PREDATOR-PREY MODELS WITH CANNIBALISM IN PREY

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DEDICATION

Special dedicate to

my beautiful mum,

my sibs,

my lovely supervisor,

my supportive friends,

and all the human kinds,

and aliens,

Thank you for never ending supports and prayers.

I love you 3000.

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ABSTRACT

Cannibalism in the predator-prey model is the study to show the interaction between prey and predator where the presence of cannibalism exists in both species in real life. Moreover, cannibalism is ubiquitous in natural communities and also among researchers who are interested in mathematical ecology. The predator-prey model system is modelled using ordinary differential equations to describe the dynamic behaviour of the systems. This study introduces the stage-structured models where the adult and juvenile prey species are considered. The purpose of the study is to analyse the effect of the stage-structured of prey cannibalism on the stability based on the concept of Lotka-Volterra in the predator-prey model. Thus, in this study, there are two cases are considered: prey cannibalism in the predator-prey model with predation on adult prey and model with predation on juvenile prey. The objectives of this research are (i) to formulate the concept of Lotka-Volterra in a predator-prey model, (ii) to analyse prey cannibalism in predator-prey model with predation on adult prey, (iii) to analyse the predator-prey in prey cannibalism with predation on juvenile prey, and (iv) to analyse the effect of stage-structured predator-prey model with cannibalism in prey on stability. In analysing the models, the stability of the equilibrium point is obtained and described by using the properties of the eigenvalues and the Routh-Hurwitz Criteria. Last but not least, numerical examples and graph analysis are given to illustrate the stability of equilibrium points.

ABSTRAK

Kanibalisme di dalam model mangsa dan pemangsa merupakan kajian interaksi di antara mangsa dan pemangsa di mana kehadiran kanibalisme wujud di kedua-dua spesies dalam dunia sebenar. Kanibalisme adalah popular dalam komuniti semulajadi dan juga di kalangan penyelidik yang mempunyai minat dalam matematik ekologi. Sistem model mangsa dan pemangsa dimodelkan dengan menggunakan persamaan perbezaan biasa untuk menghuraikan perilaku sistem dinamik. Kajian ini juga menggunakan model berstruktur di mana spesies matang dan tidak matang akan dipertimbangkan. Tujuan kajian ini adalah untuk menganalisis kesan model berstruktur dengan kanibalisme dalam mangsa terhadap kestabilan berdasarkan konsep Lotka-Volterra model mangsa dan pemangsa. Dalam kajian ini, dua kes model mangsa dan pemangsa akan dipertimbangkan: kanibalisme di dalam mangsa dengan predasi terhadap mangsa matang dan model kanibalisme di dalam mangsa dengan predasi terhadap mangsa tidak matang. Objektif kajian ini adalah (i) untuk mengkaji konsep Lotka-Volterra dalam model mangsa dan pemangsa, (ii) untuk menganalisis kanibalisme dalam mangsa pada model mangsa dan pemangsa dengan predasi terhadap mangsa matang, (iii) untuk menganalisis kanibalisme dalam mangsa pada model mangsa dan pemangsa dengan predasi terhadap mangsa tidak mangsa, dan (iv) untuk menganalisi kesan model berstruktur mangsa dan pemangsa dengan kanibalisme di dalam mangsa terhadap kestabilan. Dalam menganalisa model, kestabilan titik seimbang akan diperolehi dan dihuraikan dengan menggunakan ciri-ciri nilai eigen dan Kriteria Routh-Huritz. Dan akhir sekali, beberapa contoh berangka dan analisis graf akan diberi untuk memperlihatkan kestabilan titik keseimbangan.

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LIST OF SYMBOLS

X	-	Predator
Z_A	-	Adult prey
Z_J	-	Juvenile prey
M	-	The death rate of predator
M_1	-	The death rate of adult prey
M_2	-	The death rate of juvenile prey
K	-	The birth rate of predator
B	-	The benefit from cannibalism
S	-	Cannibalism rate
R	-	The birth rate of prey
C	-	The benefit from predation
A	-	The maturation rate of juvenile prey
V	-	The predation rate
x	-	Predator
z_1		Adult prey
z_2	-	Juvenile prey
a	-	The birth rate of predator
b	-	The death rate of predator
c	-	The maturation rate of juvenile prey
g	-	The birth rate of prey
r	-	The death rate of prey
s	-	Cannibalism rate
d	-	The birth rate of predator
e	-	The death rate of predator
f	-	The benefit from cannibalism
m	-	The birth rate of prey
p	-	The death rate of prey
v	-	Cannibalism rate
J	-	Jacobian matrix

λ	-	Eigenvalues
det	-	Determinant
I	-	Identity matrix
EP	-	Equilibrium Point
ODE	-	Ordinary Differential Equation

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Cannibalism is killing or eating of one's own kind which is a habitual ecological interaction in the animal kingdom and there are more than 1,500 species had been recorded. Due of the climate change, many populations face danger because of food shortage or even survival. Even the food availability is limited, individuals still can gain extra energy by using their conspecific individuals as an additional food supply. This clarifies that the survival rate of the cannibal has increased and give an advantage to the environment where food source is deficient. Clearly, cannibalism plays a huge role in the dynamics of specific populations, thus any communities that these populations are involves in will also be affected. This effect can be positive where predator cannibalism can lead to a higher long-term predator population size. Moreover, cannibalism in certain population will be able to survive when food supply for the adults is scarce. This is what we called a life boat effect.

