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Black Hole Information Loss Paradox and its avoidance in LQG - based models

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A Francesco

perché senza te non sarei dove sono adesso

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Introduction

If we throw a book into a black hole where does the information that was contained inside end up?
In 1975 Stephen Hawking published an article titled *"Breakdown of predictability in gravitational collapse"*. In it Hawking argued that all information that fells into a black hole is no longer recoverable, is lost forever, is destroyed. This created a deep crisis of Theoretical Physics, a crisis that lasts from forty years and that has not been resolved yet.
The first thing to do in order to understand it is defining what physicists mean when they talk about paradox.
In physics a paradox arises when, starting from a theory based on some principles, one derives from that results that violate at least one of the underlying principles. In our case, the black hole information paradox emerges when we try to build a model that involves both Einstein Theory of Gravity and Quantum Mechanics (more specifically Quantum Field Theory); this model is the black hole evaporation through Hawking radiation, and the theory that merges Quantum Field Theory and Einstein Gravity is called Quantum Field Theory in curved space-times. The model violates apparently the unitarity principle of Quantum Mechanics, and consequently the quantum information conservation.
We point out here that this paradox doesn't imply that General Relativity and Quantum Mechanics are wrong theories in their domain of validity, since the evaporation model lives, strictly, outside the domain of validity of both of them.
In the first chapter of this work we'll start by describing the black hole evaporation model, then we'll explain in detail the information paradox that such model implies, and the possible ways to avoid it.
In particular, we'll analyze the first proposed escape routes to the paradox and how these did not satisfy the scientific community; then, motivated by the fact that the paradox apparently cannot be solved within Quantum fields in curved space-times, we'll introduce in the second chapter two different models built from Loop Quantum Gravity, one of the most promising candidates as a quantum theory of gravity; we'll show in particular that

both of them avoid completely the information paradox in two radically different ways. Therefore we'll prove that the information paradox arises only if we insist to study the gravitational collapse of a star within Einstein theory of Gravity (as well as originally Hawking did), and we'll prove that if a more fundamental theory of gravity is kept in account, such paradox simply vanishes.
We'll conclude this work by comparing in the third chapter these two different quantum gravity models, focusing on their main differences and similarities focusing on the way in which they avoid the information paradox.

Chapter 1

Hawking's radiation and black hole's evaporation

1.1 Where do black holes come from?

1.1.1 Einstein Theory of Gravity in a nutshell

Before going into Black hole models of Einstein theory, let's give a brief summary of the fundamental concepts and principles of the theory. Firstly it is worth to mention that Einstein Theory of Gravity was born from the aim to extend the framework and principles of Special Relativity in a theory that includes gravitational fields and accelerated motion in presence of them.
The theory is based on two fundamental principles:

- **Principle of General Relativity:** *"All systems of reference are equivalent with respect to the formulation of the fundamental laws of physics"*.
  This principle is an extension of the Principle of Special Relativity, and holds whatever reference frame one chooses to write physical laws, whether it's inertial or not. Such requirement is realized by the tensorial formalism of the theory, since tensors are mathematical objects invariant under coordinate transformations.

- **Principle of equivalence:** *"If we look at the free falling motion of an object in a gravitational field, is always possible to consider a particular reference frame, called free falling frame, in which there is no gravity"*.
  This principle is fundamental to derive the geodesic equation of the theory, that

describes how objects move in a gravitational field.

From a conceptual point of view, the main novelty carried by Einstein theory of gravity is the redefinition of the notions of time and space: in newtonian mechanics space and time are absolute concepts, that means that time is thought as flowing in the same way everywhere, and space is absolute, in the sense that distances between objects are measured in the same way everywhere; finally, gravitational fields are physical entities living in such fixed background, as well as matter, light and whatever other form of energy or fields. Einstein perspective is radically different, and we illustrate it in two steps:

1. space and time are not independent entities, but belong to the same physical entity called space-time; such structure is not absolute, but depends on the observer: different observers with different velocities measure distances and the flowing of time in different ways (Special Relativity).

2. Gravity is not a physical entity *in* space-time, but is a property of the space-time itself: it is the curvature of space-time, and since matter generates the gravitational field, it generates also the curvature of space-time.

The interplay between matter and space-time curvature is realized in a beautiful and covariant way by the Einstein field equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1.1}$$

Let's briefly describe the previous equation; on the right-hand side we have the energy-momentum tensor $T_{\mu\nu}$ that keeps in account the energy content in space-time, and acts as source for the gravitational field; on the left-hand side we have the curvature of space-time induced by the source, captured by the Ricci curvature tensor $R_{\mu\nu}$, the Ricci scalar R and the metric tensor $g_{\mu\nu}$. This is a set of ten second order non-linear coupled partial differential equations with unknown $g_{\mu\nu}$.
The power of this mathematical equation lies in the fact that its solutions are astronomical objects and phenomena that have been found to be observable over time: historically the first experimental proof of the theory has been obtained in 1919 by the English physicist and mathematician Sir Arthur Eddington, who measured during a total solar eclipse the deflection of light from the gravitational field of the sun, in perfect agreement with the theory; anecdotally Einstein didn't participate to the experiment, since firmly convinced

of the validity of his theory.

Over the time the theory obtained many experimental proofs, like the proof of the gravitational redshift given by the Pound-Rebka experiment in 1960, and the detection of gravitational waves by the LIGO and VIRGO interferometers, observed experimentally in 2015.

In 1916 Karl Schwarzschild managed to solve Einstein's field equations exactly, whereas Einstein himself had argued that it would have been difficult to find analytical solutions, marveling and complimenting his colleague for the lucky choice. Such solution is called *eternal black hole*, and is the unique non-trivial spherically symmetric vacuum solution of the full theory.

As a consequence of his solution, in 1938 Robert Oppenheimer and Harland Snyder treated the first model of star collapse, in which the space-time exterior is described by the Schwarzschild solution, and the interior as an homogeneous and isotropic collapsing fluid. The model predicts that during the stellar collapse, once the radius of the star reaches a limiting value called Schwarzschild radius, the black hole forms and the collapsing matter reaches punctiform dimensions, namely a *gravitational singularity*. It is worth to notice here that the first black hole (the one at the center of the galaxy Messier 87, called M-87) has been observed by the Event Horizon telescope in 2019.

We have to notice that gravitational singularities, predicted by the theory lying at the center of black holes are entities that live beyond the limit of the descriptive capability of theory. This because on the physical singularity the energy density diverges, and consequently the curvature of space-time diverges as well.

The Schwarzschild solution is one of the possible solutions to Einstein's field equation. But physically what does it mean?

As anticipated before, black holes are nothing else than the final stage of a massive dying star (with mass larger than three solar masses); when a black hole forms, its density reaches rapidly values such as to create a gravitational field so strong to not allow to anything to escape its attraction, neither to light. [Stellar evolution., A K Peters. p. 105].

For a body to be called a black hole, its radius must be smaller than the Schwarzschild radius:

$$R_S = \frac{2MG}{c^2} \tag{1.2}$$

Such radius defines the so called *event horizon*, an imaginary spherical surface characterized by the fact that at each of its points the escape velocity and the speed of light

are equal. If an event occurs inside the event horizon, it cannot be seen by an outside observer: only what happens outside the event horizon is visible. If a ray of light passes close enough to the event horizon, but still outside it, the ray is bent due to the attraction of the black hole's gravitational field and is able to continue its journey. If, on the other hand, it enters the event horizon, **it will never be able to exit**.
The Schwarzschild solution is the simplest black hole solution of Einstein theory, and describes a non-rotating, uncharged black hole, but is not the only one. There is a theorem in General Relativity, namely the *No hair-theorem*, that states that black holes are characterized by three parameters: the mass M, the electric charge Q, and the angular momentum J. The classification of black hole solutions depending on the values of these parameters is the following:

- Schwarzschild black hole: $M \neq 0,\ Q = 0,\ J = 0$
- Reissner-Nordstrom black hole: $M \neq 0,\ Q \neq 0,\ J = 0$, thus a charged non-rotating black hole.
- Kerr black hole: $M \neq 0,\ Q = 0,\ J \neq 0$, thus an uncharged rotating black hole.
- Kerr-Newman black hole: $M \neq 0,\ Q \neq 0,\ J \neq 0$.

The black hole mass can take arbitrary real positive values, while its charge and angular momentum are constrained by mass as in the following relation:

$$Q^2 + \left(\frac{J}{M}\right)^2 \leq M^2 \tag{1.3}$$

When black holes saturate this inequality, they are called **extremal**.
From now on we'll focus on the simplest solution, that is the Schwarzschild solution.
As pointed out previously, black holes in Einstein theory are eternal, in the sense that if isolated, once they are formed there is no process derivable from the theory that allows them to disappear. Their only possible evolution within the theory is due to the increasing of their mass (thus their Schwartzschild radius) due to the fall of energy inside the horizon. To be precise there exists a theorem in General Relativity that states that black holes can only increase their mass during time, but not decrease. In the next paragraph we'll show that this result is no longer true if we consider a theory that includes quantum effects of matter, namely *Quantum Field theory in curved space-times*. Within this theory the Information Loss paradox arises, thus we'll describe briefly its main features.

1.2 Where do black holes end up?

In 1975, physicist Stephen Hawking published the article *"Particle Creation by Black Holes"*[1], an article that aroused astonishment among the researchers of the time and which is still at the center of debates today.

Hawking started assuming the correctness of Einstein's theory of General Relativity for gravitational collapse, thus assuming that the collapsing star, in accordance with the Penrose singularity theorem, becomes a black hole once the radius of the star is smaller than the Schwartzschild radius (as defined in the previous paragraph). However he showed in his paper that the black hole produced from the collapse is not static, as derived from Einstein theory, but its horizon turns out to be dynamical, once quantum effects of matter are kept in account. To do this, we have to consider quantum fields (for simplicity scalar and massless fields) evolving in such Schwartzschild background. The resulting model belongs to the theory *Quantum Field theory in curved space-times*, and the equations that govern this model are the following

$$\begin{cases} R_{\mu\nu} = 0 \\ \hat{\Box}\hat{\Phi} = 0 \end{cases} \tag{1.4}$$

Where the first equation is the Einstein equation in vacuum ($T_{\mu\nu} = 0$), that has Schwarzschild as solution, and the second equation is the Klein-Gordon equation for a massless scalar field in a curved background. The hatted D'Alembertian is the covariant version of the D'Alembert operator, and reads:

$$\hat{\Box} = g_{\mu\nu}\nabla^{\mu}\nabla^{\nu} \tag{1.5}$$

where $g_{\mu\nu}$ is the solution of Einstein field equations (thus the Schwarzschild metric) and the ∇ is the covariant derivative. Finally, $\hat{\Phi}$ is the quantum field operator, the unkown of the Klein-Gordon equation. The second equation shows that the quantum scalar fields evolves in space-time that is not flat; in that case in fact the D'Alembert operator would be:

$$\hat{\Box}_M = \eta_{\mu\nu}\partial^{\mu}\partial^{\nu} \tag{1.6}$$

Now, following this model he proved that a Schwarzschild black hole is not really as black as general relativity had predicted, but it radiates with a blackbody spectrum.

We will present a derivations of Hawking radiation using the tunnel effect (for more

details see [6]), a standard method to describe the emission of radiation from thermal sources. In accordance with this emission model, radiation arises in a process similar to the creation of an electron-positron pair in a constant electric field.
In Quantum Field Theory in flat space-time the fundamental state of a quantum field is interpreted physically as absence of particles; then the theory predicts the spontaneous creation from such vacuum state of couples of particles-antiparticles that annihilate after a very short period of time; this effect is called pair production from vacuum, and the particles are considered "virtual" if the process end with their annihilation. Let's see what happens if we study this effect on a Schwarzschild background; a pair can materialize with zero total energy inside (near) the horizon, after which one of the two created particles escapes from the black hole by tunneling. This scheme can be used to semiclassically derive the production of radiation from a black hole; the conservation of energy plays a fundamental role in this approach. A chargeless non-rotating black hole is described in General Relativity by the Schwarzschild line element:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{1}{\left(1 - \frac{2M}{r}\right)}dr^2 + r^2 d\Omega^2 \tag{1.7}$$

which tells how space-time distances are measured in a spherically symmetric vacuum space-time (stress-energy tensor equal to zero); it contains a gravitational singularity at $r = 0$ (we have considered the geometrized units $G = 1$, $c = 1$), that means that such singularity cannot be removed through a change of coordinates of the metric tensor, and a so called "coordinate singularity" at $r = 2M$; while a physical singularity means a divergence in the curvature of the space-time (thus an infinite deformation of space and time), a coordinate singularity arises when the coordinates that we choose to describe the space-time are ill-defined in some point, and doesn't describe any physical pathological behaviour of the gravitational field; such kind of singularities can be removed by a change of coordinates of the metric tensor; to do this, we introduce the so called Painlevé-Gullstrand coordinates; let's define the new time coordinate

$$t' = t + 2\sqrt{2Mr} + 2M \ln \frac{\sqrt{r} - \sqrt{2M}}{\sqrt{r} + \sqrt{2M}} \tag{1.8}$$

with t Schwarzschild time, the time coordinate that we use to write the metric in Schwarzschild coordinates. With this choice of the time parameter, the line element becomes

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt'^2 + 2\sqrt{\frac{2M}{r}}dt'dr + dr^2 + r^2 d\Omega^2 \tag{1.9}$$

In this way we have eliminated the singularity on the horizon.
We now recall the Hamilton-Jacobi theory and its connection with quantum mechanics, since it will be useful for the following discussion. We define Hamilton's principal function:

$$S = \int_{t_i}^{t_f} \sum_{i=1}^{n} p_i dq_i - H dt \tag{1.10}$$

where H is the Hamiltonian of the system. Applying the variational principle to the action 1.10 gives the equations of motion. If the Hamiltonian does not explicitly depend on time as in our case, we can apply the variational principle to the following reduced action

$$\hat{S} = \int_{t_i}^{t_f} \sum_{i=1}^{n} p_i dq_i \tag{1.11}$$

In Quantum Mechanics the wave function associated with a Hamiltonian system is given by

$$\Psi = \sqrt{\rho} e^{iS} \tag{1.12}$$

where ρ is the spatial probability density, that means that if the wave function depends for example on the x coordinate, $|\Psi|^2$ is the probability density to find the the system in the position x.
If there is a barrier of potential energy with amplitude greater than the energy of the system, the action assumes imaginary values and consequently we have that the probability of crossing the barrier due to the tunnel effect is proportional to

$$\Gamma \sim e^{-2Im\{\hat{S}\}} \tag{1.13}$$

Consider then an outgoing wave with positive frequency that crosses the horizon from $r_{in} < r_S = 2M$ to $r_{out} > r_S$ in radial geodesic motion. In our specific case, the potential barrier that the particle sees is given by the gravitational potential, and since the massless particle has an energy smaller than such potential energy, the action becomes imaginary. The imaginary part of its reduced action is given by

$$Im\{\hat{S}\} = Im\{S\} \int_{r_{in}}^{r_{out}} p_r dr = Im\{S\} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp'_r dr \tag{1.14}$$

Solving the integral we get

$$Im\{\hat{S}\} = 4\pi\omega \left(M - \frac{\omega}{2} \right) \tag{1.15}$$

where M is the black hole mass and ω the particle energy that moves in space-time following geodesics in the Schwarzschild space-time. This solution is valid if $r_{in} < r_{out}$; this because if $r_{in} > r_{out}$, the particle is simply free-falling, and there is no tunneling at all.

