Artificial Intelligence Review (2022) 55:6567–6605
https://doi.org/10.1007/s10462-022-10161-0

Modeling, reasoning, and application of fuzzy Petri net model: a survey

Wei Jiang[1] · Kai-Qing Zhou[1] · Arezoo Sarkheyli-Hägele[2] · Azlan Mohd Zain[3]

Accepted: 28 February 2022 / Published online: 11 March 2022

Abstract

A fuzzy Petri net (FPN) is a powerful tool to model and analyze knowledge-based systems containing vague information. This paper systematically reviews recent developments of the FPN model from the following three perspectives: knowledge representation using FPN, reasoning mechanisms using an FPN framework, and the latest industrial applications using FPN. In addition, some specific modeling and reasoning approaches to FPN to solve the 'state-explosion problem' are illustrated. Furthermore, detailed analysis of the discussed aspects are shown to reveal some interesting findings, as well as their developmental history. Finally, we present conclusions and suggestions for future research directions.

Keywords Fuzzy Petri net · Knowledge representation · Modeling · Reasoning · Industrial application

1 Introduction

Knowledge-based systems (KBSs) or expert systems are intelligent computer systems that rely on captured knowledge to perform complex tasks and programs, usually by domain experts (Valavanis et al. 1994). After analyzing basic KBS modules (including the rules, frame, and logic), Chandrasekaran (1988) proposed a generic framework for a knowledge system that consists of a series of building blocks with each block owning a function aimed at solving a specific problem. A KBS helps retain the knowledge scattered in the memories of individual domain experts and helps to find solutions by mimicking the reasoning process of domain experts (Handelman et al. 1990; Mockler and Dologite 1992). In the past few decades, KBSs have been employed to implement modeling and reasoning functions to solve different problems in various areas, such as fault diagnosis (Frank 1990), inference (Chen and Yang 2013), verification (Koriem 2000), classification (Zanni et al. 2006), and biological modeling (Chaplain 2011).

✉ Kai-Qing Zhou
jsu_computer@163.com

[1] College of Information Science and Engineering, Jishou University, Jishou 416000, Hunan, China

[2] Internet of Things and People Research Center, Department of Computer Science and Media Technology, Malmö University, 20506 Malmö, Sweden

[3] UTM Big Data Centre, Universiti Teknologi Malaysia, 80310 Skudai, Johor, Malaysia

However, as the sizes of KBSs grow, it becomes difficult to obtain individual and precise knowledge from the experience of the related experts when dealing with complex problems. This also indicates that the difficulty and complexity of KBS modeling and the corresponding inference function have increased sharply (Cardoso et al 1989; Ha et al. 2007). To handle this, a series of new methods have arisen to represent knowledge and execute automatic reasoning: fuzzy production rules (FPRs), semantic networks, G-nets, Petri nets (PNs), and fuzzy Petri nets (FPNs).

Among the mentioned methods, FPR and FPN have been successfully applied to fulfill the KBS requirements (Liu et al. 2013a, b, c; Wai and Liu 2009). FPR has been widely used to develop expert systems to represent, capture, and store fuzzy knowledge. Zhang et al. (1991) proposed fuzzy production rules based on expert systems. They developed a procedure with the Micro-PROLOG language to assist physicians in solving diseases in the field of pneumoconiosis. Based on FPR and PN correspondence, Lipp (1983) and Looney (1988) proposed that the extended PN formalism, namely, FPN, combines classical PN with fuzzy theory to represent the FPR.

FPN is a kind of high-level Petri net (HLPN) to deal with fuzzy information. Compared to other modeling methodologies, FPN has some unique features. It provides a formal and general graphical model (Hu et al. 2011) that can be visualized. Being a formal mathematical model, it allows for the verification of the properties of the modeled systems by utilizing the characteristics of Petri nets and supports knowledge reasoning in KBSs (Liu et al. 2015a, b; Li et al. 2000). Finally, FPNs can capture the dynamic nature of fuzzy rule-based reasoning with the evolution of markings and also express the dynamic behavior in algebraic forms (Lee et al. 1998). Because of the above advantages, FPNs are widely employed to represent FPRs and to model KBSs in a variety of industrial areas, such as robotic systems (Wai and Lin 2009, 2013), manufacturing systems (Mhalla et al. 2013), traffic engineering (Cheng and Yang 2009; Lee et al. 2006), and fault diagnosis (Liu et al. 2013a, b, c; Zhang et al. 2017a, b).

Although FPNs and their applications have achieved fruitful results, further utilization of FPNs will be limited due to the increasing complexity of KBSs. This is because the existing FPNs cannot adapt to the changing antecedent propositions in complex KBSs. The main disadvantage of using an FPN for modeling and reasoning consists of the following two aspects.

1. FPNs lack a learning mechanism to adapt to the growth of KBSs. Accurate values of FPN parameters (weight, threshold, and certainty factors) are increasingly difficult to obtain.
2. As with a PN, it is a huge challenge to solve the state space explosion problem of large-scale FPNs using existing analytic approaches and reasoning mechanisms.

This paper extended our previous review study (Zhou and Zain 2016) to summarize and analyze the latest research results regarding modeling and reasoning methods using FPN and the related typical industrial applications. Related articles in journals (indexed by SCI) and conferences (indexed by CPCI and Scopus) between 2014 and 2020 are gathered and reviewed. Compared with our previous paper, the specific objectives of this manuscript are as follows:

1. To explore the modeling and reasoning of KBSs using FPN under the general background and the state-explosion background.

2. To summarize and review current typical applications of FPN from 2014 to 2020 to reveal the different development lines of FPN.
3. To discuss the potential application domains of FPN in the future.

Based on the mentioned objectives, the rest of this paper is organized as follows. Basic notions of PN and FPN and the related correspondences are presented in Sect. 2. Section 3 reviews HLPN formalisms and recalls the development of the FPN family. Section 4 summarizes and analyzes existing methods for using FPN for sizeable KBS modeling. Section 5 discusses the reasoning mechanisms using FPN. Section 6 reviews and discusses FPN applications in various areas from 2014 to 2020. Section 7 concludes and points to potential future research directions.

2 Petri net and fuzzy Petri net

2.1 PN and elementary net system

The Petri net (PN) model, proposed by C. A. Petri in this thesis in 1962, is a graphical mathematical formalism used to model, analyze, and describe the control and information flows in discrete events or distributed systems with asynchronous and concurrent activities (Narahari and Viswanadham 1985; Petri and Reisig 2008). The PN model provides an intuitive way to understand systems with dynamic behaviors (Heiner 2011). According to related review papers (Murata 1989; Zurawski and Zhou 1994), a PN could be defined as follows.

Definition 1 PN is a 6-tuple:$PN = \{P, T; F, K, W, M_0\}$, where

1. $P = \{p_1, p_2, p_3, \cdots, p_n\}$ is a finite set of places. The places are represented by circles;
2. $T = \{t_1, t_2, t_3, \cdots, t_m\}$ is a finite set of transitions. The transitions are represented as rectangles;
3. $F \subseteq (P \times T) \cup (T \times P)$ is a set of flow relations or arcs. The relation (or arc) is used to illustrate the flow relationship of a place to a transition or a transition to a place;
4. $K = \{1, 2, 3, \cdots, n\}$ is the capacity of each place p_i;
5. $W = \{1, 2, 3, \cdots, n\}$ is a weight function, which indicates how many existing resources will be consumed or how many new resources will be created in the firing process; and
6. M_0 is the initial marking.

The elementary net system (EN_system) is the most fundamental model of the PN family, in which the places are considered to be conditions represented by B. The transitions are considered to be events represented by E (Thiagarajan 1986; Graubmann 1987). In general, according to related review papers (Rozenberg 1987; Pelz 1989), the EN_system formalism is defined as:

Definition 2 An EN_system is a 4-tuple: $EN\_system = \{B, E; F, c\}$, where

1. B is a finite set of places;
2. E is a finite set of transitions;
3. $F \subseteq (P \times T) \cup (T \times P)$ is a finite set of flow relations or arcs;

4. $c \in B$ is an *EN*_system case.

In every state of the EN_system, the status of conditions in B can be divided into two classes. If $M(s) = 1$, then these conditions are true; otherwise, $M(s) = 0$ and other conditions are false. Based on this finding, a subset of B (replaced by C) is used to represent all "true" conditions.

2.2 FPN

Although the PN can simulate and analyze the systems that own clearly defined states and transitions, it is difficult to endow them with uncertainty and vague information. Looney (1988) proposed a rough notion for an FPN to execute approximate reasoning. Since then, the basic framework of FPN and related applications have been discussed frequently by scholars (Chen et al. 1990; Yeung and Tsang 1994a, b; Liu et al. 2013a, b, c; Zhou et al. 2015a; Liu et al. 2017a, b) to support the structural organization of fuzzy information in KBSs, to provide visualization of diagnosis processing and to design efficient fuzzy inference algorithms. Formally, an FPN formalism structure is defined as:

Definition 3 FPN is an 8-tuple: $FPN = \{P, T, M, I, O, W, \mu, CF\}$, where

1. $P = \{p_1, p_2, p_3, \cdots, p_n\}$ is a finite set of places;
2. $T = \{t_1, t_2, \cdots, t_n\}$ is a finite set of transitions;
3. $M = (m_1, m_2, \cdots, m_n)^T$ is a vector of fuzzy markings, $m_i \in [0, 1]$ is the truth degree of $p_i (i = 1, 2, \cdots, n)$, and the initial truth degree of the vector is denoted by M_0;
4. $I : P \times T \rightarrow \{0, 1\}$ is the $n \times m$ input matrix, in which $I(p_i, t_j)$ records whether a directed arc from p_i to t_j exists ($i = 1, 2, \cdots, n; j = 1, 2, \cdots, m$), where
$$I(p_i, t_j) = \begin{cases} 1 & \textit{if there is a directed arc from } p_i \textit{ to } t_j; \\ 0 & \textit{if there is not a directed arc from } p_i \textit{ to } t_j. \end{cases}$$; and
5. $O : P \times T \rightarrow \{0, 1\}$ is the $n \times m$ output matrix. Here, $O(p_i, t_j)$ records whether a directed arc from t_j to p_i exists ($i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$), where
$$O(p_i, t_j) = \begin{cases} 1 & \textit{if there is a directed arc from } t_j \textit{ to } p_i; \\ 0 & \textit{if there is not a directed arc from } t_j \textit{top}_i. \end{cases}$$
6. $w(i, j)$ is the weight of the arc from p_i to t_j;
7. $\mu : \mu \rightarrow (0, 1]$, μ_j is the threshold of t_j; and
8. CF is the belief strength, where $CF_{ij} \in [0, 1]$ is the support strength of the arc from t_j to p_i and exists as ($i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$)

2.3 Correspondences among EN_system, PN, and FPN

Based on the existing results, the correspondences among EN_system, PN, and FPN are shown in Table 1 (Zhou et al. 2015a; Zhou and Zain 2016; Liu et al. 2018a, b).

Table 1 further indicates the main characteristics of FPN. Compared with EN_system and PN, the largest modification of FPN is to obscure the value of each parameter. Hence, the FPN could be considered a back-extensive formalism of PN without destroying the inner-dynamic structure to model and execute the inference or computation task for systems with uncertainty. It is also why the intrinsic graphical description method and

Table 1 Corresponds between EN_system, PN and FPN

	EN_system	*PN*	*FPN*
Formalism	$EN\_system = \{B, E; F, c\}$	$PN = \{P, T; F, K, W, M_0\}$	$FPN = \{P, T, M, I, O, W, \mu, CF\}$
Capacity	$K(s) = 1$	$K(s) = 1$	$K(s) = 1$
Weight	$W(i, j) = 1$	$W(i, j) = 1$	$W(i, j) \in [0, 1]$
Threshold	μ_i=1	μ_i=1	$\mu_i \in [0, 1]$
Marking	$M(s) = 1$	$M(s) = 1$	$M(s) \in [0, 1]$
Support strength	$CF(j, i) = 1$	$CF(j, i) = 1$	$CF(j, i) \in (0, 1]$

the algebra analytic technique of PN can also be employed by FPN to deal with fuzzy information.

3 High-level FPNs and their lines of development

The problems of using FPN in industrial fields and some constraints on the real systems (such as time delays and negation issues) should be considered to ensure that the obtained model fully reflects all the core properties of the corresponding system (Ribaric and Basic 1998; Wu et al. 2013; Amin and Shebl 2014; Liu et al. 2016; Yang and Li 2018). Therefore, a series of HLFPNs, such as fuzzy colored Petri nets (FCPNs) (Scarpelli et al. 1996), fuzzy time Petri nets (FTPNs) (Berthomieu and Diaz 1991), fuzzy stochastic Petri nets (FSPNs) (Molloy 1982), and intuitionistic fuzzy Petri nets (IFPNs) (Shen et al. 2009) have been proposed by researchers and practitioners to address problems resulting from different constraints.

3.1 Classical HLFPNs

To consider colored markings, Lin and Hwang (1996) developed a fuzzy colored PN model (FCPN) to verify a KBS and to classify the fuzziness of consistency. Lee and Seong (2004) utilized a novel FCPN model to analyze and model nuclear power plants and execute inference engine decision-making. Chiang et al. (2006) proposed a queueing-colored Petri nets (QCPNs) model with a fuzzy factor to check the wafer fabrication performance evaluation.

Jorge et al. (1996) proposed an HLFPN by combining fuzzy time Petri nets with G-nets for analysis of real-time distributed systems. Slobodan et al. (1999) modified the original PN model by combining FTPN with fuzzy temporal relations to implement knowledge representation and reasoning tasks. Pedrycz and Camargo (2003) proposed a modified FTPN for adding the time factor to the transitions and places. Liu et al. (2011) modified FTPN to realize a chemical abnormality monitoring function from two angles: assigning a time factor for each transition and associating a degree of reliability with each place. Yang and Li (2018) proposed a dynamic time fuzzy Petri nets (DTFPNs) modeling approach using dynamic delay time analysis. Wu (1999) proposed a fuzzy-timing colored Petri net (FTCPN) to comprehensively measure the continuous time scale by using the time scale as a linear-ordered commutative monoid.

To consider the strength of fuzzy sets in dealing with uncertain information, researchers developed the concept of a fuzzy stochastic Petri net (FSPN) by combining SPNs and fuzzy sets. Zhang and Yao (2015) analyzed multistate systems utilizing FSPN and introduced a systemic approach to investigate the adaptability and reliability of FSPN. Liu et al. (2018a, b) examined both the randomness and fuzziness of biological systems. After that, they modeled and analyzed the biological systems with uncertain kinetic parameters using FSPN. Shafiekhani et al. (2018) examined the dynamic properties of a tumor in the tumor-immune order. They captured the dynamic behavior of the tumor-immune system involving uncertain kinetic parameters using a FSPN model and calculated the steady-state response of the system.

