instruction, nor making authentic connections.

Teachers require knowledge, skills and confidence in both mathematics and music, time for effective planning of integrated tasks, and collaboration between general teachers and music specialist teachers (Bresler, 1995; Grumet et al., 2014; An et al., 2016). A need clearly exists for further research on how to support teachers in designing and implementing effective, co-equal music-mathematics integrated lessons, irrespective of whether the teachers have a background in music or not. This need provided yet further motivation for the current study.

2.7 Fractional Knowledge in Intermediate Phase

To fully participate in daily social life, people need to have a basic understanding of mathematics (Kilpatrick et al., 2001). Expanding on this notion, Siemon (2003, p. 412) argues that people need to have a sense of number and be able to engage in proportional reasoning, of which fractional understanding is a part, to "function effectively in a modern society". Fractions are a vital part of the mathematics curriculum – they are vital for developing understanding of relationships, algebra, proportional reasoning and use in daily life; and competence with fractions is often an indicator of future achievement in mathematics at higher levels (Siemon, 2003; Charalambous & Pitta-Pantazi, 2007; Azaryahu et al., 2019; Barbieri et al., 2020).

It is significant that researchers across the world and over decades (Cortina et al., 2014), have highlighted the challenges learners and teachers face with fractions in primary school mathematics curricula. Authors often describe the teaching and learning of fractions as challenging, complex, and the most difficult concepts, (Streefland, 1991; Siemon, 2003; Courey et al., 2012; Cortina et al., 2015; Azaryahu et al., 2019; Getenet & Callingham, 2021). Streefland (1991) goes so far as to say that the opinion existed that fractions are "simply too difficult for many children" (p. 6). He argues, however, that "one cannot do without fractions on account of all its applications" (Streefland, 1991, p. 23). It is no surprise then he opens his first chapter with a German quote by Goethe, "fractions hold a great secret". As I discuss in the next sub-section, research into the teaching and learning of fractions has attempted to demystify the challenges in fraction reasoning, providing some possible explanations for the difficulties faced by teachers and learners, and exploring intervention strategies for improving the experience of the teaching and learning of fractions.

Challenges in fraction teaching and learning

One of the most common challenges learners have been found to experience is that they apply the rules and properties related to whole numbers when working with fractions (Streefland, 1991; Ni & Zhou, 2005; Cortina et al., 2014; Shahbari & Peled, 2014; Siemon & Luneta, 2018; Getenet & Callingham, 2021). Learners therefore find tasks requiring them to order, compare, find equivalence, or perform operations challenging (Tanner, 2008; Cortina et al., 2014). Shahbari and Peled (2014) identified difficulties that learners generally experience when working with fractions. These include adding and subtracting fractions with unlike denominators, comparing and ordering fractions, using fractions as an operator (the fraction increases or reduces the original amount), dividing whole numbers by fractions, and working with operations when the sum of the parts is not equal to the whole (greater or less than one whole).

A further reason why teaching and learning fractions is challenging is due to the complex nature of fractions. The "meaning of fractions derives from the contexts in which they are used", according to Lamon (1999, p. 41). Freudenthal (1983) differentiated between two approaches to demonstrating fractions: "fraction as fracturer" (p. 139) and "fraction as comparer" (p. 145). Fraction as fracturer refers to cutting or splitting an object into parts. The benefit in using this approach is that it is concrete for learners; it is, however, a restricted representation as the number of equal parts is limited to making up the whole (Cortina et al., 2014). Freudenthal (1983) considered this an insufficient approach to supporting learners when it comes to fractions greater than one whole (improper fractions). In contrast, the fraction as comparer, refers to when "sizes that are separated from each other are compared" (Cortina et al., 2015, p. 9). Cortina et al. (2015) provide the example of comparing the length of a ping-pong paddle to that of a tennis racquet, where the ping-pong paddle is, for example, a third of the length of the tennis rather.

Multiple constructs of fractions

All mathematics teachers need to attend to the five interrelated constructs, namely: fraction as measure, fraction as quotient, fraction as ratio, fraction as operator and the part-whole fraction model (Kieren, 1980; Shahbari & Peled, 2014; Siemon & Luneta, 2018; Getenet & Callingham, 2021). These interrelated constructs of fractions often present teaching and learning challenges (Kieren, 1976; Charalambous & Pitta-Pantazi, 2007; Siemon et al., 2015; Getenet & Callingham, 2021). They are not separated categories, but, rather, offer multiple ways to make sense of the same situation (Lamon, 1999). Figure 2.5 shows Behr's theoretical model linking the five constructs of fractions (Charalambous & Pitta-Pantazi, 2007).

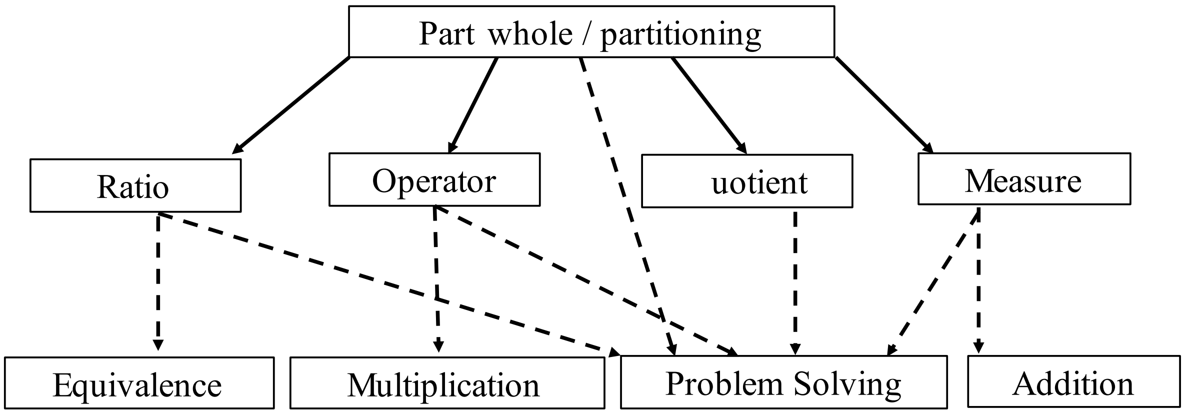


Figure 2.5: Behr et al.'s (1983) theoretical model linking the five constructs of fractions)

In Table 2.2 below I summarise the five constructs, sometimes referred to as faces of fractions. For the purposes of the current study, my focus is predominantly on just three of them: fraction as measure, fraction as ratio, and part-whole constructs (or faces) of fractions.

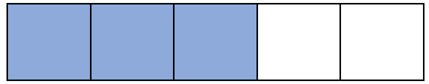
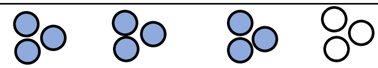
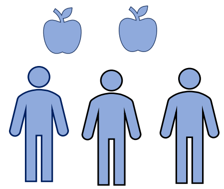
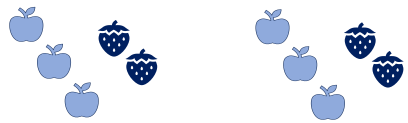
Construct of fractions	Brief explanation	Representation
Part-whole construct	A set of "discrete objects or a continuous amount that can be divided into parts of equal size" (Shahbari & Peled, 2014, p. 373).	$\frac{3}{5}$ 
Operator construct	It involves multiplication with fractions in a variety of ways or "taking part of the whole" (Getenet & Callingham, 2021, p. 204). "Operators shrink or stretch a quantity" (Shahbari & Peled, 2014, p. 373).	$\frac{3}{4}$ of 12 = 9 
Quotient construct	This is related to a division situation e.g. sharing and generating equal shares (Shahbari & Peled, 2014). Here 'quotient' refers to the "single rational number that one gets when dividing the numerator by the denominator" (Siemon & Luneta, 2018, p. 375).	$\frac{2}{3} = 2 \div 3$ 
Fraction as Ratio Construct	"A comparison or relationship between two quantities in a given order rather than being a number by itself" (Getenet & Callingham, 2021, p. 204).	2:3 
Fraction as Measure Construct	The fractions are indicated "as measures or distances from a given point (0) on the same scale" (Getenet & Callingham, 2021, p. 205), such as a number line. A common fraction is a length which is a subunit iterated a number of times (Cortina & Visnovska, 2015).	$\frac{2}{3}$ 

Table 2.2: Summary of the five constructs of fractions

- *Fractions as ratio construct:* The fraction as ratio interpretation conveys a comparison or a relationship between two quantities (Charalambous & Pitta-Pantazi, 2007; Getenet & Callingham, 2021), conveying the "notion of relative magnitude" (Shahbari & Peled, 2014, p. 373). Trimble (1949) cautioned that the "ratio idea should be presented carefully" (p. 285) and that it should be a priority of mathematics teachers from primary through to high school and tertiary studies, and should be applied to a variety of mathematical contexts, one such being fractions. In a 1949 paper, entitled *Fractions are ratios too*, Trimble critiqued the traditional manner of teaching fractions and ratios as isolated concepts in mathematics and promotes teaching which encourages learners to employ the "ratio way of thinking" about fractions (1949, p. 285). He argued that the view of fractions as ratios should be taught in order to supplement learners' understanding of fractions as a part of a whole or as a quotient. Ratio and rate are often used interchangeably in primary school. In distinguishing between ratio and rate, Lamon (1999) defines ratio as the comparison of two quantities of the same type, and rate as the comparison of two quantities of different types. Charalambous and Pitta-Pantazi (2007) further state that the ratio construct supports the understanding of equivalence in fractions. Siemon et al. (2015) explains that the ratio can either be a part-to-part relationship or a part-to-whole relationship.

- *Fraction as measure construct:* The 'fraction as measure' notion views a fraction as a number representing the size of a measured length (Cortina et al., 2019). The denominator of the fraction is therefore understood to represent the size of a subunit of measure, whereas the numerator represents the number of iterations of the subunit (Cortina et al., 2019). Freudenthal's approach of 'fraction as a comparer' (1983) can be likened to the fraction as measure construct. Siemon and Luneta (2018) explain that the sub-unit represents the equal parts of the referent unit in order to give a more precise indication of an amount. Shahbari & Peled (2014, p. 373) explain fraction as measure as "the process of counting the number of units that cover a region, derived by sub-diving a unit into smaller, equal parts". Similarly, Tzur (2016) suggests that teaching and learning of fractions can be enhanced by the notion of iteration of fractions where learners actively engage in activities requiring a whole length to be partitioned into equal parts such that the number of iterations will make a whole. Getenet and Callingham (2021, p. 204) further suggest that fraction as measure can be interpreted as "numbers that can be ordered on a number line". In this interpretation, fractions are indicated as measures or distances from a given point (0) on the same scale, such as a number line, and learners should be able to identify a number represented by a specific point on the number line (Charalambous & Pitta-Pantazi, 2007; Getenet & Callingham, 2021). Siemon et al. (2015) suggest that this interpretation of fractions helps to support learners' understanding of addition and subtraction of fractions, as it provides a "perceptual basis for renaming the parts in the same way" (p. 432), relating to fraction equivalence. However, the fraction as measure construct is not emphasised in primary school mathematics lessons (Getenet & Callingham, 2021).

- *Fraction as part-whole construct*: The part-whole interpretations of fractions can be defined as a situation where a continuous quantity or a set of discrete objects are divided into equal-sized parts (Lamon, 1999; Charalambous & Pitta-Pantazi, 2007; Shahbari & Peled, 2014; Barbieri, 2020). The fractions therefore represent the relationship between the total number of partitioned parts of the object or objects, and the selected number of parts, for example, which have been shaded (Charalambous & Pitta-Pantazi, 2007). The part-whole construct of fractions can be likened to Freudenthal's (1983) 'fraction as fracturer' approach. Learners should understand that the parts into which the original object is divided are of equal size (Getenet & Callingham, 2021). In this interpretation, the numerator of the fractions must be equal to or less than the denominator, thus not allowing for fractions greater than one whole (Charalambous & Pitta-Pantazi, 2007). While the part-whole construct is commonly used to teach fractions as it relates to children's intuitive experiences of fractions through activities of fair sharing (Getenet & Callingham, 2021), it has limitations for fraction teaching, learning and application to real-world scenarios (Siemon et al., 2015). Lamon (1999) goes as far as saying that it is "the least valuable road into the system of rational numbers" (p. 163). Barbieri et al. (2020) argue that an overemphasis on the part-whole construct can limit learners' ways of thinking about fractions, not recognising fractions as having their own numerical magnitude.

It is not sufficient to focus on one construct alone, but rather to allow learners time and experiences in developing a sense of fractions in these multiple constructs (Siemon et al., 2015; Siemon & Luneta, 2018; Barbieri et al., 2020; Getenet & Callingham, 2021). Charalambous and Pitta-Pantazi (2007), in their study, showed that a focus on the part-whole construct was not sufficient in primary school mathematics, but rather that a deep understanding of the different constructs of fractions was necessary for addition and subtraction of fractions. This supports the notion that all five constructs are relevant to learners' understanding and use of fraction equivalence, operations and problem-solving with fractions (Shahbari & Peled, 2014).

An important mechanism for enhancing learners' knowledge of fractions is through creating opportunities for learners to recognise and use the connections across the multiple, interrelated constructs of fractions (Charalambous & Pitta-Pantazi, 2007; Shahbari & Peled, 2014; Siemon et al., 2015). Getenet and Callingham (2021) found that learners' understanding, and application of fractional knowledge could be enhanced through teachers connecting the constructs of fractions. If teaching strategies provide opportunities for learners to see and use fractions beyond the typical part-whole construct, they are more likely to gain a deeper conceptual understanding of fractions which will benefit their reasoning and development in higher mathematics grades (Getenet & Callingham, 2021).

In the current study, the intent was to design tasks that allowed flexible movement between the fraction as measure, fraction as ratio and the part-whole constructs. In a subsequent conference presentation I explained how these three different constructs might apply in tasks integrating music and mathematics:

- The part-whole construct: a musical note can be divided into shorter notes of equal time value;
- The fraction as measure construct: a musical line is aligned with a triple number line (three number lines running parallel to one another) to show the relationship between the measure of distance and the measure of time;
- The fraction as ratio construct: musical 'beats *per* bar' or 'claps *per* bar' indicate ratio as rate. (See Lovemore et al., 2023.)

Realistic problem-solving activities support learners in understanding and applying fractional knowledge (Freudenthal, 1991; Streefland, 1991; Tzur, 1999; Shahbari & Peled, 2014). In my Master of Education action research study (Lovemore, 2020), though the focus was on the part-whole construct only, I found precisely this. Consistent with RME principles (Freudenthal, 1991; Cobb et al., 2008), the musical activities I used provided a real-life context in which my learners could experience fractions.

Visual representations of fractions on number lines

Number lines are a "visual, mathematically correct way to represent complex fractions" (Barbieri et al., 2020, p. 629). Linear representations, such as number lines, have the potential to support learners' understanding of fractions (Saxe et al., 2013). Monson et al. (2020) state that learners need to establish the unit, the distance from 0 to 1, which they then can partition into equal parts. The number line is a useful model of fractions as it demonstrates that a fraction can be represented as a distance from 0 and it is also a single point on the number line (Monson et al., 2020). Barbieri et al. (2020) highlight the effectiveness of using a number line to teach and learn fractions, but remark that it is a tool which is seldom used. Siemon and Luneta (2018) state that it can be challenging for learners to view equivalent fractions at the same point on a single number line. They suggest the use of fraction strips to introduce fractions on a number line. Learners often have the misconception that the full line is the unit, as opposed to the unit being the distance between 0 and 1, which can be iterated any number of times. Charalambous and Pitta-Pantazi (2007) explain that learners often incorrectly think that they should count the partition markings on the number line rather than the intervals between the partition marks. Despite it being challenging for learners, it is vital for them to develop the skills to use number lines as a support for developing a deep conceptual understanding of fractions (Siemon & Luneta, 2018). Number lines can be a "key representational tool" for teaching fractions (Soni & Okmoto, 2020, p. 1), and have been shown to be more effective than the part-whole model (such as dividing a pizza) of fractions (Barbieri et al., 2020; Soni & Okmoto, 2020). The number line also allows learners to see and work with fractions greater than one whole (Monson et al., 2020). Siemon et al. (2015) noted that learners often had difficulty when working with fractions greater than one whole. A number line could potentially be a key representation to support learners in developing deep conceptual understanding of fraction equivalence and fractions greater than 1 whole.

Expectations of fractional knowledge and skills

The South African curriculum expectations for fractional development in intermediate phase is summarised in Table 2.3, below.

These curriculum expectations appear to cohere with views such as those expressed by, inter alia, Courey et al. (2012) and Saxe et al. (2013), welcoming moves away from rote learning fractions in a rules-based manner, towards developing conceptual understanding of fractions, through problem-solving and reasoning. Streefland (1991) cautions against a superficial and brief introduction to fractions. He recommends rather that learners be given concrete opportunities to build a foundation for conceptual understanding, connecting fractions across the curriculum and looking for realistic opportunities for learners to use fractional knowledge.

<table>
<tr><th>Concept and skill</th><th>Grade 4</th><th>Grade 5</th><th>Grade 6</th></tr>
<tr><td>Describe and order fractions</td><td>• Compare and order common fractions with different denominators (halves, thirds, quarters, fifths, sixths, sevenths, eighths)
• Describe and compare common fractions in diagram form</td><td>• Count forwards and backwards in fractions
• Compare and order common fractions to at least twelfths</td><td>• Compare and order common fractions, including tenths and hundredths</td></tr>
<tr><td>Calculations with fractions</td><td>• Addition of fractions with the same denominators
• Recognise, describe and use the equivalence of division and fractions</td><td>• Addition and subtraction of common fractions with the same denominators
• Addition and subtraction of mixed numbers
• Fractions of whole numbers which result in whole numbers
• Recognise, describe and use the equivalence of division and fractions</td><td>• Addition and subtraction of common fractions in which one denominator is a multiple of another
• Addition and subtraction of mixed numbers
• Fractions of whole numbers</td></tr>
<tr><td>Equivalence</td><td colspan="2">• Recognise and use equivalent forms of common fractions (fractions in which one denominator is a multiple of another)</td><td>• Recognise and use equivalent forms of common fractions with 1-digit or 2-digit denominators (fractions in which one denominator is a multiple of another)</td></tr>
<tr><td>Solving problems</td><td colspan="3">• Solve problems in contexts involving common fractions, including grouping and sharing</td></tr>
</table>

Table 2.3: A selected summary of relevant curriculum expectations of South African CAPS curriculum (South Africa. DBE, 2011a, p. 16)

Courey et al. (2012) list the elements of fractional understanding which primary learners should develop: recognising fractions as parts of wholes, parts of collectives, as locations on a number line, and as divisions of whole numbers; fraction equivalence; and using models to judge the relative size of the fraction. When considering calculations with fractions, Dole (2005) suggests a teaching and learning sequence: (1) unit fractions (numerator is 1, for example, $\frac{1}{4}$), (2) non-unit fractions (for example, $\frac{3}{4}$), (3) identifying pairs of fractions which total 1 (for example, $\frac{1}{4}+\frac{3}{4}=1$), and (4) addition of fractions with the total being greater than one whole (for example, $\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$). Learners should first be confident and competent in calculating with fractions with like denominators before being introduced to calculations with fractions with unlike denominators (Dole, 2005). Cortina and colleagues suggested an alternative starting point for teaching fractions, by introducing the concept of unit fractions as an entity separate from the reference unit (Cortina et al., 2014; Cortina et al., 2015; Cortina & Visnovska, 2015).

My doctoral study has emphasised equivalence of fractions, working with fractions greater than one whole (improper fractions) and number line representations of fractions. Courey et al. (2012) define equivalent fractions as different fractions that represent the same rational number. Siemon and Luneta (2018) explain that equivalent fractions refer to "fractions that form the same part of a whole" (p. 270), with the numerical representations being different, while representing the same number. While equivalence is an important concept to grasp, Siemon and Luneta (2018) note that learners often have misconceptions around it. They therefore suggest the use of number lines to demonstrate the relative size of fractions and support learners in recognising equivalent forms (Siemon & Luneta, 2018).

2.8 Musical Knowledge and Notation

Curriculum expectations of musical knowledge and skills

South Africa's CAPS curriculum places music in the subject known as Life Skills and the study area of Creative Arts – Performing Arts (DBE, 2011b). The four topics expected to be covered are indicated in Table 2.4 on the following page, as well as the elements relevant to this study (taken from DBE, 2011b, pp. 12-13). Learners in intermediate phase in South Africa are exposed to both Western and African musical traditions. They are expected to be introduced to concepts of rhythm, notation and body percussion, as well as creative activities involving movement and games. From these curriculum concepts, I saw opportunities to integrate the music curriculum objectives with those of teaching the multiple constructs of fractions.

Topic	Grade 4	Grade 5	Grade 6
Warm up and play: Using games for learning.	Games exploring: • rhythm and music • creativity • direction	Games as in Grade 4, including: • spatial and group awareness • body percussion (in unison, canon and/or call & response)	Games as in Grade 5, including: • action and reaction • leading and following • story development
Improvise and create: Using arts skills individually and collaboratively to spontaneously show learning.	Rhythmic patterns using body percussion • Movement sequences • Instruments using found objects	Rhythmic patterns using body percussion, repetition, accent, call & response, echo • Movement sequences exploring elements of time and force	Musical phrases exploring dynamics, pitch and rhythmic patterns
Read, interpret and perform: Learning the language of the art form, and interpreting and performing artistic products in the classroom.	• Rhythmic patterns in meter (2/4, 3/4, 4/4) • Musical notation (stave, note values, rests, tonic solfa)	Musical notation (stave, note values, rests, clef, tonic solfa, letter names)	Rhythmic patterns using drumming techniques • Musical notation (stave, note values, rests, clef, tonic solfa, letter names, C major) • African folktale or traditional story
Appreciate and reflect: Demonstrating understanding and appreciation of own and others' artistic processes and/or products	A range of music using percussive and melodic instruments (African and Western): Individual and group performances and processes	A range of music using percussive and melodic instruments (African and Western)	Own and others' performances and processes

Table 2.4: Music curriculum expectations according to CAPS (DBE, 2011b, pp. 12-13)

Musical terminology and notation

I began Chapter 1 with a quote by the philosopher and mathematician, Leibniz (1646–1716). He observed that "Music is an unconscious exercise in mathematics in which the mind is unaware that it is dealing with numbers" (cited by Amon, 2017, p. 504). Amon himself describes music as "a period of time, with the regular recurrence of rhythmic sequences" (2017, p. 513). Schmidt-Jones (2021, p. 32) gives a similar explanation of music: something that "happens over a period of time (proportioned time), and it is organised into short time periods of the same length: beats". Pythagoras theorised about and proved many relationships between music and mathematics, which were used then, and still today, to design and create instruments (e.g. string lengths), tuning instruments and the numerical symbolism often used in musical notations (Amon, 2017, p. 513).

Musical notation, the writing down of music to indicate to the musician the pitch and rhythm, has developed over centuries to what we today know and commonly accept as Western musical notation (Westrup et al., 1991). Western music, the tradition with which I myself am most familiar, has developed its generally accepted set of 'rules' that musicians – initially from western Europe – have used since the end of the Middle Ages. These 'rules' provide what can be thought of as "a sort of grammar for the language of music" (Schmidt-Jones, 2021, p. 86). In Table 2.5 below, I give the definitions of the key musical terms used in this study[8].

Musical term	Definition
Staff or stave	"The set of five lines, with four spaces between them, each representing a pitch, on which music is written" (Westrup et al., 1991, p. 518).
Bar-line	"A vertical line drawn across one or more staves of music as a convenient sign of sub-division to guide the musician's eye when reading musical lines" (Westrup et al., 1991, p. 55).
Bar	"The spaces between bar-lines, in which the musical notes are notated, also known as a 'measure' in the United States" (Westrup et al., 1991, p. 55). "Beats are grouped into measures or bars" (Schmidt-Jones, 2021, p. 69).

[8] For the purpose of clarifying musical terms used in this study, I draw on, among other sources, Collins Encyclopedia of Music (Westrup et al., 1991). This reference book is intended as a comprehensive guide to all elements in Western music. In a review, Salter (1977) describes an earlier edition as "very reliable-succinct but mostly accurate", which leads me to believe that this 1991 edition will be just as reliable a source for the purpose of this study.

Beat	"The unit of measurement in music. The number of beats in a bar depends on the time-signature, for example in 4/4 time, there will be four beats per bar, and the beats can be made up of different note values" (eight eighth notes can fit into four beats) (Westrup et al. 1991, p. 63). A beat is a regular, predictable pulse, created when, for example, a musician hits a drum or strums a guitar (Schmidt-Jones, 2021).
Note value	The duration of a given printed note relative to other notes in a composition, considered in relation to the tempo (speed) of the musical piece, also known as time value (Collins English Dictionary, 2022). "All note values are defined based on how long they are in comparison to a whole note (e.g., if a note lasts half as long as a whole note it is referred to as a half note)" (Schmidt-Jones, 2021, p. 26).
Rhythm	"The organisation of music in respect of time" (Westrup et al. 1991, p. 456). "The basic, repetitive pulse of the music, or a rhythmic pattern that is repeated throughout the music… [or] the pattern in time of a single small group of notes" (Schmidt-Jones, 2021, p. 69).
Time-signature	"An indication at the start of a piece of music of the number and types of note-values in each bar" (Westrup et al. 1991, p. 549). For example, a waltz will be 3/4 time, with the total value per bar being that of three quarter notes (for example, a half note and a quarter note in a bar).
Percussion	Percussion is produced by instruments (or body parts, such as clapping) which produce a sound when struck or shaken. Percussion "provides a rhythmic function and instruments are unable to sustain sounds" (Westrup et al. 1991, p. 415).
Percussion line	A single staff line on which percussion symbols are notated to indicate rhythm.
Rest	"A notation indicating silence for a certain duration of time" (Westrup et al. 1991).

Table 2.5: Definitions of the key musical terms used in this study

In Figure 2.6 I show a visual representation of a musical line that I have composed and created which includes several musical terms.

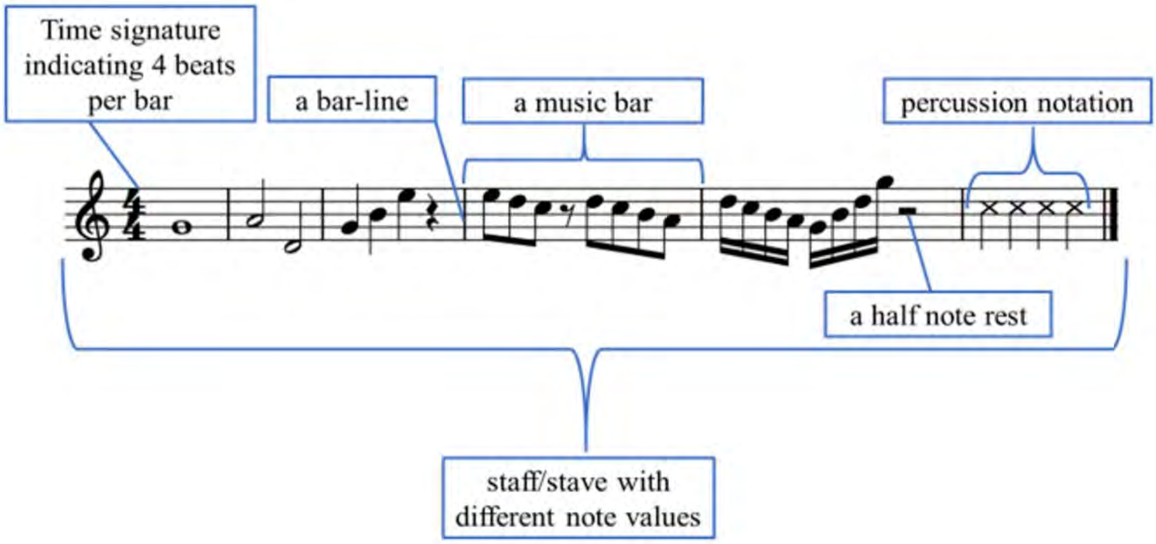


Figure 2.6: Visual representation of some musical terms, created on MuseScore®

History and critique of musical notation

McLachlan et al. (2010) describe music notation as the "use of both symbolic and spatial representation" (p. 1). They further elaborate, stating that the notation of music note values is represented on a time-line, with symbols that are shaded or unshaded, are with or without stems, dots and tails, thus indicating symbolically the relative duration of the notes (McLachlan et al., 2010). Gaare (1997) gives an account of the history of musical notation, with the oldest known example dating from around 1880 BC, from the Middle East. This was a verbal form of representation, as opposed to a graphical one. The first graphical representation, similar to what we know today as the Western staff, dates back to 1020 AD (Gaare, 1997). There has, however, been critique of the Western musical notation system.

According to Sloboda (2012), a common reason why people have difficulty learning to read music is due to its symbolic nature. Gaare (1997) believes that the need to memorise symbolic meaning and do quick computational mental mathematics while reading music is cognitively taxing on a musician. He explains that note values require "heavy math… and require processing fractions on-the-fly" (Gaare, 1997, p. 18). He therefore suggests that "the visual event must be apparent as the direct translation of the auditory event, requiring as few additional thought processes as possible" (p. 18) and calls for visually proportional note spacing to represent time. McLachlan et al. (2010) propose that a graphical notation, rather than an abstract symbolic one, could make the reading of music less cognitively taxing. Since the early 1900s, however, musicians world-wide have attempted to adapt notation, but no consensus on an improved notation system has yet been achieved (Gaare, 1997).

As an addition to the mainly Western traditions I have followed in designing the music-mathematics integrated tasks, I recognised the importance of also incorporating African indigenous music elements. Literature on African indigenous music indicates that it is difficult to define, because each culture has its own unique music (Musakula, 2014), but many common characteristics of African music are singing and dancing, call-and-response, hand-clapping and the use of percussion instruments such as drums (Dargie, 1991; Musakula, 2014; Netshivhambe, 2018). Dargie's study (1991) of the Xhosa traditional music in the Eastern Cape Province of South Africa involved collecting recordings of 43 Xhosa songs. He transcribed them in a complex notation system combining Western staff notation and a cyclic structure using an improvisatory nature of Xhosa music. As is noted, however, this type of music is ordinarily performed and passed on orally, rather than being transcribed in written notation form (Musakula, 2014; Netshivhambe, 2018). The Xhosa people, South Africa's second largest ethnic group, reside mainly in the Eastern Cape Province, the province, as noted in my introductory chapter, in which my study is located.

2.9 Community of Practice (CoP)

Lave and Wenger (1991) posit that learning frequently occurs in a space of co-participation. As Wenger (1998) notes, we learn by interacting with each other. This collective learning constitutes practices that occur within a kind of community, thus the term Community of Practice (CoP) was coined. A CoP is formed when people engage in these shared practices of learning over a period of time to achieve a particular goal (Wenger, 1998). It is a space where professionals can share approaches, solve problems and learn from one another (Pyrko et al., 2017; Bouchamma et al., 2018). The model of CoP interrelates concepts of community, identity, practice and meaning (Wenger, 1998; Ernest, 2002; Graven, 2004). A graphic representation is given in Figure 2.7.

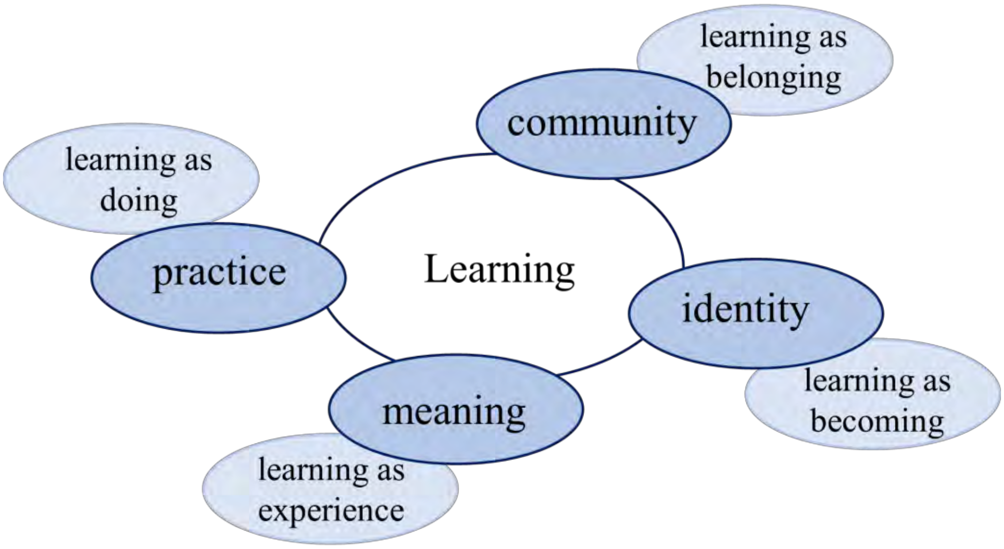


Figure 2.7: The components of a CoP (replicated from Wenger, 1998)

Community, according to Wenger (1998, p. 81) refers to developing "relations of mutual accountability", where information and resources are shared responsibly to support members. Graven (2004) explains that access to other members in the community and a wide range of "ongoing activity" (p. 182) forms an integral part of this shared accountability. Identity reflects members' ways of talking about their role in the context of the community (Wenger, 1998), for example, one's confidence as a professional mathematics teacher (Graven, 2004). Practice implies action, according to Wenger (1998), acting in a social context to give meaning to the members of the community. Wenger (1998) lastly describes the concept of meaning as the community members sharing their experiences and participation in the context meaningfully.

The idea of a CoP strongly resonates with the goals of my study. I wanted to work collaboratively with teachers, sharing, trialling, interrogating, reflecting on, and jointly learning from the various tasks that I (together with input from my supervisors) had designed for the teaching of integrated music-mathematics lessons. Horne and Makar (2013, p. 769) write of the benefits of "win-win" research projects. South Africa's Department of Higher Education and Training (DHET) (2015) explicitly asks that the country's teachers strive to be lifelong learners. Graven (1998) notes that the supportive environment of a CoP presents an ideal setting for teacher learning whereby groups of teachers can reflect on teaching practices, use of resources, assessment strategies etc., as they work towards meeting the shared goal of improving teaching and learning practices.

There is reference in the literature to the risk of a deficit perspective in which teachers are considered mere objects in research, rather than professional partners; a perspective which makes teachers feel ‘used’ instead of being treated as genuine “research collaborators” (Makar, 2021, p. 440). Setati’s distinction (2005) between research ‘with’ teachers and research ‘on’ teachers is especially useful here. Setati proposes building reciprocal relationships between researchers and teachers, with equal power dynamics. In this way the research process is more likely to be beneficial for all involved, a Horne and Makar ‘win-win’ situation through which both teachers and researchers can become empowered. To achieve this, as Horne and Makar (2013) emphasise, there needs to be a carefully planned and mutually agreed upon vision. In a conference presentation and paper Lovemore et al. (2022c) explored the importance of clear communication to maintain momentum within a CoP and of fostering equitable power relationships between the CoP facilitator/researcher and teachers so as to create a safe and productive space in which to trial and interrogate new strategies for integrated lessons.

2.10 Task Design

This study focuses specifically on mathematical tasks, defined by Watson and Ohtani (2012) as anything that learners can be actively involved in for developing their learning of a mathematical concept, whether initiated by the teacher or by the learners themselves. Tasks may require learners to classify a mathematical object, work with multiple representations, create and solve problems, and justify solutions (Swan, 2005). Mathematical tasks can take multiple forms, from a single task to a full teaching and learning sequence based on a specific mathematics concept; it can be designed for a single learner or a group of learners; tasks can be full textbooks/workbooks or single activities (Graven & Coles, 2017).

Collaboration between teachers or between teachers and researchers is a useful way to engage in task design (Choy, 2016; Jones & Pepion, 2016; Geiger et al., 2022). Researcher-designers can provide expert knowledge from theory and literature, while teachers can provide expertise in their experiences within the classroom contexts. Kaur et al. (2022) note that there is often an implied distinction between task designers, usually the researchers, and task users, the teachers who implement and adapt the task to suit their classroom needs and context. Chin et al. (2022) highlight the increased interest in studying mathematics teachers as both designers and implementers of tasks.

My study explores how researcher-designers can work *with* teachers as “research collaborators” (Makar, 2021, p. 440) to design and implement a set of music-mathematics integrated tasks. I strove to achieve such collaboration. As I explain in the Methodology chapter, I set up three separate CoPs (which I labelled ‘micro-CoPs’): one at each of the two participating schools, and a third which evolved organically out of the grappling sessions between myself and my two doctoral supervisors around how best to achieve rigour – both mathematical and musical – in the design of the various integrated tasks.

Task design refers to a careful, deliberate and purposeful process of planning engaging and meaningful opportunities for learners to learn (Ainley et al., 2015; Choy, 2016; Jones & Pepin, 2016). Models, representations and resources are carefully selected by task designers to support learners in understanding particular mathematical concepts (Sullivan et al., 2013). Sullivan et al. (2009) mention some general characteristics of meaningful tasks: tasks should be of an appropriate level of complexity for learners; learners should recognise a need to engage in the task; learners should be able to take ownership of the task, by recognising it as personally meaningful; and there should be variety within the tasks.

Researchers share multiple suggestions for engaging in the process of task design. Jones and Pepin (2017), for example, highlight the importance of task designers considering the questions, *what* to design and *which tools* to best represent the concept. Ainley et al. (2015) recommend that task designers ask themselves the question: "is this task purposeful for learners?" (p. 406). They should explicitly select which mathematical concept they intend to present through their tasks and how discussion in the lesson may contribute to the learning process (Choy, 2016). It is important too that designers of mathematical tasks are able to justify their choice of design (Pepin, 2018). I found the principles of task design, by Coles and Brown (2016), useful. They suggest using at least two contrasting examples of the mathematical concept, asking learners to discuss similarities and differences, followed by introducing representations and language from learners' distinctions, starting with a closed activity (in contrast to Sullivan et al., (2009) who suggest an open-ended activity to encourage learner investigation). Coles and Brown (2016) further suggest having challenging questions ready for learners and opportunities for them to practise new skills, as well as recognising patterns and making generalisations.

Choy (2016) argues that it is vital to design a "worthwhile task" (p. 421) that will engage learners in thinking and communicating mathematically to develop their reasoning, by deliberately selecting, adapting and designing tasks. Choy (2016) further highlights that they need to anticipate learners' possible responses and possible misconceptions, to prepare for addressing them intentionally, rather than via ad hoc responses. Three considerations researchers or teachers should be aware of when designing tasks, according to Choy (2016), are:

- Identify a clear goal of the mathematical task: The mathematical aspect of the task must be explicit and form a clear link to the task, in such a way that learners have an opportunity to engage in communicating and reasoning about that specific mathematical concept. Awareness of possible misconceptions which learners may have is important for deciding on the starting point of the task.
- Decide on the mathematical thinking required: Consider how the task will motivate learners to engage in specific mathematical thinking, such as selecting efficient problem-solving strategies.
- Allow for learners to explicitly express their mathematical thinking through multiple representations: Task designers should consider a wide variety of effective representations for developing understanding of the specific mathematical concept, as well as how learners can use the multiple representations. Thomas (2006), similarly, encourages the need for learners to "work seamlessly within and between representations" (p. 233).

Selection of tasks strongly influences how learners view mathematics (Jones & Pepin, 2017). According to Bicer et al. (2022) a possible way to adapt the routine manner in which mathematics is so often taught is through "creativity-directed tasks" (p. 492). They encourage designing tasks for mathematical creativity that provide all learners with opportunities to be creative in constructing meaning through mathematical ideas novel to them. They further suggest that engaging in open-ended mathematical tasks helps learners develop more positive dispositions towards mathematics, recognising also its creativity (Bicer et al., 2022). I see this as providing a way towards achieving the broad curriculum aims of recognising mathematics as a creative, beautiful human activity (South Africa. DBE, 2011a).

An additional consideration, from Galbraith and Stillman (2006), is to use real-world problem contexts to form part of task design. Ainley et al. (2015), however, note that the task need not be specifically real-world, but more importantly should be an "engaging challenge for the learners within the classroom context" (p. 406). This notion is similar to what van den Heuvel-Panhuizen (2003) suggests, in her description of RME principles, that tasks be experientially-real for learners to engage in, irrespective of whether they involve realistic or imaginary scenarios.

2.11 Chapter Summary

I started this chapter with a discussion around the key principles of RME which guided me in the design of integrated music-mathematics tasks. I further unpacked some key literature around the history, benefits and challenges of curriculum integration. I drew on literature explaining the importance of and difficulty in teaching multiple constructs of fractions, and I described some key musical terms and notation conventions. These elements formed part of my rationale for the study, highlighting the need to find ways to support teachers in integrating subjects, identifying the gap which exists in specifically integrating music and fractions, to extend beyond a focus on the part-whole construct, but rather to design tasks which encourage learners to move flexibly between multiple constructs, such as the fraction as measure and fraction as ratio constructs. I also briefly outlined some key points about CoPs and the importance of their representing a ‘win-win’ space, and then, lastly, I have highlighted some of the considerations for effective task design from literature that guided me in my task design grappling journey. In the next chapter I describe the methodological decisions and process of my study.

CHAPTER 3: METHODOLOGY

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3.2. Brief Overview of the Methodological Journey
3.3. A Qualitative Interpretivist Research Paradigm
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3.1 Introduction

Simon (1995) compared the planning of design-research to a 'travel plan'. This resonated strongly with me as I reflected on the methodological decisions I had made in the course of my study. This is why I have here chosen to use the following – rather lengthy – quote from him to launch this chapter detailing the methodology that underpinned my own task design journey.

> *Consider that you have decided to sail around the world in order to visit places that you have never seen. One does not do this randomly (e.g., go to France, then Hawaii, then England), but neither is there one set itinerary to follow. Rather, you acquire as much knowledge relevant to planning your journey as possible. You then make a plan. You may initially plan the whole trip or only part of it. You set out sailing according to your plan. However, you must constantly adjust because of the conditions that you encounter. You continue to acquire knowledge about what you wish to visit. You change your plans with respect to the order of your destinations. You modify the length and nature of your visits as a result of interactions with people along the way. You add destinations that prior to your trip were unknown to you (Simon, 1995, pp. 136 – 137).*

On a somewhat more pragmatic tack, Creswell (2009) describes research design as one's plans, processes and decisions which span throughout a study, from the broad philosophical assumptions to the minute details of the methods carried out. The design of this study is broadly qualitative. Maxwell (2013, p. 2) describes qualitative research design as a "do-it-yourself", cyclical and iterative approach, rather than an "off-the-shelf" menu selection of prescribed linear designs. The present study incorporated a variety of design decisions, which had to work together and often be adjusted as new developments appeared, thus resonating with Maxwell's (2013) comment that it is often necessary to "construct and reconstruct your research design" (p. 3). Many challenges, some including the influence of the global COVID-19 pandemic, meant that my proposed strategies for data collection had to be responsive to the context. Keeping with the journey analogy the flight path of this study had at times to be rerouted and there were several delays.

I begin this chapter with a brief reflective overview of the methodological route I charted. Thereafter, I describe and justify the methodological decisions that led to the actual route travelled. I discuss my choice of research paradigm and then describe how I set about incorporating task design principles into my overall design-research approach. I follow this with a discussion of the sampling and scope of the study, the data collection process and the data analysis strategies. I then provide a brief description of what I see as some of the study's limitations, and close with an outline of how I took account of ethical considerations. My sharing of the story of the rigorous documenting and analysis of data in this study may, I believe, have a methodological contribution to make: providing insights that might resonate with other researchers and teachers who wish to integrate other subjects with mathematics.

3.2 Brief Overview of the Methodological Journey

This study was not immune to the emergent nature of qualitative research, where, as Creswell (2009) describes, researchers' envisioned plans may have to be modified once they are in the field. My proposed study initially involved the setting up of a single CoP (after Lave & Wenger, 1991) with intermediate phase teachers (Grade 4 to 6) within which to trial, reflect on, modify, and extend the music-mathematics lessons I had developed in my own action-research-based Master of Education study (Lovemore, 2020). Maxwell's "construct and reconstruct" (2013, p. 2) notion was ever present, however, and a number of detours and delays were encountered along the way, three of which I outline below.

Firstly, the global COVID-19 pandemic placed immense strain on the participating teachers, with various national lockdown restrictions being put in place over the course of 2021. This meant that teachers had to take on online teaching and, upon return to in-person classes, had much catching up to do. The resultant time-constraints facing the participating teachers meant they were not able to teach and reflect on the music-mathematics lessons within my study's proposed timeframe. The lockdown restrictions also made it difficult for me to meet with them personally. This resulted in considerable ebb and flow of momentum within the CoP.

Secondly, in June 2021, I was approached by teachers in another, similar-context school. They expressed interest in also trialling my music-mathematics lessons. Having first sought and obtained ethical clearance from the Education Faculty Research Ethics Committee of Rhodes University, I was able to then initiate a second CoP with these teachers, making this an integral part of my study. This proved extremely useful in that it allowed me opportunities to make adjustments based on feedback from the teachers at the first participating school and to re-trial the music-mathematics lessons in a second iteration. This not only produced valuable data, but, given that I had initially intended forming only one CoP at one school, demonstrated how I carried out Maxwell's "construct and reconstruct" (2013, p. 2) idea. To preserve my two participating schools' anonymity, I henceforth refer to them by the pseudonyms 'Aloe School' and 'Protea School'.

A third major challenge that steered the study momentarily off course related to difficulties I encountered with my design ideas for integrating the mathematics and the music. Mindful of some of the limitations of the task design in my Master of Education study, I had numerous meetings with my doctoral supervisors. COVID-19 restrictions meant that these all had to take place over online video-conferencing platforms such as Zoom, and together we grappled with overcoming several obstacles around integrating the two subjects. It was in retrospect that my supervisors and I realised that these recorded meetings in themselves constituted a valuable source of additional data (the documenting and analysis of the actual task design process).

By adapting to the needs that arose during the study, such as those outlined above, an intricate research design was mapped out. It comprised three phases and three 'micro'-CoPs. I use the adjective 'micro-' in describing my study's three CoPs because two of my CoPs comprised only three members, and my initial CoP just nine members (the eight Aloe teachers and myself), but COVID-19 circumstances seldom afforded us the opportunity to meet as a whole group.

Data for the study were derived from the online supervisory planning meetings, where we had engaged in task design grappling, and from my in-person school visits and discussions (both formal and informal) with the participating teachers at the two schools. Initial review of data revealed multiple cycles of obstacles and resolutions as we engaged in the task design, and, in the case of the participating teachers, their implementation and subsequent review of the various tasks. Following many revisions to the task design, the following three questions emerged as useful in guiding both further design decisions and my analysis of the data (Lovemore et al., 2022a; Lovemore et al., 2023):

(i) Does the task maintain the fidelity of the mathematics, the music, and the integration of the two? How?
(ii) Does the task adequately simplify the complexity of the integration for implementation within the classroom? How?
(iii) What key representation/s would best support conceptual clarity?

I also used literature and RME framing to deductively analyse data. In the following sections I describe and justify the various methodological decisions.

3.3 A Qualitative Interpretivist Research Paradigm

Qualitative research is intended to gain deep understanding and insight about a situation or phenomenon from individuals' or groups' perspectives (Creswell, 2009; McMillan & Schumacher, 2010; du Plooy-Cilliers et al., 2014; Leavy, 2017). Creswell (2013) describes the qualitative approach to research as the "collection of data in a natural setting sensitive to the people and places under study" (p. 44). My study aimed to explore how integrated tasks could be used, guided by RME framing, to support intermediate phase teachers and learners in the process of fractional reasoning and of deepening their fractional understanding. In addition to being the researcher, I was also a participant in the study. I immersed myself in the natural setting of my fellow participants (i.e. their school settings). I had to be sensitive to the context of place (schools) and time (COVID-19) as I sought to gain rich, in-depth understanding from all the participants on how they experienced and contributed to the task design journey. I thus saw a qualitative approach as the most appropriate research route, rather than a quantitative one where researchers seek some measured, objective 'truth' (McMillan & Schumacher, 2010). My decision to employ an interpretivist paradigm is further supported by Gravemeijer and Cobb (2006), who state that interpretivism is useful when conducting design-research that involves exploring and reflecting on the "complexity and messiness" (p. 30) of a classroom or school setting in order to make sense of the events, or, in the case of my study, *ten* classrooms and *two* school settings, as well as the micro-CoP I formed with my supervisors.

Du Plooy-Cilliers et al. (2014, p. 8) point out that a researcher's assumptions "determine the chains of decisions" that take place throughout a study. My assumptions in this study were informed by a broadly interpretivist view regarding ontological and epistemological beliefs. I acknowledge that reality exists independently of individuals' beliefs and constructs (Maxwell, 2013), but I take the ontological view that individuals' experiences and contexts influence their *interpretations* of reality (Lotz-Sisitka et al., 2013). This fully aligns with Freudenthal's description of reality as a dynamic "mixture of interpretation and sensual experience" (1991, p. 17). My epistemological stance is that knowledge construction is subjective and influenced socially (Willis, 2007). While viewing the study through an interpretivist lens, I am aware of the limitations of this paradigm. Interpretivism has been critiqued for being too subjective, contextual and relative (du Plooy-Cilliers et al., 2014). I was, however, interested in understanding participants' experiences within the study as to what strategies and resources would best facilitate the integrated mathematics and music lessons. The integrated strategies had to be useful and practical in their particular contexts. I did not want to impose top-down strategies on the teachers, but rather work as a co-researcher collaboratively designing with them the various integrated tasks. Their subjective perspectives and reflections on these tasks represented a vital source of data. Makar's (2021) call for changing top-down power relations between researchers and teacher participants, and her notion of "research collaborators" (p. 440) resonated with me.

3.4 Design-Research: A Participatory Dual-Design Experiment in Task Design

This study is an example of design-research. I chose, however, to combine task design with the "dual-design experiment" (after Gravemeijer and van Eerde, 2009, p. 520) and therefore refer to my study's design as a participatory dual-design experiment in task design. My goal was to engage in the task design of an intervention for teaching fractions, guided by the framing of RME principles. My "construct[ion] and reconstruct[ion]" (after Maxwell, 2013, p. 2) of this participatory design experiment comprised four phases (the first being my Master of Education study as a preliminary phase) (four, if I include my Master of Education study as the preliminary iteration), across three micro-CoPs (three researcher-designers, and ten teachers from two schools), making it participatory in nature.

Bakker (2018) describes design-research as a "relatively new methodological genre" (p. 272). Its value as a methodology in mathematics education research continues to grow (Cobb et al., 2009; Prediger et al., 2015). Cobb et al. (2009) defined design-research as "a family of methodological approaches in which instructional design and research are interdependent" (p. 169), a definition that permeates other authors' definitions of design-research. Bakker (2018) states that design-research focuses on what education could or should be, rather than describing and evaluating education as it currently is. It can be described as the process of developing an intervention to solve an educational problem while at the same time gaining a deeper understanding of the actual intervention and the design process (Gravemeijer & Cobb, 2006; Plomp, 2007; Gravemeijer & van Eerde, 2009; Bakker 2018). Such intervention may include teaching and learning resources, materials and lesson plans, and specific sequences of tasks.

Prediger et al. (2015) list three types of outputs or products that can be achieved through design-research: "design principles, curricular products and professional development of participants" (p. 881). Design and research are intricately intertwined (Bakker, 2018). The goal, however, is not to produce ready-made, prescriptive resources, but rather to develop and share "exemplary instructional activities and materials," (Gravemeijer & van Eerde, 2009, p. 512), and additional resources and guidelines that teachers could adapt according to their classroom needs (Plomp, 2007). Prediger et al. (2015) also highlight the 'family' nature of design-research, by highlighting that many variations of design-research are possible. Although design-research as a methodology has been criticised because it does not have one defined, step-by-step method to follow, Prediger et al. (2015) argue the value in variations, provided the steps are clearly explained and justified. Bakker (2018) similarly argues that, as design-research does not follow a clear-cut method, design-researchers need to be flexible and use a variety of strategies to gain data about their designs.

Notwithstanding variation within this ‘family’ of methodologies, there are some common characteristics across different design-research studies. Design-research is interventionist, as the focus is on designing novel ways of teaching a concept in a real-world classroom (Plomp, 2007; Prediger et al., 2015; Bakker, 2018). The goal is not only to develop a product (the intervention), but also to come to understand the innovative strategy through developing or refining theories, including local instructional theories (teachers’ anticipated responses from learners based on chosen instructional activities) (Gravemeijer & Cobb, 2006; Gravemeijer & van Eerde, 2009). In design-research, theories can be generated through the research and can be used to guide the design of the instructional activities (Prediger et al., 2015). As Gravemeijer and Cobb (2006) explain, RME theory can guide the design of tasks and provide a framework from which to analyse the intervention. In the case of the current study, the goal was to use an RME framing to guide the design of an intervention product (i.e., the integrated music-mathematics lesson plans, materials and representations). Such designs can contribute to a more integrated curriculum approach where the combined experiences of the teacher participants and researcher participants can lead to a form of professional development, as – together – we grappled with fine-tuning the design of the integrated tasks.

Sometimes design-research is referred to as ‘design experiments’ (Prediger et al., 2015) or ‘teaching experiments’ (Gravemeijer & van Eerde, 2009). ‘Experiments’ here refers to exploring, interrogating, trialling and reflecting on tasks and conjectures about teaching sequences, within an experimental educational setting, to improve the design (Gravemeijer & Cobb 2006; Gravemeijer & van Eerde, 2009). Bakker (2018) describes design-research as being “open” (p. 10) in the sense that the researcher has little control of the data or situation as opposed to closed, experimental design approaches, using closed-ended questions, such as, for example, in a survey. The goal of design-research is not to conduct pre- and post-tests to prove or disprove a theory, but rather to *theorise* about best instructional sequences. In this context, I like the term “humble theories” (see Prediger et al., 2015, p. 879).

A further salient feature of design-research is its iterative nature (Plomp, 2007; Gravemijer & Cobb, 2006; Gravemeijer & van Eerde, 2009; Prediger et al., 2015; Bakker, 2018). The design of an intervention goes through several cycles: what Gravemeijer and Cobb (2006) refer to as microcycles of “thought experiments and instructional experiments” (p. 25) where ongoing reflection informs adaptations within a single design experiment. When the design experiment feeds forward into a follow-up experiment in a different context, these authors refer to this as a macrocycle. Here the reflections of the whole original process can be trialled and re-examined to gain further insight into the intervention strategy, and thus a local instructional theory may be developed. As Figure 3.1, below, illustrates, the macrocycles span across the whole teaching experiment, making up a series of microcycles (Gravemeijer & van Eerde, 2009).

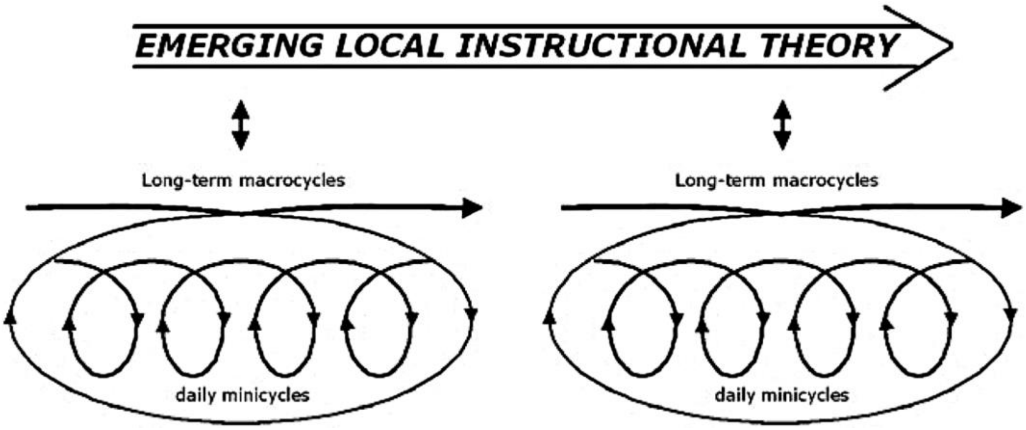


Figure 3.1: Gravemeijer and Cobb's illustration of micro- and macrocycles (2006, p. 29)

Gravemeijer and Cobb's (2006) description of micro- and macrocycles of trialling, reflecting on and adjusting an intervention can be applied to the current study. I consider my Master of Education study (Lovemore, 2020) a form of preliminary iteration, because, while it is not part of the present study, the tasks and lessons learned informed me as I started on the task design journey of the present study. For the current study, the researchers firstly worked through task design cycles of grappling with obstacles and finding resolutions. These were then shared with teachers at Aloe School. They were urged to make adaptations to the tasks to suit the needs of their classroom context as they engaged in microcycles of implementing the tasks, interrogating their effectiveness and giving feedback to me. This led to further adaptations. This represented the first iteration of implementing and trialling the designed tasks. Feedback from the Aloe teachers then fed into the design of the second iteration, this time at Protea School. The Protea teachers engaged with the tasks, and, through discussion, trialling, and reflection, contributed to further adaptations. Design-research's cyclical and iterative nature is thus clearly evident in the way my study unfolded. Figure 3.2, (adapted from Gravemeijer and Cobb's 2006 diagram), illustrates my study's micro- and macrocycles.

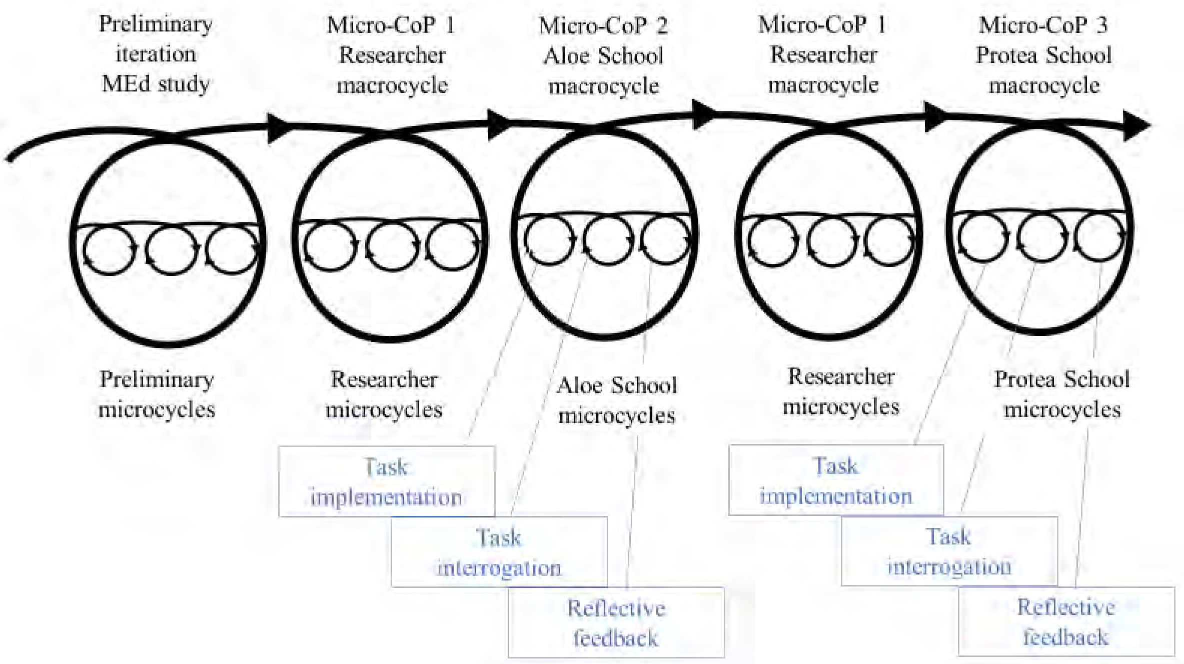


Figure 3.2: Micro- and Macrocycles of the current study (adapted from Gravemeijer and Cobb, 2006)

Authors use various labels for the three main phases in design-research. Plomp (2007, p. 15) refers, to the three phases as Preliminary Research; Prototyping Research; and then, the Assessment Phase. Gravemeijer and colleagues (Gravemeijer & Cobb, 2006; Gravemeijer and van Eerde, 2009) refer to the phases as Preparing for the Experiment, Experimenting in the Classroom, and then Conducting a Retrospective Analysis. Gravemeijer and colleagues' description of the preparation phase aligns with RME principles calling for a decision on an appropriate experientially-real starting point for learners to engage with, and from which they can develop from informal reasoning to formal mathematising as the endpoint (Cobb et al., 2008). In Table 3.1, below, I briefly capture key elements of each of the phases, as identified by these authors. The similarities between the phase divisions are clear.

<table>
<tr><td rowspan="3">Plomp (2007)</td><td>Preliminary research</td><td>Prototyping research</td><td>Assessment phase</td></tr>
<tr><td>Analysis of the context and needs.
Review of existing literature and theory, including similar projects.
Guidelines are developed from these to direct the intervention.</td><td>Development of the intervention prototypes and how to best sequence them.
Iterative cycles of trialling, evaluating and adjusting the design, to enhance the intervention.
Evaluation is done “via expert judgement” (p. 27), such as the researcher-designers or teachers.
Each cycle constitutes a part of the research.</td><td>A summative evaluation of the intervention to determine whether it met the needs of the context, which changes were made and if further trialling needs to take place.
Evaluation of the practicality of the intervention as well as the relevance and sustainability for teachers to effectively use in their classrooms.
Retrospective analysis of the whole study.</td></tr>
<tr><td colspan="3">Systematic documentation and reflection throughout all phases.</td></tr>
<tr><td rowspan="2">Gravemeijer & Cobb (2006); Gravemeijer & van Eerde (2009)</td><td>Preparing for the experiment</td><td>Experimenting in the classroom</td><td>Conducting a retrospective analysis</td></tr>
<tr><td>Initial ideas of the intervention tasks are formulated.
Designers decide on endpoints (the mathematics learning goal).
Selection of appropriate starting point from which the task will develop. Review of existing curriculum documents, literature and any evaluation on the learners’ ability.</td><td>Implementation of intended tasks takes place in the classroom.
Tasks are trialled (by teachers or researchers), interrogated, adapted and improved through the microcycles.
Reflections from the teaching experiment can then inform improvements for future classroom experiments in a different context, i.e. the macrocycles.</td><td>Thorough methodological analysis of designed intervention product or theory.
The goal is to reconstruct an improved, “potentially optimal instruction sequence” (Gravemeijer & Cobb, 2006, p. 42) which can then be used and adapted by teachers.</td></tr>
</table>

Table 3.1: Summary of key elements in the phases of design-research.

In ordering the different stages of my own study, I broadly followed this phased structure. In addition, I consider the work I did towards my Master of Education degree (Lovemore, 2020), discussed in Chapter 2, as a preliminary phase in which I started trialling music-mathematics integrated possibilities. I refer to this as Phase 0: Preliminary iteration. As researchers, my supervisors and I then engaged in initial task design of integrated lesson plans, activities and resources. I refer to this as Phase 1: Task design phase. We based this on our own experiences of teaching mathematics at this level, and more importantly on what literature on multiple constructs of fractions informed us (Kieren, 1976; Siemon et al., 2015; Getenet & Callingham, 2021). Phase 2 of my study I divide into two sub-phases, each of which can be thought of as macrocycles: the first iteration of implementing the tasks, Phase 2a (Aloe School), and the second iteration of task implementation, Phase 2b (Protea School). These macrocycles consisted of their own microcycles. Phase 3 was a reflective analysis. It included teachers' reflections of their experiences, collected through focus group interviews, as well as a retrospective analysis by the researchers (to be discussed in more detail in 3.6.3).

The process, however, was not neat and linear, but rather, as Gravemeijer and Cobb (2006) describe, complex and messy. While Gravemeijer and van Eerde (2009) discern the three phases, they also acknowledge that the boundaries between them are by no means definite or static. The cyclical and iterative design of the intervention meant that micro- and macrocycles influenced each other. Here I drew upon Gravemeijer and van Eerde's (2009) notion of 'dual-design experiment'. They refer to "dual design research" as "two learning experiments going on at the same time" (2009, p. 520): through one design experiment the learners are learning the mathematical content and skills; through a concurrent experiment, the teachers are learning about the intervention task and process. Within my own study, the three teams (the researchers in Phase 1, the teachers at Aloe School in Phase 2a, and the teachers at Protea School in Phase 2b) were all learning through interactive and cyclic design experiments, which then fed forward into each other. The dualistic nature of two concurrent design experiments here refers to (1) the teachers' interactions with the learners through implementing the intervention task and then making observations, reflections and adjustments; and (2) the researchers' interactions with the teachers' feedback and suggested alterations. Figure 3.3 provides a visual representation of the complex interaction between the different phases of my study.