The integration limits indicate that along the path of the classically forbidden trajectory, the particle/wave starts from $r = r_S - \epsilon$ inside the horizon and crosses the contracting horizon to reach $r = 2(M - \omega) + \epsilon$, just outside the final position of the horizon.

Hawking radiation can also be seen as pairing outside the horizon rather than inside, with the negative energy particle tunneling into the black hole. In both cases the semiclassical emission rate is

$$\Gamma \sim e^{-2Im\{S\}} = e^{-8\pi\omega\left(M-\frac{\omega}{2}\right)} \tag{1.16}$$

If we ignore the quadratic term in ω ($M >> \omega$), we obtain an expression in which we interpret $8\pi M$ as the inverse of the emission temperature:

$$T = \frac{1}{8\pi M} \tag{1.17}$$

Quantum mechanics therefore demonstrates that a black hole is not completely black, but that it is associated with a physical temperature with a thermal emission spectrum. Unfortunately this temperature turns out to be extremely low ($T_H \sim 10^{-8}K$ for a black hole of one solar mass) and the hopes of observing Hawking radiation are today essentially zero, also considering the fact that the cosmic background radiation has a temperature of $2.7K$, well above the estimated temperatures for black holes which of the order of nanokelvins.

But what can we say about entropy?

If a pair of particles is created inside the black hole and one is able to tunnel out, then the mass of the black hole decreases. The mass of the black hole is proportional to the area of its event horizon

$$M = \left(\frac{A}{16\pi}\right)^{\frac{1}{2}} \tag{1.18}$$

Bekenstein and Hawking have shown how to calculate the entropy of a black hole [2]:

$$S_{BH} = \frac{1}{4}\frac{Ac^3}{\hbar G} \tag{1.19}$$

and if we choose units such that $c = \hbar = G = 1$ we obtain

$$S_{BH} = \frac{A}{4} \tag{1.20}$$

As we can see, the black hole's entropy is directly proportional to the area of its event horizon. Since $S \propto A \propto M$, if its mass decreases (and it does because it "loses" a particle, indeed $M_{fin} = M - \omega$) then its entropy will also decrease. So does the total entropy decrease?

Obviously not since the one that decreases is that of the black hole but the variation of the entropy of the universe increases; calling A_{fin}, r_{fin} the area and the radius of the black hole after the emission of the particle, and A_{in}, r_{in} the area and the radius before the emission, we have:

$$\begin{aligned}\Delta S_{BH} = S_{fin} - S_{in} = S_{BH}(M-\omega) - S_{BH}(M) &= \frac{A_{fin}}{4} - \frac{A_{in}}{4} \\ &= \frac{4\pi r_{fin}^2}{4} - \frac{4\pi r_{in}^2}{4} = 4\pi(M-\omega)^2 - 4\pi M^2 \\ &= 4\pi\omega^2 - 8\pi M\omega\end{aligned} \tag{1.21}$$

So:

$$\Delta S_{BH} = 4\pi\omega^2 - 8\pi M\omega \tag{1.22}$$

If we look at the relation (1.13) we notice that the term to the exponent $-8\pi\omega\big(M - \frac{\omega}{2}\big)$ is nothing else than ΔS_{BH}. We can therefore express the emission rate terms of the Bekenstein-Hawking entropy:

$$\Gamma = e^{\Delta S_{BH}} \tag{1.23}$$

Although numerous technical subtleties are needed to make the foregoing statement precise, the underpinnings are firmly established by relatively uncontroversial theory and calculation techniques developed to deal with quantum field theory on curved spacetimes. It has become apparent that Hawking radiation is a **kinematical** as opposed to a dynamical effect, in the sense that Einstein's gravitational field equations do not play any role in deriving the effect.

It is not the fact that a black hole emits radiation that is shocking but rather the fact that the more it emits radiation, the more its mass decreases.

Following Hawking [1] there is no cut-off in the evaporation process and the process continues until the black hole has completely evaporated (fig 1.1).

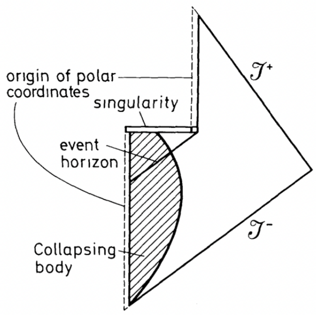


Figure 1.1: *Penrose diagram for black hole evaporation. (Credits: [1])*

The considerations made until here are done by assuming that the classical Schwarzschild space-time doesn't back-react to the particle production near the horizon. For a more realistic scenario, we should include **space-time backreaction** (space-time reacts to the presence of the particle), since the particles that undergo quantum tunneling and exit the horizon can be highly energetic, modifying locally the space-time itself (locally since the stress-energy tensor acquires a local contribution from the particle, thus the metric is modified only locally by the presence of such energetic particle). Let's describe the basic idea briefly. As anticipated before the Hawking calculation the metric is thought as a static fixed background that satisfies the vacuum Einstein equation

$$R_{\mu\nu} = 0 \tag{1.24}$$

since there is no source for the gravitational field, the stress-energy tensor is zero (static since the Schwarzschild metric is manifestly time-independent in Schwarzschild coordinates).
Now, to account for back-reaction we need a coupled system of PDE:

$$\begin{cases} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi < \hat{T}_{\mu\nu} > \\ \hat{\Box}\hat{\phi} = 0 \end{cases} \tag{1.25}$$

Let's look a bit in detail at such equations. The first equation is the Einstein equation with a source that is given by the expectation value of the quantum stress-energy tensor (i.e. the stress-energy tensor operator associated with quantum matter); the second equation is the Klein-Gordon equation for a massless scalar field in a curved space-time; the hatted D'Alembertian operator can be computed using the principle of general covariance

$$\partial_\mu = \frac{\partial}{\partial x^\mu} \longrightarrow \nabla_\mu \tag{1.26}$$

that implies

$$\Box = \partial_\mu \partial^\mu \longrightarrow \nabla_\mu \nabla^\mu \tag{1.27}$$

The second equation thus gives the dynamics of the quantum field on Schwarzschild space-time with corrections due to the presence of a non-zero stress energy tensor ; this system is coupled, in the sense that the Einstein equation contains as source the energy-momentum tensor constructed from the solution of the second equation, while the second equation describes the motion of the particles in a space-time that is solution of the first equation. Despite the fact that this system describes more realistically the radiation process, it is much more difficult to solve exactly compared to the uncoupled system. Usually one studies the simplified version as an approximation of this one, also because it captures the main qualitative features of the problem with back-reaction.

1.3 Information Loss Paradox

1.3.1 What does the evaporation of the black hole imply?

If we accept Hawking's solution for the fate of a black hole (the evaporation process continues until the black hole has completely evaporated) then we might ask: where did all the information that was inside the black hole go? According to General Relativity black holes would be cosmic containers that collect everything around them and do not allow anyone to see inside them; moreover, according to Hawking, once they have completely evaporated everything they contained disappears forever. Of course this is not possible because it violates a fundamental principle of quantum mechanics which states that every process in the universe must preserve information.

In particular the complete black hole evaporation brings to the violation of **unitarity** and its relation to the causal structure of the evaporating black hole space-time. In quantum physics, unitarity is the condition that the time evolution of a quantum state is represented mathematically by a unitary operator. This is typically taken as a basic axiom or postulate of quantum mechanics. The time evolution described by a time-independent Hamiltonian is represented by a one-parameter family of unitary operators, for which the Hamiltonian is a generator. In Schrödinger picture, the unitary operators are taken to act on the quantum state of the system, while in the Heisenberg picture, the time dependence is instead embedded in the observables. The evolution operator reads

$$\hat{U} = e^{-i\hat{H}t/\hbar} \tag{1.28}$$

where $\hat{H}$ is the quantum hamiltonian of the system, and t is the time parameter.
In quantum mechanics, every state is described as a vector in Hilbert space. When one wants to make a measurement of given physical quantity on a certain state, it is convenient to write the state in the basis of the eigenstates of the operator associated with the quantity that has to be measured, for example a momentum vector basis in case momentum is measured. The measurement operator is diagonal in this basis. The probability of obtaining a certain measurement on the state depends on the probability amplitude, which is given by the inner product of the physical state with the basis vectors diagonalizing the measurement operator. For a physical state that is measured after it has evolved over time, the probability amplitude is given by the inner product of the physical state time-evolved with its basis vectors.
Calling $|\Phi_i>$ the initial state, and $|\Psi>$ the evolved state, we can relate them using the time evolution operator:

$$|\Psi>= e^{-i\hat{H}t/\hbar}|\Phi_i> \tag{1.29}$$

where the Hamiltonian is Hermitian and the time evolution operator is unitary. The operator U is a unitary operator i.e. an operator such that

$$U^{\dagger}U = UU^{\dagger} = 1 \tag{1.30}$$

In a unitary model, the initial state of the quantum system evolves over time in such a way as to maintain the validity of a probabilistic/statistical description of the process and the possibility of making quantitative predictions about the final state. Hawking has argued that universes containing evaporating black holes can evolve from pure initial states to mixed final ones. Such evolution is **non-unitary** and so contravenes fundamental quantum principles. Physically this means that from the initial state, which is a pure state, we can predict the final state (mixed), but if we study the process in a time-reversed way, we are not able to recover the initial state from the final one: we cannot make retrodiction.
We will explain this phenomenon in the next section.
This powerful argument for non-unitary evaporation is one form of the information loss paradox, which we can usefully call **The Hawking Information Loss Paradox**.
In 1975 Hawking published an article titled *"Breakdown of predictability in gravitational collapse"* where he presented his technical reflections on the loss of information to the scientific community. He explains that gravitation introduces a new level of statistical

uncertainty into black hole physics over and above the uncertainty usually associated with quantum mechanics.

Hawking's paradox has generated an extraordinary amount of interest and controversy in the physics community. What fuels the controversy is that Hawking's result violates the Unitarity principle of Quantum Mechanics in a physical situation in which such principle should keep its validity. To quote Hawking: *"Physicists seem to have a strong emotional attachment to information. I think comes from a desire for a feeling of permanence. [...] they feel that information, at least, should be eternal."*

We will analyze Hawking's article on predictability by trying to understand precisely why the initial quantum state of the star evolves from a pure to mixed state once the black hole is formed.

1.3.2 Transition from a pure to a mixed quantum state

Let's consider the physical process of collapse of a star forming a black hole, and the following evaporation of the black hole due to Hawking radiation. Let's consider three different instants of time during this process; this means mathematically fixing three slices of the space-time describing the whole process, that we call Σ_1, Σ_2, Σ_3; the first one is chosen such that the star didn't become a black hole yet (fig 1.2), the second is an instant of time in which the black hole is already formed, and the third an instant in which the black hole is completely evaporated through Hawking radiation; let H_1 be the Hilbert space in which the state describing the system (star + surrounding vacuum) lives when the system is on Σ_1, H_2 the Hilbert space in which the state of the system lives when the system is on Σ_2, and H_3 for the system on Σ_3.

Calling ξ_C the generic basis vector for H_1, ζ_B a generic basis vector for H_2, and χ_A a basis vector for H_3 (where the indices are set to recall that they belong to three different Hilbert spaces), according to quantum theory there must exist some tensor S_{ABC} such that

$$\sum_A \sum_B \sum_C S_{ABC} \chi_A \zeta_B \xi_C \tag{1.31}$$

is the generic state on the Hilbert space $H_1 \otimes H_2 \otimes H_3$, while S_{ABC} is the amplitude to have the initial state ξ_C on Σ_1, the final state χ_A on Σ_3 and the state ζ_B on Σ_2. Let's explain better the state 1.31: the three Hilbert spaces H_1, H_2, H_3 are the spaces in which the system lives at three different times; generally in quantum mechanics a system is

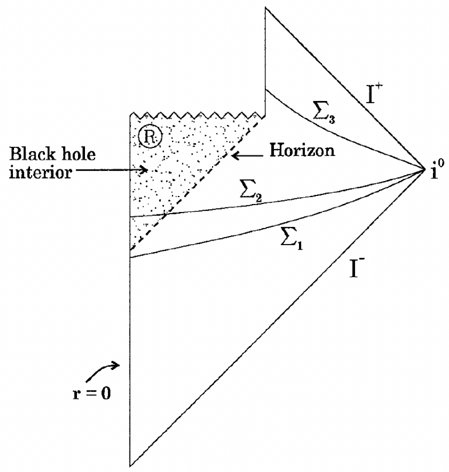


Figure 1.2: *Penrose diagram describing black hole evaporation (Credits: [5])*

described by a wavefunction, and even if the Schrodinger equation that describes the evolution of the state is deterministic (unique once the initial datum is fixed), a certain measurement on the state at a given time can give different measurement results, due to the intrinsic quantum probabilistic nature of the state; now, as for a state belonging to a certain Hilbert space H

$$\psi = \sum_A c_A \chi_A \tag{1.32}$$

the probability amplitude to obtain the χ_A state after a certain measurement is given by c_A, for a state like 1.31 the probability amplitude to obtain the state ξ_C when the system is on Σ_1, the state ζ_B when is on Σ_2 and χ_A on Σ_3 is given by S_{ABC}.

In general, given only the initial state ξ_C one cannot determine the final state but only the element $\sum_C S_{ABC}\xi_C$ belonging to the Hilbert space $H_2 \otimes H_3$. Now, we have to notice that a portion of the hypersurface Σ_2 is hidden inside the black hole horizon (see fig.1.2); now, since we are not able to make any measurement inside the horizon, the state on Σ_2 has not only an intrinsic quantum probability, but also a statistical probability; generally speaking statistical probability arises when we are ignorant about the wavefunction describing the system; thus we have two different kinds of uncertainties for the state on Σ_2: a quantum uncertainty due to its intrinsic quantum nature and a statistical uncertainty due to our ignorance about the quantum state.

Because we are ignorant of the state on the Σ_2 hypersurface we cannot find the amplitude for measurements on the final hypersurface to give the answer χ_A but we can only calculate the probability for this outcome to be $\sum_C \sum_D \rho_{CD}\overline{\chi}_C \chi_D$ where χ_D are the basis vectors of H_3 living on Σ_3, $\overline{\chi}_C$ the complex conjugates of χ_C and

$$\rho_{CD} = \sum_E \sum_B \sum_F \overline{S}_{CBE} S_{DBF} \overline{\xi}_E \xi_F \qquad (1.33)$$

is the density matrix which completely describes observations made only on the future surface Σ_3 and not on the surface Σ_2. Now, since ρ_{CD} contains the S_{DBF} tensor that regards not only the final and initial state, but also the state belonging to Σ_2 affected by statistical uncertainty, such tensor gives a statistical uncertainty to the final state.

The fact that in gravitational interactions the final state contains ρ_{CD} (technically speaking a density matrix) means that the state evolved from a pure state to a mixed state (a state with a statistical uncertainty); this kind of evolution signals the breakdown of predictability so far as a system in a pure state is a system for which the measurement of a given observables gives a well precise result, while the measurement of the same observables on a mixed state gives a given result only with a certain statistical probability. Pure-to-mixed state is a non-unitary evolution and if the evolution is not unitary there will be loss of information. The pure-to-mixed transition of the quantum state is responsible for the loss of retrodictibility and the failure of CPT-invariance. While the first statement is intuitive, we'll give an explaination of the second one in the next paragraph.