Shen et al. (2009) proposed an intuitionistic FPN (IFPN) and related reasoning algorithms. In the framework of IFPN, the credibility degree of a rule and the threshold of transition are represented by intuitionistic fuzzy numbers (with operators including membership, non-membership, and hesitancy). Liu et al. (2015a, b) proposed a new type of FPN model based on intuitionistic fuzzy sets and ordered weighted averaging operators to address the knowledge representation of uncertainty in expert systems and the incomplete reasoning results. Based on this new framework, a max algebra-based reasoning algorithm is proposed to automatically implement intuitionistic, fuzzy reasoning. This method was applied to the fault diagnosis problem of aircraft generators. Meng et al. (2017) proposed a hybrid reasoning algorithm by combining forward reasoning with backward reasoning based on the intuitionistic FPN model. Zhang et al. (2018) employed the IFPN model to forecast the influence of the uncertainty of alarm information in fault diagnosis in power grid fault systems.

Scholars have proposed various HLFPNs by using neural network theory. In 1994, Looney proposed an FPN to adjust the threshold value parameter, but the other two types of parameters still need to be adjusted by experts. Li and Lara-Rosano (2000) proposed an adaptive FPN (AFPN) model to amend the certainty factors from transition to place in the FPN model. The model not only performs knowledge reasoning, but it also enables the whole net learning process to be decomposed into several simple learning processes, which reduces the complexity of the learning algorithm. It is, in addition, a learning algorithm presented as an NN. Pedrycz and Gomide (1994) proposed an adjustable FPN to model the reasoning by combining neural network learning algorithms. However, the model was only suitable for handling the logical relationships ’OR’ and ’And’. Shen et al. (2010) developed supervised and unsupervised learning algorithms for machine learning PN (MLPN) models to make them wholly trained and to remedy the difficulties encountered by artificial neural networks (ANNs). Vahidipour et al. (2015) proposed an adaptive PN with learning automata (APN-LA) to resolve the transitions’ conflicts. Shojafar et al. (2015) proposed an adaptive stochastic Petri nets (ASPNs) model with a fuzzy factor to enhance the consumption time for tasks in economic grids. This model is based on learning automata approaches that predict the system’s next state and previous state based on current status.

3.2 Development thread of HLFPNs

The abovementioned studies indicate that HLFPNs are often formulated by combining the general FPN model with other HLPNs to enhance the descriptive ability of the original FPN. Therefore, these novel models have modified the corresponding large-size FPN models to handle specific problems, expanded the research domain of FPN theory, and improved the FPN model’s self-learning. On the other hand, FPN is also a kind of backward-extension HLPN to handle vague information without destroying the intrinsic

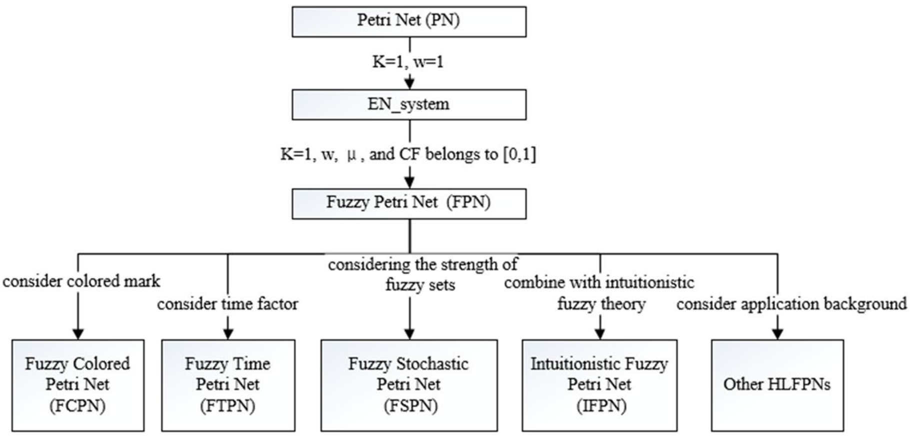

Fig. 1 Development of the FPN model family

advantages and inner structural properties of PNs. The developmental lineage of the FPN family is depicted in Fig. 1.

4 Modeling sizeable KBS using FPN

FPRs have been widely employed to represent the captured fuzzy information in a KBS by using an 'IF–THEN' form. Broadly speaking, the FPN model is also constructed to execute approximate reasoning based on descriptive graphics and parallel mathematical analysis. The first challenge of modeling sizeable KBS using FPN is to investigate the hidden associations between an FPN and FPRs.

4.1 Modeling FPR using FPN

FPR is typical for representing uncertain, fuzzy knowledge in the KBS (Negoita 1985; Chen 1988; Song and Lee 2002). In a KBS, each FPR could be expressed as a fuzzy 'IF-THEN' form. The FPR could be divided into two parts: the antecedent and the consequent. Each part is represented by fuzzy sets (Chen 1996; Awan and Awais 2011). Moreover, a composite FPR is a kind of FPR that contains either the 'AND' or 'OR' connectors (Balazinski et al. 2002; Novák and Lehmke 2006; Ting et al. 2008). The general FPR formalism is described as follows.

Definition 4 A general FPR formalism rule is presented as:

$If\ \ D(\lambda)\ \ then\ \ Q\ \ (CF, \mu, w)$, where

1. D is the antecedent part of an FPR with $D = \{D_1, D_2, \cdots, D_n\}$;
2. Q is the consequent part of an FPR with $Q = \{Q_1, Q_2, \cdots, Q_n\}$;
3. λ is the truth degree of each module of the antecedent part, $\lambda \in [0, 1]$;

4. $CF \in [0, 1]$ is the belief strength of the FPR, which is the credibility implementation of the rule after;
5. $\mu_i \in [0, 1]$ is the threshold of the rule; and
6. $w_i \in [0, 1]$ is the weight of each precondition.

According to the definitions above, the inner correspondence between FPN and FPR is listed in Table 2.

Table 3 shows that there is a strict correspondence relationship between an FPN and an FPR. In particular, the antecedent and consequent parts of each FPR could be illustrated as locations in an FPN model, and the three types of parameters also exhibit point-to-point correspondence. Hence, each FPR could be represented as an FPN, and a KBS with multiple FPRs can be depicted as a complex FPN model (Zhou et al. 2019).

Based on the above-mentioned relationships, it is easy to see that there exists a correspondence between FPR and FPN in the modeling process. Three typical FPN models corresponding to three types of FPRs, the simple rule, 'AND' rule, and 'OR' rule, are illustrated in Figs. 2, 3 and 4.

Type 1: Simple rule and corresponding FPN model

Table 2 Correspondence between FPN and FPR

Parameter	*FPN*	*FPR*
Formalism	$FPN = (P, T, I, O, M, \mu, W, CF)$	*If* D *then* Q (CF, μ, w)
Capacity function	$K(s) = 1$	$D = \{D_1, D_2, \cdots, D_n\}$
Weight function	$W(i,j) \in [0, 1]$	$W \in [0, 1]$
Threshold function	$\mu_i \in [0, 1]$	$\mu \in [0, 1]$
Marking	$M(s) \in [0,\ 1]$	null
Support strength	$CF(j,\ i) \in (0,\ 1]$	$CF \in [0, 1]$

Table 3 Comparison between reasoning by graphical method and by algebraic form

	Advantages	Disadvantages
Reasoning algorithm by graphical method	(1) Easy to execute an inference operation intuitively and visually (2) Easy to recognize and find inference path	(1) The reachability and adjacent places set rapidly increase while the structure of the FPN changes (2) In the reachability tree, it is difficult to mark the inner 'AND or OR' relationship among places in the FPN
Reasoning algorithm by algebraic method	(3) Easy to transform all relationships among places into matrices (4) Easy to implement an efficient parallel inference operation	(5) The dimensions of matrices and vectors increased with the growth of the scale of the FPN model (6) Lack of intuitive methods

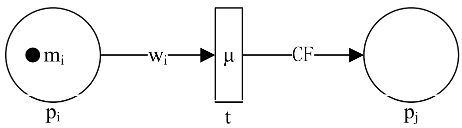

Fig. 2 FPN model of simple rule

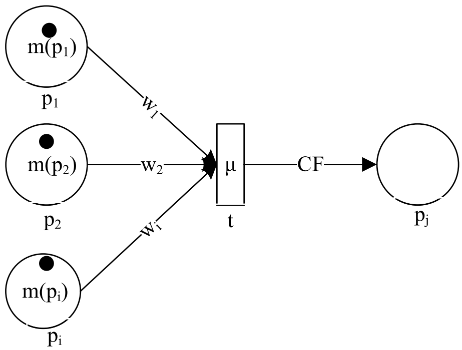

Fig. 3 FPN model of 'AND' rule

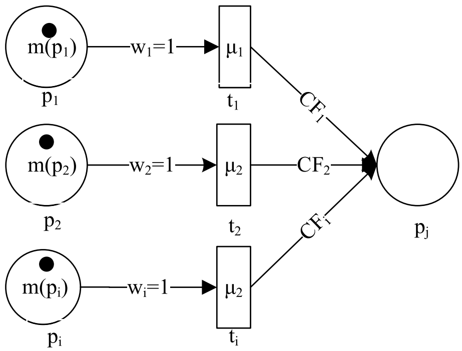

Fig. 4 FPN model of 'OR' rule

$$If\ \ D(\lambda)\ \ then\ \ Q\ \ (CF, \mu, w = 1)$$

Type 2: 'AND' rule and corresponding FPN model

$$If\ D(\lambda_1)\ AND\ D(\lambda_2)\ AND \cdots AND\ D(\lambda_n)\ then\ Q\ (CF, \mu, \sum_{i=1}^{n} w_i = 1)$$

Type 3: 'OR' rule and corresponding FPN model

$$If\ D(\lambda_1)\ OR\ D(\lambda_2)\ OR\ \cdots\ OR\ D(\lambda_n)\ then\ Q\ (CF_i, \mu_i, w_i = 1)$$

4.2 Modeling KBS using FPN

Knowledge-based systems (KBSs) are expert systems that can solve complex problems, such as modeling and verification (Koriem 2000), fault diagnosis or analysis (Paredes-Frigolett and Flavio 2016; Garcia-Crespo et al. 2011), and knowledge reasoning and representation (Fay 2000). Due to the correspondence relationship between an FPN and a KBS, various modeling algorithms of a KBS using an FPN have been widely discussed in the existing literature. Yeung and Tsang (1994b) proposed an FPN model to capture more FPRs in the KBS that maps FPRs with different threshold values in a proposition into an FPN to reflect more information in the FPR. Chen (2002) proposed a weighted FPN (WFPN) and a weighted fuzzy reasoning algorithm for rule-based systems. In this

WFPN, the truth degrees, weight values of the propositions, and certainty factors of the rules are all represented by fuzzy numbers. Shen (2006) proposed a high-level fuzzy Petri net (HLFPN) to represent the FPRs in a KBS to implement fuzzy reasoning. Ha et al. (2007) proposed a generalized fuzzy Petri net (GFPN). The GFPN model has two knowledge representation parameters, namely, input and output weights. The two kinds of weights replace the previous local weight and certainty factors of a rule used in the GFPN framework. Suraj (2012b) proposed a matrix representation of parametrized FPN (PFPN), which can quickly implement parallel firing of independent transitions in a reasoning process using matrix equations or inequalities. Liu et al. (2013a) proposed an active adaptive fuzzy Petri net (DAFPN) for knowledge representation and reasoning and developed a max-algebra-based parallel reasoning algorithm to implement the reasoning process automatically. Liu et al. (2013c) proposed a new knowledge acquisition and reasoning approach based on DAFPN and fuzzy evidential reasoning (FER). In the modified DAFPN, situations in which various rules shared the same parameters were systematically analyzed.

Furthermore, the corresponding solving strategies are also discussed. Liu et al. (2015a, b) enhanced Suraj's work (2013) to propose a new FPN based on intuitionistic fuzzy sets (IFSs) and to order weighted averaging operators. The model aimed to execute the various types of uncertain knowledge information in expert systems and presented a max algebra-based reasoning algorithm to automatically implement fuzzy reasoning, thereby improving the traditional FPN. However, there are many circumstances in which it is inappropriate to use fuzzy sets or IFSs to represent experts' knowledge. Liu et al. (2017b) proposed a linguistic Petri Nets model by combining the theory of FPN with the concept of interval clouds and the hybrid averaging operator. This model is called cloud reasoning Petri nets (CRPNs) and it is used for knowledge representation and reasoning. Li et al. (2018a, b) employed linguistic interval 2-tuples to acquire and represent default knowledge in the interval-valued intuitionistic FPN. Yue et al. (2019) proposed a self-learning interval type-2 FPN (SLIT-2FPN) for knowledge representation and reasoning. The methods introduced an interval number ordered by the weighted averaging operator to enhance the knowledge reasoning capabilities of SLIT-2FPN and extended TOPSIS to determine the parameters of each fuzzy membership function. Xu et al. (2019a, b) proposed a new type of FPN, which is called picture fuzzy Petri nets (PFPN), to represent and acquire imprecise, uncertain expert knowledge.

The existing FPN formalisms lack the self-learning ability to improve the accuracy of a relevant parameter value in the FPN model. It is not easy to accurately represent fuzzy information. Tsang et al. (1999) proposed an FPN with learning ability by combining it with an artificial neural network (ANN). Similarly, Shen et al. (2010) developed two kinds of machine learning PN (MLPN) models based on ANNs to enhance the self-learning of the Petri net (PN) model by supervised and unsupervised learning algorithms. Wang et al. (2014a, b) employed an efficient genetic particle swarm optimization (GPSO) learning algorithm to execute a given FPN model's self-learning function. However, GPSO, the proposed learning algorithm, is not suitable for more complex and large-scale FPN models. Accordingly, Jiang et al. (2019) proposed a hybrid GA-SFLA algorithm by combining the advantages of GA and SFLA to enhance the accuracy of each type of FPN parameter.

The discussed literature above indicates that the knowledge representation using an FPN could be separated into two phases. From the 1990s to 2015, most methods described fuzzy information by internally modifying the FPN model, as illustrated by WFPN (Chen 2002), GFPN (Shen 2006), and DAFPN (Liu et al. 2013a, b, c). However, since 2015, a group of hybrid frameworks combining FPN and other theories were introduced to combine vague knowledge with other factors, such as CRPN (Liu et al. 2017a, b), SLIT-2FPN (Yue et al. 2019), and PFPN (Xu et al. 2019a, b). Meanwhile, the self-learning ability of FPN has also been widely investigated to improve the accuracy of knowledge representation and reasoning.

4.3 Decomposition strategies for large-scale FPN

With the rapidly increasing KBS sizes, the number of FPRs in a rule-based system also sharply increased. As a result, the number of nodes also increased in the corresponding FPN model (Godefroid et al. 1996). This phenomenon is called the state explosion (Valmari 1998; Milinković et al. 2013). The state explosion problem severely hinders further applications of the resulting FPN model. The influences of the state-explosion problem in industrial process-based FPNs can be summarized along the following two lines:

In the forward inference process, the number of required parameters is raised in the FPN model, which results in the increase in the dimensions of the matrices and vectors involved in the associated inference algorithm. Accordingly, the time complexity of the algorithm will also increase. Hence, the accuracy of reasoning results becomes more challenging with the increasing size of FPN (Liu et al. 2017a).

