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D-Brane Inflation with Perturbative Moduli Stabilisation

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Abstract

Cosmological inflation is a period of accelerated expansion of the early universe. Nowadays there are plenty of inflationary models which agree with experimental constraints but a complete microscopic understanding of this quasi de Sitter phase is still missing.
In this thesis, after reviewing the present situation in Cosmology and Supersymmetry, we present the main possibilities to embed inflation in a UV complete framework coming from String Theory. In particular, we present the main virtues and limitations of RG-induced modulus stabilisation, focusing on $\mathrm{D3}-\overline{\mathrm{D3}}$ inflation and discussing whether slow-roll can be obtained in the regime of validity of the effective field theory.

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Chapter 1

Introduction

The universe is expanding. And it is expanding at an accelerating pace, as it had done once before in its earliest of stages.
Since Einstein published his theory of General Relativity (GR), lots of attempts have been made in order to describe our universe as a whole via the field equations which relate the curvature of spacetime to the content of matter and energy. Einstein himself tried and, in order to find a static solution, introduced for the first time the cosmological constant (Λ) term ad hoc (what he later defined as his greatest mistake). Later on, Hubble discovered experimentally that our universe is not static but is instead expanding exponentially with a positive acceleration. This process is well described at a classical level by the de Sitter solution to Einstein equations: $\Lambda > 0$ is a constant and represents physically what is called **dark energy**. From a microscopic point of view there are lots of candidates for dark energy and there is no general consensus among the community since a theory of **quantum gravity** is needed to properly address the problem. Nonetheless, string theory, being the only consistent theory able to describe gravity at a quantum level, allows to study and better understand certain issues.
Therefore, we now have evidence that our universe is undergoing a phase of accelerated expansion. What is interesting is that something very similar happened for a very short period ($\Delta t \sim 10^{-38} - 10^{-37}$ s) of the early universe. This period is called **cosmological inflation** and is the main topic of this thesis.
From Einstein equations, we know that the universe contracts going back in time, until it reaches zero size and both space and time emerge at the singularity of the standard **Big Bang**. Of course, this singularity does not have a physical meaning and we expect that an appropriate UV completion of General Relativity (e.g. via string theory) would cure this meaningless result. Moreover, a simple extrapolation of Einstein equations implies other important experimental and conceptual problems that must be addressed (e.g. the horizon problem). Inflation was introduced in order to solve these issues in the simplest and most elegant possible way. It is in great agreement with observational results and is able to make highly non trivial predictions (e.g. anisotropies in the power

spectrum of the Cosmic Microwave Background radiation). Nevertheless, we should say that inflationary models do not have an appropriate UV embedding and a microscopical description (situation analogous to the case of dark energy: on the one hand it is simpler to construct a model for the latter case since for inflation ~ 60 e-foldings are necessary, while only around a single e-folding of accelerated expansion is needed for dark energy; on the other, more experimental constraints have to be satisfied to appropriately describe the current acceleration).

Furthermore, the successes of the Standard Model of particle physics are accompanied by issues related to incompleteness, technical problems, why questions, naturalness. Nowadays, the most prominent candidate for physics beyond the Standard Model is Supersymmetry, a prediction of string theory (notice that the other fascinating theoretical prediction of string theory, i.e. extra dimensions, had already been introduced in a different context by Kaluza and Klein, immediately after Einstein wrote down the field equations of gravity, in order to unify electromagnetism and gravity).

String theory has a plethora of fields that could be candidates for the inflaton (the field that drives slow-roll inflation) and recent experimental results are constraining some of the string inflation models and ruling out others. The variety of consistent models comes from the richness of the geometries of Calabi-Yau manifolds and compactifications.

Hence, in this work, we will review the standard Hot Big Bang theory and its related issues and Supersymmetry and Supergravity in the following two chapters. Chapter 4 is dedicated to a brief introduction to string theory and to its motivations, quantising the bosonic string, the superstring and then introducing the most basics mathematical concepts and techniques necessary to better understand some geometrical aspects of this higher-dimensional theory. The chapter that follows is dedicated to the stabilisation of moduli: moduli are scalar degrees of freedom in the 4D effective action and describe low energy excitations in the extra dimensions, such as size and shape of the extra dimensions; they are gauge singlet scalars, usually with gravitational strength interactions. In the simplest supersymmetric compactifications, the potential is flat and moduli are massless. These models are ruled out because these massless moduli would mediate unobserved long-range scalar gravitational-strength interactions (fifth forces). We then focus on models of inflation in string theory and, particularly, on brane inflation (branes are extended objects in string theory). Chapters 7 and 8 are probably the core of this work, since we focus on a particular and very recent way of stabilising the moduli via renormalization group (RG) techniques evading the Dine-Seiberg problem and we apply this framework on a possible realisation of inflation with warped branes, following [1]. Then we discuss in an innovative fashion whether it is possible to obtain slow-roll in the regime of validity of the EFT using this setup and conclude introducing some aspects and some references related to reheating (the Hot Big Bang at the end of inflation, how the Standard Model degrees of freedom are excited after slow-roll), cosmic strings and eternal inflation. Our final results, remarks and comments are in Chapter 10.

Conventions

Natural units: $\hbar = c = 1$.
Reduced Planck mass: $M_p^2 = \frac{1}{8\pi G} = (2.4 \cdot 10^{18}\text{GeV})^2$.
Minkowski metric: $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$.
String scale: $M_s = \frac{1}{l_s} = \frac{1}{\sqrt{\alpha'}}$.

Chapter 2

Cosmology

The Cosmological Principle is the assumption that our universe is homogeneous and isotropic. This principle is experimentally verified for large enough scales (larger than 250 million light years) and is usually assumed to be true in most cosmological models (even relaxing only the condition of isotropy makes the models way more difficult to treat; in the following, we will always assume the Cosmological Principle to hold). Recent observations of the Cosmic Microwave Background (CMB) have detected anisotropies in about 1 part in $10^4 - 10^5$ (Fig. 2.1): inflationary models are able to predict both isotropy and the temperature fluctuations. In this chapter we will mainly focus on why and how to introduce inflation.

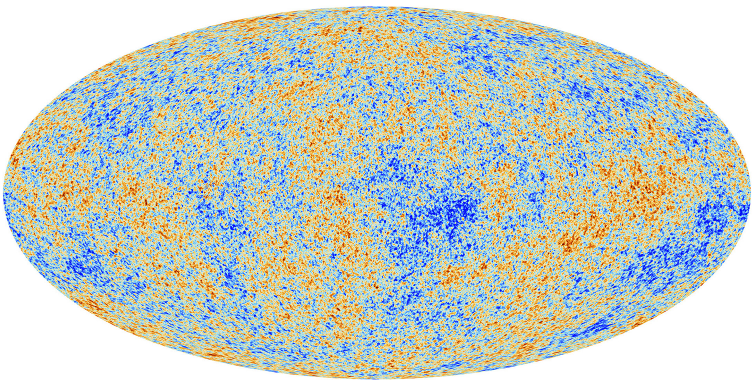

Figure 2.1: Anisotropies of the Cosmic Microwave Background radiation. Source: ESA.

2.1 Standard Big Bang Theory

Assuming that the Cosmological Principle is valid, the most general metric we can write down is the *Friedmann-Lemaître-Robertson-Walker* (FLRW) *metric*:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right], \tag{2.1}$$

where t is the proper time of an observer moving along with the homogeneous and isotropic cosmic fluid, r is a positive radial distance (notice that $r = 0$ does not have a particular physical meaning given the properties of the system), θ and ϕ are respectively the polar angle and the azimuthal angle, $a(t)$ is the *cosmic scale factor* and k is the *curvature* constant. Given that General Relativity is invariant under general coordinate transformations, we can always redefine our coordinates such that k has values 0, +1 or −1.

Flat Universe

A universe for which $k = 0$ is said to be flat: indeed we have that the spatial slice of the metric is proportional to $\mathrm{d}\sigma^2 = \mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$, where $\mathrm{d}\Omega_2^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2$.

Closed Universe

A universe for which $k = +1$ is said to be closed: choosing χ such that $r = \sin\chi$, $\mathrm{d}\sigma^2 = \mathrm{d}\chi^2 + \sin^2\chi\mathrm{d}\Omega_2^2$ (3-sphere).

Open Universe

A universe for which $k = -1$ is said to be open: choosing ψ such that $r = \sinh\psi$, $\mathrm{d}\sigma^2 = \mathrm{d}\psi^2 + \sinh^2\psi\mathrm{d}\Omega_2^2$ (3-hyperboloid).

Describing the content of the universe as a cosmic fluid with energy-momentum tensor $T^\mu{}_\nu = \mathrm{diag}(-\rho, p, p, p)$, energy and momentum conservation are expressed in the context of GR as $\nabla_\mu T^\mu{}_\nu = 0$, where ∇_μ is the covariant derivative calculated using the FLRW metric. This translates into

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \tag{2.2}$$

where a dot indicates a derivative with respect to t.

Up to now we have used only the symmetries of our model; Einstein equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2.3}$$

using the FLRW metric and the energy–momentum tensor for the cosmic fluid, imply the Friedmann equations:

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] = 8\pi G\rho \tag{2.4}$$

and

$$3\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p). \tag{2.5}$$

Defining the Hubble parameter

$$H = \frac{\dot{a}}{a} \tag{2.6}$$

and the density parameter $\Omega = \frac{8\pi G}{3H^2}\rho = \frac{\rho}{\rho_{critical}}$, where $\rho_{critical} = \frac{3H^2}{8\pi G}$, we find $\Omega - 1 = \frac{k}{H^2a^2}$, so that $\rho < \rho_{critical}$ for an open universe, $\rho = \rho_{critical}$ for a flat universe and $\rho > \rho_{critical}$ for a closed universe. We know from observations that $k \simeq 0$ and we will assume this to be true in the following.

A common assumption is the equation of state

$$p = \omega\rho, \tag{2.7}$$

where ω is a constant; in this way $\frac{\dot{\rho}}{\rho} = -3(1+\omega)\frac{\dot{a}}{a}$.

Dust

For dust $p = 0$, so that $\omega = 0$ and $\rho \propto a^{-3}$. Recalling that for a flat universe $\Omega = 1$ and $H^2 \propto \rho$, we find that $a \propto t^{2/3}$.

Radiation

For radiation the trace of the energy-momentum tensor is 0, so that $\omega = \frac{1}{3}$, $\rho \propto a^{-4}$ and $a \propto t^{1/2}$.

Vacuum or dark energy

For dark energy $\rho = \frac{\Lambda}{8\pi G}$ and $\omega = -1$, so that $a \propto e^{Ht}$.

ΛCDM

It is an experimental result the fact that our universe is nowadays accelerating ($\ddot{a} > 0$): this and other observations can be described by introducing dark energy (Λ) and cold dark matter (CDM) such that $\Omega_{visible} \simeq 5\%$, $\Omega_{CDM} \simeq 25\%$ and $\Omega_\Lambda \simeq 70\%$ and $\rho_0 \simeq \rho_{critical} \simeq 10^{-29}\text{g}/\text{cm}^3$.

2.2 Problems of the Hot Big Bang

The first issue of standard cosmology is the initial singularity given the expressions of the cosmic scale factor for dust and radiation. Some of the other issues are the flatness problem and the horizon problem.

The **flatness problem** is related to the fine-tuning of the initial conditions in order to have a so small spatial curvature of the universe [2].

The Horizon Problem

Following [2], we define the (comoving) particle horizon

$$d = \int_0^t \frac{\mathrm{d}t'}{a(t')} \tag{2.8}$$

as the greatest comoving distance from which an observer at time t will be able to receive signals travelling at the speed of light if the Big Bang singularity is at time 0 (and for 0 cosmic scale factor).
Since the size of a causally-connected patch of space is determined by the maximal distance from which light can be received and

$$d = \int \frac{\mathrm{d}a}{a\dot{a}} = \int (aH)^{-1} \mathrm{d}\ln a, \tag{2.9}$$

the comoving Hubble radius is $(aH)^{-1}$, a monotonically increasing function of time for ordinary matter sources, and d is dominated by the contributions from late times. Therefore, most parts of the CMB have non-overlapping past light cones and never were in causal contact considering that the comoving horizon at the time of recombination was much smaller than the comoving distance to the last-scattering surface.
This argument already applies to any two points in the CMB that are separated by more than 2 degrees in the sky, while we observe that two opposite directions are at almost exactly the same temperature (and, again, according to the model that we are discussing, there was not enough time to erase differences in the initial temperatures by heat transfer).

2.3 Cosmological Inflation

Historically introduced to solve the **monopole problem**, cosmological inflation is probably the most natural and simple way to solve the issues of the hot big bang theory: there was a phase before the hot Big Bang during which the homogeneity of the universe and its correlated fluctuations were generated.
Mathematically, we need a decreasing Hubble radius:

$$\frac{\mathrm{d}}{\mathrm{d}t}(aH)^{-1} < 0 \Leftrightarrow \ddot{a} > 0; \tag{2.10}$$

inflation is then a period of accelerated expansion. This latter requirement is satisfied for $\omega < -\frac{1}{3}$; from observations, $\omega \simeq -1$, which implies that ρ is almost constant and $a(t) = e^{H_i(t-t_i)}$, where H_i is the Hubble parameter during inflation (almost constant) and t_i is the time at the beginning of inflation.
Requiring $(a_0H_0)^{-1} < (a_iH_i)^{-1}$, with a_i the cosmic scale factor at the beginning of inflation and a_0 and H_0 the cosmic scale factor and the Hubble parameter now, and defining the *number of e-foldings* as

$$N_e = \ln\left(\frac{a_e}{a_i}\right), \tag{2.11}$$

where a_e is the scale factor at the end of inflation (notice that $H_e \simeq H_i$), it is possible to find that $N \gtrsim 60$ (model dependent result).

Slow-Roll parameters

Therefore, inflation is a very short period (of order 10^{-38} - 10^{-37}s) of quasi exponential expansion, during which ρ is almost constant. This translates into a condition for the dimensionless parameter

$$\varepsilon = -\frac{\dot{H}}{H^2}, \tag{2.12}$$

which is

$$\varepsilon \ll 1 \tag{2.13}$$

(notice that for $\varepsilon = 0$ exactly we get de Sitter spacetime, but inflation has to end).
On the other hand, for inflation to last enough time to have $N \gtrsim 60$, we require

$$\eta = -\frac{\dot{\varepsilon}}{2H\varepsilon} + 2\varepsilon \tag{2.14}$$

to satisfy

$$|\eta| \ll 1. \tag{2.15}$$

Scalar Inflaton

The simplest assumption we can make is that a single scalar field $\phi(t, \vec{x})$ drives inflation. Its lagrangian is

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi), \tag{2.16}$$

the action is $S = \int \mathrm{d}^3x\mathrm{d}t\sqrt{-g}\mathcal{L}$ and the energy-momentum tensor is $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$. Given the validity of the Cosmological Principle, $\phi = \phi(t)$, the spatial derivatives of the field vanish and we introduce energy density and pressure via $T_\mu{}^\nu = \mathrm{diag}(-\rho, p, p, p)$. Taking the variation of the action and imposing it to be 0, we get the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0, \tag{2.17}$$

where $3H\dot{\phi}$ is a friction term due to the expansion of the universe; moreover, it is possible to find that $\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$ and $p = \frac{\dot{\phi}^2}{2} - V(\phi)$. Since $\omega \simeq -1$, the kinetic energy has to be almost negligible with respect to the potential energy. Therefore, $H_i^2 \simeq \frac{8\pi G}{3}V(\phi)$ and $3H\dot{\phi} + V_\phi \simeq 0$. As a consequence,

$$\varepsilon = \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2 \tag{2.18}$$

and

$$\eta = M_p^2\frac{V_{\phi\phi}}{V}. \tag{2.19}$$

Moreover,

$$N_e = \int_{a_i}^{a_e} \mathrm{d}\ln a = \int_{t_i}^{t_e} H(t)\mathrm{d}t = \int_{\phi_i}^{\phi_e} \frac{H}{\dot{\phi}}\mathrm{d}\phi \simeq \int_{\phi_i}^{\phi_e} \frac{1}{\sqrt{2\varepsilon}}\frac{|\mathrm{d}\phi|}{M_p}. \tag{2.20}$$

For models of inflation in the context of string theory, see Chap. 6.

Chapter 3

Supersymmetry

The Standard Model (SM) of particle physics is probably the greatest triumph of all science, given its powerful and accurate experimental predictions. Nevertheless, it suffers from important problems that cannot be ignored, both from a theoretical and from a phenomenological point of view. In this chapter, we will briefly review why and how to introduce Supersymmetry (SUSY) and how to include general coordinate transformations in Supergravity (SUGRA).

3.1 SUSY

For this chapter we will mainly follow [3].

Motivation

Before dealing with the SUSY algebra and supersymmetric lagrangians, it is fundamental to understand from a physical point of view why SUSY and extra dimensions are introduced conceptually. The SM of particle physics is not able to address the following issues:

- **Quantum Gravity**: the SM describes electromagnetic, weak and strong interactions from a microscopical perspective; but gravity cannot be explained in the context of a Quantum Field Theory (QFT), since the Einstein-Hilbert action is non-renormalizable and therefore incapable of predictions.
- **Neutrino masses**.
- **Dark Matter**.
- **Gauge Coupling Unification**.

- $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: the SM is not elegant (it needs around 20 parameters which we should put by hand and it doesn't explain why it has its particular group structure, why there are 3 families of fermions, why there are 4 spacetime dimensions, why our world is chiral).

- **Confinement**.

- **The hierarchy problem**: $M_{ew} \simeq 10^2$GeV, $M_p \simeq 10^{18}$GeV; a so big fine tuning between the electroweak scale and the cutoff scale (notice that the cutoff scale of the SM is at most M_p since we know that we cannot neglect gravitational effects at that scale) represents a naturalness issue.

- **The strong CP problem**.

- **The cosmological constant problem**: $\Lambda \simeq 10^{-120} M_p^4$; this so small value (measured from the accelerated expansion of the universe) is another naturalness issue.

SUSY algebra

Recalling that the generators P^μ and $M^{\mu\nu}$ of the Poincaré group satisfy the algebra

$$\begin{aligned} [P^\mu, P^\nu] &= 0, \\ [M^{\mu\nu}, P^\sigma] &= i(P^\mu \eta^{\nu\sigma} - P^\nu \eta^{\mu\sigma}), \\ [M^{\mu\nu}, M^{\rho\sigma}] &= i(M^{\mu\sigma}\eta^{\nu\rho} + M^{\nu\rho}\eta^{\mu\sigma} - M^{\mu\rho}\eta^{\nu\sigma} - M^{\nu\sigma}\eta^{\mu\rho}), \end{aligned} \tag{3.1}$$

we define a graded algebra via the following equation:

$$O_a O_b - (-1)^{\eta_a \eta_b} O_b O_a = i C_{ab}^c O_c, \tag{3.2}$$

where $\eta_a = 0$ if O_a is a bosonic generator and $\eta_a = 1$ if O_a is a fermionic generator.

At this point, considering the *No-go theorem* by Coleman and Mandula [4] and its generalization by Haag, Lopuszanski and Sohnius [5], we can at most generalise the Poincaré algebra by adding the spinor generators Q_α^A and $\bar{Q}_{\dot{\alpha}}^A$, where $A = 1, ..., \mathcal{N}$, $\alpha = 1, 2$, $\dot{\alpha} = \dot{1}, \dot{2}$: if $\mathcal{N} = 1$, we talk about *simple SUSY*; if $\mathcal{N} > 1$, we talk about *extended SUSY*. In the following we will always (unless differently specified) deal with simple SUSY.