1.3.3 CPT-invariance

Some take the CPT-invariance of quantum gravity to hang in the balance of the Hawking controversy. In elementary particle physics, symmetries are of fundamental importance. We have:

- invariance of observed processes with respect to certain transformations such as charge conjugation C,
- parity P
- time reversal T

The first symmetry consists in associating an elementary particle with its antiparticle (reversing the sign of its electric charge). P is a transformation that reverses the direction of the spatial axes (a bit like seeing the world in a mirror). Finally T reverses the direction of the time axis, as if time were flowing backwards, from the future to the past.

In our daily perception many physical processes appear invariant under T and P such as marbles colliding on sand and this is because their dynamics are described by classical

physics laws.
Until the discovery of parity conservation violation in 1957 [4], there was a widespread belief that each of the three symmetries was separately preserved. However, despite this until now we have never observed a case in which the three conditions are violated **simultaneously**. We can have CP violation or CT violation but never a CPT violation. In black hole evaporation, CPT symmetry is violated, let's see why.
Consider an initial ($t \longrightarrow -\infty$) density matrix, ρ_{in}, which correspond to a pure state. Let ρ_{out} be the corresponding out state ($t \longrightarrow +\infty$), which is a mixed state in Hawking calculations. Assume that there exists a unitary, invertible quantum-mechanical CPT operator Θ such that:

$$\Theta\rho_{in} = \overline{\rho}_{out}, \quad \rho_{out} = \Theta^{-1}\overline{\rho}_{in} \quad S\overline{\rho}_{in} = \overline{\rho}_{out} \tag{1.34}$$

Where S is called S-matrix relates the initial state to the final state, and keeps in account the gravitational interaction during the dynamics. From $\rho_{out} = S\rho_{in}$ we can write:

$$\Theta^{-1}S\Theta^{-1}S\rho_{in} = \rho_{in} \tag{1.35}$$

which implies the existence of an inverse $\Theta^{-1}S\Theta^{-1}$ of the S-matrix, which as explained above does not exist as a result of the information loss in the problem. Thus, we conclude that in the evaporation of black hole case the CPT invariance cannot hold. Since CPT invariance is proved theoretically and supported by experiments for field theories in flat space-time, it is commonly assumed to be valid also for theories concerning gravity. This violation is actually seen as an issue of the Hawking model.

1.3.4 Page time and the Page curve

In this section we will briefly review the modern formulation of the paradox, due to D. Page (see [15],[17]).
In order to do this, we need to introduce some important statistical quantum mechanical notion, crucial in the following formulation. Let's consider a generic quantum mechanical state made of a certain number N of particles. As described before, such state is described by a generic density matrix ρ; then we can define for such state the so called *Von Neumann entropy* (also called fine-grained entropy):

$$S_{vN} \equiv -Tr(\rho ln(\rho)) \tag{1.36}$$

Let's give some important features of such entropy

- A pure and separable state has $S_{vN} = 0$. This means that if we have a state with zero statistical uncertainty, that can be written as a product of single-particle states

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes ... |\psi_N\rangle \tag{1.37}$$

  its Von Neumann entropy is 0.

- A pure maximally entangled state has an Von Neumann entropy given by

$$S_{vN} = ln(N) \tag{1.38}$$

  where N is the dimension of the Hilbert space of the system.

- If we start with a system A, and we divide it in two subsystems B and C, the Von Neumann entropy of the two subsystems can be computed respectively as

$$S^B_{vN} = -Tr_C(\rho ln(\rho)), \quad S^C_{vN} = -Tr_B(\rho ln(\rho)) \tag{1.39}$$

  and if the whole state is a pure state, we have

$$S^B_{vN} = S^C_{vN} \tag{1.40}$$

  thus the von Neumann entropy of the two subsystems forming a system in a pure state is equal.

From the previous features we notice that this kind of entropy measures the degree of entanglement in a certain state: if the state is a product state (no entanglement) such entropy is zero, while if is in a maximally entangled state its entropy is maximum, given by 1.38.

We use this mathematical tool to reach the Page formulation of the information paradox. Let's thus consider a collapsing star that enters in its own horizon and produces a black hole of generic mass M. The collapsing star can be described as a pure quantum mechanical state; once the black hole forms, Hawking calculations show a production of pairs of particles near the horizon, one infalling and one outgoing toward $r = +\infty$. We can consider the whole quantum mechanical system as a bipartite system, with an Hilbert space given by:

$$\mathcal{H}_{tot} = \mathcal{H}_{in} \otimes \mathcal{H}_{out} \tag{1.41}$$

We can compute then the von Neumann entropy as defined before for the two subsystems. Since the pairs are produced entangled, we have that the entropy for the system living in $\mathcal{H}_{in}$ will be equal to the entropy of the system in $\mathcal{H}_{out}$, and during the first stage of evaporation both will increase in time, since the two systems start to acquire correlations with the particles of the other system.

Before proceeding, we have to introduce the concept of *coarse-grained entropy*. Without going in details with this concept, we only state here that such entropy follows the second principle of thermodynamics (will increase over time), and in the case of a black hole it coincides with the thermodynamical Bekenstein-Hawking entropy given by 1.19. One can prove that at each time

$$S_{in}^{vN} \leq S_{BH} \tag{1.42}$$

Let's see how these quantities evolve in time. We previously noticed that the thermodynamical entropy of a black hole decreases in time, since it is proportional to the area, and the area decreases during the evaporation. By the other side, the von Neumann entropy increases in time during the evaporation, since a larger number of particles belonging to $\mathcal{H}_{in}$ are entangled with particles emitted toward infinity. At a certain point of the evaporation, the von Neumann entropy will be equal to the thermodynamical entropy of the black hole, saturating relation 1.42. This happens when half of the mass of the black hole evaporated, and the time at which this happens is called *Page time*. It is at this point that the paradox in this formulation arises; let's see why. After the Page time, the von Neumann entropy should decrease, for the simple reason that it has to be less than the thermodynamical entropy of the black hole, and the thermodynamical entropy decreases during the whole process. This however is not what comes from a direct calculation; it turns out that the von Neumann entropy of the outgoing radiation increases with time, and thus the same holds for the von Neumann entropy of the interior since they have to be equal in order to have a whole pure state. This gives rise to the paradox in its modern formulation. Page suggested that a calculation that keeps in account quantum gravity should modify this situation, and in particular should give a decreasing behaviour of the radiation von Neumann entropy after the Page time. He proposed a decreasing behaviour of the von Neumann entropy that goes to zero together with the thermodynamical entropy of the black hole. From Page's perspective, such expectation should be proved using a full quantum theory of gravity.

The situation is shown in the plot in fig.1.3. Let's give a physical interpretation of the previous formulation. After the Page time the number of outgoing partners become larger

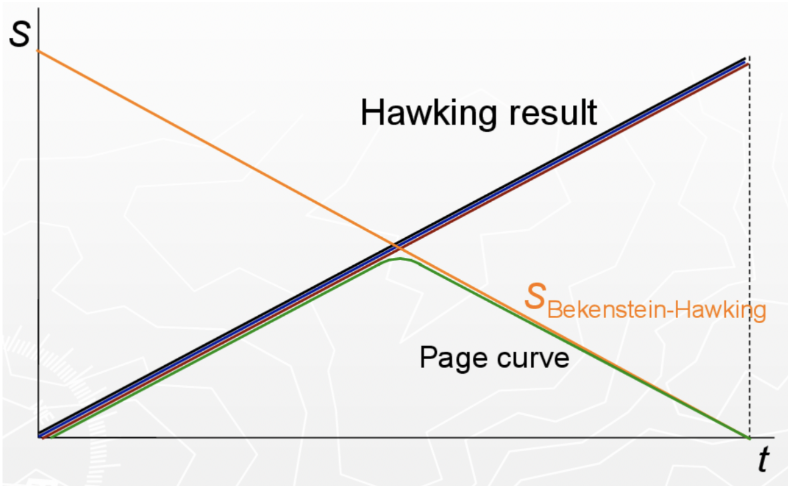


Figure 1.3: *Page curve of von Neumann entropy and Page time.*
In orange the expected Page curve, in blue the von Neumann entropy based on Hawking calculations, and in orange the thermodynamical entropy of the black hole. The page time is the time at which the Page curve changes its slope.

than the number of the ingoing particles, this because the evaporated mass becomes larger than $M/2$. This means that not all the outgoing particles can have an entangled particle in the interior, and if the entanglement entropy increases (according to the Hawking model), we would end up with particles "entangled with nothing". Thus the quantum state describing the system in this situation cannot be written as a wave function, but can be described only by a mixed density matrix, with a statistical intrinsic uncertainty. This physically brings to loss of information, and the arising of the information loss paradox. Moreover and equivalently, the transition from a pure state to a mixed state would mean the breaking of unitarity, a principle that is generally assumed to hold in any quantum theory.

1.3.5 Ways out of the paradox

Suddenly after Hawking proposed the Information paradox to the scientific community, many attempts to solve it have been formulated; all the escapes that physicists proposed during the years are united by changing some of the Hawking's assumptions; in this paragraph we won't describe all the proposals that are present in literature, but we'll describe only the more significant ones. In particular we'll focus on the last two of them, and we'll study them more in detail in the next and in the last chapter of this work.

- We should discard the theory of General Relativity where it violates the fundamental principles of physics (such as the existence of the singularity inside the black hole) in favor of a theory of gravity that allows stars to collapse without forming the event horizon. If the horizon were removed we would be able to see inside the black hole and above all the information would be able to get out (see [11]). This escape would be difficult to be trust, since black holes have been observed recently by the Event horizon telescope.

- Let's suppose we accept the theory of General Relativity as the complete theory for gravitational collapse (so black holes do form) and let's admit evaporation of black holes as a feasible solution. An escape route could be to consider the evaporation of the black hole not as in figure 1.2 but as in figure 1.3, i.e. the black hole evaporates instantaneously. In this case we would not have the problem of a time evolution of the quantum state at the initial instant and therefore it would remain a pure state while retaining all the information. Only wrong timing choices result in an apparent loss of information.
  This is unacceptable since instantaneous processes do not exist in nature: even minimal, a time interval of a transformation can never be zero.

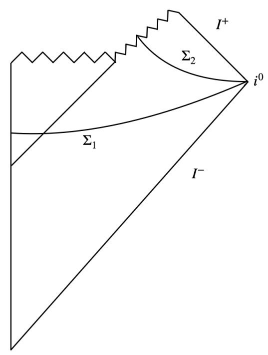


Figure 1.4: *Black hole instant evaporation*

- Let's suppose we accept the theory of General Relativity as the complete theory for gravitational collapse (so black holes do form) and they evaporate in a finite (and therefore not instantaneous) time. One solution could be to allow information entering the black hole to be *"bleached out"* on the event horizon. By virtue of the No-Hair theorem, it can be verified that the structure of space-time around a rotating black hole is uniquely determined by 3 parameters: mass, spin parameter and electric charge. So even this solution is not feasible because black holes having the same mass, same charge and same angular momentum could have "ingested" different information but from the outside they would appear completely identical to us. So the information cannot be on the surface of the horizon.

- Let's assume that the whole mass of the Black hole evaporates through Hawing radiation; this route is based on the further assumption that the outgoing photons are not really in a mixed state, but have correlations between themselves, in such a way that the final state given entirely by Hawking particles (the Black hole is completely evaporated) is pure, due to such "internal" correlations between Hawking particles. With correlations we mean that the outgoing particles form a quantum multi-particle system described by an entangled state. Radiation of this kind would not be exactly thermal (otherwise it would be described by a mixed state); the assumption here is that it appears thermal if we look at a small bunch of outgoing photons, but if we look at the whole state it is not thermal (fig.1.5).

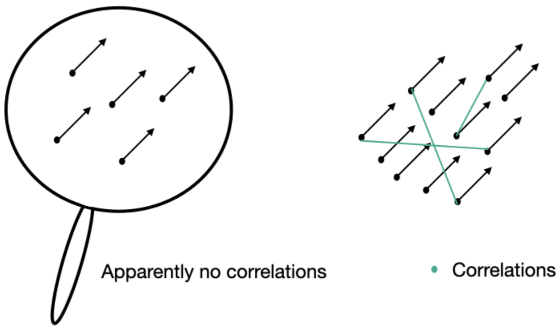


Figure 1.5: *Only a state with a large number of particles shows correlations*

Also this escape however comes with an issue; the mixed density matrix for the Hawking radiations comes from a rigorous calculation within QFT in curved spacetimes. If the final state is not mixed, this means that the calculation should be corrected, and this would mean that such theory doesn't hold in its own regime of validity (for wavelengths much larger than the Planck length), and this is something undesired.

- Here we only introduce briefly this escape, which will be discussed more extensively in the next chapter. Let's assume that the black hole radiates, but not all the mass of the black hole evaporates through the emission of Hawking radiation. Instead, assume that the evaporation stops when the internal mass of the Black Hole reaches the Planck scale:

$$M \sim m_{Pl} \tag{1.43}$$

This tiny mass survived to the evaporation is usually called *remnant*. Let's suppose as before that the initial quantum state of the star that forms the black hole during its collapse is in a pure state. As we saw in the previous section, in order to have a state that remains pure at each time, we need that each outgoing photon is correlated with some particle in the interior. However, after the black hole evaporates for half of its original mass (the Page time), in the interior there are not enough particles to be entangled with the exterior. This means that the total state (BH + Hawking radiation) from this time on cannot apparently remain pure. The way to escape from the paradox is based on the assumption that even if the number of particles that lie inside the remnant is not enough to justify the increasing von Neumann entropy of the Hawking radiation, the number of states of such interior particles keeps growing in time, allowing the von Neumann entropy to increase. This implies relaxing the assumption that $S_{vN} \leq S_{BH}$, and that the external particles can form a total pure state with the interior even if the number of particles in the exterior is larger than the internal ones. We'll look more in detail at this escape the last chapter.

- The very last escape route that we present is the clearer way to solve the Information Paradox, and will be studied extensively in the last chapter.
  Here we assume that the black hole evaporates accordingly with Hawking's calculation, but that this is not the only way in which energy comes out of the black hole. In particular, long before the Page time

  $$T_{Page} \sim \frac{M^3}{8m_{Planck}^2} \tag{1.44}$$

  where $m_{Planck} = \sqrt{\frac{\hbar}{4\pi G}}$; the non-evaporated matter inside the black hole comes out and makes the black hole horizon disappearing. In this way the state can remain pure during the whole evolution, and there is no loss of information at all. This escape is based on a collapsing star model in Loop Quantum Gravity, that will be presented as first quantum star collapse model in the last chapter.

The first five routes will not be studied in detail in this work. Instead in the next chapter we'll focus on the last two, once we introduced two different quantum gravity models that avoid the information paradox precisely in these two different ways.