In the backward inference process, the advantages of FPN model, inverse reasoning and algebraic operations are combined. The algorithm separates the part of the rules related to the problem from the knowledge base system, thereby simplifying the reasoning process and meeting real-time requirements. In a decomposition process, whether the decomposed FPN submodel destroys the inference path integrity is the first consideration (Hu et al. 2011; Liu et al. 2013b; Chun and Bien 1993).

To address the state explosion problem, many researchers have developed techniques such as state space reductions, storage size reductions, parallel and distributed computation methods, randomized techniques, and heuristics in order to decompose large-scale FPN models (Clarke and Grumberg 1987; Clarke et al. 2001; Demri et al. 2006; Camilli 2012). Garg et al. (1991) proposed a novel type of FPN utilizing three reduction rules to control the scale of FPN and to check the consistency of KBS effectiveness. Lee and Favre (1985) proposed a hierarchical reduction method to analyze the existing origin Petri net model. Zaitsev (2004) proposed a decomposition algorithm of PNs, which can divide a large-scale PN into a series of small-scale PNs using the properties of similar dynamics or structure. Nishi and Matsumoto (2015) proposed a decomposition approach of PNs to derive a near-optimal solution of deadlock-free and noncyclic scheduling of dual-armed cluster tools to reduce the computational complexity. Zhou et al. (2015a) analyzed the dynamic properties of the FPN model, such as reachability, boundedness, safeness, liveness, and fairness. The authors provided theoretical evidence for the design-related decomposition algorithm. However, as a backward-extended high-level network topology, FPN can consider fuzzy factors without destroying the original PN structure. Therefore, the PN model's original decomposition algorithm can be used for reference in the decomposition process of the FPN model.

To reduce the number of FPN nodes, Zhou et al. (2015b) proposed a decomposition algorithm of FPN by combining the backward search stage and forward strategy methods. The proposed algorithm can divide a large-scale FPN model into a sub-FPN model series using an index function and incidence matrix. All marking values could be obtained simultaneously in an inference process but without enumerating all possible paths. Zhou et al. (2018b) proposed a bidirectional reasoning algorithm by combining a forward reasoning mechanism with a backward reasoning algorithm to reduce the operational matrices' dimensions and simplify the reasoning process by removing irrelevant places and transitioning large-scale FPNs to effectively overcome the state explosion problems. Zhou et al. (2019) developed an equivalent FPN model for generating corresponding large-scale KBSs by utilizing four primary operations: separation, simplifying, generating, and merging. By executing inference operations, the authors proved that it could completely protect inner-reasoning paths among places and transitions in a large-scale FPN.

5 Reasoning algorithms using FPN

As mentioned in Sect. 2, FPN is a kind of backward-extended HLFPN designed to handle fuzzy, vague, and uncertain information without destroying the inner structure of traditional PN. This means that FPN can also describe the KBS by graphical methods (e.g., reachability trees) and algebraic methods (e.g., incidence matrices). Therefore, reasoning algorithms using FPN also have developed from two viewpoints: reasoning algorithms based on the graphical method and reasoning algorithms based on the algebraic method.

5.1 Reasoning algorithm by graphical method

The primary thinking behind the reasoning algorithm is to obtain the final inference results from the corresponding reachability tree of the FPN model.

Chen et al. (1990) introduced the reachability tree by using an immediately reachable set (IRS) and reachable set (RS). They then generated the equivalent FPN for different types of FPR and presented a forward reasoning algorithm. Kasirolvalad et al. (2004) first constructed an AND/OR graph to execute fuzzy inference operations. Similarly, Yang et al. (2002) created a backward tree inference method.

To improve the fuzzy inference ability, Chen (2000) proposed a backward reasoning algorithm based on their previous works. The proposed algorithm could automatically estimate the degree of the truth of any proposition. Chen (2002) proposed a weighted FPN model (WFPN) by adding weight to the FPN model and developed a weighted fuzzy reasoning algorithm using WFPN that automatically performs weighted fuzzy reasoning.

Yeung and Tsang (1994b) established reachability sets and adjacent places and presented an enhanced fuzzy reasoning algorithm based on Chen's previous work. Like Chen's work, this work also ignored the function of three types of FPN parameters. To improve the capability of weighted FPN, Yeung and Tsang (1998) developed a multilevel weighted fuzzy reasoning algorithm (MLWFRA). In this algorithm, the weight values are thought to improve the accuracy of reasoning results by using a new FPR evaluation method (FPREM). However, the certainty factors and the weight values of the propositions were restricted and represented by real values between [0,1].

To solve these drawbacks, Manoj et al. (1998) proposed a novel hierarchical FPN for data abstraction to generate the FPN for a system at different levels of abstraction and reduce the work of determining whether the location is reachable. At the same time, the corresponding reasoning algorithm regarding the hierarchical FPN is also introduced.

Amin and Shebl (2014) proposed adaptive fuzzy higher-order PNs (AFHOPNs) to control the arc's weight changes during the fuzzy inference process. The AFHOPN model has learning ability as a neural network (NN) and can be used for knowledge representation and reasoning. Moreover, a back-propagation (BP) learning algorithm is used in this AFHOPN model to ensure the convergence of the weights using the transition firing rules.

The traditional reachability tree used in the reasoning algorithm can adequately represent the flow relationship between place and transition in the FPN model, but the 'And' or 'Or' relationship among FPNs is challenging to deal with. Hence, Zhou et al. (2018a, b) proposed a modified reachability tree based on and/or graphs to automatically generate a corresponding reachability tree model for a large-scale FPN model. It also enhanced the application range of this kind of reasoning algorithm.

From the above discussion, it can be seen that the data structure used by this type of algorithm is complex, and the inference process is still performed using a step-by-step method of judgment. The implementation is quite complicated and does not make optimal use of the parallel reasoning capabilities of an FPN.

5.2 Reasoning algorithm by the algebraic method

Although the reachable tree algorithm using FPN have the unique advantages of a description of the flow relationship between the places and the transitions, it cannot fully realize the parallel processing capability of the FPN model. Further, logical relationships such as AND/OR graphs are ignored. Moreover, the need to enumerate all possible paths in the inference process results in relatively low reasoning efficiency.

In response, another popular approach of reasoning using the FPN model is to combine algebraic forms (matrix-operation) or vectors to execute fuzzy inference operations in a parallel way.

On the one hand, the algebra-format method makes the data structure simple and easy for computer processing. Gao et al. (2003) proposed fuzzy reasoning Petri nets (FRPNs) to execute a fuzzy reasoning algorithm using max-algebra forms automatically. Fryc et al. (2004) proposed an algebraic representation formal using FPN for performing fuzzy reasoning procedures. Suraj (2012a) proposed a generalized fuzzy Petri nets (GFPNs) model for knowledge representation and reasoning in decision support systems. This model extends the existing FPNs by introducing two operators: t-norms and s-norms, instead of min and max operators. Suraj (2012b) proposed a parameterized fuzzy Petri nets (PFPNs) model by introducing two parameterized families of sums and algebraic product operations to replace previous operators. The experimental results show that the presented model is more flexible than the GFPN model is via an application in the domain of train traffic control systems.

On the other hand, the backward reasoning method can reduce the space complexity in a KBS so that the complexity of large-scale KBSs can be reduced and the KBS can be turned into a more straightforward system. Scarpelli and Gomide (1993) developed a backward reasoning algorithm based on the HFPN model for extracting information from knowledge bases that contain fuzzy production rules. Amit and Lakhmi (2005) summarized two underlying reasoning mechanisms, forward reasoning models and backward reasoning models, and introduced a machine learning strategy in fuzzy pattern recognition from noisy training instances using FPN. Zhou et al. (2015b) proposed a decomposition algorithm of bidirectional thinking based on FPN. The algorithms include a backward search stage and a forward strategy, which can divide a large-scale FPN model into a series of sub-FPN models using an index function and an incidence matrix. Furthermore, this bidirectional thinking is also used to reduce the inference algorithm's complexity for a large-scale FPN model by Zhou et al. (2018b). Meng et al. (2016) also built an IFPN model to carry out reasoning by matrix operation.

5.3 Analysis of reasoning algorithms

As mentioned above, the main reasoning strategies using FPN could be classified into two types. On the one hand, reasoning utilizes the graphical description ability of an FPN, and on the other, reasoning utilizes the algebraic analysis ability of an FPN. In addition, the reasoning mechanisms deployed can be divided into three approaches: forward thinking, backward thinking, and bidirectional thinking.

5.3.1 Characteristics of two reasoning strategies

The characteristics of these two reasoning strategies are summarized below.

The reachability tree is a powerful visual tool to reveal the inner relationships among places of FPN and is frequently used to graphically describe the inference process. Reasoning using reachability trees was widely discussed and applied in the twentieth century to provide a clear path for each inference chain. The core goal of reachability tree-based algorithms is to implement the inference process using the graphical description of an FPN. In the inference process, the primary step is to generate the reachability tree of an FPN based on RSs and IRSs and implement different reasoning strategies within the reachability tree.

On the other hand, the reasoning algorithm using algebraic forms takes advantage of the parallel operation of FPN formalism. The core thinking of this kind of algorithm is to transform the inference process into matrix operations in order to achieve multiple places to transition transformations at the same time. Table 3 shows the comparisons between these two reasoning mechanisms.

5.3.2 Reasoning mechanisms

Based on the summarization of two reasoning strategies, there are three basic reasoning mechanisms: forward inference, backward inference, and bidirectional inference. The advantages and disadvantages of the three reasoning mechanisms are summarized in Table 4.

Table 4 shows the main advantages of the above three reasoning mechanisms. The graphical ability of the FPN is fully exploited by the first mechanism to reveal the visual reasoning process based on the inner connections among places. The second mechanism is widely applied to execute parallel operations based on algebraic forms, such as vectors and matrices. The bidirectional inference strategy is proposed to simplify the reasoning process for a complex FPN model by combining the highlights of the first two approaches.

Table 4 Comparison among forward, backward, and bidirectional inference

	Main idea	Advantages	Disadvantages
Forward inference	Input place to output place	Easy to obtain the related data structure	Algorithm complexity increases as FPN size goes up
Backward inference	Output place to input place	Easy to solve reasoning problems with imprecise results	The inference process is somewhat trickily related to the places and transitions
Bidirectional inference	Combines parallel ability and real-time operation	Reduces the dimension of the matrix and the time complexity of the inference process	The complexity of the data structure and reasoning process is increased

6 Typical applications using FPN between 2014 and 2020

As a potential graphical tool for modeling and reasoning in a rule-based system or KBSs, FPNs have been extensively applied in various industrial areas. The typical applications of an FPN from 2014 to 2020 are reviewed and summarized to reveal the latest trends in this section.

6.1 Successful applications of FPN

In the past few years (2014–2020), FPNs have been applied in various industrial domains and have gained some remarkable results, including fault diagnosis, risk assessment, manufacturing systems, health care and biological systems, automatic control systems, and so on. In the following, the papers about the FPN application field and highlight(s) are summarized in the Appendix.

6.1.1 Fault diagnosis using FPN

Zhang et al. (2014) developed an automatic reasoning algorithm using FPN to judge the transformers' legal fault cause for power systems. The method involves two main steps: building an IF–THEN structure and finding the transformer's fault reason using matrix iteration. The authors indicate that the proposed algorithm makes the diagnosis process clearer and faster. Additionally, the case study verifies the correction and feasibility of the proposed approach. He et al. (2014a, b) proposed a novel fault diagnosis method using adaptive fuzzy Petri nets (AFPNs) to implement inference for a complex power system. The proposed AFPN can achieve more accurate results by keeping the descriptive advantages of FPN and enhancing the self-learning ability of FPN. Wang et al. (2014a, b) developed a novel hierarchical fault diagnosis approach using a FPN. The method employed a directional weighted fuzzy Petri net (DWFPN) using a fault recorder and WAMS to analyze the smart power grid. The case study shows that the proposed method can effectively solve the defects and obtain correct diagnosis results in the presence of incomplete information.

Sun et al. (2015) proposed a time sequence fuzzy Petri net (TSFPN) to build a fault diagnosis model for a power system. The proposed method considers the influence of incomplete and uncertain alarm information on fault diagnosis. It also employs protective relays and circuit breakers' time constraint characteristics to eliminate conflicting time sequence information. Wang et al. (2015) developed a fault diagnosis algorithm based on FPN to address uncertain power grid elements. The discussed algorithm utilizes the features of FPN to execute the logical inference on the base of an associate tree for finding located in point of failure completely.

Zhang et al. (2016) employed the FPN model to implement a fault diagnosis of power systems. In this strategy, the FPN is used to cope with temporal constraints and fuzzy information, and the temporal attributes are assigned to the propositions in PNs to ensure that the obtained structured model can adapt to new protection schemes and topology changes. The proposed approach is suitable for online fault diagnosis in large-scale power systems compared with three existing fault diagnosis methods. To decrease the complexity of diagnosis processing, Wang et al. (2016) proposed a fault recognition method using fuzzy fault Petri nets (FFPNs) to simplify the process of fault propagation in a high-speed train traction system. The proposed method has better validity and practicability in obtaining fault recognition through forward and backward reasoning algorithms using associated failures between equipment.

Cheng et al. (2017) proposed a novel approach to implement fault diagnosis for a power grid system. A hierarchy-weighted fuzzy Petri net (WFPN) is constructed by considering the related time association character (TAC) to improve the diagnostic accuracy. Specifically, the TAC consistency check is used to correct the operation probability of the protection and circuit breaker that does not meet the time correlation consistency. This effectively reduces the failure probability of nonfaulty components and greatly improves the accuracy of fault diagnosis. Sun et al. (2017) proposed a fuzzy fault Petri net (FFPN) to address the fault propagation characteristics in power systems. The established FFPN model aims to describe the system failure occurrence, evaluate the severity of the fault and transmission path, and provide real data maintenance.

Zhang et al. (2018) proposed intuitionistic FPNs (IFPNs) to represent the certainty and uncertainty of alarm information in a power system. The reasoning mechanism using IFPNs is used to calculate the certainty and uncertainty of electrical device fault events and evaluate the robustness of real-time diagnosis in complicated circumstances where multiple components operate incorrectly. Li et al. (2018a, b) proposed a fuzzy neural Petri nets (FNPNs) method to improve a resistance strain weighing sensor's operating stability and eliminate vague factors in the fault diagnosis process. The proposed approach establishes a fault diagnosis model by FNPN to adjust the weights of models and gain the order of faults for each part using PN characteristics. Liu et al. (2018a, b) proposed a modified online fault diagnosis algorithm by using generalized mutual exclusion constraints (GMECs) with integer linear programming (ILP) and the FPN model. The algorithm further addresses the ILP problem of the POPN model with a remarkably increased DES diagnosis ability.

