COMBINATORIAL STRUCTURE OF CUBE OF FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING AND *k*-FIBONACCI SEQUENCE

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Philosophy

Faculty of Science
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NOVEMBER 2020

DEDICATION

This thesis is dedicated to my father, who taught me that the best kind of knowledge to have is that which is learned for its own sake. It is also dedicated to my mother, who taught me that even the largest task can be accomplished if it is done one step at a time.

ACKNOWLEDGEMENT

First of all, I would like to express my sincere appreciation to my research supervisor, Prof. Dr. Tahir bin Ahmad for his encouragement, patience and motivation. His guidance, advices and shared knowledge that helped me in writing this thesis and bring this study into success. I could not have imagined having a better supervisor for my research.

I also thank my family for always giving me great encouragement and support. Last but not least, I would like to thank all my friends who have provided assistance.

ABSTRACT

Fuzzy Topographic Topological Mapping (FTTM) is a model for solving neuromagnetic inverse problem. FTTM consists of four topological spaces that are homeomorphic to each other. A sequence of $FTTM_n$ is a combination of n terms of FTTM. In previous studies, FTTM are linked with three mathematical concepts namely; FTTM with Pascal's Triangle, FTTM as a graph and FTTM in relation to k-Fibonacci sequence. In this research, the relationship between graph of $FTTM_n$ and k-Fibonacci is established via Hamiltonian polygonal paths in an assembly graph of $FTTM_n$. The assembly graph is a graph with all vertices have valency of one or four. The Hamiltonian path is a path that visits every vertex of a graph exactly once. The structure of assembly graph of $FTTM_n$ including maximal assembly graph of $FTTM_n$ is introduced and its properties are investigated. The existence of Hamiltonian polygonal path in maximal assembly graph of $FTTM_n$ is proven. Several new definitions and theorems for the assembly graph of $FTTM_n$ and Hamiltonian polygonal path in maximal assembly graph of $FTTM_n$ are stated and proven, respectively. Finally, a theorem that highlight the relation between graph of $FTTM_n$ to k-Fibonacci sequence is proven.

ABSTRAK

Pemetaan Topologi Topografi Kabur (FTTM) adalah model untuk menyelesaikan masalah songsang neuromagnetik. FTTM terdiri daripada empat ruang topologi yang berhomeomorfisma antara satu sama lain. Satu jujukan $FTTM_n$ adalah gabungan FTTM sebanyak *n* sebutan. Dalam kajian terdahulu, FTTM dikaitkan dengan tiga konsep matematik iaitu; FTTM dengan Segitiga Pascal, FTTM sebagai satu graf dan FTTM dalam hubungan kepada jujukan *k*-Fibonacci. Dalam kajian ini, hubungan antara graf $FTTM_n$ dan *k*-Fibonacci telah dibina melalui laluan poligon Hamiltonian dalam graf $FTTM_n$ terhimpun. Graf terhimpun adalah graf yang semua bucunya mempunyai valensi satu atau empat. Laluan Hamiltonian adalah laluan yang melewati setiap bucu graf dengan hanya sekali sahaja. Struktur graf $FTTM_n$ terhimpun termasuk graf $FTTM_n$ terhimpun maksimum telah diperkenalkan dan ciri-cirinya diselidiki. Kewujudan laluan poligon Hamiltonian dalam graf $FTTM_n$ terhimpun maksimum telah dibukti. Beberapa takrifan baru dan teorem untuk graf $FTTM_n$ terhimpun dan laluan poligon Hamiltonian dalam graf $FTTM_n$ terhimpun maksimum, masing-masing telah dinyatakan dan dibuktikan. Akhirnya, satu teorem yang mengetengahkan hubungan antara graf $FTTM_n$ dan jujukan *k*-Fibonacci telah dibuktikan.

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LIST OF ABBREVIATIONS

BM	-	Base magnetic plane
B	-	Base magnetic plane
EEG	-	Electroencephalograph
FM	-	Fuzzy magnetic field
F	-	Fuzzy magnetic field
FTTM	-	Fuzzy topographic topological mapping
$FTTM_i$	-	Fuzzy topographic topological mapping version i
MC	-	Magnetic contour plane
M	-	Magnetic contour plane
MEG	-	Magnetoencephalogram
$FTTM_n$	-	Sequence of FTTM
$FTTM^n$	-	Sequence of n-FTTM
TM	-	Topographic Magnetic Field
T	-	Topographic Magnetic Field