The Poincaré algebra together with the commutation relations between the spinor generators and the Poincaré generators and with the anticommutation relations between the spinor generators themselves constitute the SUSY algebra:

$$\begin{aligned}[Q_\alpha, M^{\mu\nu}] &= (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta, \\ [Q_\alpha, P^\mu] &= [\bar{Q}^{\dot\alpha}, P^\mu] = 0, \\ \{Q_\alpha, Q_\beta\} &= 0, \\ \{Q_\alpha, \bar{Q}_{\dot\beta}\} &= 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu.\end{aligned} \tag{3.3}$$

Superfields

We define a general scalar superfield S via the following expansion:

$$\begin{aligned}S(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) =& \varphi(x) + \theta\psi(x) + \bar\theta\bar\chi(x) + \theta\theta M(x) + \bar\theta\bar\theta N(x) + (\theta\sigma^\mu\bar\theta)V_\mu(x) \\ &+ (\theta\theta)\bar\theta\bar\lambda(x) + (\bar\theta\bar\theta)\theta\rho(x) + (\theta\theta)(\bar\theta\bar\theta)D(x),\end{aligned} \tag{3.4}$$

where θ_α and $\bar\theta_{\dot\alpha}$ are Grassmann variables.

S transforms as

$$\delta S = i[S, \epsilon Q + \bar\epsilon\bar Q] = i(\epsilon\mathcal{Q} + \bar\epsilon\bar{\mathcal{Q}})S, \tag{3.5}$$

where $\mathcal{Q}_\alpha = -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot\beta}\bar\theta^{\dot\beta}\frac{\partial}{\partial x^\mu}$ and $\bar{\mathcal{Q}}_{\dot\alpha} = +i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \theta^\beta(\sigma^\mu)_{\beta\dot\alpha}\frac{\partial}{\partial x^\mu}$.

It is now useful to introduce a covariant derivative $\mathcal{D}_\alpha$ such that $\mathcal{D}_\alpha S$ is a superfield:

$$\mathcal{D}_\alpha = \partial_\alpha + i(\sigma^\mu)_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu, \qquad \bar{\mathcal{D}}_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\beta(\sigma^\mu)_{\beta\dot\alpha}\partial_\mu. \tag{3.6}$$

At this point we can define a **chiral superfield** Φ as a superfield that satisfies

$$\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0. \tag{3.7}$$

This implies that

$$\Phi(y^\mu, \theta^\alpha) = \varphi(y^\mu) + \sqrt{2}\theta\psi(y^\mu) + \theta\theta F(y^\mu), \tag{3.8}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta$.

A **vector superfield** V instead satisfies the condition

$$V(x, \theta, \bar\theta) = V^\dagger(x, \theta, \bar\theta). \tag{3.9}$$

Noticing that $i(\Lambda - \Lambda^\dagger)$ is a vector superfield if Λ is a chiral superfield, defining a *generalised gauge transformation* via $V \mapsto V - \frac{i}{2}(\Lambda - \Lambda^\dagger)$ and choosing appropriately the components for Λ, we write V in the *Wess Zumino gauge* as

$$V_{WZ}(x, \theta, \bar\theta) = (\theta\sigma^\mu\bar\theta)V_\mu(x) + (\theta\theta)\bar\theta\bar\lambda(x) + (\bar\theta\bar\theta)\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x). \tag{3.10}$$

4D SUSY Lagrangians

Since the D term of a general scalar superfield and the F term of a chiral superfield transform as total derivatives, the most general Lagrangian for a chiral superfield Φ is

$$\mathcal{L} = K(\Phi, \Phi^\dagger)|_D + (W(\Phi)|_F + \text{h.c.}), \tag{3.11}$$

where K is called the **Kähler potential** (arbitrary real function of Φ and $\Phi^\dagger$, it is a real superfield) and W is called the **superpotenatial** (arbitrary holomorphic function of Φ, it is a chiral superfield).
For example, for the *Wess Zumino model*, $K = \Phi^\dagger\Phi$ and $W = \alpha + \lambda\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3$.
Introducing a vector superfield in the Lagrangian means that $K = \Phi^\dagger e^{2qV}\Phi$ (to make it invariant under generalised gauge transformations) and that we should add the kinetic term $\mathcal{L}_{kin} = f(\Phi)(W^\alpha W_\alpha)|_F + \text{h.c.}$, where f is the **gauge kinetic function** and $W_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}\bar{\mathcal{D}})\mathcal{D}_\alpha V$ is the **abelian field strength superfield**, and the Fayet Iliopoulos term $\mathcal{L}_{FI} = \xi V|_D$.

We now want to only quote the important results of the **non-renormalization theorems** ([6] and [7] for details; and also notice that these are not claims about non-perturbative corrections):
K gets corrections order by order in perturbation theory;
f gets corrections only at one loop;
W and ξ are not renormalized in perturbation theory.

SUSY breaking

We speak of broken SUSY if the vacuum state $|vac\rangle$ satisfies

$$Q_\alpha |vac\rangle \neq 0. \tag{3.12}$$

From eq. 3.3, the energy $E = P^0$ satisfies $E \geq 0$ for any state and $E > 0$ for broken SUSY.
Moreover, given a chiral superfield Φ, we talk about F term breaking if $\langle F\rangle \neq 0$: in this case ψ is the goldstino.
Given instead a vector superfield V, we talk about D term breaking if $\langle D\rangle \neq 0$: in this case λ is the goldstino.

Non-linear SUSY

Let us consider a chiral superfield

$$X = X_0(y) + \sqrt{2}\psi(y)\theta + F(y)\theta\bar{\theta} \tag{3.13}$$

that satisfies a nilpotency constraint:

$$X^2 = 0. \tag{3.14}$$

This implies that the scalar component of X is not a propagating field:

$$X_0 = \frac{\psi\psi}{2F} \tag{3.15}$$

([8] for further details). We will use this construction in Chapter 7, where this sector breaks supersymmetry and the fermionic component of X is the goldstone fermion.

3.2 SUGRA

Supergravity is the extension of SUSY to a local symmetry. From a physical point of view, given eq. 3.3, local supersymmetry implies local Poincaré and so general coordinate transformations (GCT) and gravity; the superpartner of the graviton (spin 2, $g_{\mu\nu}$) is the gravitino (spin $\frac{3}{2}$, ψ_μ^α).
Using the superfield formalism [9],

$$S_{tot} = S_{SUGRA} + S[K, W, f, \xi], \tag{3.16}$$

where $S[K, W, f, \xi]$ is the SUSY action properly covariantised under GCT and

$$S_{SUGRA} = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}[M_p^2 R + \epsilon^{\mu\nu\rho\sigma}(\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma - \psi_\mu\sigma_\nu D_\rho\bar{\psi}_\sigma)] + (\text{terms with auxiliary fields}). \tag{3.17}$$

The action is Kähler invariant under the transformations

$$\begin{aligned} K &\mapsto K + h(\Phi) + h^*(\Phi^*), \\ W &\mapsto e^{-h(\Phi)}W, \end{aligned} \tag{3.18}$$

since it depends only on the invariant combination $G = K + \ln|W|^2$.
Defining the Kähler covariant derivative as

$$D_iW = \partial_iW + \frac{W}{M_p^2}\partial_iK \tag{3.19}$$

(notice that i is a field index), $K_{i\bar{j}} = \partial_i\partial_{\bar{j}}K$ as the Kähler metric and $K^{i\bar{j}}$ its inverse, the scalar potential can be written as

$$V_F = e^{\frac{K}{M_p^2}}\left(K^{i\bar{j}}D_iWD_{\bar{j}}\overline{W} - 3\frac{|W|^2}{M_p^2}\right). \tag{3.20}$$

Moreover, $F^i = e^{\frac{K}{2M_p^2}} K^{i\bar{j}} D_{\bar{j}} \overline{W}$ and the square of the gravitino mass is

$$m_{3/2}^2 = e^{\frac{K}{M_p^2}} \frac{|W|^2}{M_p^4}, \tag{3.21}$$

so that $V_F = K_{i\bar{j}} F^i \overline{F}^{\bar{j}} - 3m_{3/2}^2 M_p^2$.

SUSY breaking in SUGRA

Differently from the global case, the energy is not positive definite (note by the way that $m_{3/2} \to 0$ as $M_p \to \infty$). Therefore, if SUSY is preserved, $\langle F \rangle = 0$ and we can have AdS or Minkowski; if SUSY is broken, $\langle F \rangle \neq 0$ and we can have AdS, Minkowski or dS.[1]

[1]dS stands for de Sitter solution: $\Lambda > 0$; AdS stands for anti-de Sitter solution: $\Lambda < 0$; $\Lambda = 0$ for Minkowski spacetime.

Chapter 4

String Theory

In this chapter we will briefly introduce the theory of strings, review what are the main consequences of quantising the bosonic and the supersymmetric strings and discuss the basic concepts related to Kaluza-Klein compactification and Calabi-Yau manifolds.

4.1 Bosonic String

In this section we will mainly follow [10].

Relativistic String

The action of a relativistic point particle is $S = -m \int_\gamma \mathrm{d}s$, where γ is its worldline and $\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu$. In complete analogy, the **Nambu-Goto action** for a string measures the surface area of the worldsheet embedded in target spacetime (see Fig. 4.1):

$$S_{NG} = -T \int_\Sigma \mathrm{d}f = -T \int_\Sigma \mathrm{d}^2\xi \sqrt{-G}, \tag{4.1}$$

where T is the string tension, $\xi = (\xi^0, \xi^1) = (\tau, \sigma)$ parametrises the worldsheet, G_{ab} is the induced metric defined via $\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu = \eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu \mathrm{d}\xi^a \mathrm{d}\xi^b = G_{ab}\mathrm{d}\xi^a \mathrm{d}\xi^b$ and $G = \det(G_{ab})$.
Because of the factor $\sqrt{-G}$, it is hard to quantise the system using this action and we instead introduce the **Polyakov action**:

$$S_P = -\frac{T}{2} \int_\Sigma \mathrm{d}^2\xi \sqrt{-h} h^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}, \tag{4.2}$$

where h_{ab} is the worldsheet metric; solving its equations of motion, we find that this action is equivalent to the Nambu-Goto one for $h_{ab} = \alpha G_{ab}\ \forall \alpha$.

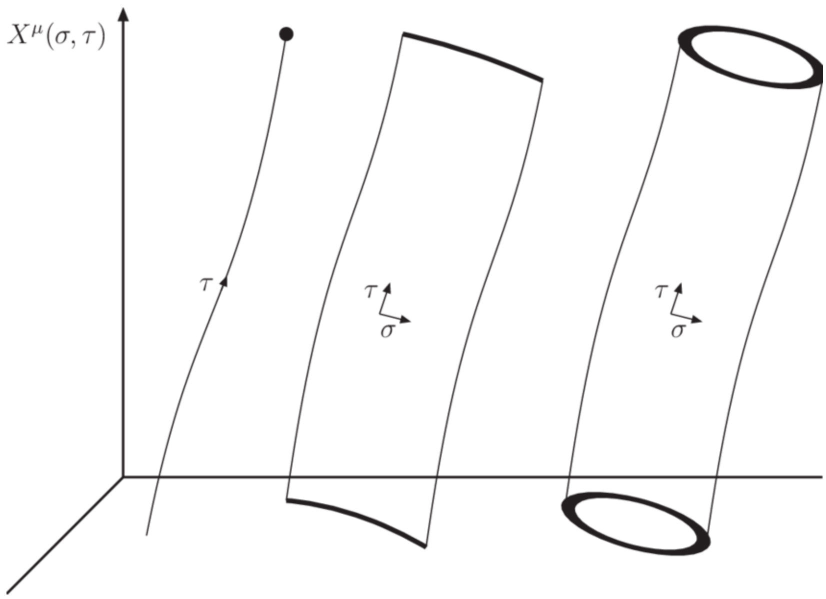

Figure 4.1: Worldline of a point particle, worldsheet of an open string and worldsheet of a closed string. Source: [11].

The worldsheet theory can be seen as a 2d QFT with metric h_{ab} and D free scalars X^μ; its symmetries are:

- *diffeomorphism*: $\xi^a \to \xi'^a(\xi^0, \xi^1)$;
- *Poincaré symmetry*: $X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu + V^\mu$, $\Lambda \in SO(1, D-1)$;
- *Weyl rescaling*: $h_{ab}(\xi) \to h'_{ab}(\xi) = e^{2\omega(\xi)} h_{ab}(\xi)$, where ω is an arbitrary real function.

We now define the **Regge slope** as

$$\alpha' = \frac{1}{2\pi T} \tag{4.3}$$

and the energy-momentum tensor on the string worldsheet as

$$T^{ab} = \frac{-4\pi}{\sqrt{-h}} \frac{\delta S_P}{\delta h_{ab}}. \tag{4.4}$$

Note that $T^a{}_a = 0$ is an identity and $T^{ab} = 0$ is the equation of motion of h_{ab}.
Since diffeomorphisms and Weyl rescalings are gauge redundancies, it is always possible to choose the flat gauge, such that $h_{ab} = \text{diag}(-1, 1)$. In this case, using light-cone coordinates $\sigma^\pm = \tau \pm \sigma$, the equations of motion read

$$(\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0, \qquad \partial_- \partial_+ X^\mu = 0. \tag{4.5}$$

The general solution to eq. 4.5 is of the form

$$X^\mu(\sigma^+, \sigma^-) = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-). \tag{4.6}$$

Imposing the constraint $X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \pi)$ and integrating we find the mode decomposition

$$\begin{aligned} X^\mu_L &= \frac{1}{2}x^\mu + \frac{l^2}{2}p^\mu\sigma^+ + \frac{il}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}^\mu_n e^{-2in\sigma^+}, \\ X^\mu_R &= \frac{1}{2}x^\mu + \frac{l^2}{2}p^\mu\sigma^- + \frac{il}{2}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n e^{-2in\sigma^-}, \end{aligned} \tag{4.7}$$

where $l = \sqrt{2\alpha'}$. Reality of X^μ implies that x^μ and p^μ are real and that $(\alpha^\mu_n)^* = \alpha^\mu_{-n}$.

Open String Quantisation

An open string is parametrised by $\sigma \in (0, \pi)$. Varying the string action, we see that it is possible to introduce two different types of boundary conditions:

- **Neumann BCs**: $\partial_\sigma X^\mu = 0$;
- **Dirichlet BCs**: $\delta X_\mu = 0$.

For Neumann BCs the string end moves freely, while for Dirichlet BCs the string end is confined to lie in a fixed hyperplane (see sect. 4.2, **D-branes**). As usual, $\Pi_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$. To quantise the system, we impose the equal-time commutation relations

$$[\hat{\Pi}_\mu(\tau,\sigma), \hat{X}^\nu(\tau,\sigma')] = -i\delta(\sigma-\sigma')\delta_\mu{}^\nu, \qquad [\hat{X}^\mu, \hat{X}^\nu] = [\hat{\Pi}_\mu, \hat{\Pi}_\nu] = 0; \tag{4.8}$$

these imply that

$$[\hat{p}^\mu, \hat{x}^\nu] = -i\eta^{\mu\nu}, \qquad [\hat{\alpha}^\mu_m, \hat{\alpha}^\nu_n] = m\delta_{m+n,0}\eta^{\mu\nu}, \qquad [\hat{\tilde{\alpha}}^\mu_m, \hat{\tilde{\alpha}}^\nu_n] = m\delta_{m+n,0}\eta^{\mu\nu}. \tag{4.9}$$

Introducing the Fourier modes of T_{++} and T_{--} as

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty}\alpha_{m-n}\cdot\alpha_n, \qquad \tilde{L}_m = \frac{1}{2}\sum_{n=-\infty}^{\infty}\tilde{\alpha}_{m-n}\cdot\tilde{\alpha}_n, \tag{4.10}$$

we can find that they satisfy the Virasoro algebra $[L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m+n,0}$, where $A(m) = (m^3 - m)\frac{D}{12}$. Moreover, $H = L_0$, where H is the hamiltonian of the system. The equation of motion $T_{ab} = 0$ has to be implemented as a constraint on the physical states as $L_m\,|phys\rangle = 0$ for $m \geq 0$. Writing L_0 as $L_0 =: L_0 : -a$ where $a = \frac{D-2}{24}$, we obtain two very important results:

- $\mathbf{D} = \mathbf{26}$ (critical dimension) in order to preserve Poincaré invariance;
- the mass-shell condition, $H\,|phys\rangle = 0$, can be written as $\mathbf{M^2} = \frac{\mathbf{N-1}}{\alpha'}$, where $N = \sum_{n>0} \alpha_{-n}\alpha_n$.

Notice the presence of a tachyon at level 0 and of a massless vector (photon) at level 1.

Closed String Quantisation

In a completely analogous fashion to what was done for the open string, for the closed string $H = L_0 + \tilde{L}_0$, $a = 1$, $\mathbf{D} = \mathbf{26}$, the physical states satisfy the condition $(N - \tilde{N})\,|phys\rangle = 0$ and $\mathbf{M^2} = \frac{\mathbf{2(N+\tilde{N}-2)}}{\alpha'}$.
At level 0 we get again a tachyon; at level 1 we have the graviton $G_{\mu\nu}$ (symmetric and traceless tensor), the Kalb-ramond field $B_{\mu\nu}$ (antisymmetric tensor) and the dilaton ϕ (trace part).

4.2 Superstring

The bosonic string theory has two main problems:

- it predicts the existence of a tachyon, hence the vacuum is unstable;
- it does not contain fermions but only bosons, in contradiction with our experience.

The solution to these issues is the introduction of worldsheet supersymmetry (to the ordinary coordinates σ^a we add fermionic coordinates θ_α). Making SUSY local (SUGRA, Sect. 3.2) and exploiting diffeomorphism and Weyl invariance (so to choose a flat metric and vielbein), we will need to quantize the action

$$S = -\frac{1}{2\pi}\int \mathrm{d}^2\sigma[(\partial_a X^\mu)(\partial^a X_\mu) - i\bar{\psi}^\mu\gamma^a\partial_a\psi_\mu]. \tag{4.11}$$

In a way similar to the bosonic case,

$$\psi = \begin{pmatrix}\psi_- \\ \psi_+\end{pmatrix}. \tag{4.12}$$

Since $\psi_\pm$ are fermionic, we can choose two different types of boundary conditions:

- **Ramond BCs**: $\psi_\pm(\sigma+\pi) = +\psi_\pm(\sigma)$;
- **Neveu-Schwarz BCs**: $\psi_\pm(\sigma+\pi) = -\psi_\pm(\sigma)$.

Therefore, there will be 4 different sectors (R-R, R-NS, NS-R, NS-NS) and

$$\psi_+^\mu = \sum_r \tilde{\psi}_r^\mu e^{-2ir(\tau+\sigma)}, \qquad \psi_-^\mu = \sum_r \psi_r^\mu e^{-2ir(\tau-\sigma)}, \tag{4.13}$$

where $r \in \mathbb{Z}$ for R BCs and $r \in \mathbb{Z} + \frac{1}{2}$ for NS BCs. Reality of ψ requires $(\psi_r^\mu)^* = \psi_{-r}^\mu$. To quantize the system, we add to the commutation relations for α the condition

$$\{\psi_r^\mu, \psi_s^\nu\} = \delta_{r+s,0}\eta^{\mu\nu}. \tag{4.14}$$

Moreover,

$$L_m = \frac{1}{2} : \left[\sum_{n\in\mathbb{Z}} \alpha_{-n} \cdot \alpha_{m+n} + \sum_r \left(r + \frac{m}{2}\right) \psi_{-r} \cdot \psi_{m+r}\right] :, \qquad G_r = \sum_{n\in\mathbb{Z}} \alpha_{-n} \cdot \psi_{r+n}. \tag{4.15}$$

Physical states are such that $(L_m - a\delta_{m,0}) |phys\rangle = 0$ for $m \geq 0$ and $G_r |phys\rangle = 0$ for $r \geq 0$.
Furthermore, $a(R) = 0$, $a(NS) = \frac{D-2}{16}$ and the critical dimension is $\mathbf{D = 10}$. The tachyonic scalar at level 0 is removed through the GSO projection [12].