Xu et al. (2019a, b) proposed a novel fault diagnosis model based on temporally constrained fuzzy Petri nets (TCFPNs) to address the uncertainty and temporal constraints of alarms for power system fault diagnosis. Similarly, Tan et al. (2019) proposed intuitionistic fuzzy Petri nets (IFPNs) based on time series matching to improve the reliability of power grid fault diagnosis. Xu et al. (2019a, b) and Tan et al. (2019) both focused on improving the accuracy of the diagnosis results of power systems by considering uncertain information and alarm information. By use of an example, the accuracy of the fault diagnosis of the proposed method is demonstrated.

Wang et al. (2020) proposed an adaptive neural fuzzy Petri network (NFPN) algorithm by combining BPNN and FPN to execute fault diagnosis for a complex system with a three-phase asynchronous motor. The established NFPN model-based motor system using the Gaussian function is employed to replace the traditional transition confidence in the NFPN model and to enhance the flexibility of NFPN by utilizing the backpropagation features of BPNN. Jiang et al. (2020) proposed a weighted fuzzy neural Petri net (WFNPN) to create a microgrid fault diagnosis model. On the one hand, the parameters of the WFNPN are trained using a fuzzy neural network (FNN) to enhance the calculation accuracy. On the other hand, WFNPN is used to reduce the computational complexity of the diagnosis process.

6.1.2 Risk assessment using FPN

Bharathi et al. (2014) developed a FPN based forecast model to predict the post-implementation risks in small and medium enterprises (SMEs). In the same year, Bharathi (2014) utilized the FPN model to assess the risks inherent in the implementation phase of SMEs.

These two projects aim to develop the prediction and assessment risk tools of ERP in SMEs, to further enhance the accuracy of the risk assessment technique and to expand the areas to which risk assessment tools can be applied. Pramod et al. (2014) utilized the FPN model to assess the risk associated with the post-implementation phase or the usage phase of enterprise resource planning (ERP) in small and medium enterprises. The method provided an effective and intuition means of representing and quantifying risks in the different phases of ERP adoption for SMEs to enhance the level of risk assessment. To expand the previous research results in 2014, Bharathi et al. (2017) addressed the problems of risk identification and assessment during the extension phase. The authors utilized the FPN model because the risks are categorized and further graded for decision support to SME stakeholders. Chiang (2015) proposed an ECG-based mental stress assessment system by combining fuzzy theory and an associated PN. This system detects mental stress and reduces subjective errors accurately by using SVD, MEPA, and APN. Moreover, this system provides diagnostic information immediately by producing a mental stress assessment model. Meng et al. (2015) proposed a local adaptive fuzzy Petri net (LAFPN) to predict the shortage of water risk values in Beijing. The LAFPN could adjust the local weights according to the real situation and respond to changes quickly to enhance the improvement of the accurate prediction of the risk value.

Guo et al. (2016) developed a comprehensive risk evaluation method using the FPN model for long-distance oil and gas transportation pipelines. The proposed model describes the propagation path of the fault and the systems' characteristics. Meanwhile, the model also provides some decision support for the risk management of oil and gas pipelines to reduce the risk. Kuchárik and Balogh (2016) developed a new form of evaluation approach by combining the FPN and Trans Place Sim tools to assess the risk of prioritizing the teaching process.

Zhou et al. (2017) proposed weighted FPNs (WFPNs) and a corresponding security risk inference method using matrix operations to evaluate the importance of risk factors and their different relationships in chemical industry areas. Majma et al. (2017) proposed a hierarchical fuzzy colored Petri-net (FCPN) to monitor device software behavior to reduce the failure risk of medical devices. Compared with the simple PN and nonhierarchical FCPN, the HFCPN has less runtime for device runtime verification.

Zakharov et al. (2018) proposed a modified FPN to represent and evaluate the fuzzy production system's ecological risk. Compared with the conventional approaches, the proposed method directly defines and solves the optimization problems by determining the complex frequency distribution of the original parameters or an unknown frequency distribution. Chang et al. (2018) developed a fuzzy reasoning algorithm using the FPN model to assess the risk degree of the deep-water drilling riser system, in which the analytic hierarchy process (AHP) and entropy method (EM) are adopted to improve the accuracy of evaluation. Yao et al. (2018) proposed a modified FPN to assess the safety risk in air traffic control. While focusing on the huge number of nodes and strong subjectivity of change weights in the assessment process, the risk level threshold, the AHP and the utilized reverse search method are used in this proposed modified FPN to simplify the complicated evaluation process to improve the accuracy of the assessment result.

Shi et al. (2019) proposed a novel failure mode and effect analysis (FMEA) method by hybrid FPN and a fuzzy evidential reasoning strategy to improve the risk assessment of traditional FMEA and determine the failure risk priority modes in FMEA. The proposed risk assessment method produces more reliable risk ranking results and aims for applications related to more fields to enhance the safety and reliability of products, processes, and services. Li et al. (2019) proposed a new FMEA method by combining linguistic term sets (PLTSs) and FPNs to assess the risk failure of reciprocating compressor failure in a marine ship system. Sachan and Donchak (2020) developed a generalized stochastic Petri Net (GSPN) with fuzzy parameters to evaluate infrastructure asset management policy and enhance the reliability and availability of infrastructural assets.

Zhou and Reniers (2020) proposed a weighted fuzzy Petri net (WFPN) with inhibitor arcs to model relationships between risk factors and establish the risk assessment structure considering veto factors. The WFPN model can properly address the risk assessment problem considering veto factors and provide useful and reasonable results. Zhang et al. (2020) proposed an interval intuitionistic integrated cloud Petri net (IIICPN) model. The improvements of the IIICPN can be summarized into two parts. On the one hand, interval-valued intuitionistic uncertain linguistic numbers have been converted into IIICs. On the other hand, related definition and reasoning rules are introduced into the IIICPN. Finally, a case study of subway fire risk assessment is used to verify the correctness of the proposed model. The experiments reveal that the proposed IIICPN can overcome the defects of FPNs and trapezium clouds.

6.1.3 Manufacturing systems using FPN

Baruwa and Piera (2014) proposed a timed colored PN (TCPN) approach to evaluate schedule flexible manufacturing systems (FMSs). Compared with traditional methods, the presented method can be parallelized on several processors with distributed memory to increase the amount of memory available. This enhances its capabilities to return solutions for large-sized problems. Li et al. (2014) proposed a colored Petri net and a hybrid meta-heuristic by combining simulated annealing (SA) and tabu search (TS) to optimize the plan schedule of manufacturing systems and thereby provided a theoretical basis for more complicated industrial cases.

Mhalla and Benrejeb (2015) proposed a monitoring-maintenance method based on FPN for maintenance (FPNM) using P-time PN and fault trees to model manufacturing systems with time constraints and analyze the behavior of diagnosis operations. Basak and Albayrak (2015) proposed object-oriented Petri nets (OOPNs) to model and analyze the shop floor scheduling problem of flexible automotive manufacturing systems (FAMSs). Case analysis illustrates that the performance evaluation problem of timed PN can be transformed into a simple linear programming problem using the proposed OOPN model.

Yu et al. (2016) proposed object-oriented Petri nets (OOPNs) and π-calculus approaches. On the one hand, OOPNs are applied to visualize the initial structure and system behaviors of reconfigurable manufacturing systems (RMSs). On the other hand, π-calculus is used to describe the reconfiguration of RMSs. Similarly, Kahloul et al. (2016) employed reconfigurable object Petri nets (ROPNs) to model, simulate, and analyze RMSs.

Shah et al. (2017) proposed a colored Petri net (CPN) with a fuzzy factor to reduce the total number of elementary circuits of flexible manufacturing systems (FMSs) and decrease the complexity of the system. Although the presented framework does not predict tool life monitoring, it can keep track of the entire machining time during a cycle. Saren et al. (2017) proposed a hierarchical colored Petri net (HCPN) to resolve conflict situations in manufacturing processes. The proposed HCPN plays an essential role in the cell-based overall performance on arrival time and part processing time in the hierarchical model and provides useful information about system behavior for this manufacturing system.

Saren et al. (2018) extended their previous work to construct a delivery date model using HCPN approaches. In this approach, three main modules of the FMS system, namely EDD, EXACT, and OVER, are defined as three sub-models. Compared with other CPNs, this HCPN model easily analyzes the behaviors of the complex FMS. Fei et al. (2018) developed 'switching the machine on/off' status approaches using FPN to reduce the idle period in manufacturing systems. Simulation experiments validate that machines' idle status can be effectively transformed into an energy-saving status to realize energy-efficient production by utilizing the proposed approach.

Wang et al. (2019) proposed a dynamic adaptive fuzzy reasoning Petri net (DAFRPN) to infer or decide on machine statues for reducing idle times by considering energy-saving aspects: real-time buffer levels and production rates in manufacturing systems. Experimental analysis shows that the proposed model can effectively analyze in parallel, assembly and disassembly structures of manufacturing systems. Chen et al. (2019) built an FPN model to analyze the fault diagnosis of manufacturing systems by executing a bidirectional reasoning algorithm. The experimental results verify the reliability and accuracy of the fault diagnosis results.

Kaid et al. (2020) proposed a different deadlock control strategy for automated manufacturing systems (AMSs) based on PNs and NNs. The strategy includes three steps: to obtain the controlled system (deadlock-free), to merge all obtained monitors into a single monitor by CPN (colored Petri net), and to detect and treat faults by combining neural networks with CPN. Al-Ahmari et al. (2020) proposed a two-step robust deadlock control strategy to analyze AMSs with unreliable resources. A live (deadlock-free) controlled system is derived by using strict minimal siphon control, and then deadlock control is solved by CPNs. The proposed approach inherits the advantages of PNs and NNs to address the deadlock problem in AMSs and to detect and treat failures. Nabi and Aized (2020) developed a compact, editable, and deadlock-free flexible manufacturing system model based on CPNs to fully represent discrete event dynamical system characteristics and build a precise and stable model of a discrete event dynamical system.

6.1.4 Health care and biological system using FPN

Tao et al. (2014) employed FPNs to replace the knowledge model of agents to handle uncertainty information and analyze and verify agents' communication with each other in collaborative health care diagnosis systems. Compared to the traditional FPN formalism, there are two main differences. On the one hand, tokens are put on arcs instead of places. On the other hand, places are permitted the capability to control the propagation of tokens. These changes make FPN more suitable for logical inference and more flexible in applications where knowledge is frequently updated. Chen et al. (2014) proposed a rule-based decision-making diagnosis system using FPN to evaluate arteriovenous shunt (AVS) stenosis for long-term hemodialysis treatment. The study employed human diagnosis work to present a fuzzy inference algorithm for a rule-based system, and the degree range was subdivided into nine scales to analyze the occlusion levels.

Hamed (2015) proposed an adaptive FPN (AFPN) reasoning algorithm for rule-based systems. The proposed algorithm predicts potential risk for esophageal cancer based on the serum concentrations of C-reactive protein (CRP) and albumin as a set of input variables. Moreover, this approach combined FPN reasoning and the capability of an adaptive learning mechanism to make qualitative inferences of a biological system. Chiang (2015) developed a rule-based reasoning model by combining fuzzy computing with associative Petri net (APN) for electrocardiogram (ECG)-based mental stress assessment. The approach combines SVD, MEPA, and the APN to build a mental stress assessment model that can address signal noise and instability and precisely identify the degree of mental stress.

Chiang and Pao (2016) proposed a hybrid inference system combining fuzzy theory and the APN model to implement early diagnosis and prediction of Alzheimer's disease. The case study shows that the proposed model could effectively detect Alzheimer's disease and use it as an objective tool for clinical diagnosis assistance. Pennisi et al. (2016) proposed colored PNs (CPNs) to model the cellular scale humoral immune system. The proposed model has a suitable level of granularity in the description of cell behavior. It permits the analysis of the biological network's fundamental properties under study, such as reachability, liveness, boundedness, place invariants, circuits, and t-invariants.

Hamed (2017) developed a quantitative framework of biological systems using FPN. Liu et al. (2017a, b) proposed a fuzzy continuous PN (FCPN) that combined CPN with fuzzy logic to model biological systems with uncertain kinetic parameters. The two studies show that FPN is a reasonable model and very successful for the information representation of intuitionistic fuzzy frameworks.

Chiang et al. (2018) proposed a diagnosis model of sleep quality based on fast Fourier transform and the FPN to explore the impact of various presleep activities on subsequent sleep quality. Experiments demonstrate that these variables are adequate for assessing the impact of different bedtime activities on sleep quality. Liu et al. (2018a, b) systematically reviewed the existing modeling methods of uncertainty with biological systems using FPNs and classified them into three categories: basic FPNs, fuzzy quantitative Petri nets, and Petri nets with fuzzy kinetic parameters. A fuzzy continuous Petri net (FCPN) is proposed by combining CPNs and fuzzy logic by viewing some kinetic parameters as missing or inaccurate. Simulation experiments show that the FCPN algorithm can be used to model and analyze biological systems with uncertain kinetic parameters. Yang et al. (2018) established a biological system model based on CPN to represent the flow of genetic information involving transcription and translation in the DNA molecular biological system. The modeling method provides an intuitive and clear description of the detailed state changes in the process of protein synthesis and establishes a theoretical foundation for intuitively modeling and analyzing other essential biological processes.

Rosdi et al. (2019) proposed a classification method using FPN to evaluate the speech intelligibility of children with speech impairments. This method can improve the discrimination ability of automatic speech detectors for children with speech impairments. Liu et al. (2019) developed a framework to simulate a biological system with uncertainties by combining continuous PNs with fuzzy theory. This method provides a suitable hybrid uncertain/certain approach to modeling and analyzing biological systems with uncertainties.

6.1.5 Control system using FPN

Weng et al. (2014) developed a parallel railroad crossing control system using TPNs. The proposed model demonstrated how to verify the level of parallel railroads from the models to critical scenarios for crossing traffic systems. Additionally, the proposed model can quickly amend some operations to handle some exceptional cases, such as interrupting traffic lights' regularity to allow emergency cars to be prioritized. Tian et al. (2014) proposed a fuzzy hybrid Petri net (FHPN) to control the traffic signal system. This method not only computes the best green light duration of each stage in a period, but it also determines a reasonable and alterable stage sequence. He et al. (2014a, b) proposed an airplane fault diagnosis model for an environmental control system (ECS) using weighted FPN (WFPN). The failure diagnosis method uses a reverse inference control strategy by considering the false alarm rate and underreporting rate to improve fault locating.