LIST OF SYMBOLS

$\{\ \}$	-	A set
γ	-	A set of Hamiltonian polygonal paths
γ_i	-	A set of Hamiltonian polygonal paths for some *i*
ψ_G	-	An incidence edge
Γ	-	Assembly graph
Γ_{G_i}	-	Assembly subgraph for some *i*
C	-	Collection of all sets of Hamiltonian polygonal paths
$\Vert\ \ \Vert$	-	Concatenation operation
$C_{i,j}FTTM_n$	-	Cube of order two
ϵ	-	Element of
$\emptyset$	-	Empty set
F_n	-	Fibonacci sequence
$\forall$	-	For all
$>$	-	Greater than
$\geq$	-	Greater than or equal
$\cong$	-	Homeomorphism
$\longrightarrow$	-	Implies
$\cap$	-	Intersection
$<$	-	Less than
$\leq$	-	Less than or equal
Γ_{FTTM_n}	-	Maximal assembly graph of $FTTM_n$
$\mathbb{N}$	-	Natural number
$\lvert\Gamma_{FTTM_n}\rvert$	-	Number of 4-valent vertices of maximal assembly graph of $FTTM_n$
$\lvert\Gamma\rvert$	-	Number of 4-valent vertices of assembly graph
$\lvert\mathrm{C}\rvert$	-	Number of collection of all sets of Hamiltonian polygonal paths
$\lvert C_{i,j}FTTM_n\rvert$	-	Number of cubes of order two

$FTTM_{4/n}$	-	Number of cubes produced by combination of any four terms in $FTTM_n$
$FTTM_{3/n}$	-	Number of cubes produced by combination of any three terms in $FTTM_n$
$FTTM_{2/n}$	-	Number of cubes produced by combination of any two terms in $FTTM_n$
$eFTTM_n$	-	Number of edges that can be generated by $FTTM_n$
$\|\Gamma_{FTTM_n}\|$	-	Number of edges visited by paths in maximal assembly graph
$\| \quad \|$	-	Number of elements in a set
$fFTTM_n$	-	Number of faces that can be generated by $FTTM_n$
$vFTTM_n$	-	Number of vertices that can be generated by $FTTM_n$
E(G)	-	Set of edges in a graph
V(G)	-	Set of vertices in a graph
$\ni$	-	Such that
$\cup$	-	Union

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CHAPTER 1

INTRODUCTION

1.1 Problem Background

Fuzzy Topographic Topological Mapping (FTTM) model was introduced in 1999 by UTM's Fuzzy Research Group (FRG). According to Ahmad et al. (2000), FTTM was developed to obtain information on the location of epileptic foci of epileptic patients from their recorded magnetoencephalography (MEG) or electroencephalogram (EEG) signals. The FTTM has four components namely, magnetic contour plane (MC), base magnetic plane (BM), fuzzy magnetic field (FM) and topographic magnetic field (TM) as shown in Figure 1.1.

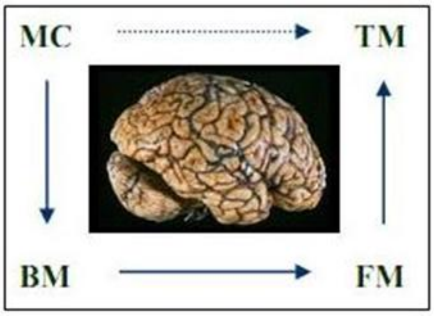

Figure 1.1 Four components of FTTM

In general, FTTM is a 4-tuple of topological spaces which are homeomorphic to each other (Ahmad, 1993; Ahmad et al., 2005) and can be represented as $FTTM = \{(M, B, F, T): M \cong B \cong F \cong T\}$. The exact arrangement for the sequential elements of FTTM is presented in Figure 1.2.

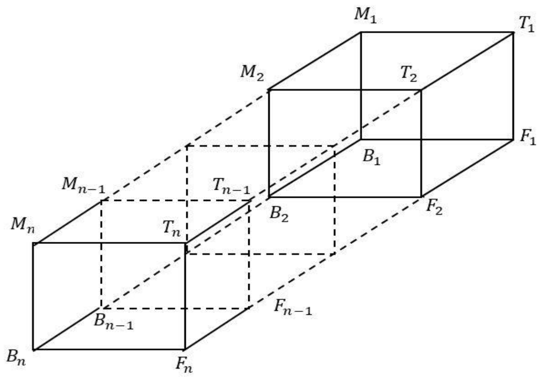

Figure 1.2 The sequence of $FTTM_n$

Yun (2006) noticed that if there are two elements of FTTM that are homeomorphic to each other component-wise, it would generate more new generated elements of FTTM, that is

$$\left[\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\right]-2=14 \text{ elements.} \tag{1.1}$$

The researcher then proposed a conjecture such that, if there exist n elements of FTTM, then the number of new generated elements are $n^4 - n$. Later, Jamaian et al. (2010) proved the conjecture and introduced several new concepts in order to achieve it.