Consistent Superstring Theories

There are 5 consistent superstring theories in 10 dimensional spacetime [13]:

- **type I**: theory of unoriented open and closed strings with $\mathcal{N} = 1$ supersymmetries;
- **heterotic** $SO(32)$: theory of closed strings (supersymmetrising only the left- or right-moving half of the worldsheet theory) with $\mathcal{N} = 1$ supersymmetries and giving rise to an $SO(32)$ Yang-Mills theory;
- **heterotic** $E_8 \times E_8$: theory of closed strings (supersymmetrising only the left- or right-moving half of the worldsheet theory) with $\mathcal{N} = 1$ supersymmetries and giving rise to an $E_8 \times E_8$ Yang-Mills theory;
- **type IIA**: theory with $\mathcal{N} = 2$ supersymmetries and stable Dp-branes for even p (Sect. 4.2, **D-branes**);
- **type IIB**: chiral theory with $\mathcal{N} = 2$ supersymmetries and stable Dp-branes for odd p (Sect. 4.2, **D-branes**).

It is now known that these theories are not different but are instead different limits of an 11 dimensional theory called M-theory [14]: M-theory is a theory of membranes (M2-branes) whose UV behaviour is not well understood yet. Figure 4.2 shows a visual representation of these relations between theories. In the following we will always deal with type IIB superstrig theory since this is chiral and hence phenomenologically viable.

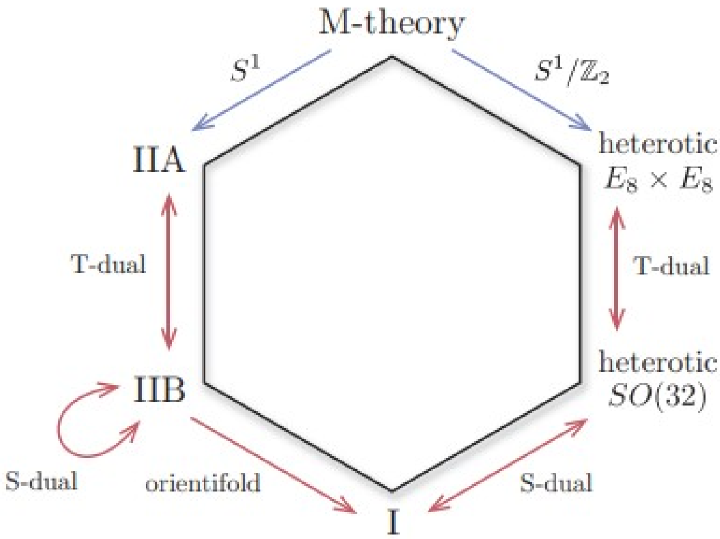

Figure 4.2: The 5 consistent 10D superstring theories and the 11D M-theory are related as depicted. Source: [15].

Type IIB String Theory

The field content of the type IIB string contains the metric G_{MN}, the dilaton Φ and the two-form B_2 for the NS-NS sector and a zero-form (scalar) C_0, a two-form C_2 and a four-form C_4 for the R-R sector (the NS-R and R-NS sectors contain spinors and vector-spinors, but we will only focus on the bosonic degrees of freedom). Following [15], we write the action as

$$S = S_{NS} + S_R + S_{CS}. \tag{4.16}$$

S_{NS} reads

$$S_{NS} = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G}e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2\right), \tag{4.17}$$

where R is the Ricci scalar, $H_3 = \mathrm{d}B_2$ and $2\kappa^2 = (2\pi)^7(\alpha')^4$. S_R and S_{CS} are instead

$$S_R = -\frac{1}{4\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right) \tag{4.18}$$

and

$$S_{CS} = -\frac{1}{4\kappa^2}\int C_4 \wedge H_3 \wedge F_3, \tag{4.19}$$

where $F_p = \mathrm{d}C_{p-1}$, $\tilde{F}_3 = F_3 - C_0 \wedge H_3$, $\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$ and we must impose the self-duality constraint $\tilde{F}_5 = *\tilde{F}_5$ (these mathematical technicalities will be discussed in the next section).

We define the **string coupling** $g_s = e^{\Phi}$ and the combinations $\tau = C_0 + ie^{-\Phi}$ and $G_3 = F_3 - \tau H_3$. In the Einstein frame, i.e. performing the Weyl rescaling $G_{E,MN} = e^{-\Phi/2} G_{MN}$ (more on Jordan frame and Einstein frame in Sect. 8.1, **String frame and Einstein frame**),

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}X \sqrt{-G_E} \left(R_E - \frac{|\partial\tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{2\,\mathrm{Im}\,\tau} - \frac{|\tilde{F}_5|^2}{4} \right) - \frac{i}{8\kappa^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\mathrm{Im}\,\tau}. \tag{4.20}$$

D-branes

D-branes are solitonic objects charged under the gauge symmetry of the R-R fields. A Dp-brane has p spatial dimensions and is charged under C_{p+1}; in type IIB, stable Dp-branes have p odd (plus the NS5-brane, magnetically charged under B_2).
The bosonic action for D-branes is

$$S_{Dp} = S_{DBI} + S_{CS}. \tag{4.21}$$

The *Dirac-Born-Infeld action* is

$$S_{DBI} = -g_s T_p \int \mathrm{d}^{p+1}\sigma e^{-\Phi} \sqrt{-\det(G_{ab} + \mathcal{F}_{ab})}, \tag{4.22}$$

where $G_{ab} = \frac{\partial X^M}{\partial \sigma^a} \frac{\partial X^N}{\partial \sigma^b} G_{MN}$ is the pullback of the metric of the target spacetime,

$$T_p = \frac{1}{(2\pi)^p g_s (\alpha')^{(p+1)/2}} \tag{4.23}$$

is the tension of the membrane and $\mathcal{F}_{ab} = B_{ab} + 2\pi\alpha' F_{ab}$ is the gauge-invariant field strength. The *Chern-Simons action* is instead

$$S_{CS} = i\mu_p \int_{\Sigma_{p+1}} \sum_n C_n \wedge e^{\mathcal{F}}, \tag{4.24}$$

where $\mu_p = g_s T_p$.
An **antibrane** is an extended object which has the same tension as the corresponding brane but with opposite R-R (Ramond-Ramond) charge.

4.3 Compactification

Superstring theory predicts that the number of spacetime dimensions is 10; since the world in which we live seems to be 4 dimensional, this means that the other 6 spatial dimensions must be compactified in some way.
The idea of extra dimensions is old almost as GR and in the following we will introduce the Kaluza-Klein idea.

Kaluza Klein theories

Following [3], let us imagine, for the sake of simplicity, our spacetime to be $\mathbb{M}_4 \times S^1$. If the extra dimension $x^4 = y$ defines a circle of radius r, then $y \equiv y + 2\pi r$. A massless scalar field $\varphi(x^M)$, $M = 0, 1, 2, 3, 4$, has action $S_{5D} = \int \mathrm{d}^5x \partial^M \varphi \partial_M \varphi$ and can be expanded as $\varphi(x^\mu, y) = \sum_{n\in\mathbb{Z}} \varphi_n(x^\mu) e^{iny/r}$. Its equation of motion $\partial^M \partial_M \varphi = 0$ implies $\partial^\mu \partial_\mu \varphi_n(x^\mu) - \frac{n^2}{r^2} \varphi_n(x^\mu) = 0$. This means that each Fourier mode φ_n is a 4D particle with mass $m_n^2 = \frac{n^2}{r^2}$ and they constitute the Kaluza-Klein tower. Since $S_{5D} = S_{4D} + ...$, keeping only the zero mode is called **dimensional reduction**, whether keeping all the massive modes is called **compactification**.

Now let us consider the graviton G_{MN} in D dimensions:

$$G_{MN} = \begin{cases} G_{\mu\nu} & \text{graviton} \\ G_{\mu n} & \text{vectors} \\ G_{mn} & \text{scalars} \end{cases}, \tag{4.25}$$

where $\mu, \nu = 0, 1, 2, 3$ and $m, n = 4, ..., D-1$. For $D = 5$, the Einstein-Hilbert action reads $S = \int \mathrm{d}^5x \sqrt{|G|}\,{}^{(5)}R$ and in vacuum ${}^{(5)}R_{MN} = 0$. Considering excitations in addition to the background metric, $G_{MN} = \phi^{-1/3} \begin{pmatrix} (g_{\mu\nu} - \kappa^2 \phi A_\mu A_\nu) & -\kappa\phi A_\mu \\ -\kappa\phi A_\nu & \phi \end{pmatrix}$;
in Fourier expansion
$S_{4D} = \int \mathrm{d}^4x \sqrt{|g|}(M_p^{2(4)} R - \frac{1}{4}\phi^{(0)} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \frac{1}{6} \frac{\partial^\mu \phi^{(0)} \partial_\mu \phi^{(0)}}{(\phi^{(0)})^2}) + (\infty$ tower of massive modes): this is a unified theory of gravity, electromagnetism and scalar fields.

From this toy model, we can now focus on the study of the 10 dimensional case; following [15], we will consider geometries of the form

$$\mathcal{M}_{10} = \mathcal{M}_4 \times X_6, \tag{4.26}$$

where X_6 is a compact six-manifold: this procedure is called compactification of string theory on X_6.
An ansatz for *vacuum configurations* (i.e. $R_{\mu\nu} = R_{mn} = 0$) is

$$G_{MN} \mathrm{d}X^M \mathrm{d}X^N = \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn} \mathrm{d}y^m \mathrm{d}y^n, \tag{4.27}$$

where y^m are coordinates on X_6, $m = 1, ..., 6$ and g_{mn} is a metric on X_6.
For *non-vacuum configurations* with maximal symmetry in the non-compact spacetime, the ansatz is

$$G_{MN} \mathrm{d}X^M \mathrm{d}X^N = e^{2A(y)} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} g_{mn} \mathrm{d}y^m \mathrm{d}y^n, \tag{4.28}$$

where $g_{\mu\nu}$ is the metric of a maximally symmetric spacetime, the **warp factor** $A(y)$ is a function on X_6 and the internal metric g_{mn} is not necessarily Ricci-flat.

We can now consider a simple example of dimensional reduction. Given the 10 dimensional geometry $G_{MN}\mathrm{d}X^M\mathrm{d}X^N = e^{-6u(x)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{2u(x)}\hat{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n$, where

$$\int_{X_6} \mathrm{d}^6y\sqrt{\hat{g}} = \mathcal{V} \tag{4.29}$$

and the factors of the exponentials of $u(x)$ are chosen so that the gravitational action in 4 dimensions will appear in Einstein frame, $S_{EH}^{(10)} = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}X\sqrt{-G}e^{-2\Phi}R_{10} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\int_{X_6}\mathrm{d}^6y\sqrt{\hat{g}}e^{-2\Phi}(R_4+e^{-8u}\hat{R}_6+12\partial_\mu u\partial^\mu u)$. If the string coupling is constant over the internal space, $S_{EH}^{(4)} = \frac{M_p^2}{2}\int \mathrm{d}^4x\sqrt{-g}R_4$ and

$$M_p^2 = \frac{\mathcal{V}}{g_s^2\kappa^2}. \tag{4.30}$$

Calabi-Yau manifolds

Let us now introduce the very basics of the geometrical and mathematical aspects necessary for compactification in string theory (we will only claim the main useful results, without proving them; [10], [16] and [17] for further details).
The very first thing that is needed to say is that Calabi-Yau manifolds are complex manifolds: *complex manifolds* are manifolds that locally look like $\mathbb{C}^n$. Local bases of tangent and cotangent space are $\frac{\partial}{\partial z^i}$, $\frac{\partial}{\partial \bar{z}^{\bar{i}}}$ and $\mathrm{d}z^i$, $\mathrm{d}\bar{z}^{\bar{i}}$, with $z^i = x^i + iy^i$.

$$J := i\mathrm{d}z^i \otimes \frac{\partial}{\partial z^i} - i\mathrm{d}\bar{z}^{\bar{i}} \otimes \frac{\partial}{\partial \bar{z}^{\bar{i}}} \tag{4.31}$$

is a tensor interpreted as a map $T_p^* \to T_p^*\ \forall p \in X$. Moreover, a Calabi-Yau manifold is a Kähler manifold: a *Kähler manifold* is a manifold with a metric (Riemannian manifold) which is compatible with J (J is covariantly constant). For a Kähler manifold, the metric can be locally written as

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial z^i \partial \bar{z}^{\bar{j}}}, \tag{4.32}$$

where K is a real function defined in every patch and $g_{ij} = g_{\bar{i}\bar{j}} = 0$.
Lowering the second index of J with the metric, we get a rank-2 lower-index antisymmetric tensor defining the **Kähler 2-form**:

$$J = ig_{i\bar{j}}\mathrm{d}z^i \wedge \mathrm{d}\bar{z}^{\bar{j}}. \tag{4.33}$$

$\forall p \in X$ and any closed curve C (beginning and ending in p), we have a linear map $R(C) : T_p \to T_p$ or $R(C) \in SO(2n)$. The set of all $R(C)$ forms the holonomy group (which does not depend on the choice of p assuming X is connected).
Therefore, we define a **Calabi-Yau 3-fold** as a compact, complex Kähler manifold with

$SU(3)$ holonomy ($SU(3)$ holonomy is equivalent to the existence of a covariantly constant spinor and so to 4D unbroken SUSY; furthermore, $SU(n)$ holonomy implies Ricci flatness).
Let us now consider the curvature 2-form $R(T_X) = \mathrm{d}z^i \wedge \mathrm{d}\bar{z}^{\bar{j}} R_{i\bar{j}}{}^k{}_l$; defining the multi-form $c(X) = \det(\mathbb{1} + R(T_X))$, we expand it as $c(X) = 1 + c_1(X) + c_2(X) + ...$; we say that the 1st Chern class is 0 if c_1 is exact (0 in cohomology). At this point, we quote *Yau's theorem*:
let X be a Kähler manifold and J its Kähler form. If the 1st Chern class vanishes, then a Ricci flat metric with Kähler form J' in the same cohomology class ($J - J'$ is exact) can be given. This so called Calabi-Yau metric is unique.

A **p-chain** is defined as the formal sum of p-dimensional submaifolds of the compact manifold X via $c_p = \sum_i \alpha_i S_{p,i}$. The boundary of each $S_{p,i}$, and so of c_p, is a $(p-1)$-dimensional submanifold. A cycle is defined to be a chain without boundary: $\partial c_p = 0$. Note that $\partial^2 = 0$. The **homology groups** are $H_p = \frac{\mathrm{Ker}(\partial_p)}{\mathrm{Im}(\partial_{p+1})}$.
p-forms are dual with respect to chains: $\omega_p(c_p) = \int_{c_p} \omega_p = \sum_i \alpha_i \int_{S_{p,i}} \omega_p$; and we introduce the exterior derivative d as $d_p : \omega_p \to \omega_{p+1} = d\omega_p$, $d^2 = 0$. The **cohomology groups** of the **de Rham cohomology** are $H^p = \frac{\mathrm{Ker}(d_p)}{\mathrm{Im}(d_{p-1})}$. A p-form ω_p is called closed if $d\omega_p = 0$ and exact if $\omega_p = d\omega_{p-1}$.
Homology and cohomology classes are dual vector spaces: $H_p(X) = H^p(X)^*$ and their dimensions coincide: $b_p(X) = \dim H_p(X) = \dim H^p(X)$ (Betti numbers). If $\dim X = n$, $[\omega_p] \cdot [\omega_{n-p}] = \int \omega_p \wedge \omega_{n-p}$ and $H^p(X) \cong H_{n-p}(X)$ (Poincaré duality). A p-form ω_p is Poincaré dual to an $(n-p)$-cycle c_{n-p} if $\int_{c_{n-p}} \omega_{n-p} = \int \omega_p \wedge \omega_{n-p}\ \forall \omega_{n-p}$.
We define the Hodge star operator $* : \omega_p \to (*\omega)_{n-p}$, $(*\omega)_{\mu_{p+1}\cdots\mu_n} = \frac{\sqrt{g}}{p!} \omega^{\mu_1\cdots\mu_p} \epsilon_{\mu_1\cdots\mu_n}$, $(\omega_p, \alpha_p) = \int_X \omega_p \wedge *\alpha_p$, the co-differential $d^\dagger = (-1)^p *^{-1} d*$ and the Laplace operator $\Delta = d^\dagger d + d d^\dagger$. A form is called harmonic if $\Delta\omega = 0$. It is possible to prove that on a compact manifold X any form has a unique decomposition in an exact, a coexact and a harmonic piece: $\omega = d\alpha + d^\dagger\beta + \gamma$ with $\Delta\gamma = 0$ (and $\beta = 0$ if ω is closed).
On a complex manifold of complex dimension n a 1-form can be expressed as $\omega(z,\bar{z}) = \omega(z,\bar{z})_i dz^i + \omega(z,\bar{z})_{\bar{i}} d\bar{z}^{\bar{i}} = \omega_{(1,0)} + \omega_{(0,1)}$ and $d = dz^i \frac{\partial}{\partial z^i} + d\bar{z}^{\bar{i}} \frac{\partial}{\partial \bar{z}^{\bar{i}}} = \partial + \bar{\partial}$, with $\partial^2 = \bar{\partial}^2 = 0$. The **Dolbeault cohomology** is defined via $H^{p,q} = \frac{\mathrm{Ker}(\bar{\partial}_{p,q})}{\mathrm{Im}(\bar{\partial}_{p,q-1})}$ with **Hodge numbers** $h^{p,q}(X) = \dim H^{p,q}(X)$ and **Hodge decomposition** $H^k = \oplus_{p+q=k} H^{p,q}$.

The Hodge numbers are usually arranged in a **Hodge diamond**, which for a Calabi-

Yau 3-fold (due to $SU(3)$ holonomy) reads

$$\begin{array}{ccccccc} & & & h^{0,0} & & & \\ & & h^{1,0} & & h^{0,1} & & \\ & h^{2,0} & & h^{1,1} & & h^{0,2} & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ & h^{3,1} & & h^{2,2} & & h^{1,3} & \\ & & h^{3,2} & & h^{2,3} & & \\ & & & h^{3,3} & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1. \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{4.34}$$

This implies the existence of a unique harmonic, holomorphic 3-form:

$$\Omega = \Omega_{ijk}(z) dz^i \wedge dz^j \wedge dz^k. \tag{4.35}$$

Let us consider the deformation of the metric $g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}} \to g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}} + \delta g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}} + \delta g_{ij} dz^i dz^j + \text{h.c.}$ (maintaining Ricci-flatness would imply the existence of moduli): a change of the metric of type $\delta g_{i\bar{j}}$ can be directly interpreted as a change of the harmonic representative of the Kähler form J. The number of these independent Kähler deformations is $h^{1,1}$. $h^{1,1} \geq 1$ since it is always possible to simply rescale the metric.
It is now useful to define a (2,1)-form representing a one-to-one map between distinct complex structure deformations and linearly independent Dolbeault cohomology classes of type (2,1):

$$\delta\chi = \Omega_{ij}{}^{\bar{k}} \delta g_{\bar{k}\bar{l}} dz^i \wedge dz^j \wedge d\bar{z}^{\bar{l}}, \tag{4.36}$$

or, equivalently,

$$\delta g_{\bar{i}\bar{j}} = -\frac{1}{||\Omega||^2} \bar{\Omega}_{\bar{i}}{}^{kl} \delta\chi_{kl\bar{j}}, \tag{4.37}$$

with $\delta\Omega = \delta\chi$, $||\Omega||^2 = \frac{1}{3!}\Omega_{ijk}\bar{\Omega}^{ijk}$ and $\delta g_{i\bar{j}} = -i\delta J_{i\bar{j}}$.