Shen et al. (2015) proposed an extended HLFPN for smartphones to analyze, develop, and identify human actions, such as regular action, exercising, and falling. A smartphone could execute fall detection using this algorithm on either side of the thigh pocket without a specific figure on a particular area. Lin and Li (2015) proposed a dynamic Petri fuzzy cerebellar (DPFC) model by integrating a cerebellar model articulation controller (CMAC) to simulate the magnetic levitation system (MLS) and a two-axis linear piezoelectric ceramic motor (LPCM) drive system. In the DPFC model, the online algorithm for learning the controller's parameters is obtained by using the Lyapunov function to guarantee the system's stability. Experimental results show that the proposed control scheme has a perfect tracking response to analyze, develop, and identify human actions, such as regular actions, exercising, and falling.

Hajduk and Wojtowicz (2016) proposed an FPN with Lukasiewicz norms to model and analyze the costs of implementing control systems. Letia and Kilyen (2016) proposed a hybrid control system (HCS) model to describe discrete event and discrete-time features. The model combined the enhanced TPN with fuzzy logic rules (FLETPNs) to describe the concurrent, synchronous, and asynchronous behavior of HCS.

Bibi et al. (2017) proposed a Petri Type 2 fuzzy neural network (PT2FNN) approximator to realize adaptive control of uncertain nonlinear systems. The PT2FNN is obtained by integrating the Petri layer into the T2FNN and replacing the K–M iterative algorithm with adaptive modulation. Compared to T1FNN and T2FNN, the PT2FNN approach has better performance. Kim and Yang (2017) developed a self-navigating robot system using FPN to simulate, evaluate, and interconnect the navigational algorithm and solve the unknown maze problem. Hansen et al. (2018) utilized an FPN model to analyze soccer ball recognition and distance prediction.

Ammour et al. (2018) proposed a partially observed stochastic PN (POSPN) to model a system with sensors for predicting the faults of discrete event systems. Sun et al. (2018) proposed a fused fuzzy Petri net (FFPN) with a shared control method to solve the control problem of a brain actuated discrete event system by combining automatic robot control (AC) with brain actuated control (BCI). The simulation test builds a software model using the FFPN to adjust the control parameter and achieve the required control.

To extend his previous work, Bibi et al. (2019) employed PT2FNN to approximate adaptive control for a class of single-input single-output nonlinear systems. Le (2019) proposed a self-organizing recurrent interval type-2 Petri fuzzy controller (SORIT2PFC) to control time-varying delay systems by hybridizing an interval type-2 fuzzy network, recurrent network, and FPN. Moreover, a self-organizing algorithm is also proposed to optimize the construction of the SORIT2PFC network, improve system performance, and reduce the computational burden. Sha et al. (2019) proposed a cooperator fault detection system using a FPN to organize several fault detectors in a smart factory that has a complex production process. Bibi et al. (2020) proposed a hybrid structure by combining type 2 fuzzy logic and PN (PT2FNN) as a new approximator to reduce uncertainties with optimal cost and time consumption.

6.1.6 Other typical applications using FPN

Shen and Chen (2015) employed a high-level fuzzy Petri net (HLFPN) to complete music segmentation and karaoke system correction. The proposed model can implement an automatic calibration system for Chinese karaoke lyrics and obtain song lyrics by GPRs and HLFPN. Cheng et al. (2015) proposed an automatic web service composition method based on a fuzzy predicate Petri net (FPPN) to model the Horn clause set and the T-invariant technique to determine the existence of composite services fulfilling the user input and output requirements. Suraj (2015) proposed a modified generalized FPN (GFPN) to model the rain traffic control system. The GFPN model combined the graphical power of Petri nets and fuzzy logic to model rule-based expert knowledge in a decision support system.

Suraj et al. (2016) proposed a flexible generalized fuzzy Petri net (FGFP-net) by GFPN to optimize decision support systems. The proposed approach can be used to control design, knowledge representation and reasoning in decision support systems. Xia et al. (2016) developed a human–machine (HM) system model using a fuzzy inference Petri net (FIPN) to realize adaptive task allocation (ATA) between human operators and machines. The FIPN constitutes a new modeling tool suitable for systems that combine (Mamdani-type) FIS and discrete logical switching. Fat et al. (2016) built a stock market price prediction strategy using fuzzy logic enhanced TPN (FLETPN). On the one hand, the GA algorithm is used to optimize the system parameters to effectively assess the change in stock market prices. On the other hand, the model can also identify asynchronous events influencing the market.

Nie et al. (2017) built a personalized learning model using FPN to analyze students' learning behaviors, characteristics, and mastery of knowledge level in education systems. To improve the precision of individualized education, students' level of knowledge and learning speed are considered essential factors. In the implementation, this is combined with the outer edge of the student's knowledge state. The simulation results show that the proposed method can dynamically plan the optimal learning path for students and compares well with other predominant personalized education methods. Shen et al. (2017) proposed a shot boundary detection (SBD) hybrid method by combining HLFPNs and keypoint matching to address the membership function of SBD in social media to adapt the transition variation of each video. On the one hand, removing all possible false shots is used to improve the precision of SBD on regular transition. On the other hand, the hybrid approach of the HLFPN model and keypoint matching can efficiently increase the precision of SBD by removing false shots and can be applied to various types of videos. To further improve the overall performance of HM systems, Zhang et al. (2017a, b) proposed a fuzzy inference PN (FIPN) to model the continuous and discrete components of an HM hybrid system. The model is uses two methods to predict operators' performance using Wang-Mendel (WM) fuzzy modeling and optimizes the membership function parameters of the fuzzy OFS model using an artificial bee colony (ABC) algorithm.

Shen et al. (2018) developed a financial investment system model as a HLFPN to decide the stock prediction system's investors, enhance profits and assist investors so that the desired investment targets are met. Furthermore, fuzzy neural reasoning or technical analysis is used to decide on investments. Yang et al. (2018) proposed novel dynamic timed fuzzy Petri net (DTFPN) approaches based on dynamic time delay analysis (e-DTA) to overcome the limitation of the TFPN field region. Compared to traditional methods, DTFPNs have a strong ability to describe dynamic correlations among process variables.

Yang et al. (2019) employed an HLFPN model to detect a threshold value in shot boundary detection. Compared with other methods, the proposed model can reduce the time in the video stream process and improper shot changes caused by motions. Zhang et al. (2019) proposed a random FPN with a time delay to model behavior reconstruction systems and control the user behavior character. Using this formalism, the behavioral flow of different user groups is constructed by the system behavior reconstruction model. Shen (2019) proposed a high-level fuzzy Petri net (HLFPN) with an abstract syntax tree to detect code plagiarism operations. The experimental results reveal that the proposed method makes a better determination of code plagiarism detection. Cheng et al. (2019) developed a fuzzy spatiotemporal ontology model using FPN to construct fuzzy spatiotemporal ontologies. The fuzzy spatiotemporal data represented by FPN models are directly upgraded to semantic web contents by using this approach so that the existing fuzzy spatiotemporal data resources in fuzzy Petri net models can be used in the context of the semantic web.

Samantra et al. (2020) proposed an FPN to optimize the ad hoc network for discovering an optimal route between the source and destination by considering different factors, such as node battery power, node mobility, and path length. Shahidinejad et al. (2020) proposed an elastic controller using CPNs and a queueing system to manage cloud infrastructures automatically. Compared with other approaches, the proposed method can significantly outperform in terms of the elasticity value, response time, and CPU utilization.

6.2 Bibliometric analysis

Based on the analysis in Sect. 6.1, 93 articles are classified into various application fields as illustrated in Fig. 5.

Figure 5 shows the extensive use of FPNs in traditional industrial fields, including fault diagnosis and manufacturing systems. Therefore, these areas have yielded significant achievements by combining HLPN and HLFPN.

There have also been academic attempts to improve the accuracy of fault diagnosis result in the fault diagnosis process by combining it with other HLFPNs and as also in the manufacturing engineering sector. Compared with the traditional applications, FPN has been widely applied in various complex systems, such as health care and biological systems and risk assessment. In the health care and biological sector, FPN is mainly used for disease assessment and diagnosis in medical engineering and the performance improvement of medical equipment. In the risk assessment sector, the main focus has been on the risk evaluation method and the accuracy of risk value estimation by combining HLFPN and other theories.

Fig. 5 Number of articles on FPN application in various fields

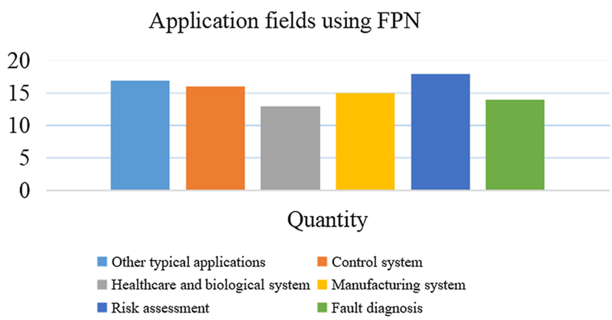

Table 5 Application of fault diagnosis using FPN

Author(s)	Application field	Highlights
Zhang et al. (2014)	Power system	Build a FFPN model using the form of IF–Then rule
He et al. (2014a, b)	Power system	Develop an AFPN model to execute fault diagnosis
Wang et al. (2014a, b)	Smart power grid system	Build a DWFPN model based on the fault recorder and WAMS to execute fault diagnosis
Sun et al. (2015)	Power system	Propose a TSFPN to build a fault diagnosis model
Wang et al. (2015)	Power system	Develop a fault diagnosis algorithm
Zhang et al. (2016)	Power system	1. Handle temporal and fuzzy information using FPN 2. Establish an online fault diagnosis framework
Wang et al. (2016)	High-speed train transition system	1. Develop a fault diagnosis method based on FFPN 2. Introduce a fault propagation flow function and the set of occurred transition based on matrix and state equation
Cheng et al. (2017)	Power system	Build a hierarchical WFPN to execute fault diagnosis
Sun et al. (2017)	Power system	Build a FFPN model and intelligent fire judgment matrix to execute fault diagnosis
Zhang et al. (2018)	Power system	Establish a power diagnostic model using IFPN
Li et al. (2018a, b)	Power system	Build a fault diagnosis model using FNPN and NN
Liu et al. (2018a, b)	Power system	Modify an online fault diagnosis algorithm based on POPN model
Xu et al. (2019a, b)	Power system	Propose a TCFPN to execute fault diagnosis
Tan et al. (2019)	Power system	Propose an IFPN model to execute fault diagnosis
Wang et al. (2020)	Power system	Propose an ANFPN algorithm to execute fault diagnosis
Jiang et al. (2020)	Power system	Establish a microgrid fault diagnosis model based on WFNPN by combine NN and WFPN

Table 6 Application of risk assessment using FPN

Author(s)	Application field	Highlights
Bharathi et al. (2014), Bharathi (2014)	ERP implementation	Formulate a rule for risk assessment and the corresponding predict model using FPN
Pramod et al. (2014)	ERP implementation	Divide two stage to build a risk assessment
Chiang (2015)	Electro-cardiograms	Introduce an HLFPN to develop a rule-based system
Meng et al. (2015)	Water shortage risk	Propose a LAFPN model to predict water shortage risk
Guo et al. (2016)	Pipeline transportation	Propose a AHP model to predict pipeline transportation risk
Kuchárik and Balogh (2016)	Teaching field	Develop a Trans Place Sim tool to evaluate the process's risk priority
Bharathi et al. (2017)	ERP implementation	Establish a risk category using FPN
Zhou et al. (2017)	Chemical industry	1. Propose two WFPN models to execute the security risk assessment 2. Develop a security risk inference method using matrix operations
Majma et al. (2017)	Medical device	Propose a HFCPN approach to test the runtime behavior of the pacemaker software
Zakharov et al. (2018)	Ecological system	Provide theoretical evidence within the fuzzy production GIS for analyzing and monitoring ecological risks
Chang et al. (2018)	Deepwater drilling riser	Propose a AHP and EM to improve the accuracy of evaluation
Yao et al. (2018)	Air traffic control system	Utilize reverse search method to simplify and complicated FPN
Shi et al. (2019)	Ship fire-safety system	Modify traditional FMEA to establish the risk priority of failure modes
Li et al. (2019)	Marine-ship system	Propose a new FMEA model based on PLTS and FPN
Sachan and Donchak (2020)	Infrastructure asset management policy	Propose a GSPN model with fuzzy parameters to evaluate management policy
Zhou and Reniers (2020)	Plants with hazardous chemicals	Propose a WFPN with inhibitor arcs
Zhang et al. (2020)	Subway Fire Accident	Propose a IIIC and IIICPN model

Table 7 Application of manufacturing engineering using FPN

Author(s)	Application field	Highlights
Baruwa and Piera (2014)	Flexible manufacturing systems	Propose a TCPN to evaluate the properties of FMS
Li et al. (2014)	Flexible manufacturing systems	Propose a CPN model and a hybrid meta-heuristic method by SA and TS to optimize the plan schedule
Mhalla and Benrejeb (2015)	Manufacturing system	Propose a FPNM to model manufacture systems with time constraints
Basak and Albayrak (2015	Flexible manufacturing systems	Propose an OOPN to model and analyze the shop floor scheduling problem
Yu et al. (2016)	Reconfigurable manufacturing system	1. Propose an OOPN to visualize the initial structure and system behaviors of RMS 2. Propose π-calculus is used to describe the reconfiguration of RMSs
Kahloul et al. (2016)	Reconfigurable manufacturing system	Employ a ROPN to model, simulate, and analyze the RMS
Shah et al. (2017)	Manufacturing system	Propose a CPN with fuzzy factor to reduce the total number of elementary circuits and decrease the complexity of FMS
Saren et al. (2017)	Flexible manufacturing system	Proposed a HCPN to resolve the conflict circumstances processes
Saren et al. (2018)	Flexible manufacturing systems	Extend their previous work to reduce the idle period of FMS
Fei et al. (2018)	Manufacturing system	Develop a switching the machines on/off statues to reduce the idle period
Wang et al. (2019)	Manufacturing system	Propose a DAFRPN to reduce the idle times
Chen et al. (2019)	Flexible manufacturing systems	Establish a bidirectional reasoning algorithm
Kaid et al. (2020)	Automated manufacturing systems	Develop a deadlock control strategy using PN and NN to detect and treat fault
Al-Ahmari et al. (2020)	Automated manufacturing systems	Develop a deadlock control strategy based on strict minimal siphon, PN and NN
Nabi and Aized (2020)	Flexible manufacturing systems	Develop a CPN to fully represent discrete event dynamical system characteristics