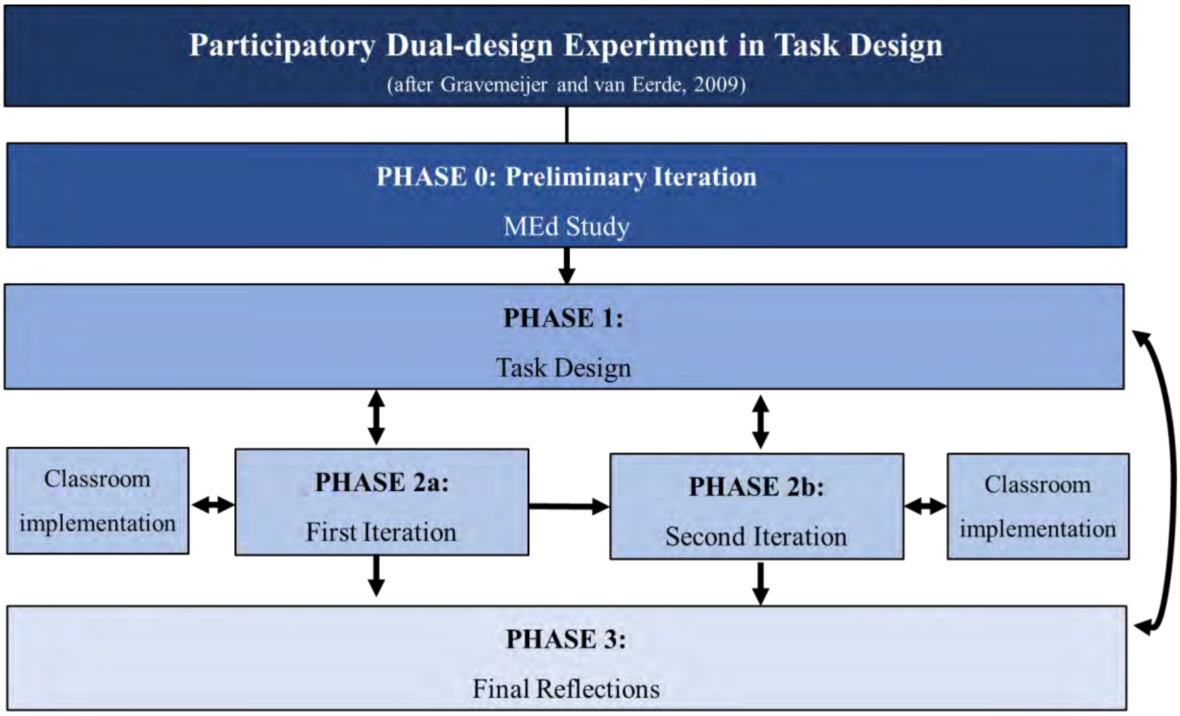


Figure 3.3: Diagram representing the relationship between the phases of the study

Prediger et al. (2015) explain that those who engage in design-research devote a great amount of thought to the design of tasks and how the tasks can encourage rich engagement and conceptual development from learners. Design-research is, therefore, well suited to explore the development of innovative teaching strategies. This resonates with principles of task design. I refer to this design-research experiment as task design, because the main purpose was to develop an effective and accessible product (i.e., the integrated music-mathematics lesson plan sequence, resources and representations). As discussed in Chapter 2, a mathematical task is anything based on a mathematical concept and requires learners to be involved, whether it be a full unit, a single task, an activity or resource. It can be designed specifically for one learner or a group of learners (Graven & Coles, 2017). Task design involves carefully selecting teaching and learning materials to support the learning of a mathematical concept, while justifying the design (Jones & Pepin, 2016). It was appropriate, therefore, to frame the task design grappling of this study as a participatory design-research experiment. My inclusion of the word ‘participatory’ here derives from points such as that made by Leavy (2017) regarding the fact that qualitative researchers often form partnerships with “nonacademic stakeholders” (p. 179) within a community to collectively carry out a research project. As Prediger et al. (2015) explain, the role of teachers in design-research has changed from passive participants to that of collaborators. Active participation of the teacher participants in the design in collaboration with the researcher-designers is, increasingly, a key characteristic of design-research (Plomp, 2007; Jones & Pepin, 2016; Geiger et al., 2022). Kelly et al. (2008) acknowledge the great investment of resources and time necessary for teachers to engage in design-research to create innovative teaching strategies, as well as teachers feeling overwhelmed. For this to be a meaningful process of exploring curriculum integration between music and mathematics, it was clear that teachers would need to be both supported, and their contributions validated.

As noted previously, the intention for this study was the initiation of a single community of practice (CoP) (after Lave and Wenger, 1991). Due, *inter alia* to the restrictions caused by the COVID-19 pandemic, three teams developed. I refer to these as micro-Communities of Practice (micro-CoPs): eight teachers from Aloe School, two teachers from Protea School, and three researchers (myself and my two doctoral supervisors). While the task design journey of creating music-mathematics integrated resources is at the forefront of the study, it is important, I believe, to highlight the relationships between the three micro-CoPs, and their influence on one another.

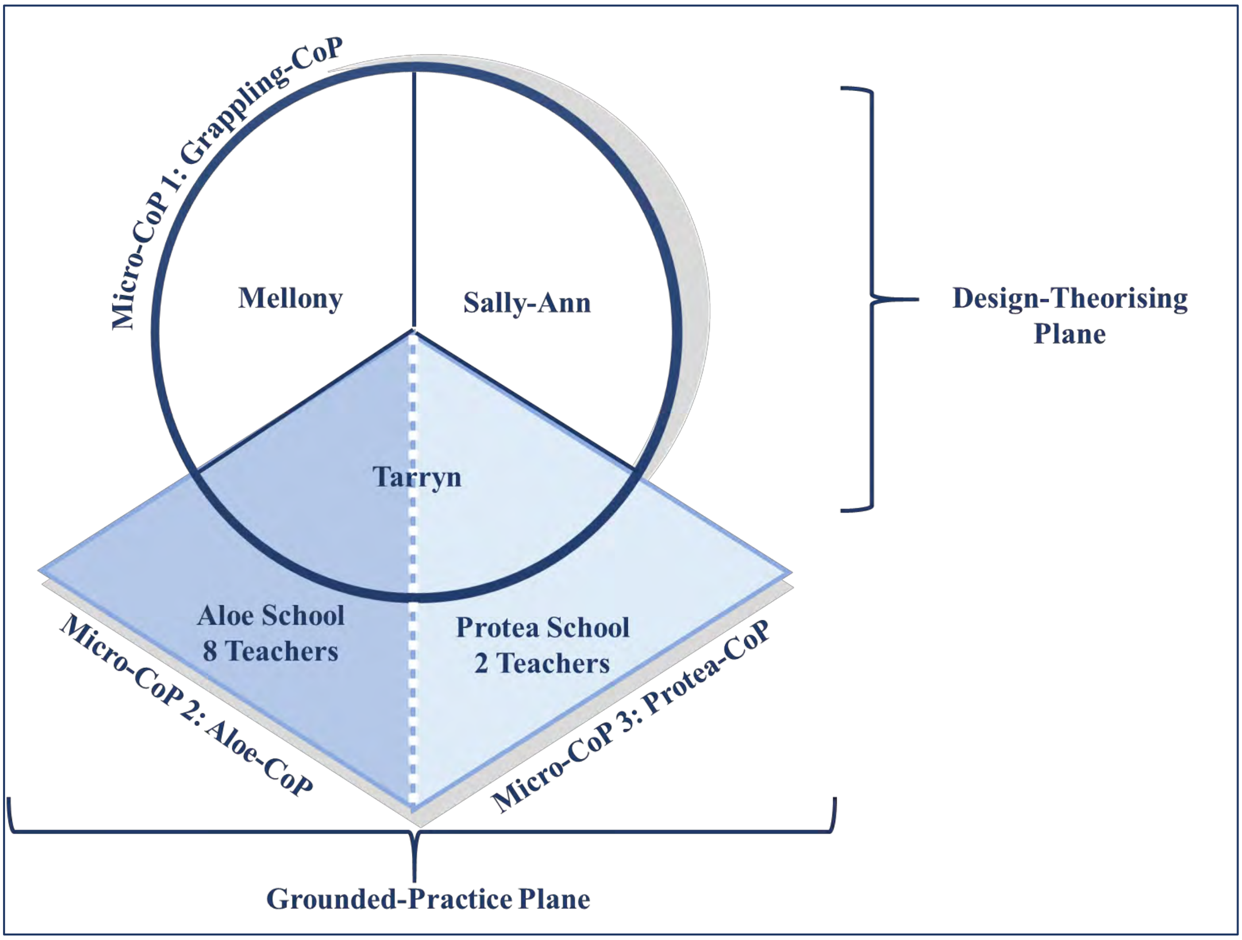


Figure 3.4: Diagram representing vertical and horizontal planes showing the relationship between the participants in the micro-CoPs

Figure 3.4, above, shows two planes – a vertical plane (the circle) representing a theorising/grappling plane (which I labelled the Design-Theorising Plane), while the horizontal dimension (two triangles) represents a grounded trialling and practice plane (labelled the Grounded-Practice Plane). I was at the centre, participating in the researcher-designer micro-CoP (labelled the Grappling-CoP), working in the Design-Theorising Plane. I also immersed myself, in the Grounded-Practice Plane, over a period of ten months as a participant in the micro-CoP at Aloe School (labelled Aloe-CoP), interviewing teachers and reflecting on ways of improving the music-mathematics lessons. There were challenges with meeting regularly as a whole group in Aloe School, but the combination of individual, small group and whole group discussions and informal interviews, constituted a productive micro-CoP. And finally, I immersed myself in the third micro-CoP (labelled Protea-CoP), which comprised myself and two teachers at Protea School. I worked with these teachers over a 12-month period, trialling, reflecting, further adjusting and even extending the tasks initially trialled at Protea School.

3.5 Sampling

The target population of this study was intermediate phase (Grade 4 to 6) teachers, who taught mathematics[9]. As this is a qualitative study, non-probability sampling techniques applied. As Du Plooy-Cilliers et al. (2014) explain, the goal of non-probability sampling is to determine how many participants are necessary to gain an in-depth understanding of the research topic, rather than randomly selecting participants for statistical data collection. Purposive sampling is an example of non-probability sampling: purposefully selected "panels" of individuals who are "uniquely able to be informative" (Maxwell, 2013, p. 97). For this study I selected intermediate phase teachers of mathematics. These teachers work at independent schools, and are well-placed (with their principals' full support) to trial innovative teaching and learning strategies. Teachers' gender, age and years of experience played no part in my selection criteria; rather it was that they were employed at the kinds of independent schools that enjoyed more autonomy and flexibility in the timeframe of curriculum planning than the average state schools where teachers are obliged to strictly follow the CAPS timeline throughout the year (South Africa. DBE, 2011a). I recognised that participation in the study would be less burdensome on teachers who had some greater measure of flexibility relative to being involved in trialling new teaching and learning strategies. My purposeful selection of the research sites and participants was thus based on what Cresswell (2007) called 'informed criteria'. While the independent

[9] In South Africa, intermediate phase teachers (Grade 4 to 6) are trained as generalist teachers. Depending on the school structure, they may teach all or a few subjects. They are not specialist teachers.

schools were a suitable site to trial the music-mathematics integrated tasks, the resources are intended to be useful to a wide range of South African schools, including township and rural schools, as the integrated tasks will be packaged into effective and accessible format, which is curriculum-aligned.

Convenience sampling is a strategy researchers use where participants are chosen based on their accessibility (McMillan & Schumacher, 2010). The accessible population for my study included teachers in the Eastern Cape Province of South Africa, in the city known as Gqeberha (formerly Port Elizabeth) and surrounds, due to their proximity for me, the researcher. While originally proposing initiating a CoP with one independent school, Aloe School, I was, as noted in Section 3.2, approached by teachers at a second independent school who were interested in trialling the music-mathematics integrated tasks. These teachers were enthusiastic about being part of the research study. As mentioned earlier, I went through the necessary steps, and ethical clearance was granted by Rhodes University's Education Faculty Research Ethics Committee to add this second site to the sample. This was especially useful, as it allowed the opportunity to have further iterations of the task intervention, acting as macrocycles, just as Gravemeijer and Cobb (2009) suggest. A further reason why I welcomed the additional school to join the study was that there had been considerable challenges in initiating the CoP and maintaining momentum at Aloe school, largely due, as noted, to the timing being during the multiple COVID-19 pandemic lockdown circumstances.

The sample comprised of all eight intermediate phase mathematics teachers at Aloe School (two Grade 4 teachers, three Grade 5 teachers and three Grade 6 teachers); and two of the four intermediate phase teachers at Protea School (one Grade 4 and one Grade 5 teacher). None of the Aloe teachers had prior experience with playing a musical instrument or with music notation. At Protea School, one of the participating teachers is a pianist and the other, although she does not play a musical instrument, is familiar with musical notation. This teacher sample spread meant that I was able not only to gain in-depth understanding of teachers' experiences of implementing the mathematics-music lessons across the full intermediate phase (Grade 4 to 6), but also from teachers who had varying backgrounds in musical knowledge.

The schools' and teachers' names are replaced with pseudonyms to maintain anonymity. I have chosen to use the names of South African musicians and composers as pseudonyms for the teachers, in honour of their contributions to the cultural heritage of the country. I have also used pseudonyms to uphold the anonymity of learners, by using the first names of various South African musicians.

School Pseudonym	Teacher Pseudonyms	Grade
Aloe School (first iteration)	Ms Makeba	4
	Mr Sontonga	4
	Ms Fassie	5
	Mr Dube	5
	Ms Ibrahim	5
	Ms Chaka	6
	Ms Cloud	6
	Mr Mandoza	6
Protea School (second iteration)	Ms Savuka	4
	Ms Clegg	5

Table 3.2: List of school and teacher pseudonyms

An unanticipated addition to the 'passenger manifest' of this study, was when we realised that my two supervisors had also become active participants in the task design aspects of the study in ways that some might see as extending beyond the supervisor norms of engagement. At each stage of the task design process I shared my ideas, samples of lesson plans and resources with my two supervisors, who then acted as a sounding board for me to review the feasibility and fidelity of the tasks. Together we grappled with key obstacles that I encountered in my task design process. This process guided me in finding resolutions and refining initial design ideas into a product that I could share with teachers.

The micro-CoP that my supervisors and I morphed into devoted 935 minutes to task design grappling. Due to the restrictions in place for combating the spread of COVID-19, this grappling took place onboard the online platform, Zoom. Having grappled with the task design of the integrated tasks, we noticed on reflection that our recordings of the online communication represented valuable data about the actual methodological process of engaging in task design, a process that in itself could be documented and analysed. Creswell (2009) explains that, in qualitative research, the researcher is a "key instrument" (p. 175), as it is the researcher who collects the data through observation, interviewing participants and analysing documents. He also states that qualitative methodology, while following protocol as guidelines, is "anything but uniform" (p. 173). Recognising the role my supervisors and I played in the data production for this study, I therefore included us (our three-person CoP engaged in the joint enterprise of grappling with task design obstacles and resolutions) in my participant sample.

3.6 Data Collection Method

Data are collected in a variety of ways in qualitative research so as to collect rich, relevant and in-depth information (Ferrance, 2000; Willis, 2007; Maxwell, 2013). My intention in this dual-design experiment in task design was to collect rich, in-depth data on the design and implementation of integrated music-mathematics strategies and materials and teachers' experiences of using them. The data collection phases took place over a period of 19 months. My supervisors and I worked in our micro-CoP, the Grappling-CoP, over the full 19-month duration. I worked with the participating teachers at Aloe School and Protea School for ten and twelve months respectively (see Gantt Chart in Table 3.3 below [adapted from Gantt, 1919]). Although the extended time working with my fellow task design journey travelers was partially due to challenges brought about by the global COVID-19 pandemic, it was largely due to the complex nature of the task design of integrating music and mathematics, and including multiple iterations.

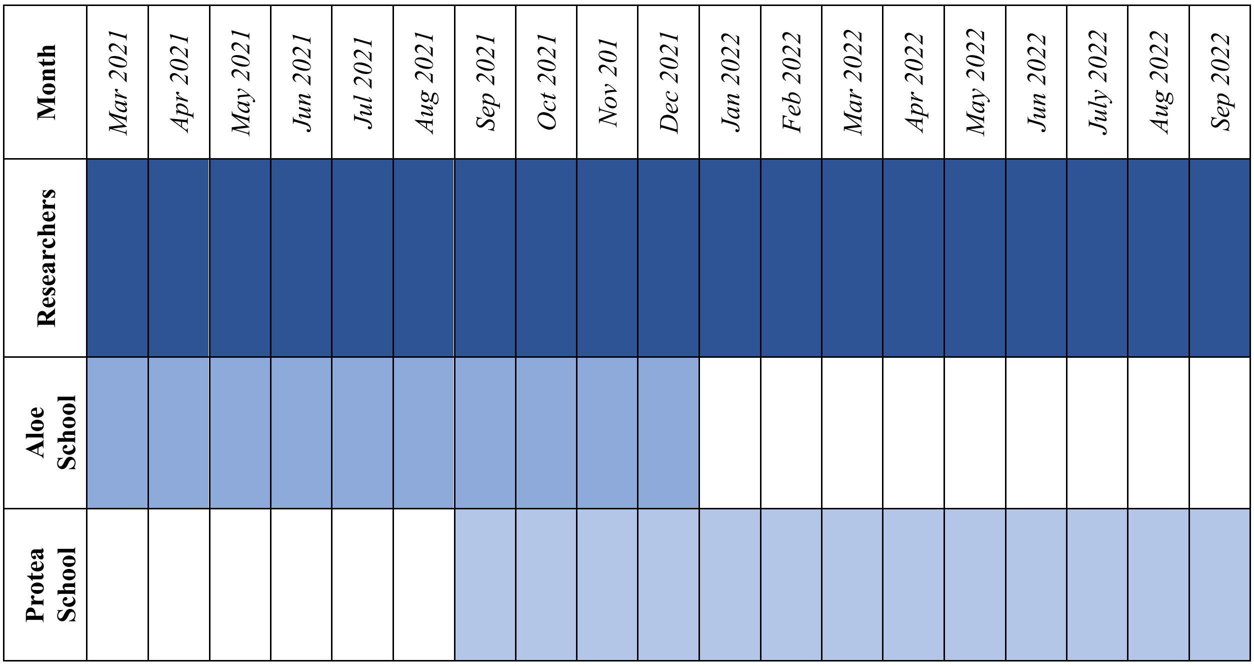


Table 3.3: Gantt Chart (Adapted from Gantt, 1919) to show data collection period

In the next section I discuss the data collection and documenting strategies per phase of the design-research experiment as summarised in the Table below:

	Data source	Platform	Quantity
PHASE 1: Task Design *Researcher-designers*	Micro-CoP meetings [Grappling-CoP]	Recorded Zoom meetings (audio and visual)	935 minutes
		Email	79 email threads
		WhatsApp communication	129 messages
PHASE 2a: First Iteration *Aloe School Teachers*	Focus group and individual interviews in micro-CoP [Aloe-CoP]	Audio recordings	476 minutes
PHASE 2b: Second iteration *Protea School Teachers*	Focus group interviews in micro-CoP [Protea-CoP]	Audio recordings	302 minutes
OVERALL	Doctoral candidate	Reflective research journal	107 handwritten pages
		Field notes	48 handwritten pages
	Artefacts	Google Drive, Dropbox	2,02GB (Gigabytes)

Table 3.4: Summary of data set

3.6.1 Data Collection of Phase 1: Task Design with Researcher-Designers

In the micro-CoP space, I (along with my doctoral supervisors) had formed, our common goal was to design tasks for integrating music and mathematics in ways that retained the fidelity of both subject areas. Starting with the tasks designed in my Master of Education (Lovemore, 2020), and feedback that emerged during various conference presentations of the work, we progressed to incorporating more mathematically complex concepts relating to the teaching of fractions; for instance, moving between the multiple constructs of fractions. We designed tasks based on our experience and informed by RME theory and literature on fraction knowledge development and music-mathematics integration. Our microcycles (Gravemeijer & Cobb, 2006) of design were informed by our own "thought experiments" (Gravemeijer & van Eerde, 2009, p. 514) around how we anticipated teachers and learners might receive the tasks. We were then further guided by the teachers' reflections on implementing the tasks and by their suggestions for possible adaptations.

Having recognised the value of our supervisory meetings and communication, I entered into the data collection (with the permission of my supervisors) the recorded Zoom meetings, our email threads, communication over the instant messaging software, WhatsApp, and the document sharing platforms, Dropbox and Google Drive. Within this micro-CoP, in the online environment we used, considerable grappling occurred on how to effectively integrate the music and mathematics. Examples of this grappling are presented in Chapter 5.

Gravemeijer and Cobb (2006) note that audio recordings of the researchers' group meetings offer an ideal opportunity to document the learning process of the design-research. Leavy (2017), similarly notes that video-conferencing interviews are a useful way to obtain data from participants who are unable to be in close proximity, while still allowing for the benefits of in-person interviews, such as seeing participants' faces and hearing their voices. The added benefits we noticed was the recording function that allowed us to fully and accurately document our meetings. As McMullin (2021) observed, the COVID-19 pandemic has resulted in an increase in qualitative data being collected over video-conferencing platforms such as Zoom, a convenient strategy she sees as likely to continue in a post-COVID-19 world. My supervisors and I argue that this shift to an increasingly online environment has led to novel opportunities for how design-research data might be collected and analysed. This is something we explored further in a paper we presented at the 2023 Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) conference (Lovemore et al., 2023). As shown in Table 3.4 above, is a record summarising the data I collected over the 19-month period. Data included audio and visual recordings, WhatsApp text messages, email correspondence, and task artefacts shared over Dropbox and Google Drive. The audio and visual recordings of Zoom meetings allowed for discussions, images of representations and resources to be collected as data. This was done via tools such as the 'share screen function', camera options, and file sharing on Zoom. The WhatsApp messaging allowed for sharing of text messages, photographs of drawn representations, and voice notes. The artefacts we shared included lesson plans, images of key representations, teachers' resources, guidance videos, music samples, worksheets and model answers.

Throughout this phase, I kept a reflective research journal for noting key discussion points and reflecting on our various task design decisions. Our micro-CoP meetings were open-ended and unstructured, because, as researcher-designer-participants in Phase 1 of the study, we identified and responded to various challenges in the task design process. Watching and re-watching the Zoom recordings, I identified key moments in the Zoom meeting recordings, and transcribed the discussions between myself and my two supervisors. Leavy (2017) refers to this form of data, where the researchers consider themselves as “a knowing subject and valuing their own experiences as worthy” for investigation, as “self-data” (p. 120).

3.6.2 Data Collection of Phase 2: First and Second Iterations with Teachers

Data collection in Phase 2 of the study predominantly involved formal and informal discussions, some of which took the form of either one-on-one, small-group, or focus group interviews with the participating teachers at the two schools.

Other data took the form of artefacts which teachers brought to the meetings, such as examples of learners’ work, photographs of activities and photographs of teachers’ whiteboard representations. I was invited by the teachers at Aloe School to teach a demonstration lesson for the Grade 6 teachers, team-teach a lesson with a newly qualified Grade 4 teacher, and teach a music-dominant lesson (in the sequence of lessons) to three groups of learners. I critically reflected on my implementation of the lessons as part of my research with the teachers. Insights from these lessons were also discussed with the teachers in the formal and informal interviews.

Through the micro-CoP meetings, I adopted focus group principles of data collection. Focus group interviews involve the researcher interviewing a small group of participants in their natural setting to obtain in-depth data about their experiences and opinions around an event, problem, programme or idea (Creswell, 2007; McMillan & Schumacher, 2010; Maxwell, 2013). The researcher acts as the facilitator in the focus group, asking open-ended questions that allow the participants to express their attitudes, opinions and preferences, in a “free exchange of ideas” (du Plooy-Cilliers et al., 2014, p. 184).

The focus group sessions for the current study were unstructured, as the participants largely directed the discussions based on their experiences of implementing the music-mathematics tasks and their reflections. Leavy (2017) suggests facilitating a focus group discussion in a ‘funnel’ order, starting from broader, general discussion and moving towards more specific, clarifying questions. I asked clarifying questions in between and facilitated the groups to allow all participants a chance to contribute. McMillan and Schumacher (2010) highlight the importance of a researcher facilitating focus group interviews with skills in interviewing (asking questions and prompting) and understanding of group dynamics. Having spent extended amounts of time at both schools, I was comfortable with facilitating the focus groups.

The nature of the interviews differed between the two schools. Aloe School, the site for the first iteration of the music-mathematics tasks (Phase 2a), started with a CoP of eight teachers. Due mainly to COVID-related challenges, but also due to challenges commonly highlighted earlier regarding CoPs, it was not always easy to maintain the momentum of the CoP meetings. I discussed these difficulties in a paper presented at CERME-12, entitled *Initiating a community of practice amongst primary school mathematics teachers: Trials and triumphs* (Lovemore et al., 2022c). The trials of initiating and maintaining the CoP included difficulty in meeting face-to-face due to lockdown restrictions. The lockdowns impacted teachers and placed heavy demands on them to teach online, and to catch-up when they returned to in-person classes. Time available to trial the intervention tasks was therefore limited, as was time for CoP meetings.

To resolve this difficulty, I immersed myself in the school environment by being at the school every Friday throughout the trialling period (20 visits in all) regardless of whether or not CoP meetings had been arranged. The participating teachers responded well to this, and took frequent opportunities to chat with me informally. I have designated these chats 'informal, unstructured individual interviews' in my data set. At times convenient for teachers, we were also able to arrange meetings in small groups within and across grades. This allowed for "free exchange[s] of ideas" (du Plooy-Cilliers et al., 2014, p. 184), and while it was not how I had originally envisioned that CoP focus group interviews would take place, I recognised that the informal, unstructured, spontaneous meetings yielded valuable qualitative data. As Bakker (2018) noted, design-research often needs to be flexible and adopt a variety of strategies to meet the needs of the design experiment.

I encountered fewer challenges with initiating the CoP at Protea school (Phase 2b: second iteration of the task design). This was due to starting in September 2021, after the major COVID-19 lockdowns in South Africa had stopped. The size of the group was also, as noted, much smaller (just the two teachers and myself) which made it easier to arrange times to meet in-person to hold the focus group interviews.

In both schools, interviews were audio recorded and transcribed: 476 minutes from Aloe School and 302 minutes from Protea School. The teachers used examples of learners' work and photographs and video recordings from their lessons to share how their implementation of the music-mathematics tasks went. These artefacts often stimulated further discussion, contributing valuable additional data on the tasks. I made field notes of key points and added them to my reflective research journal entries after each school visit.

3.6.3 Data Collection of Phase 3: Final Reflections with Researcher-Designers and Teachers

Phase three was the evaluative stage of the design-research – what Plomp (2007) labelled the Assessment Phase, and Gravemeijer and colleagues referred to as the Retrospective Analysis Phase (Gravemeijer & Cobb, 2006; Gravemeijer and van Eerde, 2009). Firstly, there were the final focus group interviews held with the participating teachers at each school. Here teachers were able to reflect on their overall experience of participating in the study and their inferred effectiveness of the music-mathematics tasks as a way of supporting learners to practise fractional reasoning in order to strengthen their fractional understanding. Both final reflective focus groups interviews were semi-structured. I had prepared a few prompting questions for points of discussion, some of which the teachers engaged with and expanded upon. For example:

- What worked well with the mathematics-music integrated lessons?
- What challenges did you experience and how did you overcome them?
- What did you notice about learner participation?
- How would you explain the learners' development of fractional knowledge through the teaching and learning sequence?
- Would you consider using a similar strategy to teach fractions in future?
- How has your view of the mathematics-music connection changed?

The reflection focus group sessions were audio recorded and transcribed. With such an enormous task ahead of me, having to transcribe 28 hours and 33 minutes of data (Zoom recordings and teacher interviews), I turned to McMullin (2021) for guidelines on transcribing in qualitative research. She calls for qualitative researchers to be explicit about the process of transcription, and to view it through an interpretive lens, involving subjective decisions, rather than a positivist view of objectively converting audio to text. She also states that there is no one correct way to go about transcribing, but that acknowledging your steps and decisions adds to the rigour of a study (McMullin, 2021).

For the purposes of the current study, I did not see it as necessary to record every single utterance, repetition, mistake or element of body language. The key content of the conversations was most important. As the researcher, I transcribed Phase 1, the Zoom recordings between my supervisors and I, and most of the interviews from Aloe School myself. I made the subjective decisions as to what I identified as critical moments in the interviews to transcribe. This McMullin (2021) refers to as selective transcription. I selected these critical moments subjectively, however, I believe that this strategy is justified, as I was present in all the interviews and can confirm the key points in line with my field notes and reflective research journal entries. I also made sure to include information that was relevant to my research question, sub-questions and objectives. I transcribed the selected critical moments with the use of MS Word.

With the (initially unexpected) addition of Protea School to the sample of participants, I decided to outsource the transcription of all the focus group interviews with the teachers at Protea School, and some of the small-group interviews from Aloe School. The selected transcriber was a professional. We worked well together as I briefed her on the goals of the study, the context and background. This is advised by McMullin (2021). I explained to the transcriber that I would need to indicate certain elements such as clapping, which we decided to represent in square brackets (e.g. [clap], [clap], [clap]). When the transcriber was unsure of an inaudible word or phrase, she contacted me and I was able to assist with filling in the missing words. I also briefed the transcriber on the focus of each audio recording, for example, 'in this interview we reflect on lesson 3 and plan for lesson 4'. I used time stamps to indicate the various speakers, so that the transcriber could identify the different voices. Also as suggested by McMullin (2021), I systematically carried out a spot check on each transcript, where I would listen to a 2-minute segment of the recording at intervals of 10 minutes. From an ethical perspective, the outsourced transcriber signed a non-disclosure agreement/confidentiality agreement, indicating that she would not discuss the contents of the interviews with anyone besides me. I shared the audio recordings with the transcriber via a password-protected OneDrive folder. I am confident that the considerations taken, based on McMullin's (2010) article, helped me in approaching the transcription of the data in a subjective, yet critical, pragmatic, and rigorous manner.

The second part of the retrospective analysis of Phase 3 involved the actual data analysis of the study. My supervisors and I recognised initial findings and reflected on the integrated music-mathematics tasks, which we discussed in a Zoom meeting. I then embarked on a methodical data analysis leg of the journey, which is discussed in the next section.

3.7 Data Analysis Method

In analysing the data I was guided by Coffey and Atkinson (1996) who wrote, "We should never collect data without substantial analysis going on simultaneously" (p. 2). Huberman and Miles' description of qualitative data analysis as being "choreographed" (1994, cited in Creswell, 2013, p. 182) resonated with me, and I agree with De Vos et al. who describe it as "messy, ambiguous and time consuming, but… also a fascinating and creative process" (2011, p. 397). These descriptions aptly capture the analysis process in the current study. Merriam (1998), however, emphasises the importance of being consistently systematic and organised in the data analysis. I found the following steps from Creswell (2013) useful as I initially planned the 'choreography' of my data analysis strategy:

(1) Organising your data
(2) Getting to know your data
(3) Describing and classifying the data
(4) Interpreting the data
(5) Reporting on the data, and finally reflecting at each stage.

As Creswell (2009) noted, however, rather than being a neat, linear approach, qualitative analysis is interactive, with various stages being interrelated. It is characterised by its cyclical an iterative nature (Creswell, 2013; Maxwell, 2013; du Plooy-Cilliers et al., 2014). I certainly found this to be the case for the analysis process of the different phases of my study. They were intertwined with one another, entirely fitting for the process of design-research, consisting as it does of continuous micro- and macrocycles of trialling, reflecting and adjusting (Gravemeijer & Cobb, 2006).

Gravemeijer and van Eerde (2009) explain that in design-research, the data analysis starts in the microcycles of design experiments, with the focus on reflecting and adjusting, following the retrospective analysis of the entire data set. During Phase 1 (task design grappling), Phase 2a (first iteration) and Phase 2b (second iteration), the reflections served as a form of initial analysis. I recorded these in my field notes and in my reflective research journal, and the reflections informed future iterations. Phase 3 then included the participants' evaluations of the overall process, as well as my systematic, retrospective analysis (as the researcher-participant) of the entire data set. In this process I could show how the design team came to the improved tasks (lesson sequence, representations and resources) based on what we learned from the microcycles, by going through an iterative process to recognise patterns (after Gravemeijer & Cobb, 2006; Gravemeijer & van Eerde, 2009).

3.7.1 Retrospective analysis of the participatory dual-design experiment in task design

As is suggested by McMillan and Schumacher (2010), I reviewed data from the full data set by immersing myself in the recordings, transcriptions, field notes, reflective research journal entries, and artefacts such as key representations and lesson plans. Relative to Creswell's (2013) first two steps, I was able to organise and get to know the data. I saved the data files in a password protected folder on an external hard drive, labeling sub-folders according to the type of data and individualising the files by date, name of participant and the main idea of the item. The variety and amount of data allowed for 'thick description' of the task design journey (Geertz, 1973).

Step three from Creswell (2013), describing and classifying the data, was a long and systematic process. Immersing ourselves in the task design journey, my supervisors and I, in our micro-CoP, recognised that in our grappling we continuously came up against challenges in the integration of the music and the mathematics. As one problem was resolved, another would appear. By re-watching the Zoom meeting recordings and reading the transcripts, I came to recognise a pattern of cycles of obstacles and resolutions. I shared my initial findings around these Obstacle-Resolution Cycles at the MERGA44 conference (Lovemore et al., 2022a). These cycles of obstacle-grappling-resolution are represented in Figure 3.5, below. They can be likened to the microcycles of Gravemeijer and Cobb (2006).

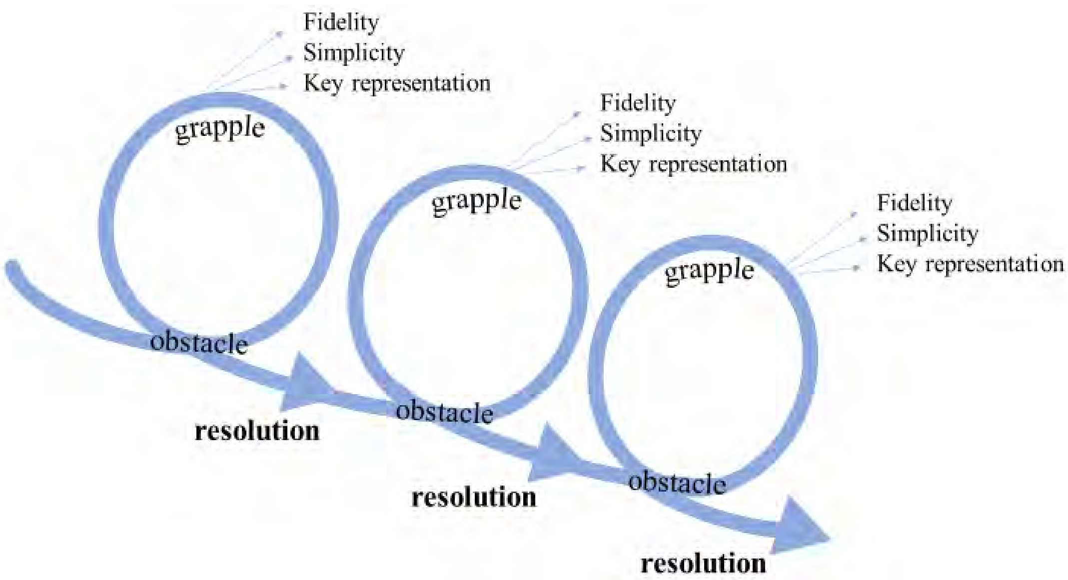


Figure 3.5: Diagrammatic representation of the task design grappling process

As my supervisors and I reflected and grappled further on ways to resolve the various obstacles, we also noticed that we had been trying to answer and meet three key questions, questions which we shared, also, in two mathematics education conference papers (Lovemore et al., 2022b; Lovemore et al., accepted for publication 25 September 2022). The questions were:

(i) Does the task maintain the fidelity of the mathematics, the music, and the integration of the two? How?

(ii) Does the task adequately simplify the complexity of the integration for implementation within the classroom? How?

(iii) What key representation/s would best support conceptual clarity?

The process of retrospective analysis, therefore, started as an inductive process, where the data were reviewed and recurring patterns were noticed (McMillan, 2010, p. 367). From these initial observations, I used categorising strategies (Maxwell, 2013) to group and compare the descriptive data, which I then captured using a matrix template (Table 3.5, below). Maxwell (2013) recommends combining qualitative data analysis strategies such as categorising strategies, connecting strategies and memos or displays. He explains that categorising strategies normally involve coding to sort the data into discrete sections and to develop themes, while connecting strategies involves finding relationships to understand the parts in context, such as a narrative analysis. Maxwell (2013) further recommends that researchers use memos to record their reflections about relationships in the data and for representing such relationships in various displays (for example, tables and matrices).

	Obstacle	Resolution of Obstacle
Fidelity		
Simplicity		
Key Representation		

Table 3.5: Template matrix for organising and comparing data

Finding Maxwell's (2013) distinction of the different analysis strategies useful, I incorporated all three. I started by categorising the data from Phase 1 Zoom meeting recordings, by recording encountered obstacles, discussions that could be grouped into the three key questions and how we resolved them. I was aware of the limitation of categorising strategies, referred to by Maxwell (2013, p. 112) as "analytical blinders". I therefore used connecting strategies to show and understand the data within context of the teacher interviews and reflections, thus showing the relationships between the categories and participants. I therefore progressed to including data from Aloe School, followed by data from Protea school, so allowing for chronological representation of the iterations. I displayed key quotations from participants and memos in a matrix by colour-coding the different participants to show the data from the three different micro-CoPs. Table 3.6 below is an abridged version of one of the matrices used for an obstacle-resolution cycle. (See Appendix 6 for a detailed example of the matrix for organising data). This strategy of manually categorising and connecting the Obstacle-Resolution Cycles data gave me the basis for an in-depth narrative analysis and subsequent presentation of the task design grappling journey towards integrating the mathematics and music in ways most likely to support learners' fractional reasoning and understanding.

	Aligning the number line and the music bars	
	Obstacle	**Resolution of Obstacle**
	Musical notation is a symbolic representation of what is heard, but does not match the time representation on a number line. The representation of the number line therefore cannot be superimposed over the music bar, as they do not perfectly overlap.	Rather than overlaying/superimposing the number line and music bar representations, align them to move flexibly between multiple constructs of fractions.
Fidelity	Cannot adjust the musical representation to be the same as the number line.	Decision made not to conflate the musical and mathematical representations, but rather to align the separate representations to show how to move flexibly between them.
Simplicity	Various attempts made to represent the number line and music bar were confusing for teachers to implement.	The aligning of the two representations would help teachers and learners recognise the links, without adjusting or learning music notation.
Key Representation	The number line and music bar representations could not be super-imposed to support understanding.	A key representation was designed with music staff line located above a triple number line (three parallel number lines), which allowed for recognising connections without overlaying the two representations. This allowed for moving flexibly between the constructs.

Table 3.6: Summarised version of the matrix used to analyse an obstacle-resolution cycle (adapted from Lovemore et al., accepted for publication 25 September 2022)

Creswell (2009) explains that qualitative researchers must delve deeper and deeper into the analysis process to understand and make sense of the data. He likens this process to "peeling back the layers of an onion" (2009, p. 183). To analyse the entire data set and help me answer the research question and sub-questions, I used NVivo software. NVivo, a Computer Assisted Qualitative Data Analysis Software (CAQDAS), has become increasingly used in qualitative studies (McMillan & Schumacher, 2010; Maxwell, 2013; McMullin, 2021). Given the amount of data, this provided a useful option for further analysis.

I used the NVivo software to make memos of my reflections as I combed through the transcriptions of the interviews and other data (such as images of key representations). I made use of coding to categorise the data. Leavy (2017) defines coding as a "process of assigning a word or phrase to segments of data" (p. 125) to summarise the key idea of that segment. Maxwell (2013), however, cautions against forgetting the connecting strategies when working with such data. I kept this in mind, as I coded the interview transcripts and reflected in the form of memos.

I started with predetermined codes drawn from Realistic Mathematics Education (RME) theory for deductive analysis of the formal Aloe-CoP and Protea-CoP meetings, and informal teacher interviews. Using the key principles identified by Cobb et al. (2008) from RME, I developed the coding system shown in Table 3.7 below.

RME Principle (according to Cobb et al., 2008)		Code
1	The starting points of an instructional sequence should be experientially-real to learners (they can immediately engage in a personally meaningful mathematical activity)	*Experientially real*
	Learners use informal ways of speaking and modelling to engage in the mathematical activity.	*Informal speaking and modelling*
2	The informal ways of speaking and modelling lead to a progressive process of vertical mathematisation.	*Vertical mathematisation*
3	With the teacher's guidance, learners' models of their informal mathematical activity can evolve through use into models for more general mathematical reasoning	*Models for general mathematical reasoning*

Table 3.7: Predetermined codes used for deductive coding based on RME theory

While meticulously going through the transcripts of the teacher interviews, I began to notice other ideas or themes, relating to the literature review, which I saw as useful for answering my research questions. This is referred to as 'open coding' (an open-ended way of approaching data to identify possible, emerging themes) (Saldaña, 2016). See Appendix 7 for a list of the

NVivo codes used to categorise the data. I then used NVivo functions to represent relationships between different codes, thus employing connecting strategies.

With the combination of manual and computer-based strategies, I was able to build up a thick description (Geertz, 1973; Maxwell, 2013) of the Obstacle-Resolution Cycles of integrating music and mathematics and how the RME framing assisted with the task design. These findings are discussed in the Section 5.2.

3.8 Trustworthiness

As this is a qualitative study, I refer to 'trustworthiness' when discussing the rigour of my methodology and extent of confidence that might be placed on my research findings, rather than terms such as 'validity' and 'reliability' which are more generally associated with quantitative studies (du Plooy-Cilliers et al., 2014; Leavy, 2017). Strategies for strengthening the trustworthiness of a qualitative study can be organised into four groups, (Guba, 1981; Creswell, 2009; Maxwell, 2013): credibility, transferability, dependability and confirmability. In the following sub-sections I discuss how I strove to attain these in my study.

Credibility

Credibility is to do with the accuracy of researchers' presentation and analyses of their data (du Plooy-Cilliers et al., 2014). One of the most common ways to enhance credibility is through member checks whereby researchers seek to make sure that they have accurately interpreted and represented participants' ideas (Guba, 1981; Willis, 2007; Creswell, 2009; McMillan & Schumacher, 2010; Maxwell, 2013). McMullin (2021) suggests sending copies of transcripts to participants to check the accuracy of the transcription. While I could not expect my teacher-participants to read through over one hundred pages of transcripts (118 pages for Aloe School and 143 pages for Protea School), I provided them with a summary of all the interviews they participated in. I offered to provide them with copies of the full transcripts, but none of them expressed an interest in seeing these, some indicating that they did not have the time to read through that amount of text. From the summarised interview notes, none of the participating teachers indicated that any information was skewed. The researcher-participants, my supervisors and I, had opportunities to access and re-watch the Zoom recordings and read the transcripts, and no discrepancies were noted.

Another strategy to strengthen the credibility of a study is by spending long periods of time with participants, to build relationships and to corroborate inferences (Guba, 1981; Creswell, 2009; McMillan & Schumacher, 2010; Maxwell, 2013). I immersed myself in the data collecting process by meeting and communicating with the researcher-participant task design team (micro-CoP) over a period of 19 months. Guba (1981) and Creswell (2009) also recommend frequent meetings between researchers to increase the credibility of a study. I met with the participating teachers at Aloe School over a period of 10 months, and those at Protea School over a 12-month period. This allowed for reflections and refining ideas via multiple interviews over an extended time.

Clarifying possible researcher bias, involving frequent self-reflection, is also seen as an important step in striving to achieve credibility (Maxwell, 2003; Creswell, 2009). This was especially important as I was part of every micro-CoP, and this was one of the reasons why I kept a reflective research journal, often setting myself reflective questions to answer. This practice of critical and reflective journaling is supported by both Friedman et al. (2018) and Willis (2007). I remained aware throughout of Siedman's comment (1998, p. 3) that researchers should constantly keep their "egos in check" while involved in fieldwork and data analysis. I acknowledged to the teachers, and also in my journaling, my bias towards the integration of music and mathematics. I myself play musical instruments and am passionate about the relationship between music and mathematics. Furthermore, because of my musical background and my preliminary/pilot iteration in my Master of Education study (Lovemore, 2020), I recognised that working with the integrated representations was much easier for me than for many of the participants, particularly those who did not have any background in music.

Gravemeijer and Cobb (2006) and Creswell (2009) write of the value of peer debriefing, where a peer in the research field can listen to and read about a study and ask questions that encourage the researcher to reflect carefully on the study as a whole. Throughout my PhD journey, while carrying out this study, I presented my work at several national and international conferences, where I had the opportunity to share the literature, methodology and preliminary findings with experts in the field and with fellow young researchers. A list of conference presentations and papers based on this study is provided on pages xi – xii. I also attended the SAARMSTE (South African Association for Research in Mathematics, Science and Technology) Research School in 2021 and 2022, where I had the opportunity to share aspects of my study with experts and peers.

Searching for and reporting on discrepant data that go against the general pattern is another way of increasing the credibility of a study (Creswell, 2009; McMillan & Schumacher, 2010). I noted the cases where teachers felt that the music-mathematics integrated tasks were not successful and I also noted discrepancies in some of the teachers' experiences of implementing the tasks. This is discussed in Chapter 4.

Transferability

My aim in this qualitative study was not to produce generalisations of one exact way in which to integrate music and mathematics to improve fraction teaching and learning. Rather, I sought transferability – the extent to which the study's findings can be useful in other, similar contexts (Guba, 1981; Maxwell, 2013; du Plooy-Cilliers et al., 2014; Leavy, 2017). This is similar to Bassey's (1981, p. 85) notion of "relatability", where a study should have sufficient details for another researcher or teacher in a similar context or situation to relate to. To achieve this, I made use of 'thick description' (Geertz, 1973; Maxwell, 2013) detailing the task design journey from the perspective of all three micro-CoPs and the obstacles encountered and resolutions sought in designing the various music-mathematics tasks.

Dependability

Dependability of a study concerns the process of translating data collection and analysis into findings (Shenton, 2004; du Plooy-Cilliers et al., 2014). Shenton (2004) explains that qualitative studies cannot be repeated with exactly the same participants in the same context to replicate results, such as can be done in some controlled quantitative studies. The goal therefore, in qualitative research, is to show what Guba termed "trackability" (1981, p. 81). Gravemeijer and Cobb (2006) explain that in design-research, the aim is to achieve trackability by reporting sufficient detail in the study that it can be retraced. This they refer to as "virtual replicability" (p. 44). In reporting details of each of the phases of the data collection process and the steps of data analysis, I aimed for readers to feel as though they were part of the task design journey.

Confirmability

Trustworthiness of a study can be enhanced through principles of confirmability. It is important to show how findings are established from the data, which can also be achieved by detailing the methods used. This allows others to scrutinise the design, so that when they look at the data they are likely to come to similar conclusions (Guba, 1981; Shenton, 2004). Guba (1981) also recommends self-reflexivity. This I practised throughout, most particularly through my reflective research journal.

Triangulation, a strategy to promote both confirmability and credibility in a study, involves collecting data from multiple sources and in multiple ways. This helps corroborate findings and reduces the risk of bias or chance associations (Creswell, 2009; McMillan & Schumacher, 2010; Maxwell, 2013). I sought to achieve triangulation by collecting data from ten teachers across two different schools, as well as from my supervisors and I as researcher-participants. My data collection methods included focus group interviews, small group and individual interviews, field notes, my own reflective research journal entries as well as the recorded Zoom meetings, emails and WhatsApp communications, along with the artefacts of lesson plans and resources. This allowed me to corroborate my findings from a variety of sources and methods.

3.9 Methodological Limitations

As this is a small-scale study involving only two schools in one province of South Africa, the results cannot be generalised. Design-researchers do not wish to generalise their findings. While they recognise that the intervention is designed for a specific context, the intention is to develop a product (tasks) and a "humble" theory (Prediger et al., 2015, p. 879), that may be applicable or transferable in some ways to various contexts (Plomp, 2016; Bakker, 2018). My study was conducted in two independent schools, both of which are well-resourced. The outcomes may not reflect the possibilities in schools with less access to resources. I also acknowledge the qualitative, interpretive nature of the study, and the possibility of personal bias of individuals (also discussed in 3.8.1). It is my hope, however, that the findings from this study may resonate with teachers and researchers in a variety of contexts who may wish to explore novel ways of teaching fractions or ways for integrating other subject areas into aspects of their mathematics teaching. I furthermore plan to share the integrated music-mathematics tasks with teachers within the South African Numeracy Project, from township and rural schools, and hope that teachers from a wide range of South African schools, as well as teachers internationally, may find it useful in their own teaching of fractions at the intermediate phase level.

3.10 Ethical Considerations

Ethical considerations were integral to every stage of my study. As both Creswell (2009) and Leavy (2017) emphasise, it is important for researchers to anticipate possible ethical issues that may arise during the study, and to be reflexive during the data collection and analysis phases in order to be responsive to any ethical concerns or needs that arise. My personal axiological beliefs are consistent with interpretivist principles. Throughout the study I sought to act with integrity and to uphold ethical practices, and I valued each participant's unique contribution and experience of the study.

In the initial planning stages I tried to anticipate ethical considerations and to plan accordingly. I was granted ethical clearance from the Education Faculty Research Ethics Committee of Rhodes University (application number: 2020-2678-4653). (See Appendix 1 for a copy of the ethical clearance letter). When the opportunity arose to include a second iteration (Protea School), I applied for and was granted this amendment.

Phase 1 of the study included my supervisors and I in the multiple roles of researchers, task designers, and participants. We agreed to the recording of the Zoom meetings and for our first names to be used in any write-up of the study, including this thesis, conference papers and journal articles.

Phase 2 and 3 of the study had multiple layers to consider in terms of ethics. Firstly, I requested and received gatekeeper permission from the schools (See Appendix 2 for blinded gatekeeper letters). The principal of Aloe School requested that I draw up a COVID-19 Protocol document to demonstrate how I would plan sessions to be in line with the health and safety regulations, which I did, and subsequently implemented at Protea School (see Appendix 3). I then arranged meetings with the intermediate phase teachers of the schools, where I invited them to participate and explained the details, goals and expectations of the study. I gave each teacher a formal letter of invitation to participate in the study and asked them to complete an informed consent document. (See Appendix 4 and 5 for templates to teachers and guardians of learners.) From Aloe School, all eight teachers voluntarily consented to participate. At Protea, two of the four teachers agreed to participate. Guided by principles from authors such as Creswell (2009), McMillan and Schumacher (2010), Maxwell (2013) and Leavy (2017), I committed to upholding participant anonymity (all teacher-participants opted for the use of pseudonyms to maintain their anonymity) and the right of my participants to withdraw at any stage. No incentives were offered for their participation.

A more complex layer of ethical considerations involved the dual-design (Gravemeijer & van Eerde, 2009) nature of the study. While I conducted research with the teachers, they carried out their own classroom research by trialling and adapting the music-mathematics tasks. Teachers would add into the data artefacts, examples of learners' work, and photographs and videos of the lessons. These served as stimuli for discussions during the focus group interviews. This, therefore, implies that the learners in their classrooms (minors under the age of 18 years) could also be considered participants in the study (albeit not direct participants). In consequence, the principals of both schools communicated with the guardians of the learners, explaining the study and requesting guardian consent. Aloe School had three guardians who did not want their children to form part of the study, so, while these children were not excluded from the integrated lessons, the teachers took care not to incorporate any of those learners' examples of work or comments in their reflections and report-back during interviews. All guardians at Protea School consented for their children to be part of the study. A further element that needed to be taken into consideration was that teachers were made aware of their rights and responsibilities with photographing and video-recording learners. Due to the commencement of the Protection of Personal Information Act (POPI Act) in July 2020 in South Africa (South African Government, 2022), both schools have policies around the photographing and video-recording of the children. Teachers were, therefore, aware of which children's guardians had not granted consent for them to be included in photographs or video-recordings, despite having granted consent for them to participate in the study. For any artefacts used, such as learners work and photographs all identifying markers are blanked out and pseudonyms used to maintain learner anonymity.

Being confident of the groundwork I had done towards carrying out an ethical study, I was also aware of the need to be reflexive throughout the study and alert to the possibility of any ethical concerns that may emerge, as suggested by Creswell (2009) and Leavy (2017). I also observed the advice of Louw (2014) to continue educating myself on ethical practices in research. I attended a webinar hosted by Rhodes University's Centre for Postgraduate Students in 2020, as well as a research design course focusing on ethics in research in 2022.

While in the data collection stage of the study, I was faced with a few ethical concerns to which I was responsive, through my own critical reflection and through the advice I sought from my two supervisors. I reflected on my role within the teacher micro-CoPs, as the researcher and task designer. I became aware of possible nuances of power relations in the first few meetings at Aloe school, for example. I sensed that teachers were apprehensive in trying to teach the first two lessons to 'get it right'. In a follow-up meeting with these teachers, I re-emphasised to them that there is no 'right' or 'wrong' way to teach these integrated lessons, and that I considered the teachers as co-researchers (after Makar, 2021). I encouraged them to make adaptations within the lessons and to share these experiences and reflections. I made it clear to the teachers that this was not a top-down, prescriptive process, and that they were free to make their own choices as to how to tackle the lessons. I found Setati's (2005) distinction between research ‘with’ teachers and research ‘on’ teachers very useful, indicative of the reciprocal researcher-teacher relationships I wished to foster.

I took everything I had learned from my work with the teachers at Aloe School with me into the second task design iteration with the Protea School teachers. Throughout the data collection phase, I remained aware of the additional demands placed on teachers during the COVID-19 pandemic. One of the things I emphasised to the participating teachers was that I did not expect them to have to create and prepare their own resources for the integrated lessons. Instead, I provided each participating teacher with a resource basket which included all the materials they would need, adding new resources as the task design journey progressed. Quite early on in my time working with the Aloe School teachers I became aware of some of them feeling anxious about integrating music into their mathematics lessons. This was especially the case for those who had no musical background knowledge. This led to my decision to spend time with the Aloe teachers in informal individual or small group settings to support them with any concerns they had, something Makar (2021) recommends when working with teachers. I also accepted the teachers’ invitation to teach one of their Grade 6 lessons so that they could see how I went about implementing the integrated tasks. When a newly-qualified teacher at Aloe School experienced some difficulties with the implementation of the music-mathematics lessons, I assured him of his right to withdraw from the study at any stage. He did consider this, but, after I spent time with him sharing ideas and suggestions for the implementation, he chose to continue with the study. He even invited me to co-teach one of his lessons with his Grade 4 class. These were just some of the ways in which I sought to support the participating teachers. I did not want my study to become a source of anxiety for them. Testament, perhaps, to the interest my study generated at Aloe School was their principal’s request that I write an article for a magazine for the Independent Education Magazine in South Africa, to which I have agreed.

Finally, as regards my observation of ethical considerations, there are two points relating to the protection of my raw data. As noted in 3.6.3, I outsourced the transcription of some of my audio recordings. I got the transcriber to sign a non-disclosure agreement and shared the audio recording files with her in a secure, password-protected file in OneDrive. This file was deleted on completion of the transcription process. With regard to data storage, Creswell (2009) suggests that data should be kept for a reasonable amount of time (ranging from 5 to 10 years). Throughout my PhD study, I have stored my research data on an external hard drive, in folders encrypted through Windows software so that they cannot be easily accessed in the case of theft.

Hardcopy data, such as examples of learners' work and teacher reflections, as well as all the consent forms, are stored in a locked cabinet at my private residence. I will store these data for a minimum of five years, whereafter the material will be destroyed. With these measures in place, the data will be secure. It will, however, be made available on request to relevant members who take part in my study, or to my examiners, should this be necessary. In such an event, the teacher-participants' identity will be masked. In the case of photographs, participants' faces will be blocked out, and, for verification of voice recordings, voice altering software (such as Voxal Voice Changer) will be used to change the unique sound of the participants' voices.

3.11 Chapter Summary

The methodological journey has been messy, complex, ever-changing and well worth the grappling. This chapter has provided a brief overview of the methodological journey of the study, followed by a discussion around the qualitative, interpretivist nature of the study. This was a participatory dual-design experiment in task design, an example of design-research. The population and sampling decisions were indicated. The study was divided into three phases, each one outlined in this chapter. The retrospective analysis method and decisions were described and justified. The chapter concluded with a discussion of the ways in which I strove to achieve trustworthiness and the ethical considerations.

CHAPTER 4: PARTICIPATORY DUAL-DESIGN EXPERIMENT IN TASK DESIGN: ARRIVING AT THE PRODUCT

4.1 Introduction

As noted in Chapter 3, design-research is used to develop an intervention *product* and to develop or refine *theories* around teaching and learning (Gravemeijer & Cobb, 2006; Gravemeijer & van Eerde, 2009). The goal of my participatory dual-design experiment in task design was to develop tasks (a product) that integrate music and mathematics for supporting teaching and learning around the multiple, interrelated constructs of fractions. The participatory nature of the task design, as described in Section 3.4, derives from the three micro-CoPs, as shown again in Figure 4.1.

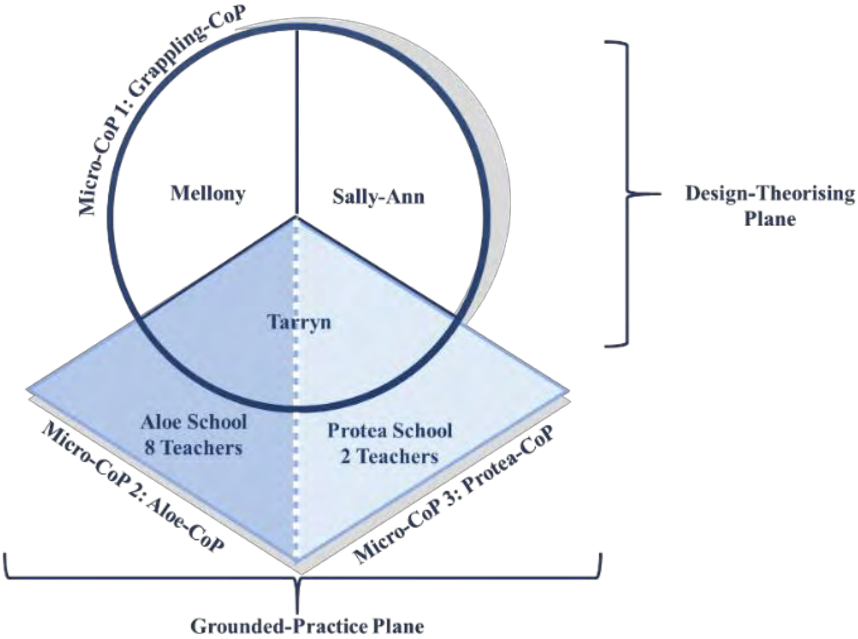


Figure 4.1: Diagrammatical representation of the interrelated planes and micro-CoPs (from Section 3.4)

The vertical plane (circle) I have labelled the Design-Theorising Plane. This is where I worked with my two supervisors (Sally-Ann and Mellony) in our Grappling-CoP in trying to arrive at the best possible designs for the music-mathematics integrated tasks. I have labelled the horizontal dimension (the rhombus constituted by the two triangles) the Grounded-Practice Plane. Here I worked with the teachers in the Aloe- and Protea- CoPs, trialling, reflecting, and where necessary, fine-tuning the integrated tasks.

This chapter is guided by the overarching research question I aimed to answer through this study:

> *In what ways can music and mathematics be integrated via realistic mathematics education task design principles so as to facilitate connections across multiple constructs of fractional understanding?*

The sub-questions guiding this chapter are:

1) *In what ways can musical and mathematical representations and concepts be integrated for developing conceptual understanding of fractions?*

2) *What realistic/meaningful tasks can integrate music and mathematics for connecting across multiple constructs of fractions?*
3) *How can integrated music and mathematics tasks support teachers in achieving the general and specific curriculum aims?*
4) *What are teachers' experiences of integrating music and mathematics for fractional understanding?*

In responding to these questions, I have divided the presentation and discussion of my findings into two parts:

Part A: The 'product' as a finding from my study – the eight lessons, plus two informal 'assessment' tasks designed to help the participating teachers gauge their learners' fractional understanding pre- and post- the eight lessons (an initial workbook and a final individual activity). These were accompanied by representations and resources, along with a description and rationale for the task design decisions (*what* was designed and *why*). Part A partially responds to the research sub-questions 1 and 2 showing the representations and tasks designed. Part A also responds partially to sub-questions 3 and 4, in which I show how the product has the potential to support developing general and specific curriculum aims, as well as some of the teachers' experiences in trialling the product.

Part B: The process of the task design grappling towards the product (*how* it was designed), including the grappling by the Grappling-CoP in the Design-Theorising Plane (answering research sub-question 1 and 2), and the trialling and reflecting by the teachers in the Aloe-CoP and Protea-CoP, in the Grounded-Practice Plane (answering research sub-question 3 and 4).

The presentation and discussion of my findings is divided across two chapters, this and the next. In this chapter I focus on Part A of the findings, the *product* of my task design. In the next chapter I focus on Part B of my findings, the *process* whereby I arrived at the various task design decisions I needed to make in the course of this research journey. In many instances Part A (Chapter 4) and Part B (Chapter 5) took place concurrently and iteratively informing each other. In Figure 4.2 I provide a visual representation of the overall structuring for the two chapters.

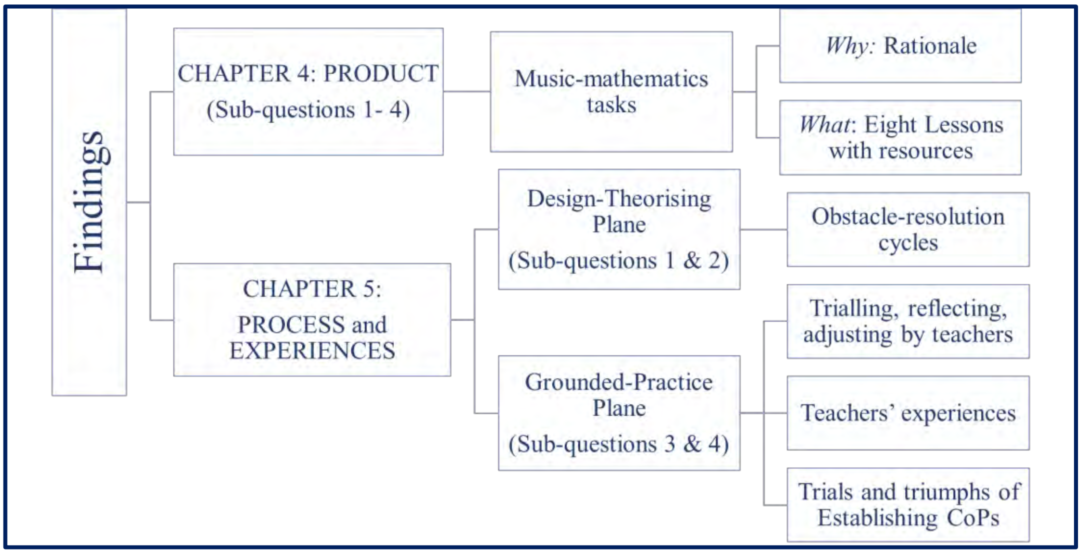


Figure 4.2: A visual representation of the structure of this chapter

4.2 Overview of the Product of the Participatory Dual-Design Experiment in Task Design

The product of this participatory dual-design experiment in task design comprises a series of eight lessons, all based on RME principles (samples and highlights of teacher lesson plans plus accompanying resources are included in Appendix 8). Part of the product included demonstration video clips that I created and posted on YouTube to share with the teachers so as to further support them with the novel music-mathematics tasks. I named the collection of music-mathematics integrated resources 'MatheMusic'[10]. I saw this as a creative name that points to the key design feature of the integration of music and mathematics.

I designed a context, which I shared with my two supervisors in our Grappling-CoP, within which teachers could introduce their learners to both the mathematical concept of fractions and the musical concept of notation and beats per bar. Following the RME principle of using a context in which resources and activities can encourage engagement of learners of all ages and abilities (Freudenthal, 1991; Streefland, 1991; van den Heuvel-Panhuizen, 2003; Cobb et al., 2008), I identified the context of African animals crossing a river as a useful starting point with which to introduce the music notation and fractional reasoning.