A real projective space $\mathbb{R}P^n$ is $\mathbb{R}^{n+1} \setminus \bar{0}$ modulo the equivalence relation $\bar{x} \sim \lambda\bar{x}$ with $\lambda \in \mathbb{R} \setminus 0$. Analogously, the *complex projective spaces* $\mathbb{C}P^n$ are the set of all $(n+1)$-tuples of complex numbers (not all zero) with the equivalence relation $(z^0, ..., z^n) \sim (\lambda z^0, ..., \lambda z^n)$ with $\lambda \in \mathbb{C} \setminus 0$ (all $\mathbb{C}P^n$ are compact). A chart $\phi_i : \{\text{class of } (z^0, ..., z^n)\} \to (\frac{z^0}{z^i}, ..., \frac{z^{i-1}}{z^i}, \frac{z^{i+1}}{z^i}, \frac{z^n}{z^i}) \in \mathbb{C}^n$ is provided for the subset U_i of all equivalence classes in which $z^i \neq 0$. $K^{(i)}(x) = \frac{1}{2}\ln\Big(1 + \sum_{j=1}^n |x^j|^2\Big)$, where $\{x^1, ..., x^n\} = \{\frac{z^0}{z^i}, ..., \frac{z^{i-1}}{z^i}, \frac{z^{i+1}}{z^i}, \frac{z^n}{z^i}\}$, is a Kähler potential in U_i and gives rise to the Fubini-Study metric.

The Kähler form can be decomposed as

$$J = t^\alpha \omega_\alpha, \tag{4.38}$$

where $\alpha = 1, ..., h^{1,1}$. The volume of the Calabi-Yau manifold is

$$\mathcal{V} = \frac{1}{6}\int_X J \wedge J \wedge J = \frac{1}{6}\kappa_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma. \tag{4.39}$$

The components of the vector t^α measure the volumes of the different 2-cycles present in the manifold. $\kappa_{\alpha\beta\gamma}$ are integers called triple intersection numbers of the 4-cycles Poincaré dual to ω_α. The volumes of the dual 4-cycles are

$$\tau_\alpha = \frac{1}{2}\int_{c_4^\alpha} J \wedge J = \frac{1}{2}\kappa_{\alpha\beta\gamma}t^\beta t^\gamma. \tag{4.40}$$

For the type IIB string,

$$c_\alpha = \int_{c^\alpha} C_4, \tag{4.41}$$

$$T_\alpha = \tau_\alpha + ic_\alpha, \tag{4.42}$$

$$K_K = -2\ln\mathcal{V}, \tag{4.43}$$

$$K_{cs} = -\ln\Big(i\int_X \Omega\wedge\bar{\Omega}\Big), \tag{4.44}$$

$$S = C_0 + ie^{-\phi} = C_0 + \frac{i}{g_s} \tag{4.45}$$

and

$$K = K_K(T^\alpha, \bar{T}^{\bar{\alpha}}) + K_{cs}(z^\alpha, \bar{z}^{\bar{\alpha}}) - \ln\big(-i(S-\bar{S})\big), \tag{4.46}$$

where S is called axio-dilaton and $z^a = \int_{A^a}\Omega$, with A^a cycles and $a = 0, ..., h^{2,1}$.

Orientifolds

A requirement for flux compactifications with D-branes is cancellation of all tadpoles associated with the charge and tension of the sources: the gravitational tadpole associated to the positive tension of a D-brane requires the presence of a negative-tension source. Orientifold planes ($O3/O7$ planes) are negative-tension extended objects that are non-dynamical and appear at the fixed point loci (points or four-cycles in X_6) of an involution which reverses the orientation of the string worldsheet.

Chapter 5

Moduli Stabilisation

Moduli are zero-energy deformations arising from the multitude of topologically different cycles in usual Calabi-Yau manifolds. In the next section, we will briefly explore why they are usually problematic [21].

5.1 No-scale structure

We talk about no-scale structure when the Kähler potential satisfies

$$\sum_{I,J=T_i} K^{I\bar{J}} \partial_I K \partial_{\bar{J}} K = 3. \tag{5.1}$$

If the superpotential is independent of the Kähler moduli, this implies that the potential becomes

$$V_F = e^K \sum_{I,J\neq T_i} K^{I\bar{J}} D_I W \overline{D_J W}. \tag{5.2}$$

It is positive semi-definite, $V_F = 0$ when $D_{I\neq T_i} W = 0$ and the minimum is not supersymmetric if $D_{T_i} W \neq 0$.

No-scale model

Let us consider a particular example of no-scale structure. Let S, T and C be three chiral superfields: S is the dilaton (closed string), $\mathrm{Re}\, S = 1/g_s$ and $\mathrm{Im}\, S$ is an axion with perturbative shift symmetry; T is the volume mode (closed string), $\mathrm{vol}(Y_{6D}) = (\mathrm{Re}\, T)^{3/2} l_S^6$ and $\mathrm{Im}\, T$ is an axion with perturbative shift symmetry; C denotes MSSM fields (open string).
For this model, working in units so that $M_p = 1$, the Kähler potential

$$K = -\ln\big(S + \bar{S}\big) - 3\ln\big(T + \bar{T} - \bar{C}C\big) \tag{5.3}$$

is at tree-level. On the other hand, the superpotential

$$W = W_0 + C^3 + Ae^{-aS} \tag{5.4}$$

is the sum of its tree-level contribution ($W_0 + C^3$, where C^3 is a Yukawa-like term) and its non-perturbative correction (Ae^{-aS}).
In this case, the no-scale cancellation reads

$$K^{T\bar{T}} K_T K_{\bar{T}} + K^{T\bar{C}} K_T K_{\bar{C}} + K^{C\bar{T}} K_C K_{\bar{T}} + K^{C\bar{C}} K_C K_{\bar{C}} - 3 = 0, \tag{5.5}$$

hence the potential is

$$V_F = e^K [(S+\bar{S})^2 D_S W D_{\bar{S}} \bar{W} + K^{C\bar{C}} W_C \bar{W}_{\bar{C}}] \geq 0; \tag{5.6}$$

the minimum is at $W_C = 0 \Leftrightarrow \langle C \rangle = 0$ and $D_S W = 0 \Leftrightarrow -aAe^{-aS} - \frac{W}{S+\bar{S}} = 0$. The first thing to notice is that if $A = 0$ the system in unstable because we get a run-away. Since in general the non-perturbative expansion is $W_{np} = \sum_{n\geq 1} A_n e^{-naS}$, if we want $W_{np} \simeq Ae^{-aS}$, we must require that $a\,\mathrm{Re}(S) \gg 1$. For $W_0 > 0$, $\langle \mathrm{Im}(S) \rangle = (2k+1)\frac{\pi}{a}$, with $k \in \mathbb{Z}$, and $a \langle \mathrm{Re}(S) \rangle e^{-a\langle \mathrm{Re}(S) \rangle} \simeq \frac{W_0}{2A}$: this means that W_0 has to be tuned to be exponentially small.
Therefore, $\langle V_F \rangle = 0$ and T is a flat direction (modulus); $\left\langle F^C \right\rangle = 0$, $\left\langle F^S \right\rangle = 0$ but $\left\langle \overline{F}^{\bar{T}} \right\rangle = -\frac{W_0}{2}\sqrt{\frac{g_s}{\langle \tau \rangle}}$, with $T = \tau + i\theta$: this implies that SUSY is broken (it is preserved in the decompactification limit); $T = (\phi_T, \psi_T, F_T)$, ψ_T is the goldstino eaten up by the gravitino and $m_{3/2} = \frac{\sqrt{g_s}}{4}\frac{W_0}{\langle \tau \rangle^{3/2}}$.
The presence of the T flat direction is a problem for the following two main reasons:

- lack of predictability: it is not possible to compute the value of $m_{3/2}$ nor the one for $\mathcal{V}$;

- T is massless and would mediate long-range unobserved fifth force:
  let us consider a stack of N D7-branes wrapped around a 4-cycle: the theory in 4D is a SUSY $SU(N)$ with gauge kinetic function $f = T$; therefore, $\mathrm{Re}(f) = \frac{1}{g^2_{SU(N)}} = \tau$. The 4D EFT lagrangian $\mathcal{L} \supset -[\mathrm{Re}(f) F_{\mu\nu} F^{\mu\nu} + \mathrm{Im}(f) F_{\mu\nu} \tilde{F}^{\mu\nu}]$. Writing $\tau(x) = \langle \tau \rangle + \hat{\tau}(x)$ and $\theta(x) = \langle \theta \rangle + \hat{\theta}(x)$ and since $\mathcal{L}_{kin} = \frac{\partial^2 K}{\partial \Phi_i \partial \bar{\Phi}_{\bar{j}}} \partial_\mu \Phi_i \partial^\mu \bar{\Phi}_{\bar{j}}$, we define the canonically normalised fields $G_{\mu\nu} = 2\sqrt{\langle \tau \rangle} F_{\mu\nu}$, $\sigma(x) = \sqrt{\frac{3}{2}}\frac{\hat{\tau}(x)}{\langle \tau \rangle}$ and $\phi(x) = \sqrt{\frac{3}{2\langle \tau \rangle}} C(x)$. Reintroducing factors of M_p, we find that $\mathcal{L}_{int} \supset \frac{1}{2\sqrt{6}} \frac{\sigma}{M_p} G_{\mu\nu} G^{\mu\nu} + \frac{m_\phi^2}{M_p} \sigma |\phi|^2$.
  This proves that σ couples gravitationally with everything and, since a fifth force has not been seen, we need $m_\tau \gtrsim 1\,\mathrm{meV}$.

Therefore, we need to develop a potential for τ by including sub-leading corrections, which could be: higher derivative corrections to K (α' corrections), loop corrections to K (g_s corrections), non-perturbative corrections to W. In the following, we will partly explore some of these possibilities.

Cosmological moduli problem

During inflation, the positive vacuum energy tends to induce instabilities of massless scalar fields, quantum fluctuations of moduli contribute to the primordial perturbations and the impact on cosmology is complex and profound: moduli can affect Big Bang nucleosynthesis (BBN), overclose the universe, comprise some of the dark matter, decay to dark radiation or mediate long-range interactions.
The equation of motion for σ is $\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2\sigma = 0$. $H_i \gg m_\sigma$ during inflation, subsequently the Hubble parameter H and the temperature T decrease, SM particles are produced, $H \sim \frac{T^2}{M_p}$ and σ starts oscillating when $H \sim m_\sigma$. Since σ interacts with radiation as stated above, we need it to decay before BBN to not spoil its successes. Considering that σ decays when $H \sim \Gamma$ and $\Gamma \sim \frac{1}{M_p^2}m_\sigma^3$, $T_{decay} \sim m_\sigma\sqrt{\frac{m_\sigma}{M_p}} \gtrsim T_{BBN} \sim 1\,\text{MeV}$ and so $m_\sigma \gtrsim 30 - 50\,\text{TeV}$, which is much more stringent than what found above to evade the fifth force issue.

5.2 KKLT scenario

In the KKLT proposal [18], perturbative corrections are not considered and instead non-perturbative contributions to the superpotential are used for constructing stabilized vacua.
The constant Gukov-Vafa-Witten flux superpotential ([19] and [20]) is

$$W_0 = \frac{c}{\alpha'}\int G_3 \wedge \Omega, \tag{5.7}$$

where c is a constant. Non-perturbative effects can be due to strong gauge dynamics (e.g. gaugino condensation) on D7-branes or to instanton contributions from Euclidean D3-branes:

$$W = W_0 + \sum_{i=1}^{h_+^{1,1}} A_i e^{-a_i T_i} + ...^{1}. \tag{5.8}$$

Under these assumptions,

$$K = -2\ln(\mathcal{V}). \tag{5.9}$$

[1] $h^{1,1} = h_+^{1,1} + h_-^{1,1}$, where the subscript denotes the parity of the corresponding two-forms under the orientifold action; Sect. 4.3, **Orientifolds**.

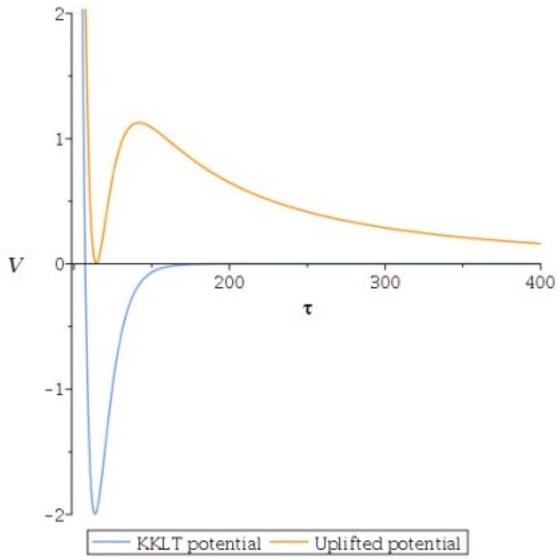

Figure 5.1: Potential in a KKLT scenario with $h_+^{1,1} = 1$. The blue line shows the potential without uplifting, the orange one with uplifting to de Sitter. Source: [21].

Considering for simplicity $h_+^{1,1} = 1$, $\mathcal{V} = (T + \bar{T})^{3/2}$, the vacuum solution is supersymmetric anti-de Sitter space (Fig. 5.1); moreover, the volume is stabilised in a controlled limit only for an exponentially small value of the flux superpotential ($W_0 \ll A$).

5.3 Large Volume Scenario

A perturbative correction to the Kähler potential comes from an $(\alpha')^3$ curvature correction in 10D; this term is part of the classical, higher-curvature 10D SUGRA theory and arises via a four-loop correction to the β-function of the worldsheet, rather than from a loop in spacetime. In the 4D EFT

$$K = -2 \ln \left(\mathcal{V} + \frac{\xi}{2 g_s^{3/2}} \right) , \tag{5.10}$$

where $\xi = -\frac{\chi(X_6)\zeta(3)}{2(2\pi)^3}$, with $\chi(X_6)$ the Euler characteristic of X_6 and $\zeta(3) \simeq 1.202$ is Apéry's constant.
In the Large Volume Scenario, stabilization of the Kähler moduli is achieved by balancing the leading α' correction to K against the non-perturbative superpotential.
Under these assumptions, the perturbative term dominates over the non-perturbative terms at very large volume; competition between these can occur if one or more cycles are exponentially smaller than the largest cycles. Requiring $h_+^{1,1} > 1$ and dividing the Kähler

moduli into two classes, those corresponding to big and small cycles $\{T_i\} = \{T_b^\rho\} \bigcup \{T_s^r\}$, with $r = 1, ..., N_s$ and $\rho = 1, ..., N_b = h_+^{1,1} - N_s$, we consider the large volume limit $\mathcal{V} \to \infty$ with $a_s^r \tau_s^r = \ln \mathcal{V}$ (a canonical class of examples of LVS vacua arise in the so called **Swiss-cheese Calabi-Yau manifolds**). All of this implies the existence of a minimum at exponentially large volume: this vacuum solution is non-supersymmetric AdS.

5.4 Uplifting to de Sitter

Summarising, in KKLT the cycles are not hierarchically different in size, the classical flux superpotential W_0 is fine-tuned to be exponentially small and the AdS vacuum is supersymmetric; in LVS some cycles are exponentially larger than others, W_0 is of order unity and the AdS vacuum is non-superymmetric. However, in both scenarios some form of uplifting effect is necessary to achieve a dS vacuum.
Therefore, both vacua are stabilised (there are no instabilities or flat directions) but are unsuitable for a realistic cosmology, since to describe the early universe (inflation) and the late universe (dark energy) we need de Sitter solutions (i.e. vacua with positive energy).
In order to have uplift, it is needed a sector that breaks SUSY dynamically in a parametrically controlled manner and makes a positive contribution to the vacuum energy without disrupting the stabilisation of the AdS vacuum (a computation of physical parameters in the original anti-de Sitter vacuum will not necessarily give an accurate prediction for these quantities in the de Sitter solution) [22].

Chapter 6

Brane Inflation

Before exploring the possible inflationary scenarios in the context of string theory (following [15]), let us recall that the fundamental scale of string theory is the string scale $M_s = (\alpha')^{-1/2}$. At energies below M_s, only the massless states of the string are excited and the theory reduces to an effective SUGRA in ten dimensions. Compactification on an internal space of volume $\mathcal{V}M_s^{-6}$ introduces the Kaluza-Klein scale $M_{KK} = M_s\mathcal{V}^{-1/6}$. It is usually assumed that $M_{KK} \ll M_s$, so that the theory is a 10D supergravity for intermediate energies, while it reduces to a 4D effective theory at energies lower than M_{KK}. The 4D Planck scale is a derived scale (eq. 4.30): $M_p \sim \frac{1}{g_s}(\frac{M_s}{M_{KK}})^3 M_s \gg M_s$ if assuming that the metric in the Einstein frame and the metric in the string frame are the same at the vacuum and $M_p \sim (\frac{M_s}{M_{KK}})^3 M_s \gg M_s$ if assuming that volumes are frame dependent also in the vacuum (Sect. 8.1, **String frame and Einstein frame**). The scale of supersymmetry breaking M_{SUSY} in the early universe is instead the highest scale of SUSY breaking that is unrelated to inflation. If $H > M_{SUSY}$, supersymmetry is only spontaneously broken during inflation and can partially protect against radiative corrections.
In this chapter we will only explore models of brane inflation, other interesting possibilities are **axion inflation**, **Kähler modulus inflation** and **dissipation inflation**.

6.1 Unwarped Brane Inflation

In this section we will consider inflationary models in unwarped regions.

D3/D7 Inflation

Let us work in a background geometry which is a compactification on the orientifold $K3 \times T^2/\mathbb{Z}_2$. At each of the four fixed points there are four D7-branes atop an O7-plane, all of which wrap the $K3$ manifold. Adding a spacetime-filling D3-brane sitting

on $T^2/\mathbb{Z}_2$, its position on $T^2/\mathbb{Z}_2$ relative to the stack of D7-branes is proposed to be the inflaton ϕ. $\mathcal{N}=2$ SUSY is preserved in 4D, there is no potential for D3-brane motion and the inflaton is massless.
But introducing the two-form flux $\mathcal{F}_2$ in the D7-brane worldvolume, when the flux is not self-dual in the worldvolume, breaks SUSY and the D3-brane feels a force: the worldvolume flux corresponds to a field dependent Fayet-Iliopoulos D-term ξ. It is possible to prove that

$$V_D(\phi) = \frac{g^2\xi^2}{2}\left(1+\frac{g^2}{16\pi^2}U(x)\right), \tag{6.1}$$

where g is the coupling of the $U(1)$ gauge field, $x=\frac{\phi}{\sqrt{\xi}}$ and $U(x)=(x^2+1)^2\ln(x^2+1)+(x^2-1)^2\ln(x^2-1)-4x^4\ln x-4\ln 2$.
Mixing of D3-brane position moduli with Kähler moduli implies that stabilisation of the volume generically leads to stabilisation of the D3-brane position; moreover, there are no inflaton mass terms from Kähler potential couplings.
A model of D-term inflation in a compactification stabilised by superpotential terms for the moduli is not a pure D-term scenario and the moduli sector introduces masses in the inflaton sector:

$$V(\phi) = V_D(\phi) - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4. \tag{6.2}$$

Inflation in the D3/D7 model is possible once moduli stabilization is properly incorporated but necessarily involves fine-tuning; the kinematical field range $\Delta\phi$ of the canonically-normalized inflaton can be super-Planckian ($\Delta\phi > M_p$) if the $T^2/\mathbb{Z}_2$ is highly anisotropic.