Table 8 Application of healthcare and biological system using FPN

Author(s)	Application field	Highlights
Tao et al. (2014)	Collaborative healthcare diagnosis system	Utilize FPN to represent the fuzzy information and verify agents' communication with each other
Chen et al. (2014)	Arteriovenous Shunt Stenosis	Develop a rule-based decision-making diagnosis system using FPN
Hamed (2015)	Assess the potential risk of esophageal cancer	Propose a reasoning algorithm using FPN
Chiang (2015)	Electrocardiograms (ECG)-based mental stress assessment	Propose a rule-based reasoning model using AFPN
Chiang and Pao (2016)	Alzheimer's disease detection	Develop a diagnosis model using FPN
Pennisi et al. (2016)	Humoral immune system	Build a framework using CPN and fuzzy factor
Hamed (2017)	Biological system	Develop a modeling technology using FPN
Liu et al. (2017a, b)	Biological system	Build a FCPN model to model and analyze biological system
Chiang et al. (2018)	Assess Sleep quality	Develop a diagnosis model of sleep quality
Liu et al. (2018a, b)	Uncertainty biological system modeling	1. Review three main methods of uncertainty biological system modeling using FPN 2. Propose a FCPN reasoning algorithm
Yang et al. (2018)	Molecular Biology	Build a novel model to represent the flow of genetic information involving transcription and translation in the DNA molecular biological system
Rosdi et al. (2019)	Speech impairment detection system	Implement FPN model to evaluate and classify the speech impairment
Liu et al. (2019)	Biological system	Develop a FCPN model to model and analyze a biological system

Table 9 Application of automatic control systems using FPN

Author(s)	Application field	Highlights
Weng et al. (2014)	Traffic Control system	Develop a parallel railroad level crossing model using TPN
Tian et al. (2014)	Traffic Control system	Propose a FHPN model to model and control traffic system
He et al. (2014a, b)	Environment Control System	Establish an airplane's fault diagnosis model using WFPN
Shen et al. (2015)	Fall detection system	Using HLFPN to analyze, identify for human fall actions
Lin and Li (2015)	System control	Develop an online-training algorithm based on DPFC
Hajduk and Wojtowicz (2016)	Traffic Control system	Propose an FPN formalism with Lukasiewicz norms based on FPGA
Letia and Kilyen (2016)	Hybrid Control System	Develop a Hybrid control system by combine PN with fuzzy logic
Bibi et al. (2017)	Uncertain nonlinear system	Propose a PT2FNN to model and analyze nonlinear system
Kim and Yang (2017)	Intelligent control system	Build a modeling and simulation of robot navigation algorithms using FPN
Hansen et al. (2018)	Intelligent control system	Utilize FPN to analyze parse pixel data of input images
Ammour et al. (2018)	Discrete event system	Build a system with sensors using SPN
Sun et al. (2018)	Intelligent control system	Propose an FFPN with shared control by AC and the BCI
Bibi et al. (2019)	Uncertain nonlinear system	Employ PT2FNN to control single input single output nonlinear systems
Le (2019)	Intelligent control system	Propose a SORIT2PFC method control the time-varying delay systems
Sha et al. (2019)	Intelligent control system	Propose a co-operator fault detection system based FPN
Bibi et al. (2020)	Uncertain nonlinear system	Develop a hybrid structure by combined PT2FNN

Table 10 Other typical applications using FPN

Author(s)	Application field	Highlights
Shen and Chen (2015)	Karaoke music system	Develop a music segmentation and karaoke music system method using HLFPN
Cheng et al. (2015)	Semantic web services system	Build a Horn clause set model using FPPN
Suraj (2015)	Rain traffic control system	Propose a modified GFPN to analyze rain traffic control system
Suraj et al. (2016)	Decision support system	Propose a FGFP-net model based on the GFPN
Xia et al. (2016)	Human machine system	Build a human machine system model using FIPN
Fat et al. (2016)	Stock market prediction system	Propose a FLETPN to model the stock market prediction system
Nie et al. (2017)	Education system	Build a personality learning model using FPN
Shen et al. (2017)	Social media	Propose an SBD hybrid method based on HLFPNs
Zhang et al. (2017a, b)	Human machine system	Develop a HM hybrid system model using FIPN
Shen et al. (2018)	Stock market prediction system	Build the financial investment systems using HLFPN
Yang et al. (2018)	Industrial process	Propose a novel DTFPN model to overcome the limitation of the TFPN field region
Yang et al. (2019)	Social media	Detect a threshold value in SBD using HLFPN
Zhang et al. (2019)	Behavior reconstruction system	Propose a random FPNs with time delay approach
Shen (2019)	Code Plagiarism Detection	Utilize a HLFPN based on AST
Cheng et al. (2019)	Semantic Web	Build a fuzzy STO model using FPN
Samantra et al. (2020)	Ad hoc network	Propose a modified FPN to simulate Ad hoc network
Shahidinejad et al. (2020)	Cloud computing	Utilizing a CPN to automatic manage cloud infrastructures

Other applications using FPNs can also be found in traffic control systems, artificial intelligence, web services, social media, system control, detection and identification, prediction, and classification.

7 Conclusion and potential future work

As a graphical, mathematical model tool, FPN provides a visual method for the reasoning process. Systematic summaries and analysis regarding the modeling and reasoning of a KBS using FPNs have been presented in this survey regarding the following aspects.

1. A series of comparisons between PN, EN_system, FPN, and HLFPNs are shown to reveal the various relationships and indicate that an FPN is a backward extension of HLPN from the EN_system model. Therefore, FPN can solve fuzzy information by using the inherent graphical description and parallel analysis ability of the PN model.
2. The correspondences between FPRs and FPNs have been discussed to explore why FPNs can fully represent the entire information of a large-scale KBS. Furthermore, the modeling techniques using FPNs have also been reviewed in detail. Specifically, the current research achievements around the decomposition algorithm of FPNs has also been mentioned.
3. A systematic recall of reasoning using PFN has been analyzed from two angles: inference by graphical method and inference algebraic form. Further, the advantages and disadvantages of these two mechanisms have been discussed.
4. Various industrial applications using FPNs from 2014 to 2020 have been reviewed and summarized to reveal the application trends of FPNs. These industrial applications using FPNs further indicate that various HLFPNs combined with different considerations have been used to implement modeling, diagnosis, assessment, and prediction from traditional manufacturing systems to recent biological systems.

Although FPNs have outstanding achievements to their credit over the last 30 years, as highlighted by the literature reviews, some possible directions for future research are:

1. The existing models using FPNs are supportive and robust enough for knowledge representation and reasoning. However, fuzzy reasoning algorithms are not suitable for parallel reasoning and they lack a learning mechanism. In other words, the inefficient self-adaptive capability of the FPN model is still a barrier to obtaining more accurate inference result(s). Therefore, how to enhance the self-learning/adaptive ability of a multioutput FPN is the first potential research hotspot.
2. Industrial applications of FPNs have received significant attention in the past few years, such as fault diagnosis, manufacturing systems, modeling systems with uncertainty, health care and biological systems, and risk assessment. However, with the rapid development of KBSs, current FPN technologies do not meet the requirements of some specific problems. Hence, how to propose HLFPNs by combining advanced techniques, such as artificial intelligence, machine learning, and deep learning, is the second potential research point.
3. The existing FPN models have limitations and are inapplicable to representation and management in the big data era. Therefore, continuously discovering and developing novel FPN models to handle the state-explosion issue in the big data era and real-time

applications in order to carry out large-scale and complex knowledge representations and reasoning is the third potential research domain.

Appendix: Various about FPN application field and highlight(s)

See Tables 5, 6, 7, 8, 9, 10.

Acknowledgements This work was supported by the National Natural Science Foundation of China (No. 62066016), the Natural Science Foundation of Hunan Province, China (Nos. 2020JJ5458 and 2019JJ40234), the Research Foundation of Education Bureau of Hunan Province, China (Nos. 18B317 and 19C1535), the Jishou University Graduate Research and Innovation Project (No. JDY19046), and the open project of the collaborative innovation center of industrial technology of manganese, zinc, vanadium (No. MXF201926).

References

Al-Ahmari A, Kaid H, Li Z et al (2020) Strict minimal siphon-based colored Petri net supervisor synthesis for automated manufacturing systems with unreliable resources. IEEE Access 8:22411–22424
Amin M, Shebl D (2014) Reasoning dynamic fuzzy systems based on adaptive fuzzy higher order Petri nets. Inf Sci 286:161–172
Amit K, Lakhmi J (2005) Distributed reasoning by fuzzy Petri nets: a review. In: Cognitive engineering advanced information and knowledge processing. Springer, London, pp 85–122
Ammour R, Leclercq E, Sanlaville E et al (2018) Faults prognosis using partially observed stochastic Petri-nets: an incremental approach. Discrete Event Dyn Syst 28:247–267
Awan MSK, Awais MM (2011) Predicting weather events using fuzzy rule-based system. J Appl Soft Comput 11(1):56–63
Balazinski M, Czogala E, Jemielniak K et al (2002) Tool condition monitoring using artificial intelligence methods. Eng Appl Artif Intell 15(1):73–80
Baruwa OT, Piera MA (2014) Anytime heuristic search for scheduling flexible manufacturing systems: a timed colored Petri net approach. Int J Adv Manuf Technol 75(1–4):123–137
Basak O, Albayrak YE (2015) Petri net-based decision system modeling in real-time scheduling and control of flexible automotive manufacturing systems. Comput Ind Eng 86:116–126
Berthomieu B, Diaz M (1991) Modeling and verification of time dependent systems using time Petri nets. IEEE Trans Softw Eng 17(3):259–273
Bharathi V (2014) A fuzzy Petri-net model for predicting the post-implementation risks of ERP in small and medium enterprises. Int Rev Compute Softw 9(11):1852–1860
Bharathi SV, Raman R, Pramod D (2014) A FPN based risk assessment model for ERP implementation in small and medium enterprises. Middle-East J Sci Res 19(6):747–759
Bharathi SV, Pramod D, Ramakrishnan R (2017) Risks assessment using fuzzy Petri nets for ERP extension in small and medium enterprises. Inf Resour Manage J 30(4):1–23
Bibi Y, Bouhali O, Bouktir T (2017) A novel Petri type 2 fuzzy neural networks approximator for adaptive control of uncertain nonlinear systems. IET Control Theory Appl 11(17):3130–3136
Bibi Y, Bouhali O, Bouktir T (2019) Robust direct adaptive controller for a class of uncertain nonlinear systems using petri type 2 fuzzy neural networks (PT2FNN) as a new approximator. J Electr Syst 15(2):181–196
Bibi Y, Seghiri M, Bouhali O et al (2020) Petri type 2 fuzzy neural networks (PT2FNN) for identification and control of dynamic systems—a new structure and a comparative study. In: Proceedings of the 4th international conference on electrical engineering and control applications constantine, Algeria, pp 319–330
Camilli M (2012) Petri nets state space analysis in the cloud. In: International conference on software engineering, pp 1638–1640
Cardoso J, Valette R, Dubois D (1989) Petri nets with uncertain markings. In: Proceedings of the 10th international conference on applications and theory of Petri nets, pp 64–78
Chandrasekaran B (1988) Generic tasks as building blocks for knowledge-based systems: the diagnosis and routine design examples. Knowl Eng Rev 3(3):183–210

Chang YJ, Wang XF, Chen GM et al (2018) Comprehensive risk assessment of deepwater drilling riser using fuzzy Petri net model. Process Saf Environ Prot 117:483–497
Chaplain MAJ (2011) Multiscale mathematical modelling in biology and medicine. IMA J Appl Math 76(3):371–388
Chen SM (1988) A new approach to handling fuzzy decision-making problems. IEEE Trans Syst Man Cybern 18(6):1012–1016
Chen SM (1996) A fuzzy reasoning approach for rule-based systems based on fuzzy logics. IEEE Trans Syst Man Cybern B 26(5):769–778
Chen SM (2000) Fuzzy backward reasoning using fuzzy Petri nets. IEEE Trans Cybern B 30(6):846–856
Chen SM (2002) Weighted fuzzy reasoning using weighted fuzzy Petri nets. IEEE Trans Knowl Data Eng 14(2):386–397
Chen CH, Yang SY (2013) A knowledge-based cooperative differential evolution for neural fuzzy inference systems. Soft Comput 17(5):883–895
Chen SM, Ke JS, Chang JF (1990) Knowledge representation using fuzzy Petri nets. IEEE Trans Knowl Data Eng 2(3):311–319
Chen YF, Li ZW, Barkaoui K et al (2014) New Petri net structure and its application to optimal supervisory control: interval inhibitor arcs. IEEE Trans Syst Man Cybern 44(10):1384–1400
Chen ZY, Xu B, Lu N et al (2019) A fault diagnosis method for flexible manufacturing tool system based on fuzzy Petri net. In: International conference on intelligent informatics and biomedical sciences, pp 374–379
Cheng YH, Yang LA (2009) A fuzzy Petri nets approach for railway traffic control in case of abnormality: evidence from Taiwan railway system. Expert Syst Appl 36(4):8040–8048
Cheng J, Liu C, Zhou MC et al (2015) Automatic composition of semantic web services based on fuzzy predicate Petri nets. IEEE Trans Autom Sci Eng 12(2):680–689
Cheng X, Lin X, Zhu C et al (2017) Power system fault analysis based on hierarchical fuzzy petri net considering time association character. Trans China Electrotech Soc 32(14):229–237
Cheng H, Yan L, Ma ZM et al (2019) Fuzzy spatio-temporal ontologies and formal construction based on fuzzy Petri nets. Comput Intell 35(1):204–239
Chiang HS (2015) ECG-based mental stress assessment using fuzzy computing and associative petri net. Med Biol Eng 35(6):833–844
Chiang HS, Pao SC (2016) An EEG-based fuzzy probability model for early diagnosis of Alzheimer's disease. Environ Syst Decis 40(5):125
Chiang TC, Huang AC, Fu LC (2006) Modeling, scheduling, and performance evaluation for wafer fabrication: a queueing-colored Petri-net and GA-based approach. IEEE Trans Autom Sci Eng 3(3):330–338
Chiang HS, Chen MY, Wu ZM (2018) Applying fuzzy petri nets for evaluating the impact of bedtime behaviors on sleep quality. Granul Comput 3:321–332
Chun MG, Bien Z (1993) Fuzzy Petri net representation and reasoning methods for rule-based decision-making systems. IEICE Trans Fundam Electron Commun Comput Sci 76(6):974–983
Clarke EM, Grumberg O (1987) Avoiding the state explosion problem in temporal logic model checking. Proc ACM Symp Princ Distrib Comput 68(3):163–172
Clarke EM, Grumberg O, Jha S et al (2001) Progress on the state explosion problem in model checking algorithms. Springer, Berlin, pp 176–194
Demri S, Laroussinie F, Schnoebelen P (2006) A parametric analysis of the state-explosion problem in model checking. J Comput Syst Sci 72(4):547–575
Fat R, Mic L, Kilyen AO et al (2016) Model and method for the stock market forecast. In: IEEE international conference on automation, quality and testing, robotics, pp 1–5
Fay A (2000) A fuzzy knowledge-based system for railway traffic control. Eng Appl Artif Intell 13(6):719–729
Fei ZC, Li SQ, Chang Q et al (2018) Fuzzy petri net based intelligent machine operation of energy efficient manufacturing system. In: Proceedings of the 14th IEEE conference on automation science and engineering, pp 1593–1598
Frank PM (1990) Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy: a survey and some new results. Automatica 26(3):459–474
Fryc B, Pancerz K, Peters JF et al (2004) On fuzzy reasoning using matrix representation of extended fuzzy Petri nets. Fundam Inform 60(1–4):143–157
Gao MM, Zhou MC, Huang XG et al (2003) Fuzzy reasoning Petri nets. IEEE Trans Syst Man Cybern A 33(3):314–324
Garcia-Crespo A, Lopez-Cuadrado JL, Colomo-Palacios R et al (2011) Sem-Fit: a semantic based expert system to provide recommendations in the tourism domain. Expert Syst Appl Int J 38(10):13310–13319