From this starting point, I designed a series of activities that the participating teachers could use in guiding their learners through the process of reinvention of mathematics (and to some extent music), as proposed by Freudenthal (1991). My intention, supported by my fellow

[10] I am currently in the process of getting a website, named MatheMusic, designed, which will become part of the teacher resource pack in future.

Grappling-CoP members, was that teachers use the activities to guide learners through their initial informal discussions and representations (horizontal mathematisation), and then through to the more formal ways of representing and justifying mathematical solutions (vertical mathematisation). In some ways, this was true for the musical concepts too, as learners would first informally clap and represent sounds, ultimately leading to more formal musical representation (notation). As Cobb et al. (2008) describe, a key tenet of RME is that the starting point of the teaching and learning sequence should be "justifiable in terms of the potential end points of the learning sequence" (p. 109). The desired endpoints in this task design for mathematics were for learners to be able to:

- work flexibly between multiple constructs of fractions (specifically fraction as measure, fraction as ratio and the part-whole constructs) in problem-solving, finding equivalence, and working with fractions greater than 1 whole;
- add fractions with unlike denominators;
- formally represent fractions on a number line;
- recognise the beauty and elegance of mathematics as a creative human activity.

The desired endpoints in this task design for music were for learners to be able to:

- develop an appreciation for percussive instruments and body percussion (from both Western and African traditions);
- clap and compose rhythms;
- recognise formal representation of note values on a music staff.

The task design was guided by multiple layers of curriculum integration (as discussed in Chapter 2, Section 2.3). The integration was partly informed by Bernstein's (1971) classification of teachers-based (ten teachers from two schools and my supervisors and I as the researcher/task-designers) integration within mathematics (moving flexibly between multiple constructs of fractions) and across subjects (connecting mathematics and music). Drake and Burns' (2004) notion of transdisciplinary integration, making use of a real-life context as the starting point, and the RME principles of an experientially-real starting point guided the selection of the initial scenario. The task design was also informed by Bresler's (1995) co-equal, cognitive style of arts integration. I wanted the mathematics and the music to be considered equally important, with equal appreciation. Bresler's (1995) affective style of arts integration also contributed to the design of tasks in terms of my wanting the tasks to foster learners' appreciation of the beauty of both mathematics and music. On reflection, I recognised that my design of tasks had not followed a neat path categorising the degree of integration, but rather that I had integrated the degrees of integration from the above-mentioned authors (Bernstein, 1971; Bresler, 1995; Drake & Burns, 2004). I use arrows in Figure 4.3, below, to show the interconnected nature of the degrees of integration.

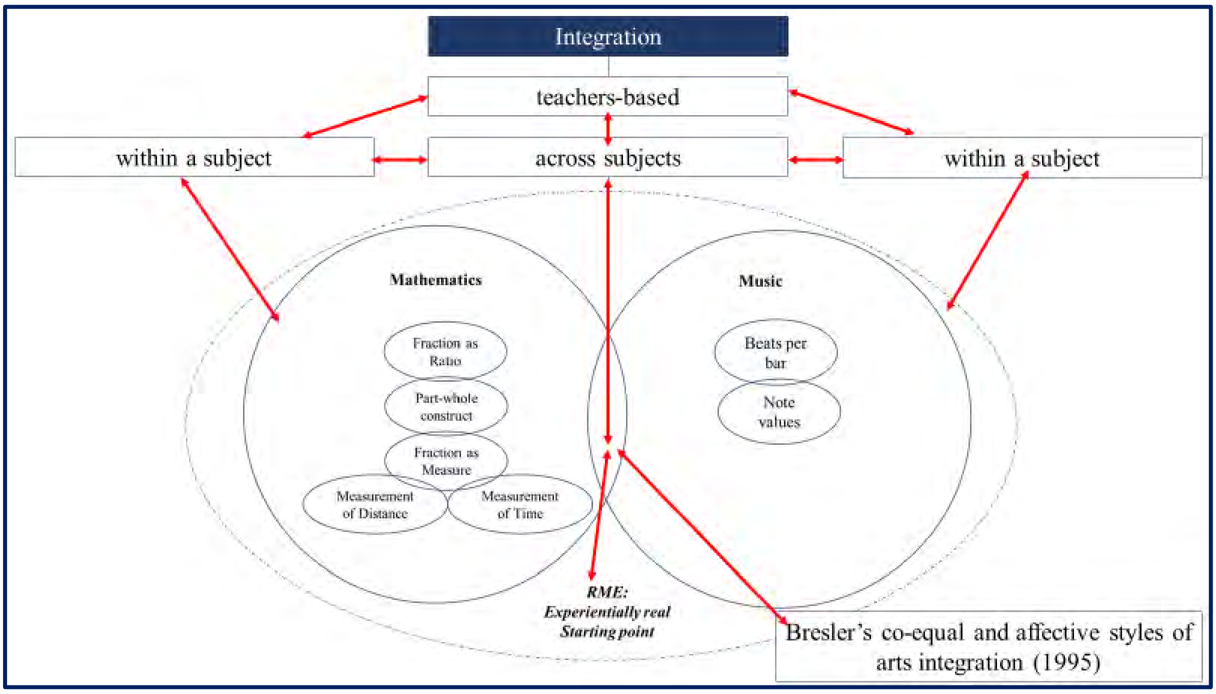


Figure 4.3: Diagrammatic representation of the interconnected degrees of integration in this design of the music-mathematics tasks

In Table 4.1, below, I provide an outline summary of the mathematical and musical focus as well as the key representations and resources used in each of the eight lessons. I also indicate the initial workbook and final individual activity that I designed as part of the product. In the Appendices, I provide samples and highlights of those lesson plans and resources that I identify as key turning points in the task design journey. The full resource pack can be accessed in my data archive, available on request.

Lesson	Key mathematical concepts	Key musical concepts	Key representation	Key Resources
Initial workbook for teachers to evaluate learners' prior knowledge and skills in mathematics (fractions), music (note values) and their dispositions towards mathematics				
1	- Informal problem-solving with fractional reasoning (distance, time) - Informal representations	- Movement - Use of and appreciation for percussion	- Informal notation of animals crossing a river	- Worksheet 1: River-crossing animal jumps template
2	- Informal problem-solving with fractional reasoning (distance, time) - Informal representations - Equivalence (a whole and fractions less than a whole)	- Movement - Use of and appreciation for percussion	- Informal notation of animals crossing a river	- Worksheet 2: River-crossing animal jumps template
3	- Informal problem-solving with fractional reasoning (measure of distance and time; fraction as ratio) - Informal representations - Equivalence (a whole and fractions less than a whole)	- Link animal river-crossing jumps to percussion notation in music - Read and clap percussion beats per bar - Create rhythms with body percussion	- Linking animal river-crossing jumps with percussion notation in music	- PowerPoint with visual and aural representations - Worksheet 3: Percussion line template
4	- Problem-solving with fractional reasoning - Equivalence - Fractions greater than 1	- Musical percussion notation - Exploring rhythmic patterns through 'body percussion orchestra'	- PowerPoint with visual and aural representations	- Worksheet 4: River-crossing animal jumps template - Worksheet 5: Group problem-solving questions
5	- Recognise the beauty and elegance of mathematics as a creative human activity	- Musical notation: from informal percussion representation to Western notation of note values - Body percussion	- Western notation of note values	- PowerPoint with visual representations

6	- Problem-solving with fractional reasoning (fraction as measure and fraction as ratio) - Equivalence - Use length or measurement models to develop the concept of fraction as measure number lines (DBE, 2011, p. 71)	- Western notation of note values - Body percussion	- Alignment of musical staff and triple number line – The river-crossing triple number line	- PowerPoint with visual representations - Note value poster - YouTube video 1 (Teacher guide) - Worksheet 6: Problem-solving
7	- Problem-solving with fractional reasoning (fraction as measure and fraction as ratio) - Equivalence - Fractions greater than 1	- Body percussion - Beats per bar	- River-crossing triple number line (printed and laminated)	- PowerPoint with visual representations - Single number line template - River-crossing triple number line template - YouTube video 2, 3 and 4 (Teacher guide) - Worksheet 7: Problem-solving
8	- Problem-solving with fractional reasoning (fraction as measure and fraction as ratio) - Equivalence - Fractions greater than 1 - Addition of fractions with unlike denominators	- Linking body percussion back to river-crossing animal jumps and the number line	- River-crossing triple number line with fraction cards	- River-crossing triple number line template - YouTube video 5 (Teacher guide) - Worksheet 8: Problem-solving
Individual activity (per grade) for teachers to make an evaluation on learners' knowledge and skills, to reflect on their perceived effectiveness of the integrated music-mathematics tasks.				

Table 4.1: Outline of the task design product

4.3 Designed Lessons

Lesson 1

Having settled on the idea of indigenous southern African animals crossing a river as my experientially-real starting point, I planned the introduction of Lesson 1. The introduction of Lesson 1 (see lesson plan in Appendix 8) started with an African folktale on *The First Music* by Dylan Pritchett (told over YouTube: https://youtu.be/EqGli-UrHPw). I selected this folktale for its relevance to the local context, but also because of van den Heuvel-Panhuizen's (2003) guidance that a *realistic* starting point could be an imaginary story (such as a fairytale), so long as it is used in such a way that learners may engage in an experientially-real activity that has meaning to them. I further saw this story as ideal for this first music-mathematics lesson as it incorporates various percussive sounds (rather than musical instruments) from different African animals, thereby sharing the message that all individuals can participate in some form of music despite having no instruments. I saw this as a good point to make at the start of the novel mathematics lessons incorporating music. I felt it would encourage all learners and teachers in the lessons irrespective of whether or not they considered themselves musical. Analysis of the teachers' reflections indicated that this had been an appropriate start. In reflecting on the first lesson, one teacher, for example, commented on the introductory story as follows:

> *Ms Makeba: We had such a valuable discussion about the story, that linked to the music and the animals.*
>
> [Aloe School, Informal interview, 2021-09-10]

Some teachers, however, expressed hesitancy towards the idea of integrating music into their mathematics lessons, feeling that they lacked the necessary music background. At the first introductory CoP meeting at Aloe School Ms Chaka, for example, in referring to her musical ability, said:

> *Ms Chaka: ... I can't even comprehend how I'm going to do this. I definitely need some sort of* [guidance]. *I haven't a clue!*
>
> [Aloe-CoP, 2021-03-12]

The folktale then led into the problem scenario from which the mathematical and musical concepts could be drawn. This scenario involved different animals having to cross a river (hereafter referred to as 'river-crossing'), of a constant distance in a constant time. The different animals, however, cross the river with a different number of jumps. Sounds can be heard by an observer as the animals jump across the river. This scenario gave learners the opportunity to experience and solve problems with various mathematical concepts, including the fraction as measure and fraction as ratio constructs, and musical concepts of time, note values and beats per bar. Below is an excerpt of the scenario:

> *This is a river which the animals need to cross for their migration. Each animal crosses the river with a different number of jumps. BUT all animals cross in the same time so they cross with a different rhythm of beats. Any animal not crossing in this time will be eaten by the crocodile.*
>
> *You are a biologist observing the migration behaviours, noticing the sounds and distance of the animal jumps on the rocks as they cross the river. You do not have a cell phone, camera or recorder, so you decide to record the jumps through drawings.*
>
> *Kudu crosses with one long jump, Ostrich with two jumps, Zebra takes four jumps, and Monkey takes eight jumps. Giraffe with her long legs only takes three river-crossing jumps, Elephant crosses with five heavy jumps, Springbok takes six jumps and jackal seven jumps.*

In the above scenario the river-crossing distance and the river-crossing time both provided constant units against which animals' number of jumps could be measured and compared. This allowed for moving flexibly between the fraction as measure and fraction as ratio constructs. For a real (embodied) experience, learners first physically jumped across a constant distance in a constant time (representing different animals) and then rhythmically clapped the jumps. This was then followed by informal representations of the jumps and the percussive claps which could, at the end point of the lesson sequence, be formally represented as fractions on a number line – timeline or distance line where the unit is the river-crossing – and notated on a musical line where the unit is the music bar.

In our Grappling-CoP, working within the Design-Theorising Plane, we reflected on the potential of the design of this first lesson.

Mellony: *What's quite powerful about what you're doing is, you are doing two of those* [constructs of fractions]. *You are doing fraction as rate and fraction as measure.*

Tarryn: *Okay, yes!*

Mellony: *You're bringing those two representations together. So the up side of the music, that's four beats per bar, which is a rate. Music has a rate of four beats per bar… and so these guys* [animals] *are running at different beats per bar. The kudu is one beat* [jump] *per bar. The other one was four beats* [jumps] *per bar. The other was eight beats* [jumps] *per bar. And so those are rates. And then later you move to fraction as measure.*

Tarryn: *With the distance?*

Mellony: *With the distance, and it's an issue of time as well. That's why the fraction as rate then links to fraction as measure of distance.*

Tarryn: *That is the connection.*

Mellony: *Ja. They all connect with these animal jumps which is both rate and measure. So both of those powerfully come together.*

Tarryn: *So this is using music and jumps to connect both fraction as rate and measure.*

[Grappling-CoP, Zoom meeting, 2021-03-25]

In the Grounded-Practice Plane in our Protea-CoP, the teachers and I discussed the value of this activity, before they implemented it. The teachers were positive about aspects which relate to RME principles, such as a realistic start to a topic, with informal discussion that would then lead to formal application.

Ms Savuka: *I love the idea of starting something that's realistic and based on real-life as well as the formal application of it.*

Ms Clegg: *Our kids love drawing those. We tell them to draw…*

Tarryn: *That would be so nice to see the different ways that they do it and to chat about it. "Why did you say this, and why did you draw it like that? Is one right, or is one wrong, and how can we show different ways?" So that's all the informal discussions, and they're starting to think fractional reasoning, but we're not talking about fractions yet. It will be the reasoning, the problem-solving that is happening informally at this stage.*

Ms Clegg: *It's very good. As soon as they see numbers, I think they go, "Uh, I can't do it," and switch off. So coming at it from that side, rather than copy down the fractions on the board.*

[Protea-CoP, 2021-09-30]

In a focus group discussion in the Protea-CoP, after trialling Lesson 1, the teachers reflected on the practical jumping activity and how their learners responded. The teachers reflected on their experience of trying to help learners understand the need for equally spaced jumps and also of taking jumps of an appropriate distance so as to fit the correct number of jumps into the whole river-crossing. I linked the teachers' reflections to Tzur's (2016) suggestion for iterating fractions, where learners have to find the correct measure of length to form a whole and then iterate the measure, thus developing a concept of fraction as measure. In the excerpt below, the teachers also recognised that the Grade 5 learners managed to get the equally spaced jumps more easily than their younger peers in Grade 4.

Ms Savuka: *They said that they loved it. The one thing that I had to really repeat and make sure that they understood, was that your jumps have to be the same* *space apart. Try and keep your jumps as evenly spaced as possible. Some did big jumps and little jumps because they tried to cross the river exactly in* [four seconds]. *In those four beats. So that was a bit of a challenge.*

Ms Clegg: *I didn't have that problem. It was almost automatic that they divided it into spaces. It was the children pointing out that if you jump on those certain bricks, then you will get across. So it was almost like them helping each other… Especially when it came to* [more]. *The monkeys that had to get across. They'd get across at seven and then they would have to like do a double step. And that's when they started helping each other, and going, "Okay, but if you divide it equally…" So I think that's just a thing of maturity.*

Tarryn: *The Grade 5s managed easier than the Grade 4s.*

Ms Savuka: *Yes, in dividing it equally.*

Ms Clegg: *They definitely realised, even the ones that were doing it, they were like "Oh they should have taken four"' So they knew that what they were doing wasn't equal. Then I'd give them another chance and say, "Try and do it and fit it all in."*

Tarryn: *And it's not about getting it right or wrong. It's about the experience. An opportunity to have something to be doing hands-on. So then you took that and you got them to represent it informally?*

[Protea-CoP, 2022-01-24]

I recognised from teacher responses that my intention to link the river-crossing jumping activity to the fraction as measure construct was successful, insofar as the jumps were distances from a given point on the same (river-crossing) scale, and that the learners were sub-dividing a unit (the river) into smaller, equal parts, while iterating the smaller jumps (Tzur, 2016; Cortina et al., 2019; Getenet & Callingham, 2021). Furthermore, learners could experience the fraction as ratio construct through my animal river-crossing scenario, as they could start informally discussing and representing the 'jumps per river-crossing' or 'claps per river-crossing'. In this case, the fraction as ratio construct referred more specifically to rate – the comparison of two quantities of different types (Lamon, 1999), that is, the *jumps* per *river-crossing*, for example, which I designed to lead to understanding the 'beats per bar' in music.

Throughout the lessons, my task designs used the problem scenario to link the various animals with specific jumps per river-crossing that learners could associate to specific beats per bar and note values. It could, however, be any animal (albeit that I had carefully selected southern African animals not only to locate the problem scenario within the learners' African context, but also to honour South Africa's wildlife heritage). I also tried to link the number of jumps per river-crossing to the real-world animals' size and agility. For example, a kudu (antelope) is known to jump long distances, hence the one jump per river-crossing; zebras, having shorter legs, would need to take more jumps to cross the river, hence four jumps per river-crossing in this problem scenario.

At Aloe School, I noticed mixed reactions to the use of an analogue clock to help learners keep a steady beat, by following the second-hand, while jumping across the river. Ms Fassie and Ms Chaka both explained the challenges with keeping a percussion beat in time with the clock and having learners jump across the river, as can be seen in their comments below.

> *Ms Fassie:* *I changed the first lesson, I used animal pictures instead of the jumping, because it was just too complicated to get a person to jump for 4 seconds.*
> [Aloe School, Informal interview, 2021-09-10]
>
> *Ms Chaka:* *I can't get it to take the four seconds. So we had someone doing the seconds, but there was no way… The jump was much faster. Maybe the zebra that did it in four. That was probably the only one that could go.*
> [Aloe School, Informal interview, 2021-09-10]

On the other hand, Ms Makeba, reflecting on her trialling of the river-crossing jumping activity and clapping in time with an analogue clock's second hand, started, herself, recognising the link between the animal river-crossing jumps, the musical rhythm and the link to fractions.

> *Ms Makeba:* *They listened to the clock, the second hand. And then we also use clapping. And something that they picked up, there seems to be an equal time between each one. I hit the sticks also, and they said, but you've got a rhythm. So I thought that would tie in with the fractions being equal parts of the whole.*
> [Aloe School, Informal interview, 2021-09-10]

The teachers at Protea School also found using an analogue clock to keep a steady beat helpful, as seen in the comments below.

> *Ms Savuka:* *The clock, it worked well. I had just one person watching. I put the clock on the chair, and he sat in front of the chair and he was hitting the drum on the beat. He could do that well.*
>
> *Tarryn:* *Yes. So, would you prefer using the clock if you were to do this again? With or without the clock for the beat and the drummer?*
>
> *Ms Clegg:* *I thought the clock was brilliant.*
>
> *Ms Savuka:* *I think so too.*
>
> *Ms Clegg:* *And my drummers liked it as well because they were literally watching. And I actually did some practice. You clapping faster* [clap] [clap] [clap] *hands. "Okay, let's try it again." In the beginning, I got everyone to clap and it definitely helped them change the jumping. So I like the clock…*
> [Protea-CoP, 2022-01-24]

I therefore chose to keep the use of the clock as part of this activity, with the option, however, for teachers to first do the activity without the clock and then progressing to some learners jumping across the river while their classmates beat in time with the second hand of the clock.

It was interesting to notice the variety of ways in which learners informally represented the animal jumps and claps. Some learners used more concrete images and other learners used more symbolic representations to show the animal river-crossing jumps. Examples of how the learners informally represented the animals' jumps and their corresponding claps are shown in Figure 4.4, overleaf.

At Aloe School, Grade 4 teacher, Ms Makeba, reflected on Lesson 1. She was pleased with the ways in which her learners represented the animal river-crossing jumps, as seen in the below excerpt from an informal discussion.

> *Ms Makeba:* *When we did this activity, I was amazed, because I gave them no guidelines. No, I gave them one guideline. I put these two lines on the board* [points]*, to indicate that's the river…* [Showing examples of learners' work]. *He's a music student. And this, see the kudu with the one big jump and then the little jumps and she's not a music student.*
>
> *Tarryn:* *Lovely! So nice to see the different ways to represent it.*
>
> *Ms Makeba:* *So this was really a successful lesson. I thoroughly enjoyed it….*
>
> [Aloe School, Informal interview, 2021-09-10]

The teachers at Protea School had a similar response to the informal representations of the animal river-crossing jumps.

> *Ms Savuka:* *I noticed that they all showed a jump of being a line. They understood that with the drum. What was really interesting is that when they were doing this activity, I was watching them. Some were doing it free-hand. Some actually took out their rulers and they wanted to space the jumps as equally as they could.*
>
> *Ms Clegg:* *So ja, I've also got curved lines. What we did was they drew the stepping stones. So quite a few of those. And some like Jesse just drew them swimming. And then just said four steps. But a lot of them you can see the stepping stones were the main focus.*
>
> *Tarryn:* *Interesting dotted lines. This is informal representation.*
>
> *Ms Clegg:* *And then Miriam, just the same thing but used the numbers. So she actually numbered the steps.*
>
> *Ms Savuka:* *You can see the emphasis on the jumps on the landing. He did the jump and then he said for the landing he did a little splash.*
>
> [Protea-CoP, 2022-01-24]

From the teacher reflections on Lesson 1, I was satisfied that the problem scenario allowed for real experiences for the learners from which they could informally discuss, reason and represent in a form of horizontal mathematisation that would allow for subsequent progression to formal vertical mathematisation (Treffers, 1986; Freudenthal, 1991; Cobb et al., 2008). Figure 4.4 below shows some examples of the children's informal representations.

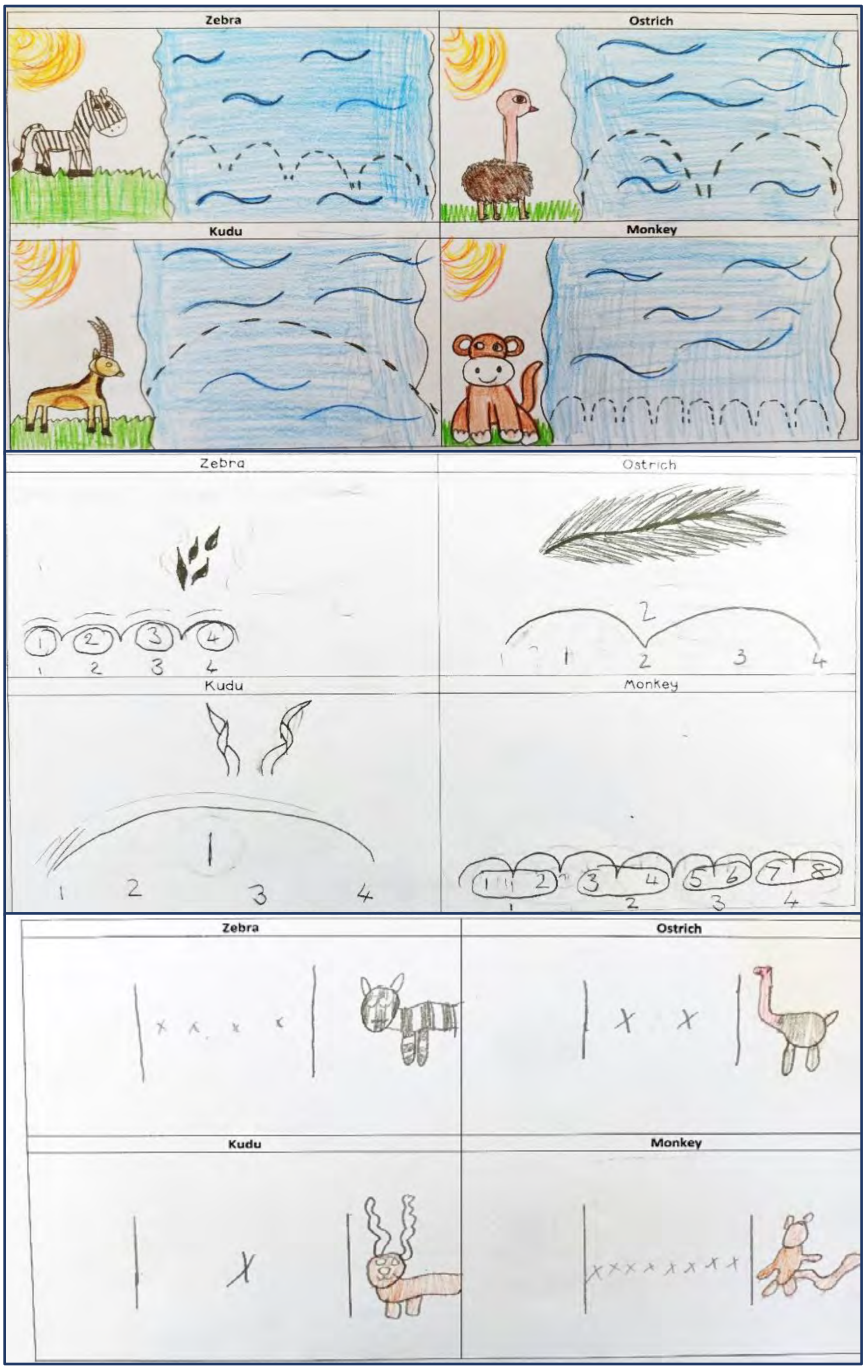


Figure 4.4: Examples of learners' informal animal river-crossing representations

Lesson 2

Lesson 2 was a continuation of Lesson 1, with the same river-crossing scenario as the starting point. In the first iteration of the task design at Aloe School, due to challenges around the COVID-19 pandemic, quite some time had passed between teachers' trialling of Lesson 1 and being ready to trial Lesson 2. In some cases the time between lessons was just two weeks, but in others it was as much as three months, depending on when a teacher had managed to implement his or her Lesson 1. At a small group meeting with Grade 6 teachers from Aloe School, the teachers suggested that Lesson 2 be similar to Lesson 1 as a form of revision lesson for the learners. They suggested another animal river-crossing activity. I used this opportunity to introduce more animals, whose 'jumps per river-crossing' would include numbers other than one, two, four or eight jumps per river-crossing.

The decision to include these other jumps derived from my supervisors' and my grappling in the Design-Theorising Plane. We felt concern about the limited fractions used in music note values: notation mostly comprising only values that allowed each time for doubling and halving (whole note, half note, quarter note, eighth note, etc.). We recognised that this limited opportunity for linking music notes to fractions with other denominators (e.g. thirds, fifths, or sixths). We therefore decided we needed to include other animals in the problem scenario which would allow for linking to a range of different fractions when reaching the vertical mathematisation stage of the task design. A summary of our communication around the addition of animal jumps, as well as the Aloe School teachers getting involved in the task design, is captured in the following excerpt from our email correspondence.

> *Tarryn:* *The adjusted lesson 2, as decided upon by the teachers in the CoP, will recap the animals jumping, and introduce jumps of 3, 5, 6 and 7. The idea of 'equivalent jumps' will then be explored, with physical jumping and drawings. The teachers came up with some of the suggested questions/problems. This was so exciting as a CoP (smaller groups, unfortunately not all 8 teachers together).*
>
> [Grappling-CoP, 2021-09-14]

Aloe's Ms Makeba gave detailed feedback on how her Grade 4 class engaged in Lesson 2. Her reflection shows that the scenario did provide opportunities for learners to be actively engaged in a hands-on experience which could lead from informal to formal fractional reasoning while also connecting the animal river-crossing jumps to the musical beats.

> *Ms Makeba:* *We did the animal that took the three steps and we did the animal that took the six steps. And one pupil was able to comment and come up with, "Oh, for every one step taken by the giraffe, the springbok* [antelope] *had to*

take two steps". So then we also linked that, we went back to the animals that take 4 steps and 8 steps, and worked with that as well. Another pupil brought in terms such as 'doubling and halving', so the springbok's steps are double to the giraffe. So that was quite good. Then I did some clapping and beating on the drum, and then children did the beating on the buckets and clapping, and what was mentioned here was that the claps and the beats were evenly spaced, so there weren't two quick claps and then a break and then a third clap. So that was also quite nice to see that they realised the claps are evenly spaced, and that's when somebody said, "Oh, that's like with fractions when we work with equal parts". So that was absolutely fantastic.

[Aloe School, Informal interview, 2022-09-30]

Aloe's Grade 5 teacher, Mr Dube similarly, commented positively on the engaging, 'fun' nature of Lesson 2. What I found interesting from his reflections on the lesson was his inclusion of other numbers of animal jumps per river-crossing, such as 10 or 11 jumps per river-crossing. This was a good way to pre-empt various fractions, as well as demonstrating the inverse order relationship of fractions (Tzur, 2016): the more jumps per river-crossing, the smaller they become, or the more claps per river-crossing, the shorter they are.

Mr Dube: *What we did also do, was like you said with the different numbers, we did some of those numbers but then every now and then I would have thrown in a different number… Just that as the extra thing. It was seven, six, five, three. So we did those ones. And then I did one or two extra ones. It was like a 10 and 11. Just to, also, then they see it's the quicker claps.*

[Aloe School, Informal interview, 2022-10-20]

Having received this positive feedback from the Aloe School teachers' first iteration of Lesson 2, I then shared the lesson with the Protea School teachers for the second iteration. The Protea teachers reports on their experiences with Lesson 2 were similar to those of the Aloe teachers. In addition, though, they reported a greater emphasis on the idea of equivalence. This is captured in the following excerpt from the Protea-CoP focus group interview:

Ms Clegg: *I wanted to show them that at some stage the animals are all gonna jump on the same stepping stone… What I was very surprised by at the end of this, is that not even having to draw it, and then asking them, "If the ostrich now had to jump and pause in the middle of the river, how many jumps would it take, for example, the zebra? How long would it take the zebra to catch up to where the ostrich is?", and they all said, "two". They understood. And then after understanding, I actually had them do it. I had one person on one side, the ostrich, on the other side the zebra. And have them jump in those four beats, that they actually land on.*

Tarryn: *Okay, so it will be like that ju-ump and land together.*

Ms Savuka: *And jump, ja. I was very surprised that they understood that… And then afterward when we went back inside then I just showed them on the board* *how that worked. Then used some other examples with the three and the six. The giraffe and the springbok.*

[Protea-CoP, 2022-01-24]

I noticed, however, that the focus was still on a part-whole construct of fractions (i.e., breaking up the river-crossing unit into different, equally spaced fractions). This led me to recognise the need to more explicitly emphasise the fraction as measure and fraction as ratio constructs. To highlight the fraction as ratio concepts, I therefore decided to introduce the musical 'beats per bar' in Lesson 3, linking the animal jumps to percussion.

Lesson 3

After establishing the experientially-real starting point that would provide a context from which to use musical and fractional reasoning in the first two lessons, I intended to link the animal river-crossing jumps to a musical representation in Lesson 3. The Aloe teachers were hesitant, however, about working with Western musical notation of note values in the first iteration of the task design, as can be seen in some of their reflective comments below.

Ms Ibrahim: *The music scares me… I can clap, that's fine, and I can show it on a number line, but the music scares me…*

Ms Makeba: *Music still terrifies me… I had to sit before doing these lessons and teach myself first. It did help the children, but for me, I do get nervous with music.*

Tarryn: *If I were to do something to help take the nervousness away, or to adapt the lesson for other teachers, what could I do?*

Ms Makeba: *Don't put those little music notes on a bar.*

Ms Cloud: *Because if we had the jumps…*

Ms Makeba: *Ja! I was happy with those jumps and the claps.*

Ms Cloud: *Ja, if you say its jumps and claps, that's fine.*

Tarryn: *So you're saying that the jumps and the claps were okay. What about the Xs on the line?*

Mr Dube: *Ja!*

Ms Makeba: *Ja, yes. That was also good.*

Ms Fassie: *I think it was not so much the music notes, but the idea that it was music.*

Ms Cloud: *We can do it, but it was just the idea that I have to do music.*

Ms Fassie: *But when you actually get to it, it was quite easy to do.*

[Aloe-CoP, 2021-12-03]

This discussion with the Aloe-CoP was enlightening. It showed me the teachers' lack of confidence in their music knowledge and skill. This insight is in line with Kneen et al.'s (2020) finding that music was the art form in which teachers were the least confident about integrating into their other subjects. The teachers were, however, confident about implementing the jumping, clapping and informal percussion notation (of Xs and squares). In light of the teachers' responses, I concluded that it was not necessary that the teachers and learners should have to learn to 'read music', but that they could rather work flexibly with the concepts of beats per bar and percussion (by, for example clapping) and that this would still maintain the fidelity of the musical concepts. I noted this, earlier on in the first iteration, in my reflective research journal as follows:

> *The focus does not have to be on formally teaching Western musical notation of note values, but rather to allow for informal opportunities to experience the musical beats per bar and the clapping/percussion, and then at a later stage the learners can be exposed to the formal notation of musical note values, with the music teacher if necessary and these lessons will help support that. The focus here should be on the clapping and beats per bar and the informal representations which we can relate to fractions.*
>
> [Reflective research journal, 2021-07-12]

This realisation informed my decision to support the teachers with the musical aspects whereby I myself, as someone with a musical background, would have to teach the lesson where the informal representations were aligned to the formal representations. With the teachers' and their principal's go-ahead, I therefore arranged for this to happen as Lesson 5. (Such a lesson could be taught by either a class teacher who felt confident to do so, or by a music teacher at the school, and it could be considered an enrichment/supplementary lesson in the task design sequence. This I discuss in more detail in Section 4.7.)

In the Grappling-CoP, my supervisors and I therefore made two decisions regarding the task design for Lesson 3. Firstly, we decided the task design should focus on percussion, especially body percussion, so as to eliminate any anxiety that may go with playing a musical instrument, and also because using body percussion is a curriculum aim. Secondly, we decided it would be premature to introduce teachers and learners to Western musical notation of note values. Instead, an alternative, informal form of musical notation needed to be offered.

Lesson 3, therefore, started with introducing learners to the ideas of body percussion, through the use of YouTube videos (see Lesson 3 plan in Appendix 8). The first video was a demonstration of a clapping composition by Steve Reich, which teachers can use to stimulate discussion around body percussion also being part of music. The second was a body percussion 'karaoke' of the popular song *Uptown Funk* by pop-artist Bruno Mars. I selected this video as it is familiar to children, and uses images to informally notate which body percussion to do (for example, clapping your hands, stomping your feet, clicking your fingers etc.). I felt this provided an ideal introduction to informal representation of musical bars and body percussion. I also hoped that it might ease any anxiety that teachers might have had regarding the musical aspects of the lessons.

Our decision, in the Grappling-CoP, to use an adapted musical representation, as shown in Figure 4.5 overleaf was, to some extent, guided by the RME principle of using informal representations for learners to reinvent a mathematical concept before guiding them towards the formal, abstract models and representations (Freudenthal, 1991; van den Heuvel-Panhuizen, 2003; Cobb et al., 2008). I thought that this would be useful in the musical notation as well ('realistic *musical* education'). It would also allow for the musical beats per bar, as mentioned in Lesson 2, that include beats beyond one (whole note), two (half note), four (quarter note) or eight (eighth note) beats per bar (namely, three, five, six, seven etc. beats per bar).

In the Design-Theorising Plane, my supervisors and I grappled with finding the best way to represent the musical beats. As is discussed in more detail in Section 5.2, we eventually settled on the use of Xs to represent a clap (which is very similar to the formal notation for non-pitched percussion instruments), and a square to represent a musical rest (when the beat is counted but there is no clap). In addition to the music beat symbols being simplified, I decided to use a more minimalist notational approach. Rather than the five-line Western music staff, I opted for the simplicity of a percussion line (a single staff line, which closely resembles a mathematical number line) (see Figure 4.5). We considered the musical percussion bar to be the mathematical unit 1, with each bar being divided into the required number of beats (or counts), which would later be representative of the denominator of a fraction, rather than the symbolic representation of note values.

I intended for this adapted, informal representation to link to the animal river-crossing jumps, and to allow for teachers and learners to engage in fractional and musical reasoning without being distracted by having to learn the musical note value notation. This, as Gaare (1997) explains, can be a mentally taxing process. Figure 4.5 shows a single percussion line with six beats in the bar, of which five beats are clapped and one beat is a rest ('*shh*').

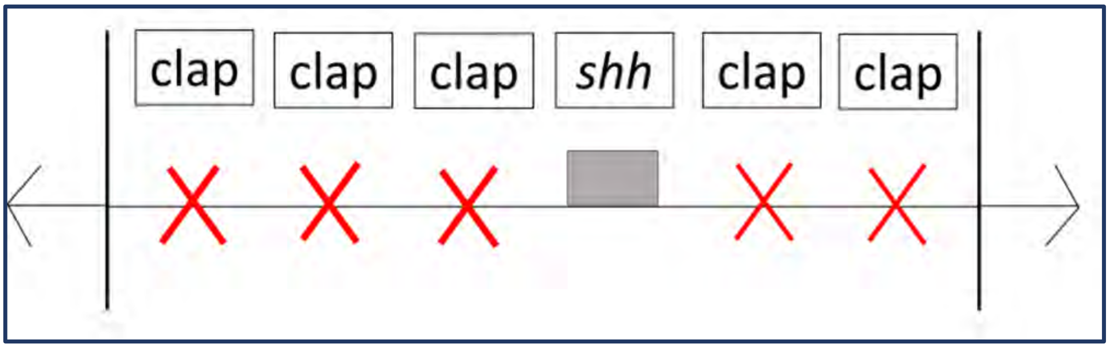


Figure 4.5: Adapted, informal representation of musical beats

I designed a PowerPoint Presentation (see Appendix 8) which teachers could use to demonstrate to their learners the move from the river-crossing unit (constant distance and time) to the percussion music bar. The following Figure 4.6 shows how I visually represented this relationship.

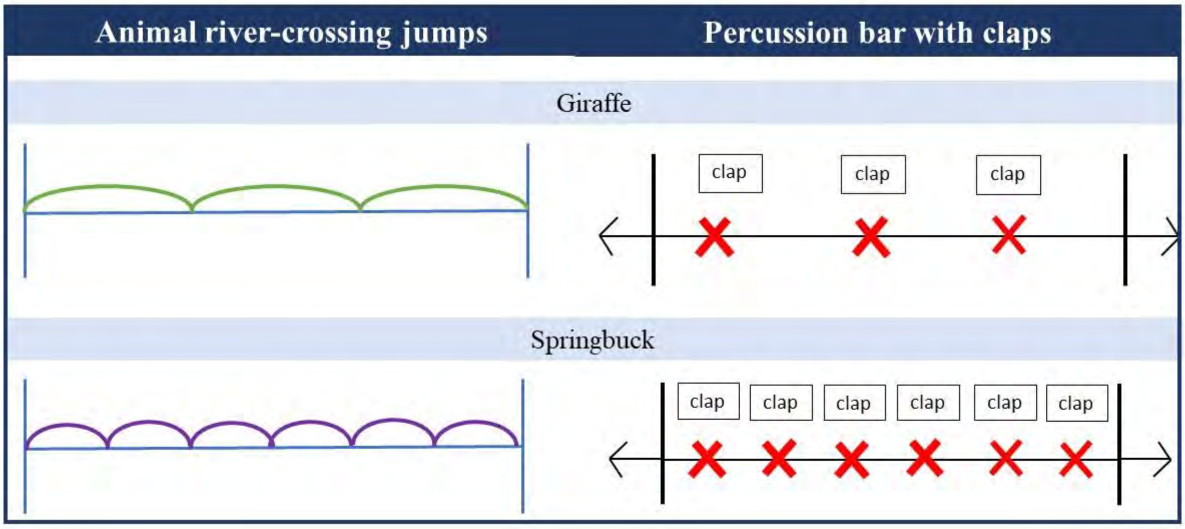


Figure 4.6: Visual representation linking the animal river-crossing jumps and the percussion bar

Protea School's Ms Savuka commented on how her Grade 4 learners made the connection between the animal river-crossing jumps and the informal percussion notation:

> *Yes, that was one thing that really surprised me that they could make that correlation very easily and they understood that the number of jumps that they jump over the river could be counted as beats. So they needed it for the clapping.*
>
> [Protea-CoP, 2022-02-10]

Lesson 3 progressed to a task that required learners to create their own percussion 'song' which they could represent with Xs and squares, to show their chosen number of claps and rests per bar. Based on teachers' responses from both schools, this activity worked well. Some teachers allowed their learners to use a variety of informal symbols (such as in the introductory percussion YouTube karaoke video based on Bruno Mars' popular song), to show the different types of body percussion (claps, taps, clicks etc.). Figure 4.7 below shows some of the learners' 'percussion compositions'.

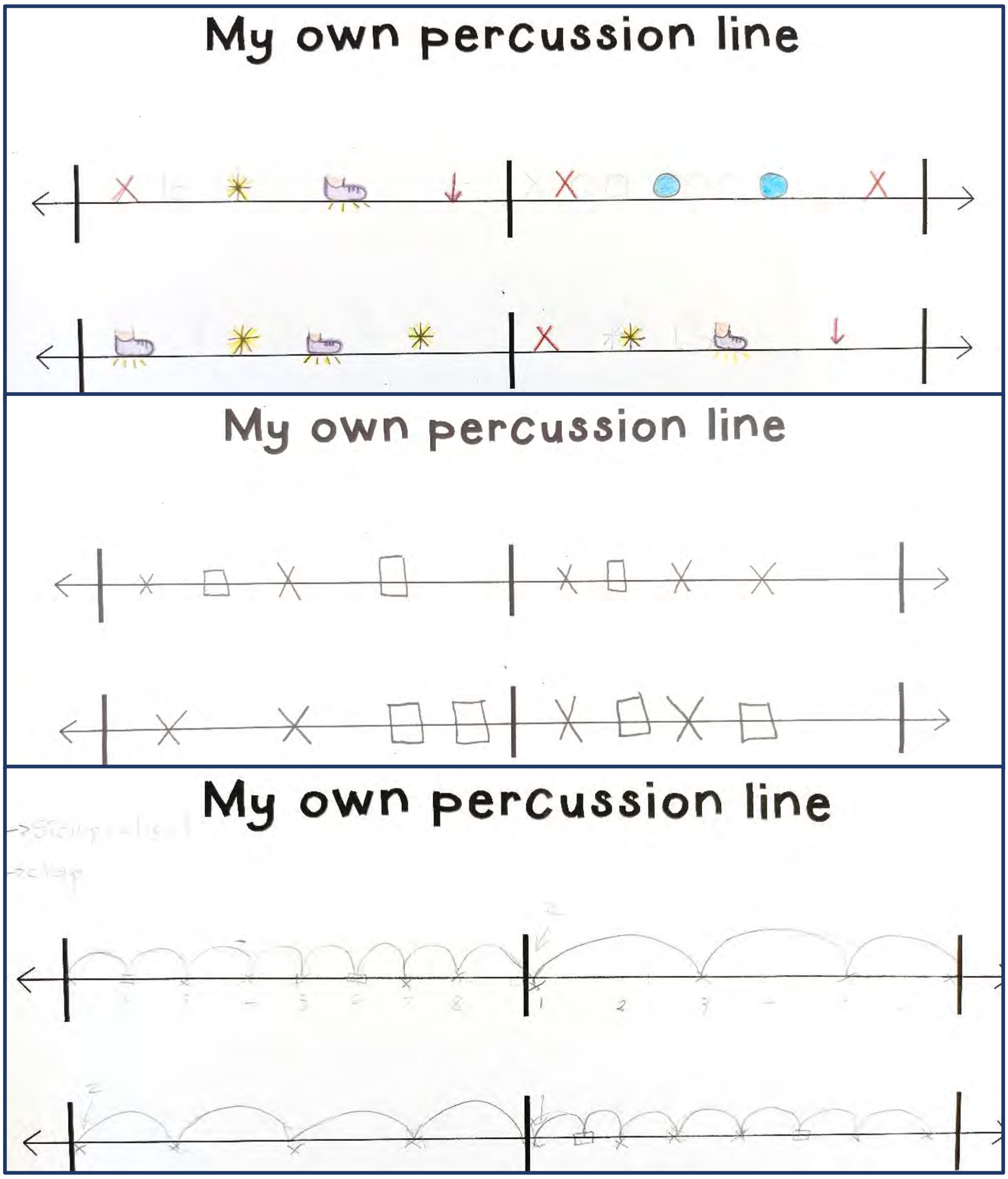


Figure 4.7: Some examples of learners' percussion compositions

The Aloe School teachers shared with me their reflections on the activity and of how their learners informally represented the percussion sounds.

> *Ms Makeba: I found with the Grade 4's that they overcomplicated their own percussion line, they didn't just use the X and the square. They were drawing sticks and stones.*
>
> *Ms Ibrahim: Mine* [Grade 5 learners] *were fine, they loved it.*

Mr Mandoza: For this activity, I put mine into groups. And then as a class, we decided how many sounds we want to put between here and here, and we agreed on four.

Tarryn: Okay, four beats per bar.

Ms Fassie: Mine also mostly chose four. It's far easier.

Mr Mandoza: And then they made their own songs and they had to pass it around and play the others'.

Tarryn: Yes, it is easier to use the 4/4 time-signature.

Mr Mandoza: And for the first one, I said they have to use the Xs and the shh squares, but then with the second one, I said they could bring in their own sounds and use different symbols. Like some of them use a circle for a chest bump. So they made different songs.

Ms Ibrahim: It was just nice to see that okay, we're staying with four beats, so it was a constant rhythm, but with a nice sound.

[Aloe-CoP, 2021-11-05]

The teachers at Protea School had similarly positive feedback on the percussion line activity, and discussed the differences they had noticed between Ms Savuka's Grade 4 class and Ms Clegg's Grade 5 class.

Ms Savuka: But what I also noticed is that some of them, they linked it when we did the jumping across the river, keeping their spaces the same. So in the beginning when they jumped they weren't spacing evenly, especially in the drawings as well. Some of them did struggle. But after we'd pointed it out, they tried to make the jumps equal. They tried to do that with the percussion line as well. So they did a lot of different variations. Some did the symbols that they saw in the example PowerPoint, the X and the rest. And some were very creative. They used little images. And some of them wanted to write above what each one represented.

Ms Clegg: Now, funnily enough, you could definitely tell the difference between the two age groups. So mine, you will see very set crosses and squares. And they linked it immediately to what they were doing in music. So almost straight away they said, "This is like the bar line that we learned in music, and are these like the notes?" So almost immediately they recognised it. So we also did the body percussion line.

Ms Savuka: It was nice cos the clapping video also showed what a rest was. That it counts as a beat but there is no sound.

Ms Clegg: They also linked it back to the jumping on the stones. I didn't redraw the jumping on the stones. We went through the PowerPoint and we discussed each slide. But I didn't actually draw… But because they had clicked so well… And interestingly enough in music at the moment, they are busy making recyclable percussion instruments, and they have to play different beats. That is without us talking to the music teacher. It just happened to

> *coincide… So what I did with my group was they wrote their own, then they swapped with the friend next to them, and they had to do each other's, and then they could perform it.*
>
> [Protea-CoP, 2022-02-10]

It was interesting to see that Ms Clegg had recognised a link to what her Grade 5 learners were doing in their class music lessons with the music teacher. This gave me more confidence that the problem scenario and task design could benefit both the mathematics and the music, exactly as was recommended by Bresler (1995) in her co-equal cognitive style of arts integration. I was satisfied that this step in the teaching and learning sequence was appropriate to guide learners towards the more formal mathematical and musical reasoning.

Lesson 3 concluded with some questions which the teachers could pose to their learners:

- → How many beats did you have per bar?
- → How many claps would you need to fill a whole bar?
- → How many claps would your friend need to fill a whole bar on their percussion line? (Implicit link to equivalent fractions).
- → How many beats would you clap to fill two whole bars on the percussion line?

Lesson 4

My key aim for Lesson 4 was to introduce learners to working with fractions greater than one whole (i.e., improper and mixed fractions). It was challenging to find a realistic scenario in which animals would continue jumping in order to cross the river. My supervisors and I discussed in our Design-Theorising Plane different ways in which we might introduce the fraction greater than one whole. We decided to expand on the animal river-crossing scenario, by asking learners whether the animals might continue jumping after crossing the river, and to discuss some possible reasons why this might happen. This was to continue with the RME principle of using the experientially-real problem scenario from which learners could informally reason to support progressive formal mathematisation. We also decided to keep the river-crossing as the unit, which animals could repeat after one whole river-crossing. In Figure 4.8 below I share an image from our grappling over this during one of our Zoom meetings, followed by an excerpt from our discussion about deciding to use the river-crossing as a constant unit in relation to the problem scenario.

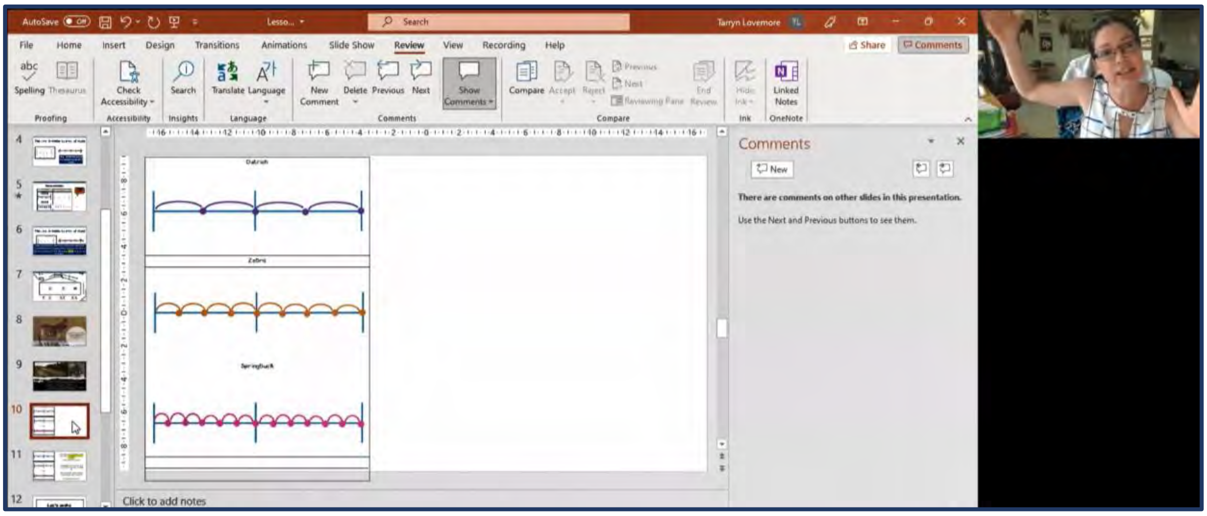


Figure 4.8: Example of grappling over the Zoom platform [Grappling-CoP, 2021-10-20]

Some key decisions which we made are captured in an excerpt of the Zoom meeting below:

> *Tarryn:* *And then I gave them a new problem scenario, so I wanted to say this, this equal distance and equal time so it's one river-crossing, now a second river-crossing.*
>
> *Mellony:* *I would just keep the one river-crossing as your unit of measurement. So don't complicate it, if there's any more river-crossings. There was one river-crossing which we have now taken as our standard unit of measurement with all these animals. Because every single jump they did we considered in relation to that river-crossing. So we're not going to bring in a different river-crossing. But everything, every run, every distance, every time that they do, we are going to consider in relation to this lovely unit of measurement which is: time of river-crossing distance. Just like in the modern world, we have time as a standard minute or we have a measure as a standard metre. The standard here for these animals is that river-crossing that's the standard of measurement because we didn't have these other forms of measurement with these animals. It was the river-crossing. If you've got one unit, how many river-crossings? He's done two and a half. We can now once we think about it like that, we can put a naught and a one because it's one river-crossing.*
>
> [Grappling-CoP, Zoom meeting, 2021-10-20]

I kept in mind van den Heuvel-Panhuizen's (2003) explanation that the scenario need not be real-life, but – more importantly – *real* in the learners' experience. I therefore found the problem scenario suitable to assist with the development of the fractional and musical concepts. The task then required learners to represent such jumps of different animals, iterated past one whole, as shown in Figure 4.9, below. This was the introduction to the use of a river-crossing unit.

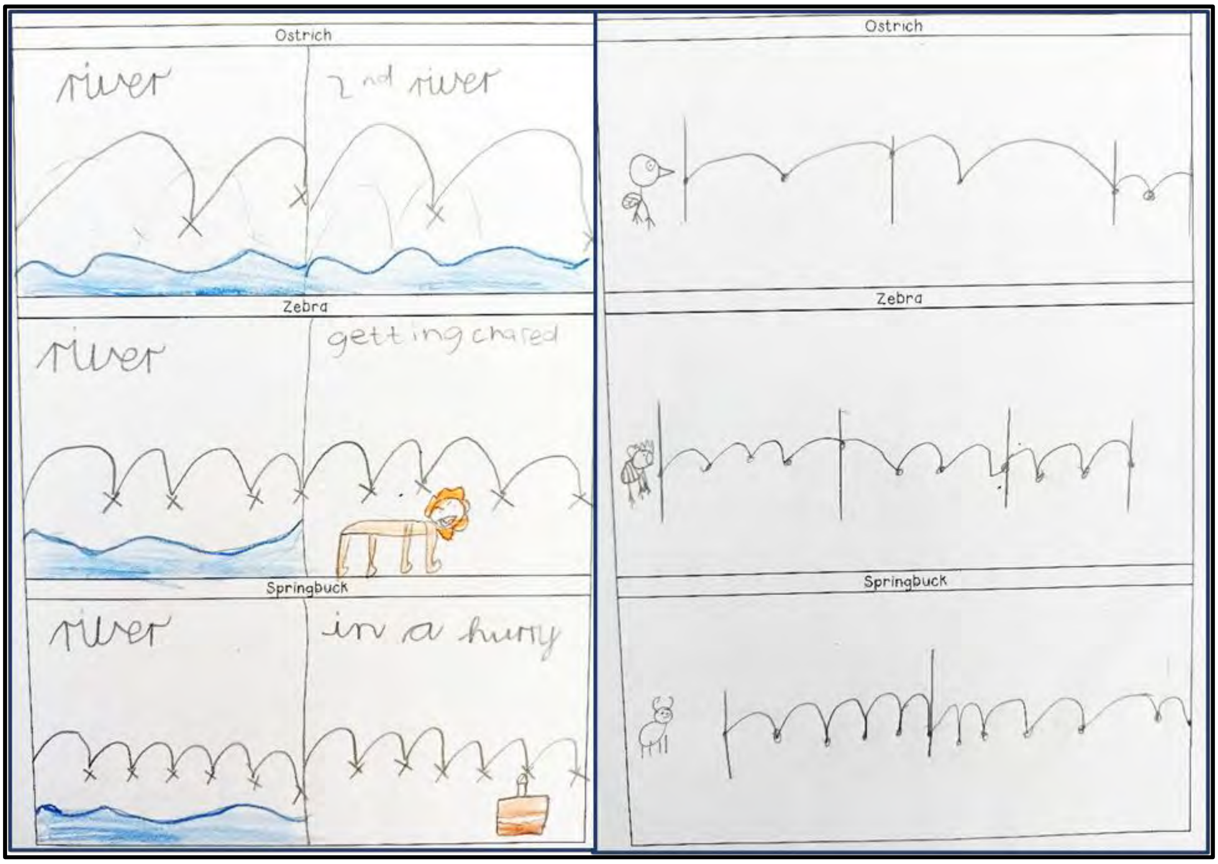


Figure 4.9: Learners' examples of continuing river-crossing greater than one whole river-crossing

A key idea, though, was to link the river-crossing which continued beyond one whole, to music bars which continue beyond one whole bar (i.e. for a whole song or composition). I, furthermore, designed this task to give learners opportunities to recognise *equivalence* of fractions greater than one whole in the river-crossing and the musical clapping contexts.

I designed an 'animal percussion orchestra' activity in which a class of learners could be divided into two groups, where one group would clap the ostrich two beats per bar and the other group would clap the zebra four beats per bar. Sometimes the claps would be at the same time, just as sometimes the animals would land on the same spot in the river-crossing. I intentionally kept the link between the jumps and claps linked to the same animals, to help learners recognise and remember the beats per bar. Initially, the percussion activity was rather challenging for some of the teachers, which is why I decided to create a 'How To' guide[11] which teachers then used to better understand the activity, and to even show to their learners. This was a worthwhile activity relating to equivalence and the reflections from the teachers. Below is an excerpt from the first iteration where the Aloe-CoP teachers reflect on their

[11] The video clip is accessible on YouTube with the following link: https://youtu.be/11Yc5uvaY4E

experiences in trialling the animal jumping and animal percussion activities to show equivalence.

Ms Makeba: I put the river crosses on the board for them, and when I posed the problem questions you gave us, it was so nice because the children could come indicate on the whiteboard exactly where the one animal had landed… One of the children's responses was, "Oh the zebra landed on the ostrich's last jump, and the zebra and the ostrich had both landed on the same spot." And when I asked them where did they think the springbok would have landed, they were able to do that with ease. The pupils were able to represent the jumps with clapping with ease. It was also quite nice that they were keeping the rhythm and the beat with one another. So the animal percussion activity was also successful… And then the other one was that the ostrich got stuck at one and a half river-crossings, "okay show where he would be," and then we had a look at, "okay where had they landed up?"

Tarryn: The important thing here is to realise that in the time of my two claps, for example, you play one clap. In the time that you do six claps, I do three claps. So it's the time duration that it takes.

Ms Fassie: They found the two and the four easy, then I made three groups. So it was 2, 4, 6, which they couldn't do.

Tarryn: So the music claps could have been 2, 4 and 8. That would be easier.

Ms Fassie: I said they could use the 8 then, yes.

[Aloe-CoP, 2021-11-05]

Ms Makeba's strategy to project the jumping representations on the whiteboard for the learners to solve the equivalence problems can be seen in Figure 4.10 below.

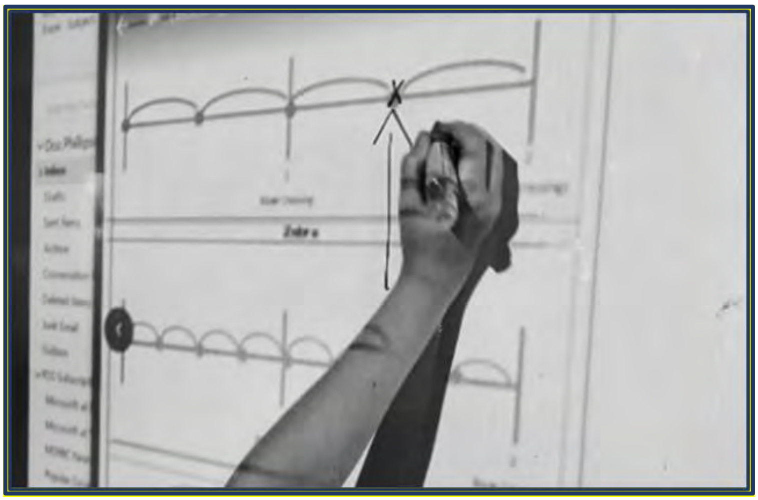


Figure 4.10: Example of learners using the informal jumping notation to solve the problems of equivalence

Outcomes from the second iteration of Lesson 4 (Protea School) were similar. Learners were able to recognise equivalence in both the jumping and the clapping. Ms Savuka also referred to using informal representations on the board to help her Grade 4 learners. Ms Clegg similarly noticed the use of mathematical terms such as ‘doubling and halving’.

> *Ms Savuka:* *We also spent a lot of time in Lesson 4 discussing and drawing on the board, and them coming up and doing their jumps. And then when they started looking at how, when I ask them, “Can the animal continue jumping after the river end?”, again I had a small group saying, “No.” They said, “Once the animal jumps across the river, they will start walking normally. And they won’t jump anymore.”*
>
> *Tarryn:* *That’s an interesting point.*
>
> *Ms Savuka:* *And then some said, “But wait, what if the animal’s being chased?” But they understood that as the animal jumping, there could be another animal that’s landing the same time as them, even though they are taking more or less jumps. And then it came out again through the clapping exercises how that is possible.*
>
> *Tarryn:* *Sometimes you clap together, where my one clap is two of your claps.*
>
> *Ms Savuka:* *Yes, so they made that link as well.*
>
> *Tarryn:* *So that’s kind of leading us to equivalence.*
>
> *Ms Clegg:* *But then as soon as they realised there’s a pattern to it, the animals jumping double the amount of the others. So that language came up. I was saying, “Okay, we can just double if this animal…” I can’t even remember the animal… “If the animal’s jumping twice, then it would double that, we will find out what the other animal is without physically drawing it.” But ja, they came up and drew it.*
>
> *Ms Savuka:* *And they also using the half… I think it was, “If the ostrich jumps halfway, how long will it take this animal to catch up?”*
>
> [Protea-CoP, 2022-03-02]

The animal percussion orchestra proved to be a positive, fun way to conclude the lesson. It succeeded in encouraging learners to create their own rhythms with beats per bar that could be overlayed to show the equivalence. This activity also met the musical curriculum aim of “using percussive instruments from found objects and/or body percussion” (South Africa. DBE, 2011b, p. 13). Ms Chaka shared with me her recording of her Grade 6 class ‘composing’ an interesting percussive piece with hand clapping, foot stomping, and making percussive sounds from sticks and stones beating a rock.

Lesson 5

After reflecting and grappling within the Design-Theorising Plane, I planned Lesson 5 as a supplementary and enrichment lesson focusing primarily on musical notation. (See Lesson 5 plan in Appendix 8.) The goal for this lesson is to link the animal jumps and informal percussion notation with more formal notation symbols used in traditional Western music. My teaching this lesson arose out of the Aloe teachers' concerns about their ability to convey the musical concepts accurately. In our Grappling-CoP, my supervisors and I agreed that, in the circumstances, I should teach Lesson 5 at Aloe School. The lesson could be taught by a music teacher at a school, so allowing for teachers-based integration and collaboration, a strategy identified in literature as a way of maintaining quality in the individual subjects across the integrated lessons (see, for example, Bernstein, 1971; Adler et al., 2000; McPhail, 2018; Pluim, 2020). If, however, class mathematics teachers felt confident to use the resources they could themselves teach this lesson in the sequence of the task design. It is also possible to skip over this lesson, should a school not have the resources to teach it, especially given that the link between the informal and formal musical representations is picked up in the following lesson (6).

To support the Aloe teachers in trialling the novel integrated lessons without putting additional expectations on them, I set up Lesson 5 as a demonstration lesson. It was too late in the academic school year, unfortunately, to collaborate with the school's music teacher. I taught Lesson 5 three times to groups of Grade 4, 5 and 6 learners respectively. I did it in their afternoon extra-curricular sessions so as not to interfere with their daily schedule. The Aloe teachers expressed their appreciation for my supporting them in this way with this lesson, Ms Makeba, for example, saying, "*I would never have been able to teach that lesson*" [Aloe-CoP, 2022-12-03].

The lesson started with a discussion around the animal river-crossing jumps. The next activity was a 'spot the difference' activity, where I asked the learners to identify what they noticed between the informal percussion notation with Xs and squares (*shh*), and the formal musical percussion and Western Staff notation. Below is an example of the resource used for this lesson.