Fluxbrane Inflation

A possibility to obtain inflation with D-branes is to consider a pair of branes separated in the compact space, almost parallel and misaligned by a small relative angle θ. θ leads to controllably small SUSY breaking and to a force that draws the branes together, merging and reheating the universe.
Generically, the potential for the interaction of a brane-antibrane pair is too steep for successful inflation in an unwarped compact space. Weak SUSY breaking by $\theta \ll 1$ would diminish the Coulomb force so that the interaction potential could drive slow-roll for non-compact internal dimensions, but the effect of compactification is to make the interaction potential for the branes comparable to the vacuum energy.
In a compactification on an O3/O7 orientifold of a Calabi-Yau three-fold X_6, let us suppose that there is a continuous family Σ_4 of four-cycles, on any representative of which a D7-brane can be wrapped. The inflaton coordinate is the effective separation of a pair of intersecting D7-branes:

$$V(\phi) = V_F(\phi) + V_D(\phi), \tag{6.3}$$

where $V_F(\phi)$ results from moduli stabilisation and

$$V_D(\phi) = V_0 \left(1 + \alpha \ln\left(\frac{\phi}{\phi_0}\right)\right), \tag{6.4}$$

with V_0, α and ϕ_0 constants.
The D7/D7 interaction potential due to flux can be made flat enough for inflation to happen.

6.2 Relativistic Brane Inflation

Inflation in systems driven by slowly moving D-branes suffers from the η problem (i.e. $|\eta| \sim \mathcal{O}(1)$). Therefore, in this section we will consider D-branes that move relativistically.

Dirac-Born-Infeld Inflation

Let us imagine a framework with a spacetime-filling D3-brane probing $AdS_5 \times S^5$:

$$\mathrm{d}s^2 = \left(\frac{r}{L}\right)^2 \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \left(\frac{L}{r}\right)^2 (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{S^5}), \tag{6.5}$$

where $\frac{L^4}{(\alpha')^2} = 4\pi g_s N$ and N is the total D3-brane charge of the background. The D3-brane lagrangian reads

$$\mathcal{L} = -\frac{\phi^4}{\lambda}\left(\sqrt{1 + \frac{\lambda}{\phi^4}(\partial\phi)^2} - 1\right) - V(\phi), \tag{6.6}$$

with

$$\phi = \sqrt{T_3} r \tag{6.7}$$

and $\lambda = T_3 L^4 = \frac{N}{2\pi^2}$.
In this spacetime, a probe D3-brane does not feel any force, but a potential for its motion is generated by physical effects related to the IR and UV deformations of the spacetime (see Sect. 6.3 for more details on the Klebanov-Strassler geometry). Both SUSY breaking in the IR (for example by an anti-D3-brane) and SUSY breaking and moduli stabilisation in the UV lead to a potential for the D3-brane position.
Considering warped backgrounds,

$$\mathcal{L} = -T(\phi)\left(\sqrt{1 + \frac{(\partial\phi)^2}{T(\phi)}} - 1\right) - V(\phi), \tag{6.8}$$

where $T(\phi) = T_3 e^{4A(\phi)}$ and $e^{4A(\phi)}$ is the warp factor. Generalizing the metric in eq. 6.5, we use Calabi-Yau cones approximating finite warped throat regions:

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(r)}(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{X_5}), \tag{6.9}$$

where X_5 is an arbitrary Einstein manifold, $e^{-4A(r)} \simeq \frac{L^4}{r^4}$, $\frac{L^4}{(\alpha')^2} = \frac{4\pi^4 g_s N}{\mathrm{Vol}(X_5)}$ and $\mathrm{Vol}(X_5)$ is the volume of X_5 in string units. In this case, as in eq. 6.32,

$$\frac{\Delta\phi}{M_p} \leq \frac{2}{\sqrt{N}}. \tag{6.10}$$

Defining the Lorentz factor

$$\gamma = \left(1 - \frac{\dot{\phi}^2}{T(\phi)}\right)^{-1/2}, \tag{6.11}$$

relativistic brane dynamics occurs for $\gamma \gg 1$. From the lagrangian in eq. 6.6, we compute the stress-energy tensor and it is the one of a perfect fluid with $\rho = (\gamma - 1)T + V$ and $p = (\gamma - 1)\frac{T}{\gamma} - V$. Therefore, $3M_p^2 H^2 = (\gamma - 1)T + V(\phi)$ and $\dot{\phi} = -\frac{2M_p^2 H'}{\gamma}$, where $H' = \frac{\mathrm{d}H}{\mathrm{d}\phi}$.
We can now find that $\varepsilon = -\frac{\dot{H}}{H^2} = \frac{2M_p^2}{\gamma}(\frac{H'}{H})^2$ and $\tilde{\eta} = \frac{\dot{\varepsilon}}{H\varepsilon} = \frac{2M_p^2}{\gamma}[2(\frac{H'}{H})^2 - 2\frac{H''}{H} + \frac{H'\gamma'}{H\gamma}]$. The slow-roll parameters are suppressed for large γ. Accelerated expansion occurs if the potential energy dominates over the kinetic energy: $\frac{V}{\gamma T} \gg 1$. From these relations we find that $\gamma^2 = \frac{M_p^2}{3}(\frac{V'}{V})^2 \frac{V}{T} \gg 1$ (notice that, since the D3-brane is relativistic and is accelerated by the potential, it will emit gravitational and scalar synchrotron radiation into the compact dimensions; losses due to bremsstrahlung dominate the dynamics in a significant fraction of parameter space, including the regime of weak string coupling and large volume).
DBI inflation is a particular case of $P(X)$ theories:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_p^2}{2}R + P(X,\phi)\right), \tag{6.12}$$

where $X = -\frac{1}{2}(\partial\phi)^2$ and P is an arbitrary function. For DBI inflation

$$P(X,\phi) = -T(\phi)\left(\sqrt{1 - \frac{2X}{T(\phi)}} - 1\right) - V(\phi). \tag{6.13}$$

A non-trivial warp factor can lead to a field dependence of the sound speed. DBI inflation in a CY cone cannot have both detectable non-Gaussianity and detectable tensors.

6.3 Warped Brane Inflation

The positions of localized sources in a string compactification correspond to scalar fields in the 4D EFT. Recall that a D3-brane and an anti-D3-brane attract each other both gravitationally and through the C_4 potential. The Coulomb interaction of the brane-antibrane pair decreases with increasing distance: in order to drive slow-roll inflation, the distance between the branes should be larger than the size of the compact space [25]. The **KKLMMT proposal** [26] consists in noticing that warping of extra dimensions suppresses the Coulomb force between the brane-antibrane pair (flattening the potential even for small separations) and that the leading contributions to the curvature of the inflaton potential come from the physical effects that stabilise the moduli.

AdS

For a stack of N D3-branes in 10D Minkowski spacetime, the metric is

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(r)}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{S^5}), \tag{6.14}$$

where $e^{-4A(r)} = 1+\frac{L^4}{r^4}$ and $L^4 = 4\pi g_s N(\alpha')^2$. For $r \ll L$, this metric reduces to $AdS_5\times S^5$ (eq. 6.5). For a D3-brane

$$S_{D3} = -T_3\int \mathrm{d}^4\sigma\sqrt{-\det(G^E_{ab})} + \mu_3\int C_4. \tag{6.15}$$

The D3-brane is filling the spacetime and r is its radial location in AdS. Therefore,

$$\mathcal{L} = -T_3 e^{4A(r)}\sqrt{1+e^{-4A(r)}g^{\mu\nu}\partial_\mu r\partial_\nu r} + T_3 e^{4A(r)}. \tag{6.16}$$

The canonically normalised field is

$$\phi = \sqrt{T_3}r \tag{6.17}$$

and, for small velocities ($\dot{r}^2 \ll e^{4A(r)}$),

$$\mathcal{L} \simeq -\frac{1}{2}(\partial\phi)^2. \tag{6.18}$$

The electrostatic repulsion from the four-form background cancels the gravitational attraction in the AdS background: a single D3-brane does not experience any force.[1]

[1]For the antibrane the force exerted by gravity and the four-form field are of the same sign and add.

Conifold

A 6D Calabi-Yau cone X_6 for which, in $\mathbb{C}^4$,

$$\sum_{A=1}^{4} z_A^2 = 0 \tag{6.19}$$

is called *singular conifold*; it describes a cone, topologically equivalent to $S^2 \times S^3$, whose base is the Einstein manifold[2] $T^{1,1}$:

$$T^{1,1} = [SU(2) \times SU(2)]/U(1), \tag{6.20}$$

$$\mathrm{d}\Omega^2_{T^{1,1}} = \frac{1}{9}\left(\mathrm{d}\psi + \sum_{i=1}^{2} \cos\theta_i \mathrm{d}\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\right), \tag{6.21}$$

where $\theta_i \in [0, \pi]$, $\phi_i \in [0, 2\pi]$ and $\psi \in [0, 4\pi]$ [27].
Therefore,

$$k(z_\alpha, \bar{z}_\alpha) = \frac{3}{2}\left(\sum_{A=1}^{4} |z^A|^2\right)^{2/3}. \tag{6.22}$$

A stack of N D3-branes at $z_A = 0$ backreacts on the geometry:

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(r)}(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{T^{1,1}}), \tag{6.23}$$

where $e^{-4A(r)} = 1 + \frac{L^4}{r^4}$ and $\frac{L^4}{(\alpha')^2} = \frac{27\pi}{4} g_s N$.

The base manifold in the singular conifold has zero size for $z_A = 0$. To avoid this curvature singularity, we introduce the *deformed conifold* via

$$\sum_{A=1}^{4} z_A^2 = \epsilon^2. \tag{6.24}$$

Far from the tip of the cone the metric is approximately that of the singular conifold, at the tip the S^3 is finite and S^2 has zero size.

The deformed conifold contains two independent three-cycles, i.e. the S^3 at the tip (A-cycle) and the Poincaré dual three-cycle (B-cycle). The background three-form fluxes are quantized:

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_3 = M \tag{6.25}$$

[2] An Einstein manifold is a manifold satisfying $R_{ab} \propto g_{ab}$.

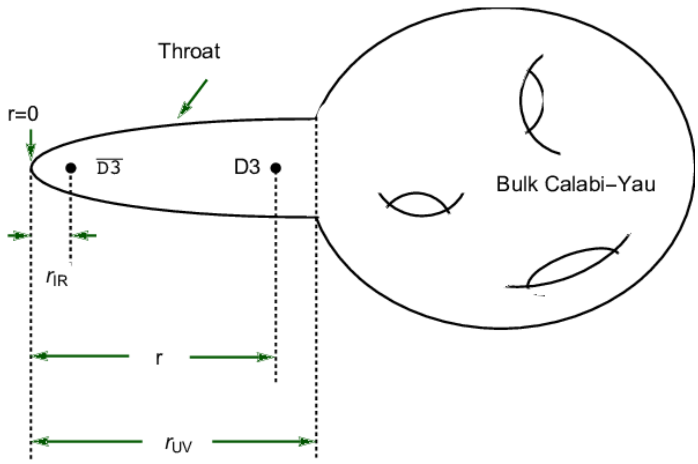

Figure 6.1: Klebanov-Strassler is the part of the warped throat in this geometry. Source: [28].

and

$$\frac{1}{(2\pi)^2\alpha'}\int_B H_3 = K, \tag{6.26}$$

with M and K integers such that $M \gg 1$, $K \gg 1$. Under these conditions,

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(r)}\mathrm{d}\tilde{s}^2, \tag{6.27}$$

where $\mathrm{d}\tilde{s}^2$ is the metric of the deformed conifold: this is the **Klebanov-Strassler geometry**.
This 10D solution involving a non-compact warped deformed conifold does not give rise to dynamical gravity when dimensional reducing to 4D; hence, we study a flux compactification with a finite warped throat region approximated by a finite portion of the KS solution: from $r = r_{IR}$ (the tip) to $r = r_{UV}$ (an ultraviolet cutoff). The throat attaches to a bulk space (Fig. 6.1).
The IR geometry is smooth and the A-cycle has radius $r_A = \sqrt{g_s M \alpha'}$. Far from the tip

$$e^{-4A(r)} = \frac{L^4}{r^4}\left(1 + \frac{3g_s M}{8\pi K} + \frac{3g_s M}{2\pi K}\ln\left(\frac{r}{r_{UV}}\right)\right), \tag{6.28}$$

with $\frac{L^4}{(\alpha')^2} = \frac{27\pi}{4} g_s N$ and $N = MK$. The warp factor is minimal for $r = r_{IR}$:

$$e^{A_{IR}} = e^{-\frac{2\pi K}{3 g_s M}}. \tag{6.29}$$

The total compactification volume is

$$\mathcal{V} = \mathcal{V}_B + \mathcal{V}_T, \tag{6.30}$$

where $\mathcal{V}_B$ is the volume of the bulk space and $\mathcal{V}_T$ is the throat volume:

$$\mathcal{V}_T = \int \mathrm{d}\Omega^2_{T^{1,1}} \int_{r_{IR}}^{r_{UV}} \mathrm{d}r r^5 e^{-4A(r)} = 2\pi^4 g_s N (\alpha')^2 r_{UV}^2. \tag{6.31}$$

From eq. 4.30, $M_p^2 > \frac{N r_{UV}^2}{4(2\pi)^3 g_s (\alpha')^2}$ and, since $\Delta\phi^2 < T_3 r_{UV}^2 = \frac{r_{UV}^2}{(2\pi)^3 g_s (\alpha')^2}$,

$$\frac{\Delta\phi}{M_p} \leq \frac{2}{\sqrt{N}} \tag{6.32}$$

(see eq. 6.10). Considering that $N \gg 1$ for the SUGRA approximation to be valid, observable gravitational waves are not allowed because of the Lyth bound, i.e. $\Delta\phi \gtrsim \left(\frac{r}{0.01}\right)^{1/2}$.

Potential

An anti-D3-brane added to the compactification minimises its energy being stabilised at the tip of the conifold; it perturbs the background SUGRA solution and the D3-brane experiences a force described by the Coulomb potential

$$V_C(\phi) = D_0 \left(1 - \frac{27}{64\pi^2} \frac{D_0}{\phi^4}\right), \tag{6.33}$$

where

$$D_0 = 2T_3 e^{4A_{IR}}, \tag{6.34}$$

with $D_0 \ll 2T_3$. According to the KKLMMT proposal, this potential is flat enough for slow-roll inflation to happen even for modest separations. To couple the system to dynamical gravity, we need to include

$$V_R(\phi) = \frac{1}{12} R\phi^2. \tag{6.35}$$

In 4D dS, $R = 12H^2$. Therefore,

$$V(\phi) = V_C(\phi) + V_R(\phi) + ... \simeq V_0 + H^2\phi^2 \tag{6.36}$$

and $\eta \simeq \frac{2}{3}$: the curvature coupling causes the eta problem.
Notice that the volume of the compactification is pertubed by the D3-brane and so depends on its position: $\mathcal{V} = \mathcal{V}(\phi)$.

Backreaction

The backreaction of a D3-brane on the compactification volume leads to

$$\mathcal{V} = (T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha))^{3/2}, \tag{6.37}$$

where $\gamma = \frac{T_3}{6}(T + \bar{T})|_{r_{IR}}$, and

$$K(Z^I, \bar{Z}^I) = -3\ln\big(T + \bar{T} - \gamma k(z_\alpha, \bar{z}_\alpha)\big). \tag{6.38}$$

On the other hand,

$$|\Delta W| \propto e^{-\frac{2\pi}{N_c}\mathcal{V}_4}, \tag{6.39}$$

under gaugino condensation on a stack of N_c D7-branes, with $\mathcal{V}_4$ the volume wrapped by the D7-branes.

Quasi-Single-Field Inflation

The natural mass scale of the EFT for D3-brane inflation is the Hubble parameter. The D3-brane position is parametrised by six real scalar fields that hence will in general have masses of order H. For inflation to occur, a scalar field should have, for some reason, a mass $m \ll H$. If the others have masses $m \gg H$, inflation is effectively governed by single-field dynamics, but generically their evolutions and fluctuations are not negligible and therefore quasi-single-field inflation happens.

Phenomenology

In all explored models, inflation is confined to a small part of the throat, where the potential is tuned flat, and the tensor amplitude is much smaller than the maximum permitted by the geometric bound. Therefore, in warped D-brane inflation, gravitational waves are unobservable.

As already announced, a proper description of warped brane inflation needs us to take into account multi-field effects. We now want only to state three important points: (i) multi-field effects are usually exponentially suppressed after 10 e-folds but are significant before; (ii) the lightest field is often tachyonic, while one field has mass $m \sim H$ and the others have masses $m > \frac{3}{2}H$. So there is one instability, two fields fluctuate and five entropic perturbations decay exponentially after exiting the horizon (adiabatic limit); (iii) if $N_e \gg 70$ for a model, single-field is a valid approximation; if $N_e < 70$, multi-field effects influence the observable anisotropies.

Chapter 7

RG-Induced Modulus Stabilisation: Perturbative de Sitter Vacua

Mainly following [1], in this chapter we will adapt to string theory a perturbative method for stabilising moduli without leaving the domain of perturbative control. The standard renormalization-group resummation of leading logarithms allows us to work at fixed order in the perturbative parameter α and to all orders in $\alpha \ln \tau$, where τ is an extra-dimensional modulus, minimising the potential at $\tau \sim e^{1/\alpha}$. This formalism can be applied to warped $\mathrm{D3} - \overline{\mathrm{D3}}$ inflation, evading the η-problem and providing a dynamical mechanism whereby inflationary scales are much larger than late-time physical (for instance SUSY breaking) scales.

7.1 The Dine-Seiberg problem

Weak string coupling is an expansion in powers of the string dilaton $e^{\hat{\phi}} = 1/s$ and the extra-dimensional size is expressed via the volume modulus $\mathcal{V} = \tau^{3/2}$ (it measures its overall volume in string units). Fields like s and $\mathcal{V}$ are moduli and need to be stabilised to make practical predictions (since particle masses and couplings depend on the stabilised values). The potential to be minimised is generically an expansion of the form

$$V(s, \tau) = \sum_{n,m} A_{nm} s^{-n} \tau^{-m}. \tag{7.1}$$

As pointed out by Dine and Seiberg in [24], this leads to a problem:
if the leading term is positive, the scalar potential goes to zero as $1/s \to 0$ and $\tau \to \infty$, but this corresponds to 10 dimensional flat spacetime (it does not describe what we see around us and lies beyond the reach of the 4D effective theory). Dine and Seiberg claimed in their paper that, if the potential has a non trivial minimum (required to avoid the runaway), different orders in the expansion must compete with one another, but this

would signal the breakdown of the perturbative expansion itself. Therefore, the generic weak-coupling situation is a runaway without a non trivial minimum and, on the other hand, if it exists, a non trivial minimum should generically be at strong coupling with an extra-dimensional volume of order the string scale ($s \sim \tau \sim \mathcal{O}(1)$).
Let us now consider, instead, the expansion

$$V(\tau) = \sum_n A_n(s)\tau^{-n}, \tag{7.2}$$

where A_n depends on all of the other moduli. The key observation is that A_n is not independent on τ but generically can depend logarithmically on it (quantum corrections introduce anomalous scalings into effective interactions that, in a perturbative regime, become logarithmic dependences on ratios of particle masses and imply logarithmic dependences on τ, since in string theory particle masses depend on it).
Therefore, standard renormalization group (RG) methods can be used to resum leading-log effects. For weak coupling, τ is stabilised at exponentially large values (explaining large hierarchies) without losing perturbative control.