Chapter 2

Fate of black holes in Loop Quantum Gravity

In paragraph 1.2 we introduced the Schwarzschild solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{1}{\left(1 - \frac{2M}{r}\right)}dr^2 + r^2 d\Omega^2 \tag{2.1}$$

and we gave a brief description about the two singularities that affect such metric.
In particular we see that in $r = 0$ the first component of the metric diverges and the second component becomes zero while in $r = 2M$ the first component becomes zero and the second infinite. As noticed before these are two different kind of singularities: the one at $r = 2M$ can be removed through a coordinate transformation like 1.8 while the second one at $r = 0$ cannot be removed and is a coordinate-independent feature of the line element. In General Relativity in order to distinguish between a coordinate and a physical singularity we compute the so called *curvature scalars* that are quantities independent from the coordinates that we choose. Some examples of curvature scalars are the Ricci scalar ($R_{\mu\nu}g^{\mu\nu}$) and the double contraction of the Ricci tensor ($R_{\mu\nu}R^{\mu\nu}$). If we compute these scalars for the Schwarzschild solution in an arbitrary coordinate system we obtain that they diverge at $r = 0$ while are perfectly finite at $r = 2M$. The points on which the curvature scalar diverge are called gravitational singularities; therefore on gravitational singularity the curvature of space-time diverges (or equivalently the space-time deformation becomes infinite). Although physical singularities are historically considered as physical features of Einstein solutions, it is a common belief between theoretical physicists that such singularities have to be seen as limits of Einstein theory; let's give a brief argument for this position: looking at 1.1 if $R(r = 0) = \infty$ we necessarily

need $T_{\mu\nu}(r=0)=\infty$ that means in particular that the source of the gravitational field described by the Schwarzschild solution is focused in a geometrical point of space; this is seen as unphysical for two main reasons: firstly this implies that the energy density of the source is infinite at $r=0$, and infinite quantities belong to the mathematical realm and not to the physical reality; secondly (and most importantly) a geometrical point has a size much smaller (infinitely smaller) than the size at which quantum mechanical effects are expected to play a role for the gravitational field, i.e. the Planck size ($l_p \sim 1.6 \cdot 10^{-35} m$); there are various heuristic arguments that suggest that this is the scale at which quantum gravity effects become relevant, linked with black hole thermodynamics, cosmology, and the problem of divergences in classical general relativity (for a brief review see 12).

The presence of gravitational singularities in Einstein theory has been one of the main reasons that brought theoretical physicists to look for a quantum theory of gravity: a valid quantum theory of gravity is expected to resolve the singularity problem of General Relativity and avoid the physical singularities with some non diverging physical alternatives.

The singularity resolution is actually one of the successes of the Loop Quantum Gravity program, one of the most promising quantum theory of gravity; in this chapter we will describe briefly Loop Quantum Gravity and then we'll show how the singularity avoidance in black hole models built within this theory allows to avoid completely the information loss paradox.

2.1 Loop Quantum Gravity in a nutshell

2.1.1 Spin Network states and discreteness of geometry

Constructing Loop Quantum Gravity from Einstein theory of Gravity is an extremely technical and rigorous path; since it is beyond the scopes of this work we won't describe it here (for a complete derivation see [16]); to have a physical insight about the theory and to acquire the tools that we need for our scopes, we'll limit ourself by introducing directly the quantum states of the theory.

In ordinary quantum mechanics the fundamental entities are quantum particles like electrons, protons,.. and using them as building blocks we can define a generic state with N particles belonging to the Hilbert space $\mathcal{H}^N$; similarly in Loop Quantum Gravity the fundamental objects are **quanta of space**, that can be thought as atoms of space with volume $\sim l_P^3$ and mathematically described by quantum states called spin-networks. Let's give a pictorial example of such quantum states: a spin network can be depicted as a mathematical graph in which each node of the graph represents (is dual to) an atom of space and each link carries a certain quantum number l, that can take only half-integer values $l = \frac{1}{2}, 1, \frac{3}{2}, ...$; in order to understand the physical meaning of the links and the

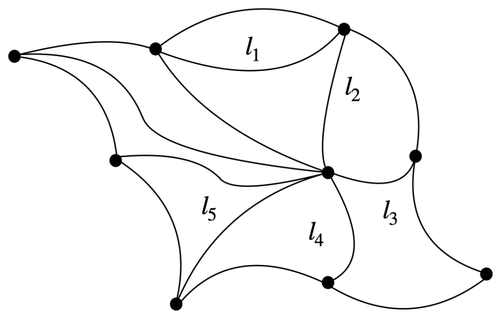


Figure 2.1: *Picture of spin networks*

quantum numbers l's associated to each link, we have to imagine that each node of the graph is the center of a polyhedron, with a number of faces given by the number of links that emanate from it; for example, if four links emanate from a node we have a tetrahedron, for six links we have an hexahedron,..; thus each link emanating from the node

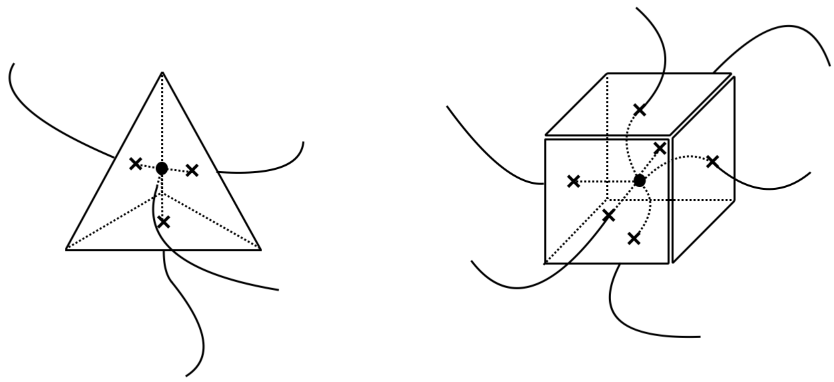


Figure 2.2: *On the left a tetrahedron is depicted as dual to a four-valent node; on the right an hexahedron dual to a six-valent node*

describes a face of the polyhedron associated with the node; this means that if a link emanates from a node and ends in a different node, the two atoms of space are attached through the faces relative to that link. Given a link, the relative value l determines the area of the face associated with the link itself, through the following:

$$A = 8\pi l_P^2 \gamma \sqrt{l(l+1)} \tag{2.2}$$

where γ is called Immirzi parameter, and is a free parameter of the theory. Such spectrum gives the possible eigenvalues of the LQG area operator acting on a given link $\tilde{l}$ of the state:

$$\hat{A}\left|\psi\right\rangle_{\tilde{l}} = 8\pi l_P^2 \gamma \sqrt{\tilde{l}(\tilde{l}+1)}\left|\psi\right\rangle_{\tilde{l}} \tag{2.3}$$

If we make $\hat{A}$ acting on the whole spin network state, we will obtain the total area, given in general by:

$$A = \sum_i 8\pi l_P^2 \gamma \sqrt{l_i(l_i+1)} \tag{2.4}$$

where the sum is performed over all the links of the state. We notice from 2.2 some important features: firstly the faces of the polyhedron have areas proportional to the Planck area, thus are Planckian-size; then there is a minimum possible value admitted for the area, given by

$$A_{min} = 4\sqrt{3}\pi l_P^2 \gamma \tag{2.5}$$

for $l = \frac{1}{2}$; this is the minimum area in Loop Quantum Gravity; we notice that it is proportional to the Planck area $A_{Planck} = l_{Planck}^2$. Finally, the spectrum of the operator $\hat{A}$ is discrete: this is in opposition with the value of area computed in General Relativity for a generic surface, that in general can take continuous values; such discreteness can be seen as a genuine quantum feature of LQG, as well as the discreteness of the energy levels in the hydrogen atom or in an harmonic oscillator. Near the quantization of areas, one can derive the quantization of volumes of the atoms of space in LQG; if we construct the volume operator (that acting on a state measures the physical volume of the state) and we make it acting on a certain spin network state $\left|\psi\right\rangle$

$$\hat{V}\left|\psi\right\rangle = V\left|\psi\right\rangle \tag{2.6}$$

it turns out that the spectrum of $\hat{V}$ is discrete; this physically means that each atom of space can assume only discrete volumes. Moreover, as well as the area spectrum, the volume spectrum is bounded from below, i.e. each atom cannot reach a volume smaller than the smallest eigenvalue of $\hat{V}$, that is

$$V_{min} \propto l_P^3 \tag{2.7}$$

This discreteness, that is present already at the kinematical level, so it regards also states that are not solutions of the quantum EOM, has a deep impact in the singularity problem of General Relativity: the macroscopic space-times derived as solutions of Einstein theory can be thought as containing an enormous amount of such grains of space, as well as a beach contains an enormous amount of sand grains; if such atoms of space cannot be reduced to a size smaller than l_p^3, then matter (or more generally energy) cannot be confined in a space smaller than l_P^3, thus the quantum theory avoids completely the singularity problem that affects General Relativity. The derivation of the Area and Volume spectra is one of the most fascinating results of Loop Quantum Gravity, and due to singularity resolution, the cosmological and astrophysical models based on this theory don't contain gravitational singularities. The derivation of the spectra is rigorous, and comes from the quantization of the classical area and volume operators of General Relativity; moreover, differently from other quantum gravity approaches, the theory doesn't postulate the fundamental granularity of space, or the discretization of the operators, but derives it from first principles (see i.e. [18], [19]).

We clarified how physical singularities are avoided in Loop Quantum gravity; the question now is: what does replace them?

The answer to this question is crucial and is at the core of the avoidance of the information loss paradox in LQG-based models.

2.1.2 Effective equations and quantum bounce

Before looking in detail at the singularity avoidance in LQG-based models, we need to introduce a fundamental concept, useful for the next development: the concept of effective model.

To describe it let's start with a classical parallelism: suppose that we want to study the dynamics of a macroscopic fluid, like the motion of water in a sea or the evolution of oxygen in the atmosphere; we know that such fluids are made of an enormous amount of particles and to study its evolution we don't use the equations of motion of single particles but the fluid dynamics equations that are macroscopic equations: this because the microscopic details of the system like the dynamics of single particles are irrelevant for the behaviour of the fluid. Similarly the dynamics of single quanta of space (or of a bunch of them) is irrelevant for the dynamics of the space-time generated by a huge amount of them. Since the equations of motion of Loop Quantum Gravity describe the microscopic dynamics of space-time we cannot use them to study the evolution of astrophysical and cosmological systems, like black holes or the universe itself; with this we don't mean that this cannot be done with Loop equations but that is almost impossible and useless, as well as studying the dynamics of a gas using a system of $N \propto N_A$ (Avogadro number) of Schrödinger equations. In order to extract equations that described the collective behaviour of a huge amount of quanta of space for models like evolving black holes or an evolving universe we adopt the so called **effective approach**. To understand it properly we have to recall the features of a quantum state in ordinary quantum mechanics: a quantum state is described by wave function that allows to compute the probability of the output in a certain measurement on the state itself; the spreading of the wave function means a quantum uncertainty on the results of measurements and such uncertainty is small for sharply-peaked states. Let's consider now a system of N particles with unit mass in ordinary quantum mechanics; the canonical commutation relations between positions and velocities of single particles are given by

$$[\hat{x}_i, \hat{\dot{x}}_j] = i\hbar\delta_{ij} \tag{2.8}$$

let's study now the commutation relation between position and velocity of the center of mass of the system: recalling that $X = \frac{1}{N}\sum_N x_N$ we have

$$[\hat{X}, \hat{\dot{X}}] = \frac{i\hbar}{N} \tag{2.9}$$

which goes to zero for $N \to \infty$; this means that in the limit of large N the center of mass becomes a classical (or more precisely a semi-classical) variable; at the level of wave function this means that the wave function of the whole system is sharply-peaked on precise values of position and momentum of the center of mass at the same time. Similarly we can extract from a quantum state of N atoms of space collective variables that behave semiclassically, i.e. the quantum state of the system is sharply-peaked on precise values of such variables at each time. This gives rise to the effective equations of Loop Quantum Gravity (for references about the validity of the effective approach see i.e. [20]). The evolution of such variables is simpler than the evolution of the system in terms of microscopic variables, as well as the evolution of variables like temperature, pressure and density for a fluid is much simpler than the evolution of the microscopic variables describing the fluid; the effective approach can be seen as a thermodynamic limit of the system. We won't give a precise derivation of the effective Loop Quantum Gravity equations but we will give some important features and application. The effective equations extracted from the full theory (the quantum equations for single quanta of space) turn out to be the Einstein equations with quantum corrections and such equations do not contain solutions affected by gravitational singularities. This means in particular that the collective

variables extracted from the fundamental microscopic theory are the metric variables, or more precisely the Ashtekar-Barbero variables that is a set of variables introduced in the loop quantization procedure and equivalent to the original Einstein variables (the components of the metric tensor).
An important example of effective Loop Quantum Gravity equations comes from cosmology and we introduce it here since turns out to be extremely useful to modelize the dynamics of a black hole.

2.1.3 Classical vs Loop Quantum Cosmology

In cosmological models based on Einstein theory of gravity the space-time describing the whole universe is generally assumed to be expanding, homogeneous and isotropic; the simplest metric tensor describing a space-time with such symmetries is given by the Friedmann-Robertson-Walker flat line element:

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\theta^2 + r^2 sin^2(\theta) d\phi^2) \tag{2.10}$$

where r, θ, ϕ are the usual spherical coordinates, t is a time coordinate and $a(t)$ (that is the only unknown of the system) is called **scale factor**. The function $a(t)$ describes how spatial lengths are deformed during the evolution of the universe, and it can be found by solving the so called Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho \tag{2.11}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\rho \tag{2.12}$$

that are the Einstein equations (without cosmological constant) once homogeneity and isotropy are imposed; we notice here that the assumption of homogeneity on the solution makes the original set of partial differential equations a system of ordinary differential equations, simpler to be solved. The function $\rho(t)$ is the energy density of the fluid generating the curvature of space-time; in these equations the fluid is assumed to be pressureless (technically called dust).
It is easy to see that for $a(t) \to 0$ the FRW line element becomes singular; if one computes the curvature scalars finds that they diverge in such limit. This means that the metric is

affected by a gravitational singularity if $a(t) = 0$. The solution of the system 2.12 is

$$a(t) \propto t^{\frac{2}{3}} \tag{2.13}$$

where the proportionality factor depends on the initial condition. It is clear that the solution is actually affected by a gravitational singularity at $t = 0$; notice that for $a(t) \to 0$, $\rho \to \infty$, that means that the energy density diverges on the gravitational singularity.
In classical cosmology this singularity is called **big bang singularity** and for many years has been considered by cosmologists as the physical beginning of our universe. However as we pointed out many times a physical singularity is not a prediction of Einstein theory but a limit of the theory that is supposed to be removed by each reasonable quantum theory of gravity. Let's see now how the Friedmann equations are modified in Loop Quantum Gravity, and how they remove such singular behavious; if we apply the effective approach described before to an homogeneous and isotropic metric one finds that the Friedmann equations are corrected by terms proportional to the Planck area $\Delta \equiv l_P^2 = \hbar G$ (here and for the rest of this work we'll assume $c = 1$), that is proportional to the minimum eigenvalue of the area operator of the full theory.
In particular we obtain the following system

$$\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{8}{3}\pi G\rho\Big(1 - \frac{\rho}{\rho_c}\Big) \tag{2.14}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\rho\left(1 - \frac{5}{2}\frac{\rho}{\rho_c}\right) \tag{2.15}$$