Garg ML, Ahson SI, Gupta PV (1991) A fuzzy Petri net for knowledge representation and reasoning. Inf Process Lett 39(3):165–171

Godefroid P, van Leeuwen J, Hartmanis J et al (1996) Partial-order methods for the verification of concurrent systems: an approach to the state-explosion problem. Springer, Berlin

Graubmann P (1987) The construction of EN systems from a given trace behavior. In: Proceedings of the 8th European workshop and theory of Petri nets, pp 133–153

Guo Y, Meng X, Wang D et al (2016) Comprehensive risk evaluation of long-distance oil and gas transportation pipelines using a fuzzy Petri net model. J Nat Gas Sci Eng 33:18–29

Ha MH, Li Y, Wang XF (2007) Fuzzy knowledge representation and reasoning using a generalized fuzzy petri net and a similarity measure. Soft Comput 11(4):323–327

Hajduk Z, Wojtowicz J (2016) Hardware implementation of fuzzy Petri nets with Lukasiewicz norms for modelling of control systems. Asian conference on intelligent information and database systems. Springer, Berlin, pp 449–458

Hamed RI (2015) Esophageal cancer prediction based on qualitative features using adaptive fuzzy reasoning method. J King Saud Univ Comput Inf Sci Arch 27(2):129–139

Hamed RI (2017) Quantitative modelling of gene networks of biological systems using fuzzy Petri nets and fuzzy sets. J King Saud Univ Sci 30(1):112–119

Handelman DA, Lane SH, Gelfand JJ (1990) Integrating neural networks and knowledge-based systems for intelligent robotic control. IEEE Control Syst Mag 10(3):77–87

Hansen P, Franco P, Kim SY (2018) Soccer ball recognition and distance prediction using fuzzy Petri nets. In: Proceedings of 2018 IEEE international conference on information reuse and integration, pp 315–322

He C, Wang L, Yan J et al (2014a) Environment control system fault diagnosis expert system. In: Proceedings of the first symposium on aviation maintenance and management-volume, Springer, Berlin, pp 453–462

He ZY, Yang JW, Zeng QF et al (2014b) Fault section estimation for power systems based on adaptive fuzzy Petri nets. Int J Comput Intell Syst 7(6):605–614

Heiner M (2011) Preface: Petri nets for systems and synthetic biology. Nat Comput 10(3):987–992

Hu H, Li ZW, Al-ahmari A (2011) Reversed fuzzy Petri nets and their application for fault diagnosis. Comput Ind Eng 60(4):505–510

Jiang W, Zhou KQ, Mo LP (2019) Parameter optimization strategy of fuzzy petri net utilizing hybrid GA-SFLA algorithm. Simulation tools and techniques. Springer, Cham, pp 416–426

Jiang T, Du C, Guo S et al (2020) Microgrid fault diagnosis model based on weighted fuzzy neural Petri net. In: Proceedings of the IEEE 4th information technology, networking, electronic and automation control conference, pp 2361–2365

Jorge CAF, Perkusich A, Chang BSK (1996) Faults and timing analysis in real-time distributed systems: a fuzzy time Petri-net-based approach. Fuzzy Sets Syst 83(2):143–168

Kahloul L, Bourekkache S, Djouani K (2016) Designing reconfigurable manufacturing systems using reconfigurable object Petri nets. Int J Comput Integr Manuf, pp 1–18

Kaid H, Al-Ahmari A, Nasr EA et al (2020) Petri net model based on neural network for deadlock control and fault detection and treatment in automated manufacturing systems. IEEE Access 8:103219–103235

Kasirolvalad Z, Motlagh MRJ, Shadmani MA (2004) An intelligent modular modelling approach for quality control of CNC machines product using adaptive fuzzy Petri nets. In: IEEE control, automation, robotics and vision conference control, pp 1342–1347

Kim SY, Yang YL (2017) A self-navigating robot using fuzzy Petri nets. Robot Auton Syst 101(2):153–165

Koriem SM (2000) A fuzzy Petri net tool for modeling and verification of knowledge-based systems. Comput J 14(3):206–223

Kuchárik M, Balogh Z (2016) Evaluation of fuzzy Petri nets with the tool trans place sim. In: Proceedings of the IEEE international conference on application of information and communication technologies, pp 1–5

Le TL (2019) Self-organizing recurrent interval type-2 Petri fuzzy design for time-varying delay systems. IEEE Access 7:10505–10514

Lee KH, Favre J (1985) Hierarchical reduction method for analysis and decomposition of Petri nets. IEEE Trans Syst Man Cybern 15(2):272–280

Lee SJ, Seong PH (2004) Development of automated operating procedure system using fuzzy colored petri nets for nuclear power plants. Ann Nucl Energy 31(8):849–869

Lee J, Liu K, Chiang W (1998) Fuzzy Petri nets for modeling rule-based reasoning. Int J Artif Intell Tools 7(4):463–485

Lee YK, Lee JH, Lee SK et al (2006) Nano manufacturing using fountain pen nano-lithography with active membrane pumping. Key Eng Mater 326–328:425–428
Letia TS, Kilyen AO (2016) Evolutionary synthesis of hybrid controllers. In: Proceedings of the IEEE international conference on intelligent computer communication and processing, pp 133–140
Li X, Lara-Rosano F (2000) Adaptive fuzzy petri nets for dynamic knowledge representation and inference. Expert Syst Appl 19(3):235–241
Li X, Yu W, Lara-Rosano F (2000) Dynamic knowledge inference and learning under adaptive fuzzy Petri net framework. IEEE Trans Syst Man Cybern C 30(4):442–450
Li LL, Li CB, Tang Y (2014) A color petri net-based scheduling model for remanufacturing system with stochastic process routing. In: IEEE international conference on automation science and engineering, pp 474–479
Li H, You JX, Liu HC et al (2018a) Acquiring and sharing tacit knowledge based on interval 2-tuple linguistic assessments and extended fuzzy petri nets. Int J Uncertain Fuzz 26(1):43–65
Li J, Zhu XL, Cheng XZ (2018b) Sensor fault diagnosis based on fuzzy neural Petri net. Complexity. https://doi.org/10.1155/2018/8261549
Li WJ, He M, Sun YB et al (2019) A novel layered fuzzy Petri nets modelling and reasoning method for process equipment failure risk assessment. J Loss Prev Proc 62:103953
Lin CP, Hwang CN (1996) Modeling and verification of fuzzy knowledge base with fuzzy colored Petri nets. In: Proceedings of the IEEE international conference on systems man and cybern, pp 586–590
Lin CM, Li HY (2015) Dynamic petri fuzzy cerebellar model articulation controller design for a magnetic levitation system and a two-axis linear piezoelectric ceramic motor drive system. IEEE Trans Control Syst Technol 23(2):693–699
Lipp HP (1983) The application of a fuzzy Petri net for controlling complex industrial processes. In: Proceedings of the IFAC conference on fuzzy information control, pp 471–477
Liu ZJ, Li HG, Zhou PJ (2011) Towards timed fuzzy Petri net algorithms for chemical abnormality monitoring. Expert Syst Appl 38(8):9724–9728
Liu HC, Lin QL, Mao LX et al (2013a) Dynamic adaptive fuzzy petri nets for knowledge representation and reasoning. IEEE Trans Syst Man Cybern Syst 43(6):1399–1410
Liu HC, Lin QL, Ren ML et al (2013b) Fault diagnosis and cause analysis using fuzzy evidential reasoning approach and dynamic adaptive fuzzy Petri nets. Comput Ind Eng 66(4):899–908
Liu HC, Liu L, Lin QL et al (2013c) Knowledge acquisition and representation using fuzzy evidential reasoning and dynamic adaptive fuzzy petri nets. IEEE Trans Cybern 43(3):1059–1072
Liu HC, You JX, You XY et al (2015a) Fuzzy Petri nets using intuitionistic fuzzy sets and ordered weighted averaging operators. IEEE Trans Cybern 46(8):1839–1850
Liu HC, You JX, You XY et al (2015b) Linguistic reasoning Petri nets for knowledge representation and reasoning. IEEE Trans Syst Man Cybern: Syst 46(4):499–511
Liu XR, Gao YW, Wang ZL (2016) Method of power distribution network fault diagnosis based on improved time fuzzy petri net. J Northeast Univ 37(2):1526–1529
Liu HC, Xue L, Li ZW et al (2017a) Linguistic Petri nets based on cloud model theory for knowledge representation and reasoning. IEEE Trans Knowl Data Eng 30(4):717–728
Liu HC, You JX, Li ZW et al (2017b) Fuzzy Petri nets for knowledge representation and reasoning: a literature review. Eng Appl Artif Intell 60(4):45–56
Liu F, Chen SY, Heiner M et al (2018a) Modeling biological systems with uncertain kinetic data using fuzzy continuous Petri nets. BMC Syst Biol 12(4):42
Liu JF, Zhou ZH, Wang ZS (2018b) Online fault diagnosis in discrete event systems with partially observed Petri nets. Int J Control Autom Syst 16(4):217–224
Liu F, Sun W, Heiner M et al (2019) Hybrid modelling of biological systems using fuzzy continuous Petri nets. Brief Bioinform 22(1):438–450
Looney CG (1988) Fuzzy Petri nets for rule-based decision making. IEEE Trans Syst Man Cybern 18(1):178–183
Majma N, Babamir SM, Monadjemi A (2017) Runtime verification of pacemaker functionality using hierarchical fuzzy colored Petri-nets. J Med Syst 41(2):1–21
Manoj TV, Leena J, Soney RB (1998) Knowledge representation using fuzzy Petri nets-revisited. IEEE Trans Knowl Data Eng 10(4):666–667
Meng DX, Liu SF, Han L (2015) The application of local adaptive fuzzy Petri net in result prediction of multiple-factor issues. In: International conference on human centered computing, pp 512–521
Meng FX, Lei YJ, Zhang B et al (2016) Intuitionistic fuzzy Petri nets for knowledge representation and reasoning. J Digit Inform ManagE 14:104–113
Meng FX, Lei YJ, Lei Y et al (2017) Hybrid reasoning using intuitionistic fuzzy Petri nets. Acta Electron Sin 45(8):1937–1946

Mhalla A, Benrejeb M (2015) A monitoring-maintenance approach based on fuzzy petri nets in manufacturing systems with time constraints. In: Azar A, Vaidyanathan S (eds) Computational intelligence applications in modeling and control. Springer, Cham, pp 205–228
Mhalla A, Jenhani O, Dutilleul SC (2013) Contribution to the monitoring of manufacturing systems with time constraints: application to a surface treatment line. In: Proceedings of the 14th international conference on sciences and techniques of automatic control and computer engineering, pp 243–250
Milinković S, Marković M, Vesković S et al (2013) A fuzzy Petri net model to estimate train delays. Simul Model Pract Theory 33:144–157
Mockler RJ, Dologite DG (1992) Knowledge-based systems to support strategic planning decision making. In: Proceedings of the 24th annual Hawaii international conference on system sciences, pp 173–180
Molloy MK (1982) Performance analysis using stochastic Petri nets. IEEE Trans Comput 31(9):913–917
Murata T (1989) Petri nets: properties, analysis and applications. Proc IEEE 77(4):541–580
Nabi HZ, Aized T (2020) Performance evaluation of a carousel configured multiple products flexible manufacturing system using Petri net. Oper Manage Res 13:109–129
Narahari Y, Viswanadham N (1985) A Petri net approach to the modelling and analysis of flexible manufacturing systems. Ann Oper Res 3(8):449–472
Negoita CV (1985) Expert systems and fuzzy systems. Proc IEEE 75(10):1438–1439
Nie Y, Su GP, Dai JY et al (2017) Research on the application of fuzzy Petri net in personalized education. In: Proceedings of the 2017 2nd international conference on materials science, machinery and energy engineering, pp 385–390
Nishi T, Matsumoto I (2015) Petri net decomposition approach to deadlock-free and non-cyclic scheduling of dual-armed cluster tools. IEEE Trans Autom Sci Eng 12(1):281–294
Novák V, Lehmke S (2006) Logical structure of fuzzy IF-THEN rules. Fuzzy Sets Syst 157(15):2003–2029
Paredes-Frigolett H, Flavio AMGL (2016) A novel method for rule extraction in a knowledge-based innovation tutoring system. Knowl-Based Syst 92(15):183–199
Pedrycz W, Camargo H (2003) Fuzzy timed Petri nets. Fuzzy Sets Syst 140(2):301–330
Pedrycz W, Gomide F (1994) A generalized fuzzy Petri net model. IEEE Trans Fuzzy Syst 2(4):295–301
Pelz E (1989) About the concurrent behaviour of EN systems: definability and closure results. In: Proceedings of an advanced course Bad Honnef, pp 354–373
Pennisi M, Cavalieri S, Motta S et al (2016) A methodological approach for using high-level Petri nets to model the immune system response. BMC Bioinform 17(498):91–105
Petri CA, Reisig W (2008) Petri Net. Scholarpedia 3(1):133–136
Pramod D, Bh Arathi S, Raman R (2014) A fuzzy petri-net model for predicting the post-implementation risks of ERP in small and medium enterprises. Int Rev Comput Softw 9(11):1852–1860
Ribaric S, Basic BD (1998) Fuzzy time Petri net primitives for processing fuzzy temporal knowledge. In: Proceedings of 9th Mediterranean electrotechnical conference, pp 549–553
Rosdi F, Salim SS, Mustafa MB (2019) An FPN-based classification method for speech intelligibility detection of children with speech impairments. Soft Comput 23(1):2391–2408
Rozenberg G (1987) Behaviour of elementary net systems. Adv Course Petri Nets 254:60–94
Sachan S, Donchak N (2020) Generalized stochastic Petri-net algorithm with fuzzy parameters to evaluate infrastructure asset management policy. In: Proceedings of 2020 IEEE international conference on fuzzy systems, pp 1–8
Samantra A, Panda A, Das SK et al (2020) Fuzzy Petri nets-based intelligent routing protocol for ad hoc network. Design frameworks for wireless networks. Springer, Singapore, pp 417–433
Saren SK, Blaga F, Dzitac S et al (2017) Decision based modeling of a flexible manufacturing cell based on hierarchical timed colored Petri nets. Proced Comput Sci 122:253–260
Saren SK, Blaga F, Vesselenyi T (2018) Implementation of fuzzy system using hierarchical colored Petri nets to model flexible manufacturing cell. IOP Conf Ser Mater Sci Eng 400(4):42–50
Scarpelli H, Gomide F (1993) Fuzzy reasoning and fuzzy Petri nets in manufacturing systems modeling. J Intell Fuzzy Syst 1(3):225–241
Scarpelli H, Gomide F, Yager RR (1996) A reasoning algorithm for high-level fuzzy petri nets. IEEE Trans Fuzzy Syst 4(3):282–294
Sha W, Li W, Jiao G et al (2019) A co-operative fault detection system with multiple detectors for smart factory based on fuzzy Petri net. In: Proceedings of 2019 international conference on artificial intelligence and advanced manufacturing, pp 778–783
Shafiekhani S, Rahbar S, Akbarian F et al (2018) Fuzzy stochastic Petri net with uncertain kinetic parameters for modeling tumor-immune system. In: Proceedings of the 25th national and 3rd international Iranian conference on biomedical engineering, pp 1–5
Shah SA, Bohez ELJ, Shah K et al (2017) Colored Petri net model for significant reduction of invariants in flexible manufacturing systems. Int J Adv Manuf Technol 88:1775–1787