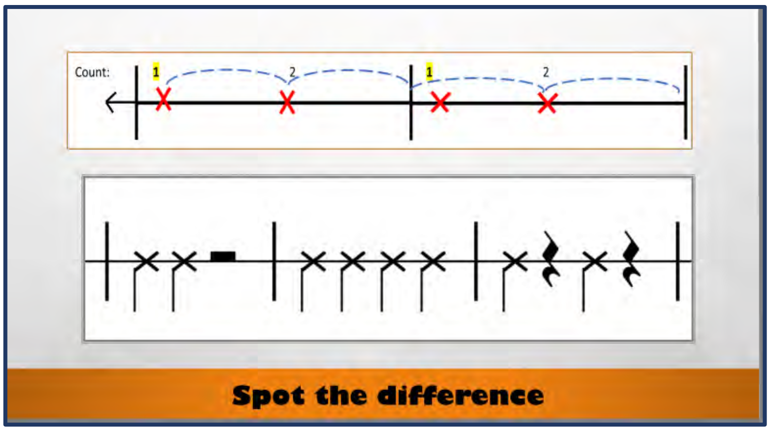


Figure 4.11: An example of the resources used to introduce formal Western Staff notation in Lesson 5

I designed the next part of the lesson to give learners an opportunity to sing, and use pitched instruments such as a Xylophone. The music note values I chose were simple, only using whole notes, half notes or quarter notes. I also only introduced notes of three pitches (Middle C, D and E), which, played in sequence, sound like the start of the well-known children's song *Three Blind Mice*. Reflecting on my own trialling of Lesson 5 in my reflective research journal, I wrote:

> *The music lessons went well… The main idea of linking the representations worked. They were able to recognise the differences, and to clap the rhythms and to play them on the instruments, quite quickly. It went quicker with the Grade 6s… Overall, I feel positive about the music lessons, and I think that there is definitely opportunity for the maths teacher and the music teacher to draw links across the contexts of jumping and clapping and representing the Xs and notes and linking it to fractions.*
>
> [Reflective Research Journal, 2021-11-16]

In the second iteration the two participating Protea teachers felt confident to teach Lesson 5. Both had some form of prior musical knowledge. Some of their reflections on their trialling of the music-based lesson are shared below:

Tarryn: *So, that* [Lesson 5] *was intended to be for a music teacher to do, but you were willing to do it because you've got the musical knowledge.*

Ms Clegg: *So that one went pretty well actually, because the children, my Grade 5s have just started to touch on it in music. So when they saw them they did recognise… They couldn't remember the names but they recognised what they were. And we kind of looked at the* [staff] *and when we spoke about the bar and spoke about the times notes in the front, one of my kids piped up that it was a fraction, and we were like, "Well, that's exactly what it is. It's three claps out of four or whatever." And so there was actually some discussion round that. Then we looked at 'Three Blind Mice.' I actually just like hummed it… And then we spoke about, "Okay, well when we clap, something unpitched." And then we brought up melody versus just rhythm and a steady beat. So we brought in some musical language. And then we just followed it along with your PowerPoint… So I'd say that's how we kind of linked it in that, and then they remembered the clapping from the previous sessions. So they quite enjoyed doing that. And then we just took that across when we did Lesson 6.*

Ms Savuka: *So, for Lesson 5, it worked very well with my Grade 4s. They were able to see the correlation between the percussion line and then now placing notes on the bar of music. What I did, is I, because you did ask now about linking the jumping of animals. I showed them when we did this, "If we look at putting the animals jumps on… If it was a percussion line and if we had to clap the jumps and putting that, and clapping the notes, how long do notes last? What animals would you link it to? So, if we looked at the semibreve* [whole note]*, that you hold it for four counts and that one clap is one jump, 'What animal took one jump to cross the thing?" And then when we did the* *minims* [half note] *and if we had to clap a minim, it's count to two, and the clap one, two, and they hold the note. "What animal would jump in the…?"*

Tarryn: *Yes, in our story. And they could link the animals?*

Ms Savuka: *They could link the animals. And also when they saw, a few, not everybody, but when they saw this they also linked to fractions. They saw half and they saw eighth. And they linked the two to fractions.*

[Protea-CoP, 2021-04-12]

It was encouraging to see that the music-based lesson could be beneficial, either taught by a music teacher or by the class teachers if they felt confident to do so. Based on the demonstration lesson I taught at Aloe School and the Protea teachers' reflections, there is potential to use the problem scenario to link animal river-crossing jumps to note values and to fractions. Depending on the context, it may be a collaborative integration or an individual teacher's integration of the musical notation and the mathematical representations. The feedback I got from the teachers also guided me in my design of Lesson 6.

Lesson 6

In the first iteration at Aloe School, the task design ended with Lesson 6. The lesson focused on explicit fraction problem-solving questions. The feedback was mainly around learners' use of a fraction wall linking to the fraction as measure concept, rather than modelling fractions with a 'pizza' diagram where the focus is on the part-whole construct. There was, however, little discussion around the fraction as ratio construct. Some of the Aloe School teachers' reflections on the final lesson are captured below.

Ms Fassie: *They were able to answer the* [fraction] *questions on the number line.*

Mr Mandoza: *Mine* [Grade 6 learners] *could do the maths questions, but when I asked them to explain their answers they struggled. So most of them drew pictures.*

Tarryn: *And what did they draw? What was their modelling?*

Ms Fassie: *Like long bars.*

Mr Mandoza: *Like a fraction wall.*

Ms Fassie: *Fraction wall, hmm.. because I don't think you can compare two pizzas, whereas a fraction wall, you put them next to each other.*

Ms Cloud: *And you can put another one next to.*

[Aloe-CoP, 2021-12-03]

Ahead of the second iteration of the task design at Protea School my supervisors and I worked in the Design-Theorising Plane, reflecting on the feedback, including the challenges we encountered in the first iteration. We grappled with the best ways to represent the integration of the musical staff and the number line. Details of this grappling process are discussed in Chapter 5. We arrived at a representation in which, rather than super-imposing the musical staff and the number line, the two representations were aligned in a way that showed the similarities and the differences. This lesson could, in future iterations, be taught straight after Lesson 4, if a school were not to have a music teacher available to teach Lesson 5.

I designed the lesson (see Lesson 6 plan in Appendix 8) to start with a reminder of the experientially-real animal river-crossing problem scenario, including visuals represented on the whiteboard or a poster format of the animal jumps. Figure 4.12, below, shows the first activity. It involved learners using the key resource I developed, with a music staff line and a number line, to 'compose' their own animal river-crossing song first using a combination of their choice of different animal jumps on a number line representing river-crossing units, where the river-crossing unit will have a constant time and distance.

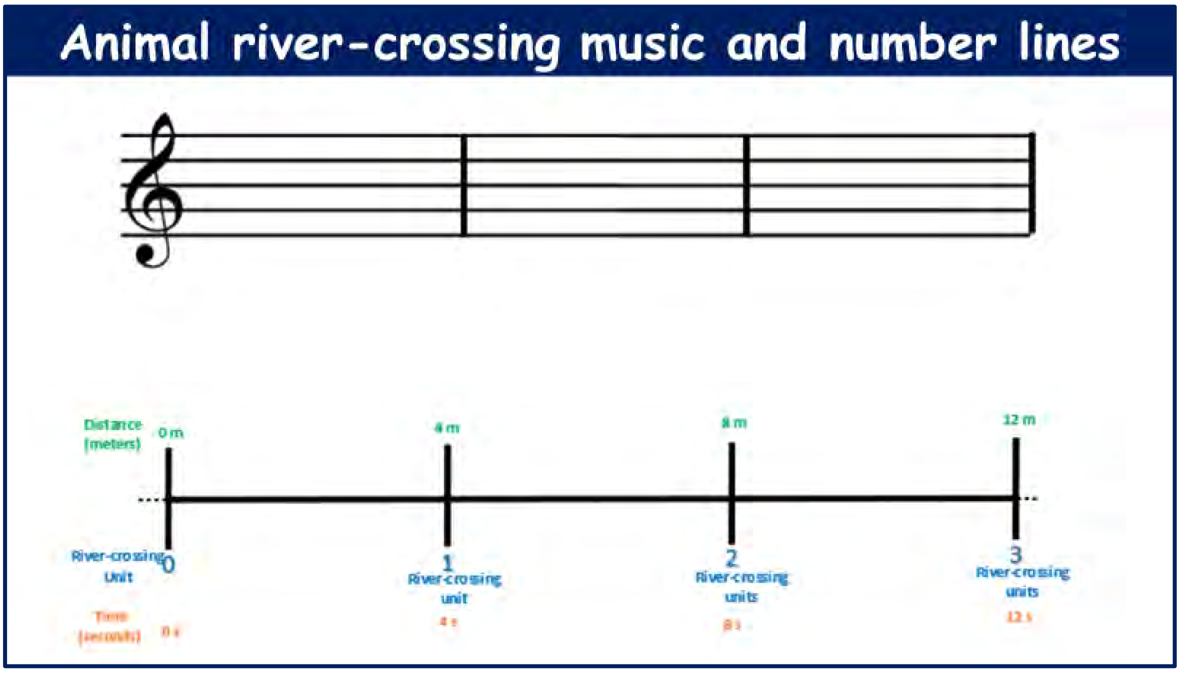


Figure 4.12: Resource aligning the animal river-crossing number line and a music staff to be printed on A3 and laminated

Learners then matched the animal jumps to note value cards. These were printed on transparencies, so as to show the music staff, in such a way that they would fit exactly into a music bar. I made the music bar the same visual distance as the river-crossing unit. Placing carefully measured cards over the music staff and number line also links to the fraction as measure construct. The cards model measures that are iterated to fill a music bar and show the relationships between the different note values. The advantage of introducing the note value symbols in this way was that teachers and learners did not need to have a background knowledge in musical notation, nor did they need to commit the notation to memory. I had eliminated the cognitively taxing aspect of reading music that Gaare (1997) refers to. I wanted this activity to be an experientially-real opportunity for learners to work with formal notation of music note values and beats per bar. While acknowledging that the scenario of different animals jumping and stopping at certain points in the river, was not real-world, the learners enjoyed the opportunity to participate in this imaginary scenario. The teachers reported that the learners came up with possible imaginary stories that could account for the various animal jumps. This coheres with van den Heuvel-Panhuizen's point (2003) that the scenario need not be real-life, but experientially real for the learners. Below is an example of this matching activity (Figure 4.13). It is the rough version of the one I shared with my supervisors in our Grappling-CoP Zoom meeting.

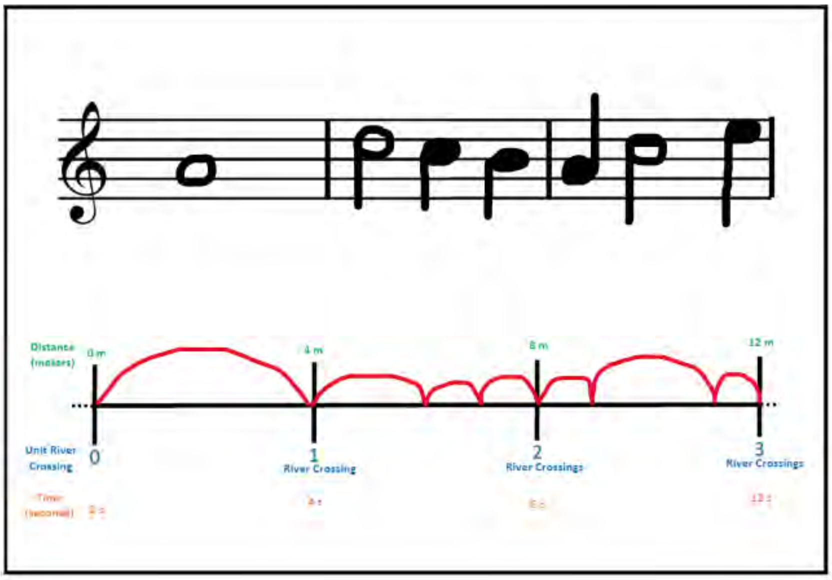


Figure 4.13: An example of the use of the resource aligning the music staff and number line, as trialled over our Zoom meeting [Grappling-CoP, 2022-02-01]

I also recorded a demonstration video for teachers[12]. As part of my task design decisions, I highlighted, in the demonstration video, some key points for teachers to notice in this example. I explained to the teachers that in this scenario, the river-crossing is the unit, which is linked to the music bar unit. For the sake of keeping the musical concepts easily accessible and manageable for teachers with non-musical backgrounds to implement in the classroom, I chose only to use music in a 4/4 time-signature. This means that the examples only work with four

[12] This can be seen on YouTube at the following link:
https://youtu.be/ftI9zG_C3XA?list=PLsv7tdlUM0iPE0pbCJIPdDU7RVCr9wxFZ

beats per bar (four quarter notes per bar to make up one whole note). I therefore also only used whole notes, half notes, quarter notes and eighth notes in this example. I chose to use the animal jumps which would link to these note values (i.e., the kudu, ostrich, zebra, and monkey jumps in this scenario). (It is possible to do a similar activity with other time-signatures and rhythms, including the use of dotted note values – the notation of a dotted note implies adding half the value of the note).

In the Design-Theorising Plane Zoom meeting that preceded this Lesson, Mellony summarised the 'Aha' moment from our grapplings around my design of this animal jumping river-crossing song activity resource:

> *I think we had a breakthrough with this. You've got the double, triple number line, you've got the proportions, you've got the fact that units can be different and then you've got the link to the music above.*
>
> [Grappling-CoP, Zoom meeting, 2022-02-01]

I suggested to the Protea teachers that they might follow on from the animal jumping river-crossing song activity with some problem-solving questions requiring learners to use the key resource to practise their fractional reasoning and so strengthen their conceptual understanding of fractions. Some of the questions can be seen in the excerpt below.

→ How long does it take to play your song?
(should be 12 seconds = 3 bars x 4 seconds each).
→ If one river-crossing took 8 seconds, how long would it take to play your song?
(3 bars x 8 seconds = 24 seconds).
→ If one river-crossing took 2 seconds, how long would it take to play your song?
(3 bars x 2 seconds = 6 seconds).

I also designed the following open-ended worksheet as a way of concluding Lesson 6 (Figure 4.14, on the following page). It required learners to think creatively and practise fractional reasoning. I designed the worksheet so as to expose learners to the note value without either they or their teachers having to memorise the notation but rather, simply to recognise the relationships between the note values and their links to fractions.

Worksheet 6: Problem solving

Did you Know?!

Note value symbol	British name	American name	Counts
𝅝	Semibreve	Whole note	4
𝅗𝅥	Minim	Half note	2
𝅘𝅥	Crotchet	Quarter note	1
𝅘𝅥𝅮	Quaver	Eighth note	$\frac{1}{2}$

1. Why do you think the Americans named the notes the way they did?

2. IMAGINE that musicians in the land of Wakanda called the **crotchet** a '**whole note**'. What would they call a…

   quaver?

   minim?

   semibreve?

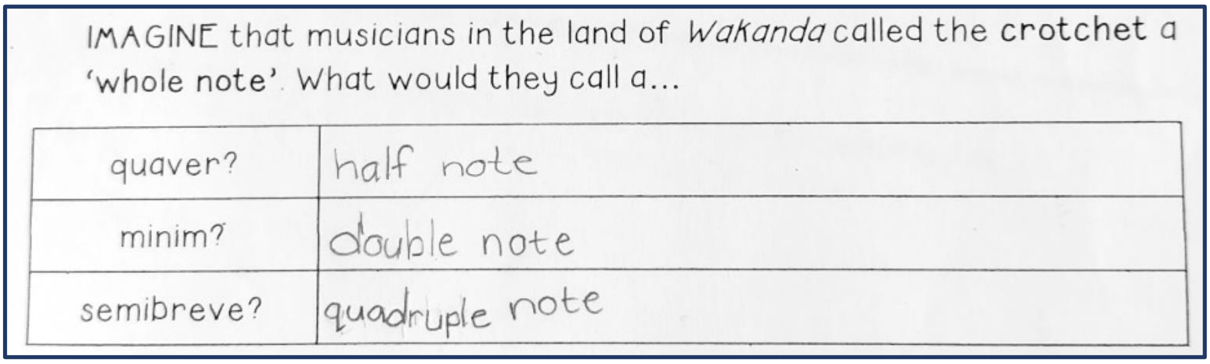
3. What do you notice about music and mathematics?

Figure 4.14: Worksheet 6 used for Lesson 6 to expose learners to musical note value symbols and naming conventions

I close this sub-section with some feedback on the worksheet task. The feedback comes both from the learners and their teachers. It starts with one of Ms Clegg's learners' answers to the first question about why the American naming convention refers to note values in terms of whole note, half note etc.:

> *Because a semibreve is 4 counts and a bar of music is 4 counts so it's a whole note a minim is 2 counts and half of 4 counts is two counts so a Minim is a half note.*

For the second question I designed an imaginary scenario – as van den Heuvel-Panhuizen (2003) suggested is also useful in RME-based activities – that requires learners to imagine they are in a different land '*Wakanda'* where the music note values are named differently. In this scenario the quarter note now became the whole, which allowed for fractional reasoning where the unit changes and required learners to practise proportional reasoning. I chose to include this activity to show learners that the unit of a fraction depends on the context in which it can be used and that the unit changes based on the context (Lamon, 1999; Siemon et al., 2015). Below is an example of one learner's response (Figure 4.15).


IMAGINE that musicians in the land of *Wakanda* called the crotchet a 'whole note'. What would they call a...

quaver?	half note
minim?	double note
semibreve?	quadruple note

Figure 4.15: Example of a learner's new names for note values in '*Wakanda'*

Learners' responses to the open-ended question asking them what they noticed about music and mathematics included:

- → Because music notes have fractions.
- → Half is in Math and half note is in music so Math is in music.
- → They are similer (sic).
- → You can't use one without using the other.
- → They are kinda the same music and mathematics use numbers to the beats.
- → They use the number line in the music bar.

The Protea teachers' reflections on this activity showed much enthusiasm. They explained how the learners used the carefully measured note value transparency cards to link the animal river-crossing jumps to the note values. Ms Clegg, in her Grade 5 class, however, explained that her stronger learners could recognise the link without comparing the size of the concrete transparencies. This shows that the music-mathematics integrated tasks have the potential for differentiation in the mathematics class.

Ms Savuka: *So then I showed them in a bar of music, if each animal had to jump individually, what would it look like? And they were able to recognise that. And then when we brought in different animals jumping together, "If one animal had to jump and stop halfway across the river, and another animal had to take over and jump, what would it look like?" So thinking back to that image helped them with completing this part of the activity. And then they were able to match the lengths of the jumps to the notes.*

Tarryn: *Did they use the visuals of the transparency to match how it fitted?*

Ms Clegg: *There I would say that my top group didn't really think about the size of the transparency. They just kind of had them laid over and they were just picking them up. Because by then they really had a good connection between the note values… Whereas my lower group, they were measuring. They were like, "No, this one doesn't fit in. The paper goes over the line. So we have to change it." Which was great. It was a bit of like self-regulation. Like they were checking their own work. So the size value of that transparency was a great support for my lower group, which essentially would mean that it worked really well…*

[Protea-CoP, 2022-04-12]

The teachers reported also on their learners having noticed the link between music and fractions, and that this prompted considerable discussion in class.

Ms Savuka: *…because they are essentially writing music… They are composing their own music and I think that has a little bit of satisfaction for them. You know, they were very chuffed when they looked and they were like, "This looks like real music."*

Ms Clegg: *Well, it is real music… Neil says they* [music and mathematics] *are very similar. He can't use one without the other.*

Tarryn: *The maths and the music? I like that.*

Ms Clegg: *I thought that was a very interesting link. He says, "Well obviously you can't play music without." So I said, "Well that is why music is so helpful in maths."*

[Protea-CoP, 2022-04-12]

I interpret these reflections as linking to the broad curriculum aims of recognising mathematics as a creative human activity. This leads me to believe that the task design does hold the potential to meet the broader curriculum aims, as well as the useful synergies between music and mathematics which hold powerful potential for curriculum integration opportunities.

In the following lessons which I designed, learners could progress to working more formally with the fraction as measure (measure of distance and time) and fraction as ratio (beats per bar and jumps per river-crossing) constructs, by aligning the musical staff and the number line. This resource, I believe, would thus allow for learners to move flexibly between the multiple constructs of fractions and across mathematics and music integration and within mathematics, as described by Bernstein (1971).

Lesson 7

As I indicated earlier, circumstances prevented my going beyond Lesson 6 at Aloe School, meaning that the seventh lesson which I now discuss was only trialled at Protea School (i.e. it formed part of only the second iteration of my participatory dual-design experiment in task design). Teachers' feedback from *both* schools contributed, however, to my supervisors' and my grappling in the Design-Theorising Plane around how best to design a lesson which would lead more explicitly to using fractional reasoning.

The introductory activity for Lesson 7 involved revising animal jumps per river-crossing and their related beats or claps per bar. I selected to use the zebra (four jumps per river-crossing/four claps per bar) and the ostrich (two jumps per river-crossing/two claps per bar), to keep with the doubling relationship, which would be easier to demonstrate through clapping. Other relationships would, however, also work, for example, three of the giraffe jumps to six of the springbok jumps. In the activity, learners were then given the opportunity to clap each of the jumps, as indicated with Xs on a percussion line (shown on a PowerPoint Presentation, but which could also have been printed out as a poster). I designed some questions for the teachers to pose to learners, based on the animal claps per bar and then linking this to indicating the fraction of the bar that would have been clapped in different scenarios (Figure 4.16, overleaf).

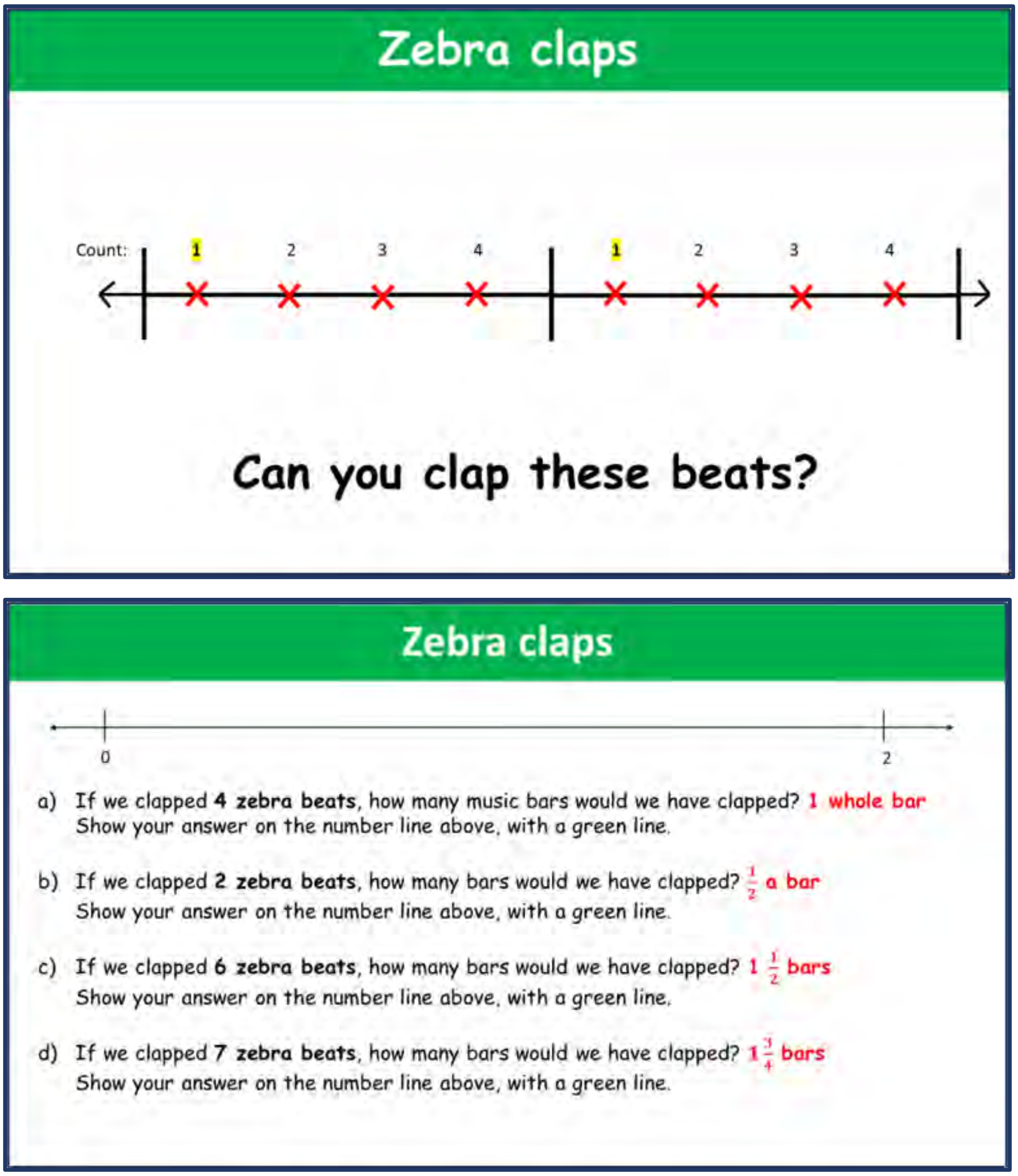


Figure 4.16: Example of linking animal claps and the number line

In our grapplings in the Design-Theorising Plane, my supervisors and I had recognised the potential of the previous Lesson 6 to build on the fraction as ratio and fraction as measure constructs with the key representations of a number line. In fact I had already designed a key representation of a triple number line (see Figure 4.17, below). Building on from Freudenthal's (1991) suggestion of using a double number line, I saw the use of three number lines placed parallel to one another as a useful representation linking the river-crossing unit to the measure of distance and time.

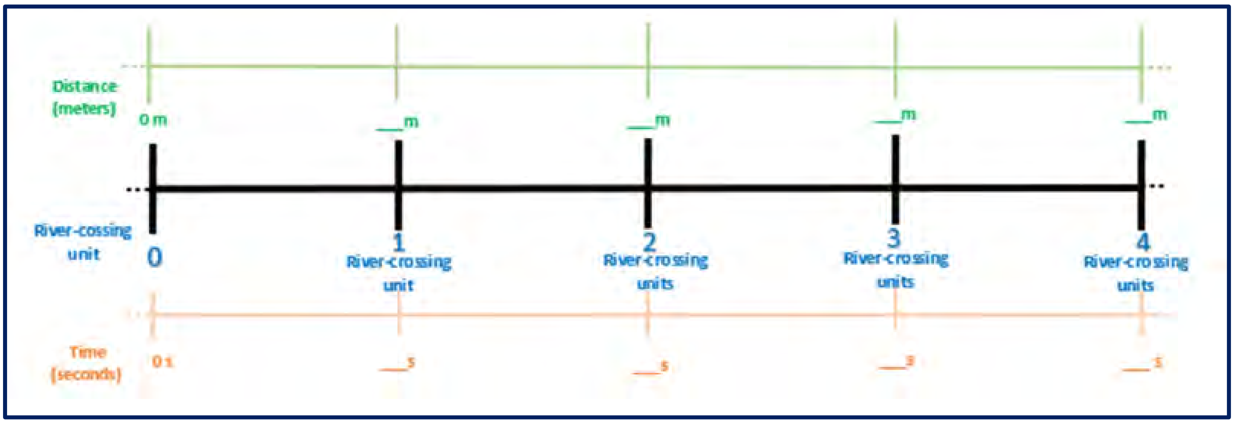


Figure 4.17: Key representation of triple number line for supporting learning in solving problems related to fraction as measure

For this scenario, I referred to a river-crossing as the unit (which I earlier related to the bar of music). Just as a bar of music can have a certain number of beats per bar, so too the river-crossing unit could have a certain number of jumps per bar. I used the number line above the river-crossing unit line to indicate measurement of distance and the number line below, as a timeline to indicate measurement of time. The units of the distance number line and the time number line could indicate different variables creating meaningful opportunities for problem-solving (e.g., If the river is 20 metres wide and it takes all animals 8 seconds to cross then… etc.). With this resource, I, along with my fellow Grappling-CoP, members believed learners could work flexibly between the multiple constructs of fractions in solving complex problems. Below are some of the questions, from Lesson 7, that I designed for learners to solve while using the triple number line.

One river-crossing is 10 metres, and it takes 4 seconds to cross.

a) If Kudu jumps 2 and a half river-crossings, how far has he jumped? How long did it take for Kudu to get there?
b) If Ostrich jumped 5 river-crossings, how far has she jumped?
c) How long did it take her to get there?
d) If Zebra jumped 10 river-crossings, how far did he jump? How long did it take him to jump all that way?
e) If it takes one minute to cross the river, how long does it take Ostrich to get halfway?

In our Grappling-CoP we reflected on how the triple number line could support teachers and learners in moving flexibly between multiple constructs of fractions, rather than simply focusing on the part-whole construct, as suggested in literature (for example by Siemon et al., 2015). Figure 4.18 below shows a screenshot from the Zoom meeting where we grappled with ways in which the triple number line could be used for such fractional problem-solving. Following this is an excerpt from our discussion around my key representation.

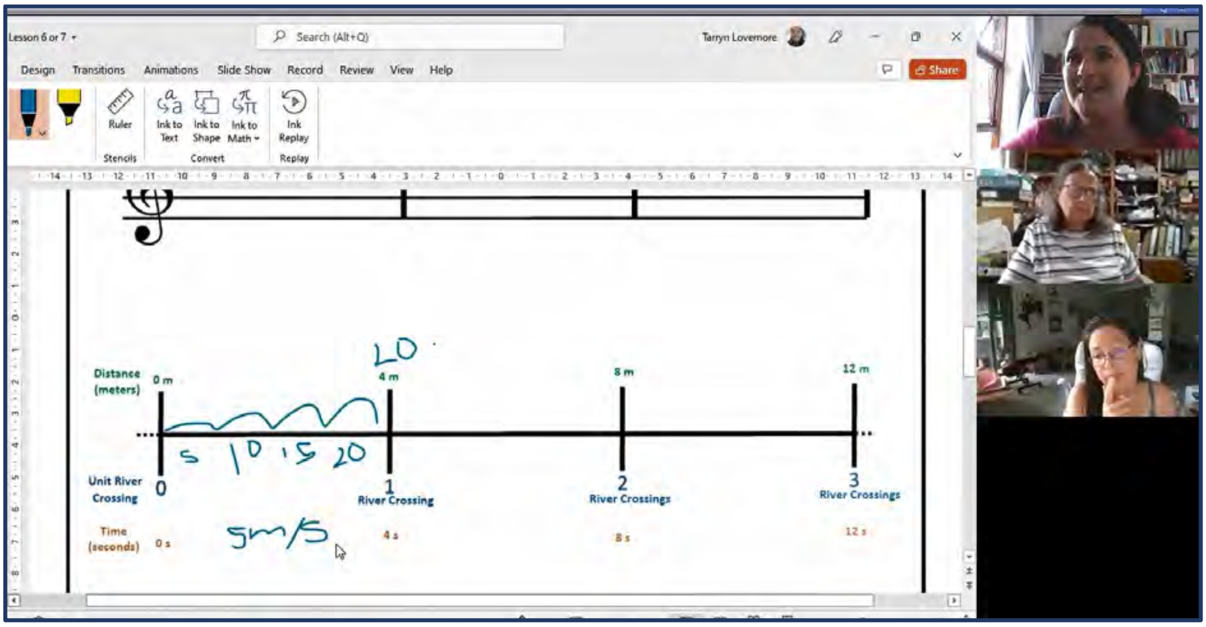


Figure 4.18: Screenshot from Grappling-CoP Zoom meeting to explore ways to use the triple number line

> Mellony: *When we get to problem-solving, that's when we're really getting to the powerful fraction stuff, because we can change that. We will say, if the river is 10m wide... now what? So the river-crossing is the main unit, completely in the foreground in our head, and then the other two variables change.*
>
> Tarryn: *Yes.*
>
> Mellony: *The river-crossing is the unit, and then we vary.*
>
> Tarryn: *And we're getting to developing the deep conceptual understanding.*
>
> Mellony: *So the teachers have to know that the kids are not expected to answer any of those questions without the image of the triple number line. And bringing it all together is that complex fractional problem-solving. With that visual crutch they're going to do very well. The key representational resource there is that triple number line… where as we change it, they can change the distance to 20, 40, 60m etc. and when they draw the animal jumps, each time, they will draw onto that… so it would be 20m per 4 seconds, 5m per 1 second. And they can see everything of the proportion. So speed is coming alive, ratio and proportion is coming alive. The unit can change…*
>
> [Grappling-CoP, Zoom meeting, 2022-02-01]

I decided that demonstration video clips would be helpful for the teachers to understand how I intended the key representation to be used to support learners in solving complex problems involving fractional reasoning. Teachers could also use them as a demonstration for their learners. The YouTube video clips can be accessed using the links[13].

The Protea teachers reflected on how they used this triple number line in supporting their Grade 4 and Grade 5 learners solving the complex fractional problems. Ms Clegg used the story of the animals being eaten by a crocodile or falling into the water to explain to her learners why the animals might stop before completing a whole river-crossing and what language they used in discussing the scenario.

> *Ms Clegg: Ja, so if they don't get to the other side they get eaten. So we looked at where the ostrich would be eaten.*
>
> *Tarryn: And when you say where they stopped, did you ask fraction-type questions?*
>
> *Ms Clegg: I, ja, so we used the language, so it's halfway. The jump is only once, then whereabouts in the river do… "No, he fell in halfway." And then we said, "What's the next one?" and then, they said, "Fifteen."*
>
> [Protea-CoP, 2022-08-15]

The teachers also reported on how useful they had found the laminated triple number line explaining how many of their learners relied on the visual representation of the number line to support them in their understanding and then solving of the problems. They reported on some variations in how the learners used the visual number line representation. It is interesting to note that more of Ms Savuka's Grade 4 learners used the number line as 'crutch' as opposed to Ms Clegg's Grade 5 class, where some learners did not rely on the visual representation of the number line at all. This points to progression across the grades.

> *Ms Clegg: They like to write on them since they could scribble out afterwards.*
>
> *Tarryn: They like the laminated sheets?*
>
> *Ms Clegg: And you can see like Enoch, he drew it, and then did the answer next to it. A couple did that, and then some just answered.*
>
> *Tarryn: So the idea is we're using the river-crossing as a unit, and then we had some problems where they had to solve the distance in metres. And then we had some problems where they had to solve with the time. And then we combined them. So working with this triple number line is quite higher-order thinking. And so that's what I wanted to ask you how did you find this resource coming from the music composition one and the story of the jumps through to this as your resource?*
>
> *Ms Clegg: I don't think they would have managed this without.*

---

[13] YouTube video 2: https://youtu.be/VTdGNR_zPO8
YouTube video 3: https://youtu.be/EiIQ9iYO8WU
YouTube video 4: https://youtu.be/y-YYSP5N3SY

Ms Savuka: They definitely needed it, and so for some questions, yes. For others, I could see they knew what to do. What I thought going into this, I was a bit apprehensive. I thought, now introducing distance and time, how are they going to be able to answer problem-solving questions, and I was blown away.

Tarryn: With the Grade 4 class?

Ms Savuka: With the Grade 4's. I was expecting a lot of hands to go up to ask for help, but I also let them work in pairs. But they managed to do it really well, and the one thing that, especially in Grade 4, problem-solving and word sums is something that so many of them struggle with. And I just saw the benefit of having a visual representation. How important it is to have something in picture form or in front of you, where solving a word problem, how impactful it is. Not to just work with the sentence and the numbers, but to actually see it. That really helped them a lot.'

Ms Clegg: And you know, maybe that's something that we can look at just in our general maths. Because I agree, there is a step missing in maths where we go from foundation phase [Grades 1 to 3]*, where they go from very concrete, and then we should be semi-abstract, and suddenly we jump into, now they must read an entire paragraph, know what all the fancy maths words mean, and there's no pictures.*

Ms Savuka: Nothing they can use to answer the questions.

Ms Clegg: Yes, whereas you know, having a basis like a number line, or a picture that forms a number line. Or something like that, that they can actually draw on.

[Protea-CoP, 2022-08-15]

Both teachers' reflections on the triple number line were the sorts of things we had anticipated in our Design-Theorising Plane. Mellony referred, for example, to the triple number line as being a "visual crutch" which the learners would use to solve the problems where the unit can change. These reflections resonate too with what is mentioned in the literature about the value of using number lines when modeling fractions. Firstly, the fraction as measure construct is when fractions are indicated as measures or distances from a given point (0) on the same scale, such as a number line (Charalambous & Pitta-Pantazi, 2007; Getenet & Callingham, 2021). Secondly, a number line representation of fractions is also useful for developing deeper understanding of fractions greater than one whole (Monson et al., 2020). I used both these ideas in the animal river-crossing problem scenario. As I discussed in Section 2.7, number lines are an effective way of representing complex fractions and their relationships, and have the potential to support learners (Saxe et al., 2013; Barbieri et al., 2020; Monson et al., 2020). Although an initially challenging representation to work with, number lines are a key representational tool with which to practise fractional reasoning in order to deepen learners' understanding of fractions (Siemon & Luneta, 2018; Soni & Okmoto, 2020). From the Protea teachers' reflections, I interpreted that they too found the triple number line representation a useful way to support their learners. The triple number line representation is fully consistent also with RME principles. The Protea learners progressed from their informal representations of the animal jumps and the musical claps, to a more formal representation of fractions on a number line, thus working within the formal, abstract mathematical representations, discovering mathematical relationships between concepts (Treffers, 1986; van den Heuvel-Panhuizen, 2003).

I designed Lesson 7 to conclude with a fun body percussion/clapping game – a game commonly known as *Sevens!* In the first iteration, I had trialled this activity with the group of Grade 6 teachers at Aloe School as part of Lesson 5. Although this clapping game was intended to be a fun way to end a lesson, on reflection I recognised its potential to be used to stimulate further problem-solving. It served as a realistic context to stimulate problem-solving requiring fractional reasoning where fraction as a ratio is considered and when working with fractions greater than one whole. Below is my entry in my reflective research journal on the experience in the first iteration with the Grade 6 teachers and learners at Aloe School.

> *We discussed 'beats per bar', with seven beats per bar. So it was a good context to introduce that idea, and then we spoke about "if there's 7 beats in a bar, if we had 14 beats, how many bars would it be? If we had 3 bars, how many beats would it be? 21". So we asked question like that. And then I asked them, "If we had 9 beats, how many bars would it be?". The one girl said, "one", and then some learners said, "no, you're wrong!". And she felt embarrassed, and I said, "she's not wrong. We have got one whole bar". And then we discussed if we had, out of 9, we've got 7 beats which would make one bar, how much is left? And then I drew the bars and the Xs in the middle, 7 Xs to show one bar, and two more Xs. And so we said "well, that means that if we're counting the bars, we would have 1 whole bar and 2 sevenths". So we looked at an improper fraction of 9 over 7, because we had 9 sevenths in terms of our clapping. Each clap is a seventh, each beat… then we said that that means that there's one whole bar and two sevenths.*
>
> [Reflective Research Journal, 2021-11-16]

Figure 4.19 below is a photograph of the visual representation that I drew on the whiteboard, during my trialling of the task at Aloe School, to support learners in the problem-solving discussion.

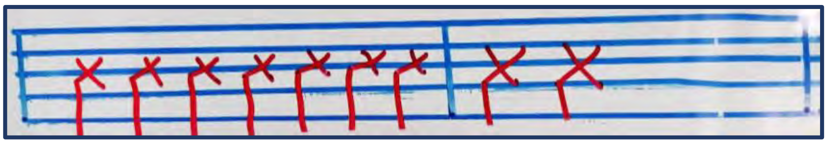


Figure 4.19: Photograph of visual representation of nine claps when the music is seven beats per bar

The Protea teachers used a YouTube video clip (which can be accessed in the archived lesson plan) to teach learners the *Sevens!* clapping game, and used it as a fun way to end the lesson with their Grade 4 and Grade 5 learners.

Lesson 8

In the Grappling-CoP, I had indicated to my supervisors that my original plan was to complete the task design with Lesson 7. The teachers at Protea School, however, in our planned final CoP meeting started discussing a possible lesson which they wanted to do as a follow-up. I felt greatly encouraged by the fact that Lesson 8 stemmed not from the Design-Theorising Plane but from the Grounded-Practice Plane of the Protea-CoP. In this section I discuss how the Protea teachers and I, in our micro-CoP, together conceptualised Lesson 8, which I then designed based on the CoP discussion.

The teachers highlighted the desirability of one more lesson because of a gap they had identified between the novel music-mathematics integrated sequence of lessons and what their learners were expected to do in their school textbooks and workbooks. Ms Savuka expressed it thus:

> *Cos this feels like an amazing intro, and then you've got the textbooks. And there's this little gap in between. So looking at how we're going to link this.*
>
> [Protea-CoP, 2022-08-15].

I found this feedback from the teachers, experts in the contexts of their classrooms and now working with me in the Grounded-Practice Plane, extremely valuable. It reminded me of Setati's (2005) distinction between research 'on' teachers versus research 'with' teachers, and Makar's (2021) notion of researchers and teachers working as "research collaborators" (p. 440). The conceptualisation of Lesson 8 was an example of just this. It was also a good example of working as a CoP. We were engaged in a *shared practice* with the *common goal* (after Wenger, 1998) of designing an integrated music-mathematics lesson for deepening fractional reasoning and understanding. In discussing the planning of Lesson 8 the teachers mentioned that they felt learners needed to practise working with equivalence and fractions greater than one in an explicit manner. I interpreted their comments as an indication that they had taken shared ownership of the task design's onward journey.

> *Ms Savuka:* *And I think maybe after all the seven lessons are done, then I would love to go back to the river-crossing and then show them that this can be represented as fractions.*
>
> *Ms Clegg:* *I agree, that would be a very nice…*

Ms Savuka: *…and going past into mixed fractions as well, past the first river-crossing.*

Ms Clegg: *Use two fractions, two jumps. So like an ostrich would represent half, and almost do an activity where now the animals are fractions.*

Ms Savuka: *And bringing in also equivalent fractions. But we did do that without telling them indirectly, and then just bringing them back, and almost like revealing, this is what it is. That's actually what we did.*

Tarryn: *We can absolutely do that.*

Ms Clegg: *Cos they quite battle with the equivalent fractions, so that would be quite nice to say, "One ostrich would need to be friends with two zebras," kind of thing. And do it more visually as like an extra thing. I just like the idea of the animals being equated to the fractions.*

Ms Savuka: *Representing the jumps as fractions, or animals as fractions.*

Ms Clegg: *You know you did the music notes? You know where they fitted the music notes onto the thing. If we had instead of music notes, two ostriches that fitted within the river…*

[Protea-CoP, 2022-08-15]

Ms Savuka thought that it would be a useful exercise to link all seven previous lessons integrating music and mathematics directly to formal fractional representations in ways similar to how learners would ordinarily come across them in their normal curricula work. This shows how the task design of the eight lessons links to RME principles. The experientially-real starting point allowed first for informal discussion and representations and this then progressed to vertical mathematisation where, in Lesson 7 and more so in Lesson 8, learners used formal, abstract representations of fractions. In my task design the starting point therefore allowed for progression to a formal endpoint (Cobb et al., 2008), exactly as I had originally hoped would be the case.

The Protea teachers and I, therefore, decided Lesson 8 should start with a reminder to learners of all the animals which had been used in Lesson 1 and Lesson 2. This would be shared with the class via a poster or a drawing on the whiteboard. We then decided to replicate the activity from Lesson 6 with the A3 laminated musical staff and river-crossing number line. Instead, however, of matching transparency cards with musical note values on to the animal jumps, we would allow learners to match fraction cards to the animal jumps. The animal jumps would then each represent a fraction. Figure 4.20 below shows an example of how this might look.

I created another demonstration video[14] for teachers, one which they could use both to familiarise themselves with the resource and to show to their learners.

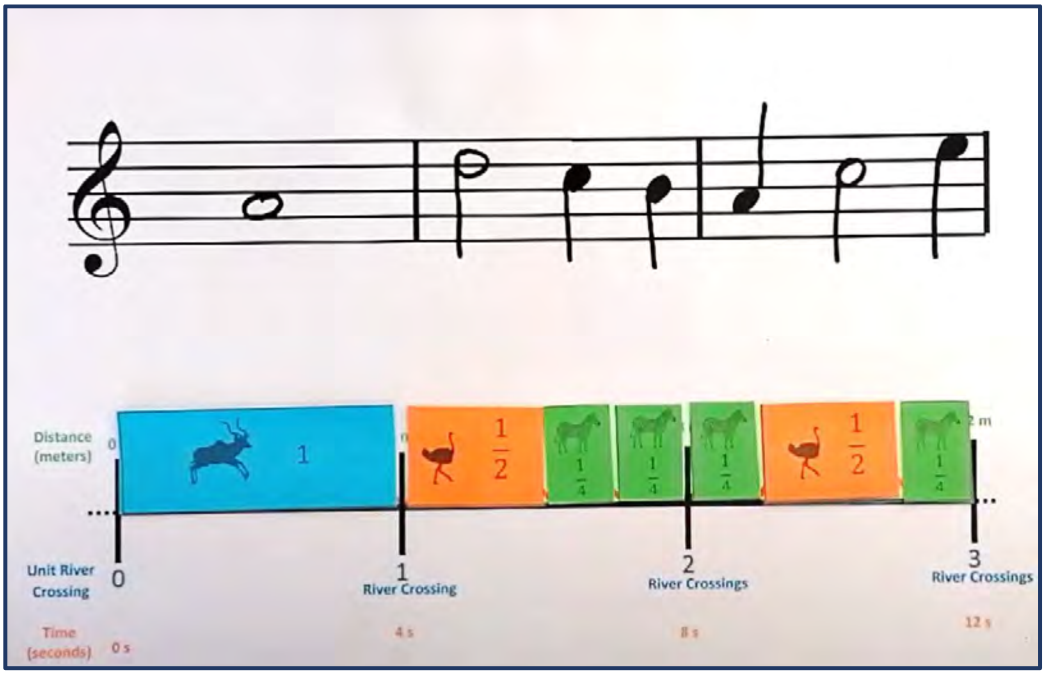


Figure 4.20: Example of animal river-crossing jumps representing animals on a river-crossing number line

We also discussed how the last lesson could integrate the visual representations of the animal jumps per river-crossing, the musical notes and bars, and the formal representation of fractions:

Tarryn: *Perhaps a strip, like a blue strip and then it's got a picture of a kudu on. And then you would want two orange strips with ostrich jumps?*

Ms Clegg: *With the same size as your music transparency cards. And we could even use it on that same card. Cos now you've already done…*

Tarryn: *Then you have the notes on the top and you will have the fractions at the bottom.*

Ms Savuka: *That will be a nice visual for them.*

[Protea-CoP, 2022-08-15]

We then decided that after the hands-on experience with linking the animal river-crossing jumps to fractions, learners should solve problems requiring fractional reasoning. The teachers specifically identified the need for their learners to practise working with equivalent fractions, fractions greater than one whole, and adding fractions with unlike denominators (the latter

[14] This can be seen on YouTube at the following link: https://youtu.be/9q1PINF14u4

especially for the Grade 5 learners, as per curriculum requirements). Below are some of the questions we developed to put to the learners.

1. (a) Jackal and Elephant each took 4 jumps. Who would be the furthest across the river?

   (b) Which fraction is greater, $\frac{4}{7}$ or $\frac{4}{5}$?

2. Arrange these fractions from smallest to biggest (ascending order). Use the animal fraction cards to help you:

   $$\frac{4}{5} \qquad 1\frac{1}{2} \qquad \frac{5}{6} \qquad \frac{3}{4} \qquad \frac{5}{3}$$

3. Which animal fractions could you combine to make $2\frac{1}{2}$ river-crossings?

In our planning of Lesson 8 the teachers and I explored the benefits of using the fraction cards on a number line, as opposed to their experiences of just using a fraction wall/chart. It was interesting to hear of the teachers' experiences comparing the animal river-crossing number line in my task design to their normal use of a fraction wall. Again, I see these reflections from the teachers as aligning with the literature on representing fractions on a number line (for example, Siemon & Luneta, 2018; Soni & Okmoto, 2020; Barbieri et al., 2020; Monson et al., 2020). This also showed the benefits of the South African curriculum's expectation to use "length or measurement models such as the number line" to develop the concept of fraction as measure (DBE, 2011, p. 71). Some of the discussion around this key resource is captured in the following excerpt.

Ms Clegg: Yes, so almost like instead of a fraction wall, a fraction line. And so you could have the animals. We could do them in different sized cards, showing that, "Okay, now we gonna build… So the river. We've got two ostriches that fit in here, or else we've got four zebras that fit in here. And then what happens if we've got six ostriches next to each other?"

Ms Savuka: And it's also showing how many fractions equal to a number that's greater than a whole, as well. They can go past one river-crossing.

Ms Clegg: It's almost like a picture animal version of a fraction wall. Because the fractions are quite intimidating, jumping straight into that. Cos that would be quite cool, because from there you could go into, "How many halves in…?" You know, you can then make it more abstract. And then eventually take away the animals and just be left with the fractions… Ja, equivalence is a big thing to try get through to them, like with the fraction wall. Like right in the very beginning, they suddenly told to put fractions from biggest to smallest, smallest to biggest. And then they're all different denominators.

Ms Savuka: But then above the worksheet you could have a table showing what fraction each animal's jump is actually representing. And then you can also say, "Linking the fraction of the animals jumps to the river-crossings itself." So the fraction, in say one and a half river-crossings. So do equivalent fractions.

Ms Clegg: Yeah.

Ms Savuka: And then they're answering it in a fraction form….It's gonna be actually so nice to put it on a number line because a fraction wall seems so like finite. It seems like it only stops after a certain time. So for them to see now that it can actually continue. To actually continue counting.

Ms Clegg: I must be honest, I rarely use a fraction wall because I find for some of my children, it is completely overwhelming, because if you're working in twelfths and sixes, for example, take all those other fractions in between…Ja, they almost get a bit overwhelmed. Where something like this, it actually would be a very good resource for us to make.

Ms Savuka: Cos that's the main tool that we have to use in Grade 4 for them, comparing fractions as well. And some work well with fractions, and others struggle, like with everything in between. They just can't see clearly where the fractions lie.

[Protea-CoP, 2022-08-15]

The teachers then went on to discuss the challenges learners frequently encounter when adding fractions, especially in Grade 5 when they need to start adding fractions with unlike denominators.

Ms Clegg: If we're talking about addition. You could talk about if you had two ostriches and you add the zebra. What would it be? How many would it be kind of all together? Because then they'd have to first figure out how many zebra would be equal to ostriches. Especially for my Grade 5s.

Ms Savuka: Like they did with the jumps of the animals to make their river-crossing song… Cos that's the main tool that we have to use in Grade 4 for them, comparing fractions as well. And some work well with fractions, and others struggle, like with everything in between. They just can't see clearly where the fractions lie.

[Protea-CoP, 2022-08-15]

The closure activity the teachers and I chose for this lesson was a game suggested by Ms Clegg. It resembles the well-known children's game called '*Shipwrecks*'. Instead, however, of the teacher calling out parts of a ship which learners should run to, the teacher would call out a whole number or a fraction (mixed number or improper fraction). The learners could then represent the animal jumps and animal fractions that can be added together to make the number or fraction the teacher called out. This would be a physically embodied way for learners to see addition of fractions with unlike denominators and working with fractions greater than one. Below is an excerpt from the discussion we had in designing this activity.

> *Ms Clegg: Do you remember that game, 'Shipwrecks,' where they had to…? They'd have to get into groups of three, or whatever. You could play… You could make a game of this where they have to get together first, in their own animals, but then you could start bringing in a bit of an equivalence. And so then you could make like, if you want a whole, then there'd have to be one ostrich and two zebra in a group.*
>
> *Ms Savuka: I like that.*
>
> *Tarryn: So you'd have four zebra, or two ostrich.*
>
> *Ms Clegg: … and two ostrich.*
>
> *Tarryn: Then what about saying, "Okay, now we need to get to one and a half river-crossings. Form your groups." And then it would be a kudu and two zebras, or…*
>
> *Ms Clegg: Yes.*
>
> *Ms Savuka: That's really cool.*
>
> [Protea-CoP, 2022-08-15]

Although time constraints in their academic year planning prevented the teachers from trialling this jointly-designed lesson, they have committed to trialling it at a later stage, and to inviting me back for a post-lesson interview. My own (thesis-writing) time constraints prevent me from waiting for this however. I will therefore need to share findings from this final implementation via a different platform, either a conference presentation or a future journal publication.

4.4 Initial Workbook and Final Individual Worksheet

The sequence of my task design began with an initial workbook that learners could do either individually or in small groups, depending on what teachers deemed appropriate in their classes, and ended with a final worksheet for each learner to tackle independently. These were not intended to function in a quantitative way as some sort of pre- and post-test. Rather, they were intended to support teachers in making qualitative evaluations of their learners' knowledge, skills and attitudes before and after the implementation of the music-mathematics integrated lessons. My goal was that the tasks would enable the teachers to reflect on and share perceptions on the effectiveness or otherwise of the integrated music-mathematics tasks. Precisely because it was not my intention to collect quantitative data on the learners' abilities, no numerical scores were assigned to the questions in the initial and final tasks.

Working with my supervisors in the Design-Theorising Plane, I shared my decision to label the first task an 'initial workbook' rather than a pre-test, because I wanted to avoid creating any anxieties for the teachers and the learners. I designed a 'workbook' made from PowerPoint slides that could be printed, cut and stapled together. I felt this would make the design of the task look less like a test and less intimidating. In the course of the micro-CoP meetings at both schools, I emphasised to the participating teachers that the initial workbook was low stakes, designed simply for them to qualitatively evaluate their learners' prior knowledge, skills and beliefs towards mathematics. The initial workbook was also the same across Grades 4 to 6. I suggested to teachers that they explain to learners that they might find some of the questions challenging and that they should try their best to answer them, but that it did not matter if they could not manage them all. Teachers at both the schools were appreciative of this point, most particularly the Grade 4 teachers.

In the Design-Theorising Plane, my supervisors and I decided that I should focus my design of the initial workbook on three key areas:

- Fractional knowledge: fraction as measure, fraction as ratio, and the part-whole construct.
- Musical knowledge: informal and formal/Western notation of music.
- Learners' dispositions towards mathematics.

The fractional knowledge questions that I designed guided by literature, were around the multiple constructs of fractions. Cortina et al.'s (2015) examples of problems where the fraction is used as a comparer, based also on RME principles, guided my own design of problems that would encourage fraction as measure and fraction as ratio constructs. Siemon et al.'s (2015) description of fraction as ratio further guided my questions on this construct of fractions. The literature on the importance of representing fractions on a number line also influenced my design of the initial workbook (for example, Saxe et al., 2013; Barbieri et al., 2020; Monson et al., 2020). In Figure 4.21, below, I share two examples of the initial workbook problems.

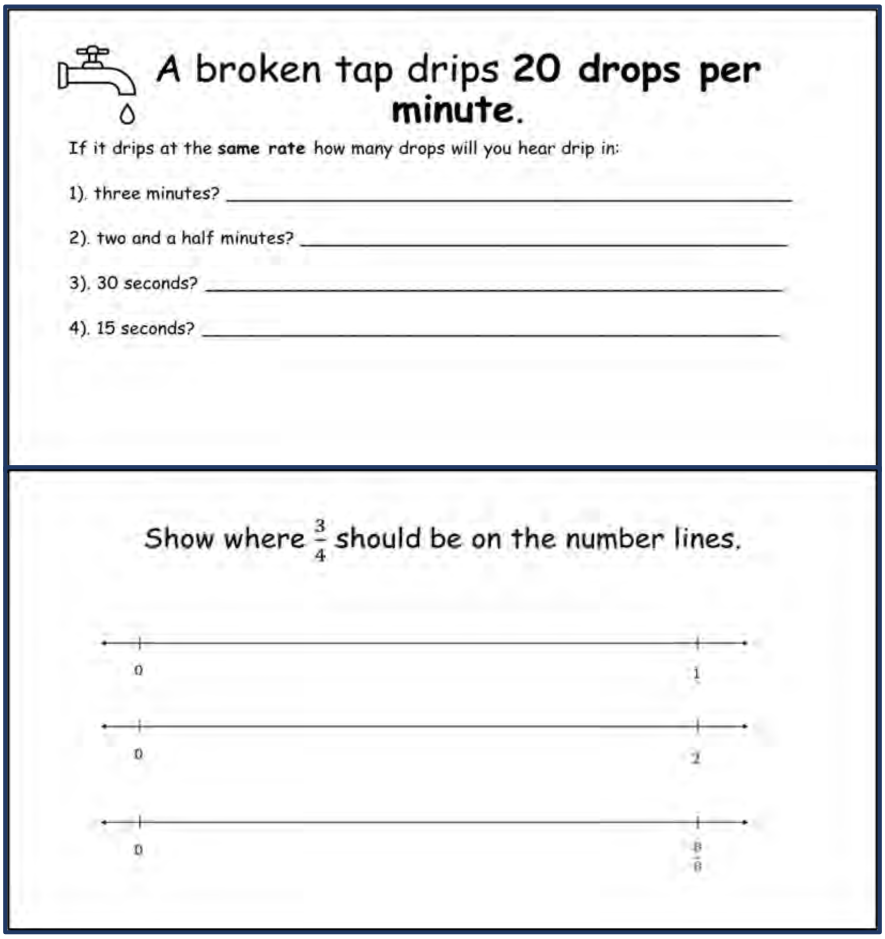


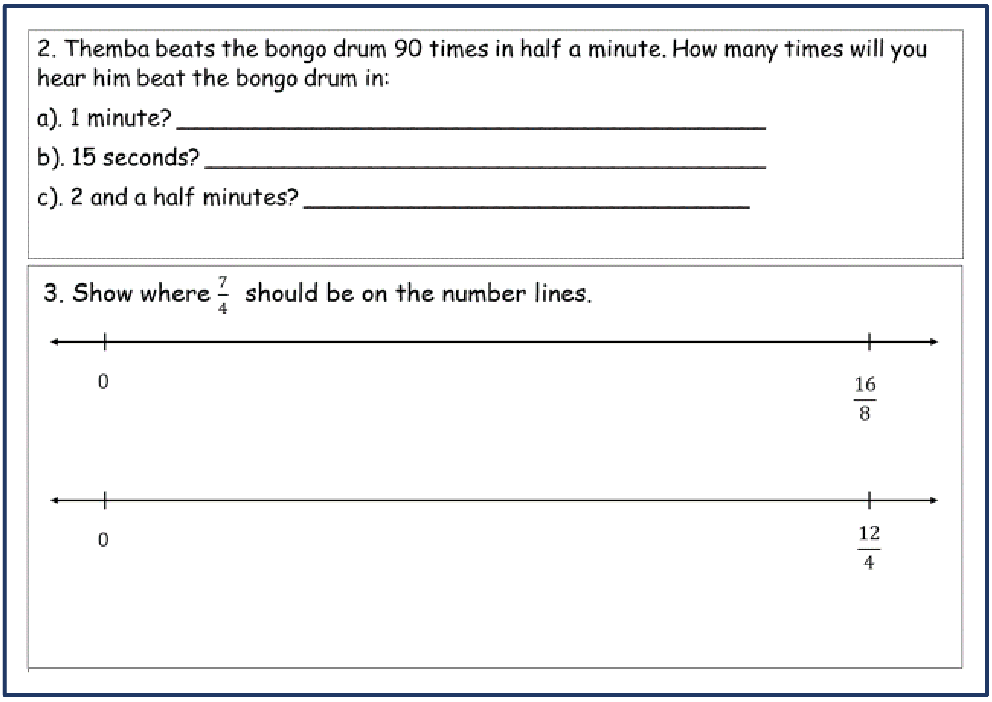
Figure 4.21: Examples of problems in the initial workbook

The questions I designed around establishing learners' prior knowledge and skills were focused primarily on musical notation. It could be Western notation, one of the curriculum aims (South Africa. DBE, 2011b), or it could be learners' informal notation. There was also a group activity to establish whether learners could read Western notation of note values.

The design for the third part of my pre- and post- tasks was aimed at helping the teachers identify possible changes in their learners' beliefs and dispositions towards mathematics. Literature around teachers' and learners' attitudes, dispositions and beliefs about mathematics being narrow and disconnected (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017; Jojo, 2019) prompted me to get learners to complete the sentence: 'Maths is….' (also used in Graven & Heyd-Metzuyanim, 2014). Based on this question, I further developed a continuum with phrases from the curriculum's definition of mathematics and the broad aims of recognising mathematics as a creative, beautiful human activity (South Africa. DBE, 2011a). I saw this as an important element to explore with teachers. One of the driving motivations of the study is finding ways in which this integration can support teachers and learners in meeting not only the specific curriculum aims of fractional knowledge, but also of the general aims of recognising mathematics as a creative, elegant human activity (South Africa. DBE, 2011a).

In our Grappling-CoP, working within the Design-Theorising Plane, I explained to my supervisors that I intended the final activity to be an 'individual activity' (but again emphasising that it was low stakes and thus not something the learners and the teachers need become anxious over). I designed grade-appropriate individual questions on the fraction as measure, fraction as ratio and part-whole construct of fractions. I also included questions where the fraction is greater than one whole. Through our grappling and reflection, however, my supervisors and I decided that it should not include any questions requiring that learners remember musical notation as such. Instead, so as to incorporate the musical element in a realistic scenario, I designed a problem based on the context of playing a bongo drum.

I ended the task by again asking learners to complete the sentence 'Maths is…'. This would allow teachers to establish if and how learners' dispositions might have changed over the course of implementing the integrated music-mathematics lessons. Two examples of the Grade 6 individual activity are shown in Figure 4.22 below.


2. Themba beats the bongo drum 90 times in half a minute. How many times will you hear him beat the bongo drum in:

a). 1 minute? ____________

b). 15 seconds? ____________

c). 2 and a half minutes? ____________

3. Show where $\frac{7}{4}$ should be on the number lines.

Figure 4.22: Two examples of the Grade 6 individual activity

I end this subsection on the initial and final activities by briefly sharing some of the teachers' reflections and evaluations of their learners' knowledge, skills and attitudes based on the initial workbook as compared with how learners responded in the final individual activity once they had gone through the integrated lessons experience.

Different teachers shared their insights on various aspects of these tasks in different ways. Some teachers were explicit about their learners' progress, while other teachers referred more specifically to individual questions. Despite these discussions in micro-CoP meetings at both schools, where some teachers expressed that there had been positive changes in their learners' fractional understandings and mathematical dispositions, it is impossible for me to make any substantive claim regarding learning gains from the integration tasks. Nonetheless, the teachers' perceptions and evaluations of the learners' development of fractional reasoning and understanding and their disposition of mathematics gave me a means of exploring whether, and if so, in which ways, the task design of the music-mathematics lessons appeared to have met the curriculum aims. I acknowledge, at both the theorising and the grounded plane levels, that, because the implementation of the integrated lessons took place over extended periods, learners' progression could also be related to the normal expected progression in the school year. I do, however, believe that the participating teachers' insights greatly contributed towards my making ongoing refinements to my task design.

Fractional knowledge and skills

Below are excerpts of some of the teachers' qualitative evaluations on their learners' fractional understanding based on the initial workbook.

> *Ms Chaka: When I first looked at the booklet I thought that my Grade 6s would have no difficulty. Some are quite competent, but others are really having difficulty… I can see that there really is a need for this intervention for understanding fractions*.
>
> [Aloe School, Informal interview, 2021-05-21]

> *Ms Makeba: The Grade 4s, my class, really struggled. They wanted me to explain everything… I just let them do what they were able to do.*
>
> [Aloe School, Informal interview, 2021-05-28]

> *Ms Fassie: The first stuff, they could all do easily, but this on the number line got a little more difficult, uhm, towards the end.*
>
> [Aloe School, Informal interview, 2021-09-10]

> *Ms Ibrahim: Not so much the music, they eventually got it right. It was the fractions that they struggled with.*
>
> [Aloe School, Informal interview, 2021-10-01]

Teachers discussed their evaluations of the learners' fractional understanding after the final individual activity, too. Below is an excerpt from the final Aloe-CoP meeting.

Mr Dube: *I found the question about netball court, easier for the children than when we jumped across the river. And then the number line was easier for them.*

Ms Fassie: *They were able to answer them. The number line, I didn't have to do anything.*

Ms Makeba: *I thought it was better* [compared to the initial workbook]. *With the Grade 4s, they were thrown off by the eight eighths on the number line. I did have to point out, what is eight eighths? It is the same as one whole. And then they could plot the three quarters on the number line more accurately. But otherwise they did really well.*

[Aloe-CoP, 2021-12-03]

The responses from the initial workbook were consistent with claims in the literature that fractions are a challenging concept to teach and learn (Streefland, 1991; Siemon, 2003; Courey et al., 2012; Cortina et al., 2015; Azaryahu et al., 2019; Getenet & Callingham, 2021). Encouragingly, the teachers' feedback on their learners' fractional understanding suggests they could see positive signs of progression having happened between the start and the finish of the integrated music-mathematics lessons.

Musical knowledge and skills

Some of the participating teachers discussed the samples of learners' work in the workbook during the interviews. They also shared some of their learners' responses. It was interesting to notice that all the artefacts we discussed showed learners using informal representation of the musical notation. Many learners represented the audio clip with lines indicating change in pitch or volume. One learner in the shared examples represented the duration of notes with lines. Two of these examples can be seen below in Figure 4.23.