7.2 de Sitter vacua

As already anticipated, in this section we will show how RG techniques can be used to stabilise moduli maintaining perturbative control and how accidental approximate scale invariance allows, in this context, SUSY breaking and dS solutions with large hierarchies.

4D perspective

The minimal supersymmetric and accidental approximate scaling symmetric low energy 4D EFT requires the gravity multiplet and a chiral superfield $\mathcal{T}$ containing the complex scalar

$$T = \frac{1}{2}(\tau + i\mathfrak{a}), \tag{7.3}$$

where τ is the dilaton and $\mathfrak{a}$ is an axion. This SUGRA effective theory is described at the two-derivative level by the Kähler potential, the superpotential and (if there are gauge multiplets) the gauge kinetic function. We consider for the moment

$$W = w_0 \tag{7.4}$$

to be a constant and, according to what already stated,

$$e^{-K/3} = \tau - k + \frac{h}{\tau} + ..., \tag{7.5}$$

where we neglect higher order in $1/\tau$ and we do not track the s dependence; we will search for minima of the potential where this approximation is valid, i.e. for $\tau \gg 1$. In a string context, the field τ is the extra-dimensional volume (a Kähler modulus) and hence the effective theory condition is equivalent to having geometries that are much larger than the string scale. Notice also that we are working in units such that $M_p = 1$ and we will reinsert the appropriate factors when useful for an appropriate physical interpretation. Since quantum effects could complicate the scaling properties of subdominant terms in the lagrangian, we consider that $k = k(\ln \tau)$ and similarly for the terms that we are now going to neglect. The scalar potential is

$$V = e^K (K^{\bar{T}T} \overline{D_T W} D_T W - 3|W|^2), \tag{7.6}$$

where $D_T W = W_T + K_T W \simeq -\frac{3}{\tau} w_0$. Therefore

$$V \simeq -\frac{3k_{T\bar{T}}}{\mathcal{P}^2}|w_0|^2 \simeq \frac{3(k' - k'')}{\tau^4}|w_0|^2, \tag{7.7}$$

where we neglect terms of order $\mathcal{O}(\tau^{-5})$, define

$$\mathcal{P} = e^{-K/3} \tag{7.8}$$

and denote differentiation with respect to $x = \ln \tau$ with primes. Note that, if h and other subdominant terms (first arising at order $\mathcal{O}(\tau^{-5})$) vanish and k is independent on T, V has a no-scale structure and becomes zero. Notice also that the kinetic terms are given by

$$-\frac{\mathcal{L}_{kin}}{\sqrt{-g}} = \frac{1}{2}R + K_{T\bar{T}}\partial_\mu T \partial^\mu \bar{T} \simeq \frac{1}{2}R + \frac{3}{\tau^2}\partial_\mu T \partial^\mu \bar{T}. \tag{7.9}$$

We can then write

$$V(\tau) \simeq \frac{U(\ln \tau)}{\tau^4} \tag{7.10}$$

and let us assume that k acquires its dependence on $\ln \tau$ via a perturbative expansion in a running dimensionless coupling $\alpha_g \ll 1$:

$$k = k_0 + k_1 \alpha_g + \frac{k_2}{2}\alpha_g^2 + ... \tag{7.11}$$

In general, the RG evolution introduces logarithms of mass ratios which could develop a dependence on $\mathcal{P}$ (e.g. particles localised on D3 and D7-branes have masses that depend differently on the volume modulus in IIB compactifications) and therefore the running of α_g is expressed through

$$\tau \frac{\mathrm{d}\alpha_g}{\mathrm{d}\tau} = \beta(\alpha_g) = b_1 \alpha_g^2 + b_2 \alpha_g^3 + ... \tag{7.12}$$

Neglecting the sub-leading terms,

$$\alpha_g(\tau) = \frac{\alpha_{g0}}{1 - b_1 \alpha_{g0} \ln \tau}, \tag{7.13}$$

where α_{g0} is an integration constant. This implies that

$$U = U_1 \alpha_g^2 - U_2 \alpha_g^3 + U_3 \alpha_g^4 + ... \tag{7.14}$$

We now assume that U_1, U_2 and U_3 are positive and satisfy

$$\frac{U_1}{U_2} \sim \frac{U_2}{U_3} \sim \epsilon, \tag{7.15}$$

where $\epsilon \ll 1$ is a numerical parameter. In this case $\frac{\partial V}{\partial \tau}|_{\tau_0} = 0$ for $\alpha_0 \sim \epsilon$ and $\tau_0 \sim e^{1/\epsilon}$ if $\epsilon \ll \alpha_{g0}$ and $b_1 < 0$, validating the expansion in $1/\tau$ a posteriori. There is a local minimum for the potential at τ_0 and a local maximum at $\tau_1 > \tau_0$ if $9U_2^2 > 32 U_1 U_3$; the potential is positive at the minimum for $U_2^2 < 4U_1 U_3$ (this is the case in which we are interested). Moreover, $U(\tau_0) \sim \epsilon^5$ and SUSY is broken since $F^T \neq 0$ (notice that $W_T = 0$ and $\frac{K_T W}{M_p^2} \to 0$ as $M_p \to \infty$: SUSY is not broken in the global case).

10D perspective

The action for the massless bosonic fields in the 10D SUGRA below the string scale, at the two-derivative level, is

$$S = \int \mathrm{d}^{10}x \sqrt{-\tilde{g}} \left(\tilde{R} - \frac{|\partial \mathcal{S}|^2}{(\mathrm{Re}\, \mathcal{S})^2} - \frac{|G_3|^2}{\mathrm{Re}\, \mathcal{S}} - \tilde{F}_5^2 \right) + \int \frac{C_4 \wedge G_3 \wedge \overline{G}_3}{\mathrm{Re}\, \mathcal{S}}, \tag{7.16}$$

where $\mathcal{S} = s - iC$, C is an axionic scalar, $G_3 = H_3 + i\mathcal{S}F_3$, $H_3 = \mathrm{d}B_2$, $F_3 = \mathrm{d}C_2$ and $\tilde{F}_5 = \mathrm{d}C_4 + \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$.
This action has an accidental $SL(2,\mathbb{R})$ symmetry under which $\mathcal{S} \to \frac{a\mathcal{S} - ib}{ic\mathcal{S} + d}$ and $G_3 \to \frac{G_3}{ic\mathcal{S}+d}$ with $ad - bc = 1$ and an accidental approximate scale invariance under which $\tilde{g}_{MN} \to \lambda \tilde{g}_{MN}$, $\mathcal{S} \to \mathcal{S}$, $B_2 \to \lambda B_2$, $C_2 \to \lambda C_2$ and $C_4 \to \lambda^2 C_4$ (the tree-level action scales like $S \to \lambda^4 S$ and the volume modulus scales like $\mathcal{V} \to \lambda^3 \mathcal{V}$). These symmetries are broken by α' (in 4D the α' expansion is in inverse powers of $\mathcal{V}$) and loop (in 4D the string loop expansion is in powers of $e^{\hat{\phi}}$) corrections to the effective action (the volume modulus and the string dilaton are their pseudo-Goldstone dilaton modes).
The leading form for K is

$$K(T, \bar{T}) = -2 \ln \mathcal{V} = -3 \ln \tau. \tag{7.17}$$

The no-scale structure ($K^{\bar{A}B} K_{\bar{A}} K_B = 3$) and the condition $W_T = 0$ (due to the axionic shift symmetry of $\mathfrak{a}$; notice that W could depend on T exponentially if this symmetry is anomalous) imply that the scalar potential does not depend on τ, which is then a modulus not fixed at leading order in α' and g_s. Stabilisation can occur via mechanisms such as those described in Chapter 5, but we here consider only perturbative corrections (in powers of $1/s$ and $1/\tau$) to K and no dependence on T for W:

$$e^{-K/3} = s^{1/3}\tau \sum_{n,m,r} \mathcal{A}_{nmr} \left(\frac{1}{s}\right)^n \left(\frac{s}{\tau}\right)^{(m+r)/2}, \tag{7.18}$$

with $\mathcal{A}_{nmr}$ functions of all the other moduli and of $\ln\tau$. Being interested only in the volume dependence,

$$K(T,\bar{T}) = -3\ln\mathcal{P}, \tag{7.19}$$

with

$$\mathcal{P}(\tau) = \tau\left(1 - \frac{k}{\tau} + \frac{h}{\tau^{3/2}} + \mathcal{O}\left(\frac{1}{\tau^2}\right)\right), \tag{7.20}$$

where we assume $g = 0$ in the expansion $\mathcal{P} = \tau(1+\frac{g}{\sqrt{\tau}}-\frac{k}{\tau}+...)$ since no known Calabi-Yau produces these terms and the corresponding corrections to the scalar potential, which are of order $\tau^{-7/2}$, are not generically present at leading order in string loops. Note that if k were T independent, $K^{\bar{A}B}K_{\bar{A}}K_B = 3$ for $\mathcal{P} \simeq \tau - k$, which is the extended no-scale property; in this case, the leading contribution to the scalar potential would come from h at order $\tau^{-9/2}$ ($(\alpha')^3$ corrections).
In this approach, instead of balancing (and hence possibly ruining the validity of the expansion) different powers of $1/\tau$, powers of $\alpha_g \ln\tau$ are balanced to obtain the minimum via the requirement that $k = k(\ln\tau)$. This gives an exponentially large volume and $1/s$ acquires naturally the role of α_g; hence, similarly to LVS, a second expansion modulus is required (but this does not need to be a Kähler modulus and an uplifting mechanism to dS is not necessary).

Yoga models

As explored in [29], in order to have the minimum of the scalar potential at the value of the observed dark energy density, $\tau_0 \gtrsim 10^{26}$ is needed and so the τ field and its axionic partner are light enough to be cosmologically active in the recent universe (notice also, from eq. 7.9, that the axion decay constant is of order $\frac{M_p}{\tau_0}$). Moreover, so large values for τ_0 are problematic in the present context since

$$M_s \sim \frac{M_p}{\mathcal{V}^{1/2}} \sim \frac{M_p}{\tau^{3/4}} \tag{7.21}$$

and

$$M_{KK} \sim \frac{M_s}{\mathcal{V}^{1/6}} \sim \frac{M_p}{\mathcal{V}^{2/3}} \sim \frac{M_p}{\tau}. \tag{7.22}$$

The constraint from M_{KK} can be evaded if not all extra dimensions have the same size because the lower bound on M_{KK} can be much smaller than 10^4GeV if these extra dimensions can only be probed using gravitational strength interactions. On the other hand, $M_s \gtrsim 10$ TeV implies $\tau_0 \lesssim 10^{20}$.

Supersymmetry breaking

As already seen, in order to avoid the cosmological modulus problem, we must require $m_\tau \gtrsim 30$ TeV, where

$$m_\tau \sim \left(\tau^2 \frac{\partial^2 V}{\partial \tau^2}\right)^{1/2} \sim \frac{\epsilon^{5/2}|w_0|}{\tau^2}. \tag{7.23}$$

Given the Kähler potential and the superpotential,

$$m_{3/2} \sim \frac{|w_0|}{\tau^{3/2}}, \tag{7.24}$$

where $|w_0| \lesssim \tau_0^{1/2}$, since for a 4D SUGRA EFT the gravitino mass has to be smaller than the Kaluza-Klein scale. If a Standard Model multiplet ψ^i appears via $k(\psi, \bar{\psi})$ (e.g. for states sequestered in local D3 or D7 branes), soft SUSY breaking masses are

$$m_\psi = (m_{3/2}^2 - F^i F^{\bar{j}} \partial_i \partial_{\bar{j}} \ln Z_\psi)^{1/2} \sim \frac{w_0}{\tau^2}, \tag{7.25}$$

where $Z_\psi \sim \partial_{i\bar{j}} K \sim -\frac{k_{i\bar{j}}}{\tau}$ and $F^T = e^{K/2} K^{T\bar{T}} K_T W \sim \frac{w_0}{\tau^{1/2}}$ and $F^S \simeq e^{K/2} K^{S\bar{S}} K_S w_0 \sim \frac{w_0}{\tau^{5/2}}$; gaugino masses are

$$M_G = \frac{F^i \partial_i f}{\mathrm{Re}\, f} \sim \frac{w_0}{\tau^{5/2}}. \tag{7.26}$$

Therefore, gauginos are lighter than scalars and both of them and τ are lighter than the gravitino. Considering the constraint for τ coming from the cosmological modulus problem, $M_G \sim \mathcal{O}(1)$TeV for $\tau_0 \sim 10^6$ (notice anyway that the contribution from the Standard Model cycle could ruin the sequestering making all soft terms of order the gravitino mass).

Field expansions, perturbation theory, runaway

As already seen at the beginning of this chapter, a potential of the form $V(\tau) = \sum_n \frac{V_n}{\tau^n}$ leads to the Dine-Seiberg problem, since $V'(\tau_0) = 0$ needs at least two terms of the series to compete in size. Nonetheless, this does not necessarily mean that solutions only occur outside the perturbative domain because some coefficients of the expansion could be unusually small (this is essentially equivalent to the requirement that $|w_0|$ must be tuned to be small in KKLT with a single modulus).

Differently, in the LVS context, the potential is a multiple expansion in two small functions of the two independent moduli, so that the minimum can be found through balancing different terms without spoiling the validity of the perturbative control or assuming hierarchies for some coefficients.
On the other hand, RG-induced modulus stabilisation allows to use a fixed order in $1/\tau$, $V(\tau) \simeq \frac{U(\ln\tau)}{\tau^4}$, where U acquires its dependence on $\ln\tau$ via an expansion in a second parameter $\alpha_g(\tau)$. A minimum for V exists if different orders in α_g balance, consistently with $\alpha_g(\tau_0) \ll 1$. The renormalization group assures the reliability of the solution for $\alpha_g \ln\tau \sim \mathcal{O}(1)$.

Let us now note very briefly that the expansion in $1/\tau$ (and analogously for $1/s$) is valid only for large enough τ, but the limit $\tau \to \infty$ corresponds to 10 dimensional flat spacetime and the tower of the higher dimensional Kaluza-Klein modes descend into the low energy theory, ruining its validity, since $M_{KK} \to 0$. This means that it is never a good approximation to include some KK states in the 4D theory and neglect the heaviest ones, because this would correspond to an expansion in $\frac{n}{n+1}$ for example if these states have masses $M_n = \frac{n}{L}$, with n an integer and L an extra dimensional length. But this problem is not there if we only include moduli ($n = 0$ states) in the 4 dimensional EFT, since neglecting all the other states is justified for $\tau \gg 1$. Therefore, the EFT is reliable for large but finite τ.

7.3 Inflaton potential and Inflationary evolution

In order to obtain inflation in the scenario of RG-induced modulus stabilisation, a large source of positive potential energy, which breaks SUSY (Sect. 3.2, **SUSY breaking in SUGRA**), is needed. The minimal such sector at low energy is the goldstone fermion G, incorporated into a nilpotent chiral superfield X to remove any scalar superpartners of G (Sect. 3.1, **Non-linear SUSY**):

$$X^2 = 0. \tag{7.27}$$

We call ϕ the would be inflaton, the field that slowly evolutes between the regime where the large SUSY breaking energy is dominant and the one where it is not. A chiral superfield Φ such that

$$\bar{\mathcal{D}}(X\bar{\Phi}) = 0 \tag{7.28}$$

represents a non-supersymmetric scalar ϕ, removing the fermionic and auxiliary field components of Φ.

Scalar potential

Using the superfields $\mathcal{T}$, X and Φ in this supersymmetric framework with approximate accidental scale invariance[1], the Kähler potential reads

$$e^{-K/3} = \tau - k + \frac{h}{\tau} + ..., \tag{7.29}$$

where

$$k = \kappa(\Phi, \bar{\Phi}, \ln \tau) + (X + \bar{X})\kappa_X(\Phi, \bar{\Phi}, \ln \tau) + \bar{X}X\kappa_{X\bar{X}}(\Phi, \bar{\Phi}, \ln \tau) \tag{7.30}$$

and we neglect higher powers of $1/\tau$. Moreover,

$$W \simeq w_0(\Phi) + Xw_X(\Phi, \bar{\Phi}), \tag{7.31}$$

recalling that $X\bar{\Phi}$ is left chiral.
Defining $z^I = \{\mathcal{T}, \Phi\}$, since the scalar component of X does not propagate,

$$\begin{aligned} &-\frac{\mathcal{L}_{kin}}{\sqrt{-g}} = K_{I\bar{J}}\partial_\mu z^I \partial^\mu \bar{z}^J \\ &\simeq \frac{3}{\mathcal{P}^2}\left(1 + \frac{k'' - 2k'}{\mathcal{P}}\right)\partial_\mu T \partial^\mu \bar{T} - \left(\frac{3}{\mathcal{P}^2}(k_\phi - k'_\phi)\partial_\mu \phi \partial^\mu \bar{T} + \text{h.c.}\right) + \frac{3}{\mathcal{P}}\left(k_{\phi\bar{\phi}} + \frac{k_\phi k_{\bar{\phi}}}{\mathcal{P}}\right)\partial_\mu \phi \partial^\mu \bar{\phi}. \end{aligned} \tag{7.32}$$

We now instead denote $z^A = \{\mathcal{T}, X\}$, so that

$$V = e^K(K^{\bar{A}B}\overline{D_A W} D_B W - 3|W|^2) \tag{7.33}$$

because Φ has no auxiliary field and where $X = 0$ must be chosen after differentiation. Noticing that each T derivative of k costs a power of $1/\mathcal{P}$ given that $k = k(\ln \mathcal{P})$, with $\mathcal{P} \simeq \tau - k$, the scalar potential is

$$V = \frac{A|w_X|^2}{\mathcal{P}^2} - \frac{2\,\text{Re}(B\bar{w}_X w_0)}{\mathcal{P}^3} + \frac{C|w_0|^2}{\mathcal{P}^4}, \tag{7.34}$$

where

$$A \simeq \frac{\kappa^{\bar{X}X}}{3}, \quad \frac{B}{\mathcal{P}} \simeq \kappa^{\bar{X}X}\kappa_{X\bar{T}}, \quad \frac{C}{\mathcal{P}^2} \simeq -3(\kappa_{T\bar{T}} - \kappa^{\bar{X}X}\kappa_{T\bar{X}}\kappa_{X\bar{T}})^2. \tag{7.35}$$

Note that both $\kappa_{X\bar{T}}$ and $\kappa_{T\bar{T}}$ are $\mathcal{O}(\alpha_g^2)$. For the leading term to be positive, we require $A > 0$.

[1]Although scale invariance can be sufficient for no-scale supersymmetry, it is actually not necessary. Accidental scale invariance is incorporated by assuming the theory has an expansion in powers of $1/\tau$.

[2]Notice that the denominator for the C term in eq. (3.9) in [1] is actually not present. Some typos in Appendix A of [29] led to this mistake.