where ρ_c is a constant called **critical density** and is given by $\rho_c = \frac{3}{8\pi G\gamma^2\Delta}$. By solving these equations numerically one finds that the scale factor behaves almost in the classical way for t large while differs significantly from 2.13 near $t = 0$; if by a side the classical solution $a(t)$ decreases for decreasing t reaching $a(t = 0) = 0$, the effective solution instead reaches a minimum value $a(t = 0) = a_{min}$ and then starts to increase.
Thus instead of having a physical singularity at $t = 0$ the effective solution undergoes a **quantum bounce** that allows the universe to have a non singular dynamics before such bounce. This conceptually means pushing backward in time the beginning of the universe, which does not start at the big bang as classically supposed, but has a dynamics infinitely extended in the past of $t = 0$. The solution describes a Universe that starts at $t = -\infty$ collapsing until $t = 0$, at which point reaches the maximum (non singular) curvature then starts to expand until $t = +\infty$; the solution is singularity-free and well

defined at each time. The dynamics is symmetric under the transformation $t \to -t$ and replaces the classical notion of big bang with the quantum notion of **bounce**. The energy density of the solution reaches the maximum value at the bounce $\rho_{Bounce} = \rho_c$ and is an increasing function of time in the pre-bounce phase and a decreasing function in the post-bounce phase. Notice that in this brief analysis we didn't speak about the spatial size of the universe during the dynamics; this because the metric 2.10 is defined for $r \in [0, +\infty)$, or in other words describes a space-time infinitely spatially extended. This means in particular that the whole universe doesn't reach a minimum size at the bounce since according with this solution it has to be intended to have an infinite volume at each time. Therefore what really bounces is not the volume of the whole universe but its energy density and its curvature. Nonetheless we can follow the dynamics of a certain arbitrary physical volume:

$$V(t) = V_0 a(t)^3 \tag{2.16}$$

and through the dynamics of $a(t)$ we obtain easily that such portion of universe shrinks in the collapsing phase, reaches a minimum at the bounce and then starts to re-expand. V_0 in 2.16 determines the initial size kept in consideration. In order to keep the validity of the effective approach we need to look at a large initial volume, since the thermodynamic limit needed in the effective theory for this system means taking a large number of quanta, that means a large initial volume. It is worth to underline here that depending on the size of the initial volume kept in consideration, the volume at the bounce can be arbitrarily large, much larger than the Planck volume: each initial volume will shrink and bounce when the related energy density reaches the critical one (also called Planckian density), independently from the size of the volume at the bounce. This can be seen as contradictory with the fact that quantum gravity effects are expected to arise at the Planck length scale; this however is wrong, since the Planck regime in quantum gravity regards the curvature of space-time, or equivalently the energy density generating such curvature. Let's give an argument to support the previous statement. If we consider the Minkowski space-time, we can in principle describe it with Loop equations, and we can take an extremely small size of space to describe the dynamics of the atoms of space belonging to it; if quantum gravity would depend strongly on the spatial size of the system, one would expect that the dynamics of the quanta at that level would affect significantly the whole space-time, since the whole space-time can be thought as an enormous bunch of such quanta; by the other side, the Minkowski space-time is not expected to undergo quantum effects, since the gravitational field is zero. This can give an hint about the fact

that strong quantum gravity effects are linked with the curvature of space-time, and not on the size under consideration.
By the other side, if we consider an initial small spatial portion of Universe, it cannot be described by the effective equations, since the thermodynamic limit would not be longer valid (thus the effective approach would fail), and the correct dynamics can be predicted only by the full LQG theory. Numerical simulations show that qualitatively each initial volume, independently from the size follows the same kind of dynamics with a contracting pre-bounce phase, the bounce and the post-bounce phase, but the details of the evolution depend on the initial size of the volume kept under consideration. For references about cosmological models in effective Loop Quantum Gravity see e.g. [22],[23],[24].
Even if the cosmological solution seems to be unrelated with the topic of this work, we'll see briefly that can be extremely useful for our purposes. When we modelize a collapsing star becoming eventually a black hole, we can assume in first approximation that the interior of the star is homogeneous and isotropic, with matter made of a pressureless fluid. This mathematically means that the interior of the star will follow the effective Friedmann equations 2.14 and 2.15, so that the solution for the interior can be assumed to be the cosmological solution, with the difference that the volume in this case is not infinite, but given by the physical volume of the star. In the next sections we'll describe two different astrophysical models for a collapsing star based on effective Loop Quantum Gravity that start both from this assumption for the interior, and we see how they completely avoid, in two radically different ways, the information loss paradox that affects the classical model.

2.2 White hole shock wave

We ended the previous paragraph introducing the idea behind the modelization of the collapse of a star in effective Loop Quantum Gravity; this approach mimics the path followed by Oppenheimer and Snyder in the classical framework [21] with the substantial difference that here the model is not based on Einstein theory of gravity but on effective LQG. As a consequence of this the equations that govern the collapse are quantum equations and the dynamics of the system differs significantly from the classical results. In this paragraph we will present a recent model of star collapse based on the effective LQG approach that successfully solves the two fundamental problems that we described before: the singularity problem in General Relativity that affects both the Schwarzschild and FRW metric, and the information loss paradox that derives from the Hawking calculation

based on quantum field theory in curved space-times.
Before looking in detail at the full collapse model, we'll give a brief description of the spherically symmetric vacuum solution derived from the effective equations. We introduce the vacuum solution separately since it allows us to enlight some important features of the collapse model. The collapse model will describe the collapse of a star with an exterior given by such vacuum solution.

2.2.1 Quantum corrected Schwarzschild metric

In chapter one we introduced the Schwarzschild line element as a spherical symmetric solution of the vacuum Einstein equations (the Einstein equations with vanishing energy momentum tensor); similarly the effective LQG equations produce a spherical symmetric solution that is the classical Schwarzschild metric plus quantum corrections, which vanish in the large radius limit. We won't give here a derivation of such line element, since is out of the scopes of this work, but we'll limit ourself to present it and show important features that make it significantly different from its classical counterpart (for a detailed derivation see [25]). For the ones familiar with the theory, we only remark that the model is derived by the canonical formulation of Loop Quantum Gravity, with spherical symmetry and the areal gauge imposed at the classical level. The quantum corrected effective Schwarzschild metric in Painlevè-Gullstrand coordinates reads:

$$ds^2 = -\left(1 - \frac{R_S}{r} + \frac{\gamma^2 \Delta R_S^2}{r^4}\right)dt^2 + 2\sqrt{\frac{R_S}{r}\left(1 - \frac{\gamma^2 \Delta R_S}{r^3}\right)}drdt + dr^2 + r^2 d\Omega^2 \tag{2.17}$$

where Δ is the minimum area eigenvalue in LQG introduced in the previous section. We immediately notice that it reduces to the classical Painlevè-Gullstrand metric if we send $\Delta \to 0$ (or we consider equivalently $r >> \sqrt{\Delta}$). This is a fundamental feature expected by a quantum metric, since it has to reproduce the classical result in a regime in which quantum gravity effects are negligible: sending Δ to zero means that we consider radii much larger than the Planck length, thus we study the space-time far away from the singularity; technically speaking this metric has the correct classical limit.
The first remarkable feature of such metric is that it contains two singularities (the two

zeroes of the first component of the metric) respectively in

$$r_{in} \sim (\gamma^2 \Delta R_S)^{\frac{1}{3}} + \frac{1}{3}\left(\frac{\gamma^4 \Delta^2}{R_S}\right)^{\frac{1}{3}} \tag{2.18}$$

$$r_{out} \sim R_S - \frac{\gamma^2 \Delta}{R_S} \tag{2.19}$$

if one computes the curvature scalars for such metric finds that such singularities are coordinate singularities, that as we explained extensively means that are points in which the coordinate system fails, but don't describe pathological behaviours of the metric tensor. For a reason that will be clear soon, we call such singularities respectively *inner horizon* and *outer horizon*. Moreover, we notice that the singularity 2.19 is located on the Schwartzschild horizon, up to corrections proportional to Δ at the leading order. This means that the quantum metric has the Schwarzschild horizon almost in the same position of the classical metric. This is also something expected, since quantum gravity corrections to the classical metric are assumed to be irrelevant far away from the classical gravitational singularity, or equivalently where the curvature of space-time is small. By the other side, the coordinate singularity 2.18 is a genuine quantum feature of the metric, and gives an extremely important departure from the classical solution. In the classical Schwarzschild metric (both in Painlevè-Gullstrand or Schwarzschild coordinates) the spatial region $0 < r < R_S$ is technically called **trapped region**, since the curvature is so strong that no form of energy (matter, radiation,..) can escape from inside the horizon. This can be easily seen if one draws the light cones of the classical Schwarzschild metric (fig. 2.3):

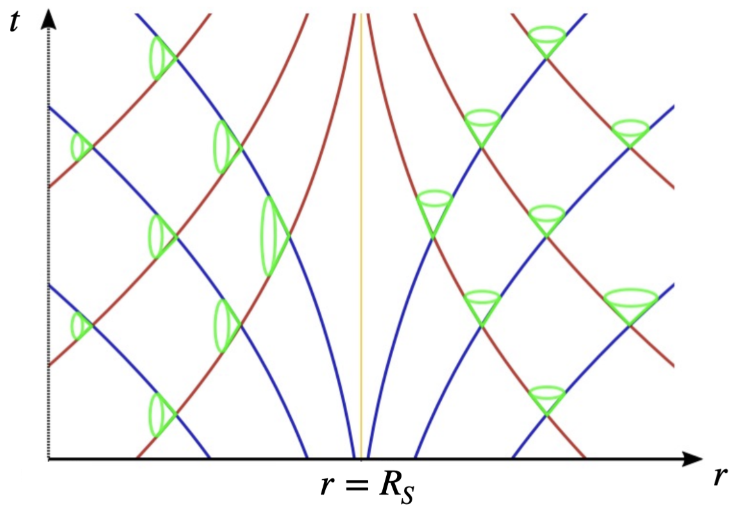


Figure 2.3: *Light cone structure of the classical Schwarzschild metric*

since particles cannot move faster than light, whatever kind of motion is a motion toward the classical physical singularity. Although this picture has been assumed to describe a physical black hole for decades, it turns out to be significantly modified in the quantum case. Also the effective metric has a trapped region within the exterior horizon (that as we noticed is almost located in the classical position), but it doesn't extend up the center $r = 0$. Instead, here the trapped region is bounded from below by the inner horizon, and the region inside $r = r_{in}$ is non-trapped. This means that if a particle moves inside r_{in}, is not forced to fall toward $r = 0$, but in principle can have also an outgoing motion. This pure quantum feature is fundamental in the fate of a collapsing star, and as we'll see in the avoidance of the information loss paradox. Its physical meaning is intuitive: quantum effects produce a repulsion conceptually similar to the repulsion that in the cosmological model brings to the bounce of energy density, and in the deep inner region significantly counteracts the trapping attraction of classical gravity, producing a region in which particles are not forced to fall in.

Before showing the light cones of this effective space-times we have to introduce the second important feature of 2.17; we notice that the non diagonal term of the metric becomes imaginary if the argument of the square root becomes negative; this mathematically happens for

$$r < (\gamma^2 \Delta R_S)^{\frac{1}{3}} \equiv r_{min} \tag{2.20}$$

Therefore the metric tensor is not defined for $r < r_{min}$, and in $r = r_{min}$ the metric is flat. The flatness of the metric on the inner boundary can be physically understood by the fact that on that point the attractive nature of gravity is perfectly balanced by a quantum repulsion of the space-time, completely absent in the classical setup. Thus in the point $r = r_{min}$ the metric behaves fine and the curvature scalars vanish. The existence of the region $0 < r < r_{min}$ can be well understood in the collapsing star model, that we'll introduce in the next paragraph. We only anticipate here that the existence of such region requires the existence of extended matter producing such kind of vacuum space-time. The conceptual picture of this effective space-time is now complete, and we can show its light cone structure (fig. 2.4) where the shaded grey region represents the region in which the metric is ill-defined. We notice from the previous picture that the light cones are flipped in the region $r_{min} < r < r_{in}$, that means that particle geodesics don't necessarily satisfy $\frac{dr}{dt} < 0$ (are not necessarily in-falling). This region, strongly affected by quantum gravity effects play a central role in the fate of a collapsing star entered in its own Schwarzschild horizon.

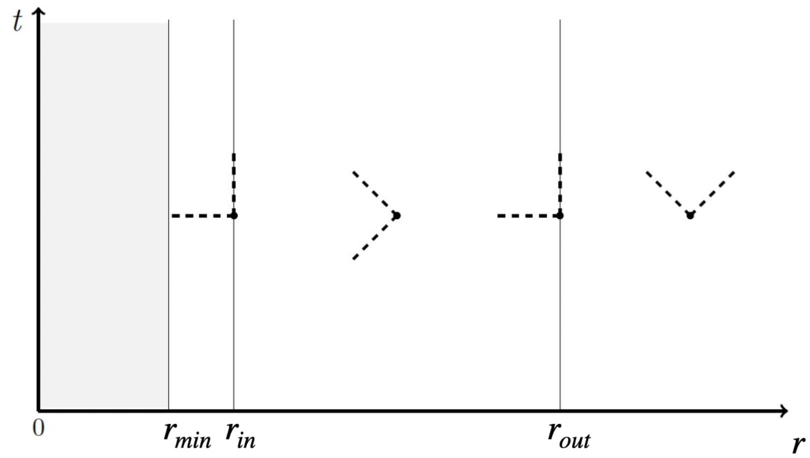


Figure 2.4: *Light cone structure of the effective Schwarzschild metric. (Credits: [25])*

It is worth to underline that such kind of black hole would be indistinguishable from its classical counterpart, as seen by an external observer; this is not the same when collapsing matter is kept in account. Finally, we want to emphasize that the line element 2.17 is not built as an ad hoc metric with some external inputs coming from physical intuitions, but is an exact analytical solution of the effective LQG equations of motion, as well as the Schwarzschild classical line element is solution of Einstein equations.
Let's look now at the complete picture of a collapsing star producing such external vacuum.