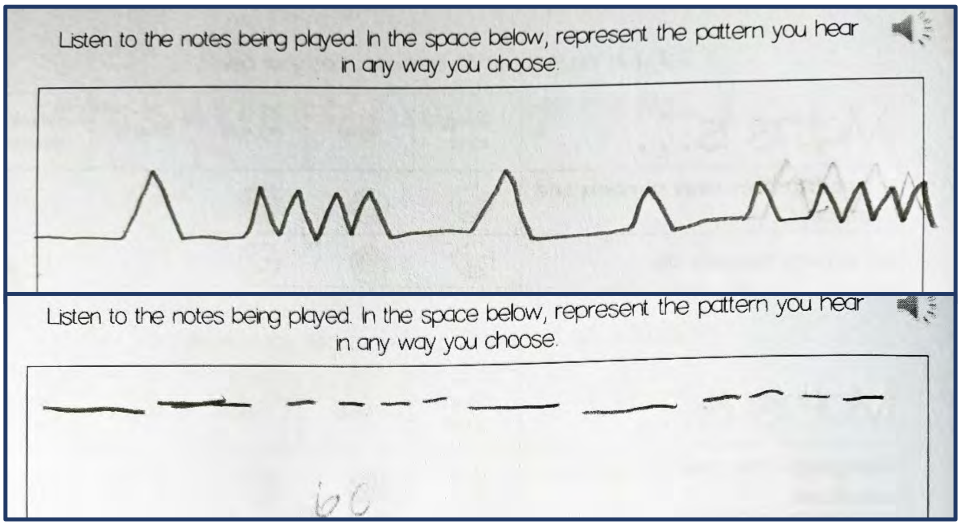


Figure 4.23: Examples of how learners informally notated a musical audio clip in the initial workbook

In the initial workbook, with regards to the reading of the musical notation, teachers had mixed responses. Ms Chaka explained that it was easier for her to follow when the learners' clapped the notes as opposed to when they hummed or whistled [Aloe School, Informal interview, 2021-05-21]. It was this, and similar responses from other teachers that added to my decision to focus the lessons on body percussion such as clapping.

Learners' dispositions towards mathematics

Some of the teachers shared their learners' descriptions of mathematics in answering the 'Maths is…' prompt. As the examples cited below show, most of these descriptions were indicative of a narrow view of mathematics mentioned in the literature (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017; Jojo, 2019). For example, Ms Chaka shared the example from one of her Grade 6 learners,

> *"Maths is difficult but I promise I will keep trying my best"*
>
> [Aloe School, Individual interview, 2021-05-21].

Ms Clegg shared some examples from her Grade 5 learners,

> *"Maths sucks. I don't like maths… Maths is numbers and sums."*
>
> [Protea-CoP, 2022-01-24].

The next section of the initial workbook was the continuum. Here learners could select their views on mathematics. Figure 4.24 below provides an example of one learner's response.

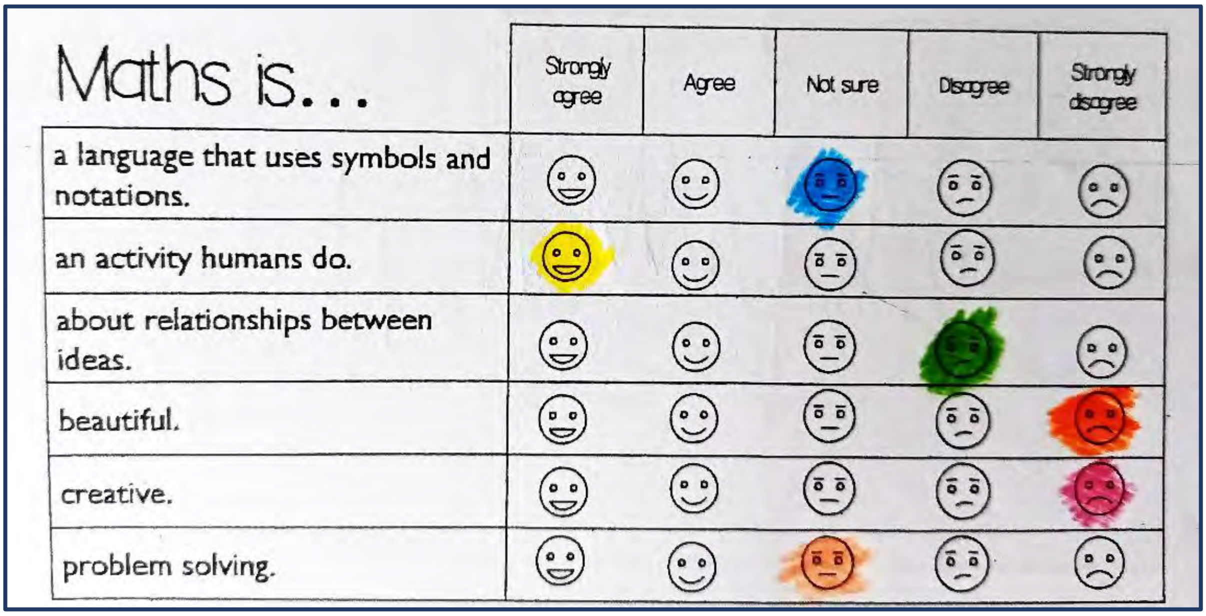


Figure 4.24: Example of learner response describing mathematics in the initial workbook

Ms Savuka relayed a discussion she had with her Grade 4 learners about the creativity in mathematics. It showed how her learners struggled to recognise how mathematics could be creative, particularly when compared with other learning areas such as art and music.

> *Ms Savuka: And something that I realised was a very common thread is when we did this, and I talked about maths as being creative. They couldn't find the creativity. They said, "How? Art is creative. Music is creative. I don't see how maths can be creative". So that made them think a little bit. How can it be creative?*
>
> [Protea-CoP, 2022-01-24]

The final individual activity completed at the end of the music-mathematics integrated lesson sequence indicated that there had been a broadening in some of the learners' views of mathematics. For example, two of Ms Makeba's Grade 4 learners wrote,

> *"Maths is so much fun I Love it is my favret subjeket! (sic)."*
>
> *"Maths is important"*
>
> [Aloe-CoP, 2021-12-03].

Only one learner's example that a teacher shared identified mathematics as having a 'creative' element. Mr Dube reported that one of his Grade 5 learners said,

> *"Maths is amazing, creative, fun and sometimes challenging which is good as a challenge will make us stronger"*
>
> [Aloe-CoP, 2021-12-03].

The main point emerging from our Aloe- and Protea-CoP discussion was that learners appeared to be more positive about mathematics; recognising that, although challenging, it can also be fun. This demonstrates that integrating music and mathematics has the potential to develop the broader curriculum aims of recognising mathematics as a creative, beautiful and elegant human activity (South Africa. DBE, 2011). It also relates to Freudenthal's (1991) notion of mathematics being a rich and natural human activity.

4.5 Chapter Summary

This chapter, Part A of the findings, explained and gave examples of the integrated music-mathematics 'product'. I developed this product based on my working within three micro-CoPs and between the Design-Theorising Plane and the Grounded-Practice Plane. In the next chapter I describe the grappling process that unfolded as the product was being designed.

CHAPTER 5: PRESENTATION AND DISCUSSION OF FINDINGS: PROCESS

5.1 Outline of the Task Design Journey

In Chapter 4, I presented and discussed the *product* of the participatory dual-design experiment in task design, (i.e., the eight-lesson sequence, initial workbook and final individual worksheet, and accompanying resources and representations). In Chapter 5 I share the *process* of task design grappling that, together with my two supervisors, I followed in the Design-Theorising Plane as I worked to develop the product. I also share here the participating teachers' feedback on their experiences of, and reflections on, the implementation of this product.

It is not possible within the space of a thesis to tell the full story of the task design journey chronologically as it unfolded. I therefore made certain decisions about how best to communicate the key ideas and findings from the task design journey. The task design grappling was a 'messy', iterative process. Rather than a chronological sequence, I have chosen to organise the presentation and discussion of these findings according to the phases and groupings of Obstacle-Resolution Cycles. I have divided the discussion according to the phases of the study (Phase 1, Phase 2 and Phase 3) and refer to the three micro-CoPs within each of these. In Figure 5.1 below, I show how the micro-CoPs aligned to the phases of the study, as well as my role as the central member, acting as a 'cog' working between the phases.

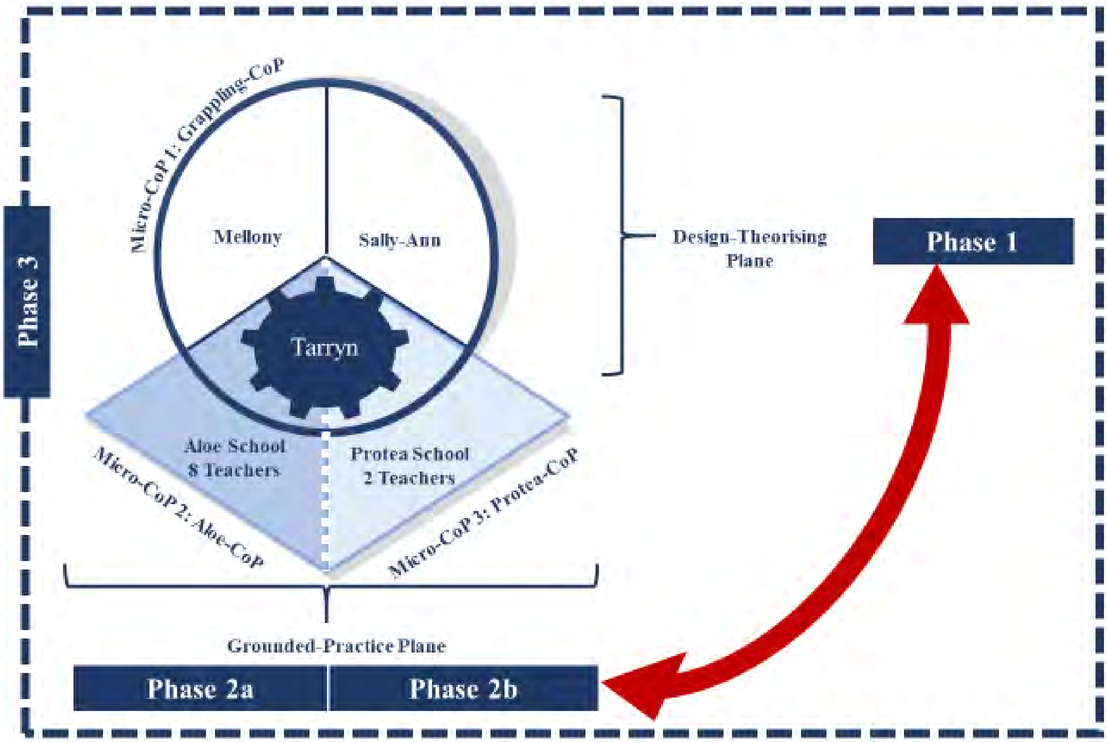


Figure 5.1: Diagrammatic representation of the micro-CoPs alignment with the phases of the study

Phase 1 involved the process I followed with my fellow Grappling-CoP members while we worked in the Design-Theorising Plane. Here we wrestled with key obstacles that arose in my designing of the integrated music-mathematics lessons, seeking to find appropriate resolutions. The pattern of Obstacle-Resolution Cycles and three key questions from the design process is something I recognised in my retrospective analysis of our Zoom recorded meetings. From this I developed a matrix (reproduced in Table 5.1, this time highlighting the three key questions) that I used to analyse the task design process.

	Obstacle	**Resolution of Obstacle**
Fidelity	**Does the task maintain the fidelity of the mathematics, the music, and the integration of the two? How?**	
Simplicity	**Does the task adequately simplify the complexity of the integration for implementation within the classroom? How?**	
Key Representation	**What key representation/s would best support conceptual clarity?**	

Table 5.1: Template matrix for organising and comparing data with three guiding questions

I identified ten groupings of Obstacle-Resolution Cycles using the above matrix. In Figure 5.2, below, is an example diagrammatic representation of the task design grappling process's cycles of Obstacle and Resolutions (Obstacle-Resolution Cycle 5).

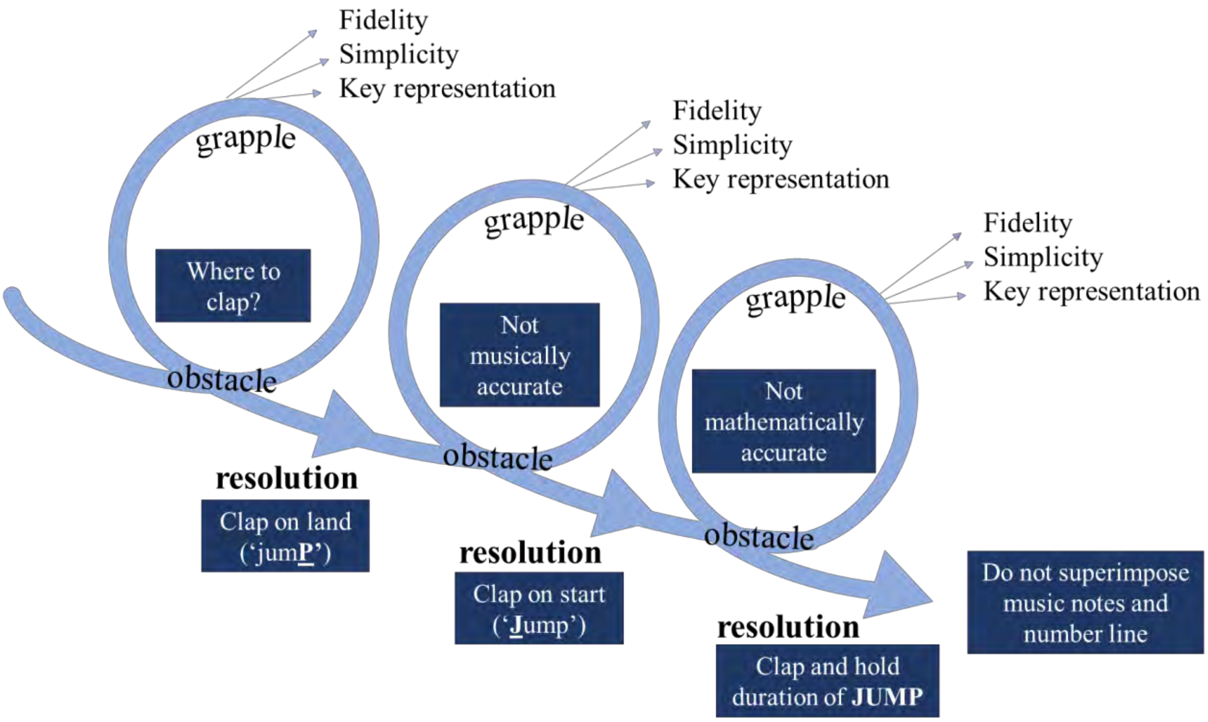


Figure 5.2: Diagrammatic representation Obstacle-Resolution Cycle 5

Phase 2 involved my work within the two micro-CoPs (Aloe and Protea), as, in the Grounded-Practice Plane, the participating teachers trialled the music-mathematics integrated lessons. I discuss the teachers' reflections and feedback that contributed to the grappling in the Design-Theorising Plane. I need therefore to discuss Phase 1 and Phase 2 (2a and 2b) together, to show the nature of the participatory dual-design experiment in task design. The red arrow in Figure 5.1, earlier, shows this interrelatedness between Phases 1 and 2. I placed my name in the shape of a cog to indicate the central role I played in working between the Planes and Phases. Figure 5.3, below, shows this interrelatedness again as I progressed through each of the Obstacle-Resolution Cycles. The Design-Theorising Plane grapplings (of my two supervisors and I) guided my design decisions. These were then trialled and interrogated by the participating teachers. Their feedback fed forward into further grappling.

Finally, Phase 3 involves my sharing of the overall teachers' and researchers' experiences of the participatory dual-design experiment in task design. I include brief discussion of some of the trials and triumphs I encountered around trialling the integrated tasks. These relate most particularly to the initiating (and sustaining) of the second of my micro-CoPs (Aloe School), and to adapting to the circumstances around COVID-19.

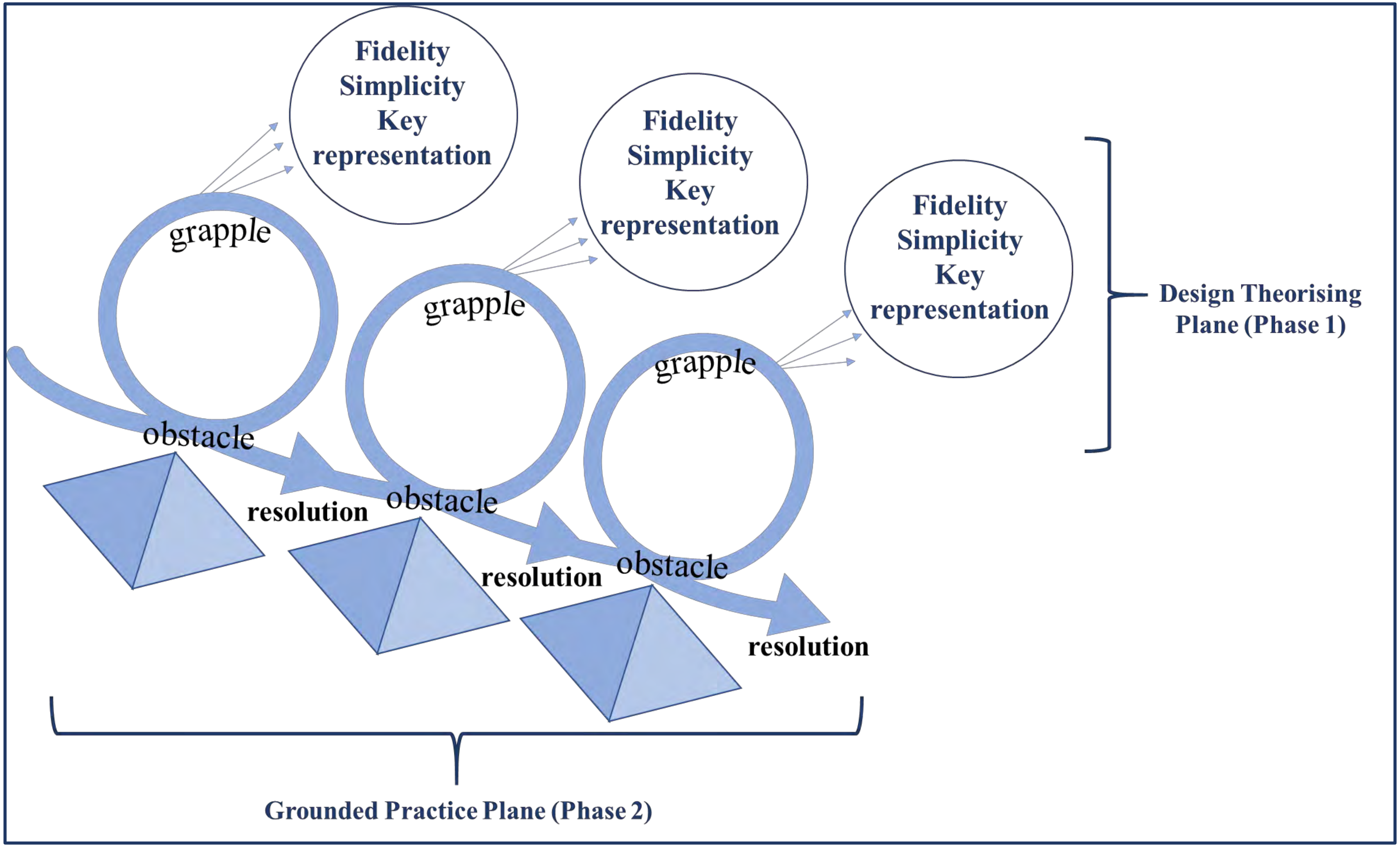


Figure 5.3: Diagrammatic representation of the Obstacle-Resolution Cycles working between the Design-Theorising Plane and the Grounded-Practice Plane

5.2 Phases 1 and 2: Obstacle-Resolution Cycle Groupings

I organised the Obstacle-Resolution Cycles into ten groupings (as summarised in Table 5.2, below). By grouping the cycles I do not intend to imply they are discrete ideas: all ten cycles do relate to one another and sometimes overlap. I have grouped them according to the element/aspect which we focused on in the Grappling-CoP. As mentioned earlier, the full scope of the grappling process that took place over 19 months is too extensive to delve into in the space of this thesis. It is because I recognise Obstacle-Resolution Cycles 4, 5 and 6 as having been the most challenging and significant groupings in my task design journey that I have chosen to discuss these three groupings in greater detail. Then, more briefly, I discuss the other seven obstacle-resolution cycles, some of which I intend elaborating on in more detail in future publications.

	Obstacle	Resolution
1	Selecting a problem scenario that is experientially-real.	Animal river-crossing jumps can link to percussion claps and music notes, or a number line (time and distance)
2	Deciding on what the unit will be (including fractions greater than one whole).	The unit is the river-crossing or the music bar, but the whole/unit can change (i.e. the distance and time of one river-crossing can vary). The animals keep jumping after crossing the river, or musical beats continue to the next musical bar.
3	Preventing confusion about speed. All animals cross the same distance in the same amount of time, therefore, their speed (distance/time) is equal, yet they have a different rhythm of jumps.	Speed will be constant for all animals, but *how* the animals cross will be different. I will discuss this with teachers, but it will not be the focus of the lessons.
4	Musical notation limitations and challenges.	First introduce percussion line and informal notation using Xs. Progress to exposing learners to formal Western note value notation, but without expecting them to memorise it.
5	Visual placement of animal river-crossing jumps linked to the percussion claps	Focus on the animal jump rather than on the start or landing. Place music notes in the middle of the bar. Cannot conflate all aspects of the music and the mathematics; separate fraction as

		measure and fraction as ratio to maintain fidelity of music concepts.
6	Cannot super-impose the number line and music line (staff).	Align the music line (staff) with the number line (double and triple number line).
7	Difficulty with visual representation of note values that could be mistaken for a zero.	Careful notation with resource template, not expecting teachers and learners to write the notation, but to use the designed resources.
8	Uncertainty about including the musical rest.	Include the musical rest just for creativity, but not as a focus for notation.
9	Finding progression across grades.	Start with same problem scenario and informal representations. Problem-solving questions can progress in complexity.
10	Questioning the value of integrating music.	To meet the general curriculum aims and for a context which allows opportunities for moving flexibly between multiple constructs of fractions

Table 5.2: Brief outline of the ten Obstacle-Resolution Cycles

5.2.1 Obstacle-Resolution Cycle 1: Selecting an experientially-real problem scenario

The first obstacle of the task design grappling journey was to select a problem scenario that would link the musical concepts of note values and beats per bar to the mathematical concept of multiple constructs of fractions. As noted, I was guided throughout by the RME principle of selecting an experientially-real starting point that would allow for progression from informal, horizontal mathematisation to formal, vertical mathematisation (Cobb et al., 2008). I recognised the need to have a scenario that would highlight a constant time (such as in a music bar). The scenario I selected was of wild animals commonly found in southern Africa, as discussed in Section 4.3, jumping across a river in a constant time. I then saw the opportunity this provided for measure of distance as well as time, and also for the fraction as ratio construct (jumps per river-crossing or beats per bar).

I felt some concern about the 'realistic' nature of this problem scenario, as animals do not realistically jump across a river in a constant time. River-crossings are seldom a constant distance. Van den Heuvel-Panhuizen's (2003) explanation of the implication of 'realistic' was helpful in this respect. As she explained, a problem scenario need not be accurately real-world.

Rather it should be a real experience for learners and should have meaning to them. Fairytales could be an imaginary yet meaningful problem context. We therefore decided in the Design-Theorising Plane that my animal river-crossing scenario would meet these requirements. Learners would be actively involved in the jumping activity, and this imaginary story could be considered a type of indigenous African folktale. Via email communication, in our Grappling-CoP, we discussed the value of this problem scenario:

> *Sally-Ann:* *I love the 'African-ness' and also the move away from whole-to-part division.*
>
> *Mellony:* *I think this is then quite a powerful investigation of how to cross connect different representations and facets of fractional reasoning and to explicitly bring attention to these connections through the music/ animal jumps rhythm introduction.*
>
> [Grappling-CoP, e-mail communication, 2021-03-24].

5.2.2 Obstacle-Resolution Cycle 2: Deciding on what the unit will be

In our Design-Theorising Plane we grappled also with the question of what the unit should be. We wondered whether we should focus on a musical bar as a unit or on one musical beat (for example, the quarter note) that gets iterated:

> *Tarryn:* *There's been something bothering me about the note values and the bar… we were always saying the bar must be the unit, but it only works in certain cases.*
>
> *Mellony:* *You know what I like about you grappling, about whether the bar should be the unit or the crotchet* [quarter note]*should be the unit. In fact, that's exactly what kids get confused with… that's what I like what you're grappling with now, it's opening up. Nobody can say this is the correct unit, you choose the unit then you work with it. So what you're trying to figure out now is, what's going to be the best unit for your pedagogical purpose… you can look at it from different perspectives, and then the unit changes. What you're trying to figure out is what's the best pedagogical choice for your aim.*
>
> [Grappling-CoP, Zoom meeting, 2021-07-12].

I resolved that in the animal river-crossing scenario, the river-crossing distance and the river-crossing time would both be constant units against which other distances and times can be measured and compared with other animals. The unit, in terms of time, would be the constant time it takes all animals for one river-crossing (River-Crossing Unit of Time - RCUT). The unit, in terms of distance, would be the constant distance of one river-crossing (River-Crossing Unit of Distance - RCUD). This grappling in our Design-Theorising Plane led to my decision that the unit in this scenario would be either the river-crossing (distance and time) or the music bar. However, the distance over which the river-crossing stretches or the time it takes the animals to jump across could be varied, just as the visual representation of distance of a musical bar or the time a bar takes to be played could. In the musical context, the sounds that occur within the music bar could all occur within the same time, but the duration of each individual sound (each beat) might vary. In the Design-Theorising Plane we established that the music bar would be divided into equal beats which could be likened to the equally-spaced jumps in the river. I designed a task where the beats per bar would further explore the constructs of 'fraction as ratio'. The note values (music measure of time) or animal river-crossings (measure of distance) would be the 'fraction as measure' construct. Each beat would therefore need to be iterated a certain number of times to fill the whole music bar and could continue beyond one bar. The iteration of these fractional time and distance jumps (i.e. 'fraction as measure') thus easily allowed for working with fractions greater than one whole, something that, along with equivalence of fractions, the participating teachers had identified as a challenge in their teaching of fractions.

Tarryn: *I'm going to be focusing on equivalence and fractions greater than 1.*

Ms Chaka: *Okay. That fractions greater than will be great.*

Ms Cloud: *That's what they struggle with.*

Ms Fassie: *That will be nice, ja, I agree.*

[Aloe-CoP, 2021-03-12].

In our Design-Theorising Plane we also had to think of a way to explain to learners that the unit could be the river-crossing, but that this could continue past one whole.

Mellony: *I would just keep the one river-crossing as your unit of measurement. So don't complicate if there's any more river-crossings, there was one river-crossing which we have now taken as our standard unit of measurement with all these animals. Because every single jump they did we considered in relation to that river-crossing so we're not going to bring in our different river-crossing. But everything, every run, every distance, every time that they do, we are going to consider in relation to this lovely unit of measurement which is, time of river-crossing, distance of river-crossing, just like in the modern world, we have time as a standard minute or we have a measure as a standard metre. The standard here for these animals is that river-crossing that's the standard of measurement because we didn't have this, these other forms of measurement with these animals, it was the river-crossing…*

Tarryn: *So with this idea, will this game still be able to work?*

Sally-Ann: *Yes, I think you're now sticking to the same river.*

Mellony: *Yes, no extra river.*

[Grappling-CoP, 2021-10-20].

5.2.3 Obstacle-Resolution Cycle 3: Preventing confusion about speed.

In my problem scenario the animals cross the river of the same distance in the same amount of time. I decided on this as I wanted the constant time to reflect the constant time of a music bar. This meant that the animals would cross the river at the same speed (distance/time). The speed at which the animals crossed the river is therefore constant. Rather than considering speed in 'metres per second', in my scenario it could be thought of as 'river-crossing unit of distance' per 'river-crossing unit of time' (i.e. RCUD/RCUT). Each animal, however, would cross the river with a different number of jumps with a different distance and time taken *per* jump.

Early on in the task design grappling journey, we were concerned about the possibility of confusion for teachers and learners in this scenario with regard to the different jumps versus the constant speed. This was an important aspect to address in order to maintain fidelity of the mathematics and the music.

> *Sally-Ann:* *What is happening here, is that they're crossing the same distances (your fraction as measure), but they doing it with smaller steps and maybe it's complicating it to bring in the speed… bring the time factor in as separate… it's trying too many things in one.*
>
> *Mellony:* *My concern is that teachers are going to get confused because they're going to start to think about speed as distance over time and learners might even...*
>
> *Tarryn:* *The thing is, with music, rhythm is always going to have the same time, in a bar…so, the rhythm changes, but the constant is the time in the bar. You're always going to have a bar that is four seconds long, but I can make that in many different rhythms.*
>
> *Mellony:* *The only thing that I'm nervous of them talking about is the speed, because they all get across that river, they all get across the ravine, at exactly the same speed… see this is why we'll talk about the rhythms being at different speeds, but then that's not in relation to distance in terms of distance over time. They're all the same speed but in terms of rhythm they different paces. You have four seconds to cross and how we're going to do it in this four seconds depends on what animal you have.*
>
> [Grappling-CoP, Zoom meeting, 2021-03-25].

I made the decision not to change the scenario, but to rather make this link to speed explicit to the participating teachers. With the goal of simplifying the task for practical implementation by the teachers, I would explain to them that the focus is not initially on the speed of all the animals but rather on the rhythm at which they jump across the river. I recognised that through percussion activities, this scenario could powerfully and rhythmically be clapped out by the learners. The claps and jumps were later represented as fractional representations on timelines and distance lines where the unit is the distance or time of a single unit crossing. Teachers could then use this as an opportunity for subsequent progression or enrichment if they saw it as appropriate in their Grounded-Practice Plane context. This forms part of Lesson 7 (see Section 4.9). Our Grappling-CoP then considered possible problem questions that could be posed to learners to conclude this teaching and learning lesson sequence, as seen, for example, in Mellony's comment, below:

> *So here, our bridge* [river-crossing] *and a bar is the same. You specially chose it so that the crossing of the bridge and the bar is the same, timewise. And so right at the end of this you can talk about speed…. How might we describe the speed of these animals? Well for all of these animals the speed is one bridge in 4 seconds… If the bridge was 4m, the speed of these animals is that they are all covering 4m in 4sec, so they're travelling a metre per second. All of them.*
>
> [Grappling-CoP, Zoom meeting, 2021-04-12].

We were in fact pleased with the opportunities that this obstacle had presented in the problem scenario. We found the resolution of using the speed for complex problem-solving valuable as is captured, five months later, in the following comment from Mellony:

> *Quite a powerful AHA, mathematically and pedagogically, is not giving a specific distance or time of the river of the bar, because it is an abstract unit. We can later say, "What if the river is 10m wide? If it takes 8 seconds to cross, etc."*
>
> [Grappling-CoP, Zoom meeting, 2021-09-24].

We were thus able to maintain the fidelity of the mathematics and the music, simplify the task for implementation by teachers, and use key representations for powerful problem-solving across multiple constructs of fractions.

5.2.4 Obstacle-Resolution Cycle 4: Musical notation limitations and challenges

As traditional Western music notation involves symbols which indicate, among other information, the pitch and the measure of time (note value), it can be cognitively taxing for the music reader (Gaare, 1997; McLachlan et al., 2010). I experienced a similar problem to what is identified here in the literature. Teachers, especially those with no musical background, were not only nervous about having to teach (and read) musical notation to their learners but were also concerned about adding yet another unfamiliar symbolic language into their teaching of fractions. So, for example, in the first Aloe-CoP meeting, teachers expressed concern about their having to read the music:

> *Ms Chaka: I definitely need some sort of* [guidance]. *I haven't a clue!*
>
> *Ms Fassie: And I know zero about music!*
>
> [Aloe-CoP, 2021-03-12]

After the second meeting with the Aloe School teachers I shared the following reflection in an email with my fellow Grappling-CoP members.

> *I'm considering using a percussion line (one line) rather than the whole stave (5 lines) as it takes away the distraction of the note names (A, B, C ...) and focuses only on the note value. It also looks similar to the number line.*
>
> [Grappling-CoP, email correspondence, 21-03-24].

To simplify having to distinguish the pitch of notes (A, B, C… or Do, Re, Mi…) in our Design-Theorising Plane, my supervisors and I discussed the benefit of not using the five-line musical staff and rather using a single-line percussion line. Figure 5.4, below, is a screenshot from one of our Grappling-CoP meetings, where we were trying to find the best way to represent note values on a percussion line and indicated the time duration of each note value.

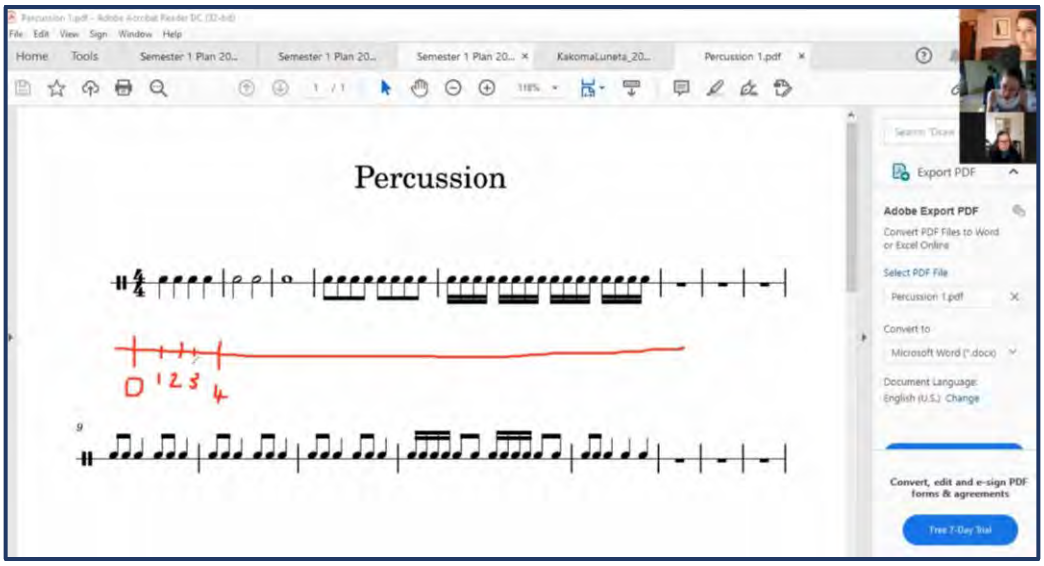


Figure 5.4: Screenshot of Zoom meeting screenshare discussing the use of the percussion line to simplify the musical notation [Grappling-CoP, Zoom meeting, 2021-04-12]

In our Design-Theorising Plane, however, we still encountered obstacles with representing the note values. In our Grappling-CoP, I shared my concerns around the notation consisting only of values which are halved each time (whole note, half note, quarter note, eighth note, etc.) thereby excluding the possibility of linking the music notes to fractions with other denominators (e.g. thirds, fifths, or sixths). A further concern I shared was that my design, by breaking up a whole note into two half notes or four quarter notes, was emphasising the part-whole construct of fractions. This would not be sufficient for developing deep conceptual understanding of the multiple interrelated constructs of fractions. I recognised that using only Western note values would not be sufficient for moving flexibly between multiple constructs of fractions.

Tarryn: *I'm thinking let's ditch the note values, and rather focus on the beats, or the claps, or the jumps across the river, and we use this to show fraction as rate and measure, with no misconceptions or getting stuck in the fraction as part-whole construct.*

[Grappling-CoP, 2021-07-12].

Upon reflection, some six months down the line, Mellony reflecting on the difficulty we experienced when only working with the Western note values, observed, "But the minute we had the music on the music lines, it didn't work…" [Grappling-CoP, 2022-01-25].

A further concern I had about using only the Western note values was that the link to fractions would *only* be suitable in a musical piece which is in a 4/4 time-signature (four quarter notes making up a whole bar). If, however, one were to use the 3/4 time-signature, such as in a waltz, then three quarter notes would make up a whole bar. Three quarter notes could be translated to the fraction three quarters ($\frac{3}{4}$), which in mathematical terms would not equal one whole. I recognised that the naming of the note values might therefore result in misconceptions.

Tarryn: *I felt that the notes and the bars together was creating possibilities for misconceptions… and I felt maybe this is why the teachers in the CoP are hesitant. And they are grappling – so I'd like to make it more simple…It can only work, if it is in 4/4 time-signature…*

Sally-Ann: *But what about the waltz?*

Tarryn: *If we had a waltz, in 3/4-time, we would have a whole bar, made of three quarters [quarter notes]. But for a child to think I've got 3 quarter notes, it looks like that [3/4] as a fraction, but that is not equal to a whole. It doesn't mathematically work. So that could also lead to confusion.*

Mellony: *So the issue of the changing bars is that they need too much understanding of music now to be able to link the music and the maths.*

[Grappling-CoP, 2021-07-21].

From these reflective grapplings in the Design-Theorising Plane, I recognised that I had to re-establish what elements of music would be most conducive to supporting a deep conceptual understanding of fractions and would be least likely contribute to misconceptions. I therefore decided to include additional animals in the problem scenario that would allow for linking to a number of different fractions when reaching the vertical mathematisation stage of the task design. In the Design-Theorising Plane, we decided that first introducing teachers and learners to percussion on a music percussion line would help in simplifying the design for implementation in the classroom. We decided also that, rather than focusing on note value representations, I would use an adapted notation of percussion. First, as shown in Figure 5.5

below, we trialled using a percussion convention of using an X on a stem, to represent claps. We subsequently noted that this could become confusing for teachers and learners in trying to distinguish the claps from the number line markings.

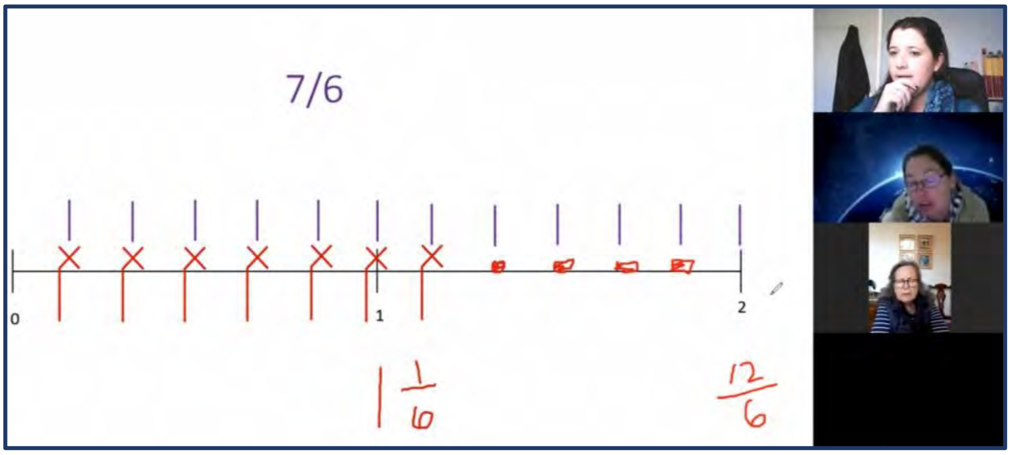


Figure 5.5: Screenshot of grappling over best way to represent percussion [Grappling-CoP, Zoom meeting, 2021-07-12]

We then opted to use an X (without a stem) to indicate hitting an unpitched instrument, for example, clapping hands, and a square to indicate a musical rest (a quiet), as shown in Figure 5.6 below.

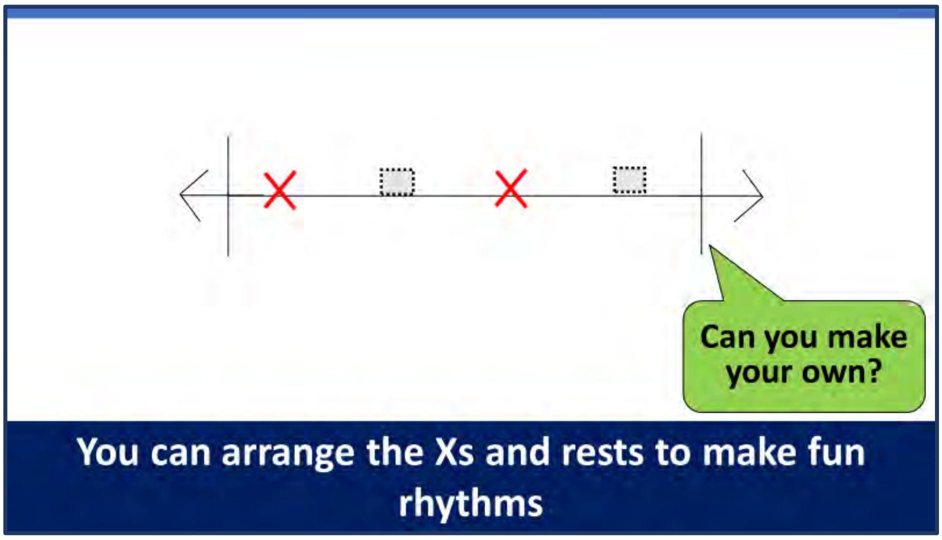


Figure 5.6: Adapted informal representation of percussion [Grappling-CoP resources]

We resolved that this representation would suffice for linking animal jumps to the percussive claps, and meeting the key questions of simplicity and designing a useful key representation. To uphold the fidelity of the music, I decided that at a later stage in the teaching and learning sequence (Lessons 5 and 6), I could expose learners to the formal Western staff notation of musical note values without making the tasks too cognitively taxing and without taking the focus away from the fractional reasoning.

5.2.5 Obstacle-Resolution Cycle 5: Visual placement of animal river-crossing jumps linked to the percussion claps

Obstacle-Resolution Cycle 5 derived, in a sense, from the previous Obstacle-Resolution Cycle 4 because the resolution of the adapted informal musical notation led to fresh obstacles of fidelity in the matching of musical and mathematical representations. I linked the animal jumps and the percussion claps, and (in Obstacle-Resolution Cycle 6) progressed to linking them both to the mathematical number line. My first key concern in this cycle was the animal river-crossing jumps. In the Design-Theorising Plane, my supervisors and I grappled with when to clap and how to notate the animal jumps. In the problem scenario the animals jumped on rocks across the river and a bystander would hear sounds as each animal jumped. Would we, however, notate the jumps at the starting point of the jump (the take-off), the landing of the jump, or the full duration of the jump? We now confronted the challenge of selecting the option that would best link to the fidelity of the subsequent key musical and mathematical representations.

I had observed from the reflective feedback I had got from the participating teachers that the children informally drew the jumps in a variety of different ways (as shown, for example, in Figure 5.7, below).

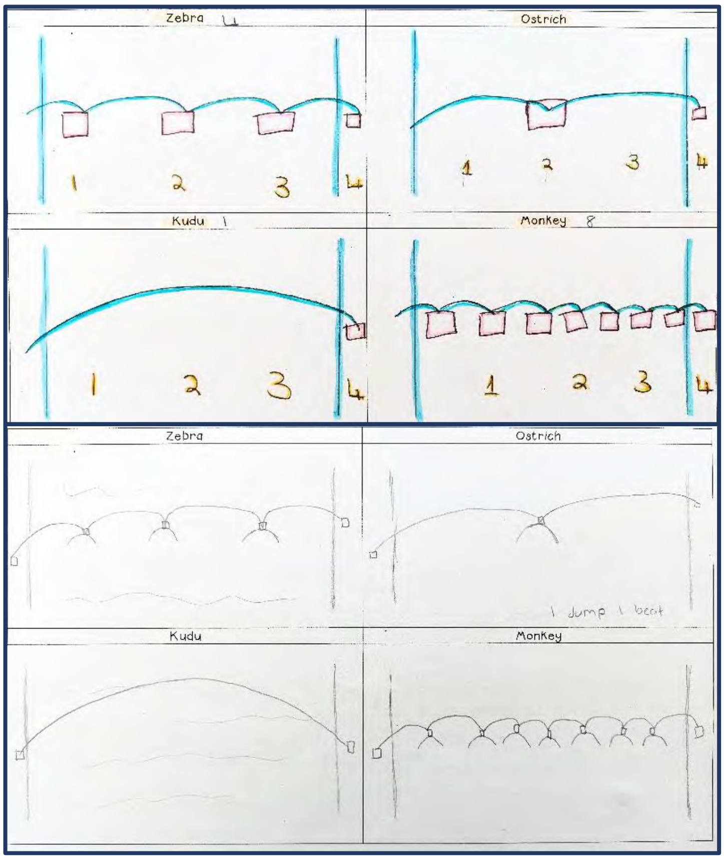


Figure 5.7a

Figure 5.7b

Figure 5.7a and 5.7b: Examples of learners' representations of animal river-crossing jumps

In the first representation (Figure 5.7a), the learner represented the duration of the jumps with curved lines and the animal landings with a square. This meant that the animals would land on the opposite riverbank in the correct number of jumps. Some learners, however, represented the animal jump starting point (take off), and then the animals would not land on the bank in the required number of jumps. In the second representation (Figure 5.7b), a learner represented the animal's starting and landing points, which could lead to misconceptions. For example, the zebra (with its four jumps) made five sounds. We quickly recognised in our Grappling-CoP how this sort of inaccuracy would threaten the usefulness of the initial problem scenario and the fidelity of both the music and the mathematics, especially when linking the informal animal river-crossing jumps to the informal musical representations.

I share some of our grappling in the Design-Theorising Plane in the excerpt of a Grappling-CoP Zoom meeting below.

> *Tarryn:* *What I was concerned about when I first started working on this idea was that if you look at notes in a music bar, you will have the notes here, in the centre of the bar… and then the number line is the next whole. The fourth quarter note is before the bar ends, which is why I've placed the jump right on the line here, each time to show that the fourth jump takes us to the whole. The four quarters, is the whole, it is the end of the bar, it is the riverbank.*
>
> *Mellony:* *...you need to start at the zero line, you've got your start 0 line and you would need to start on that line.*
>
> *Tarryn:* *To show that it's the whole, but when you have the music bar, it is going to be before the bar line.*
>
> *Mellony:* *So put the lines, so that the first line is the line that they jump off. It's the zero, it's the river bank… And then you've got four jumps 1234, so yeah you've got your start line, you've got your finish line and you've got your three lines in between, and then your music notes on the land. It's got to be drawn like this so that exactly four of them sit when you draw the notes that's where they land.*
>
> [Grappling-CoP, 2021-03-25].

In light of this example, we decided that in the practical activity, as the learners jump (like animals jumping the river-crossing), they should clap on the land, on the 'p' of the 'juuum**P**'. We shared possible visual representations of the animal jumps over electronic communication such as WhatsApp and e-mail. Below is a rough representation Mellony sent our Grappling-CoP WhatsApp group, followed by my attempt at representing the animal jumps with the informal percussion X on the landings of the animal jumps.

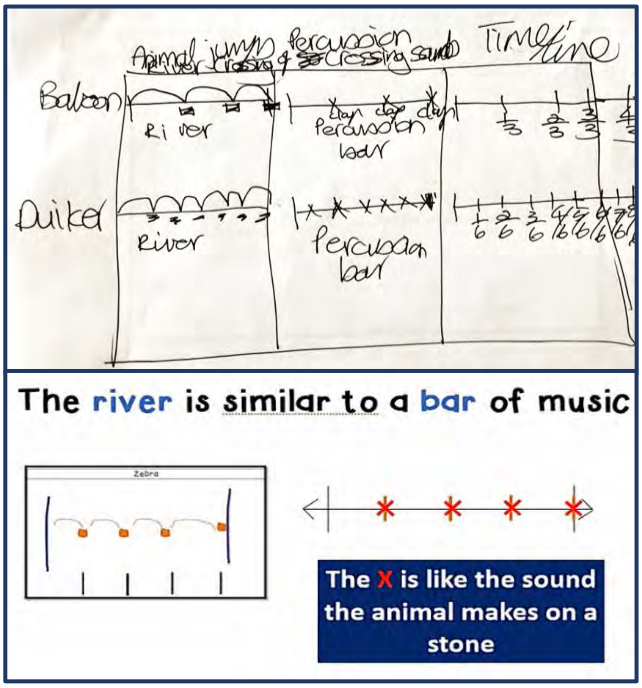


Figure 5.8: First attempt at representing the animal jumps and musical percussion claps with fidelity

This representation focusing on the landings of the animal river-crossing jumps led, however, to fresh obstacles of fidelity in the musical representation. I was concerned from the musical aspect about where learners doing this activity would clap together. In music, if two musicians clapped rhythms of different note values (e.g., if Musician A were to clap two beats per bar and Musician B, four beats per bar), the first time they clapped together would be on count 1. However, in mathematics, the notion of equivalence should mean that Musician A's first clap (of two) would align with Musician B's second clap (of four), because $\frac{1}{2} = \frac{2}{4}$. Here, yet again, I recognised the potential for misconceptions to occur. I subsequently tried a representation where the focus was on an animal's starting point (take off) and on the start of a percussive clap, as shown in Figure 5.9 below.

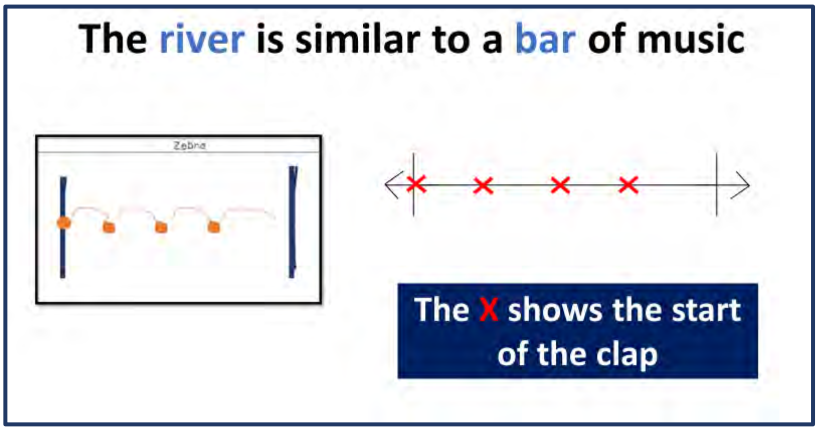


Figure 5.9: Second attempt at representing the animal jumps and musical percussion claps with fidelity

According to this representation learners would clap on the ‘j’ of the ‘**J**uump’:

> *Mellony:* *Juump, Juuump… but now we’re going to clap on ‘J’, we’re not going to clap on the land.*
>
> *Tarryn:* *…and so you would start together.*
>
> [Grappling-CoP, Zoom meeting, 2021-03-25].

This representation cohered with neither the problem scenario story, nor with musical notation, nor with the mathematical representation on a number line that was to follow. We continued this grappling in the Design-Theorising Plane for over six months, trying to establish a key representation that would stay true to the animal jumping scenario and the musical representation without possibly causing misconceptions for the mathematical representation.

> *Mellony:* *But where this doesn’t then link up is that we’ve got the sounds based on the animal jumps, and the animal doesn’t have a sound when he leaves the bank. The first sound is when he lands. We are clapping animal percussion, it’s not percussion when we all start clapping at the same time.*
>
> *Tarryn:* *The sound line would only start there.*
>
> *Mellony:* *But you can start with a quiet.*
>
> *Tarryn:* *Does it have to match perfectly, visually?*
>
> *Sally-Ann:* *You’re trying to do too much at one time. Early on in our discussion, we said that in musical notation, the sound starts at the beginning of the bar, on the zero. But I quite like the idea of a rest, the jump starts on a silent point, a rest in music. Would that resolve the infidelity?*
>
> *Tarryn:* *No…because it will be adding in a beat. A rest is also a beat… if we look at the animals jump rather than the landing or the start, it will help.*
>
> *Mellony:* *That’s why the music notes are in the middle.*
>
> *Tarryn:* *If we move away from the sound of the land, and think that each jump in itself is a duration of sound.*
>
> *Mellony:* *Jump, jump, jump [clapping]… What’s problematic is we have the clap start on the zero line, because at 0 it is 1 clap, and it’s problematic when we go to the number line…. Okay, so at that line, is the end of the sound of the first clap…*
>
> *Tarryn:* *…and the beginning of the sound of the first one.*
>
> [Grappling-CoP, Zoom meeting, 2021-09-24].

As I show in Figure 5.10, below, I then tried adding in dotted lines to show that the musical clap starts at the first line (that is, at zero seconds on a timeline) and is then held for one count (which would be one second on a timeline).

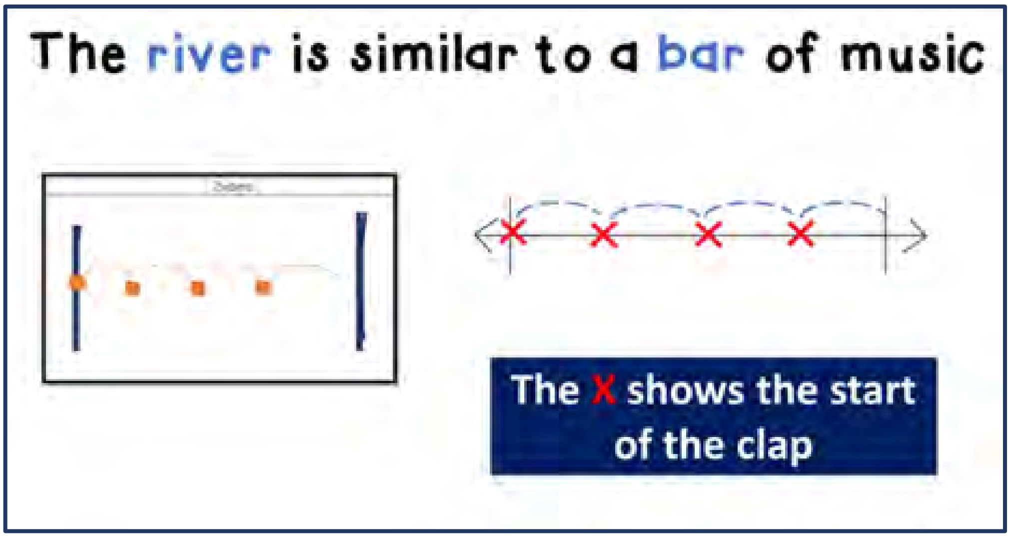


Figure 5.10: Third attempt at representing the animal jumps and musical percussion claps with fidelity

Part of our grappling, in the Design-Theorising Plane, for finding the best representation that maintains the fidelity and is appropriately simplified for classroom implementation, is captured in the following excerpt.

Mellony: *It's not the count, but it's the end of the sound…. The line needs to show the time. What if the cross had a line going across, horizontal? Almost like an arrow, so that this clap had the duration of the time interval. So that there's a continuity… so that's the time that it lasts.*

Tarryn: *I also want to say, that on the percussion line we should have the zero or the one. In other words we are looking at the duration, more than the claps.*

Sally-Ann: *And then whether you do duration or you do just the sound of each jump is also a question at issue, because we've not got a piano key where you can hold the sound.*

[Grappling-CoP, Zoom meeting, 2021-09-24].

A month later, Mellony, in our Design-Theorising Plane, raised something that would lead us to the next Obstacle-Resolution Cycle (6). In our discussions and grapplings, she pointed out that when we refer to jumps and claps, we are, in fact, focusing on the fraction as ratio construct, in other words, a rate of jumps per river-crossing or claps per bar.

Mellony: *The problem is representing percussion because we're not going to go jump and clap on the land we're actually going to start clapping together and now we're talking beats per bar and everything that we do differently. They are simply showing claps per bar and now we're not talking number lines anymore now we're talking ratio.*

[Grappling-CoP, Zoom meeting, 2021-10-20].

The Protea School teachers identified the misconceptions that their learners had when it came to reading a number line, so we discussed how this related to representing the claps and the animal jumps:

> *Tarryn:* *We're not going to clap on the land, we want to clap for the jump in the air. Which is what we do in music. It's not there you clap, you hold the note. And it's easier if you sing, or if you play the violin, or play the piano than if you're doing percussion. So we're going to try and just clap and hold, clap and hold.*
>
> *Ms Savuka:* *...what we do now in maths was reading a scale. You don't count the intervals, the little lines, you count the spaces.*
>
> *Tarryn:* *Yes, exactly.*
>
> *Ms Savuka:* *You don't hear how many spaces there are.*
>
> *Ms Clegg:* *And they really battle with that. Because they end up counting the lines and then it doesn't match.*
>
> [Protea-CoP, 2021-10-28].

The teachers at Aloe School, similarly, shared their reflections with me on the difficulty their learners experienced with the mathematical and the musical notation linked to these jumps. This was confirmation of the changes that we had to make in the task design if we were to maintain the fidelity of both the mathematics and the music while also simplifying our key representations for classroom implementation.

> *Mr Sontonga:* *In Grade 4, I asked them, "Where have you seen this before, the 0 and a 1 on a number line?" They said in their NumberSense books* [Mathematics Workbooks]. *So they could see where we are going with this. But because they saw four quarters they were drawing 4 jumps or claps in between the bars, as opposed to, 1, 2, 3, 4....*[pointing].
>
> *Tarryn:* *Yes, that's why we're changing to clap the jump rather than on the land, to show the spaces.*
>
> *Ms Fassie:* *Because that's where the note is.... Because my music children had difficulty today, because the X on the line is not like it is in music. So I said to them, "It's similar to music, but it's not exactly the same." And so they struggled with the change over from Music to the way it's now written.*
>
> [Aloe-CoP, 2021-11-05].

The realisation that Mellony alerted us to, namely that the percussion claps per bar focused on the fraction as ratio construct, which in mathematical notation, is *not* represented on a number line, helped move my design thinking forward. As I explain in my discussion of the next cycle, it was an important turning point in my task design grappling journey.

5.2.6 Obstacle-Resolution Cycle 6: Aligning the musical and mathematical linear representations (rather than super-imposing them)

Musical notation is a symbolic representation of pitch and measure of time. It has, as I discussed in Obstacle-Resolution Cycle 4, limitations. What I had initially seen as an easy connection between the musical line (percussion line or Western staff) and the mathematical number line proved to be more challenging than I had expected. I experienced an obstacle in integrating the symbolic representation of the musical line and the number line in a way that would uphold the fidelity of both the music and the mathematics. In formal musical notation, notes (either note values or percussion symbols) are visually placed in the middle of the bar as illustrated in Figure 5.11, below (created on MuseScore®). There is a discrepancy between what one hears in music and how one traditionally notates the musical note values: the note does not line up with the markers on the number line. This could cause confusion if, for example, I were to show time on a number line.

Figure 5.11: Visual representation of some musical notes, created on MuseScore®

Visually notated, the note values are centre-aligned, between the musical bar lines, rather than left-aligned against the starting bar line. In Figure 5.11's example, above, a whole note is held for four beats/counts and is notated in the middle (half way) of the music bar. The whole note is, however, played (held or sustained) from the start of the bar (time 0 seconds) to the end of the bar (time 4 seconds). Similarly, the two half notes in the next bar would be played and sustained from time 0 seconds to 2 seconds, and time 2 seconds to 4 seconds. This would, however, be positioned at the visual distance of one third and two thirds of the notated music bar. This visual placement at thirds does not represent that the first note is played from the start (time 0 seconds) of the bar to halfway (2 beats is half of 4 beats per bar) and the second is played from the middle to the end of the bar (for another 2 beats).

The Aloe teachers and I had already identified this obstacle in the second of our CoP meetings. As the excerpt below shows, despite having little background in musical notation, and looking at the musical representations from a mathematics teacher's perspective, the teachers had seen that there was a potential problem here. We thought we had solved it, by placing the note values in the middle of the number line markers, as Figure 5.12 below shows, a photograph of my drawing on the whiteboard.

Tarryn: *Just like I drew the zebra jumps, I can also show it in music. So when you did the 4 jumps for the zebra, we have four notes in the bar or 4 jumps in the river and we call that a quarter note. We've got 4 quarter notes and that's what it looks like.*

Ms Chaka: *Sorry, now I'm confused, because there are 4 in between the river there* [points at music bar and notes]. *I would have thought that's 5?*

Ms Cloud: *Ja.*

Tarryn: *So that is where we're going to have to look at the idea of a fraction on a number line compared to the music notes. So if I were to draw a number line* [draws on white board], *and I have my zero and my one. I would have three lines: one quarter, two-quarters, or a half, three quarters and my whole would be the four quarters.*

Ms Chaka: *Ja.*

Tarryn: *So if I were to superimpose the two, because this* [points] *isn't a 0 and a 1. In music we divide our notes in a bar. The bar is the whole. That's how we represent it in music* [points], *but if I were to draw it together, it would be like this* [points].

Ms Chaka: *Ohh… Can I take a photo of that?*

Tarryn: *I thought this could be a possible misconception. Perhaps what you could do is go back to your image with the jumps and let them put the notes in between the jumps?* [Draws on the whiteboard (Figure 5.12, below).]
[Aloe-CoP, 2021-03-26].

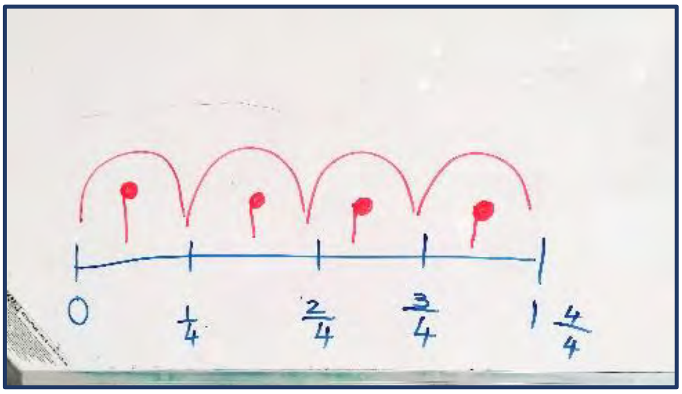


Figure 5.12: Early attempt at aligning the musical and mathematical representations in an Aloe-CoP meeting [Aloe-CoP, 2021-03-26]

In the Design-Theorising Plane, we grappled with how to address this placement of note values and the possible misconceptions it could lead to. The screenshot in Figure 5.13, below, is of our Grappling-CoP Zoom meeting. The example shows the placement of four quarter notes that could be misinterpreted as fifths if looking at the stems of the note values.

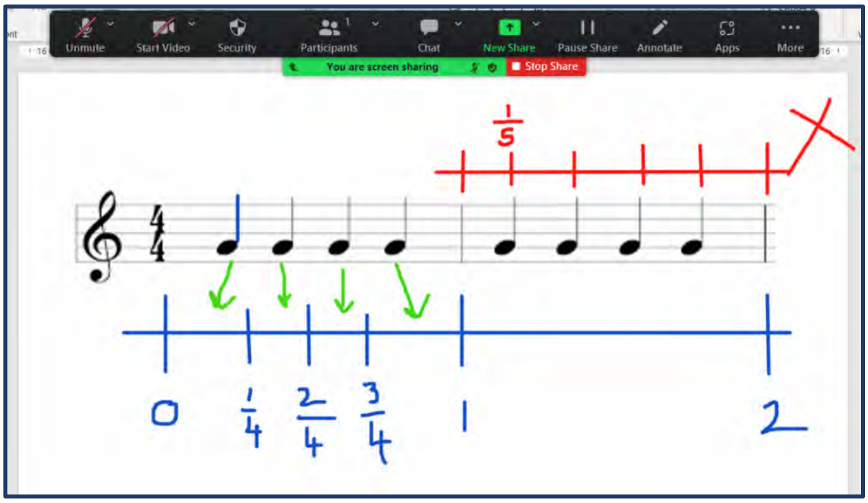


Figure 5.13: Screenshot from Zoom meeting of recognising the obstacles in placement of note values in musical notation [Grappling-CoP, 2021-04-12]

A further complexity in the musical notation is that composing and playing involves working with ‘beats per bar’ to ‘keep time’. Often one hears musicians counting ‘1234, 1234,’ for example. In terms of a time progression (measurement of duration of time), a musician starts at the zero second time and holds a note for, for example, one second (0 to 1 seconds on the timeline). However, this holding of the note would be counted as ‘beat 1’, with the 1 being verbally said at the start of the note or the clap (the mathematical 0 point in time). However, zero is not equal to one ($0 \neq 1$). This mirrors Gaare’s (1997) point that in musical notation “the visual event must be apparent as the direct translation of the auditory event, requiring as few additional thought processes as possible” (p. 18). He also emphasised the importance of visually proportional note spacing to represent time. My use of informal musical notation for Lessons 1 to 4 (shown here again in Figure 5.14 below) adhered to McLachlan et al.’s (2010) suggestion that a graphical notation, rather than an abstract symbolic one, can make the reading of music less cognitively taxing. This did not, however, resolve the issue I faced of matching my visual representations of the music line and the number line.