The inflationary regime is the one where the A term of the scalar potential dominates. Defining

$$\delta = \frac{|w_X|}{|w_0|} \tag{7.36}$$

and assuming for simplicity B, w_x and w_0 to be real, we extremise the potential with respect to $\mathcal{P}$, since, as will be shown later, inflation occurs when ϕ evolves and τ sits at a local minimum of the potential, finding that

$$\frac{1}{\mathcal{P}_\pm} = \frac{D_\pm w_X}{w_0}, \tag{7.37}$$

where

$$D_\pm = \frac{3B}{4C} \pm \sqrt{\frac{9B^2}{16C^2} - \frac{A}{2C}}. \tag{7.38}$$

For the physics we are interested in, we need to require both A/C and B/C to be positive and $\frac{8}{9}AC < B^2$ for $\mathcal{P}_+$ to be a local minimum and $\mathcal{P}_-$ to be a local maximum and to require $B^2 < AC$ for $V(\tau_+) > 0$. Recalling that $\frac{B}{A} \propto \alpha_g^2 \sim \epsilon^2$ and $\frac{C}{A} \propto \alpha_g^2 \sim \epsilon^2$ and given that it is needed to have $\frac{B^2}{C^2} \sim \frac{A}{C}$, we assume that $\kappa_{T\bar{T}}$ is numerically suppressed so that $\kappa_{T\bar{T}} \sim \alpha_g^2\epsilon^2$. Therefore

$$\mathcal{P}_\pm \sim \frac{\epsilon^2}{\delta} \tag{7.39}$$

and $\mathcal{P}_\pm \gg 1$ for $\delta \ll \epsilon^2$.
From eq. 7.32, the mixing for τ and ϕ in the kinetic terms is subleading and, for $k_{\phi\bar{\phi}} \sim \mathcal{O}(1)$, the canonical fields are $\mathrm{d}\chi \sim \frac{\mathrm{d}\tau}{\tau}$ and $\mathrm{d}\varphi \sim \frac{\mathrm{d}\phi}{\sqrt{\bar{\tau}}}$ near a semiclassical background $\tau = \bar{\tau}$.
As already anticipated, we imagine that the A term of the scalar potential dominates during the early universe, and so

$$\epsilon_\tau \sim \left(\frac{V_\chi}{V}\right)^2 \sim \left(\tau\frac{V_\tau}{V}\right)^2 \sim \mathcal{O}(1) \tag{7.40}$$

and

$$m_\tau^2 = \left(\frac{\partial^2 V}{\partial \chi^2}\right)_{\tau_+} \sim \tau^2 \frac{\partial^2 V}{\partial \tau^2} \sim V \sim H_I^2. \tag{7.41}$$

This suggests that inflation can be studied using single-field dynamics.

Let us analyse this model from a string theory point of view: we consider warped $D3 - \overline{D3}$ inflation and the inflaton field is the separation between the two. At large distances, the potential is the sum of the brane tension and of the brane-antibrane Coulomb interaction. In the past, the challenge to obtain slow-roll from such a scenario was that the separation between the branes needed to be bigger than the size of the

extra dimensions, rendering this model unusable.
The solution to this problem is considering warped geometries (KKLMMT proposal): fluxes backreact on the metric so that the compactification is a conformal CY threefold:

$$\mathrm{d}s^2 = \left(1 + \frac{e^{4\mathcal{A}}}{\mathcal{V}^{2/3}}\right)^{-1/2} \mathrm{d}s_4^2 + \left(1 + \frac{e^{4\mathcal{A}}}{\mathcal{V}^{2/3}}\right)^{1/2} \mathrm{d}s_{CY}^2, \tag{7.42}$$

where $\mathcal{A}(y)$ is a function of the coordinates of the extra dimensions and the warp factor is $\mathcal{W} = (1 + \frac{e^{4\mathcal{A}}}{\mathcal{V}^{2/3}})^{-1/2}$ (there are some little differences for certain conventions with respect to the preceding chapter). Highly warped regions are defined via $e^{4\mathcal{A}} \gg \mathcal{V}^{2/3} \gg 1$. Moreover, to have a reliable EFT, the KK scale should be smaller than the warped string scale [30], that is

$$e^{\mathcal{A}} \lesssim \mathcal{V}^{2/3}. \tag{7.43}$$

Due to supersymmetric BPS cancellation of bulk forces, a space filling D3 brane does not experience position dependent forces and is free to move in the CY space. On the other hand, an anti-D3 brane energetically prefers to minimise the warp factor and moves to the tip of the throat, where

$$e^{4\mathcal{A}_{tip}} := e^{4\rho} = e^{8\pi K/(3g_s M)}, \tag{7.44}$$

with K and M integers. The brane tension (eq. 4.23) reads

$$T_3 = \frac{1}{8\pi^3 g_s \alpha'^2} \tag{7.45}$$

and its contribution to the potential is what allows a dS solution. The canonically normalised would be inflaton field is

$$\varphi = \sqrt{T_3} y, \tag{7.46}$$

where y is the brane separation and, from eq. 6.33,

$$V = 2T_3 e^{-4\rho} \mathcal{V}^{2/3} \left(1 - \frac{27}{64\pi^2} \frac{2T_3 e^{-4\rho} \mathcal{V}^{2/3}}{|\varphi|^4}\right). \tag{7.47}$$

It is now a good point to recall that the number of inflationary e-foldings between horizon exit and the end of inflation is

$$N_e = \int_{\varphi_{end}}^{\varphi_*} \frac{\mathrm{d}\varphi}{\sqrt{2\varepsilon}} = \int_{\varphi_{end}}^{\varphi_*} \mathrm{d}\varphi \frac{V}{V_\varphi}, \tag{7.48}$$

the amplitude of primordial scalar density perturbations is

$$\delta_H = \frac{1}{\pi\sqrt{75}} \left(\frac{V^{3/2}}{V_\varphi}\right)_{\varphi_*}, \tag{7.49}$$

the spectral index is

$$n_s = 1 + 2\eta_* - 6\varepsilon_* \tag{7.50}$$

and the tensor to scalar ratio is

$$r = 16\varepsilon_*; \tag{7.51}$$

experimentally, from [1], $\delta_H = 1.9 \times 10^{-5}$ and $\frac{1}{2}(n_s - 1) \simeq -0.015$.

Let us now compare the potential in eq. 7.47 with the A term of the one in eq. 7.34. Recalling that $\mathcal{P} \simeq \mathcal{V}^{2/3}$, we choose κ as

$$\kappa(\phi, \bar{\phi}, \ln \mathcal{P}) \simeq \gamma(\phi, \bar{\phi}) + \hat{\kappa}(\ln \mathcal{P}) \tag{7.52}$$

and coordinates such that $\gamma \simeq \bar{\phi}\phi$. In order to describe warped brane inflation within our framework, we use

$$w_X(\Phi, \bar{\Phi}) = \mathfrak{t} - \frac{\mathfrak{g}}{|\Phi|^4} + ... \tag{7.53}$$

Therefore, the A term of the scalar potential reads

$$V = \frac{\kappa^{\bar{X}X}}{3\mathcal{P}^2} \left(|\mathfrak{t}|^2 - \frac{2\,\mathrm{Re}(\bar{\mathfrak{t}}\mathfrak{g})}{|\phi|^4} + ... \right). \tag{7.54}$$

Assuming $\mathfrak{t}$ and $\mathfrak{g}$ to be real and rescaling X to have $\kappa^{\bar{X}X}$ warping independent, $\mathfrak{t} \propto e^{-2\rho}$ and $\mathfrak{g} \propto e^{-6\rho}$.

The eta problem

Generically, let us consider the Kähler potential $K = -3\ln\big(\tau - k(\phi, \bar{\phi}) + ...\big)$ with $k(\phi, \bar{\phi}) \simeq \bar{\phi}\phi$ and that τ is fixed via $W_{np}(T)$:

$$V = e^K \hat{V}_0 \simeq \frac{\hat{V}_0}{(\tau - \bar{\phi}\phi)^3} \simeq \frac{\hat{V}_0}{\tau^3}\left(1 + \frac{3\bar{\phi}\phi}{\tau}\right) \simeq \frac{\hat{V}_0}{\tau^3}(1 + \bar{\varphi}\varphi). \tag{7.55}$$

This shows that $H_I^2 \simeq V \simeq \frac{\hat{V}_0}{\tau^3}$ and $m_\phi^2 \sim \frac{\hat{V}_0}{\tau^3} \sim H_I^2$, which implies that $\eta = \frac{V_{\varphi\varphi}}{V} \simeq \frac{m_\phi^2}{H_I^2} \sim \mathcal{O}(1)$. In the standard construction it is necessary to include a large unwarped $\bar{\phi}\phi$ contribution into $\hat{V}_0$ and to tune it against the term coming from e^K to obtain slow-roll. This fine tuning problem arises because K depends only on $\mathcal{P}$, but W, being a holomorphic function, can depend only on T and ϕ separately. This problem is evaded if, instead of considering corrections to the superpotential, modulus stabilisation arises from corrections to the Kähler potential, stabilising $\mathcal{P}$ (instead of τ).

Annihilation and the tachyon superpotential

Including a Higgs scalar field $\mathcal{H}$, the superpotential is modified through

$$w_X = \mathfrak{t} - \frac{\mathfrak{g}}{|\Phi|^4} - \lambda|\mathcal{H}|^2. \tag{7.56}$$

The A term of the scalar potential becomes

$$V = A\left(\frac{(\mathfrak{t}|\phi|^4 - \mathfrak{g})^2}{\mathcal{P}^2|\phi|^8} + \frac{2\lambda(\mathfrak{t}|\phi|^4 - \mathfrak{g})|\mathcal{H}|^2}{\mathcal{P}^2|\phi|^4} + \frac{\lambda^2|\mathcal{H}|^4}{\mathcal{P}^2}\right). \tag{7.57}$$

When $\mathfrak{t} - \frac{\mathfrak{g}}{|\phi|^4} > 0$, $\mathcal{H}$ has a positive mass and the potential is minimised at $\mathcal{H} = 0$; on the other hand, if it were possible for ϕ to be small enough to have $\mathfrak{t} - \frac{\mathfrak{g}}{|\phi|^4} < 0$ in the regime of validity of the effective field theory, $\mathcal{H}$ would have a tachyonic mass

$$m^2 = \frac{2\lambda}{\mathcal{P}}\left(\mathfrak{t} - \frac{\mathfrak{g}}{|\phi|^4}\right). \tag{7.58}$$

This phenomenon would be what we expect from a string theory point of view: a mode of an open string stretching between the antibrane and the brane becomes lighter as the branes approach one another until it becomes tachyonic at a critical distance (of order the string length). This tachyon could give rise to topological defects such as cosmic strings (Sect. 9.2).[3]

Equations of motion

In presence of more than one scalar, the lagrangian generically reads

$$\mathcal{L} = -\sqrt{-g}\left(\frac{1}{2}G_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j + V(\phi)\right) \tag{7.59}$$

and, defining the Christoffel symbol as

$$\Gamma^i_{jk} = \frac{1}{2}G^{il}(\partial_j G_{kl} + \partial_k G_{jl} - \partial_l G_{jk}), \tag{7.60}$$

[3] Let us briefly say that the superposition of a Dp-brane and an anti-Dp brane is a non-BPS system whose instability is the existence of a complex tachyonic mode in the open strings stretched between the pair. If its phase acquires a winding number when the tachyon rolls down to its true minimum, a magnetic vortex soliton is created because of its coupling to the $U(1)$ vector field. This vortex solution carries D$(p-2)$-brane charge. A D$(p-2)$-brane is left as a topological soliton because of charge conservation: through the Higgs mechanism, the $U(1)$ vector field acquires a mass by eating the phase of the tachyonic field (removed from the low energy spectrum). The overall $U(1)$ vector field, under which the tachyon is neutral, remains however unbroken, posing a puzzle [32].

the classical equations of motion are

$$\ddot{\phi}^i + 3H\dot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + G^{ij}\partial_j V = 0. \tag{7.61}$$

Furthermore,

$$\rho = \frac{1}{2}G_{ij}\dot{\phi}^i\dot{\phi}^j + V(\phi), \quad p = \frac{1}{2}G_{ij}\dot{\phi}^i\dot{\phi}^j - V(\phi), \quad H^2 = \frac{\rho}{3}. \tag{7.62}$$

In a slow roll situation, $\frac{1}{2}G_{ij}\dot{\phi}^i\dot{\phi}^j \ll V(\phi)$ and so $H^2 \simeq \frac{V}{3}$ and $3H\dot{\phi}^i + G^{ij}\partial_j V \simeq 0$.

For the supersymmetric case, the target space is a Kähler manifold with complex coordinates ϕ^a and $\phi^{\bar{a}}$: $G_{ab} = G_{\bar{a}\bar{b}} = 0$ and $G_{a\bar{b}} = \partial_a\partial_{\bar{b}}K$. Therefore,

$$\ddot{\phi}^a + 3H\dot{\phi}^a + \Gamma^a_{bc}\dot{\phi}^b\dot{\phi}^c + G^{\bar{b}a}\partial_{\bar{b}}V = 0, \tag{7.63}$$

where

$$\Gamma^a_{bc} = \frac{1}{2}G^{\bar{d}a}(\partial_b G_{c\bar{d}} + \partial_c G_{b\bar{d}} - \partial_{\bar{d}}G_{bc}) = G^{\bar{d}a}\partial_b\partial_c\partial_{\bar{d}}K. \tag{7.64}$$

In our context, $\phi^a = \{T, \phi\}$, $K = -3\ln\mathcal{P}$, $\mathcal{P} = T + \bar{T} - \bar{\phi}\phi$ and $\tau = T + \bar{T}$, therefore $K_T = -\frac{3}{\mathcal{P}}$, $K_{\bar{T}} = -\frac{3}{\mathcal{P}}$, $K_\phi = \frac{3\bar{\phi}}{\mathcal{P}}$, $K_{\bar{\phi}} = \frac{3\phi}{\mathcal{P}}$, $K_{T\bar{T}} = \frac{3}{\mathcal{P}^2}$, $K_{T\bar{\phi}} = -\frac{3\phi}{\mathcal{P}^2}$, $K_{\phi\bar{T}} = -\frac{3\bar{\phi}}{\mathcal{P}^2}$, $K_{\phi\bar{\phi}} = \frac{3\tau}{\mathcal{P}^2}$, $K^{\bar{T}T} = \frac{\tau\mathcal{P}}{3}$, $K^{\bar{T}\phi} = \frac{\mathcal{P}\phi}{3}$, $K^{\bar{\phi}T} = \frac{\mathcal{P}\bar{\phi}}{3}$, $K^{\bar{\phi}\phi} = \frac{\mathcal{P}}{3}$, $K_{TT\bar{T}} = -\frac{6}{\mathcal{P}^3}$, $K_{TT\bar{\phi}} = \frac{6\phi}{\mathcal{P}^3}$, $K_{T\phi\bar{\phi}} = -\frac{3}{\mathcal{P}^2} - \frac{6\bar{\phi}\phi}{\mathcal{P}^3}$, $K_{\phi T\bar{T}} = \frac{6\bar{\phi}}{\mathcal{P}^3}$, $K_{\phi\phi\bar{T}} = -\frac{6\bar{\phi}^2}{\mathcal{P}^3}$, $K_{\phi\phi\bar{\phi}} = \frac{6\tau\bar{\phi}}{\mathcal{P}^3}$, $\Gamma^T_{TT} = -\frac{2}{\mathcal{P}}$, $\Gamma^\phi_{T\phi} = -\frac{1}{\mathcal{P}}$, $\Gamma^T_{T\phi} = \frac{\bar{\phi}}{\mathcal{P}}$, $\Gamma^\phi_{\phi\phi} = \frac{2\bar{\phi}}{\mathcal{P}}$, $\Gamma^\phi_{TT} = 0$ and $\Gamma^T_{\phi\phi} = 0$. The equations of motion are then

$$\ddot{T} + 3H\dot{T} - \frac{2\dot{T}^2}{\mathcal{P}} + \frac{2\bar{\phi}\dot{\phi}\dot{T}}{\mathcal{P}} + \frac{\mathcal{P}\tau V_{\bar{T}}}{3} + \frac{\mathcal{P}\bar{\phi}V_{\bar{\phi}}}{3} = 0 \tag{7.65}$$

and

$$\ddot{\phi} + 3H\dot{\phi} + \frac{2\bar{\phi}\dot{\phi}^2}{\mathcal{P}} - \frac{2\dot{\phi}\dot{T}}{\mathcal{P}} + \frac{\mathcal{P}\phi V_{\bar{T}}}{3} + \frac{\mathcal{P}V_{\bar{\phi}}}{3} = 0. \tag{7.66}$$

Notice that for the potential in eq. 7.34, considering the first of the two equations above, the $V_{\bar{\phi}}$ term is smaller than the $\tau V_{\bar{T}}$ term; this and the absence of $\dot{\phi}^2$ terms mean that an initially motionless T will evolve towards the zero of V_T, justifying a single field treatment.

Chapter 8

Improved $\mathrm{D3}-\overline{\mathrm{D3}}$ Inflation

Observations have become more constraining on theoretical models [33]. According to the latest Planck results [34], we will consider

$$n_s = 0.9649 \pm 0.0042 \tag{8.1}$$

(68%, Planck TT,TE,EE+lowE+lensing) and

$$\delta_H = 1.9 \times 10^{-5}. \tag{8.2}$$

In this chapter, the inflationary model introduced in Chap. 7 will be analysed in further detail, taking into account all of the numerical details and describing the EFT constraints and their physical meaning from a string theory point of view.

8.1 Slow-roll and EFT constraints

Recalling the discussion following eq. 7.38, we can choose $\kappa_{X\bar{X}}$, $\kappa_{X\bar{T}}$ and $\kappa_{T\bar{T}}$ so that

$$\kappa_{X\bar{X}} = 1, \quad \kappa_{X\bar{T}} = b\frac{\alpha_g^2}{\mathcal{P}}, \quad \kappa_{T\bar{T}} = a\frac{\epsilon^2\alpha_g^2}{\mathcal{P}^2}, \tag{8.3}$$

where a and b are numerical parameters to be fixed below and $\epsilon = \alpha_0$ at the minimum of the potential in the τ direction, where $\mathcal{P} \sim \frac{\epsilon^2}{\delta}$. Working near this point (as we will always do in the following unless differently specified), $\alpha_g \simeq \epsilon$ and, from eq. 7.35,

$$A \simeq \frac{1}{3}, \quad B \simeq b\epsilon^2, \quad C \simeq -3\epsilon^4(a-b^2) \tag{8.4}$$

and

$$\mathcal{P}^{-1} = \frac{\delta}{\epsilon^2}\left(\frac{3b}{12(b^2-a)} + \sqrt{\frac{9b^2}{144(a-b^2)^2} - \frac{1}{18(b^2-a)}}\right), \tag{8.5}$$

as can be seen from eq. 7.37 and eq. 7.38.
The condition $\frac{8}{9}AC < B^2 < AC$, required to have a local positive minimum for the potential in the $\mathcal{P}$ direction, now translates into $-\frac{b^2}{8} < a < 0$. Without any lose of generality, we choose $a = -\frac{b^2}{10}$ and impose $b = \frac{1}{3}$.

From the kinetic terms of the lagrangian in eq. 7.32 and for $\kappa \simeq \bar{\phi}\phi$, the canonically normalised fields read

$$\varphi = \sqrt{\frac{3}{\mathcal{P}}}\phi \tag{8.6}$$

and

$$\chi = \sqrt{3}\ln\tau. \tag{8.7}$$

Since the field φ (the would be inflaton) represents physically the brane-antibrane separation and since $\varphi = \sqrt{T_3}y$, where y is a physical distance (of dimensions of length when reintroducing factors of M_p), we must require

$$l_s < y < l_{KK}. \tag{8.8}$$

The brane-antibrane separation has to be smaller than the Kaluza-Klein length

$$l_{KK} = \frac{1}{M_{KK}} \tag{8.9}$$

because this represents the size of the extra dimensions, while y has to be bigger than the string length

$$l_s = \frac{1}{M_s} \tag{8.10}$$

to avoid stringy corrections: otherwise, the massive states of the string are excited and must be integrated in. Notice that the first condition is a physical requirement, while the second one is an assumption made in order to trust our effective 4-dimensional supergravity description.
Moreover, the Kaluza-Klein scale is related to the string scale via

$$M_{KK} = 2\pi\frac{M_s}{\mathcal{V}_s^{1/6}},\,^{1} \tag{8.11}$$

where $\mathcal{V}_s$ is the extra-dimensional volume in the string frame. We define y_* via $\varphi_* = \sqrt{T_3}y_*$, where φ_* is the field φ at horizon exit.
The gravitino mass $m_{3/2} = e^{K/2}|W|$ is

$$m_{3/2} = \frac{\sqrt{g_s}w_0}{\mathcal{P}^{3/2}} \tag{8.12}$$

[1] For an isotropic compactification, $M_{KK} = 1/R$, with $R = R_s l_s$ and $\mathcal{V}_s = (2\pi R_s)^6$.

for $e^{-K/3} \simeq s^{1/3}\tau$ and

$$m_{3/2} \lesssim M_{KK} \tag{8.13}$$

needs to be satisfied in order to have a reliable low energy effective supersymmetric 4D description.