2.2.2 Effective Oppenheimer-Snyder collapse

Pre-bounce phase

As we said before, the classical Oppenheimer-Snyder collapse describes the collapse of an homogeneous, isotropic pressure-less star, with an interior given by the cosmological FRW metric 2.10 satisfying the classical Friedmann equations 2.12, and an exterior given by the classical Schwarzschild metric. The fate of a classical collapsing star is becoming a physical singularity when the volume of the star reaches $V(t) = 0$. Once the star is shrinked to a point, the metric describing the full space-time is simply the Schwarzschild metric, in which the total mass of the star is placed in $r = 0$, the classical gravitational singularity of the Schwarzschild metric. Since the metric 1.7 doesn't depend on t, the classical black hole is sometimes called eternal, in the sense that once is formed, there is no classical effect that can modify the metric, thus the space-time. The only effect that affects such space-time is the Hawking radiation, that allows the black hole to gen-

erate thermal particles, producing a shrinking of the horizon, or in physical terms the evaporation of the black hole itself. This dynamics is drastically modified in the effective quantum gravity model (see [27], [28], [29]).
The classical interior of the star (the classical FRW metric) is replaced by the effective FRW metric, that has precisely the classical form 2.10, but satisfies the effective Friedmann equations 2.14, and not their classical counterpart. By the other side, the classical Schwarzschild exterior is replaced by the effective counterpart 2.17. Let's see what happens to the interior. As well as in the cosmological case the collapsing star will not produce a physical singularity at the end of the collapse, but will bounce, once the critical energy density $\rho = \rho_c = \frac{3}{8\pi\gamma^2\Delta}$ is reached, then will start to expand. This bounce gives consistency to the effective approach, that avoids the physical singularity also in a spherical symmetric model like the star collapsing, a model that is extremely more complicate to loop quantize with respect to the cosmological models.
Another remarkable point is that such solution can be studied in a separate way, looking at the interior and the exterior independently, but the result is exactly the same for the pre-bounce phase if we study the system as a PDE system, in which there is no distinction between the interior and the exterior. This to say that the cosmological behaviour of the interior in the pre-bounce phase is not assumed a priori, but is a result of the effective equations of the model. This makes the identification with the cosmological solution non trivial.
We have to point out here that at each time during the collapse the exterior is the effective Schwarzschild one, that extends from the radius of the star $L(t)$ up to $r = +\infty$; this means that the collapsing star during the collapse enters both in the outer horizon r_{out} and in the inner horizon r_{in} of the effective metric. The significant departure from the classical collapse arises when the star reaches the inner horizon ($r_{min} < L(t) < r_{in}$), i.e. the deep quantum region of the black hole. In this region the radius of the star moves in a non trapped region, and is not forced to decrease toward $r = 0$. What one finds both analytically and numerically is that the radius of the star decreases up to r_{min}, and then inverts its motion. Therefore the star undergoes a quantum bounce, analogous to the cosmological quantum bounce described in the previous paragraph. This physically means that the physical volume of the star reaches a minimum given by $V_{min} = V_{Bounce} = \frac{4}{3}\pi r_{min}^3 = \frac{4}{3}\pi\gamma^2\Delta R_S$, and its energy density a maximum $\rho_{max} = \rho_c = \frac{3}{8\pi G\gamma^2\Delta}$. It is worth to notice here that the energy density reached at the bounce is equal to the one reached by the collapsing universe in the cosmological sector,

but this is not imposed as an input to the model, but is an analytical result. This suggests that such limit in the energy density is an intrinsic feature of the effective theory, independently from the particular model analyzed.
Another important observation regards the volume of the star at the bounce: for a large Schwarzschild radius (that means large mass of the star), the minimum volume reached by the star can be significantly larger than the Planck volume $V_P = l_P^3$. The reason for this is exactly the same that we explained in the previous paragraph within the cosmological sector: the planckian regime doesn't regard the lengths, but the curvature of the space-time and the energy density of the source. As well as in the cosmological sector, the effective dynamics has to be considered valid for stars with large masses, while for small (planckian) masses one has to consider the dynamics extracted from the full theory. For our purposes, since we are interested in stars with masses equal or larger than the solar mass (more precisely masses much larger than the Planck mass $m_P = \sqrt{\frac{\hbar}{4\pi G}}$), the effective approach is considered valid and representative of the dynamics eventually extracted from the full Loop Quantum Gravity theory.
This concludes the analysis of the pre-bounce phase; since the post-bounce phase is much more rich in terms of physical features, we'll describe it in a separate section.
We conclude this section with a consideration about the Hawking radiation that is supposed to be present from the formation of this quantum black hole to the bounce point: since the Hawking radiation (and in particular the Hawking temperature) depends strongly on the classical location of the horizon, and since the outer horizon is almost in the same position of the classical one (up to a quantum correction proportional to Δ), even if no computations are yet computed in this effective background we expect that the thermal spectrum is kept almost intact by quantum gravity corrections. In the next section, once we introduced the full dynamics of the star, we'll come back to the information loss paradox, and show explicitly how it is automatically avoided in this LQG-based model.

Post-bounce phase: the shock wave arising

In this section we'll analyze the post bounce phase of the model. Differently from what one expects looking at the cosmological solution, the post-bounce phase doesn't occur as an expansion symmetric to the contracting phase; in other words, we don't have a time symmetry $t \to -t$ with respect to the bounce point. This is a central difference with other LQG-based models, as the one that we'll analyze in the last section. Starting from the effective equations of the model, one finds both analytically and numerically that after the bounce the star becomes a shock wave of matter that expands ideally forever. Since the shock wave motion is an outward motion, and is physically impossible to change its travel direction, such shock wave is sometime called *white hole shock wave*, due to the fact that also a classical white hole in General Relativity describes the space-time generated by outgoing matter that cannot invert its motion.

Before looking in detail at the outgoing motion, let's give more physical details regarding the shock wave: firstly we have to point out that the post-bounce dynamics preserves the spherical symmetry of the model, that means that the shock wave has to be seen as a spherical expanding thin shell, that contains almost all the mass of the initial star. The internal space-time is an effective expanding FRW universe, in which the mass rapidly decays, making increase the mass of the shock wave. After a small amount of time from the bounce, almost all the mass of the star is accumulated in the shock wave, which can be seen as a boundary between two space-times: the interior is an expanding effective FRW that rapidly becomes Minkowski (when $\rho \to 0$), and the exterior is the effective Schwarzschild metric given by 2.17. During its outgoing dynamics the shock wave firstly reaches the inner horizon r_{in}, then the outer horizon r_{out}. A natural question that can arise at this point is how the shock can overcome the trapped region between r_{in} and r_{out}, in which we previously said that the light cones are flipped and matter can only in-fall; such outgoing motion can happen because the inner horizon (differently from the outer horizon) is not static, and when the shock wave reaches r_i, the inner horizon starts to be dragged by the shock, and follows its outgoing motion until r_{out} is reached; at this point, the two horizons merge and disappear. This means a disappearance of the black hole, and the shock wave keeps to expand indefinitely. Thus, as measured from the interior metric, the trajectory of the shock is time-like, as expected for a physical motion. However, if the trajectory is computed using the exterior metric, it turns out to be space-like. To understand better this point we have to describe why the shock wave arises, thus the reason why the outgoing motion of the star is not the expected FRW one.

Looking at the evolution of the gravitational field, one can numerically see that after the bounce the gravitational field develops a discontinuity on the boundary of the star. Such discontinuity is interpreted as a physical discontinuity of the solution, and such discontinuity produces a shock wave in the energy density; this is a common feature of discontinuous solutions for PDE's (or more precisely of PDE's in the weak form); for references, see [26]).

The discontinuity in the gravitational field between the interior and the exterior produces a substantial difference in the computation of lengths, and in particular in the evaluation of the kind of trajectory followed by the shock: as anticipated before, the outgoing shock follows a space-like trajectory, if computed with the exterior effective metric 2.17, and a time-like trajectory if computed with the interior metric. It is worth to notice here that no external observer can see the outgoing space-like motion of the shock wave; this because during the space-like (as measured by the exterior metric) motion across the trapped region, no signals starting from the shock can reach an external observer, so none from the exterior can make experience of this space-like motion of the shock wave.
For the sake of clarity let's present a diagram showing the dynamics of a star with mass $M = 5$ (in Planck units $\Delta = 1$) and the location of the horizons at different times, as computed numerically: (fig 2.5). Let's give some comment about the previous plot: in the first column we can notice that the star increases its energy density during the collapsing phase (first and second raw); then, soon after the bounce the internal energy density rapidly decreases to zero (third and fourth row). The peak in the last three frames is the outgoing shock wave. In the right column, the expansion term is showed; without going in technical details, the zeroes of such function describe the location of the two horizons: in the first frame, there are no zeroes, and the black hole is not formed yet; in the second, third and fourth row the horizons are formed, and we can notice that if by a side the outer horizon is static (and located at $r = 2M$ in Planck units), the inner horizon is dynamical. In the last frame the black hole disappears, and the two horizons merge. Let's notice finally that even if in the pre-bounce dynamics the boundary of the star should be located at a certain $L(t)$, the plot shows a tail in the energy density; this is simply due to numerical error, and doesn't affect the dynamics significantly.
Let's give a physical insight about the nature of the shock wave. The white hole shock wave can be seen in comparison with the supernovae, which are explosions produced by collapsing stars that become then black holes (in models in which matter more physically accurate than dust is kept in account). Well, even if the shock wave can be seen qualitatively as a violent explosion of the star, an important difference between the two processes is that the white hole shock wave emits *all* the matter that constituted the original star, while instead during supernovae explosions only a fraction of the matter of the star is emitted during the process. In a physically more realistic model than the dust star, and under particular physical conditions, these two processes are both expected to happen during the death of a star (firstly the supernova, followed by the black hole appearance,

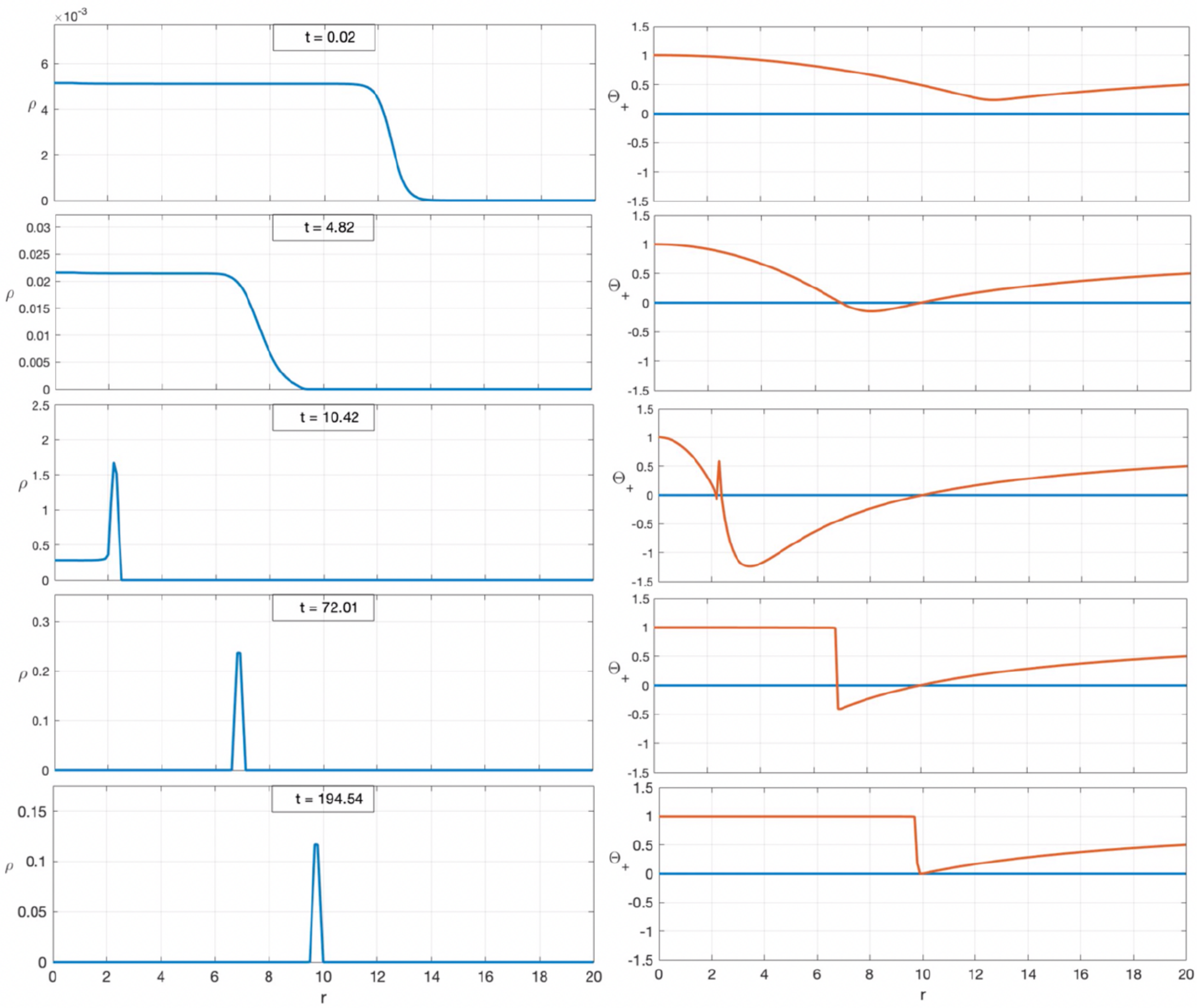


Figure 2.5: *In the left column is showed the dynamics of the energy density at different frames (the first two rows are pre-bounce frames, and the last three are post-bounce frames); on the right the evolution of the expansion term* Θ*, which zeroes give the location of the horizons. (Credits: [29])*

then the white hole shock wave), even if no model has been constructed yet to test this expectation. Thus the white hole shock wave can be seen as an explosion, conceptually similar to a supernova or a sonic boom in fluid dynamics. We also point out here that no remnant survives once the shock crosses the outer horizon, and the space-time that the traveling shock leaves behind is the flat Minkowski space.

Even if a lot of work has to be done to understand the implications of this model and possible more realistic extensions, we want to notice here that this is the first LQG-based effective collapse model derived from effective equations, and it has the merit to not be constructed from physical inputs based on mere intuition; instead, the solution is mathematically and numerically derived from the effective equations of motion. We also have to underline that this is not the only possible LQG-based model describing a spherically symmetric dust collapse: as well as loop quantum cosmological models, to derive the effective equations one has to make some mathematical choices, and it is possible that different choices can bring to different quantitative results; without going into details (that are extremely technical and outside the aim of this work), we conclude by saying that the expectation is that different choices are expected to bring to the same results, at least qualitatively.

It's worth to underline here that the Hawking evaporation is supposed to happen also in the post bounce phase, and it would eventually end once the shock wave reaches the outer horizon; thus, even if the production of thermal particles is supposed to happen during the whole dynamics, it is completely neglected in this model, i.e. the effective equations are not corrected keeping in account the dynamical consequences of the Hawking radiation. This is a simplification performed in the model, and the physical reason behind this is because the evaporation is supposed to be a subleading effect, not affecting qualitatively the full dynamics of the star. The reason behind this expectation will be clear in the next section.

An extremely interesting feature of this model is the quantitative prediction of the black hole life-time, defined as the time measured by a distant observer from the instant in which the star enters in its own outer horizon, to the instant in which the shock wave exits from r_{out}; the life-time T can be computed both analytically and numerically, and with an high agreement between the two methods one finds:

$$T \sim \frac{M^2}{m_{Planck}} + O(M) \tag{2.21}$$

assuming $G = 1$. The first term is actually the time as measured from the production of the shock to its crossing of the outer horizon, while the second term $O(M)$ is the pre-bounce time, thus the time between the appearance of the black hole up to the bounce point. This result is of fundamental importance for the information loss paradox, in particular in the modern Page formulation. We'll describe this point in a separate section, since is crucial for the purposes of this work.

2.3 Quantum black-to-white hole transition

The second effective model that we present is also based on Loop Quantum Gravity, but is significantly different from the shock-wave model described before. The black-to-white hole model has been explored for many years [30],[31],[32], and reached a quite sophisticated formulation recently, see [34]

. Since the idea that brought to such line of research comes from classical considerations, in order to understand it properly we need to introduce some concept regarding the classical theory of General Relativity. The first important physical concept required is the *white hole* solution in classical General Relativity, and we'll introduce it briefly in the next section.

2.3.1 Classical white holes

Let's come back to the classical Schwarzschild metric 1.7. We recall that such metric contains a coordinate singularity at $r = 2M$. As we explicitly showed, one can introduce a change of coordinates to remove such singularity, as for example the Painlevè-Gullstrand coordinates. Even if such coordinate system could be used to derive and study the white holes nature, we will introduce a different coordinate system, that shares many features with the P-G one. Starting from the Schwarzschild line element, we can introduce the following coordinate transformation:

$$v = t + r^*, \quad with \quad r^* = r + 2Mln\left|\frac{r}{2M} - 1\right| \tag{2.22}$$

where t and r are the Schwarzschild time and radius. Under the previous transformation, the Schwarzschild solution takes the form

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dv^2 + 2dvdr + r^2 d\Omega^2 \tag{2.23}$$

This metric is called *Eddington-Finkelstein advanced metric*, and is immediate to notice that it is not diverging at $r = 2M$: it is a regular metric that is affected by the physical singularity at $r = 0$. The v coordinate is called advanced Eddington-Finkelstein time, and plays the role of a time coordinate for the vacuum space-time.