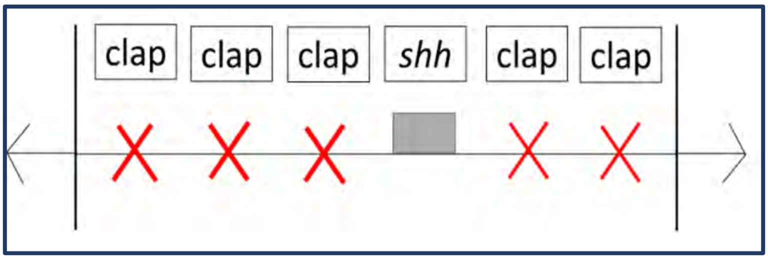


Figure 5.14: Adapted, informal representation of musical beats

As I shown in Figure 5.15, below, we attempted in our Grappling-CoP to adjust the notation of the musical bar to line up with the number line (distance and time). We first tried representing a note value visually on the zero of the timeline with an arrow to indicate the duration (for example, that the whole note would be sustained for four seconds).

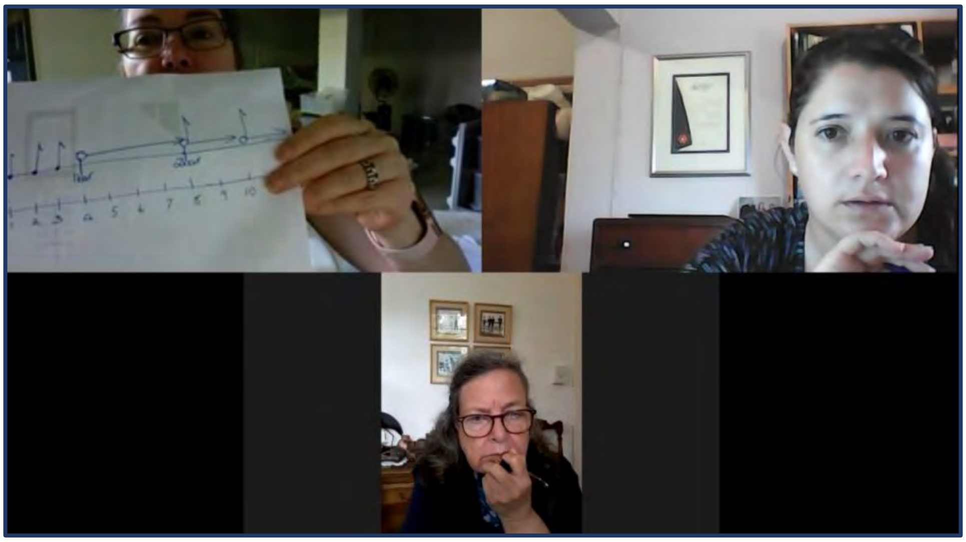


Figure 5.15: Screenshot showing our grappling with adjusting the musical notation to match the mathematical number line [Grappling-CoP, 2021-04-12]

I voiced my concern that this sort of adapted notation would move too far from the fidelity of music, possibly causing later misconceptions also for learners furthering their musical studies. I was also mindful of Bresler's (1995) co-equal, cognitive style of arts integration: upholding the importance of both the art and the 'academic' subject.

Tarryn: *What I'm seeing here, Mel, is where you are placing the crotchet* [quarter note] *or the note, it's fitting in between the lines on the timeline.*

Mellony: *So that we need to think about… Sticking it between could be confusing. So I think you have to actually do it on the seconds.*

Tarryn: *But in music we don't. In music we draw them in the middle…*

Mellony: *Uhm, when we've got our second timeline… and when we play our music we play our crotchet* [quarter note] *at the start and it lasts one second…*

Tarryn: *So are you starting on the zero of the timeline? The first* [note]*?*

Sally-Ann: *It gives musical insight as well as time as a measure insight. You measure the second after its done. The note you count as it starts.*

Mellony: *So in relation to the timeline, we start at zero, instantly we start, 1234. And then it lasts that long.*

Tarryn: *So are we moving away from this idea where we place it in the middle?*

Mellony: *Yes.*

Sally-Ann: *So you can't exactly align the two number lines.*

[Grappling-CoP, Zoom meeting: 2021-04-12]

This grappling with the obstacles of integrating music and mathematics to uphold the fidelity is not new to the literature. As mentioned in Section 2.7, various authors describe reluctance for curriculum integration precisely because of this challenge of upholding the quality and standards of the subjects being integrated and ensuring they not be diluted at the expense of another subject (Drake & Burns, 2004; Naidoo, 2009; Fraser, 2013; Barnes, 2015; McPhail, 2018; Kneen, 2020; Pluim et al., 2020).

Almost a year after the Aloe-CoP discussion and the start of our Grappling-CoP discussions, we eventually reconciled ourselves to the fact that the traditional, formal notation of music could not be superimposed over the number line (representing time and distance). Through our various grapplings with confluences and contrasts, we reached the decision to not overlay the representations. I realised that we did not have to superimpose the representations in order to meaningfully show the similarities of the notation conventions. The synergies between the musical and mathematical representations were still a powerful context for problem-solving allowing for moving flexibly between multiple constructs of fractions. The revelation my supervisors and I had is captured in the excerpt below.

> *Sally-Ann:* *I think something's happening here like Bresler's co-equal thing. You're trying to hold onto it. And I think what our grapplings have led us to is that there is this distinction that has emerged that has created problems for us. So the co-equivalence is creating problems, which does not deny the value of integrating music with maths to get some of the concepts across. Then you're recognising, because there are these distinctions between rate and measure and duration, it's becoming more complex…*
>
> *Tarryn:* *Yes, and we're trying to represent what we hear, to make it look the same, but it's not. We don't actually have to put the note, the X, on the number line.*
>
> *Mellony:* *So what we need to do here, is to draw some distinction between what we're going to do with the number line, when we're thinking distance, time, where there was a starting point of zero, whereas with the percussion we stick it in the middle of the line… We can see the similarity but we can see what's different as well… because we keep looking at this bar line and we see the similarities. But we're conflating two concepts.*
>
> *Sally-Ann:* *We were moved to percussion because of making it accessible to teachers, compared to music note values. It may be that the duration will take you into note value but the percussion won't necessarily. You need to show that there are different stages towards a clear conceptualisation of fractional knowledge, so you start simply with the percussion, whether you move beyond that, maybe that is where your progression happens?... So I think your challenge now is making our struggle a little bit invisible and then the parts that we want to make very clear you make very accessible… explicit.*
>
> [Grappling-CoP, Zoom Meeting: 2022-01-25].

I therefore decided to shift away from my intention of superimposing the musical and mathematical representations to rather showing the possible alignment of the two. This alignment, rather than overlaying, would maintain the fidelity of the mathematics and of the music, in a way that supports teaching and learning of fractions. Our Grappling-CoP's AHA-moment allowed me to move forward. I now focused on aligning the original problem scenario of animals crossing a river (constant distance, constant time) with different numbers of jumps. I then separated the fraction as measure construct (measure of distance, time of river-crossing, and the music bar) and the fraction as ratio construct (jumps per river-crossing and the beats or claps per bar). I designed tasks that allowed for moving between these concepts, which is what is recommended in literature (Lamon, 1999; Siemon et al., 2015; Siemon & Luneta, 2018; Barbieri et al., 2020; Getenet & Callingham, 2021). Mellony's comment, below, captures all this.

> *I think a big AHA is here, when we're dealing with percussion claps, we're dealing 4 beats per bar - that is a rate* [emphasis]. *We don't normally put rate on a number line. This is a different concept of fraction as ratio to what we're going to do on the number line. When we say how long does it take to jump? That's a length of time, that is fraction as measure. If the river-crossing is 1 unit that's fraction as measure.*
> [Grappling-CoP, Zoom meeting: 2022-01-25].

From the first iteration in the Grounded-Practice Plane at Aloe School, I therefore adjusted the sequencing of the task design and shared this with the Protea teachers for the second iteration. Lessons 1 to 4 focused on linking the animal river-crossing jumps to the fraction as ratio construct. Just as a zebra would make four jumps per river-crossing, so too learners could clap four zebra claps per musical bar. Progressing from the fraction as ratio construct, and based on our Design-Theorising Plane discussions, I designed a new Lesson 6. As I described in detail in Section 4.8, the lesson introduced learners to the alignment of the number line and the musical Western staff with note values, moving to the fraction as measure construct. (See also Lovemore et al. (2022a) for further discussion of this key representation aligning the number line and musical line). I chose not to adapt the musical notation. Using the 4/4 time-signature was the most suitable for linking fractions and music notes. I therefore decided I would ask the participating teachers to keep the focus on animal jumps that corresponded to the note values whole, half, quarter and eighth notes. I acknowledge, however, that more complex options are possible. I intend exploring how this could best be achieved for future lessons where learners will have progressed in their mathematical and musical knowledge. For this Lesson 6 though, I stipulated that only the following animal jumps be used:

- Kudu jumps (1 whole jump per river-crossing corresponds with whole notes in music)
- Ostrich jumps (2 jumps per river-crossing correspond with half notes in music)
- Zebra jumps (4 jumps per river-crossing correspond with quarter notes in music)
- Monkey jumps (8 jumps per river-crossing correspond with eighth notes in music)

As I described in Section 4.3, I designed the key representation of a river-crossing number line with the river-crossing as the unit. The Protea learners used this to compose their own animal river-crossing song with a combination of the different animals jumps. They then matched the note values printed on transparent cards cut to exact size where one whole note card fits into one whole music bar, two half note cards fill the measure of one whole bar, thus satisfactorily showing the similarity between the animal jumps and the music note values, where the measure of time is considered. Figure 5.16 below shows the actual resource with the music note values on transparency cards.

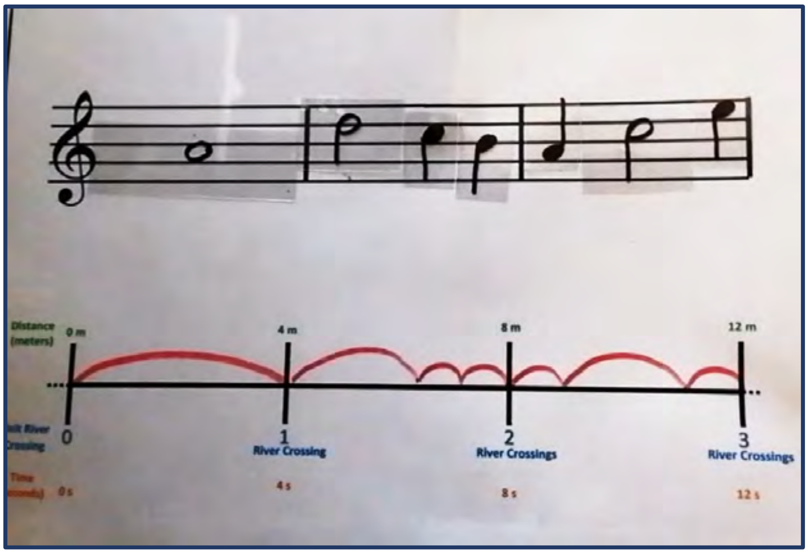


Figure 5.16: Screenshot from the demonstration video of how to use this resource

With help from my two supervisors in the Design-Theorising Plane, I then developed the aligned resources into a triple number line (three parallel number lines). This resource allowed for the fraction as measure construct to be further developed, and formed part of Lesson 7 (see Section 4.9). The river-crossing number line, where the unit is the river-crossing, remained constant, and, above it, I put a number line indicating distance. Below it, I put a timeline indicating the duration of the animal jumps or the music notes. The units of the distance number line and the time number line signified different variables, so creating powerful opportunities for problem-solving (e.g., If the river is 10 metres wide and it takes all animals 4 seconds to cross then… etc.). In our Grappling-CoP we trialled various ways in which learners could use this key resource to solve complex problems, moving flexibly between multiple constructs of fractions. See Figure 5.17, below.

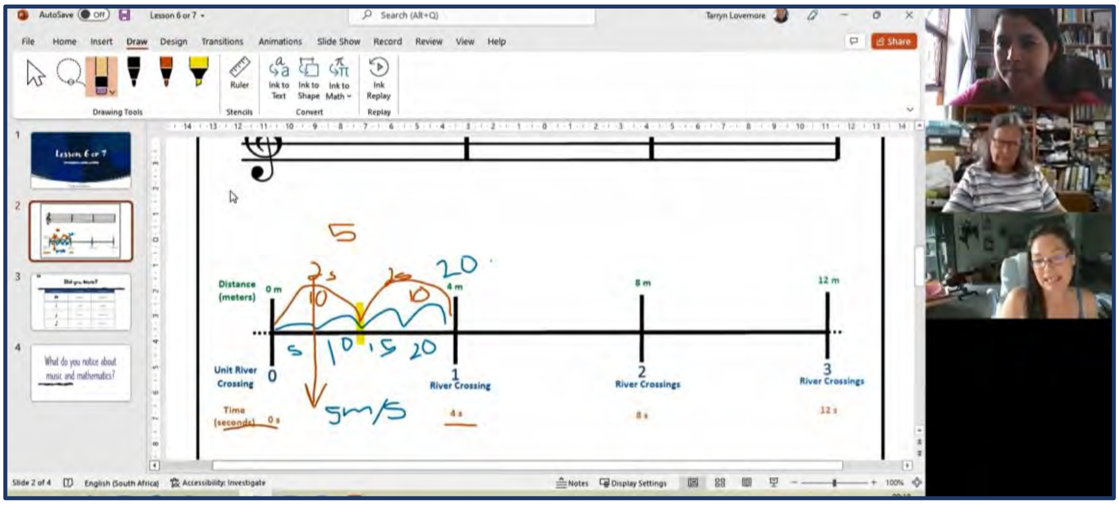


Figure 5.17: Screenshot of screensharing over Zoom: Grappling with design of questions to promote complex problem-solving using the key representation [Grappling-CoP, Zoom meeting, 2022-02-01]

I then designed questions for the Protea teachers that required that learners use this key representation, for example:

> *Zebra does four jumps per river-crossing. If one river-crossing is x metres, and it takes y seconds to cross, and Zebra jumped six jumps, how far would he be in metres? How long would it take Zebra to get there? At what speed did Zebra travel?*

In the Design-Theorising Plane, we recognised the potential value of this key resource to support learners in solving complex problems requiring fractional reasoning with the goal to deepen learners' conceptual understanding of fractions. Mellony, in our Grappling-CoP, summarised this realisation as follows:

> *Mellony:* *The power here is that you bring multiple aspects of fractional reasoning into play. The learning here is about shifting attention between different wholes and proportional ways of working. That's the power of this and integrating into music.*
>
> [Grappling-CoP, Zoom meeting, 2022-01-25].

I shared these resources with the teachers at Protea-School. Their experience in the Grounded-Practice Plane was invaluable. Their reflections, after trialling this key resource are discussed in detail on Section 4.8 and 4.9, but I share below some key comments from their highlighting of how this new resource had supported their learners.

> *Ms Clegg:* *I had to remind them, "So remember now, one river-crossing is 10 metres. And now if the ostrich had to jump five of these, how many would there be? If one river-crossing is 10, how many would 5 be?" So that's what I said… I think this is where it comes in so handy. Have them at the start of*

each question, to set out the distances, and then use it for each, to answer the question. It really is a great tool.

Ms Savuka: *I saw it now with one of the groups that was struggling, was Yvonne and Brenda who are working together. They're not the greatest at maths but they wanted to work together and help one another, and they didn't understand the question at first. We read the question and I said, "Let's try and draw the kudu jumps." It was the question with the two and a half. "Let's jump and let's do it on the number line." And they did it. And it was amazing that they could see that each one was now 10, and they saw where they landed and how many metres it was, and then they answered correctly. But it was just really wonderful to see, that they could find the answer on their own.*

[Protea-CoP, 2022-08-15].

It is interesting that my final key representation aligning the mathematical number line and the musical line is very like what was first discussed in the Aloe-CoP in the early stages of the task design process. This points to the cyclical, iterative nature of design-research described by various authors (for example, Plomp, 2007; Gravemijer & Cobb, 2006; Gravemeijer & van Eerde, 2009; Prediger et al., 2015; Bakker, 2018).

5.2.7 Obstacle-Resolution Cycle 7: Difficulty with visual representation of note values

In one of our Grappling-CoP Zoom meetings, guided by teacher reflections from the Grounded-Practice Plane, we recognised that the musical notation used in Lesson 6 to represent a whole note (***O***), might be mistaken for a zero in mathematics (0) (see Figure 5.18, below). We again explored how best to represent the musical notation.

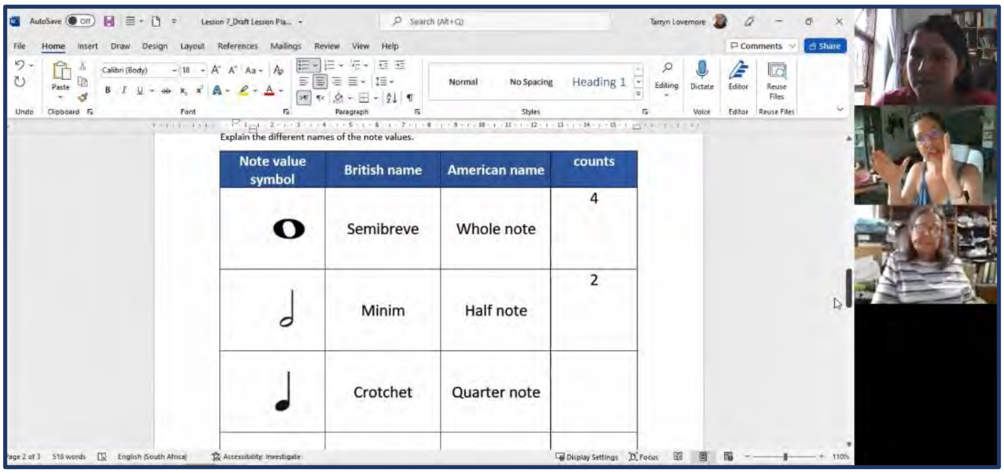


Figure 5.18: Screenshot of our grappling around the musical notation symbols [Grappling-CoP, Zoom meeting, 2022-02-01]

To maintain the fidelity of the music, we decided not to adjust the notation. We decided instead that our designed resources would clearly show the musical note value notation. I carefully designed the resources in Lesson 6 (as shared in Section 4.8 of the previous chapter and in

Appendix 8) with a musical notation software programme, MuseScore® so as to accurately represent the note value. I gave the note values to the teachers as a ready-made resource where learners would just have to match the correct size cards with the corresponding animal river-crossing jumps. They would not have to notate the music note values accurately according to conventions. This also helped simplify the implementation of the task design in the Grounded-Practice Plane. Our Grappling-CoP discussion below shows our thinking around making the resources and representations accessible for teachers without a musical background, and ensuring the music-mathematics integrated lessons were less cognitively taxing (Gaare, 1997; McLachlan et al., 2010).

Tarryn: *…but how do we teach them the note values?*

Sally-Ann: *Don't teach them, just expose them to it, and it's right at the end bringing it back to the lovely integration… I can see a little pack of music cards and then they must choose to make up the equivalent of one whole.*

Tarryn: *And they can place it so it links and we can even link it to the animal jumps… so they can overlay it on top, but without bogging them down with the names of the note values.*

Mellony: *Let them put it on the music staff, because this is at the end. Then the music teacher can play it.*

Sally-Ann: *You can have a 'Did you know?' and tell them the names of the note values. In America they call it this… the other names are…*

[Grappling-CoP, Zoom meeting, 2022-01-25].

I was satisfied that my task design would uphold the fidelity of the music in a way that, although simplified for classroom implementation, would support fractional reasoning and understanding.

5.2.8 Obstacle-Resolution Cycle 8: Uncertainty about including the musical rest

I wanted to introduce learners to the musical rest where time continues but there is no sound. Musical rests have their own notation. In our Design-Theorising Plane, we discussed that rests would be useful in the percussion tasks. They would provide learners with variety beyond simply clapping. I was uncertain of how to include the idea of the rest in the animal river-crossing scenario. I discussed this in the Grappling-CoP, as seen in an excerpt of our discussion below.

Sally-Ann: *So we're thinking, what would be an alternative sound, so I actually suggested a splash noise as an alternative sound .*

Mellony: *I think, for now, just keep it simple… And they're always clapping… and we're dealing with fractions iterating… The rest idea must go in the* *animal jumps but include in the rhythm playing… Okay, and we hear it every time in your crossing, if you want, you can have a jump but on the next jump it can just be a splash.*

Tarryn: *We need to show them that there's still time, it is still happening…*

Mellony: *So I think in the beginning, they just cross the river and hit the rocks okay. Then in the percussion they have rests, yes, and then that gives them the creativity to play with.*

[Grappling-CoP, Zoom meeting, 2021-10-20].

I decided, from discussions in the Design-Theorising Plane, that I did not need to add other, possibly confusing elements to the scenario. Instead I thought I would rather introduce learners to the idea of a rest through my own informal percussion notation, by using a square on the percussion line (as shown earlier in Figure 4.5). This allowed learners to compose their own percussion rhythms with some variety. I also decided to first introduce the rest with learners making a '*shhh*' sound, to show them that a rest means that the time is still passing, something is still taking place, but that the sound was quiet. As their learners progressed, the teachers could decide if they wanted the children to stop saying '*shhh*' on the rest count and just keep the time with no sound. Ms Savuka, in the Protea-CoP, commented on the usefulness the YouTube video body percussion 'karaoke' of the popular song *Uptown Funk* by pop-artist Bruno Mars that we had used in Lesson 3 to informally introduce learners to the musical rest idea.

Ms Savuka: *It was nice cos the clapping video also showed what a rest was. That even counts… It counts as a beat but there is no sound. So it was nice to show them that as well.*

[Protea-CoP, 2022-02-10].

Based on these teacher responses and on my grappling with my two supervisors, I then felt confident about including the informal representation of the musical rest in the percussion activities. I see this as something that could be explored further also for situations where learners might have begun to use the formal, traditional Western notation.

5.2.9 Obstacle-Resolution Cycle 9: Finding progression across grades

My next task design challenge was to consider how progression could take place across the grades (Grades 4-6). Teachers, in the Grounded-Practice Plane reflected on the differences in how the learners managed with the activities. For example, the Grade 5 learners at Protea School managed with the animal river-crossing jumps and informal notations more easily than the Grade 4 learners. Reflections from Aloe School showed that the Grade 6 learners were able to complete the lessons and activities in a shorter time than the learners in the Grades 4 and 5.

Early on in the task design journey, I realised that the problem-solving with the triple number line and linking the distance and the time of the animal river-crossing jumps would allow for more exploration of the idea of speed (as mentioned in Obstacle-Resolution Cycle 3). In the Design-Theorising Plane we saw this as an opportunity for teachers to extend their learners with complex problem-solving requiring fractional reasoning and understanding.

> *Mellony:* *Yes, the speed of crossing the river is the same… So if the Grade 6 teachers at the end of this, wanted to connect this to speed, they can.*
>
> *Tarryn:* *So we are just making them aware that this can come up and we can build on it at a later stage.*
>
> [Grappling-CoP, Zoom meeting, 2021-03-25].

My problem scenario context clearly allowed for extension. In discussions about this in the Grappling-CoP members, Mellony commented:

> *Mellony:* *In terms of progression, of deep mathematical progression of concepts, and of linking and connecting different powerful mathematical fraction ideas, like fraction as partition or fraction as rate, instead of you know, breaking or whatever... So I think that this allows you, especially for the learners in the slightly older grades to do more powerful and connected mathematics.*
>
> [Grappling-CoP, Zoom meeting, 2021-04-12].

Aloe's Grade 6 teachers also had suggestions for how the problem scenario could be used to advance their learners' understanding of fractions:

> *Mr Mandoza:* *With the 1, 2, 4 and 8* [as denominators] *it was easier for them to see the equivalence, and then with the others, 3, 5 and 6 we focused more on what they do in their workbooks and spoke about greater than, less than, comparing.*
>
> *Ms Cloud:* *For the Grade 6s, an odd number is more difficult for them to break up the bar. So that was good for them to see that the sixths are between the fifths and the sevenths. So that was more difficult for them.*
>
> *Tarryn:* *So that's an opportunity for progression.*
>
> *Ms Cloud:* *Ja, maybe the Grade 6s can just do those.*
>
> *Ms Chaka:* *It's quite difficult in Grade 6 now, because we've actually moved on to addition and subtraction of fractions. I have found that subtraction works really nicely on the number line, so counting backwards on the number line...*
>
> [Aloe-CoP, 2021-11-05].

These suggestions were valuable, especially in relation to my planning of the final problem-solving lessons and the final individual activity (post-test). Guided by curriculum requirements, I designed the final individual activity at grade-specific levels. So, for example, the younger grades could attempt questions about speed as a 'Bonus ue stion', but were not obliged to do so. What this indicated to me was that my designs for the music-mathematics integrated tasks had the potential for activities that required moving flexibly between constructs of fractions at grade-appropriate levels, but that the tasks could also be differentiated for supporting learners and/or for enriching and extending learners.

5.2.10 Obstacle-Resolution Cycle 10: Questioning the value of integrating music

After presenting at a mathematics education conference, I was asked a pertinent question: "Why do you need to incorporate music? Why not simply focus on the animal river-crossing jumps and the number line?" I considered this carefully and, in reflecting with my supervisors in our Design-Theorising Plane, arrived at the conclusion: it *is* possible to carry out the tasks without the musical aspect. But what the musical integration allows for is (1) it helps meet broader curriculum aims of recognising mathematics as a creative, elegant and beautiful human activity (South Africa. DBE, 2011a), and (2) the musical aspects create contexts where learners can engage in problem-solving that requires moving flexibly between the fraction as ratio, fraction as measure and part-whole constructs. Some reflections from two different Grappling-CoP meetings are captured in the excerpts below. They show that our belief in the value of the musical integration with the mathematics throughout the journey is justifiable.

Tarryn: *I'm using the music as a context for the RME, for an opportunity for the learners to experience mathematics and fractions.*

Mellony: *It's quite a powerful bridge, a bridge to other mathematical concepts. You have the whole is one music bar and in the metaphor of the kudu jumping a distance.*

Sally-Ann: *Music is a vehicle for strengthening the children's conceptual grasp of fraction as measure and that measure can be time and it can be distance or it can be both simultaneously.*

[Grappling-CoP, Zoom meeting, 201-03-25].

Sally-Ann: *It's a symbiosis.*

Tarryn: *People ask, why are you doing the music? You can do it without.*

Mellony: *You're aiming at the broad curriculum outcomes.*

Sally-Ann: *Creativity, beauty.*

Mellony: *We're not saying this is the best, perfect way to teach fractions. We're saying if you want to integrate, be creative, give this a go. We think this is a powerful way of looking at both of these [fraction as measure and ratio]. They're using maths creatively. Fractions are part of human creative endeavour. Maths describes the musical creation. Here they are going to*

> *experience how mathematics describes animal jumps, and percussion, and note values.*
>
> [Grappling-CoP, Zoom meeting, 2022-01-25].

My argument, therefore, is that the integration of mathematics and music has the potential to support teachers and learners in the challenging concept of fractional reasoning to deepen understanding. I believe my example of curricular integration may resonate with other teachers who would like to find ways of integrating music or other art areas, or any other subjects into their teaching of mathematics.

5.3 Phase 3: Overall Teachers' and Researcher's Experiences of the Participatory Dual-Design Experiment In Task Design

In Phase 3, I respond to the research sub-question 4: "What are teachers' experiences of integrating music and mathematics for fractional understanding?" In responding to this question I organised the discussion according to three key themes: meeting curriculum aims; challenges; opportunities. I end my Phase 3 discussion by looking at some of the trials and triumphs I experienced in fulfilling my triple role: researcher, designer and facilitator. Included here is feedback given by the teachers who worked with me in the Grounded-Practice Plane of my research.

5.3.1 Meeting Curriculum Aims

A driving motivation for this study was that I wanted to find ways in which integrating mathematics and music could support teachers in meeting broad and specific curriculum aims. The specific curriculum goals which I hoped to achieve related to developing deep conceptual understanding of multiple constructs of fractions, with a specific focus on fractions greater than one whole and equivalence of fractions (South Africa. DBE, 2011a). The broad curriculum aims I hoped to contribute to involved supporting learners in using creative and critical thinking skills to solve problems and in developing in them a spirit of curiosity about mathematics while also recognising it as a creative, beautiful and elegant human activity (South Africa. DBE, 2011a). I used open coding (Saldaña, 2016) to analyse the formal whole group CoP meetings at my two participating schools as well as the various informal teacher meetings that took place in the Grounded-Practice Plane. The coding revealed that the teachers' reflections and feedback were in alignment with both the specific and the broad aims for the curriculum.

Developing deep conceptual understanding of fractions

The teachers from both schools recognised fractions as a challenging topic to teach and to learn, as the literature confirms (Siemon et al., 2015, for example). I noted in my reflective research journal entry following on from my first Aloe-CoP meeting:

> *The teachers immediately acknowledged that equivalence and fractions greater than 1 are areas that need support. This could be because the normal focus is on the part-whole structure. So from initial discussion, it seems that literature is confirmed about the constructs of fractions in the classroom i.e. the focus on the part-whole construct and challenges with other areas of using fractions.*
>
> [Reflective Research Journal, 2021-03-12].

The problem of fractions greater than one whole was often discussed in the micro-CoPs. Teachers in Grade 5 and Grade 6 (in line with curriculum expectations) specifically mentioned that their learners had difficulty when working with mixed fractions and improper fractions in problem scenarios. Ms Clegg gave the following example about her Grade 5 learners:

> *As soon as you've got 1 kilogram and 125g, then you often have to put it into a fraction, and then they leave off the one. Every single time. They just look at the grams.*
>
> [Protea-CoP, 2021-10-28].

Ms Chaka gave a similar example from her Grade 6 class where she wanted them to link fractions greater than one whole to decimal fractions and percentages.

> *… I have, because of your input, I've changed the way I've taught fractions. So today, we had to do a table on common fractions, to decimal fractions, to percentages. And for the first time ever, I've said "Okay, now this common fraction is five-fourths." Because I've always just given them three-fourths, or two-fourths, or one-fourth. But today, I gave them five-fourths, because you can take an extra jump. And then they were fascinated because they said, "But then you can't get a percentage." And I said, "Well, you actually can. It's actually 125%." And then we had a very interesting conversation because of how would you use it? You can't get 125% for a test. I said, "Yes. But you can have 125% mark-up"… and I would never have thought to do that without that jumping exercise and talking about a fraction as a unit, rather than part of a whole. I'm definitely changing the way I'm seeing things by having seen that little piece. But I think you taught me how to teach fractions better.*
>
> [Aloe School, Informal interview, Ms Chaka, 2021-09-10].

This feedback from Ms Chaka was encouraging as it indicates that she found my problem scenario and focus on the fraction as measure construct, rather than only on the part-whole construct of fractions valuable. I took this feedback as an indication that my task design had the potential to extend to further areas of the mathematics curriculum, such as decimal fractions and percentages in the Grade 6 curriculum. Similarly, Ms Clegg's example pointed to how my tasks could further be integrated into measurement topics from the curriculum.

In the Grounded-Practice Plane, I grappled with the teachers on what other problem scenarios might best support the learners in working with fractions greater than one whole. Ms Chaka suggested the possibility of a scenario of a game where children have to bounce a ball over different sections of a 'court'. We discussed the possibility of jumping across netball courts or car parking spaces. For the final product, however, I decided I would stay with the animal river-crossing scenario with one river-crossing considered the unit. It was a comment from Mr Dube, reflecting on his teaching of Lesson 3 in the first iteration, that reinforced me in this decision. Here is how he explained the conversation he had with his Grade 5 learners.

> *"When it jumps over the river, is the journey finished?" type of thing. And they* [the learners] *said, "They just jumped over the river." Couldn't they carry on going? They can jump again. It can carry on. Because then I asked, "You know that fractions don't only go to one?", to try and get them to see that that's not the end. Then it's a new one. So we looked at that, and then we had the kudu jump for one whole. Then the zebra, I think it was, jumped halfway…*
>
> [Aloe School, Informal interview, Mr Dube, 2021-10-29].

Equivalence was another aspect of fractions which I intended to develop through the music-mathematics integrated tasks. Teachers from both participating schools explained that some of their learners had difficulty in identifying and working with equivalent fractions. This they found was especially important for preparing children for addition and subtraction of fractions with unlike denominators. The teachers recognised the animal river-crossing jumping activities and the musical activities as possible ways to support their learners in recognising equivalent fractions. Ms Makeba, in her reflection on Lesson 2, gave me feedback on the value of the physical animal river-crossing jumping activity for her Grade 4 learners. She saw this as a useful hands-on activity for her learners to start informally recognising equivalence of fractions. The example she refers to is matching the one giraffe jump (three jumps per river-crossing) to two springbok jumps (six jumps per river-crossing). She then linked this back to the animals in Lesson 1 and got her learners to explore the equivalence between halves and quarters. Below is a photograph that Ms Makeba entered into the data set to show how she carried out the physical jumping activity in her classroom. The photograph is followed by an excerpt from her reflection.

Figure 5.19: Photograph showing example of the animal river-crossing activity in Ms Makeba's class

Ms Makeba: So we did the animal that took the 3 steps and the animal that took 6 steps. And one pupil was able to comment and say "Oh but for every one step taken by the giraffe, the springbok had to take two steps." So then we went back and linked it to the animal that takes 4 steps and the animal that takes 8 steps. However, we didn't work with the animal that took 5 steps and 7 steps.

[Aloe School, Informal interview, Ms Makeba, 2021-09-30].

Some of the teachers, in Grades 4 and 5 particularly, noted that the animals with five and seven jumps per river-crossing were more difficult for their learners. In her Grade 5 class, Ms Ibrahim used the following challenging example (working with fifths and sevenths), to discuss equivalence with her learners. I recognised from these comments the potential for deep discussion and problem-solving in these tasks.

Ms Ibrahim: The learners recognised that they could not find equivalent jumps between the elephant, with five, and the jackal's seven. One learner recognised that the animal jumps are close but not the same.

[Aloe School, Informal interview, Ms Ibrahim, 2021-10-01].

The animal river-crossing activity is in line with RME principles. It was a *real* experience for the learners, and allowed also for informal talking and reasoning about equivalence. It is also in line with curriculum expectations at the Grade 5 level, learners should "recognise and use equivalent forms of common fractions (fractions in which one denominator is a multiple of another)" (South Africa. DBE, 2011a, p. 16).

In the second iteration at Protea School, Ms Clegg commented that she had found the use of animals associated with specific numbers of jumps per river-crossing and beats per bar a useful context for her Grade 5 learners to deepen their understanding of equivalent fractions.

Ms Clegg: Cos they quite battle with the equivalent fractions, so that would be quite nice to say, "One ostrich would need to be friends with two zebra," kind *of thing. And do it more visually as like an extra thing. I just like the idea of the animals being equated to the fractions.*

[Protea-CoP, 2022-08-15].

A further indication that the teachers recognised my integrated music-mathematics lessons as having potential to support recognising equivalence of fractions is shown in the following reflection from Mr Dube. He explained how he had used the idea of clapping rhythms to show equivalence, posing to his learners the challenge of finding rhythms (or beats per bar) that would align to show how they are equivalent.

Mr Dube: *It was quite interesting. It made a challenge in one or two of them because they didn't use fractions that could become equivalent. So the one used twenty-fourths, for example, and fifteenths. Just cos they tried to go big even though I said don't go big. So we just chatted about it…. We chatted about it and said, "But now if you do it like this, can you find a rhythm that might help you to do both of them?" And then she had to think about it for a bit. But then one or two of them did it with thirds, and sixths, and all of that, and then they were able to see that, "Yes, I can do it because…"*

[Aloe School, Informal interview, Mr Dube, 2021-10-29].

I was satisfied that my design of the problem scenario with a focus on multiple constructs of fractions was useful in supporting the teaching and learning of fractions greater than one whole and equivalence of fractions. The problem scenario linking animal river-crossing jumps and musical claps or beats per bar was a *real* opportunity for learners to experience these fractional concepts in informal ways before moving to formal vertical mathematisation, thus meeting the principles set out in RME.

The literature shows that number lines are a useful representation for developing fractional understanding, especially when working with fractions greater than one whole and when developing the fraction as measure construct (Siemon et al., 2015; Barbieri et al., 2020; Monson et al., 2020; Soni & Okmoto, 2020). When analysing the teachers' reflections from both participating schools, I noticed that they too recognised the value of the number line as a key representation. I had intended that the informal discussion and modeling of the animal river-crossing jumps and the music clapping and notes would lead to formal, abstract representations of fractions (guided by RME principles). I found, from the teachers' feedback, that this was a useful key representation that supported learners in visualising fractions, and that it became a tool to support them in solving problems involving fractions. Mr Mandoza, for example, explained how his Grade 6 learners used the number line to compare fractions:

Mr Mandoza: It's like how number lines work. So when they place them on the number lines on the top, then below that, it is, "Which is bigger? Use the number line above to answer." So from that, they were able to use this to answer the questions.

[Aloe School, Informal interview, Mr Mandoza, 2021-11-26].

Ms Makeba similarly commented on how her Grade 4 learners had used the animal river-crossing jumps and the number line to solve the mathematical problems in Lesson 6:

Ms Makeba: It did help the children when they were solving these problems, they were using the number lines and the jumps.

[Aloe-CoP, 2021-12-03].

The positive first iteration feedback from the Aloe School teachers regarding the value of a number line when working with fractions further guided me in the adjustments I made for Lessons 6 to 8 for the second iteration at Protea School. Here is part of our discussion in the Protea-CoP about the value of representing fractions on a number line.

Ms Savuka: They asked me… "Is this jumping on a number line?"

Tarryn: And fractions are numbers on a number line. It doesn't have to be whole numbers.

Ms Savuka: And that's, I think, going to be a nice representation for them, specially for those who struggle conceptualising what a fraction is. So I think it's going to be nice when they see that. Those problem-solving. "So this one jumps, how many would this one?" So I decided to show it and then have them see, and then ask the question again… Ja, I think the whole number line thing is something they need to get used to using. It could be a great idea just for general maths in future to have some blank number lines that they can actually physically use when they do activities.

[Protea-CoP, 2022-04-12].

From the teachers' responses, I recognised how useful the number lines as a key representation was in Lessons 6 to 8, and how my task design had achieved the goal of moving from the informal representations to the formal representations. This met the goals of RME principles, and was consistent with other findings in the literature (for example, Saxe et al., 2013; Barbieri et al., 2020; Soni & Okmoto, 2020). I therefore see my music-mathematics integrated tasks as having the potential to support teachers in meeting the mathematics curriculum aim of developing deep conceptual understanding of fractions.

Attitudes and disposition towards mathematics in the integrated lessons

I found that my integration of mathematics and music also created opportunities for learners to be creative, to have fun and be motivated to participate in the lessons, thus also meeting the mathematics curriculum aims of developing an appreciation for mathematics as a creative, elegant and beautiful human activity (South Africa. DBE, 2011a). I noticed that teachers' reflections often referred to the learners' creativity. They often also even expressed surprise at how creative their learners were in the music-mathematics tasks. I also noticed that the tasks created opportunities for learners to think creatively when solving problems. Ms Chaka, for example, remarked on how impressed she was when she had used a group percussion activity (with found objects) for learners to align the beats per bar (fraction as ratio construct).

> *Ms Chaka: But, I must say, I was quite impressed with their creativity, because I had the drums, and I had the sticks and the drum, and the clapping, and the rice, the shaker, and everything. And some of them were absolutely adamant that you can't do the four beats with the rice-shaker, cos that must be the one that goes 'shh'.*
>
> [Aloe School, Informal interview, Ms Chaka, 2021-10-20].

Teachers were impressed with their learners' body percussion activities, such as making their own rhythms on the percussion line with informal notation, and their use also of a variety of ways to make percussive sounds. In the second iteration, the percussion progressed to introducing learners to formal Western musical notation. Ms Savuka, in her feedback on Lesson 6 was surprised by the creativity her Grade 4s showed in using the animal river-crossing jumps to 'compose' their own music.

> *Ms Savuka: Just to spark that bit of creativity because they are essentially writing music. They are composing their own music and I think that has a little bit of satisfaction for them. You know, they were very chuffed when they looked and they were like, "This looks like real music!"*
>
> [Protea-CoP, 2022-04-25].

The creative aspects of the integrated music-mathematics lessons also appeared to have increased learner motivation and participation. This finding is consistent with the literature (for example, McPhail, 2018; Pluim et al., 2020; Tytler et al., 2021), including the findings from my own MEd action research study (Lovemore, 2020). Many teachers referred to the lessons being 'fun' and expressed how exciting the learners found the novel music-mathematics integrated lessons. They reported that their learners would ask them when they would be doing the next animal jumping or clapping lesson. Here, for example, is what Ms Makeba said about her learners' excitement when they saw that they would be doing one of the integrated lessons:

> *Ms Makeba: It was actually great. There was much excitement when the pupils walked into the class and saw the river and there was much excitement and enthusiasm from the children ...*
>
> [Aloe School, Informal interview, 2021-09-30].

I expected the learners to be motivated in the initial lessons especially, as these were novel activities. However, I found comments, such as Ms Cloud's, Mr Mandoza's and Ms Chaka's below, especially encouraging. They indicate that learners' attitudes were positive throughout the integrated music-mathematics lesson series, even in the case of learners who were not particularly confident in mathematics. Ms Cloud, for example, reflected on one of her learners who did not have a good conceptual understanding of fractions, but noted that the integration of music into the lessons would be appealing to this learner who enjoys musical activities.

> *Ms Cloud: Jesse is like dreadful with fractions, but I mean, she would get something to do with music because she's just naturally musical. It will appeal to her where normally she switches off.*
>
> [Aloe School, Informal interview, Ms Cloud, 2021-09-10].

Ms Chaka and Mr Mandoza discussed the increased participation of their Grade 6 learners during the integrated music-mathematics lessons.

> *Ms Chaka: Learner participation? High, it was very high. They all enjoyed it and they loved it.*
>
> *Mr Mandoza: Even the children that normally switch off in maths time, when it was these lessons, they were fully involved. I normally have kids that when we do fractions or any maths, "Ag I'm so bored". But now with the music, they are the ones participating and fully involved in the fraction lessons.*
>
> [Aloe-CoP, 2021-12-03].

I recognised such integrated lessons as having made a contribution to counteracting the negative narrow and disconnected views that Venkat and Graven (2017) described learners as having about mathematics, and instead contributing to their developing increasingly positive dispositions towards it. It is my hope that such positive dispositions may carry over into future mathematics lessons.

5.3.2 Challenges in Implementing the Integrated Music-Mathematics Lessons

As noted earlier, neither the design process of the integrated lessons nor their implementation were without challenges. I focus in this sub-section specifically on some of the challenges impacting the first iteration in the Grounded-Practice Plane. The challenges were directly music-related. For this first iteration, none of my participating mathematics teachers had a background in music, and consequently felt somewhat 'out of their comfort zones' in incorporating music into their lessons. One of these teachers (Ms Ibrahim) even went so far as to indicate that music 'terrified' her.

Given the Aloe teachers' lack of background in music, it is unsurprising that they indicated in most of the meetings how they felt a lack of confidence about the inclusion of music into their mathematics lessons. This was 'out of their comfort zone'. Early on in his implementation of the tasks, Mr Dube, for example, reflected,

> *Mr Dube:* *I didn't feel like in my mind, I knew what I was doing. So that was my mistake. I think I did it, fine, but in my mind I still, I wasn't 100% confident.*
> [Aloe School, Informal interview, Mr Dube, 2021-09-10].

As the researcher/task-designer, I tried to support the teachers in the design and also in the implementation of the lessons. Some of the Aloe teachers invited me into their classrooms and asked me to co-teach with or demonstrate a lesson for them. As I felt it was my ethical responsibility to support these teachers, I agreed, with support from the school Principal, to assist in this way. The teachers were appreciative of my demonstration lessons. From teachers' reflections, in the first iteration at Aloe School, it seemed that they were anxious not only about being able to teach the lesson, but also about being able to answer learners' questions.

> *Mr Dube:* *Stepping out of my comfort zone... I felt like sometimes, I've got a basic understanding, but if anyone asked me an in-depth question I don't know if I could explain it.*
>
> *Ms Cloud:* *And some of us have music pupils in our classes!*
>
> *Mr Dube:* *So you're feeling challenged, because you want to learn a new way of teaching, and if the kids are enjoying this way of teaching and it's benefitting them, now I have to challenge myself now to try and do it. But now I have to step away from the formulas and I had to change the way I was going to teach. I found that quite challenging.*
>
> *Ms Cloud:* *And we're used to being in control.*
>
> *Mr Mandoza: I had the same challenge as well, but then I got the music students to help me. I asked them, to incorporate them in helping us.*
> [Aloe-CoP, 2021-12-03].

These reflections demonstrated their willingness to try the novel music-mathematics integrated lessons, despite their anxiety around not being in control. They also recognised that it would be possible to include learners' who do have a background in music to assist the class. The teachers' responses are consistent with the literature that explains how teachers integrating subjects that are not their expertise were often anxious (see, for example, Adler et al., 2000; Pluim et al., 2020). Music was also noted by Kneen et al. (2020) as the most intimidating art form to integrate into other subjects.

My analysis of the Aloe School teachers' reflections indicated that a prime reason they were anxious about stepping out of their comfort zone by integrating music into their mathematics lessons was the musical notation. This symbolic notation was new to them and they felt that they were learning with the learners, which somewhat dented their confidence. Below are some of the comments they made in our final CoP meeting.

Ms Ibrahim: *Music still terrifies me.*

Mr Sontonga: *I was very nervous, because music is foreign to me*

Mr Mandoza: *I think Lesson 1 and 2 were more mathematical, but then when I saw Lesson 3 is more musical, that's when I got nervous. But now I taught it and it goes to a more mathematical link also, and now I feel more confident.*

[Aloe-CoP, 2021-12-03].

The teachers at Aloe School were, however, more confident with the use of the informal notation of music, such as the Xs. This feedback was important and guided my adaptations of the task design for the second iteration at Protea School. As explained in Chapter 4, I introduced the musical notation informally in the second iteration before then progressing to the resources requiring learners to match the formal Western musical notation to animal jumps. I made these decisions so that teachers would not have to memorise, read and write formal musical notation. The Protea School teachers, having a background in musical knowledge, made the point that the music-mathematics integrated lessons could be challenging for teachers without a musical background. They also reflected on the (adapted) resources I had given them.

Ms Clegg: *I would think the big thing would be for teachers who aren't musical because they might feel anxious because notation does look scary. But I think if they understand beforehand like it's not knowing the names of the notes or anything like that. And when we do the basic names, it's all on that piece of paper. It's not like they have to teach it.*

Tarryn: *The resources that I gave you?*

Ms Clegg: *Yes, the resources were there to support you, whether you knew music or not. Like I say, I don't play a musical instrument, but when I taught it, you know... But off hand in class now, I've forgotten what goes where. So it's not like you can't teach it without that knowledge.*

[Protea-CoP, 2022-08-15].

I recognised the challenge that teachers without musical background faced, especially in the first iteration of implementing the music-mathematics lessons. I do, however, believe that the adapted resources would make it easier for teachers with or without musical knowledge to implement, without any anxiety.

5.3.3 Opportunities from Implementing the Integrated Music-Mathematics Lessons

Analysis of teachers' experiences and reflections helped me recognise opportunities that arose from my integrated music-mathematics task design. In particular, I recognised a number of practical recommendations from teachers, opportunities for supportive teacher resources, for differentiation, and for teacher learning and collaboration, each of which I now briefly discuss.

Based on their challenges, teachers made several recommendations. Some of the more practical implementation challenges in the Grounded-Practice Plane involved the time that it took for the lessons to be implemented. Teachers at both schools indicated that the lessons often went over the allocated mathematics class period, and sometimes had to be split into two lessons.

The teachers also mentioned that the animal river-crossing jumping activity was difficult to implement in the limited space of a classroom, and that it was better to do the activity outside. This however posed a problem with the paper plates that were meant to represent rocks in the river, as they would blow away in the wind or learners would sometimes slip on them. Another challenge with practically implementing the animal jumping activity was that learners could not focus on the jumping and the timing with the second hand on a clock. For example, it was not possible for them to do one long Kudu jump, staying in the air for four seconds. Ms Fassie suggested using pictures of animals that learners could hold and demonstrate the imaginary jumps across the river. The teachers' feedback was invaluable in my adjusting of the lessons and design of the 'final' product.

With regard to opportunities for future teaching and learning resources, the challenges that teachers experienced guided me to developing demonstration videos to share with teachers. They used these either to get a better understanding of an activity or to present in their classes as a resource. Ms Cloud's comment indicated how useful the videos were. These video clips became part of my teacher resource pack.

> *Ms Cloud:* *All those video clips... Once I watched your examples, I was like "Oh, okay!". So that made a big difference.*
>
> [Aloe-CoP, 2021-12-03].

In the final Aloe-CoP meeting, I asked the teachers whether they would consider teaching a similar music-mathematics integrated sequence of lessons in future. They indicated that they would consider this if they did not have to teach the musical notation. Ms Fassie said she "Could definitely use a variation of some of the ideas" from these lessons [Aloe-CoP, 2021-

12-03]. The teachers also discussed that they would need to wait a few years, as all the Grade 4 to 6 learners had similar experiences while we were trialling the lessons, and that in future they would make more use of opportunities to show progression across the grades. Mr Mandoza discussed using the problem scenario and music integration as a way to introduce fractions at the beginning of a school year,

> *Mr Mandoza: I would definitely use this lesson again, but at the start of the year to introduce fractions. I think it's a good way to get the children excited about fractions.*
>
> [Aloe-CoP, 2021-12-03].

The teachers at Protea School similarly suggested that this would be a good unit to teach at the beginning of a school year to get learners excited about mathematics and to refer back to when teaching fractions throughout the year.

I further noticed, from teacher reflections, that my design of music-mathematics integrated lessons has the potential for differentiation in a classroom. Ms Clegg, for instance, indicated that the resources used in Lessons 6 to 8 to support the musical and the mathematical concepts helped her learners who had difficulty (shown below), whereas her stronger learners relied less on the resources to solve the problems.

> *Ms Clegg: So the size of that transparency was a great support for my lower class, which essentially would mean that it worked really well.*
>
> [Protea-CoP, 2022-04-12].

In the Protea-CoP, we also discussed how learners could use the integrated tasks for enrichment problem-solving questions. Ms Savuka, for example, referred to the bonus questions that motivated her learners, saying that "Some were really excited to try and answer the bonus question" [Protea-CoP, 2022-08-15]. I therefore believe that teachers can use these integrated lessons to support their learners at the level they are at and to deepen their mathematical and musical understanding.

Upon reflection, teachers indicated that they too recognised opportunities to continue learning. This resembles the expectation of the South African Department of Higher Education and Training (2015) that teachers strive to be lifelong learners. When requesting gatekeeper permission, the Principal of Aloe School shared his hope with me that the teachers' participation in the study would be a form of professional development. Mr Dube, discussed above, mentioned how he was challenged to step out of his comfort zone to trial ways that would interest learners in fractions. Ms Chaka also acknowledged that she changed the way she viewed the teaching of fractions (discussed in Section 5.3.1). She further used this opportunity in the study to reflect on her own mathematics teaching in general, as shown in the excerpt below.

> *Ms Chaka:* *I was a little bit disappointed with them. But I've realised that I probably, when we do an example, I as a teacher, I possibly guide them too much. Because I didn't give them any guidance with that. Now, had I worked through it, had I read it with them, I think they would have fared better. So perhaps I must try and take a step back a little bit and let them do more of it on their own. Because when we do our* [workbooks], *I'm only ticking, but perhaps I give them too much before the lesson, you know before the activity actually has to happen. And they, you know, they're not having to think. They know what to do.*
>
> [Aloe School, Informal interview, Ms Chaka, 2021-09-10].

Ms Makeba also reflected on her newly gained knowledge on music, despite being nervous of the musical notation, she acknowledged this as an opportunity to learn, saying, "But I also do have a better, a very basic understanding when you talk about a bar or a beat" [Aloe-CoP, 2021-12-03]. I therefore recognised that my study of trialling integration strategies has potential for teachers wanting to learn and grow in their own professional development.

My music-mathematics integrated tasks created opportunities for learner and teacher collaboration. Teachers' reflections showed that they frequently let the learners work in groups to carry out the activities. They reflected on the group skills that developed and how the learners supported one another in small groups and pairs. Teachers' reflections also indicated the possibility of collaboration with music teachers in their schools. They firstly recognised some overlaps with what their learners had been learning in their class music lessons. Ms Makeba, for example, recognised that her Grade 4 learners had done some examples of body percussion in their music lessons. She appreciated this as it supported her in teaching the lessons.

> *Ms Makeba:* [A Grade 4 learner said,] "*Ma'am, that's called body percussion." And I said, "Where did you get that from?" She said, "We did it in music at the beginning of the year." So that was great. And then I thought, "Okay, we're on the right track. We're going somewhere."*
>
> [Aloe School, Informal interview, Ms Makeba, 2021-10-29].

Similarly, the teachers at Protea School told me that their learners were starting to be introduced to the formal Western notation of music note values. They thought that my integrated music-mathematics lesson would be mutually beneficial to the music and the mathematics teachers.

The Aloe teachers even suggested including the music teacher in these music-mathematics integrated lessons. We discussed this possibility, in the Aloe-CoP, as shown in the excerpt below.

> *Ms Fassie:* *I would love to have one music class that the music teacher does, just to see. As we all are experienced teachers, we have preconceived ideas about fractions, but somebody who came in new… I know it's not your study, but it would have been interesting to see, can they get the children to get the concept.*
>
> *Ms Cloud:* *Can they link their music with maths? If they could do this music lesson, it would be good… couldn't they just put that as a lesson in the music curriculum?*
>
> [Aloe-CoP, 2021-03-26].

The teachers' suggestions were helpful and led to my design of Lesson 5 that could be taught either by a mathematics or a music teacher. Ms Clegg's comment, below, with regard to the possibility of collaboration between teachers, resonated with Bernstein's (1971) notion of teachers-based integration and blurring the boundaries between subjects.

> *Ms Clegg:* *This [teacher collaboration] would also make the children more aware that nothing is in isolation.*
>
> [Protea-CoP, 2021-09-10]

I therefore suggest that my task design, guided by RME principles, integrating mathematics and music, provides opportunities for developing deep conceptual understanding of multiple constructs of fractions and for recognising the creativity in mathematics, thus promoting positive views of the subject. I have endeavoured to show that the challenges experienced by participating teachers informed my adjusting and improving on the task design and that opportunities exist for teacher learning, learner learning, collaboration and differentiation.

5.3.4 Initiating a Community of Practice: Trials and triumphs in the face of a global pandemic

I end off this presentation and discussion of my findings on the process aspects of my task design journey by briefly sharing insights from one particular source of trials and triumphs encountered in initiating and sustaining the Grounded-Practice Plane CoPs. These CoPs were a space to interrogate tasks, reflect on experiences and suggest improvements on the task design journey. The participating teachers and I shared various trials and triumphs in the CoP space as a result of the COVID-19 pandemic that affected our communication and meeting times. I briefly discuss some of these.

The literature notes that, inevitably, challenges arise in initiating and sustaining a CoP (Bouchamma et al., 2018; Pyrko et al., 2017). For my own research, the global COVID-19 pandemic made maintaining the momentum of our CoP meetings particularly challenging. In 2021, most South African schools were forced to close for periods of time during the national lockdown. Teachers had to isolate in quarantine if they had been in contact with someone infected with the virus. Our CoP meetings were often postponed or cancelled. Teachers also had to adjust the online teaching space and my integrated music and mathematics lessons were another addition to their teaching expectations. This resulted in the lessons being spread out over months, rather than taking place over consecutive weeks as we had originally planned in our CoPs.

Due to the challenges with meeting in the whole group CoP, the participating Aloe teachers and I jointly made the decision to meet informally when possible. I therefore immersed myself in the school and took every opportunity to meet with teachers individually or in small groups. When possible, we then met in the whole group CoP. This was less challenging during the second iteration at Protea School, as there were only two participating teachers and we found ways to always meet together in our micro-CoP.

Furthermore, in responding to these emerging challenges, the participating teachers and I resorted to using the social media platform of WhatsApp to arrange meeting times, or for teachers to ask me questions and share examples of artefacts from their lessons. This was useful, as we would then discuss the points from the WhatsApp communication the next time we saw each other face to face. Ms Clegg, for example, found this communication helpful as she explains below,

> *Ms Clegg:* *Ja that's fine because we can reflect. Like you say, it is easier. I do forget, and especially if the children have said something, and those key little things are always nice. A child said this, or whatever… And then also if you think of anything else that you think we can add, then that will help us as well.*
>
> [Protea-CoP, 2021-09-30].

Despite these COVID-19-related challenges, my analysis of the teachers' experiences and reflections showed that teachers collaborated and supported one another at school and we jointly found ways to continue the study. The aspect of CoPs also contributed to the task design journey, even though it did not happen the way we had intended it to go from our initial meetings. For further elaboration on this see Lovemore et al. (2022c).

5.4 Chapter Summary

As the preceding discussion shows, I was faced with numerous obstacles along the way in my task design journey. Working between the two planes was paramount to finding resolutions in the task design: the researcher-designer expertise informed by literature in the Design-Theorising Plane, and the teachers with their expertise in the classroom context in the Grounded-Practice Plane. Through joint discussion, grappling and reflection, within the three micro-CoPs, I was able to follow a *process* that led to resolutions and a music-mathematics integrated *product* that I see as having a great deal of potential. Teachers' experiences and reflections, in particular, contributed significantly to my ability to successfully navigate my way along this task design journey's route.

CHAPTER 6: IMPLICATIONS AND CONCLUSION

6.1 Introduction

In this concluding chapter I highlight the key findings of my task design journey in integrating music and mathematics for supporting fractional understanding. I firstly reiterate the goals of the study. To show how I achieved the research objectives and answered the research questions, I summarise the product of the task design followed by the process that unfolded. I also summarise the participating teachers' experiences of implementing the integrated music-mathematics tasks. Thereafter, I share some of the implications of my findings as a whole. I also reflect on the overall quality of my study: its strengths and limitations, and end the discussion with future opportunities for sharing the findings from my study and continuing the research.

6.2 Reiteration of the Goals for the Study

My goal for this study was to answer the overarching research question,

> *In what ways can music and mathematics be integrated via realistic mathematics education task design principles so as to facilitate connections across multiple constructs of fractional understanding?*

My objectives involved wanting to trial ways to integrate music and mathematics for developing deep conceptual understanding of fractions. I planned to design real and meaningful tasks which integrate musical and mathematical representations and concepts for connecting across multiple constructs of fractional understanding, thus meeting the specific South African curriculum aims for the teaching and learning of fractions. Exploring the integration of music and mathematics for meeting general curriculum aims of appreciating mathematics as a creative and elegant human activity was a further research objective. It was important too for me to understand and describe teachers' experiences of integrating music and mathematics for fractional understanding, hence the following research sub-questions:

1) In what ways can musical and mathematical representations and concepts be integrated for developing conceptual understanding of fractions?
2) What realistic/meaningful tasks can integrate music and mathematics for connecting across multiple constructs of fractions?
3) How can integrated mathematics and music tasks support teachers in achieving the general and specific curriculum aims?
4) What are teachers' experiences of integrating music and mathematics for fractional understanding?

In the following section I show how my findings from Chapter 4 and 5 answer these questions and respond to my research objectives.

6.3 Discussion of How the Key Findings Respond to the Research Questions

I divided my discussion of the key findings into three main categories. In the first category, I summarise the product of my participatory dual-design experiment in task design (i.e., the eight-lesson teacher resource pack). The second category is a summary of the 'messy', iterative and cyclical process through which I grappled to reach this end product. In the last category, I summarise the teachers' experiences of implementing the music-mathematics integrated tasks.

6.3.1 The Product of the Participatory Dual-design Experiment in Task Design

To answer Research Sub-Questions 1 and 2, I discuss the key representations and concepts I used to integrate mathematics and music for supporting fractional reasoning and subsequent understanding. The real and meaningful tasks I designed connected multiple, interrelated constructs of fractions.

A mathematical task includes anything – ranging from a single activity to a full teaching and learning sequence – that requires learners to be actively engaged in order to learn a specific mathematical concept (Watson & Ohtani, 2012; Graven & Coles, 2017). The product of my study is a sequence of eight lessons, including teacher resources and key representations, along with the informal initial workbook and final individual worksheet. The mathematical concept I designed these resources around was three constructs of fractions (fraction as ratio, fraction as measure and the part-whole construct of fractions), with an emphasis on fractions greater than one whole and equivalence of fractions. I integrated this mathematical concept with the musical concepts of 'beats per bar' and note values, with the goal that this integration would not only support the teaching and learning of fractions, but also achieve the broad curricular aim of recognising that mathematics can be experienced as both a beautiful and a creative human activity. I paid careful attention to finding what I thought were the 'best' research-informed representations and resources to support this, as suggested by Sullivan et al. (2013).

Pepin (2018) highlights the point that task designers should be able to justify their design decisions. Realistic Mathematics Education (RME) principles guided my design decisions. I met some of the key tenets of RME (as identified by Cobb et al., 2008): the starting point in my teaching and learning sequence was experientially-real; and my designed tasks created opportunities for learners to informally engage in mathematical problem-solving and representations, and then had the potential to lead to vertical mathematisation using formal, abstract representations and symbols.

The starting point of my task design was a problem scenario set in a southern African context. Different animals had to cross a river of a constant distance in a constant time. They would, however, do so with a different number of jumps per river-crossing (introducing both the fraction as ratio and fraction as measure constructs). I designed the lessons so that learners could act out this jumping in a real, embodied manner. This met the RME principle of a starting point that is an experientially-real, meaningful activity for the learners. From the physical activity, learners then had to informally represent the jumps per river-crossing according to the different animals, thus meeting the requirement for informal representation.

My tasks then linked the animal river-crossing jumps to musical beats per bar in the form of body percussion such as claps. For example, a Zebra takes four jumps per river-crossing and, therefore, four claps per river-crossing. The teacher would then guide learners in various body percussion activities and informal notation, again, a *real* and *meaningful* introduction to the fraction as ratio construct. Freudenthal (1991) stressed that fractions and ratio should be taught together from the beginning; "… and I do mean the very beginning", he emphasised (p. 118). My task design focused specifically on 'rate', as the comparison was of units of two different quantities (beats per bar or jumps per river-crossing). I designed informal problem-solving questions that the participating teachers posed to learners with the context of the animal river-crossing jumps and the body percussion. From the 'beats per bar' concept, my lesson sequence progressed from informal, adapted percussion notation to the formal Western note value conventions and notation. I designed the final product to expose teachers and learners to this formal musical notation in ways that did not require that they had to memorise, read or write the notation, but rather to match carefully designed note value cards to animal river-crossing jumps. From here the tasks focused on the fraction as measure construct, exploring music bars that then extend past one whole bar, and using the river-crossing as a unit.

The final two lessons I designed for the sequence involved progressing to formal representations of fractions on a number line, specifically a triple number line. This number line became a key representation in task design as learners could use it to solve complex problems involving the multiple constructs of fractions. The triple number line had the river-crossing as the main unit, and then the measure of distance and time could be varied allowing for this moving flexibly between the multiple constructs of fractions to solve complex problems. My design aligned with Freudenthal's (1991) suggestion of using a double number line to develop these skills in proportional reasoning with fractions. Literature (for example, Siemon et al., 2015) also describes the benefits of formally representing fractions on a number line. Teachers used the initial workbook and final individual worksheet to make qualitative evaluations about their learners' knowledge of, skills in and attitudes towards mathematics prior to and after their implementing and trialling the various integrated mathematics/music tasks. In the course of each iteration I received feedback from teachers via both formal and informal interviews. Their feedback helped me make in-process adjustments to my design, and will also contribute to further on-going refinement of the product.

Arriving at this product of task design was no easy journey. In the next sub-section, I summarise the unfolding process that led to the eventual design of the integrated music-mathematics teaching and learning sequence and resources that were used.