Adults consuming on juveniles from the same species have been widely studied nowadays which is cannibalism phenomenon has been a popular mathematical literature subject, and several mathematical modelling approaches have been proposed. Zhang (2019) found that without cannibalism, the dynamic behavior of predator-prey model is fully determined by the number of basic reproductions of the predator species which is the average number of offspring produced by the adult over its adult stage and turning into the adult without predation and cannibalism. If the basic reproduction number of predator population is less than 1, both species still exists and if the basic reproduction number of predator species is more than 1, the prey population tends to infinity and the predator population leads to extinction.

Kunal Chakraborty (2013) stated that the dynamical behavior analysis of the predator-prey model has a major role in mathematical ecology. They also pointed out that the sexual cannibalism phenomenon is very ordinary in spiders and scorpion species where cannibalism leads to a trophic structure and feedback loops within a population which it gives a huge impact on the dynamics and structure of the population. It is well learning in mathematical literatures that cannibalism can have either a stabilizing or a destabilizing effect on predator-prey system.

1.2 Background of Research

Hang Deng et. al. (2019) found that cannibalism has both positive and negative effect on the stability of predator-prey system depends on the dynamic behavior of predator-prey model system itself. If the predator population without cannibalism in the system, then the cannibalism may cause to the coexistence of both populations. In this case, cannibalism in predator stabilize the system. High cannibalism rate leads the prey population to the extinction. In the meantime, the predator population still exists. If both species coexistence in the stable state in the predator-prey system, then cannibalism in predator may be driven to the extinction of prey population. Thus, cannibalism has an unstable effect.

Zhixin Zhang et. al. (2017) showed direct evidences that intermolt E. Japonica individuals cannibalized conspecifics under a few conditions. Firstly, cannibalism in E. Japonica was size-dependent and directional where larger size of crabs succeeded to consume the smaller crabs only when the size difference between opponents was large, so that the cannibals would be at less risk of injury. Second, the sexual differences that took place in cannibalism of E. Japonica showed that males have a stronger cannibalistic tendency rather than the females because male crabs have a higher propensity to act aggressively since they enlarged chelipeds to attack conspecifics and also as a protection. The third one is both alternative food and shelters reduces the cannibalism rate. Their results showed that the alternative food was more effective than shelters in diminishing cannibalism in E. Japonica. This is might be because alternative food and shelters worked in different ways in mitigating cannibalism such as reducing the hunger levels of predator or affecting predator's foraging strategies while shelters reduced the encounter rate of different-size individuals or increasing predator handling time. Rudolf (2007) stated that cannibalism has major impacts on a dynamic community, and the temperature effects may be developed these outcomes on other species in the predator-prey model system. Increasing cannibalism is presumed to debilitate the effect of predator on prey which is letting the prey to increase in abundance.

Magnusson (1999) proposed predator cannibalism on a stage-structured predator-prey system where $X(t)$, $Y(t)$ and $Z(t)$ are the predator, the adult prey and the juvenile prey respectively at a fixed time, t. Their study can be said a simplification of the predator-prey system studied by Van Den Bosch & Gabriel (1997) where there were two simplifications had been made: (i) Assume that all juveniles are vulnerable to predations by the adult predators, and (ii) immediate maturation of juveniles into adults is proportional to the present juvenile biomass. Based on three coupled ordinary differential equations, there were several assumptions had been made:

1) Predator population is divided into two stage-structured which is adults and juveniles, with adults feeding on juveniles.
2) There is only inter-class cannibalism and no intra-class cannibalism involved.
3) The per capita maturation rate is constant which implies some juveniles can mature instantly.
4) The interaction terms of the Lotka-Volterra type are linear.
5) There is no prey density dependence in the stability analysis of equilibrium.