It can be shown that the region $0 < r < 2M$ is a trapped region, as well as in Painlevè-Gullstrand coordinates and in Schwarzschild ones; this means that such metric describes a classical black hole, as well as the metric in the other coordinate systems we introduced before. Let's now introduce a different coordinate transformation, mathematically similar to the former but essentially different: starting from 1.7, we perform the following transformation:

$$u = r^* - t, \quad r^* = r + 2Mln\left|\frac{r}{2M} - 1\right| \tag{2.24}$$

the u coordinate is called *retarded time*; the metric becomes

$$ds^2 = -\left(1 - \frac{2M}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 \tag{2.25}$$

and is called *retarded Eddington-Finkelstein metric.* As well as the advanced form, it is regular across $r = 2M$, but the fundamental difference is that the region $0 < r < 2M$ is not a trapped region, but *anti-trapped.* Even if this feature is not manifest in the line element 2.25, one can check it solving the geodesic equations for such kind of background. This implies that if we locate a particle in the anti-trapped region, instead of falling toward the physical singularity $r = 0$, it is forced to move toward increasing r. For this reason such metric is considered describing a **white hole** solution. To understand properly the peculiarity of this metric we can look at its light cone structure in Fig. 2.6. Looking

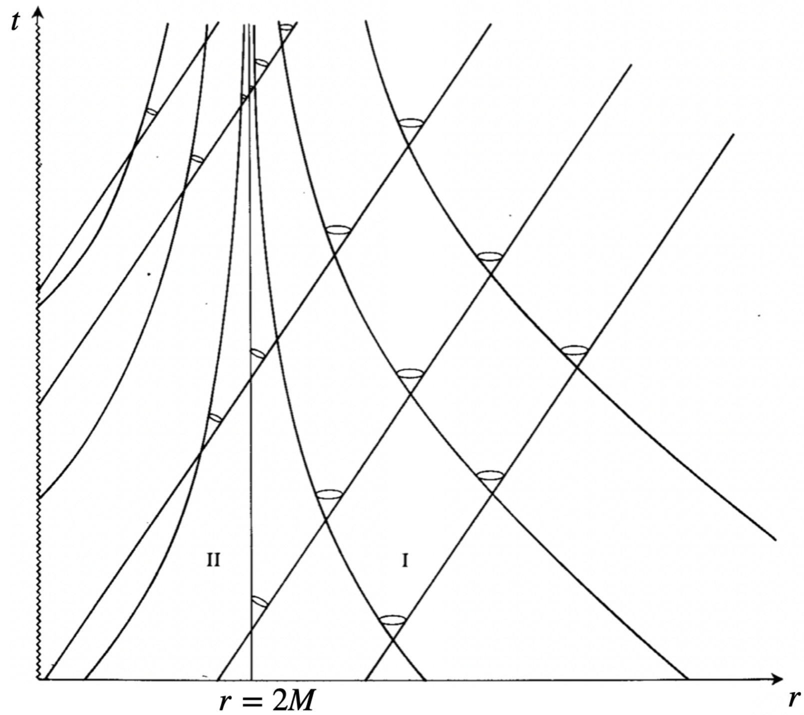


Figure 2.6: *Light cone structure of a white hole in retarded Eddington-Finkelstein coordinates*

at the light cone structure of the region $r < 2M$ is clear that starting from the interior the only possible motion is outgoing; in particular as light cannot move outward in a trapped region, in a similar way cannot move inward in an anti-trapped region. Such kind of metric can be better understood if one notices that the advanced Eddington-Finkelstein metric 2.23 is not invariant under time reverse $v \to -v$, differently from the Schwarzschild line element 1.7. If we perform the transformation $v \to -v$, and we call $v = u$, we obtain exactly the retarded form 2.25; therefore the white hole metric can be seen as a time reverse of the black hole metric. Physically, if a black hole captures everything that falls behind the Schwarzschild horizon, a white hole rejects everything beyond the Schwarzschild horizon. An important classical difference between the two space-times is that if by a side a black hole is eternal, a white hole instead disappears rapidly, once all the matter that contains is expelled across the horizon. Moreover, from the experimental ground, if black holes have been "observed" (although only recently) by the Event Horizon telescope, white holes seem to not have left tracks (if they existed), at least for our current experimental knowledge.

During last decades many researchers analyzed in detail the main properties of white holes, and possible consequences of their existence. We won't go in detail into describing white holes, but a classical model based on such solution. Since this will be crucial to understand the quantum model, we'll analyze it in a separate section.

2.3.2 Classical black-to-white hole transition

During the last decades theoretical physicists started to argue that white holes could be the very last step of a collapsing star, after it became a black hole. One of the main motivations under such considerations is the possibility to recover the information apparently lost (avoiding an eventual complete evaporation), through its emerging from the white hole.

An example of such hypothetical process can be described by a Penrose diagram in which we glue the collapsing dynamics of the star (with the consequent black hole formation) with its time reverse (thus an expanding star rejected by a white hole), at the singularity $r = 0$. The associated Penrose diagram is in fig.2.7

Such classical model, even if extremely intriguing, suffers of some important mathematical and physical issues; we'll describe here the most important ones. Firstly, as one can see explicitly from the diagram 2.7, on the gluing there is the physical singularity that affects both the classical black hole metrics 2.23 and 2.25; this means physically that collapsing matter would reach the singularity, and then from the singularity should emerge

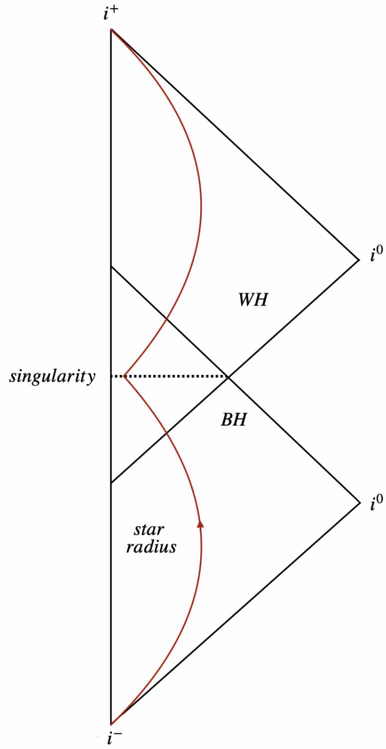


Figure 2.7: *Penrose diagram of a classical black-to-white hole transition*

and expand toward the anti-trapped region of the white hole. The existence of the classical singularity is an important conceptual issue, since as we pointed out many times on physical singularities the classical Einstein equations of motion loose their validity. Moreover, even if we admit the existence of the singularity, a singularity in the past of the white hole region means a complete loss of predictability of the future of such hypersurface. In fact, if a physical trajectory starts on the singularity moving toward the white hole region, its form cannot be determined by the Einstein geodesic equations (the equations that govern the motion of particles on a given space-time), since the solution would be ill-defined at the initial time. Finally, if the star ends in a singularity, all the information contained in it would be destroyed once the singularity is reached: the infinite curvature would destroy completely whatever kind of information brought by the matter forming the star, and the information paradox would not be avoided.

The second important issue about this glued classical solution regards the exterior of the trapped region. Consider in fact and observer that remains at a certain distance r_O from the horizon; his trajectory can be depicted in the previous Penrose diagram:

we notice that in his far future is not able to see how the black hole evolved in a white hole. From his perspective, the black hole would be eternal, and such solution would be completely equivalent to the eternal classical black hole; this would be true for *each*

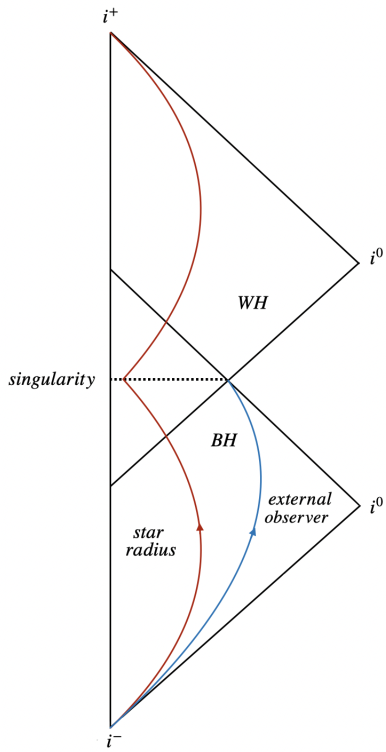


Figure 2.8: *Penrose diagram of a classical black-to-white hole transition with an external observer*

observer that eventually remains outside the horizon. Therefore, if 2.7 is the physical Penrose diagram of the evolution of a star, it would be impossible to be distinguished by a more simple black hole Penrose diagram. Moreover, even if the information contained in the star would be able to not be destroyed by the singularity for some exotic effect and reaches the white hole region, it would be lost forever for observers living in the universe in which the black hole has been created. This model is affected by the same issues of the ordinary classical star collapse: the physical singularity and the information loss paradox. Both these fundamental problems are solved by the second LQG-based effective collapse model that we present in the following section.

2.3.3 Effective black-to-white hole model

Before introducing the effective model it's worth to emphasize here that the following is not the only black-to-white hole effective LQG-based model present in literature. However we decided to present it since contains a clear resolution of the information paradox, that is at the core of this work. For references about different models see ([30],[31],[32],[33]). This effective black-to-white hole model [34] is born following the research line introduced in the previous paragraph, therefore its aim is to build a black-to-white hole transition

that describes hopefully the end of the life of a black hole generated by a collapsing star. The important improvement with respect to the classical attempts is that it both removes the singularity issue of classical GR, and avoids the information loss paradox that as explained before keeps affecting classical black-to-white hole models. Such improvements are possible using features that come from Loop Quantum Gravity, and some mathematical technique that we'll describe briefly and allow to avoid the presence of two different asymptotic flat regions, or in more physical terms allow the black-to-white hole process to be seen by an observer external to the horizons.
As well as the shock-wave model, the black-to-white hole model that we present here describes the Oppenheimer-Snyder collapse of a pressure-less (dust) star in an effective LQG framework. However, an important difference from the first one is that the space-time is not derived rigorously from effective equations. Therefore it has to be seen as an attempt to describe the effective dynamics of the star and space-time using lessons learned from other sectors of Loop Quantum Gravity, as the cosmological sector and the full theory; finally, some important considerations are based on physical intuitions regarding the quantum nature of space-time and its classical region. Let's proceed now to introduce the model. Since we don't have equations of motion (that would be Einstein equations modified by some quantum corrections) supporting this model, the working assumption is that the space-time characterized by the collapsing star can be safely divided in the region of space-time internal to the star, and the external one; the interior and exterior space-times can be treated separately and constructed through physical considerations and lessons from other sectors of the theory. This assumption is needed since we don't have equations of motion from which deriving a dynamical solution, as in the shock-wave model, and the easiest way to construct a metric without EOM is to study the interior of the star and the exterior separately. We have to point out here that such assumption is strongly consistent with the particular dynamics that the model describes, that is the Oppenheimer-Snyder collapse; in fact, in the O-S collapse the boundary of the star is not spread but well defined at each time, and this allows to describe the interior and the exterior of the star separately. Also in the shock-wave model one can work with this assumption, and instead of studying the full space-time dynamics we can focus on the interior and the exterior separately: in the particular O-S model, the result would be the same (the same can be done in the classical setup). The main difference between the two models in this sense is that the shock-wave model can be studied looking at different initial collapsing stars with a boundary arbitrarily spread, while the black-to-white hole

model is designed precisely on the Oppenheimer-Snyder collapse, that assumes a well defined boundary at each time. For this reason we'll describe the interior and the exterior separately.

The interior

In the Oppenheimer-Snyder model the interior is assumed to be made of homogeneous, isotropic and pressure-less matter. This means that in the effective background it evolves as the cosmological effective FRW solution 2.10, following the effective Friedmann equations 2.14 and 2.15. This implies a bounce point in which the star reaches a minimal radius, and an expansion that can be seen as the time-reverse of the contracting phase. The energy density reached by the star at the bounce is the usual ρ_c characterizing both the cosmological solution and the shock-wave model. A crucial difference with the shock-wave model is the assumption of symmetry under time-reverse around the bounce point, that gives a post-bounce dynamics completely different from the first model presented. Such symmetric behaviour realizes the idea behind the classical black-to-white hole transition, with important differences that we'll underline during the treatment.
As we will see later, the dynamics will not be really time-reversal, since the Hawking radiation is assumed to take a predominant role during the black-to-white hole transition: another important assumption of the model is in fact that well before the bounce the star looses almost all its original mass through the radiative process, and only an extremely tiny fraction actually undergoes the quantum bounce. Therefore, even if the bouncing star expands in a way similar to the time-reverse of the contracting phase, the expanding mass is much smaller than the initial collapsing one. We'll come back to this point later, since is central for the complete picture and the way in which the model avoids the information loss paradox. Let's focus now on the external region.

The exterior

Let's focus now on the portion of space-time external to the star; the vacuum metric proposed by the authors for the exterior is extremely similar to the one introduced in the shock-wave model, and reads:

$$ds^2 = -\left(1 - \frac{R_S}{r} + \frac{\gamma^2 \Delta R_S^2}{r^4}\right)dt^2 + \frac{1}{\left(1 - \frac{R_S}{r} + \frac{\gamma^2 \Delta R_S^2}{r^4}\right)}dr^2 + r^2 d\Omega^2 \tag{2.26}$$

that is fundamentally the classical Schwarzschild metric in Schwarzschild coordinates with quantum corrections. It can be easily shown that this metric and 2.17 can be linked by a coordinate transformation that mimics the classical transformation 1.8, and in this sense can be thought to describe the same physical space-time. This however is not the case, in the sense that such metric is not solution of the effective equations governing the shock-wave model. Since this is an extremely tricky point, still under examination of the authors, we make some considerations about. In classical General Relativity, once we have a metric tensor, we can always perform a coordinate transformation of kind

$$x^{\mu} \longrightarrow x'^{\mu} = f^{\mu}(x^{\nu}) \tag{2.27}$$

this transformation produces a metric tensor that describes physically and mathematically the same kind of space-time, but in other coordinates. The transformation law of the metric reads:

$$g'_{\mu\nu} = \frac{\partial x^{\rho}}{\partial x'^{\mu}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} g_{\rho\sigma} \tag{2.28}$$

The property of covariance of the Einstein theory (invariance under coordinate transformations) guarantees that the metrics g and g' describe the same physical space-time. Well, the effective model governing the shock-wave solution is also covariant, but the coordinate transformations allowed by the model are deformed by quantum corrections; this means that two apparently different solutions can describe the same physical space-time, but the transformation of the metric produced by the coordinate transformation 2.27 is not of the form 2.28. Even if the exact generic transformation is not yet derived, its expected generic form is the following

$$g'_{\mu\nu} = \frac{\partial x^{\rho}}{\partial x'^{\mu}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} g_{\rho\sigma} + (o(\Delta))_{\mu\nu} \tag{2.29}$$

where the adding terms contain functions of the metric proportional to Δ and eventually higher powers of Δ. This means that even if the effective theory is still covariant, classical coordinate transformations with the generic form 2.28, like the one that brings from the effective Schwarzschild metric in Schwarzschild-like coordinates 2.26 to the P-G coordinates 2.17 are not allowed. This is the mathematical reason why the metric 2.17 is solution of the effective equations, and 2.26 not. Despite this difference between the two metrics, they share important features that we'll underline here.