String frame and Einstein frame

In the string frame the gravitational part of the action is not in the canonical Einstein-Hilbert form, while in the Einstein frame the Ricci scalar does not couple to anything other than $\sqrt{-G^E}$ and the dilaton is canonically normalized. Following [35] and [37], the two frames are related via a conformal transformation of the 10-dimensional metric:

$$G^E = e^{2\Omega} G^S, \tag{8.14}$$

where

$$\Omega = -\frac{\Phi - \Phi_0}{4}. \tag{8.15}$$

The conformal transformation is fixed up to a constant Φ_0.

A choice to fix this constant is to require that the metric in the Einstein frame and the metric in the string frame are the same at the vacuum:

$$\Phi_0 = \langle \Phi \rangle \,. \tag{8.16}$$

This implies that

$$\mathcal{V}_s = \mathcal{V}_E, \tag{8.17}$$

where $\mathcal{V}_E$ is the extra-dimensional volume in the Einstein frame,

$$M_s = g_s \sqrt{\frac{\pi}{\mathcal{V}}}, \tag{8.18}$$

$$M_{KK} = 2\pi g_s \frac{\sqrt{\pi}}{\mathcal{P}}, \tag{8.19}$$

$$T_3 = \frac{g_s^3}{8\pi \mathcal{V}^2} \tag{8.20}$$

and $m_{3/2} \lesssim M_{KK}$ implies

$$w_0 \lesssim 2\pi \sqrt{\pi g_s \mathcal{P}}. \tag{8.21}$$

Comparing (and requiring them to be equal) the potential in eq. 7.47 with the one in eq. 7.54, we obtain that

$$\mathfrak{t} = \sqrt{\frac{3\kappa_{X\bar{X}}}{4\pi}} g_s^{3/2} e^{-2\rho} \tag{8.22}$$

and

$$\mathfrak{g} = \frac{1}{128\pi^2 \kappa_{X\bar{X}}} \mathfrak{t}^3. \tag{8.23}$$

Another possible choice is to assume that volumes are frame dependent also in the vacuum, that is

$$\Phi_0 = 0. \tag{8.24}$$

Therefore,

$$\mathcal{V}_s = \mathcal{V}_E g_s^{3/2}, \tag{8.25}$$

$$M_s = \sqrt{\frac{\pi}{\mathcal{V}}}, \tag{8.26}$$

$$M_{KK} = \frac{2\pi\sqrt{\pi}}{g_s^{1/4}\mathcal{P}}, \tag{8.27}$$

$$T_3 = \frac{1}{8\pi g_s \mathcal{V}^2} \tag{8.28}$$

and $m_{3/2} \lesssim M_{KK}$ becomes [36]

$$w_0 \lesssim \frac{2\pi\sqrt{\pi\mathcal{P}}}{g_s^{3/4}}. \tag{8.29}$$

Comparing eq. 7.47 with eq. 7.54,

$$\mathfrak{t} = \sqrt{\frac{3\kappa_{X\bar{X}}}{4\pi g_s}} e^{-2\rho} \tag{8.30}$$

and

$$\mathfrak{g} = \frac{1}{128\pi^2 \kappa_{X\bar{X}}} \mathfrak{t}^3. \tag{8.31}$$

Notice that these two possible choices are not just a matter of convention but have a precise physical interpretation, therefore only experiments and observations could allow to make this situation clearer.

8.1.1 $M_s \propto g_s$

Let us first consider the case $\Phi_0 = \langle\Phi\rangle$. Fixing the coefficients A, B, C, a and b as discussed above, we use the entire potential given in eq. 7.34. The parameters that we choose to vary so to match the experimental values for n_s and δ_H are ϵ, δ and w_0, since

$\mathfrak{t} \sim \delta w_0$ (notice that δ is technically a function of the field ϕ, so that, every time that we will claim that δ has a fixed numerical value, that is actually to be referred to $\frac{\mathfrak{t}}{w_0}$, i.e. to the warping). Moreover, from now on we will consider

$$g_s = \epsilon \ll 1, \tag{8.32}$$

that is to say, $1/s$ plays the role of α_g, as already suggested in [1]: the condition $\alpha_g \ll 1$ from the previous chapter is equivalent to weak string coupling. It is needed to underline the fact that to avoid α' corrections, as can be seen from eq. 5.10,

$$(\mathcal{P} g_s)^{3/2} \gg 1. \tag{8.33}$$

These initial considerations will apply without any change for the case $M_s \propto g_s^0$.
For the next paragraphs, it will be useful to remember that $\delta \ll \epsilon^3$ and $w_0 \lesssim 2\pi\sqrt{\pi g_s \mathcal{P}}$. Therefore, we parametrise δ as

$$\delta = 10^{-x}\epsilon^3, \quad x > 0 \tag{8.34}$$

and, since for our choice of a and b, $2\pi\sqrt{\pi g_s \mathcal{P}} \simeq 13\sqrt{\frac{\epsilon^3}{\delta}}$, w_0 is parametrised as

$$w_0 = 13 \times 10^{-y+x/2}, \quad y > 0. \tag{8.35}$$

Furthermore, in order to find the values for ϵ, δ and w_0 that allow slow-roll inflation, we will require $\delta_H \simeq 1.9 \times 10^{-5}$ and $n_s \simeq 0.965$.

$M_s \propto g_s : y_* \lesssim l_{KK}$

When fixing $y_* \simeq l_{KK}$, it is possible to find that $\delta_H \simeq 7.8 \times 10^{-11} \frac{\sqrt{\delta}}{w_0\sqrt{\epsilon}}$ and $1 - n_s \simeq 90\left(4.9 \times 10^8 \frac{w_0^2\delta}{\epsilon} - 1.2 \times 10^{14} \frac{w_0^4\delta^3}{\epsilon^3}\right)$, which, finding suitable values for x and y, are solved by $\epsilon \simeq 0.49$ (we could discuss whether this allows perturbative control or not, but we present all best and worst scenarios to show that our conclusions will always be the same), $\delta \simeq 0.11$ and $w_0 \simeq 1.9 \times 10^{-6}$. But this implies that $e^\rho \simeq 900$ and $\mathcal{P} \simeq 2.8$, which would mean that the Kaluza-Klein scale is higher than the warped string scale, spoiling the validity of the treatment.

$M_s \propto g_s : y_* \gtrsim l_s$

When fixing $y_* \simeq l_s$ (notice that this is an extreme case, inflation cannot start at this point since it would mean that the would be inflaton cannot roll in the EFT; we study this case only to prove that the situation described above for $M_s \propto g_s$ and $y_* \lesssim l_{KK}$ is happening for the whole range of y), it is possible to find that $\delta_H \simeq 5.7 \times 10^{-7} \frac{\delta^{7/4}}{w_0\epsilon^3}$ and

$1-n_s \simeq 90\left(1.1\times 10^4 \frac{w_0^2\epsilon^2}{\sqrt{\delta}} - 2.4\times 10^6 w_0^4\sqrt{\delta}\epsilon^2\right)$. A possible solution is given by $\epsilon \simeq 0.17$, $\delta \simeq 0.0031$ and $w_0 \simeq 2.5\times 10^{-4}$. But this implies that $e^\rho \simeq 210$ and $\mathcal{P} \simeq 12$. Considering the lower bound and the upper bound for ϵ, here, the latter is $\epsilon \simeq 0.41$, so that $\delta \simeq 0.01$ and $w_0 \simeq 1.4\times 10^{-4}$. This means that $e^\rho \simeq 300$ but $\mathcal{P} \simeq 21$.

8.1.2 $M_s \propto g_s^0$

For the choice $\Phi_0 = 0$, $\delta \ll \epsilon^3$ and $w_0 \lesssim \frac{2\pi\sqrt{\pi\mathcal{P}}}{g_s^{3/4}}$. Therefore, we parametrise δ as

$$\delta = 10^{-x}\epsilon^3, \quad x > 0 \tag{8.36}$$

and, since for our choice of a and b, $\frac{2\pi\sqrt{\pi\mathcal{P}}}{g_s^{3/4}} \simeq 13\sqrt{\frac{\epsilon^{1/2}}{\delta}}$, w_0 is parametrised as

$$w_0 = 13\times 10^{-y+x/2}\frac{1}{\epsilon^{5/4}}, \quad y > 0. \tag{8.37}$$

Furthermore, in order to find the values for ϵ, δ and w_0 that allow slow-roll inflation, we will require again $\delta_H \simeq 1.9\times 10^{-5}$ and $n_s \simeq 0.965$.

$M_s \propto g_s : y_* \lesssim l_{KK}$

When fixing $y_* \simeq l_{KK}$, it is possible to find that $\delta_H \simeq 7.8\times 10^{-11}\frac{\sqrt{\delta}}{w_0\epsilon^{17/4}}$ and $1-n_s \simeq 90\left(4.9\times 10^8 w_0^2\delta\epsilon^{7/2} - 1.2\times 10^{14} w_0^4\delta^3\epsilon^{9/2}\right)$, which, finding suitable values for x and y, are solved by $\epsilon \simeq 0.26$, $\delta \simeq 7.3\times 10^{-3}$ and $w_0 \simeq 1.1\times 10^{-4}$. But this implies that $e^\rho \simeq 1100$ and $\mathcal{P} \simeq 12$, spoiling again the validity of the treatment.

$M_s \propto g_s : y_* \gtrsim l_s$

For $y_* \simeq l_s$, it is possible to find that $\delta_H \simeq 5.7\times 10^{-7}\frac{\delta^{7/4}}{w_0\epsilon^8}$ and $1-n_s \simeq 90(1.1\times 10^4\frac{w_0^2\epsilon^8}{\sqrt{\delta}} - 2.4\times 10^6 w_0^4\sqrt{\delta}\epsilon^{12})$, which are solved by $\epsilon \simeq 0.38$, $\delta \simeq 2.6\times 10^{-3}$ and $w_0 \simeq 2\times 10^{-3}$. But this implies that $e^\rho \simeq 390$ and $\mathcal{P} \simeq 71$.

Chapter 9

Inflation and its end

Thse remaining sections are dedicated to a very brief introduction to reheating, cosmic strings and eternal inflation from a general perspective in the context of $D3-\overline{D3}$ inflation.

9.1 Reheating

Since warped brane inflation occurs in a warped throat, we have the freedom to choose whether to situate the Standard Model on D-branes in the unwarped bulk region or in another warped throat because, if it was in the inflationary throat, relic cosmic strings would disintegrate when at contact with SM branes (Sect. 9.2). Following [15] and [31], in this section we will briefly discuss what happens at the end of inflation and how the SM is excited for generic models of warped brane inflation.

Accelerated expansion ends when the D3-brane falls towards the tip of the throat, where the anti-D3-brane is: when their separation is sufficiently small, a tachyon develops; this instability is the reason for the decay of the pair of branes into massive KK excitations of massless string modes in the inflationary throat. The wavefunctions of these KK modes peak exponentially in the IR and their mutual interactions (Kaluza-Klein modes are self-interacting) are suppressed by $e^{A_{IR}} M_p \ll M_p$; moreover, their coupling to KK zero modes (e.g. the graviton) are suppressed by M_p (the coupling to 4-dimensional gravitons is universally small deep inside the throat). Massive particles are confined to the IR region because of the gravitational potential barrier created by the warping, so that tunnelling through the bulk of the compactification can allow access to other throats. Therefore, denoting the thermalization time for KK modes of the inflationary throat τ_{therm}, the timescale for decay to gravitons $\tau_{graviton}$ and the tunnelling timescale τ_{tunnel},

$$\tau_{therm} \ll \tau_{graviton} \ll \tau_{tunnel}. \tag{9.1}$$

Once excited strings have decayed into excited KK modes, the energy of the previous

inflaton is mainly in the inflationary throat. As can be seen from the previous equation, if Kaluza-Klein modes were to decay to 4D gravitons more quickly than to other channels, the universe would be dominated by gravitational radiation, spoiling BBN. Since the heaviest KK modes have the largest tunnelling probability, we then have to require their lifetime τ_{KK} to satisfy $\tau_{KK} \gtrsim \tau_{tunnel}$.
If it is supposed that one of the throats has approximate angular isometries (e.g. the $SU(2) \times SU(2)$ isometry of the KS solution), the associated angular momentum is approximately conserved and KK modes with this charge can decay only via interactions violating the symmetry. Therefore, charged KK modes produced during reheating could overclose the universe being long lived KK relics. It is then necessary to study the perturbations breaking the isometry to understand if KK relics decay fast enough.
If an additional intermediate throat (e.g. where SUSY is broken) whose warp factor is between the one of the inflationary throat and the one of the visible sector throat is assumed, tunnelling leads to its KK excitations; their transfer to the visible sector throat is slow and constitutes a problem since they could dominate the energy density of the universe.
In a throat much more strongly warped than the inflationary one (e.g. addressing the electroweak hierarchy by warping of the visible sector throat), the reheating temperature can exceed $\frac{e^{A_{IR}}}{\sqrt{\alpha'}}$: reheating can induce a large production of excited strings in the strongly warped throat.
The cascading energy from inflaton to radiation is summarised in Fig. 9.1.

9.2 Cosmic Strings

Cosmic strings are cosmologically relevant if they are produced after inflation, if they remain stable over cosmological times and if they could be observable without already being excluded; these requirements can be satisfied in our inflationary model.
At the end of inflation, tachyonic condensation produces cosmic F-strings, D-strings and (p, q) string bound states (their stability depends on the presence of D-branes in the inflationary throat). (p, q) strings are not BPS ($B_{\mu\nu}$ and $C_{\mu\nu}$ are projected out by the orientifold action) and a string can break apart by coming into contact with its orientifold image. If there are no orientifold fixed planes in the throat, a string has to fluctuate out to meet its image in the image throat (exponentially slow process since the strings are confined to the bottom of their respective throats and breakage via the orientifold image can be neglected). On the other hand, the presence of D3-branes or anti-D3-branes (for the SM or for SUSY breaking) causes cosmic superstrings to fragment and be irrelevant (but are stable if there are only D7-branes). The spectrum of tensions is

$$T_{(p,q)} \simeq \frac{e^{2A_{IR}}}{2\pi\alpha'}\sqrt{\frac{q^2}{g_s^2} + \left(\frac{bM}{\pi}\right)^2 \sin^2\left(\frac{\pi(p - qC_0)}{M}\right)}, \tag{9.2}$$

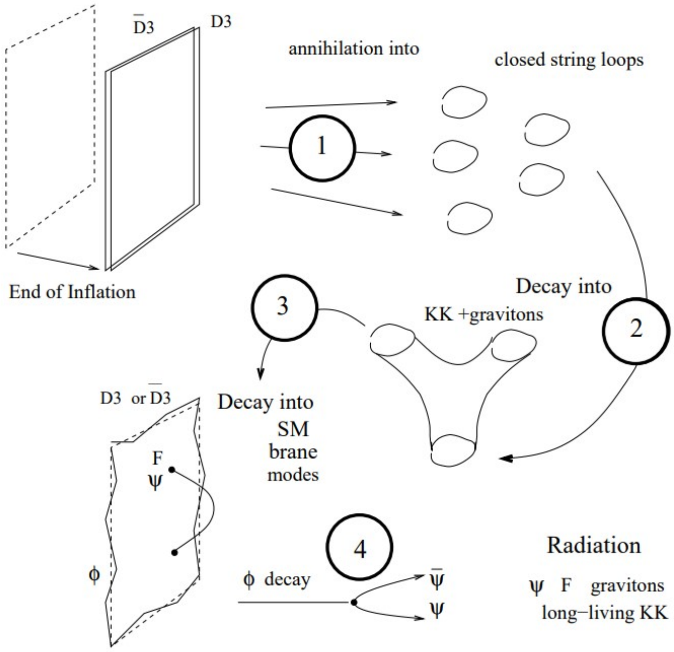

Figure 9.1: Channels of D-brane decay. Source: [31].

with $b \simeq 0.93$ a constant from the KS solution. The warp factor determines the scale of the inflaton potential and is constrained by the normalization of the scalar fluctuations, hence allowing for the possibility to predict the cosmic string tensions. For details, [38].

9.3 Eternal Inflation

For very flat potentials, the force pushing the inflaton down is very small and the amplitude of inflationary fluctuations remains essentially constant. Therefore, the motion of the inflaton at large values is predominantly controlled by quantum jumps. This effect leads to eternal inflation.
Eternal inflation causes the formation of a fractal structure of the universe on large scales; it occurs for values of the inflaton such that the post-inflationary amplitude of perturbations of the metric would exceed unity [26].

Chapter 10

Conclusions

In this thesis, after presenting the modern situation in Cosmology, hence exploring the issues of the Standard Big Bang Theory and their resolutions via inflationary scenarios, we introduced the formalisms of Supersymmetry and String Theory necessary for the project. In particular, we focused on moduli stabilisation, exploring the KKLT proposal and the LVS stabilisation, and on String Inflation.
The last chapters were dedicated to moduli stabilisation via renormalization group techniques: this framework allows to evade both the Dine-Seiberg problem and the η problem, obtaining de Sitter solutions with supersymmetry breaking without any form of uplifting. Concentrating on a possible embedding of inflation in this scenario, and in particular on warped $D3-\overline{D3}$ inflation, we analysed whether slow-roll is possible in the regime of validity of the effective field theory.

From a string theory point of view, quite a few constraints have to be satisfied to have control over our theory, i.e. (for $g_s = \epsilon$):

$$\epsilon \ll 1, \quad (\mathcal{P}\epsilon)^{3/2} \gg 1, \quad l_s \lesssim y \lesssim l_{KK}, \quad e^{\rho} \lesssim \mathcal{P}, \quad m_{3/2} \lesssim M_{KK} \tag{10.1}$$

and

$$\delta_H = 1.9 \times 10^{-5}, \quad n_s = 0.965 \pm 0.004. \tag{10.2}$$

This allowed us to show that it is very difficult to embed a phenomenologically suitable inflationary scenario into this UV completion for $M_s = g_s\sqrt{\frac{\pi}{\mathcal{V}}}$, that is to say if the metric in the Einstein frame and the metric in the string frame are the same at the vacuum, or for $M_s = \sqrt{\frac{\pi}{\mathcal{V}}}$, i.e. volumes are frame dependent also in the vacuum.

To conclude, since these results are of a no-go type, it is not possible to talk about a speculative minimum for the potential in the ϕ direction, which would suggest, from a physical point of view, a brane-antibrane bound state and it is not possible to consistently require $w_X(\phi_0) = 0$, for some ϕ_0 in the regime of validity of the EFT, to consider minimisation at late times (applicable to present day dark energy) with ϕ being a relaxation field which dinamically minimises the $|w_X|^2$ term.

Given these results, the present context could be worth further exploration if new ingredients could be introduced in a consistent way.

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