Besides letting X, Y and Z be the biomasses of adult predators, juvenile predators and prey respective, their research considers the three classes' biomasses and the changes due to deaths and individual growth for three populations only. Since $X = Nw$ where N is the number of adults and w is the average individual weight, we have

$$\frac{dX}{dt} = \frac{dN}{dt}w + \frac{dw}{dt}N = \left(\frac{1}{N}\left(\frac{dN}{dt}\right) + \frac{1}{w}\left(\frac{dw}{dt}\right)\right)X \tag{1.1}$$

They assumed that the mature predators die at a constant per capita rate, M_1 while the specific individual growth rate is linearly related to the biomass of the prey and the biomass of the juvenile predators.

$$\frac{dX}{dt} = -M_1X + (-b_0 + BY + CZ)X = -M_{mat}X + BXY + CXZ$$

where b_0, B, and C are constants while $M_{mat} = M_1 + b_0$. Since they assumed that the juvenile predators are recruited to adult predators at a constant specific rate, A had been denoted to the equations determining the biomass of adult predator, they have

$$\frac{dX}{dt} = -M_{mat}X + AY + BXY + CXZ \tag{1.2}$$

For the equation of juvenile biomass, it can be derived by assuming that the specific growth rate is a constant and the per capita death rate is $m_0 + SX$, where m_0 and S are constants, meanwhile the birth rate is proportional to the adult biomass which is R, we get

$$\begin{aligned}\frac{dY}{dt} &= \big(h - (m_0 + SX)\big)Y \\ &= (-(m_0 - h) - SX)Y\end{aligned}$$

$$= RX - AY - M_{juv}Y - SXY \tag{1.3}$$

where $M_{juv} = m_0 - h$ and since they assumed that the juvenile predators are recruited to adults at constant specific rate, $-A$.

For the third equation, they assumed that the birth rate of prey is t, the specific growth rate, $s_0 - s_1 Z$ and the per capita death rate is $v_0 + v_1 Z + VX$, we have

$$\frac{dZ}{dt} = \big(t + (s_0 - s_1 Z) - (v_0 + v_1 Z + VX)\big)Z$$

$$= \big((t + s_0 - v_0) - (s_1 + v_1)Z - VX\big)Z$$

$$= TZ - UZ^2 - VXZ \tag{1.4}$$

where $T = t + s_0 - v_0$ and $U = s_1 + v_1$.Combining the three equations, the full model is given below:

$$\begin{aligned}
\frac{dX}{dt} &= -M_{mat}X + AY + CXZ + BXY \\
\frac{dY}{dt} &= RX - AY - M_{juv}Y - SXY \\
\frac{dZ}{dt} &= TZ - UZ^2 - VXZ
\end{aligned} \tag{1.5}$$

where

X	denotes as the adult predator
Y	denotes as the juvenile predator
Z	denotes as the prey
M_{mat}	denotes as the death rate of adult predator
M_{juv}	denotes as the death rate of juvenile predator

A	denotes as the maturation rate of juvenile predator into adult predator
V	denotes as predator attack on prey
S	denotes as the cannibalism rate
C	denotes as the growth rate of predators due to preying
B	denotes as the benefit from cannibalism
R	denotes as the birth rate of predator
T	denotes as the birth rate of prey

From Magnusson's study, he found when the death rate of juvenile predator is low and/or the recruitment rate to the mature population is high, therefore all three positive population sizes have a stable equilibrium. In the meantime, if the death rate of juvenile predator is high and/or the recruitment rate to the mature population is low, therefore the low cannibalism rate has a stable equilibrium, but a loss of stability of a Hopf bifurcation took place as the cannibalism rate increases.

1.3 Problem Statement

Cannibalism in the predator-prey model has been studied by many mathematician and researchers. However, there are always some new findings in the study on the predator and prey populations interactions especially when it comes to the existence of cannibalism in both species. This study is focused on how the cannibalism in prey population affects the stage-structured predator-prey model.

1.4 Objectives of Research

The objectives of this research are:

1) To formulate the concept of Lotka-Volterra in a predator-prey model.

2) To analyze prey cannibalism in the predator-prey model with predation on adult prey.
3) To analyze prey cannibalism in the predator-prey model with predation on juvenile prey.
4) To analyze the effect of a stage-structured predator-prey model with cannibalism in prey on stability.

1.5 Scope of Research

The main scope of this research is to analyze the effect of the stage-structured predator-prey model with cannibalism in prey where we focus on three populations which are juvenile prey, adult prey and predator. We will formulate two cases of predator-prey models where there is cannibalism in juvenile prey and cannibalism in adult prey, and also find the factors that affect the stability of equilibrium points in the predator-prey model.

1.6 Significant of Research

The findings of this study are very functional and useful for the mathematician researchers and ecologists who have interest in the ecology fields because this study will give us more understandings and comprehension about the stage-structured predator-prey model with cannibalism in prey and the effect of prey cannibalism in the predator-prey model on stability. The result that we achieved can be a guide by applying another population model in the ecology field such as forestry and wildlife.

1.7 Conclusion

This chapter contains the introduction of this research. Moreover, this chapter discusses the research background, problem statement, objective of research, scope and significance of this research. For this study, there are two models will be analyzed. The first model is a stage-structured predator-prey model with cannibalism in prey where predator preying on adult prey. And the second model is a predator-prey model with stage-structured in prey cannibalism where predator preying on juvenile prey. For the next chapter, some literature reviews on the stage-structured predator-prey model with cannibalism in prey will be discussed.

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