6.3.2 The Grappling Process to Reach the End Product

I labelled my design-research study a participatory dual-design experiment in task design, because throughout, participants were involved in the process of designing the product. I initiated three small CoPs, rather than one large CoP with all the participants, which is why I refer to them as micro-CoPs. I worked with my two doctoral supervisors in the Grappling-CoP and with the teachers at the two schools, Aloe-CoP and Protea-CoP, respectively. The literature describes a CoP as a space where professionals can jointly solve problems, share approaches and learn from one another (Wenger, 1998; Bouchamma et al., 2018; Pyrko et al., 2017). This aligned with my experience of working in such a space for this study: collaborating with my supervisors and the participating teachers to share, trial, interrogate, and reflect on my designed music-mathematics tasks. I was the central cog of all three micro-CoPs, sharing experiences and reflections between the groups.

I further referred to my design process as being a "dual-design experiment" (after Gravemeijer & van Eerde, 2009, p. 520). This involves two experiments taking place simultaneously: learner learning and teacher learning. In my study, two planes worked dually: the Design-Theorising Plane and the Grounded-Practice Plane. In the Design-Theorising Plane I grappled, as the researcher and the task designer, with input from my two supervisors. I strove, to find optimal, user-friendly ways of integrating the mathematics and the music. These I shared with teachers in the Grounded-Practice Plane who then trialled the lessons and their subsequent feedback informed continued grappling, adjusting and further design decisions. In this context, I, as the researcher-designer, supported by my two supervisors, provided the theoretically-based expertise, and the teachers provided the practically-grounded expertise of the classroom context. Furthermore, given that the participating teachers would reflect on the learners' learning and participation feedback, my design could be considered a *triple*-design experiment. Working within the Grounded-Practice Plane, the teachers were "research collaborators" (after Makar, 2021, p. 440). Through this collaboration in the micro-CoPs, I wanted to honour Setati's (2005) point about doing research *with*, rather than *on* teachers.

The iterative nature of design-research means that an intervention task goes through cycles of trialling, reflecting, adapting and re-trialling (Gravemeijer & Cobb, 2006; Plomp, 2007; Gravemeijer & van Eerde, 2009; Prediger et al., 2015; Bakker, 2018). The trialling of the integrated music-mathematics tasks started in my MEd study, and then progressed to my initial design and trialling in the Design-Theorising Plane. This was followed by two iterations in the Grounded-Practice Plane at Aloe and Protea Schools respectively, with adaptions taking place between the iterations. The process I followed links to Gravemeijer and Cobb's (2006) micro- and macrocyles: the microcycles referring to the grappling within each micro-CoP, and the macrocycles referring to the iterations trialled at the two schools over time.

Upon a reflective analysis of the Grappling-CoP meetings, working in the Design-Theorising Plane, I noticed that we continually faced obstacles in integrating the mathematical and musical concepts. We grappled with three key questions to eventually come to a resolution for each obstacle:

1) Does the task maintain the fidelity of the mathematics, the music, and the integration of the two? How?
2) Does the task adequately simplify the complexity of the integration for implementation within the classroom? How?
3) What key representation/s would best support conceptual clarity?

These questions could provide a guide for design-researchers looking to design tasks that integrate mathematics with other subjects. Task design is a careful, deliberate and purposeful process, requiring designers to anticipate learners' possible responses and possible misconceptions (Ainley et al., 2015; Choy, 2016; Jones & Pepin, 2016). Grappling through the Obstacle-Resolution Cycles was certainly deliberate and intentional. I had to consider the 'best' ways to integrate the mathematics and music so as not to create possible misconceptions for learners. I wanted to ensure that the fidelity of both subjects was maintained, in line with

Bresler's (1995) co-equal cognitive style of arts integration. Because this requires expertise in both subject areas, Bresler (1995) described this style of arts integration as being the most challenging and least prevalent. My findings from my own task design grapplings corroborate Bresler's findings. Similarly, findings from other authors resonated with my experience, such as McPhail (2018). He stated that curriculum integration is "much harder than it looks" (p. 66) and remains a challenge still, even for experienced teachers.

I had to design tasks in a way that the complexity was simplified for classroom implementation by teachers who may not have a background in music. I also trialled various representations to carefully select the best representation to align the music and the mathematics to support fractional reasoning and understanding. The aligned Western music staff and the triple number line was the key representation that I designed to support learners in working flexibly between multiple constructs of fractions and solving complex problems.

I created Figure 6.1 (Chapter 5) to represent the Obstacle-Resolution Cycles and how we grappled in the Design-Theorising Plane (informed by reflections from the teachers in the Grounded-Practice Plane and by our own mathematical, musical and pedagogical insights). Below I re-show this Figure as a way of highlighting the culmination of the process of the three micro-CoPs working together through the Obstacle-Resolution Cycles from their respective planes.

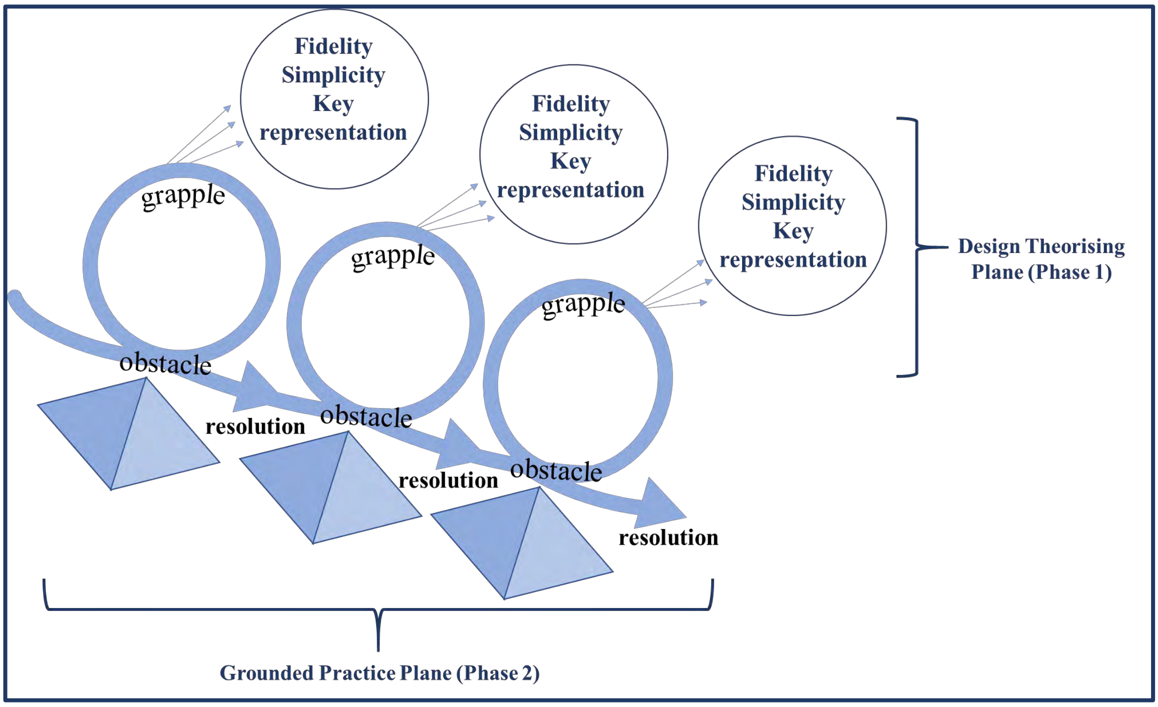


Figure 6.1: Diagrammatic representation of the combined Design-Theorising Plane and Grounded-Practice Plane working through the Obstacle-Resolution Cycles

I then designed an analysis matrix where I organised the various obstacles and how the three key questions contributed to arriving at the resolution. I found this matrix useful in analysing my data from the Grappling-CoP Zoom recorded meetings. I arranged the obstacle-resolution cycles into ten groupings. Table 5.3 summarises the outcome of each cycle grouping.

Obstacle-Resolution Cycle Grouping	Outcome of grappling in the Obstacle-Resolution Cycles
1	An animal river-crossing problem scenario is used as an experientially-real starting point.
2	The river-crossing is the constant unit.
3	Teachers are aware that the animals jump at a constant speed, and this can be used as an extension problem-solving discussion.
4	First introduce informal percussion notation, thereafter expose learners to formal Western note values through carefully designed note value cards (not having to memorise the notation).
5	Visually place the note values in the middle of the music bar and focus on the animal jumps rather than on the landing.
6	Cannot super-impose the number line and music line (staff), rather align the two representations to show similarities and differences.
7	Use designed note value representations to avoid mistaking the whole note for a zero.
8	Informally include the musical rest notation for variety in the percussion activities only.
9	Use problem-solving questions for progression across grades.
10	Integrating the music with the animal river-crossing jumps and the number line allows for working flexibly across multiple constructs of fractions, while showing the creativity and beauty of mathematics.

Table 5.3: Outline of the Obstacle-Resolution Cycle outcomes

One further challenge outside my control was that I had to work with the micro-CoPs during the global COVID-19 pandemic. My supervisors and I met and worked in the Design-Theorising Plane over the online conferencing platform, Zoom. This meant that I could video- and audio-record our meetings. With this challenge also came the opportunity to rigorously document and analyse the task design process itself, over and above grappling with implementation challenges. Zoom allowed me to take screenshots of rough work that my supervisors and I shared in our Grappling-CoP, as well as collecting the recordings of our 935 minutes of trialling, adjusting and reflecting on the integrated tasks.

The pandemic also created challenges around the school CoP meetings. Bouchamma et al. (2018) explain that there are challenges in sustaining a CoP in normal conditions. The pandemic made it especially difficult to maintain momentum with the schools and teachers. Finding time and communication was difficult in the lockdown context. This resulted in many smaller informal interviews at Aloe School rather than regular meetings with the full group of eight participating teachers. I formed a micro-CoP with the Protea teachers, in the second iteration, as only two teachers from this school participated in the study, making us a group of three. As the central 'cog' in this system linking the three micro-CoPs together towards achieving our shared goal, I found other ways to support and get feedback from the teachers during their implementation of the integrated music-mathematics lessons. We resorted also to communicating over the social media platform, WhatsApp, and I created some mini-videos as a way to stay in contact and communicate ideas. Teachers shared their reflections with me and asked me questions when difficulties were encountered. I suggest that my experiences in initiating, facilitating, and maintaining momentum and communication in micro-CoPs provides helpful insights for other researchers and teachers.

6.3.3 Exploring Teachers' Experiences of Implementing the Music-Mathematics Integrated Tasks

In responding to Research Sub- ue stions 3 and 4, I now summarise the participating teachers' experiences in meeting the curriculum aims, the challenges they experienced and the opportunities they recognised.

Teachers at both participating schools confirmed that their experiences resonated with the literature on fraction teaching and learning: that it is challenging, that learners have difficulty in working with fractions greater than one whole and equivalence, and that using a number line for fraction representation was useful. They found the fraction as measure construct especially helpful in their own teaching of fractions. In particular, they recognised that it is not sufficient to focus only on the part-whole construct. From teachers' feedback and reflections, it was clear that the task design had the potential to support learners in meeting the curriculum aim of developing deep conceptual understanding of fractions.

My interpretation of teachers' evaluations and reflections on the learners' initial workbooks was that the learners had a narrow view of mathematics as being 'numbers and sums'. This echoes findings by other authors on the dispositions of primary mathematics learners in South Africa (Graven et al., 2013; Graven & Heyd-Metzuyanim, 2014; Venkat & Graven, 2017; Jojo, 2019). The teachers, however, reflected on learners' creativity, increased motivation and participation while implementing the music-mathematics integrated tasks. Many authors report similar findings on the benefits of curriculum integration including increased attention, interest and engagement (for example, Naidoo, 2010; McPhail, 2018; Pluim et al., 2020; Tytler et al., 2021). Teachers too noticed more broad views of mathematics in their learners' final individual worksheets. These included comments such as: Maths is "amazing, creative, fun and sometimes challenging which is good as a challenge will make us stronger". This broader view speaks to the curriculum aim of developing learners who appreciate mathematics as a creative human activity (South Africa. DBE, 2011a), as well as to Freudenthal's (1991) view of mathematics as a *meaningful* human activity. I therefore see my music-mathematics integration as holding the potential to highlight the creativity in mathematics - "one of humanity's great achievements … of great sophistication and beauty" (Kilpatrick et al., 2001, p. 1).

Participating teachers experienced challenges in their trialling of the integrated music-mathematics lessons, especially those teachers who felt they lacked the confidence to integrate music into their lessons given their limited background knowledge in music. It was, as one teacher said, 'out of their comfort zone'. According to the literature, this is not unusual where teachers are asked to integrate subjects in which they do not have expertise (Adler et al., 2000; Naidoo, 2010; McPhail, 2018; Pluim et al., 2020). While the musical notation was undoubtedly more challenging for the Aloe School teachers, the adapted resources in the second iteration better-supported the Protea School teachers in exposing their learners to music notation. The adaptations meant teachers and learners were not expected to memorise the notation or learn any new symbolic language.

Challenges notwithstanding, feedback from teachers at both schools was encouraging in that all teachers recognised opportunities within my music-mathematics integrated lessons. They noted the potential of the task design to be a useful resource pack for other teachers who might want to trial such integration strategies as a unit, at the beginning of the school year for example, and they themselves said they would try similar strategies in future. Opportunities for differentiation, supporting weaker learners and extending stronger learners, was remarked upon by several of the teachers. They saw their participation in the task design implementation process as an opportunity to learn something new in their own teaching styles as well as in mathematics and in music. Future collaboration between mathematics teachers and music teachers was another opportunity that teachers identified as potentially beneficial. This is in line with findings elsewhere (Bernstein, 1971; McPhail, 2018; and Pluim, 2020, for example).

6.4 Some Implications of the Study

I see my study as making a contribution in terms of providing a ‘product’ of music-mathematics integrated tasks that can be shared with teachers who may wish to trial teaching fractions through integration with music. This ‘product’ could support teachers and learners in achieving topic-specific curriculum aims and also in achieving the broader aims of recognising the beauty and elegance in mathematics. Teachers could use these resources as a unit to introduce fractions at the beginning of the year and to get their learners excited about mathematics.

My study’s further contribution is a methodological one. I see the explication and summary of the task design grappling journey as providing an example for teachers and researchers who want to integrate either music or other subjects with mathematics. My example of the Obstacle-Resolution Cycles and the three key questions to consider (how to maintain fidelity of the mathematics and the music; how to simplify the complexities for classroom implementation; and what key representations to use), could support teachers and researchers in identifying and grappling through their own obstacles when trying to integrate subjects.

Furthermore, my study points to adaptations made due to the unprecedented global COVID-19 pandemic. This resulted in rigorous documenting and analysing of the task design process, enabled by recording features on the Zoom online conferencing platform. This form of documentation across different communication platforms could inform task design in future studies. The analysis matrix I developed could be used by other researchers in analysis of their own task design process. The initiation of micro-CoPs, of which I was the central member, may be a useful way to get rich data from various sources in different contexts, when not being able to meet as a whole group. An important contribution has been illuminating how researchers and task designers in the Design-Theorising Plane can share with and be informed by teachers working in the Ground-Practice Plane, thus truly becoming “researcher collaborators” (Makar, 2021, p. 440).

6.5 Reflections on the Study

My reflections on the overall quality of my study involves considering the strengths and limitations of the study. This study, firstly, was conducted over a relatively long period of 19 months. I collected a large amount of data from a variety of sources (935 minutes of Zoom meeting recordings, 778 minutes of recorded interviews, 79 email threads, 129 WhatsApp messages, 107 handwritten pages of reflective research journal entries, 48 pages of field notes, and 2,02 Gigabytes of artefacts such as lesson plans, resources and examples of learners’ work).

Participants included ten teachers with whom I worked at two schools, across Grades 4 to 6. Eight of the teacher participants had no background knowledge of music, and two had some background knowledge and skills in music. Teacher reflections are therefore from varying perspectives. My two doctoral supervisors and I also formed part of the participants for the study. This sort of triangulation adds to the trustworthiness of my data (Creswell, 2009; McMillan & Schumacher, 2010; Maxwell, 2013).

A particular advantage for the study is that I have sound knowledge in both mathematics and music. I am a qualified Intermediate Phase teacher, with a master's degree in mathematics education, and I am an amateur musician, with a strong theoretical background in Western musical conventions. My doctoral supervisors are also experts in the mathematics education field, and having their expertise in the Design-Theorising Plane was invaluable as we grappled through key obstacles. Working through these obstacles and finding resolutions made the research stronger.

The fact that the concepts I explored in my task design are part of the national curriculum expectations was advantageous. Schools would need to cover the topics regardless of my study. However, the trialling of the tasks could add value to teachers' normal teaching and learning of fractions and musical concepts.

Conducting the research at two independent schools was beneficial. While these schools follow the national curriculum, they are not bound to following the DBE's prescribed weekly planners. This allowed for flexibility in their time frame for the implementation and trialling of the music-mathematics tasks. The tasks were, however, not trialled in a set period of time as a unit, but due to the cyclical, adaptive nature and limitations from the COVID-19 pandemic, were spread out across months. I could not make clear-cut assumptions from data of the informal pre- and post-tests on learning gains of the learners.

I recognise that carrying out the research at the two independent schools could be seen as a possible limitation, as the two schools are of similar context and both are well-resourced. I did not trial the integrated tasks in government schools as access to resources in many cases is not optimal and these schools have less opportunity to depart from the prescribed weekly planners. The small-scale and circumscribed nature of my study means I cannot generalise from the findings (Willis, 2007). My goal of this study was not to make generalisations, however. Rather it was that my findings may resonate with other teachers and researchers wanting to explore the integration of music and mathematics, or of some other subject and mathematics. My use of thick description (Geertz, 1973; Maxwell, 2013) to describe the product and process of the task design journey, I believe, contributes to my study's "relatability" (Bassey, 1981, p. 85). The details I provide in this thesis allow for other researchers and teachers in a similar context to make decisions on whether my findings are relatable to them. I believe that the potential *transferability* (Guba, 1981; Creswell, 2009; Maxwell, 2013) of my study to other teachers and researchers in similar contexts is a strength.

6.6 Future Opportunities for Sharing Findings and Further Research

With the findings from this study come future opportunities. Firstly, I have packaged the product of the task design into a user-friendly teacher guide consisting of lesson plans, key representations, worksheets and other resources including the demonstration videos. I have posted these on YouTube, but I intend also to share these with teachers in a wide range of South African schools, including township and rural schools. Having carried out my research within the ambit of the South African Numeracy Chair Project at Rhodes University, one possible forum in which I plan to share these resources is with the teachers involved in various of the Chair's activities.

I have commenced with the creation of an interactive website. This will be accessible to teachers and learners in both well- and under-resourced contexts. I intend for this website to house open-source resources, such as the packaged teacher resources and demonstration videos. The website is intended to be an online interactive one that will make the use of musical notation less intimidating for teachers and learners with no prior musical knowledge. I am in the process of encapsulating the animal river-crossing scenario linking the number line and musical staff line in the form of a children's picture storybook. I intend sharing this as an open-source resource to teachers. The storybook will be available in both colour and black and white print options so as to be printable at low costs. I hope to have this resource translated and available in multiple South African languages in future, so as to reach a diverse group of teachers and learners.

In terms of future research opportunities, I would like opportunities for ongoing trialling of the implementation of my music-mathematics integrated lessons at schools of different contexts. It would be useful also to research how teachers and learners respond to the tasks being implemented as a whole unit over a two-week period, perhaps at the beginning of an academic year, as suggested by some of the participating teachers. This may facilitate more formal pre- and post-test data to be collected and analysed.

I intend sharing my findings from this study, along with possible additional findings from the jointly-designed Lesson 8 at Protea School, via various national and international conference presentations as well as through journal articles. I look forward to future engagement with academics and teachers on my study topic.

6.7 Concluding Reflective Remarks

Having reached the final destination for this particular task design journey, I look back on the path I navigated, along with my fellow travelers, as well as the unexpected layovers and detours with a sense of having achieved the most important aspects of my research goals: to explore ways in which I could integrate mathematics and music to support connecting across multiple constructs of fractional understanding, and to do so with RME principles guiding the choices and decisions I made on this task design journey.

Embarking on this ‘messy’, cyclical and iterative journey in design-research, I developed skills as a task designer, a CoP facilitator, and a researcher. My own conceptual understanding of the multiple constructs of fractions and of musical notation has deepened, along with my pedagogical insights into task design. Through this process, I too have developed an even greater appreciation for the elegance and beauty of the “mental art” (Freudenthal, 1991, p. 2) that we call mathematics.

REFERENCES

Adler, J., Pournara, C., & Graven, M. (2000). Integration within and across mathematics. *Pythagoras*, *53*, 2–13. https://doi.org/10.15700/saje.v28n2a170

Ainley, J., Gould, R., & Pratt, D. (2015). Learning to reason from samples: Commentary from the perspectives of task design and the emergence of "big data". *Educational Studies in Mathematics, 88*(3), 405-412. https://doi.org/10.1007/s10649-015-9592-4

Amon, R. (2017). Music and mathematics. In G. Glaeser (Ed.), *Math Tools: 500+ Applications in Science and Arts* (pp. 501-518). https://doi.org/10.1007/978-3-319-66960-1

An, S. A, Capraro, M. M., & Tillman, D. (2013). Elementary teachers integrate music activities into regular mathematics lessons: Effects on students' mathematical abilities. *Journal for Learning Through the Arts, 9*(1), 1-20. https://doi.org/10.21977/D99112867

An, S. A., & Tillman, D. (2014). Elementary teachers' design of arts based teaching: investigating the possibility of developing mathematics-music integrated curriculum. *Journal of Curriculum Theorizing, 30*(2), 20-38. https://journal.jctonline.org/index.php/jct/article/view/511

An, S. A., Tillman, D., Lesser, L., Siessen, A., & Tinajero, J.V. (2016). Learning to Teach Music-Themed Mathematics: An Examination of Preservice Teachers' Beliefs about Developing and Implementing Interdisciplinary Mathematics Pedagogy. *Mathematics Teacher Education and Development, 18*(1), 20-36. https://mted.merga.net.au/index.php/mted/article/view/273

Ariba, O., & Luneta, K. (2018). Nurturing creativity in early years' mathematics via art-integrated mathematics lessons. *The International Journal of Early Childhood Learning, 25*(2), 31-48. https://doi/org/10.18848/2327-7939/CGP/V25I02/31-48

Azaryahu, L., & Adi-Japha, E. (2020). "MusiMath" - a music-based intervention program for learning patterns and symmetry. *The Journal of Experimental Education, 90*(2), 319-343. https://doi.org/10.1080/00220973.2020.1799316

Azaryahu, L., Courey, S. J., Elkoshi, R., & Adi-Japha, E. (2019). 'MusiMath' and 'Academic Music' – Two music-based intervention programs for fractions learning in fourth grade students. *Developmental Science, 23*(4), https://doi.org/10.1111/desc.12882

Bakker, A. (2018). *Design research in education: a practical guide for early career researchers*. Routledge.

Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2020). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. *Journal of Educational Psychology, 112*(3), 628–648. https://doi.org/10.1037/edu0000384

Barnes, J. (2015). An Introduction to Cross-Curricular Learning. In P. Driscoll, A. Lambirth, & L. Roden. *The Primary Curriculum: A creative approach* (pp. 260-283). Sage.

Bassey, M. (1981). Pedagogic research: On the relative merits of search for generalisation and study of single events. *Oxford Review of Education, 7*(1), 73-94. https://doi.org/10.1080/0305498810070108

Bautista, A. (2021). STEAM education: Contributing evidence of validity and effectiveness. *Infancia Y Aprendizaje, 44*(4), 755-768. https://doi.org/10.1080/02103702.2021.1926678

Bautista, A., Tan, L. S., Ponnusamy, L. D., & Yau, X. (2016). Curriculum integration in arts education: Connecting multiple art forms through the idea of 'space'. *Journal of Curriculum Studies, 48*(5), 610-629. https://doi.org/10.1080/00220272.2015.1089940

Behr, M., Lesh, R., Post, T., & Silver E. (1983). Rational Number Concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and Processes* (pp. 91-125). Academic Press.
Bernstein, B. (1971). On the classification and framing of educational knowledge. In B. Bernstein (Ed.), *Knowledge, Education, and Cultural Change: Papers in the Sociology of Education* (pp. 363-392). Routledge.

Bicer, A., Bicer, A., Perihan, C., & Lee, Y. (2022). Pre-service teachers' preparations for designing and implementing creativity-directed mathematical tasks and instructional practices. *Mathematics Education Research Journal, 34*(3), 491–521. https://doi.org/10.1007/s13394-022-00409-x

Bickley-Green, C. A. (1995). Math and art curriculum integration: A post-modern foundation. *Studies in Art Education, 37*(1), 6-18. https://doi.org/10.2307/1320488

Bouchamma, Y., April, D., & Basque, M. (2018). How to Establish and Develop Communities of Practice to Better Collaborate. *Canadian Journal of Educational Administration and Policy, 187*, 91-105.

Bresler, L. (1993). Music in a double-bind: Instruction by non-specialists in elementary schools. *Bulletin of the Council for Research in Music Education, 115*(Winter, 1993), 1-13. https://www.jstor.org/stable/40318744

Bresler, L. (1995). The subservient, co-equal, affective, and social integration styles and their implications for the arts. *Arts Education Policy Review, 96*(5), 31-37. https://doi.org/10.1080/10632913.1995.9934564

Bright, G. W., Behr, M. J., Post, T. R., & Wachsmuth, I. (1988). Identifying Fractions on Number Lines. *Journal for Research in Mathematics Education, 19*(3), 215-232. 10.2307/749066

Broudy, H.S. (1972). *Enlightened cherishing: An essay on aesthetic education.* University of Illinois.

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics, 64*(3), 293-316. https://doi.org/10.1007/s10649-006-9036-2

Chin, S. L., Choy, B. H., & Leong, Y. H. (2022). Overlaps and shifts of instructional goals in the design of a set of mathematics tasks. *Mathematics Education Research Journal, 34*(3), 523-549. https://doi.org/10.1007/s13394-022-00425-x

Choutou, C. (2019). Supporting collaboration between visual art and mathematics teachers. In H. Borko & D. Potari (Eds.), *Proceedings of the 25th ICMI Study: Teachers of mathematics working and learning in collaborative groups*, 3-7. Lisbon, Portugal: ICMI.

Choy, B. H. (2016). Snapshots of mathematics teacher noticing during task design. *Mathematics Education Research Journal, 28*(3), 421-440. https://doi.org/10.1007/s13394-016-0173-3

Civil, M. D. (2007). *Using music to improve learning in mathematics*. (Publication No. 272) [Master's thesis, State University of New York College]. SUNY Brockport Digital Commons. https://digitalcommons.brockport.edu/ehd_theses/272

Cobb, P., Zhao, Q., & Dean, C. (2009). Conducting design experiments to support teachers' learning: A reflection from the field. *The Journal of the Learning Sciences, 18*(2), 165-199. https://doi.org/10.1080/10508400902797933

Cobb, P., Zhao, Q., & Visnovska, J. (2008). Learning from adapting the theory of realistic mathematics education. *Education & Didactique*, *2*(10), 154 -124. https://doi.org/10.4000/educationdidactique.276

Coffey, A., & Atkinson, P. (1996). *Making sense of qualitative data: complementary research strategies*. Sage.

Coles, A., & Brown, L. (2016). Task design for ways of working: making distinctions in teaching and learning mathematics. *Journal of Mathematics Teacher Education, 19*(2-3), 149-168. https://doi.org/10.1007/s10857-015-9337-4

Collins English Dictionary. (2022). Collins. https://www.collinsdictionary.com/

Cortina, J. L., Visnovska, J., & Zuniga, C. (2014). Unit fractions in the context of proportionality: supporting students' reasoning about the inverse order relationship. *Mathematics Education Research Journal, 26*(1), 79-99. https://doi.org/10.1007/s13394-013-0112-5

Cortina, J., Visnovska, J., Graven, M., & Vale, P. (2019). Instructional design in pursuing equity: The case of the 'fraction as measure' sequence. In N. Govender, R. Mudaly, T. Mthethwa, A. Singh-Pillay (Eds.), *Proceedings of the 27th Annual Conference of Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE)*, 16-18. Durban, South Africa: SAARMSTE.

Cortina, J.L., & Visnovska, J. (2015). Supporting students to reason about the relative size of proper and improper fractions. *In M. Marshman, V. Geiger, & A. Bennison (Eds.), Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia*, 181–188. Sunshine Coast: MERGA.

Cortina, J.L., Visnovska, J. & Zúñiga. C. (2015). An alternative starting point for fraction instruction. *International Journal for Mathematics Teaching and Learning,* 1- 28. http://www.cimt.org.uk/ijmtl/index.php/IJMTL/issue/archive

Courey, S. J., Balogh, E., Siker, J. R., & Paik, J. (2012). Academic music: Music instruction to engage third-grade students in learning basic fraction concepts. *Educational Studies in Mathematics, 81*(2), 251-278. https://doi.org/10.1007/s10649-012-9395-9

Creswell, J. W. (2007). *Qualitative inquiry & research design: choosing among five approaches* (2nd ed.). Sage.

Creswell, J. W. (2009). *Research design: qualitative, quantitative, and mixed methods approaches* (3rd ed.). Sage.

Creswell, J. W. (2013). *Qualitative inquiry & research design: choosing among five approaches* (3rd ed.). Sage.

Dargie, D. (1991). Umngqokolo: Xhosa overtone singing and the song Nondel'ekhaya. *African Music, 7*(1), 33-47. http://www.jstor.org/stable/30249985

De Vos, A. S. (2011). *Research at grass roots: For the social sciences and human services professions* (4th ed.). Van Schaik.

Dewey, J. (1902). *Child and the curriculum*. University of Chicago.

Dewey, J. (1938). *Education and experience*. Macmillan.

Dole, S. (2005). *Mental computation: A strategies approach, Module 2 – Fractions and decimals*. (2nd ed.). https://research.usc.edu.au/esploro/outputs/99450184602621

Dole, S. (2010). Making connections to the big ideas in mathematics: Promoting proportional reasoning. In *Teaching Mathematics? Make it count: What research tells us about effective teaching and learning of mathematics.* [Paper presentation]. The Research Conference for Australian Council for Educational Research, 71-74.

Dowden, T. (2011). Locating curriculum integration within the historical context: Innovations in Aotearoa New Zealand state schools, 1920s-1940s. *History of Education Review, 40*(1), 47-61. https://doi.org/10.1108/08198691111140802

Drake, S. M., & Burns, R. C. (2004). *Meeting standards through integrated curriculum*. ASCD.

Du Plooy-Cilliers, F., Davis, C., & Bezuidenhout, R. (2014). *Research matters*. Juta.

Edelson, R. J., & Johnson, G. (2003). Music makes math meaningful. *Childhood Education, 80*(2), 65–70. https://doi.org/10.1080/00094056.2004.10521259

Eisner, E. W. (1996). *Cognition and curriculum reconsidered* (2nd ed.). Stanford University.

English, L. D. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM Education, 3*(3), 1-8. https://doi.org/10.1186/s40594-016-0036-1

Ernest, P. (2002). Book Review: Communities of practice: Learning, meaning, and identity. *British Journal of Educational Psychology*, *72*(3), 460 - 464.

Ferrance, E. (2000). *Action research.* Brown University.

Fraser, D. (2013). Curriculum integration. In Whyte, B., Fraser, D., & Aitken, V. (2013). *Connecting Curriculum, Linking Learning*. NZCER Press.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Kluwer Academic.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Kluwer Academic.

Friedman, V.J., Gray, P., & Aragon, A.O. (2018). From doing to writing action research: A plea to ARJ authors. *Action Research Journal, 16*(1), 3-6. https://doi.org/10.1177/1476750318763041

Gaare, M. (1997). Alternatives to traditional notation. *Music Educators Journal, 83*(5), 17–23. https://0-doi.org.wam.seals.ac.za/10.2307/3399003

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM*, 38(2), 143–162. https://doi.org/10.1007/BF02655886

Gantt, H.L. (1919). *Organizing for work*. Harcourt.

Geertz, C. (1973). *The interpretation of cultures*. Basic Books.

Geiger, V., Galbraith, P., Niss, M., & Delzoppo, C. (2022). Developing a task design and implementation framework for fostering mathematical modelling competencies. *Educational Studies in Mathematics, 109*(2), 313-336. https://doi.org/10.1007/s10649-021-10039-y

Geist, K., Geist, E., & Kuznik, K. (2012). The patterns of music: Young children learning mathematics through beat, rhythm, and melody. *Young Children*, *67*(1), 74 – 79. https://eric.ed.gov/?id=EJ975497

Getenet, S. & Callingham, R. (2021). Teaching interrelated concepts of fraction for understanding and teacher’s pedagogical content knowledge. *Mathematics Education Research Journal, 22*, 201-221. https://doi.org/10.1007/s13394-019-00275-0

Giles, A. M., & Frego, R. J. D. (2004). An inventory of music activities used by elementary classroom teachers: An exploratory study. *Update: Applications of Research in Music Education*, *22*(2), 13–22. https://0-doi.org.wam.seals.ac.za/10.1177/87551233040220020103

Gravemeijer, K. P. E. (1994). Educational development and developmental research in mathematics education. *Journal for Research in Mathematics Education, 25*(5), 443-471. https://doi.org/10.2307/749485

Gravemeijer, K., & Cobb, P. (2006). *Design research from a learning design perspective*. Routledge.

Gravemeijer, K., & van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *The Elementary School Journal, 109*(5), 510-524. https://doi.org/10.1086/596999

Graven, M. & Heyd-Metzuyanim, E. (2014). Exploring the limitations and possibilities of researching mathematical dispositions of learners with low literacy levels. *Journal of Sciencia Educacione, 5*, 20-35.

Graven, M. (1998). Are mathematics high school teachers ready for Curriculum 2005? In *Proceedings of the 6th Annual Meeting of the Southern African Association For Research in Mathematics and Science Education, 1*, 172-178).

Graven, M. (2002). The effect of the South African curriculum change process on mathematics teacher roles. In P. Valero & O. Skovsmose (Eds.), *Proceedings of the 3rd International MES Conference*, 1 - 10. Centre for Research in Learning Mathematics.

Graven, M. (2004). Investigating mathematics teacher learning within an in-service community of practice: The centrality of confidence. *Educational Studies in Mathematics*, *57*(2), 177-211. https://www.jstor.org/stable/4150270

Graven, M. (2012). Accessing and assessing young learner's mathematical dispositions. *South African Journal of Childhood Education, 2*(1), 49–62.

Graven, M. (2015). Strengthening maths learning dispositions through 'math clubs'. *South African Journal of Childhood Education, 5*(3), 1–7. http://dx.doi.org/10.4102/sajce.v5i3.342

Graven, M., & Coles, A. (2017). Resisting the desire for the unambiguous: Productive gaps in researcher, teacher and student interpretations of a number story task. *ZDM, 49*(6), 881-893. https://doi.org/10.1007/s11858-017-0863-7

Graven, M., Hewana, D., & Stott, D. (2013). The evolution of an instrument for researching young mathematical dispositions. *African Journal of Research in Mathematics, Science and Technology Education, 17*(1-2), 26-37. https://doi.org/10.1080/10288457.2013.826968

Greene, M. (1991). Texts and margins. *Harvard Educational Review, 61*(1), 27–39.

Grumet, M.R., Randolph, D., & Stanley, F. (2014). Arts integration: Reunion for a creative curriculum. In L. Bresler, M. Flemming, & J. O'Toole. (Eds.), *The Routledge International Handbook of the Arts and Education* (pp. 268–279). Routledge.

Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational Communication & Technology, 29*(2), 75-91. https://doi.org/10.1007/BF02766777

Gunawan, S., & Shieh, C. (2020). Effects of the application of STEM curriculum integration model to living technology teaching on business school students' learning effectiveness. *Contemporary Educational Technology, 12*(2), 1-7. https://doi.org/10.30935/cedtech/8583

Harney, K. (2020). *Integrating music across the elementary curriculum*. Oxford University. https://doi.org/10.1093/oso/9780190085582.001.0001

Hilton, A., Hilton, G., Dole, S., & Goos, M. (2016). Promoting middle school students' proportional reasoning skills through an ongoing professional development programme for teachers. *Educational Studies in Mathematics, 92*(2), 193-219. https://doi.org/10.1007/s10649-016-9694-7

Holmes, S., & Hallam, S. (2017). The impact of participation in music on learning mathematics. *London Review of Education, 15*(3), 425-438. https://doi.org/10.18546/LRE.15.3.07

Holmlund, T. D., Lesseig, K., & Slavit, D. (2018). Making sense of "STEM education" in K-12 contexts. *International Journal of STEM Education, 5*(1), 32-18. https://doi.org/10.1186/s40594-018-0127-2

Horne, R., & Makar, K. (2013). Building relationships between stakeholders and researchers: People, persistence and passion. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia*. Melbourne: MERGA.

Hough, S., & Gough, S. (2007). Realistic Mathematics Education. *Mathematics Teaching*, 203, 34-38.

Jojo, Z. (2019). Mathematics education systems in South Africa. In G. Porto (Ed.), *Education Systems Around the World. IntechOpen.* https://doi.org/10.5772/intechopen.85325

Jones, K., & Pepin, B. E. U. (2016). Research on mathematics teachers as partners in task design. *Journal of Mathematics Teacher Education, 19*(2-3), 105-121. https://doi.org/10.1007/s10857-016-9345-z

Julie C., & Gierdien F. (2020) Reflections on Realistic Mathematics Education from a South African Perspective. In van den Heuvel-Panhuizen M. (Ed.), *International reflections on the Netherlands didactics of mathematics. ICME-13 Monographs.* (pp.71-82). Springer. https://doi.org/10.1007/978-3-030-20223-1_5

Kaur, B., Leong, Y.H. & Attard, C. (2022). Teachers as designers of instructional tasks. *Mathematics Education Research Journal, 34*(3), 483–489. https://doi.org/10.1007/s13394-022-00437-7

Kelley, T. R., & Knowles, J. G. (2016). A conceptual framework for integrated STEM education. *International Journal of STEM Education, 3*(1), 1-11. https://doi.org/10.1186/s40594-016-0046-z

Kelly, A. E., Lesh, R. A., & Baek, J. Y. (2008). *Handbook of design research methods in education: innovations in science, technology, engineering, and mathematics learning and teaching*. Routledge.

Kieren, T. E. (1976). On the mathematical, cognitive and instructional foundations of rational numbers. In R. Lesh (Ed.), *Number and Measurement. Papers from a Research Workshop* (pp. 101–144). ERIC/SMEAC.

Kieren, T. E. (1980). The rational number construct – its elements and mechanisms. In T. E. Kieren (Ed.), *Recent research on number learning* (pp. 125-149). ERIC/SMEAC.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: helping children learn mathematics.* National Academy Press. http://www.nap.edu/catalog/9822.html

Kneen, J., Breeze, T., Davies-Barnes, S., John, V., & Thayer, E. (2020). Curriculum integration: The challenges for primary and secondary schools in developing a new curriculum in the expressive arts. *Curriculum Journal, 31*(2), 258-275. https://doi.org/10.1002/curj.34

Lamon, J. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Erlbaum.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University.

Leavy, P. (2017). *Research design: Quantitative, qualitative, mixed methods, arts-based, and community-based participatory research approaches*. Guildord.

Lotz-Sisitka, H., Fien, J., & Ketlhoilwe, M. (2013). Traditions and new niches: An overview of environmental education curriculum and learning research. In R. Stevenson, M. Brody, J. Dillon, & A. Wals (Eds.), *International Handbook of Research in Environmental Education* (pp. 194-205). Routledge.

Louw, M. (2014). Ethics in Research. In F. du Plooy-Cilliers, F. Davis & R-M. Bezuidenhout (Eds.), *Research Matters* (pp. 262-273). Juta.

Lovemore, T. (2020). *Enriching my teaching around the inverse order relationship in unit fractions at the grade 5 level through the inclusion of musical activities: An action research case study*. (Publication No. 38079) [Master's thesis, Rhodes University]. http://hdl.handle.net/10962/142429

Lovemore, T., Robertson, S-A., & Graven, M. (2021). Enriching the teaching of fractions through integrating mathematics and music. *South African Journal of Childhood Education 11*(1), a899. https://doi.org/10.4102/sajce.v11i1.899

Lovemore, T., Robertson, S-A., & Graven, M. (2022a). Aligning mathematical and musical linear representations to support fractional reasoning. In N. Fitzallen, C. Murphy & V. Hatisaru (Eds.), *Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia*, 354-361. Launceston: MERGA.

Lovemore, T., Robertson, S-A., & Graven, M. (2022b). Task design grapplings in integrating music and fraction representations. In K. R. Langenhoven & C. H. Stevenson-Milln (Eds.), *Book of Proceedings of the 30th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education*, 49-61. Western Cape: SAARMSTE.

Lovemore, T., Robertson, S-A., & Graven, M. (2022c). Initiating a community of practice amongst primary school mathematics teachers: Trials and triumphs. [Paper presentation]. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of European Research in Mathematics Education (CERME12)*. (pp. 3418-3425). ERME / Free University of Bozen-Bolzano.

Lovemore, T., Robertson, S-A., & Graven, M. (2023). Integrating mathematics and music to support fractional reasoning across multiple constructs: A task design journey. In *Proceedings of the 31st Conference of the Southern African Association for Research in Mathematics, Science and Technology Education.*

MacDonald, A., Wise, K., Tregloan, K., Fountain, W., Wallis, L., & Holmstrom, N. (2020). Designing STEAM education: Fostering relationality through design-led disruption. *The International Journal of Art & Design Education, 39*(1), 227-241. https://doi.org/10.1111/jade.12258

Makar, K. (2021). Engaging in longitudinal research with classroom teachers: Theoretical and practical considerations. In J. Novotná & H. Moraová. (Eds.*), International Symposium*

Elementary Maths Teaching SEMT '21 Proceedings (pp. 439– 440). Faculty of Education. ISBN 978-80-7603-260-6.

Maxwell, J.A. (2013). *A realist approach for qualitative research*. Sage.

McKinnon, D. H., Nolan, P. C. J., Openshaw, R., & Soler, J. (1991). New Zealand curriculum innovation in historical and political context: The Freyberg integrated studies project and parallel projects of the 1940s. *Journal of Curriculum Studies, 23*(2), 155-175. https://doi.org/10.1080/0022027910230205

McLachlan, N. & Greco, L. & Toner, E. & Wilson, S. (2010). Using spatial manipulation to examine interactions between visual and auditory encoding of pitch and time. *Frontiers in Psychology*, *1*. 233. https://doi.org/10.3389/fpsyg.2010.00233

McMillan, J. H., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry*. (6th ed.). Pearson.

McMullin, C. (2021). Transcription and Qualitative Methods: Implications for Third Sector Research. *Voluntas*. 1-14. https://doi.org/10.1007/s11266-021-00400-3

McPhail, G. (2018). Curriculum integration in the senior secondary school: A case study in a national assessment context. *Journal of Curriculum Studies, 50*(1), 56-76. https://doi.org/10.1080/00220272.2017.1386234

Merriam, S.B. (1998). *Qualitative research and case study applications in education.* Jossey-Bass.

Monson, D., Cramer, K., & Ahrendt, S. (2020). Using models to build fraction understanding. *The Mathematics Teacher, 113*(2), 117-123. https://www.jstor.org/stable/10.5951/mtlt.2019.0105

Musakula, F.M. (2014). *African music in music education: An exploration into the teaching of African music in two primary colleges of education in Zambia*. [Master's Thesis, Oslo and Akershus University College of Applied Sciences]. https://oda.oslomet.no/oda-xmlui/bitstream/handle/10642/2048/Musakula.pdf?sequence=2&isAllowed=y

Naidoo, D. (2009). Case studies of the implementation of "progression and integration" of knowledge in South African schools. *Education as Change, 13*(1), 5-25. https://doi.org/10.1080/16823200902933297

Naidoo, D. (2010). Losing the "purity" of subjects? understanding teachers' perceptions of integrating subjects into learning areas. *Education as Change, 14*(2), 137-153. https://doi.org/10.1080/16823206.2010.518001

Netshivhambe, E. (2018). The importance of documenting indigenous African sheet music. *Proceeding of the 4th International Conference on Arts and Humanities, 4*, 107-117. https://doi.org/10.17501/icoah.2017.4111

Ni, Y. & Zhou, Y. (2005). Teaching and learning fractions and rational numbers: The origins and implications of whole number bias. *Educational Psychology, 40*(1), 27 – 52. *https://doi.org/10.1207/s15326985ep4001_3*

Papadopoulos, A. (2014). Mathematics and group theory in music. In L. Ji, A. Papadopoulos, & S.-T. Yau. *Handbook of Group actions, 2*. https://doi.org/10.48550/arXiv.1407.5757

Pepin, B. (2018). Enhancing teacher learning with curriculum resources. In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), *Research on mathematics textbooks and teachers' resources: Advances and issues* (pp. 359-374). Springer.

Plomp, T. (2007). Educational design research: An introduction. In T. Plomp, & N. Nieveen. *An Introduction to Educational Design Research* (pp. 9-35). Netherlands Institute for Curriculum Development. https://research.utwente.nl/files/14472302/Introduction_20to_20education_20design_20research.pdf

Pluim, G., Nazir, J., & Wallace, J. (2020). Curriculum integration and the semicentennial of Basil Bernstein's classification and framing of educational knowledge. *Canadian Journal of Science, Mathematics and Technology Education, 20*(4), 715-735. https://doi.org/10.1007/s42330-021-00135-9

Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes: An overview on achievements and challenges. *ZDM, 47*(6), 877-891. https://doi.org/10.1007/s11858-015-0722-3

Presmeg, N. (2009). Mathematics education research embracing arts and sciences. *ZDM, 41*(1-2), 131-141. https://doi.org/10.1007/s11858-008-0136-6

Pyrko, I., Dörfler, V., & Eden, C. (2017). Thinking together: What makes communities of practice work? *Human relations, 70*(4), 389-409. https://doi.org/10.1177/0018726716661040

Reddy, V., Isdale, K., Juan, A., Visser, M., Winnaar, L., & Arends, F. (2016). *TIMSS 2015: Highlights of Mathematics Achievement of Grade 5 South African Learners*. HSRC Press. https://www.iea.nl/sites/default/files/2019-04/TIMSS%202015%20Grade%205%20Highlights%20South%20Africa.pdf

Reddy, V., Winnaar, L., Arends, F., Juan, A., Harvey, J., Hannan, S., & Isdale, K. (2022a). *The South African TIMSS 2019 grade 9 results: Building achievement and bridging achievement gaps.* HSRC Press. https://www.timss-sa.org/wp-content/uploads/2022/03/FINAL-HSRC-TIMSS-Gr9-report_-ELECTRONIC.pdf

Reddy, V., Winnaar, L., Harvey, J., Hannan, S., Isdale, K., Arends, F. & Juan, A. (2022b). *The South African TIMSS 2019 grade 5 results: Building achievement and bridging achievement gaps*. HSRC Press. https://www.timss-sa.org/wp-content/uploads/2022/03/TIMSS-2019-Grade-5-National-report-FINAL.pdf

Saldaña, J. (2016). *The coding manual for qualitative researchers* (3rd ed.). Sage.

Salter, L. (1977). Review: Facts, feats, fictions. *The Musical Times, 118*(1612), 476. https://www.jstor.org/stable/958715

Samsudin, M. A., Bakar, K. A., & Noor, N. M. (2019). The benefits of music and movement in early mathematics. *Creative Education, 10*, 3071-3081. https://doi.org/10.4236/ce.2019.1012231

Saxe, G. B., Diakow, R., & Gearhart, M. (2013). Towards curricular coherence in integers and fractions: A study of the efficacy of a lesson sequence that uses the number line as the principal representational context. *ZDM, 45*(3), 343-364. https://doi.org/10.1007/s11858-012-0466-2

Schmidt-Jones, C. (2021). *Understanding basic music theory*. http://cnx.org/content/col10363/1.3/

Seidman, I. (1998). *Interviewing as qualitative research: A guide for researchers in education and the social sciences.* Teachers College Press.

Setati, M. (2005). Researching teaching and learning in school from “with” or “on” teachers to "with" and "on" teachers. *Perspectives in Education, 23*(1), 91–101. https://0-hdl.handle.net.wam.seals.ac.za/10520/EJC87300

Shahbari, J.A. & Peled, I. (2014). Modelling in primary school: Constructing conceptual models and making sense of fractions. *International Journal and Science and Mathematics Education, 15,* 391-391. https://doi.org/10.1007/s10763-015-9702-x

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information, 22*(2), 63-75. https://doi.org/10.3233/EFI-2004-22201

Siemon, D. (2003). Partitioning: The missing link in building fraction knowledge and confidence. *Australian Mathematics Teacher, 59*(3), 22-24. https://doi.org/10.3316/aeipt.130543

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2015). *Teaching mathematics: Foundation to middle Years*. (2nd ed.). Oxford.

Siemon, D., & Luneta, K. (2018). *Teaching mathematics: Foundation to senior phase* (2nd ed.). Oxford University.

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145. https://doi.org/10.2307/749205

Sloboda, J. A. (2012). *Exploring the musical mind: cognition, emotion, ability, function*. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780198530121.001.0001

Soni, M., & Okamoto, Y. (2020). Improving children's fraction understanding through the use of number lines. *Mathematical Thinking and Learning, 22*(3), 233-243. https://doi.org/10.1080/10986065.2020.1709254

South Africa. Department of Basic Education. (2011a). *Curriculum and Assessment Policy Statement: Mathematics Intermediate Phase Grade 4 - 6*. Department of Basic Education. https://www.education.gov.za/LinkClick.aspx?fileticket=dr7zg3CFCr8%3d&tabid=572&portalid=0&mid=1568

South Africa. Department of Basic Education. (2011b). *Curriculum and Assessment Policy Statement: Life Skills Intermediate Phase Grade 4 - 6*. Department of Basic Education. https://www.education.gov.za/LinkClick.aspx?fileticket=OX12620gNnw%3d&tabid=572&portalid=0&mid=1568

South Africa. Department of Basic Education. (2011c). *Curriculum and Assessment Policy Statement: Natural Science and Technology Intermediate Phase Grade 4 - 6*. Department of Basic Education. https://www.education.gov.za/LinkClick.aspx?fileticket=IzbFrpzoQ44%3d&tabid=572&portalid=0&mid=1568

South Africa. Department of Basic Education. (2019). *Diagnostic report 2018 – Part 1.* Department of Education. https://www.education.gov.za/Portals/0/Documents/Reports/FINAL%20DIAGNOSTIC%20REPORT%20NON%20LANGUAGES%202018.pdf?ver=2019-01-24-125928-000

South Africa. Department of Basic Education. (2020). *National Senior Certificate: School Subject Report*. Department of Education. https://www.education.gov.za/Portals/0/Documents/Reports/2019%20NSC%20School%20Subject%20Report.pdf?ver=2020-01-07-135604-000

South Africa. Department of Basic Education. (2022). *2021 National Senior Certificate: Diagnostic Report Part 1: Content Subjects*. Department of Basic Education. https://www.education.gov.za/LinkClick.aspx?fileticket=H1cFr8xeEh0%3d&tabid=92&portalid=0&mid=4359

South Africa. Department of Education. (2002). *Revised National Curriculum Statement Grades R-9 (Schools)*. https://www.education.gov.za/Portals/0/Documents/Policies/GET/NCS2002.pdf?ver=2007-08-22-083040-000

South Africa. Department of Education. (2003). *Revised National Curriculum Statement Grades R-9 (Schools): Teacher's Guide for the Development of Learning Programmes*. Government Printers. https://pdf4pro.com/amp/view/revised-national-curriculum-statement-grades-r-9-4b9f62.html

South Africa. Department of Higher Education and Training. (2015). Revised Policy on the Minimum Requirements for Teacher Education Qualifications, Government Gazette 38487. DHET. https://www.careerhelp.org.za/sites/default/files/Revised_Policy_Minimum_requirements_for_teachers.pdf

South African Government. (2022). *Protection of Personal Information Act 4 of 2013*. https://www.gov.za/documents/protection-personal-information-act

South African Numeracy Chair Project (SANCP). (2021). https://www.ru.ac.za/sanc/

Spangenberg, E. D., & Myburgh, C. (2017). Comparing South African female and male pre-service teachers' beliefs about the nature of mathematics. *Africa Education Review, 14*(2), 140-155. https://doi.org/10.1080/18146627.2017.1292828

Spaull, N. (2019). *The education problem as it stands in January 2019*. https://nicspaull.com/2019/01/?blogsub=confirming#blog_subscription-3

Stevenson-Milln, C. (2018). *Researching the development of a programme that merges mathematics and music in Grade R*. [Master's Thesis, Rhodes University]. http://hdl.handle.net/10962/61928

Stott, D., Graven, M., Baart, N., Hebe, G., & Mofu, Z. (2017). After school maths clubs: Investigating learner progression in an expanding intervention model. In T. Penlington & C. Chikiwa (Eds.), *Proceedings of the 23rd Annual National Congress of the Association for Mathematics Education of South Afric*a, 1, 313–324. Port Elizabeth, South Africa: AMESA.

Streefland, L. (1991). *Fractions in realistic mathematics education: A paradigm of developmental research*. Kluwer Academic.

Sullivan, P., Clarke, D., & Clarke, B. (2009). Converting mathematics tasks to learning opportunities : an important aspect of knowledge for mathematics teaching. *Mathematics Education Research Journal, 21*(1), 85-105. https://doi.org/10.1007/BF03217539

Sullivan, P., Clarke, D., & Clarke, B. (2013). *Teaching with Tasks for Effective Mathematics Learning*. Springer New York. https://doi.org/10.1007/978-1-4614-4681-1

Swan, M. (2005). *Improving learning in mathematics: Challenges and strategies*. DfES.

Tanner, K. (2008). Working with students to help them understand fractions. *Australian Primary Mathematics Classroom, 13*(3), 28-31.

Tasik, W. F. & Edo, S. I . (2019). Design research on applied Realistic Mathematics Education (RME) approach in teaching math for vocational college. *Jurnal Pendidikan Vokasi*, *9*(3), 294-306. https://doi.org/10.21831/jpv.v9i3.27839

Thomas, M. O. J. (2006). Developing versatility in mathematical thinking. In A. Simpson (Ed.), *Proceedings of Retirement as Process and Concept: A festshcrift for Eddie Gray and David Tall* (pp. 223–241). Charles University.

Treffers, A. (1986). *Three dimensions: a model of God and theory description in mathematics instruction - the Wiskobus project*. Reidel.

Trimble, H. C. (1949). Fractions are ratios too. *The Elementary School Journal*, 49(5), 285-291. https://doi.org/10.1086/459044

Tytler, R., Mulligan, J., Prain, V., White, P., Xu, L., Kirk, M., Nielsen, C., & Speldewinde, C. (2021). An interdisciplinary approach to primary school mathematics and science learning. *International Journal of Science Education, 43*(12), 1926-1949. https://doi.org/10.1080/09500693.2021.1946727

Tzur, R. (1999). An integrated research on children's construction of meaningful, symbolic, partitioning-related conceptions and the teacher's role in fostering that learning. *The Journal of Mathematical Behavior, 18*(2), 123-147. https://doi.org/10.1016/S0732-3123(99)00025-5

Tzur, R. (2016). On natural numbers and fractions: A reorganization (no bias) stance. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education, 1*, 399-403. Hungary: University of Szeged.

van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9-35. https://doi.org/10.1023/B:EDUC.0000005212.03219.dc

van den Heuvel-Panhuizen, M. (2020). Seen through other eyes—opening up new vistas in realistic mathematics education through visions and experiences from other countries. In M. van den Heuvel-Panhuizen (Ed.), *International reflections on the Netherlands didactics of mathematics*. ICME-13 Monographs. (pp.1-20). Springer, Cham. https://doi.org/10.1007/978-3-030-20223-1_1

Venkat, H. & Graven, M. (2017). Changing teaching through a resources approach. In M. Graven & H. Venkat (Eds.), *Improving Primary Mathematics Education, Teaching and Learning Research for Development in Resource-Constrained Contexts* (pp. 163–178). Macmillan.

Virtue, D. C., Wilson, J. L., Ingram, N. (2009). L.E.S.S can be more! *Middle School Journal*, *40*(3), 4−11.

Watson, A., & Ohtani, M. (2012). Task design in mathematics education discussion document. ICMI study 22 announcement and call for papers. http://ncm.gu.se/media/ncm/dokument/ICMI_Study_22_announcement_and_call_for_papers.pdf

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University.

Westrup, J. A., Harrison, F. L., & Wilson, C. (1991). *Collins encyclopedia of music* (Revised ed.). Chancellor.

Whitaker, N. L. (1996). Elusive connections: Music integration and the elementary classroom. *Bulletin of the Council for Research in Music Education,* (130), 89-99.

Willis, J., W. (2007). *Foundations of qualitative research: Interpretive and critical approaches*. Sage.

APPENDICES

APPENDIX 1: ETHICAL CLEARANCE CONFIRMATION LETTER

RHODES UNIVERSITY
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28/09/2020

Ms Tarryn LOVEMORE

Education Department

g18l4923@campus.ru.ac.za

Dear Ms Tarryn LOVEMORE and Dr Sally-Ann Robertson

Re: Intermediate Phase teachers' experiences of the use of music note values to support their teaching of fractions

APPLICATION NUMBER: 2020-2678-4653

This letter confirms that your research ethics application has been reviewed and **APPROVED** by the Education Faculty Research Ethics Committee (EF-REC). Your permission letter from the principal has been received and you are free to proceed with your study.

Approval is granted for 1 year. An annual progress report is required in order to renew approval for an additional period. You will receive an email at this address, notifying you when the progress report is due.

Should any substantive change(s) be made during the research process, that may have ethical implications, you should notify the Education Faculty REC Chair via email. This includes changes in investigators. The REC Chair will advise as to whether a new application is necessary.

Do keep this clearance letter secure and accessible throughout your study and after its completion. It will be needed when a thesis is examined and when publications are submitted to journals.

Please also submit a brief report to the REC Chair on the completion of the research. This can be done via email. The purpose of this report is to indicate whether the research was conducted successfully and whether any ethics-related matters arose that the committee should be aware of, in order to guide future studies.

Sincerely,

ERosenberg

Prof Eureta Rosenberg

Chair: Education Faculty Research Ethics Committee

APPENDIX 2: COPY OF GATEKEEPER PERMISSION REQUEST LETTERS

Aloe School

[Address]
21 September 2020

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT XXX

Dear Principal

I am a PhD student in Mathematics Education student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my doctoral full thesis will involve me initiating a Community of Practice, consisting of 6 to 8 intermediate phase (IP) teachers. In this space I intend on sharing the teaching strategies and resources which I have developed in integrating mathematics and music for the teaching and learning of fractions. The teachers will be given the opportunity to interrogate, trial and adjust these strategies and resources and then provide reflections on their perceptions on the use of music to tech fractions. I intend for this research to be conducted during semester 1 of 2021. This research will be conducted under the supervision of Dr Sally-Ann Robertson and Prof Mellony Graven.

This letter serves to seek formal consent to approach the IP teachers in your school, to invite them to participate in this Community of Practice. I request your permission to conduct my research as outlined in my research proposal.

I attach a copy of my research proposal, which includes copies of the consent forms to be used in the research process. Once I have received ethical clearance from Rhodes University, I will provide you with the ethical clearance letter. As part of this, I undertake to ensure that the name of the school and all participants will be replaced with pseudonyms and that all the material I collect as part of the research will be accessible only to myself and my supervisor.

Upon completion of the study, I undertake to provide you and the participants access to the research findings. If you require any further information, please do not hesitate to contact me on [cell phone number] or at [e-mail address].

Thank you for your time and consideration in this matter.

Yours sincerely

Miss Tarryn Shirley Lovemore (18L4923)

Rhodes University

Protea School

[Address]
19 August 2021

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT XXX

Dear Principal

I am a PhD Mathematics Education student at Rhodes University (RU) in Makhanda, South Africa. The research I wish to conduct for my doctoral full thesis will involve me initiating a Community of Practice, consisting of intermediate phase (IP) teachers. In this space I intend on sharing the teaching strategies and resources which I have developed in integrating mathematics and music for the teaching and learning of fractions. The teachers will be given the opportunity to interrogate, trial and adjust these strategies and resources and then provide reflections on their perceptions on the use of music to teach fractions. I intend for this research to be conducted during term 4 of 2021 and term 1 of 2022. This research will be conducted under the supervision of Dr Sally-Ann Robertson and Prof Mellony Graven.

This letter serves to seek formal consent to approach the IP teachers in your school, to invite them to participate in this Community of Practice. I request your permission to conduct my research as outlined in my research proposal. If you agree to give permission, would you kindly complete the attached template gatekeeper permission letter.

I attach a copy of my research proposal, which includes copies of the consent forms to be used in the research process. Once I have received ethical clearance from Rhodes University, I can provide you with the ethical clearance letter. As part of this, I undertake to ensure that the name of the school and all participants will be replaced with pseudonyms and that all the material I collect as part of the research will be accessible only to myself and my supervisors.

Upon completion of the study, I undertake to provide you and the participants access to the research findings. If you require any further information, please do not hesitate to contact me on [cell phone number] or at [e-mail address].

Thank you for your time and consideration in this matter.

Yours sincerely

Miss Tarryn Shirley Lovemore (18L4923)

Rhodes University

APPENDIX 3: COVID-19 PROTOCOL DOCUMENT

Research Study: Covid-19 Protocols to be followed

Tarryn Lovemore, PhD Student, Rhodes University

In line with regulations set by the Department of Health* I shall endeavour to follow protocols to decrease the risk of the spread of Covid-19, while conducting research at XXX School. I intend to do so by following these guidelines:

- Ensure to wear a mask at all times
- Request research participants to wear their masks at all times
- Set up the venues for meetings in such a way that social distancing can be maintained
- Open windows and doors whenever possible to make sure the venue is well ventilated
- Provide alcohol-based hand sanitizer for research participants to use
- Sanitize surfaces of objects we may come into contact with
- If necessary, the meetings can be conducted online on platforms such as Zoom

I trust that by maintaining these measures research participants will be at minimal risk of spreading the virus.

*Department of Health. (2020). *COVID-19 Disease: Infection Prevention and Control Guidelines Version* 2. Available from: https://www.nicd.ac.za/wp-content/uploads/2020/05/ipc-guidelines-covid-19-version-2-21-may-2020.pdf [Accessed 24-02-2021].

APPENDIX 4: COPY OF INVITATION TO PARTICIPANTS (TEACHERS) AND INFORMED CONSENT FORM

Dear Teacher

Re: Invitation to participate in a research study

You are invited to participate in a research study entitled, *Integrating Music and Mathematics for Connecting Across Multiple Constructs of Fractional Understanding: An RME Task Design Journey*. The aim of this study is to trial, interrogate and reflect on some strategies and resources which I have developed for the teaching and learning of fractions through the integration of music note values. I envision forming a Community of Practice (CoP) in which we share reflections on the resources and strategies and your experiences. This is intended to be a supportive space.

The first phase will involve a workshop style meeting where I share with you the strategies and resources I have already designed. I will provide you with some resources which you may make use of. I will also share with you basic knowledge of music note value representation, (You do not need to have any prior experience or knowledge in music in order to participate). Thereafter, you will be able to make decisions on how you feel the use of these strategies and resources could benefit your teaching of fractions. I intend for this research to take place during Semester 1 of 2021. We will make joint decisions on the frequency and times of meetings within the CoP to give regular feedback and to adjust and improve the strategies and resources. I will meet with you at your school during times which best suite you. I would also request and individual interview with you at the end of the intervention period.

I am fully aware of the heavy workload placed on teachers, and I intend to ensure that your participation in this study does not negatively affect your teaching workload. I will be available to support you throughout the journey, in person and via electronic communication. Your participation would be very helpful to me in terms of receiving feedback on your perceptions of the integration of music note values to teach fractions. Your participation in the research would be anonymous.

If you are willing to participate, I will explain in more detail what would be expected of you and provide you with the information you need to understand the research. These guidelines would include potential risks, benefits, and your rights as a participant. This study has been approved by Rhodes University's Ethics Committee (Faculty of Education) and I can send you a copy of their letter of ethical approval.

Participation in this research is voluntary. A positive response to this letter of invitation does not oblige you to take part in this research. To participate, I will ask you to sign a consent form to confirm that you understand and agree to the conditions, prior to any interview commencing. Please note that you would have the right to withdraw at any given time during the study.

Thank you for your time. I hope that you will respond favourably to my request.