1.2 Problem Statement

In previous studies, FTTM are linked with three other mathematical concepts namely; FTTM with Pascal's Triangle (Jamaian et al, 2010), FTTM as a graph (Sayed and Ahmad, 2013) and FTTM in relation to *k*-Fibonacci sequence (Ahmad et al., 2015) (see Figure 1.3). About the same year, Bolat and Köse (2010) hinted that generated functions for the *k*-Fibonacci number have wonderful applications in graph theory. These linkages are illustrated in Figure 1.3.

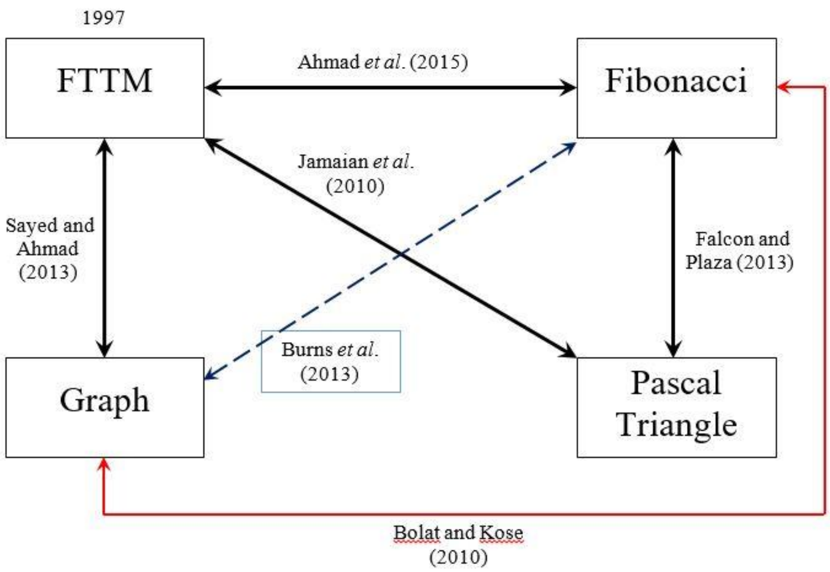

Figure 1.3 Three mathematical concepts which are linked to FTTM

However, Burns et al. (2013) established a linkage between graph to Fibonacci number by an important theorem. This particular theorem will be discussed further in Chapter 2. Finally, the theorem has led to the establishment of the relation between graph of $FTTM_n$ to k-Fibonacci which will be elaborated in Chapter 5. Prior to that, Chapter 3 and Chapter 4 contain all the necessary developed items in the form of definitions, theorems and corollaries that are needed to comprehend the claimed relation.

1.3 Objectives

The objectives of this research are as follows.

1. To prove the mathematical relation between graph of FTTM and k-Fibonacci sequence.
2. To establish the structure of FTTM combinatoric-wise.

1.4 Scope of Research

The goal of this research is to prove the conjecture made by Bolat and Köse (2010) by establishing the relation of the graph of FTTM to the k-Fibonacci sequence. In order to achieve this goal, the two mathematical structures are studied comprehensively. Besides, the theorem developed by Burns et al. (2013) which is focusing on Hamiltonian polygonal paths in assembly graph will be studied. Thus, the characteristics of assembly graph of FTTM and Hamiltonian polygonal paths in it are investigated in order to accomplish the goal.

1.5 Significance of Research

As mentioned earlier in this chapter, Bolat and Köse (2010) hinted that the graph and k-Fibonacci are somehow linked to each other. By proving the conjecture, new concepts of graph of $FTTM_n$ are expected. Therefore, the relation between two mathematical structures, graph and number theory, in particular, the graph of FTTM and k-Fibonacci sequence is established by a well-developed theorem. The study of characteristics of assembly graph and Hamiltonian polygonal paths are carried out in order to achieve the goal. Additionally, the findings of this research are great worth for the future.

1.6 Framework of Research

The thesis consists of six chapters. The first chapter first chapter is the background of the research, problem statement, objectives, scope, significance and framework of the research.

Chapter 2 consists the literature review of the research. It presents the concept and characteristics the graph of sequence of FTTM. The assembly graph and Fibonacci sequence are covered in this chapter too.

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