Shahidinejad A, Ghobaei-Arani M, Esmaeili L (2020) An elastic controller using colored Petri nets in cloud computing environment. Cluster Comput 23:1045–1071
Shen VRL (2006) Knowledge representation using high-level fuzzy Petri nets. IEEE Trans Syst Man Cybern A 36(6):1220–1227
Shen VRL (2019) Novel code plagiarism detection based on abstract syntax tree and fuzzy Petri nets. Int J Eng Educ 1(1):46–56
Shen VRL, Chen HC (2015) An automatic calibration system for Chinese karaoke lyrics based on high-level fuzzy Petri nets. In: IEEE international conference on machine learning and cybernetics, pp 544–549
Shen XY, Lei YJ, Li CH (2009) Intuitionistic fuzzy Petri nets model and reasoning algorithm. In: Proceedings of the 6th international conference on fuzzy systems and knowledge discovery, pp 119–122
Shen VRL, Chang YS, Juang TTY (2010) Supervised and unsupervised learning by using Petri nets. IEEE Trans Syst Man Cybern A 40(2):363–375
Shen VRL, Lai HY, Lai AF (2015) The implementation of a smartphone-based fall detection system using a high-level fuzzy Petri net. Appl Soft Comput 26:390–400
Shen RK, Lin YN, Shen VRL et al (2017) Automatic detection of video shot boundary in social media using a hybrid approach of HLFPN and key point matching. IEEE Trans Comput Soc Syst 5(1):210–219
Shen RK, Yang CY, Shen VRL et al (2018) A stock market prediction system based on high-level fuzzy Petri nets. Int J Uncertain Fuzziness Knowl-Based Syst 26(5):771–808
Shi H, Wang L, Li XY et al (2019) A novel method for failure mode and effects analysis using fuzzy evidential reasoning and fuzzy Petri nets. J Ambient Intell Hum Comput 11:2381–2395
Shojafar M, Pooranian Z, Meybodi MR (2015) ALATO: an efficient intelligent algorithm for time optimization in an economic grid based on adaptive stochastic Petri net. J Intell Manuf 26:641–658
Slobodan R, Dalbelo BB, Pavešić N (1999) A model for fuzzy temporal knowledge representation and reasoning. In: Proceedings of the IEEE international fuzzy systems, pp 216–221
Song S, Lee S (2002) A Strategy of dynamic reasoning in knowledge-based system with fuzzy production rules. J Intell Inf Syst 19(3):303–318
Sun YB, Chen YP, Zhang B et al (2015) Fault diagnosis for power system using time sequence fuzzy Petri net. In: Proceedings of the 3rd international conference on mechanical engineering and intelligent systems, pp 729–735
Sun FF, Cheng YC, Cheng CH et al (2017) Intelligent evaluation of fault status in power transformer based on fuzzy fault Petri net. In: Proceedings of the IEEE electrical insulation conference, pp 104–107
Sun FC, Zhang WC, Chen JH et al (2018) Fused fuzzy Petri nets: a shared control method for brain computer interface systems. IEEE Trans Cogn Dev Syst 2(11):188–199
Suraj Z (2012a) Knowledge representation and reasoning based on generalized fuzzy Petri nets. In: Proceedings of the 12th IEEE international conference on intelligent systems design and applications, pp 101–106
Suraj Z (2012b) Parameterised fuzzy Petri nets for approximate reasoning in decision support systems. In: Proceedings of the international conference on advanced machine learning technologies and applications, pp 33–42
Suraj Z (2013) Matrix representation of parameterised fuzzy Petri nets. In: Proceedings of the international workshop on rough sets fuzzy sets data mining and granular-soft computing, pp 200–207
Suraj Z (2015) Modified generalised fuzzy petri nets for rule-based systems. In: Proceedings of the international workshop on rough sets, fuzzy sets, data mining, and granular computing, pp 196–206
Suraj Z, Grochowalski P, Bandyopadhyay S (2016) Flexible generalized fuzzy petri nets for rule-based systems. In: Proceedings of the international conference on theory and practice of natural computing, pp 196–207
Tan MY, Li JM, Chen XQ et al (2019) Power grid fault diagnosis method using intuitionistic fuzzy Petri nets based on time series matching. Complexity 4:1–14
Tao XH, Miao Y, Zhang YC et al (2014) Collaborative medical diagnosis through fuzzy Petri net based agent argumentation. In: Proceedings of the IEEE international conference on fuzzy systems, pp 1197–1204
Thiagarajan PS (1986) Elementary net systems. In: Proceedings of the advanced course on petri nets: central models and their properties, advances in Petri nets, pp 26–59
Tian Z, Zhang Z, Dong H et al (2014) Real-time control for traffic signal based on fuzzy hybrid petri net. J Comput Inf Syst 10(5):2079–2090
Ting Y, Lu WB, Chen CH et al (2008) A fuzzy reasoning design for fault detection and diagnosis of a computer-controlled system. Eng Appl Artif Intell 21(2):157–170
Tsang ECC, Yeung DS, John WTL (1999) Learning capability in fuzzy Petri nets. In: Proceedings of the IEEE conference on systems man and cybernetics, pp 355–360

Vahidipour SM, Meybodi MR, Esnaashari M (2015) Learning automata-based adaptive Petri net and its application to priority assignment in queuing systems with unknown parameters. IEEE Trans Syst Man Cybern Syst 45(10):1373–1384

Valavanis KP, Kokkinaki AI, Tzafestas SG (1994) Knowledge-based systems in engineering applications: a survey. J Intell Robot Syst 10:113–145

Valmari A (1998) The state explosion problem. In: Reisig W, Rozenberg G (eds) Lectures on petri Nets I: basic models. Springer, Berlin, pp 429–528

Wai RJ, Lin YW (2013) Adaptive moving-target tracking control of a vision-based mobile robot via a dynamic Petri recurrent fuzzy neural network. IEEE Trans Fuzzy Syst 21(4):688–701

Wai RJ, Liu CM (2009) Design of dynamic petri recurrent fuzzy neural network and its application to path-tracking control of nonholonomic mobile robot. IEEE Trans Ind Electron 56(7):2667–2683

Wang WM, Peng X, Zhu GN et al (2014a) Dynamic representation of fuzzy knowledge based on fuzzy petri net and genetic-particle swarm optimization. Expert Syst Appl 41(4):1369–1376

Wang YN, Ye JF, Xu GJ et al (2014b) Novel hierarchical fault diagnosis approach for smart power grid with information fusion of multi-data resources based on fuzzy petri net. In: Proceedings of the IEEE international conference on fuzzy systems, pp 1183–1189

Wang L, Chen Q, Gao ZJ et al (2015) A new fuzzy Petri net model for power grid fault diagnosis. Appl Mech Mater 700:61–66

Wang YH, Lu YJ, Li M et al (2016) Fault identification method based on fuzzy fault Petri net. In: Proceedings of the international conference on electrical and information technologies for rail transportation, pp 125–133

Wang JF, Fei ZC, Chang Q et al (2019) Energy saving operation of manufacturing system based on dynamic adaptive fuzzy reasoning Petri net. Energies 12(11):2216

Wang C, Li J, Zhu X et al (2020) Adaptive neural fuzzy Petri net algorithm for motor fault diagnosis. IOP Conf Ser: Earth Environ Sci. https://doi.org/10.1088/1755-1315/446/4/042063

Weng YS, Huang YS, Pan YL et al (2014) Design of traffic safety control systems for railroads and roadways using timed Petri nets. Asian J Control 17(2):626–635

Wu F (1999) Fuzzy time semirings and fuzzy-timing colored Petri nets. Int J Intell Syst 14(8):747–774

Wu W, Wen F, Xue Y et al (2013) Power system fault diagnosis with a weighted fuzzy time Petri net. In: International conference on sustainable power generation and supply, pp 1–7

Xia JJ, Zhang JH, Wang RB (2016) Modeling of adaptive human–machine systems based on fuzzy inference Petri nets. In: Proceedings of the 5th international conference on cognitive neuro dynamics, pp 493–499

Xu B, Yin X, Yin X et al (2019a) Fault diagnosis of power systems based on temporal constrained fuzzy Petri nets. IEEE Access 7:101895–101904

Xu XG, Shi HX, Xu DH et al (2019b) Picture fuzzy Petri nets for knowledge representation and acquisition in considering conflicting opinions. Appl Sci 9(5):12–21

Yang R, Leung WS, Heng PA (2002) Improved algorithm on rule-based reasoning systems modeled by fuzzy Petri nets. IEEE International Conference on Fuzzy Systems, pp 1204–1209

Yang B, Li H (2018) A novel dynamic timed fuzzy Petri nets modeling method with applications to industrial processes. Expert Syst Appl 97(12):276–289

Yang JL, Pu HT, Lian J et al (2018) Modeling and analysis of protein synthesis and DNA mutation using colored Petri nets. IEEE Access 6(2):22386–22400

Yang SH, Lin YN, Chiou GJ et al (2019) novel shot boundary detection in news streams based on fuzzy Petri nets. Appl Artif Intell 33(12):1035–1057

Yao DK, Wang QH, Gan XS (2018) Safety risk assessment on the air traffic control via the improved fuzzy Petri net. J Saf Environ 18:413–417

Yeung DS, Tsang EC (1994a) Fuzzy knowledge representation and reasoning using Petri nets. Expert Syst Appl 7(2):81–289

Yeung DS, Tsang ECC (1994b) Improved fuzzy knowledge representation and rule evaluation using fuzzy petri nets and degree of subset hood. Int J Intell Syst 9(12):1083–1100

Yeung DS, Tsang ECC (1998) A multilevel weighted fuzzy reasoning algorithm for expert systems. IEEE Trans Syst Man Cybern A 28(2):149–158

Yu ZH, Guo F, Ouyang J et al (2016) Object-oriented Petri nets and -calculus-based modeling and analysis of reconfigurable manufacturing systems. Adv Mech Eng 8(11):456–480

Yue WC, Gui WH, Chen XF et al (2019) Knowledge representation and reasoning using self-learning interval type-2 fuzzy Petri nets and extended TOPSIS. Int J Mach Learn Cyber 10:3499–3520

Zaitsev DA (2004) Decomposition of Petri nets. Cybern Syst Anal 40(5):739–746

Zakharov S, Taganov A, Gusev S et al (2018) The analysis and monitoring of ecological risks on the basis of fuzzy Petri nets. In: Proceedings of the 3rd Russian-pacific conference on computer technology and applications, pp 1–5

Zanni C, Goc ML, Frydman CS (2006) A conceptual framework for the analysis, classification and choice of knowledge-based diagnosis systems. Int J Knowl-Based Intell Eng Syst 10(2):113–138

Zhang XJ, Yao SZ (2015) Fuzzy stochastic Petri nets and analysis of the reliability of multi-state systems. Software IET 9(3):83–93

Zhang Y, Liang FC, Su F et al (1991) A fuzzy production rule based expert system. Fuzzy Sets Syst 44(3):391–403

Zhang WH, Yuan JS, Zhang K et al (2014) A reasoning method of transformer fault causes based on fuzzy Petri Net. Appl Mech Mater 631–632:537–542

Zhang Y, Zhang Y, Wen FS et al (2016) A fuzzy Petri net-based approach for fault diagnosis in power systems considering temporal constraints. Int J Electr Power Energy Syst 78(6):215–224

Zhang JF, Reniers G, Zhang L (2017a) A weighted fuzzy Petri-net based approach for security risk assessment in the chemical industry. Chem Eng Sci 174:136–145

Zhang JH, Xia JJ, Garibaldi JM et al (2017b) Modeling and control of operator functional state in a unified framework of fuzzy inference petri nets. Comput Methods Programs Biomed 144:147–163

Zhang X, Yue S, Zha XB (2018) Method of power grid fault diagnosis using intuitionistic fuzzy Petri nets. IET Gener Transm Distrib 12(2):295–302

Zhang Z, Ge L, Wang P et al (2019) Behavior reconstruction models for large-scale network service systems. Peer-to-Peer Netw Appl 12:502–513

Zhang C, Tian G, Fathollahi-Fard AM et al (2020) Interval-valued Intuitionistic uncertain linguistic cloud petri net and its application to risk assessment for subway fire accident. IEEE Trans Autom Scie Eng 44(9):1–15

Zhou J, Reniers G (2020) Modeling and application of risk assessment considering veto factors using fuzzy Petri nets. J Loss Prev Process Ind 67:104216

Zhou KQ, Zain AM (2016) Fuzzy Petri nets and industrial applications: a review. Artif Intell Rev 45(4):405–446

Zhou KQ, Zain AM, Mo LP (2015a) Dynamic properties of fuzzy Petri net model and related analysis. J Central South Univ 22(12):4717–4723

Zhou KQ, Zain AM, Mo LP (2015b) A decomposition algorithm of fuzzy Petri net using an index function and incidence matrix. Expert Syst Appl 42(8):3980–3990

Zhou KQ, Gui WH, Mo LP et al (2015c) A bidirectional diagnosis algorithm of fuzzy Petri net using inner-reasoning-path. Expert Syst Appl 42(8):3980–3990

Zhou J, Reniers G, Zhang L (2017) A weighted fuzzy Petri-net based approach for security risk assessment in the chemical industry. Chem Eng Sci 174:136–145

Zhou KQ, Mo LP, Ding L et al (2018) An automatic algorithm to generate a reachability tree for large-scale fuzzy Petri net by and/or graph. Symmetry 10(10):454

Zhou KQ, Mo LP, Jin J et al (2019) An equivalent generating algorithm to model fuzzy Petri net for knowledge-based system. J Intell Manuf 30(4):1831–1842

Zurawski R, Zhou MC (1994) Petri nets and industrial applications: a tutorial. IEEE Trans Ind Electron 41(6):567–583