Yours sincerely,

Miss Tarryn Shirley Lovemore

Informed consent form

Research Project Title:	Integrating Music and Mathematics for Connecting Across Multiple Constructs of Fractional Understanding: An RME Task Design Journey
Principal Investigator:	Miss Tarryn Shirley Lovemore

Participation Information
• I understand the purpose of the research study and my involvement in it. • I understand the risks and benefits of participating in this research study. • I understand that I may withdraw from the research study at any stage without any penalty. • I understand that participation in this research study is done on a voluntary basis. • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name. • I understand that data collection requirements particular to this research (personal information, video recording, photographs, additional artefacts) may be used. • I understand and agree that the interviews will be recorded electronically. • I understand that I will be given the opportunity to read and comment on the transcribed interview notes. • I confirm that I am not participating in this study for financial gain.

Information Explanation
The above information was explained to me by Miss Tarryn Shirley Lovemore.
The above information was explained to me in English and I am in command of this language:

Voluntary Consent	
I, ____ hereby voluntarily consent to participate in the above-mentioned research.	
Signature:	Date: / /

Investigator Declaration	
I, Miss Tarryn Shirley Lovemore, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked of me by the participant.	
Signature:	Date: / /

APPENDIX 5: COPY OF INVITATION TO PARTICIPANTS (GUARDIANS OF LEARNERS) AND INFORMED CONSENT FORM

(communication via principal)

Research Project Title:	Integrating Music and Mathematics for Connecting Across Multiple Constructs of Fractional Understanding: An RME Task Design Journey
Principal Investigator:	Miss Tarryn Lovemore

Participation Information
• I understand the purpose of the research study and my child's involvement in it. • I understand the risks and benefits of my child participating in this research study. • I understand that my child may withdraw from the research study at any stage without any penalty. • I understand that participation in this research study is done on a voluntary basis. • I understand that while information gained during the study may be published, my child will remain anonymous and no reference will be made to him/her by name. • I understand that data collection requirements particular to this research (comments, photographs, additional artefacts such as examples of learners' work) may be used. • I understand that I will be given the opportunity to see and comment on any artefacts involving my child. • I confirm that my child is not participating in this study for financial gain.

Information Explanation
The above information was explained to me in written communication by [Principal of School] and I have been made aware that I can communicate any further questions or concerns with Miss Tarryn Lovemore via electronic or face to face discussion.
The above information was explained to me in writing in English and I am in command of this language.

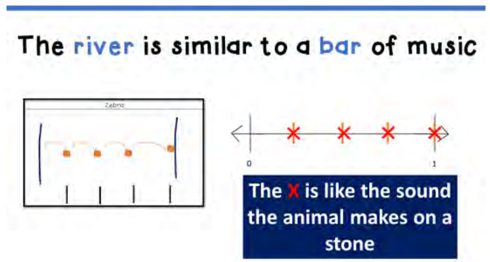
Voluntary Consent (Guardian 1)	
I, ____ hereby voluntarily consent to allow my child, ____________________________ , to participate in the above-mentioned research.	
Signature:	Date: / /
Voluntary Consent (Guardian 2)	
I, ____ hereby voluntarily consent to allow my child, ____________________________ , to participate in the above-mentioned research.	
Signature:	Date: / /

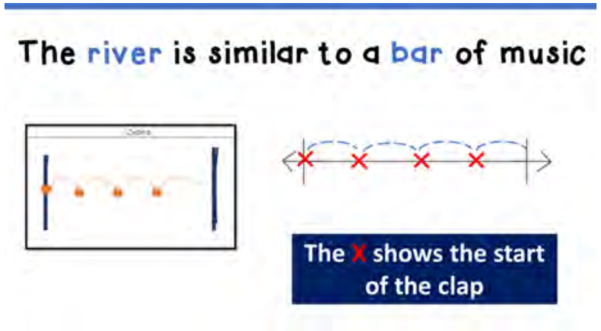
OR

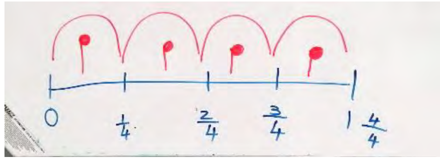
I, ____ would prefer my child, ____, NOT to be part of the above-mentioned research.	
Signature:	Date: / /

APPENDIX 6: SAMPLE OF ANALYSIS MATRIX

Obstacle-Resolution Cycle 5	**Obstacle** **Visual placement of animal river-crossing jumps linked to the percussion claps**	**Resolution of obstacle** **Focus on the animal jump rather than on the start or landing. Place music notes in the middle of the bar. Cannot conflate all aspects of the music and the mathematics; separate fraction as measure and fraction as ratio to maintain fidelity of music concepts.**
*How does the task maintain the **fidelity** of the mathematics, the music, and the integration of the two?*	*"What I was concerned about when I first started working on these ideas was that if you look at notes in a music bar, you will have the notes here, in the centre of the bar… and then the number line is the next whole. The fourth quarter note is before the bar ends, which is why I've placed the jump right on the line here, each time to show that the fourth jump takes us to the whole. The four quarters, is the whole, it is the end of the bar, it is the riverbank." (TL_21-03-25).* **Mathematics:**"…you need to start at the zero line you've, you've got your start 0 line and you would need to start that line" (MG_21-03-25). **Music:** *"To show that it's the whole, but when you have the music bar, it is going to be before the bar line" (TL_21-03-25).*	*"So put the lines, so that the first line is the line that jump off. It's the zero, it's the river bank… And then you've got four jumps 1234, so yeah you've got your start line, you've got your finish line and you've got your three lines in between, and then your music notes on the land. Your music notes sit in the middle of the lines, yes… it's gotta be drawn like this so that exactly four of them sit when you draw the notes that's where they land." (MG_21-03-25).*
*How does the task **simplify** the complexity of the integration*	The question of **where to place the jumps** (or lands) of the animals' river-crossings was challenging, as notes are	21-03-25: Notes in the middle… notes start on 1st bar line, notes end on last bar line…**notes in the middle (Xs and staff)**

for implementation within the classroom?	normally in the middle. We did not know where to place the jumps/landings and the notes so that they would align. A further obstacle was to decide **where to start the jumps vs the notes** – from zero?	Focus on jumps, focus on Jump, **focus on jumP**; draw landings starting from 1st riverbank, ending on last river bank. Explain to teachers that the music and mathematical representations are similar but not the same.
*What **key representation** would best support conceptual clarity?*	“But mathematically the image of same jumping a distance, you get that image of yours, like jumping across the river and they jumping at this rhythm” (MG_21-03-25).  **2021-09-24** *MG: But where this **doesn’t then link up** is that we’ve got the sounds based on the animal jumps, and the animal doesn’t have a sound when he leaves the bank. The first sound is when he lands. We are clapping animal percussion, it’s not percussion when we all start clapping at the same time.* *TL: If you interpose it, the sound line would only start there.* *MG: but you can start with a quiet.* *TL: does it have to match perfectly, visually? If we’re not going to super-impose them?* *SR: You’re trying to do too much at one time. Early on in our discussion, we said that in musical notation, the sound*	 **2021-09-24** MG: *It’s not the count, but it’s the end of the sound…. The line needs to show the time. What if the cross had a line going across, horizontal? Almost like an arrow, so that this clap had the duration of the time interval. So that there’s a continuity… so that’s the time that it lasts. Then the tails align.*

	starts at the beginning of the bar, on the zero. But I quote like the idea of a rest, the jump starts on a silent point, a rest in music. Would that resolve the infidelity? *TL: No…because it will be adding in a beat…* ***if we look at the animals jump rather than the land, it will help.*** *MG: that's why the music notes are in the middle.* *TL: if we move away from the sound of the land, and think that each jump in itself is a duration of sound.* *MG: jump, jump, jump….* *MG:* ***it's problematic is we have the clap start on the zero line, because at 0 it is 1 clap, and it's problematic*** *when we go to the number line…. Okay, so at that line, is the end of the sound of the first clap* *TL: and the beginning of the sound of the first one.*	*TL: I also want to say, that on the percussion line we should have the zero or the one. You just have bars… in other words, we are looking at the arrows, the duration, more than the claps.* MG: the  **21-10-20** MG: the way we going with it now is that it's all about the animals in this lesson. In a later lesson you'll look at the percussion… Animals are jumping across the river, there are different little stones or different you know sounds they make as they jump. They jump these different distances… jump jump jump, so that the **claps are happening on when they land**. Okay, because that then links up that style of percussion with the time of the jump jump… **So the clap is the land**… In terms of these animals crossing the River. Okay, and the crosses represent a percussion sound and we can clap as they land. … MG: But now we're going to look at how actually musicians do it in percussion and then they're going to teach that percussion land where you start clapping all on the on the start. TL: put in the middle. … MG: The problem is representing percussion because we're not going to go jump and clap on the land we're actually going to start clapping together and now we're talking beats per bar and everything that we do differently.

	21-09-24 SR: and then whether you do duration or you do just the sound of each jump is also a question at issue, because we've not got a piano key where you can hold the sound. You only get the sound on the land. MG: Ja TL: Yes.	…. MG: we're going to clap on the **land** of the jump… Then we're going to go to the music teacher and say okay, but in music, we all start clapping together. …In music it's not jump, and then the sound, as long as the jump takes. In music we start immediately with the sound and it's all about how long the sound last… They are simply showing claps per bar and now we're not talking number lines anymore now we're talking ratio . 21-10-21 MG: Juump, Juuump… but now w**e're going to clap on 'J', we're not ging to clap on the land**. … TL: and so you would start together 

APPENDIX 7: NVIVO CODES USED FOR ANALYSIS

CoP		
	Communication Channels	
	Covid-19	
	Teacher Co-researchers	
	Teachers adapt	
	Teachers' comfort zone	
	Time	
Grappling		
Integration		
	Music-Maths integration	
Mathematics		
	Evaluation of prior SKAV	
		Attitudes and beliefs
		Fractions greater than 1
	Fraction Constructs	
		Measure
		Part-whole
		Ratio
	Number line	
Musical Knowledge		
Obstacle-Resolution Cycles		
	ORC 1 Selecting problem scenario	
	ORC 10 Why music	
	ORC 2 What is the unit	
	ORC 3 Speed	
	ORC 4 Musical notation limitations	
	ORC 5 Visual placement of jumps and claps	
	ORC 6 Aligning rather than superimposing	
	ORC 7 Mistake visual notes	
	ORC 8 Rest	
	ORC 9 Progression	
RME		
	Experientially real	
	Informal speaking and modelling	
	Models for general mathematical reasoning	
	Vertical mathematisation	
Teacher Experiences		
	Challenges	
	Collaboration	
	Notation	
	Positive	
		Creativity
		Enrichment
		Fractional reasoning
		Motivation
		Opportunity for differentiation
		Resources
		Teachers' own learning
	Recommendations	
		Progression

APPENDIX 8: PRODUCT OF PARTICIPATORY DUAL-DESIGN EXPERIMENT IN TASK DESIGN - SAMPLES AND HIGHLIGHTS

Integrated music-mathematics lessons and resources

Lesson 1

Curriculum aims	
Mathematics	**Life Skills - Performing arts: Music**
• Problem-solving with fractional reasoning • Equivalence (DBE, 2011a: 16) • Focus is on Fraction as a Ratio (Rate) and Fraction as a Measure.	• Movement (jumps) • Spatial awareness • Exploring rhythmic patterns • Use of percussive instruments from found objects • African traditional story • Reflection of social, culture, environmental issue. • Appreciation for percussive instruments (African and Western) • Read and interpret African folktale or traditional story (DBE, 2011b: 12-13)

Lesson Outcomes

By the end of the lesson, learners should be able to:

- Share opinions from the traditional story relating to music.
- Beat a percussion instrument (drum, sticks, clapping) to the time of a second.
- Walk to the beat of a second.
- Jump across a space in time with a beat (4 jumps, 2 jumps, 1 jump, 8 jumps).
- Draw (notate) the beat at which they jumped to cover the distance.
- Recognise their ability to participate in a music activity.

Resources

- Video clip *The First Music* by Dylan Pritchett: https://youtu.be/EqGli-UrHPw
- Laptop, projector, speakers
- String
- Analogue clock
- Percussion instruments (from found objects, e.g. bucket)
- Drawing/notation template sheet

Introduction

1. Show learners the video (African folktale): https://youtu.be/EqGli-UrHPw
2. Ask guiding questions to facilitate class discussion:
   Which animals could be part of making music?
   Where do we hear music around us?
   What lesson can you take from the story?

Body

1. Set up the space which will indicate the 'river', using string.

2. Provide learners with the scenario: ***This is a river which the animals need to cross for their migration. Each animal crosses the river with different rhythmic beats/jumps. However, IF the animal does not cross the river in the allocated 4 beats (on the percussion instrument), the animal will be eaten by the crocodile.***
   NOTE: Animals will cross the river at the same speed i.e. all animals will take 4 seconds to cross the river.

3. Divide learners into groups:
   - Different animals
   - Drummers/percussionists

4. Guide learners in beating the percussion instruments to a constant 4 beat rhythm, by following the second hand on an analogue clock (1 beat = 1 second). [This step may initially be left out and added in at a later stage].

5. Allow learners to cross the river within the 4 beats (4 seconds)
   → In any way they choose
   → Now with only 4 jumps, as a Zebra
   → Now as an ostrich with only 2 jumps (start encouraging equal distanced jumps)
   → Now as a kudu, with one long jump
   → Next, as a monkey with 8 jumps
   → Possibly add: as a rabbit with 16 jumps

6. The second part of the scenario is that the learners want to record the different ways in which the animals cross the river:
   You are a biologist observing the migration behaviours, noticing the sounds as the animals jump. You do not have a cell phone, camera or recorder, so you decide to record the jumps through drawings (notation).

7. After each jump, instruct learners to draw (notate) their jumps on the template sheet.

Conclusion

1. Facilitate class discussion about the drawings (notations). Possible questions to ask:
   Why did you decide to draw/notate the jumps in this way?
   How did you decide where to place the jumps/symbolic representations?
   How did the 4 beats from the percussion instrument help you to make the jumps?

Worksheet 1: River-Crossing Animal Jumps

Show the how the animals crossed the river

Zebra	Ostrich
Kudu	**Monkey**

Lesson 3

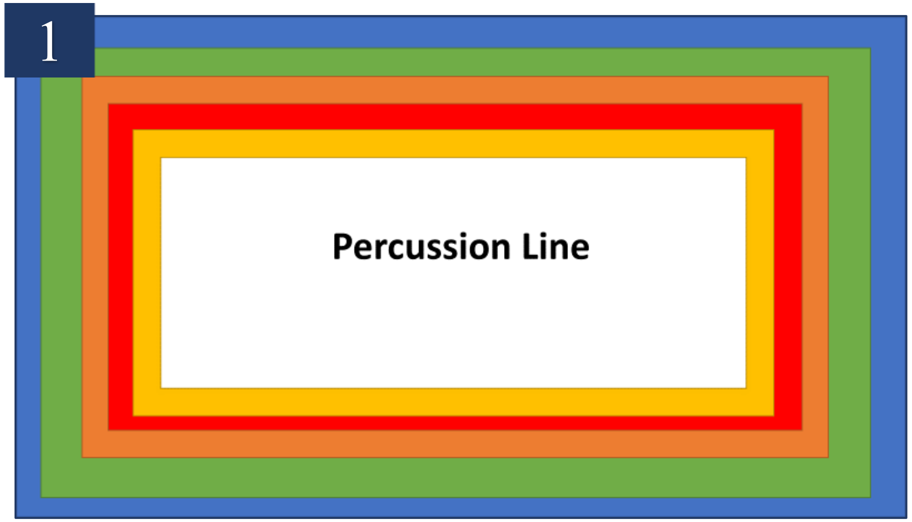
<table>
<tr><th colspan="2">Curriculum aims</th></tr>
<tr><th>Mathematics</th><th>Life Skills - Performing arts: Music</th></tr>
<tr><td>• Problem-solving with fractional reasoning
• Equivalence
(DBE, 2011a: 16)
• Focus is on Fraction as a Ratio (Rate) and Fraction as a Measure.</td><td>• Musical notation (adjusted percussion representation)
• Exploring rhythmic patterns
• Use of percussive instruments from found objects/body percussion
• Appreciation for percussive instruments (African and Western)
• (DBE, 2011b: 12-13)</td></tr>
<tr><th colspan="2">Lesson Outcomes</th></tr>
<tr><td colspan="2">By the end of the lesson, learners should be able to:
• Beat a percussion instrument (drum, sticks, clapping) / body percussion
• Relate notation of jumps to percussion notation of music.
• Recognise a percussion line and musical notation of beats in a bar.
• Read the beats in a bar and play them on a percussion instrument.
• Create rhythms with body percussion.
• Solve problems using fractional reasoning</td></tr>
<tr><th colspan="2">Resources</th></tr>
<tr><td>• Percussion instruments (from found objects, e.g. bucket)
• Laptop, projector, speakers
• PowerPoint with music percussion representation and sound clips</td><td>• Video clips for body percussion karaoke
• Worksheet 3: Percussion line template.</td></tr>
<tr><th colspan="2">Introduction</th></tr>
<tr><td colspan="2">Listening activity
1. Play the Clapping piece by Steve Reich (https://youtu.be/liYkRarIDfo)
2. Ask questions:
3. What do you hear? What instruments did you hear?
→ Did you know that body percussion is a type of music. We can use our hands and other body parts to make music.
→ Can you give some examples? (e.g. hand clapping, tapping, clicking, lap, stamp feet).

Body Percussion Karaoke
4. Play video: Uptown Funk by Bruno Mars (https://youtu.be/vkf5f-jmuZg)
5. Instruct learners to follow with the instructions, paying attention to the symbols</td></tr>
</table>

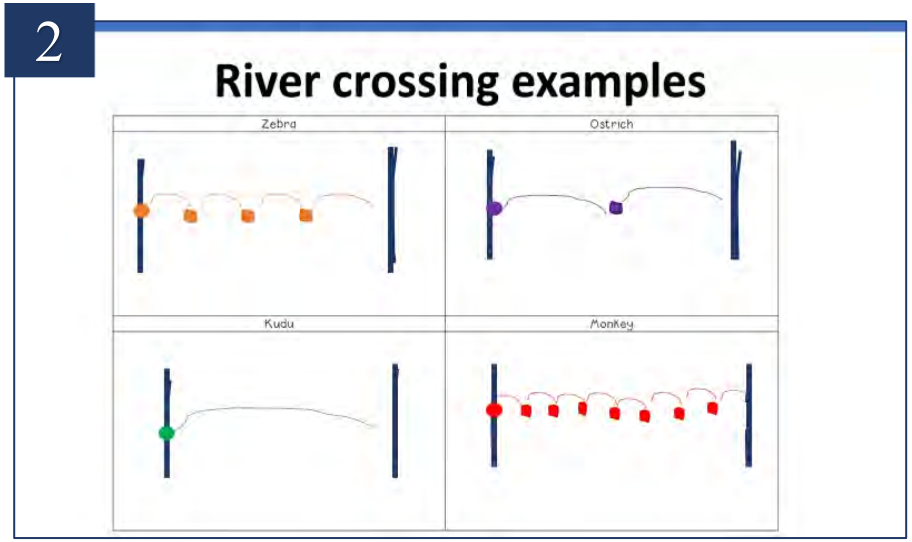
Body
Linking river-crossing to percussion line 1. Display examples of learners' drawings of the various animals' jumps across the river from the previous lesson. 2. Explain to learners that the sounds of the animals' jumps can be thought of as 'beats' in music. 3. Show learners the PowerPoint Presentation with visuals linking the river-crossing to a percussion line with music bars. 4. Discuss the notation of Xs to represent claps and squares to represent rests (shhh). 5. Discuss different ways in which one could arrange the claps and rests (shhh) to create an interesting rhythm. 6. Present examples of the beats per bar, visually and aurally. 7. Give the learners examples of percussion lines to clap and to play on percussion instruments (on PowerPoint Presentation). **Composition of percussion rhythms** 1. Place learners in groups (*optional) 2. Provide learners with the percussion line template (Worksheet 3). 3. Guide learners to create and notate their own percussion rhythms: → Decide on how many beats per bar you will have. → Draw these beats in with small vertical lines (even spacing). → Decide on how you will organise the claps (Xs) and the rests (□) to create your own rhythm. → Perform your percussion line with body percussion or any percussion instruments. → Share your notated percussion line with another group and let them perform your piece! → Perform another group's percussion line.
Conclusion
Questions on learners' notated percussion line 1. Ask learners questions about their own percussion line (start to link implicitly to fractions): → *How many beats did you have per bar?* → *How many claps would you need to fill a whole bar?* → *How many claps would your friend need to fill a whole bar on their percussion line? (Implicit link to equivalent fractions).* → *How many beats would you clap to fill two whole bars on the percussion line?*

PowerPoint with visual and aural representations

1



2



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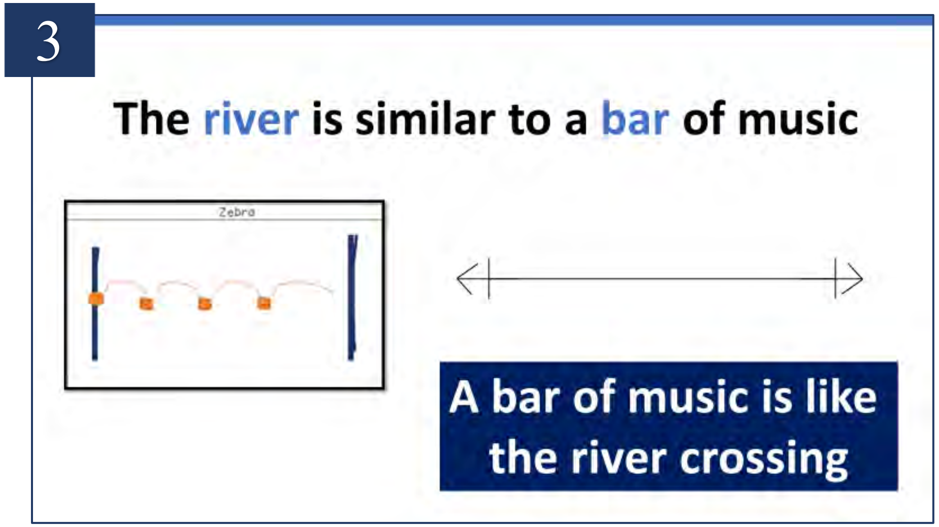

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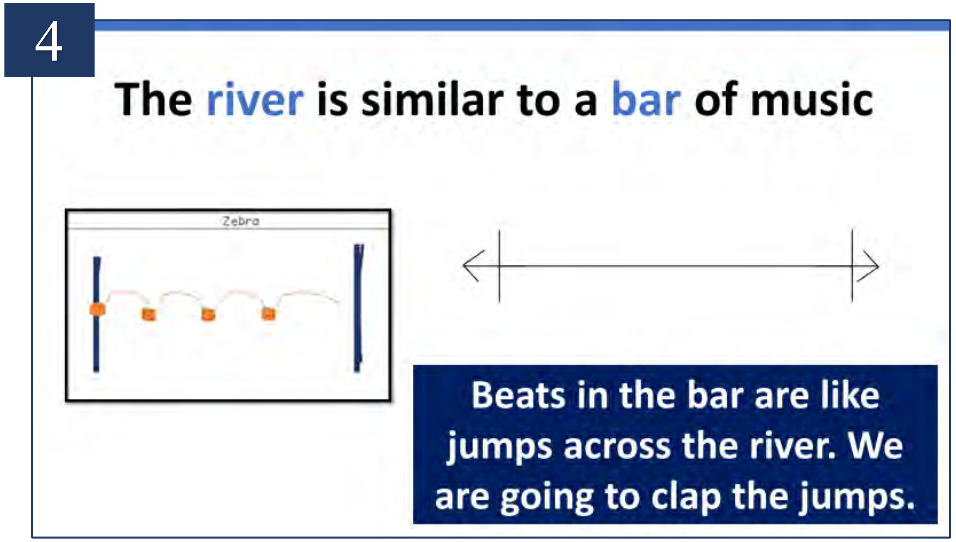
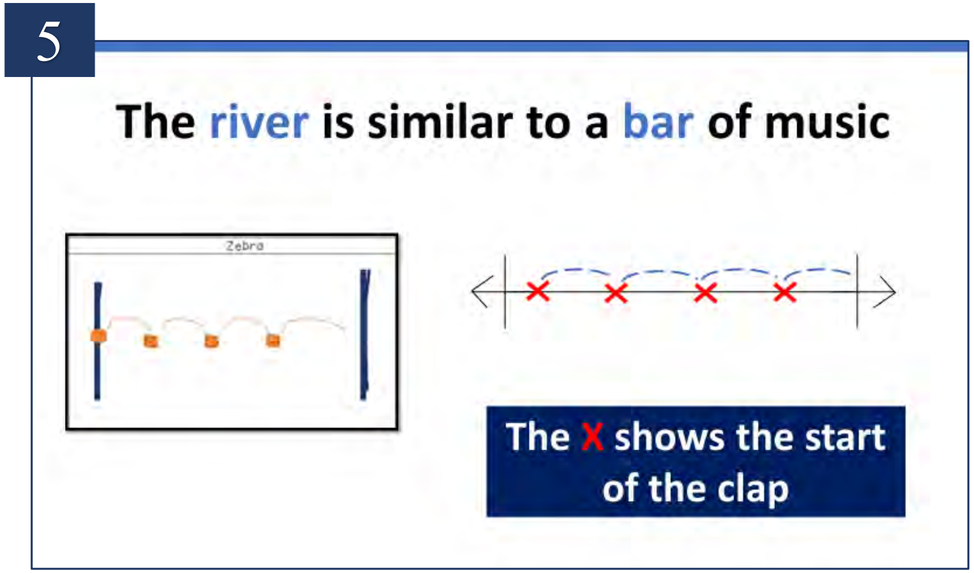
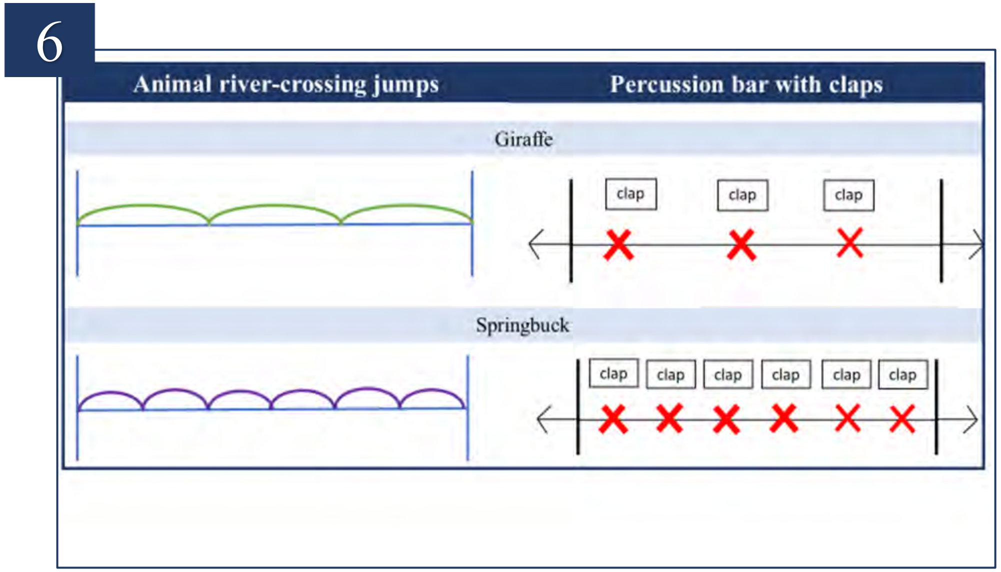
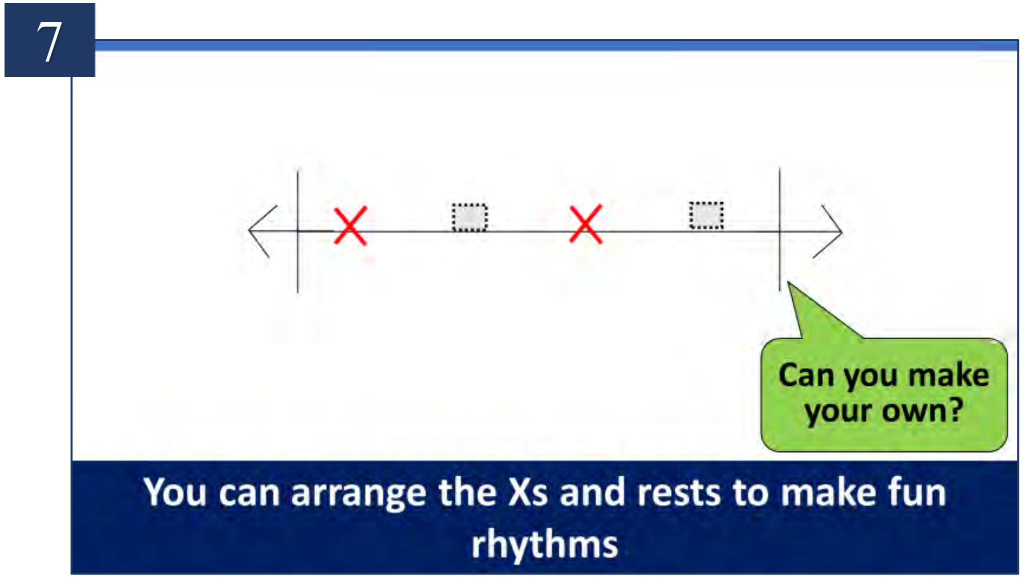
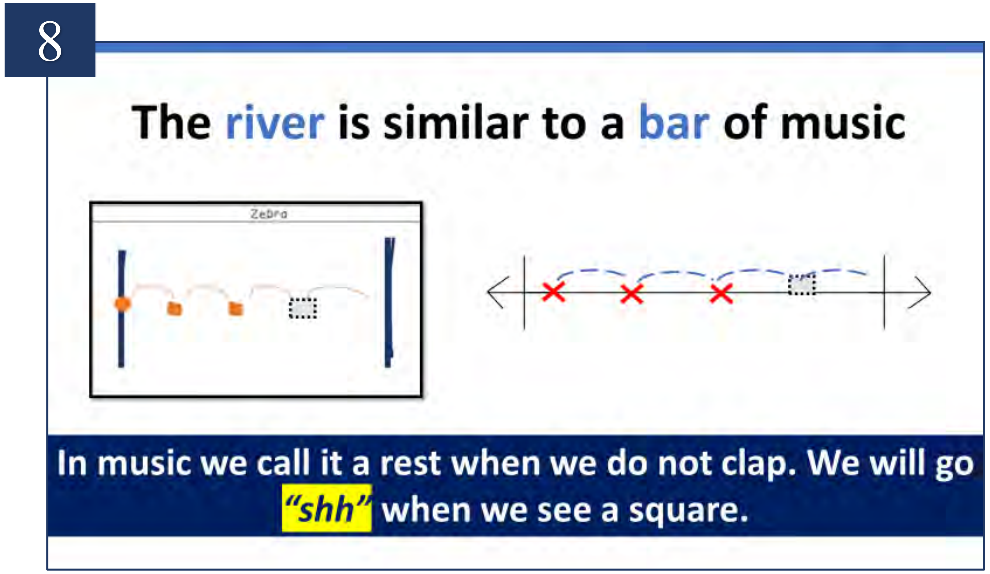


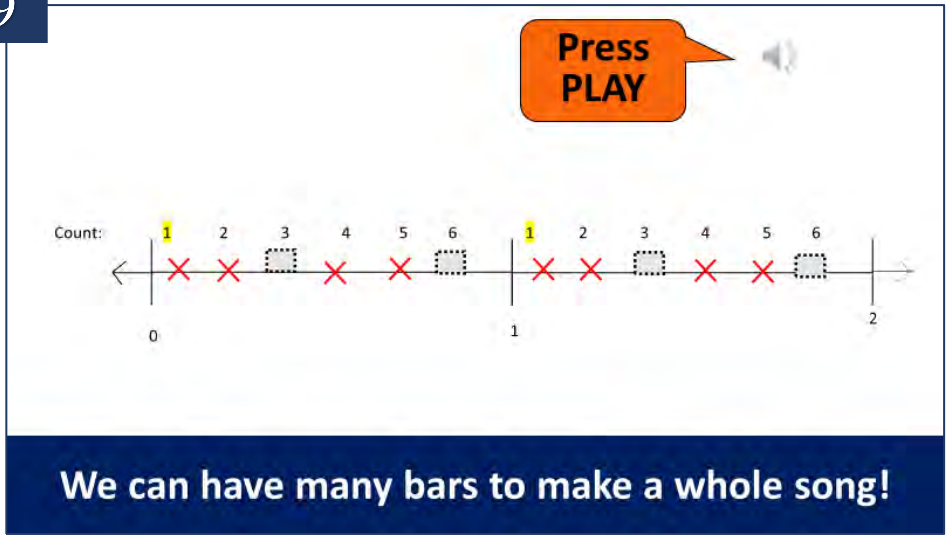

5
The river is similar to a bar of music
Zebra
The X shows the start of the clap


6
Animal river-crossing jumps
Percussion bar with claps
Giraffe
clap
clap
clap
Springbuck
clap
clap
clap
clap
clap
clap

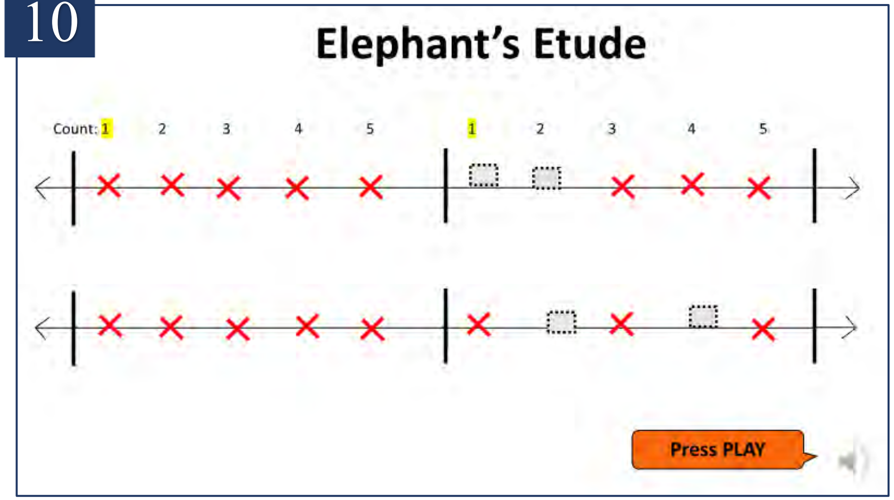

7
Can you make your own?
You can arrange the Xs and rests to make fun rhythms


8
The river is similar to a bar of music
Zebra
In music we call it a rest when we do not clap. We will go "shh" when we see a square.

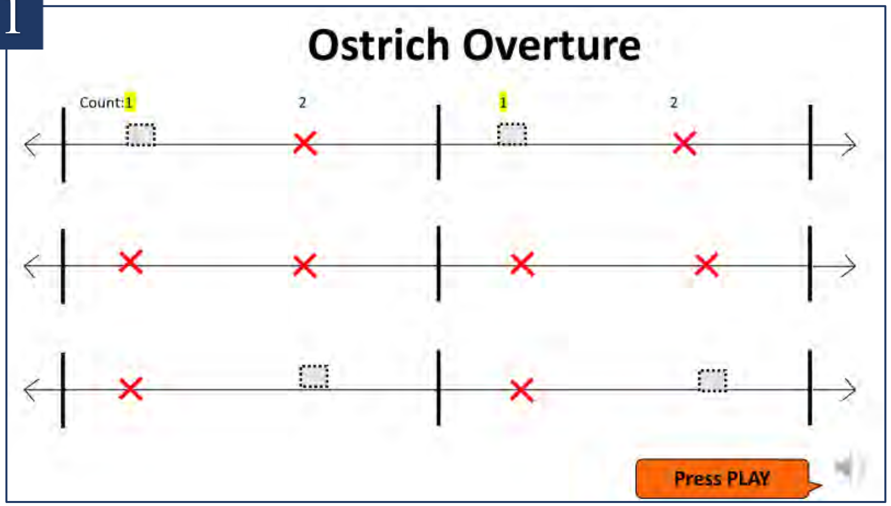
9


Press PLAY
Count:
1 2 3 4 5 6 1 2 3 4 5 6
0
1
2
We can have many bars to make a whole song!

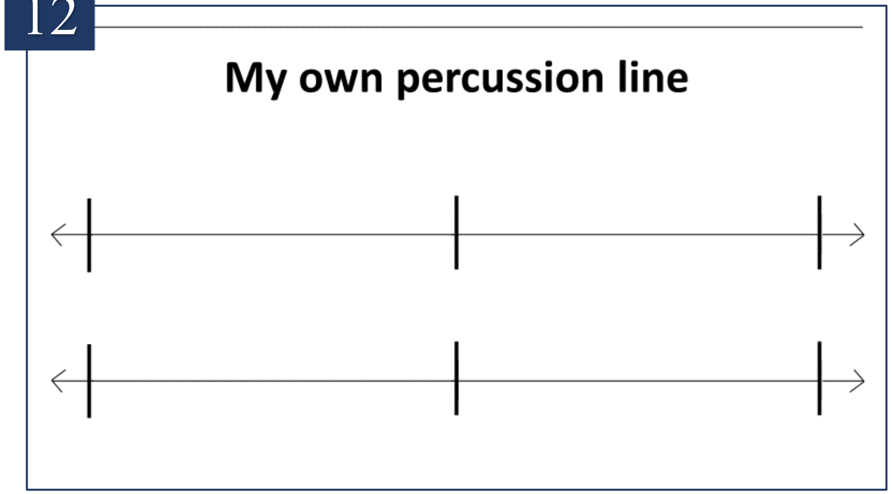
10


Elephant's Etude
Count: 1 2 3 4 5 1 2 3 4 5
Press PLAY

11


Ostrich Overture
Count: 1 2 1 2
Press PLAY

12


My own percussion line

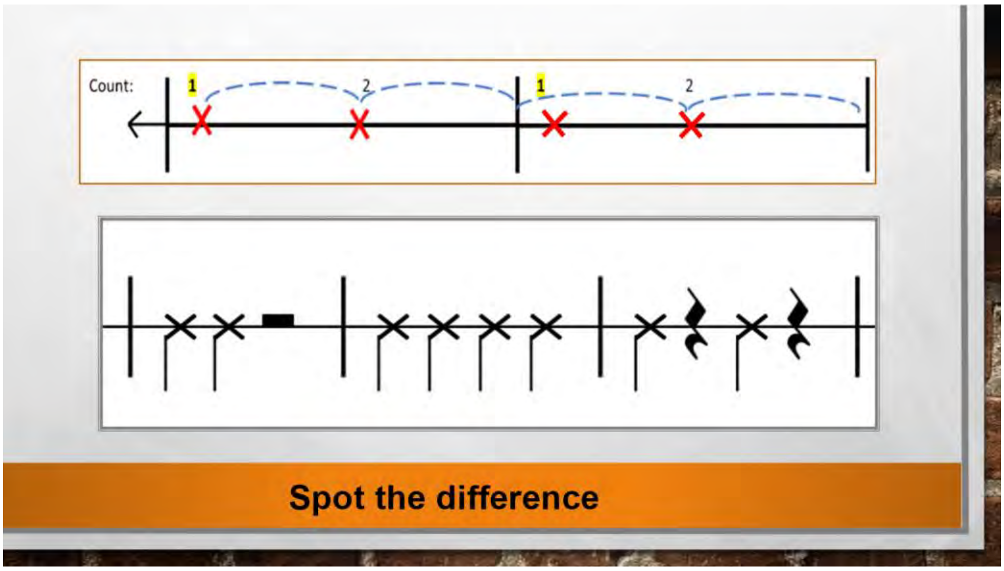
Lesson 5

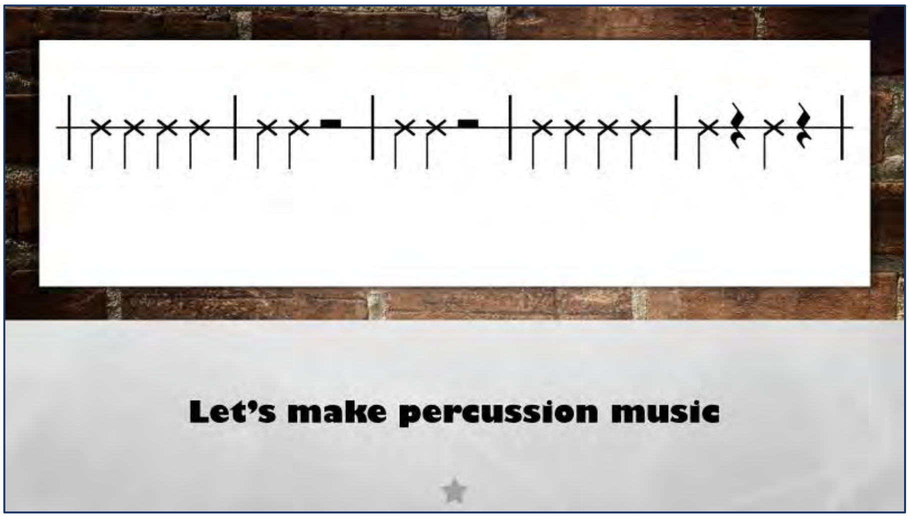
This supplementary/enrichment lesson can be taught by a music teacher, or it can be left out

<table>
<tr><th colspan="2">Curriculum aims</th></tr>
<tr><th>Mathematics</th><th>Life Skills - Performing arts: Music</th></tr>
<tr><td>Recognising the beauty and elegance of mathematics
(DBE, 2011a: 8)</td><td>Musical notation
Exploring rhythmic patterns
Use of percussive instruments and body percussion
Musical phrases exploring dynamics, pitch and rhythmic patterns
Musical notation (stave, note values, rests, clef, letter names)
(DBE, 2011b: 12-13)</td></tr>
<tr><th colspan="2">Lesson Outcomes</th></tr>
<tr><td colspan="2">By the end of the lesson, learners should be able to:
Link the informal percussion notation to musical notation
Identify differences between the informal percussion notation and musical notation
Read and clap rhythms from a percussion line.
Sing a note with correct note value on the Western Staff.
Read and play a 3-note song notated on the Western Staff on a pitched percussion instrument (xylophone etc.)</td></tr>
<tr><th colspan="2">Resources</th></tr>
<tr><td>PowerPoint with visuals
Laptop, projector
Percussion instruments (Xylophone, Marimba, Piano etc.)</td><td>YouTube link (body percussion karaoke): https://youtu.be/92gf8dAlhUw</td></tr>
<tr><th colspan="2">Introduction</th></tr>
<tr><td colspan="2">1. Recap on previous jumping activities and adjusted percussion notation.
2. Use the PowerPoint visuals to stimulate discussion.</td></tr>
<tr><th colspan="2">Body</th></tr>
</table>

Spot the difference Activity 1. Show the visuals of the informal percussion representation musical percussion line and the adjusted percussion notation. 2. Facilitate discussion around what the differences and similarities are. → Notes (Xs) have a "tail" (stem) → There are 5 lines (the Western Staff) → No dotted line to show the animal jumps → There is a Treble Clef (right hand) → There is a Time signature → There are round notes called note values: whole note (4 counts), half note (2 counts), quarter note (1 count) → There are different symbols for different rest counts 3. Facilitate the learners singing Middle C ('*Doh*') with correct note values.
Let's play a song **1.** Explain the note names and facilitate learners finding the notes on a pitched instrument (e.g. Xylophone, marimba, piano) 2. Facilitate learners playing the tune. 3. Ask learners whether they recognise the tune? (*Three blind mice*)
Conclusion
4. Play the body percussion karaoke over YouTube and let learners follow the instructions (https://youtu.be/92gf8dAlhUw).

PowerPoint Presentation




Let's make percussion music


Count:
1
2
1
2
Spot the difference


E D C
E D C
E D D C
Let's play a song!

Lesson 6

<table>
<tr><th colspan="2">Curriculum aims</th></tr>
<tr><th>Mathematics</th><th>Life Skills - Performing arts: Music</th></tr>
<tr><td>• Problem-solving with fractional reasoning
• Equivalence
(DBE, 2011a: 16)
• Use length or measurement models to develop the concept of fraction as measure number lines (DBE, 2011, p. 71)</td><td>• Exploring rhythmic patterns
• Rhythmic patterns using body percussion
• Use of percussive instruments from found objects
(DBE, 2011b: 12-13)</td></tr>
<tr><th colspan="2">Lesson Outcomes</th></tr>
<tr><td colspan="2">By the end of the lesson, learners should be able to:
• Draw different combinations of animal jumps on the river-crossing number line.
• Link the river-crossing animal jumps to cards representing musical note values.
• Use the note value cards to ‘compose’ an animal river-crossing song.
• Solve questions relating to the animal river-crossing song.
• Recognise that different note values have different names.
• Use the note value names resources to solve problems (proportional reasoning).</td></tr>
<tr><th colspan="2">Resources</th></tr>
<tr><td>• River-crossing animal jump representations (Poster)
• A3 laminated resource aligning the animal river-crossing number line and a music staff</td><td>• Whiteboard markers
• Transparent music note value cards
• Note value poster
• Worksheet 6: Problem-solving
• Video demonstration:</td></tr>
<tr><th colspan="2">Introduction</th></tr>
<tr><td colspan="2">1. Remind learners of the River-crossing animal jump representations (with poster or on whiteboard).</td></tr>
<tr><th colspan="2">Body</th></tr>
<tr><td colspan="2">Animal river-crossing song
1. Divide learners into groups. Each group should have an A3 laminated resource aligning the animal river-crossing number line and a music staff.
2. Give learners the instructions below and facilitate the activity.
3. Show teacher example (draw example on board – project PDF blank copy OR show video example).</td></tr>
</table>

→ You are going to cross the river
→ Imagine you can do different animal jumps: you can use the kudu, ostrich, zebra or monkey jumps. You can use a combination of these.
→ Use the river-crossing number line.
→ Use a whiteboard marker to draw your combination of animal jumps on the river-crossing number line.
→ Now, match the note value cards with your jumps, by placing them on the music staff. Make sure your cards fit in to the bars!
→ You can place the note on any line or space.
→ See if you can sing your song. Ask your teacher to play your animal jumping song on the piano or any instrument.

NOTE for teachers: In this example…

- The number line shows unit of river-crossing (1 to 3),
- as well as distance (every river-crossing takes 4 metres → 0m, 4m, 8m, 12m)
- and time (every river-crossing takes 4 seconds → 0s, 4s, 8s, 12s).
- The music line will consist of 3 bars. There are 4 beats per bar (4 counts), that can be made up of different combinations of note values.

Problem-solving discussion questions:

1. Ask learners the following questions and facilitate group or class discussion:

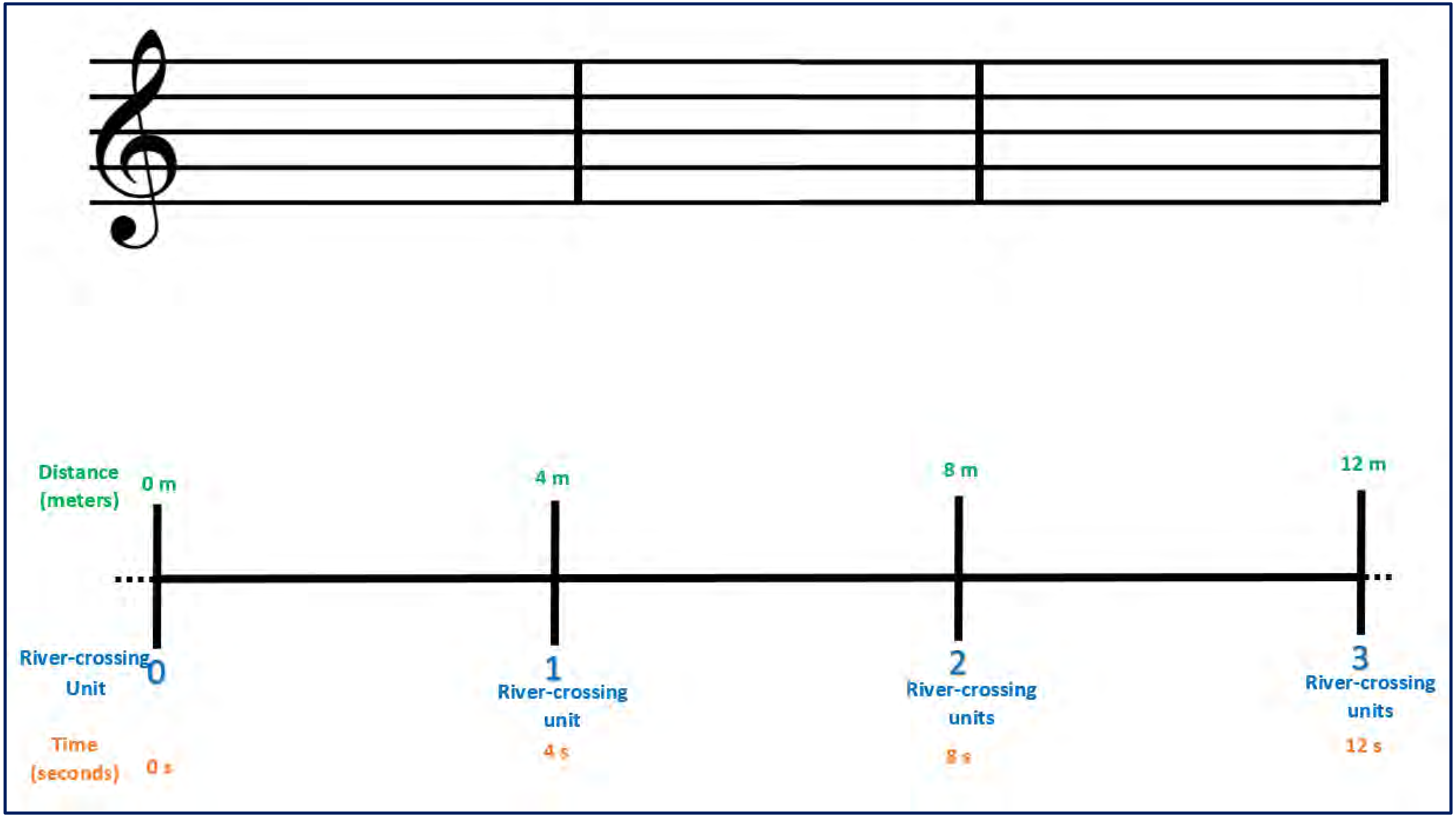
→ How long does it take to play your song?
(should be 12 seconds = 3 bars x 4 seconds each).
→ IF one river-crossing took 8 seconds, how long would it take to play your song?
(3 bars x 8 seconds = 24 seconds i.e. double).
→ IF one river-crossing took 2 seconds, how long would it take to play your song?
(3 bars x 2 seconds = 6 seconds i.e. half).

Conclusion

Didi you know?!

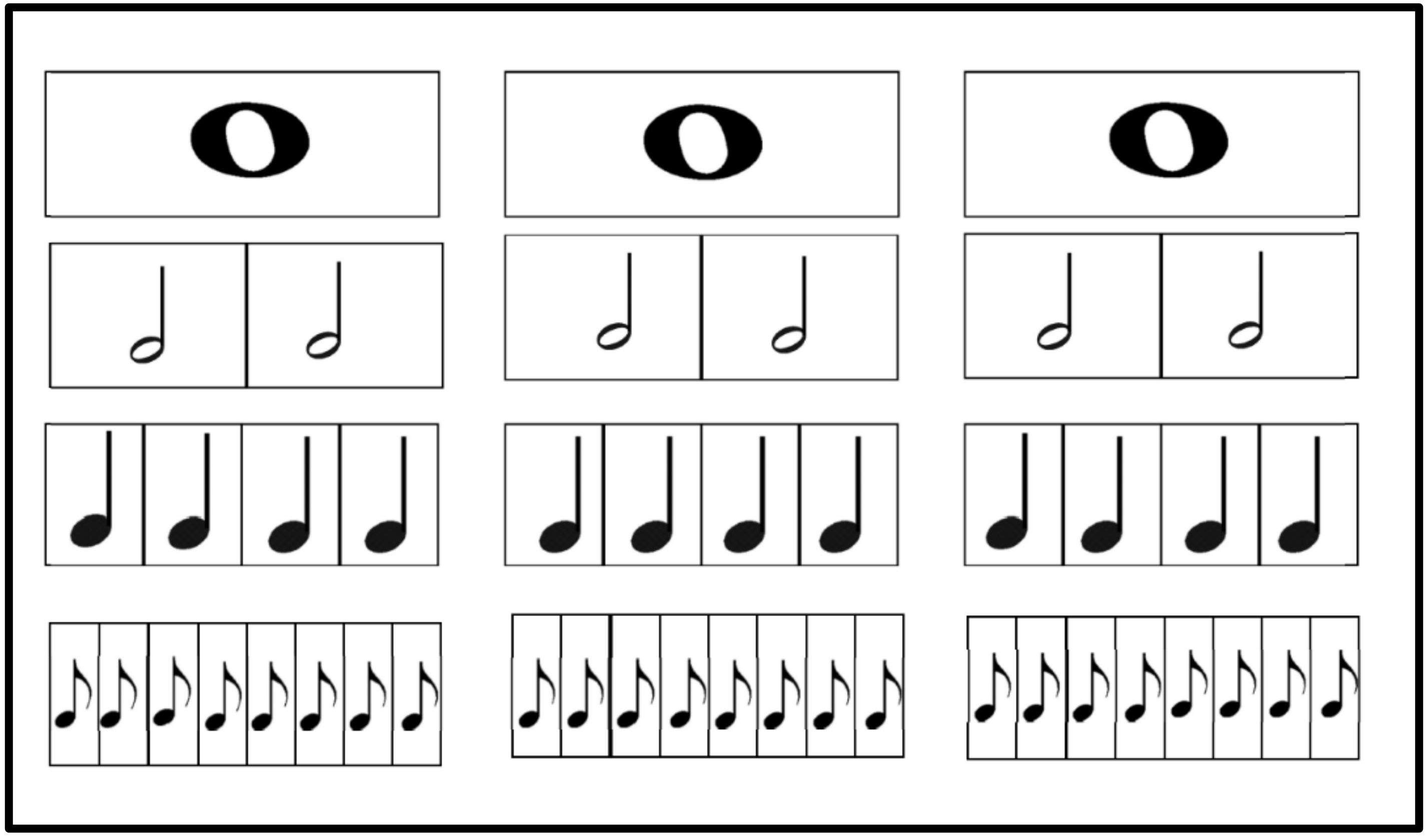
1. Use the note value Poster or PowerPoint Presentation to show learners the symbols and names of different note values.
2. Facilitate learners working in pairs to solve the problems on Worksheet 6.

Aligning the animal river-crossing number line and a music staff[16]



[16] Print on A3 and laminate

Note value cards*



*Print on transparencies. Must fit exactly into the music bar and the number line unit.

Lesson 7

<table>
<tr><th colspan="2">Curriculum aims</th></tr>
<tr><th>Mathematics</th><th>Life Skills - Performing arts: Music</th></tr>
<tr><td>• Problem-solving with fractional reasoning
• Equivalence (DBE, 2011a: 16)
• Use length or measurement models to develop the concept of fraction as measure number lines (DBE, 2011, p. 71)</td><td>• Develop skills such as… problem-solving… (DBE, 2011, p. 10)
• Rhythmic patterns using body percussion
• Exploring rhythmic patterns (DBE, 2011b: 12)</td></tr>
<tr><th colspan="2">Lesson Outcomes</th></tr>
<tr><td colspan="2">By the end of the lesson, learners should be able to:
• Answer questions linking beats per bar in music with fractions on a number line.
• Indicate fractions on a number line.
• Use the river-crossing (triple) number line to solve problems requiring fractional reasoning.
• Participate in a body percussion game and solve problems relating to fractional reasoning.</td></tr>
<tr><th colspan="2">Resources</th></tr>
<tr><td>• PowerPoint with number line and zebra/ostrich jumps
• Single number line template
• River-crossing (triple) number line (printed and laminated)
• Worksheet 7 (problem-solving)</td><td>• Demonstration YouTube videos:
• Video 2: https://youtu.be/VTdGNR_zPO8
• Video 3: https://youtu.be/EiIQ9iYO8WU
• Video 4: https://youtu.be/y-YYSP5N3SY
• Body percussion game YouTube video: https://youtu.be/DjoZbgxg4ZY</td></tr>
<tr><th colspan="2">Introduction</th></tr>
<tr><td colspan="2">1. Revise the zebra (4) and ostrich (2) claps per river-crossing, using the PowerPoint Presentation.
2. Ask learners questions relating to the animal beats per bar and fractions.
3. Let learners represent the fractions on their on single number line template.
4. Share answers and discuss as a class.</td></tr>
<tr><th colspan="2">Body</th></tr>
</table>

River-crossing number line problem-solving

1. Hand out Worksheet 7 and the river-crossing (triple) number line.
2. Optional: Show learners the demonstration YouTube video clips.
3. Facilitate learners solving the problems individually or in small groups using the river-crossing (triple) number line.
4. Share and discuss answers to the problems.
5. Allow learners to create their own animal river-crossing problems and solve each other's problems.

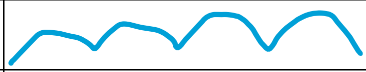
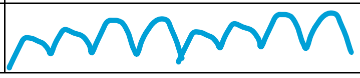
Conclusion

Sevens! Clapping game

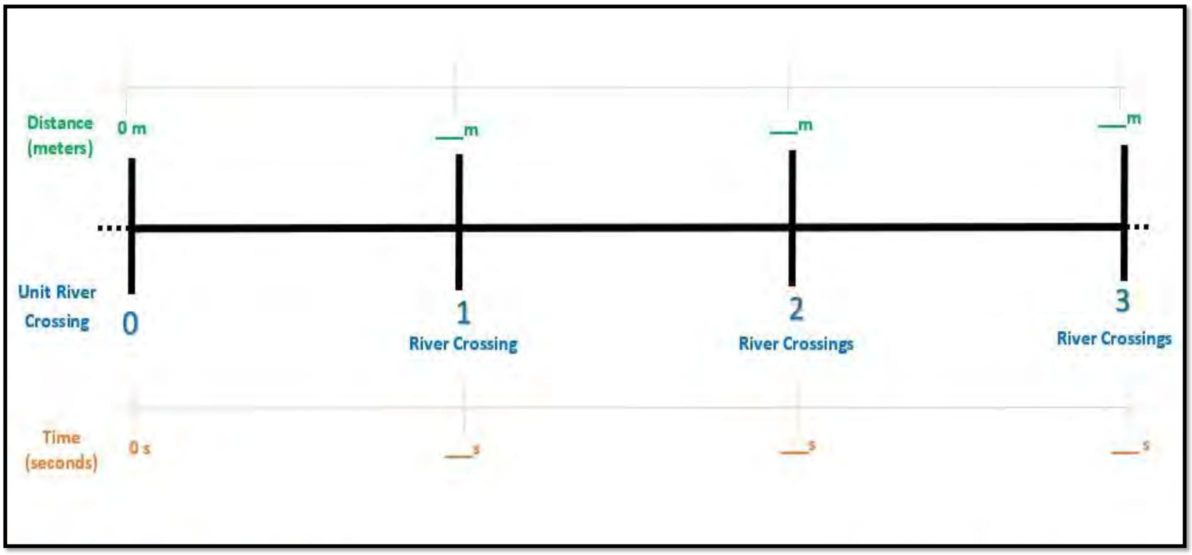
1. Use the YouTube video to teach learners the Sevens! Clapping game.
2. Ask and discuss questions about the game (which has 7 beats per bar):
   - → How many beats are in a bar? (seven)
   - → If I clapped for 2 bars, how many claps would that be? (14 claps)
   - → If I clapped 21 claps, how many bars of the song did I play? (3 bars)
   - → If I clapped 9 claps, how many bars of the song did I play? ($\frac{9}{7}$ or $1\frac{2}{7}$).

Worksheet 7: Problem-solving

Do you remember the animal jumps per river-crossing?

Kudu	1 jump per river-crossing	
Ostrich	2 jumps per river-crossing	
Zebra	4 jumps per river-crossing	
Monkey	8 jumps per river-crossing	

Use the river-crossing number line to help you solve the questions.



1). If one river-crossing is 20 metres, how far (in metres) is…

1 Ostrich jump?	
$1\frac{1}{2}$ Ostrich jumps?	
1 Zebra jump?	
5 Zebra jumps?	
4 Monkey jumps?	
3 Kudu jumps?	

BONUS QUESTION. This one is REALLY tricky – try your best!

If one river-crossing takes 4 seconds, what is the speed (in metres per second) that the animals cross the river? ______________________________

2). One river-crossing is 10 metres, and it takes 4 seconds to cross.

a) If Kudu jumps 2 and a half river-crossings, how far has he jumped?
____

How long did it take for Kudu to get there? _______________________

b) If Ostrich jumped 5 river-crossings, how far has she jumped?
____

c) How long did it take her to get there? __________________________

d) If Zebra jumped 10 river-crossings, how far did he jump?
____

How long did it take him to jump all that way? ______________________

3. If it takes one minute to cross the river, how long does it take Ostrich to get halfway? ____

4. Create your own word problem about the animal river-crossings.

APPENDIX 9: DATA ARCHIVE CONTENTS

Data archive available to examiners on request

1. **Reflective research journal entries:**

2021/03/12	2021/09/10	2021/11/26
2021/04/12	2021/09/16	2021/12/02
2021/04/27	2021/09/16	2022/01/10
2021/05/06	2021/09/24	2022/01/18
2021/05/21	2021/09/25	2022/01/19
2021/05/23	2021/09/30	2022/01/24
2021/05/28	2021/10/01	2022/01/25
2021/06/01	2021/10/18	2022/01/26
2021/06/02	2021/10/20	2022/02/01
2021/06/18	2021/10/21	2022/02/08
2021/06/24	2021/10/29	2022/03/02
2021/07/12	2021/11/10	2022/03/08
2021/07/23	2021/11/15	2022/03/08
2021/07/23	2021/11/16	2022/03/09

2. Grounded-Practice Plane: Field notes and interview transcripts from school visits

2.1 Aloe-CoP:

2021/03/12	2021/10/29
2021/03/26	2021/11/02
2021/04/27	2021/11/05
2021/05/21	2021/11/10
2021/06/18	2021/11/12
2021/09/10	2021/11/15
2021/09/16	2021/11/16
2021/10/01	2021/11/26
2021/10/01	2021/12/02
2021/10/20	2022/03/08

2.2 Protea-CoP:

2021/09/30
2021/10/28
2022/01/24
2022/02/10
2022/03/02
2022/04/12
2022/08/15

3. **Design-Theorising Plane**

3.1 Grappling-CoP Zoom meeting recordings and transcripts:

2021/03/25
2021/04/12
2021/05/24
2021/07/12
2021/07/23
2021/09/24
2021/10/20
2021/10/21
2022/01/25
2022/02/01
2022/03/09

3.2 Grappling-CoP e-mail and WhatsApp communication:

2021/03/04
2021/03/04
2021/03/23
2021/03/23
2021/03/24
2021/03/26
2021/03/26
2021/03/30
2021/04/01
2021/04/09
2021/04/16
2021/04/19
2021/04/19
2021/04/19
2021/04/20
2021/04/21
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2021/05/06
2021/05/20
2021/05/20
2021/06/01
2021/06/01
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2021/07/23
2021/07/28
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2021/08/03
2021/08/23
2021/08/25
2021/08/26
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2021/09/14
2021/09/21
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2022/01/19
2022/01/20
2022/02/04
2022/02/07
2022/02/10
2022/02/10
2022/02/23
2022/03/08
2022/03/15

4. Product of participatory dual-design experiment in task design:

Initial workbook
Lesson 1
Lesson 2
Lesson 3
Lesson 4
Lesson 5
Lesson 6
Lesson 7
Lesson 8
Final individual worksheet

5. Data analysis matrix:

Obstacle-Resolution Cycle 1
Obstacle-Resolution Cycle 2
Obstacle-Resolution Cycle 3
Obstacle-Resolution Cycle 4
Obstacle-Resolution Cycle 5
Obstacle-Resolution Cycle 6
Obstacle-Resolution Cycle 7
Obstacle-Resolution Cycle 8
Obstacle-Resolution Cycle 9
Obstacle-Resolution Cycle 10