The diagonal effective metric 2.26 is significantly different from its classical counterpart. As well as 2.17 it contains two horizons: an inner horizon located at 2.18 and an outer horizon located at 2.19. Therefore it contains a trapped region for $r_{in} < r < r_{out}$, and a non-trapped region that extends toward $r = 0$, thus for $0 < r < r_{in}$. Differently from the P-G effective metric however it has also an anti-trapped region, as well as the Reissner-Nordstrom metric for a charged black hole; we come back to this point when we look at its conformal diagram. This means that this metric doesn't have a minimum radius r_{min} as the effective P-G metric, but is defined in the whole space $(0 < r < \infty)$ except for $r = 0$, in which a physical singularity at r=0. The presence of a physical singularity in this metric however is not dramatic (differently from its classical counterpart) for a simple reason: if we consider the full picture, in which the vacuum is generated by a physical collapsing star, the interior region is described by the FRW metric, that is not singular at $r = 0$; once the star collapses, differently from the classical case doesn't shrink until a point, but bounces. This means that the physical singularity of the pure vacuum metric 2.26 never arises during a dynamics in which matter is kept in account. Even though one expects that a quantum gravity theory avoids whatever kind of physical singularities, the singularity of such metric is never produced during the dynamics of a collapsing star that has such vacuum as exterior. In other words, one can argue that the vacuum metric 2.26 is only a mathematical metric, and has no physical sense if one doesn't consider an extended distribution of matter that produces it. Once matter is kept in account, the metric makes physical sense and is coherent with the fundamental features of Loop Quantum Gravity.

We notice that the line element 2.26 is not well defined everywhere in $r \in (0, +\infty)$. In particular, on the two horizons r_{in} and r_{out} the metric is singular, as well as the classical Schwarzschild metric is singular in $r = R_S$; such singularities are coordinate singularities, and one can compute the curvature scalars to check that they don't diverge on the horizons. To remove such singularities, the authors perform a series of classical coordinate transformations (of kind 2.28), similarly to what one does in the classical theory to regularize the Schwarzschild metric. The result is a metric similar to the classical Kruskal metric, but we won't introduce it here explicitly since is complicate to be interpreted, and it would give no insights about our discussion. In the next paragraph we'll show however its associated Penrose diagram, crucial to understand the physics behind the model.

The complete picture

We can glue the interior FRW metric 2.10 and the exterior metric, derived as a maximal extension of the effective Schwarzschild line element 2.26. The resulting Penrose diagram is the following:

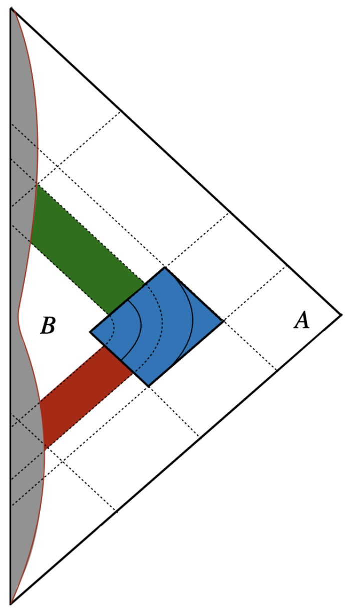


Figure 2.9: *Penrose diagram of an effective black-to-white hole transition*

Since this diagram has an extremely rich structure, we'll divide it in regions and study them separately.
Firstly we notice that there is a trapped region (depicted in red) and an anti-trapped region (depicted in green). Secondly, we notice that the external region (in white) that we called A contains only an asymptotic flat region. This is an extremely important feature of the diagram and allows to solve the issue that affects the classical black-to-white hole model: an observer that remains at a certain radius $R > r_{out}$ can see both the collapsing star that enters in its horizon, and in the future the same star that exits from the white hole horizon. The information can in principle be restored in the model, and the black hole doesn't look eternal for an external observer. We'll see how this happen in detail later. Let's look now at the internal region B in white. Such region is made of points of space at $L(t) < r < r_{in}$, where $L(t)$ is the dynamical radius of the star. This means that when the radius of the collapsing star overcomes r_{in} such region forms, and lasts until the radius of the bouncing star reaches r_{in} in its outgoing motion. In such region particles can move not necessarily inward, and this is possible because as we pointed out many times such region is non-trapped. The existence of such region allows the bounce of the star (depicted in dark grey) and its outgoing motion. This is a pure quantum region that has no classical counterpart, and is fundamental to avoid the singularity problem of the classical model. Firstly its existence allows the physical singularity of the vacuum

region to be time-like (in particular a vertical line at $r = 0$) instead of space-like as in the classical model (the horizontal line between the trapped and anti-trapped region); then, as we pointed out before, the presence of matter allows the complete avoidance of the physical singularity at $r = 0$. In this way, the evolving matter never reaches a singularity, and the space-time can be studied without loss of predictability. The last crucial region of the diagram is the one in blue, because it contains the transition from the trapped region (in red) to the anti trapped region (in green).

The trapped-anti trapped transition is assumed to be driven by quantum gravity effects, and the calculations about this process can be found in [35], [36], [37]. The calculations performed for such transition are based on a path integral formulation of the full theory, namely covariant Loop Quantum Gravity. Let's give here only the basic idea behind the calculations.

In the path integral formulation of quantum field theories, physical processes can be studied by computing the probabilities that given a certain quantum state α, the system evolves in another quantum state β. Each final state carries a certain quantum probability to be reached by the system, and the square root of such probability is called transition amplitude. The same kind of approach can be performed in Loop Quantum Gravity; thus, given an initial spin network state and a final spin network state, the transition amplitude is the square root of the probability that given the spin network state α, we obtain a final state β. The probability is computed keeping in account all the possible *histories*, thus all the possible processes that have α as initial state and β as final state. A generic spin network history is also called *spin foam*, and can be depicted pictorially as in 2.10. Starting from an initial spin-network state (at the bottom of the diagram),

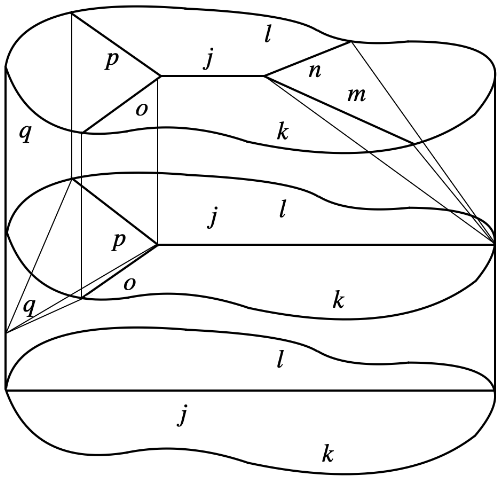


Figure 2.10: *a generic Spin foam in covariant LQG*

the spin foam describes one of its possible histories (attaching a quantum probability to it), as well as the Feynman diagram allows to compute quantum probabilities between initial and final states in QED or QCD. For references about the spin-foam formalism, and covariant loop quantum gravity see [38],[39],[40], [41]. The latin letters in the diagram are the quantum numbers associated with the links of the graph describing a spin network.
Well, using this approach one can compute the transition amplitude between an initial state given by a spin network describing a trapped region, and a final state given by a spin network describing an anti-trapped region. The calculation is quite involved but many progresses are made in this direction, and the approximate result is the following:

$$P \sim e^{-\frac{M^2}{m_P^2}} \tag{2.30}$$

where m_P is the usual Planck mass and M is the mass of the star. This means that the probability of the process is non zero for each possible mass of the star, but is significantly different from zero only for masses $M \sim m_P \sim 2.17 \cdot 10^{-8} kg$. This could seem contradictory at first sight, since here we are studying stars with masses more or at most equal to one solar mass. However, this would be completely reasonable if we keep in account the Hawking radiation as a fundamental process of the model. Such behaviour of the quantum probability can be understood if one realizes that for macroscopic black holes the trapped region is far away from the bouncing radius (that also in this model is $r_{min} = (\gamma^2 \Delta R_S)^{\frac{1}{3}}$), thus quantum gravity effects are not supposed to be relevant there for large masses. This is not the same for microscopic black holes, in which the inner and outer horizons *live* in the quantum region, and the region between them is expected to be significantly affected by quantum gravity effects.
Therefore, in order to understand why quantum gravity effects reach such region, which for large masses is supposed to be an almost classical region without dominant quantum gravity effects, we have to consider the Hawking radiation, that differently from the shock-wave model (in which the radiative process is found to be subdominant) here is supposed instead to play a crucial role. When we initially introduced the model we described it as symmetric process under time-reverse around the bounce point. This is actually right only qualitatively; in fact, as we anticipated briefly before, is true by a side that the expanding bounced star moves across the anti-trapped region in a way that is the time-reverse of the collapsing motion in the trapped region, but when the star bounces it already lost almost all of its original mass through Hawking radiation. In particular at the bounce the star is supposed to have reached the planckian mass, in such a way that

the trapped region outside the star can be affected by a quantum tunneling, becoming an anti-trapped region. The tiny mass survived by evaporation is also called *Planck star* in literature.

For the sake of clarity, we point out that such tunneling is expected to happen on the horizontal line (that is a three-dimensional spatial surface) that links the two lateral wedges of the blue diamond in fig. 2.9, and is a quantum tunneling that allows the trapped region to become an anti-trapped region. To conclude the analysis of the model we have to look at the whole time of the process, as seen by an external observer. The life-time of the black hole (the time between the production of the black hole outer horizon and the emergence of the remnant from the white hole outer horizon) as seen by a stationary observer at $R >> r_{out}$ can be computed analytically. The trajectory of the observer is depicted in fig.2.11 We can estimate the life-time as the proper time

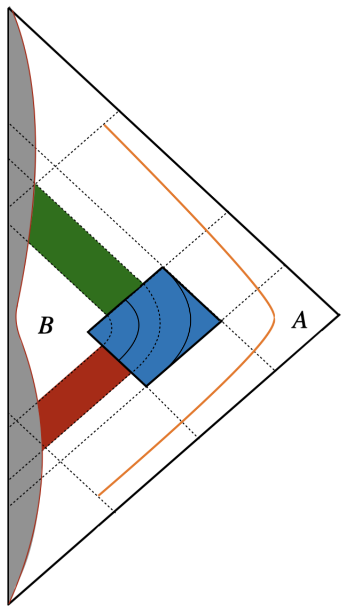


Figure 2.11: *World line of a stationary observer at* $R >> r_{out}$

of the observer from the initial point of the orange line, to the final point. This would be the time between the instant in which the observer sends an in falling photon that reaches the star at the moment of the outer horizon formation, to the instant in which a photon produced by the emerging remnant reaches the same observer at the end of the full process. The result is the following

$$T \sim 2R + 4Mln(R - 2M) - 4Mln(\delta) \tag{2.31}$$

where δ is a free parameter of the model. The first two terms in the expressions are classical terms, and depend on the position R of the observer, while the last term is a global feature of the model, independent on the particular position of the external observer. We call it

$$\mathcal{T} \equiv -4Mln(\delta) \tag{2.32}$$

Notice that for arbitrarily small δ, $\mathcal{T}$ is arbitrarily large. As a technical insight, we underline that δ is a parameter coming from the construction of the maximal extension avoiding the existence of two different asymptotic regions (for the black hole and white hole space-time). For more details about the construction see [37].

It's worth to notice here also that the first two classical terms don't describe actually the life-time of the process, but is twice time required by the infalling photon to reach $r = 0$: the first term is the time travel in a Minkowski space-time, and the second is the relativistic correction due to the curved space-time. This means that only the last term describes the intrinsic duration of the process. We remark that such quantity is a free parameter of the model, and the authors suggest that it has to be determined by the quantum theory, even if no direct calculations are proposed yet. δ (and thus $\mathcal{T}$) is expected to depend on the mass M of the initial star and $\hbar$, since has a quantum origin. Moreover, in the limit $\hbar \to 0$ (classical limit) $\mathcal{T}$ is supposed to diverge, obtaining the classical limit in which black holes are eternal.

We conclude by observing that $\mathcal{T}$ has to be of order (or eventually larger than) M^3, since the model predicts that almost all the mass of the original star has to evaporate during the process, and as we pointed out in the first chapter the full evaporation requires a time proportional to M^3. This gives a prediction for the black hole life-time significantly different from the first model proposed, and at this stage only experimental evidences can eventually determine which model is more accurate to describe the full process. This concludes the qualitative analysis of the model. Let's see now how the information loss paradox is tackled by the model. We conclude this section by anticipating that this model avoids the paradox in a way completely different from the shock-wave model, and we'll discuss the comparison between the two in the very last section of this work.

Chapter 3

Avoidance of the Information Loss Paradox in details

In this chapter we analyze more in details the differences between the two quantum models presented before with a particular focus on the way in which they avoid the information loss paradox.

3.1 Avoidance in shock-wave model

The M^2 dependence of the effective black hole lifetime is an extremely important result for black hole physics.
Firstly we would like to clarify why the Hawking process is considered subleading: the lifetime predicted keeping in account only the black hole evaporation is proportional to M^3, and even if the Hawking computation is assumed to be valid (up to small corrections proportional to Δ) in this effective model, it can be safely neglected for the qualitative description of the full dynamics. This because for astrophysical black holes, and in particular for supermassive black holes $M^3 >> M^2$. This means that during the whole process only a small amount of mass is lost through Hawking evaporation, and such decreasing in the mass doesn't affect at least qualitatively the dynamics.
The star during the collapse, bounce and shock wave production looses a tiny amount of mass, and this means that the outgoing shock wave will have a mass given by $M_{Shock} = M - \delta M$, where δM is the mass lost through the radiation process.
Apart from this correction, the qualitative picture that we gave in the previous chapter is assumed to remain valid. The information loss paradox is thus completely avoided by this model, without the need of introducing ad hoc assumptions on the physics of the

process, as many escape routes that we presented at the end of chapter one require.
This can be easily seen looking at the Page formulation of the information paradox. As we explained before, in this formulation the paradox arises only when the black hole radiates half of its mass; however, using the Hawking result we have that this happen only after a time $T_{page} \sim \frac{M^3}{8m_{Planck}^2}$, that is much larger (for massive black holes) than M^2, the time required for the whole process. In fact, if

$$T_{effective} \sim \frac{M^2}{m_{Planck}} \tag{3.1}$$

$$T_{Page} \sim \frac{M^3}{8m_{Planck}^2} \tag{3.2}$$

then

$$T_{Page} >> T_{eff.} \Longleftrightarrow M >> 8m_{Planck} \tag{3.3}$$

where the Planck mass is $m_{Planck} \sim 10^{-8}kg$. This means that for macroscopic black holes the life-time of the effective black hole is much smaller than the Page time, and the information loss paradox in the Page formulation is *completely avoided.* We want to stress here that this result is a mathematical consequence of the model, and no ad hoc assumptions have been made to obtain it.
This is the first effective model that we propose to support the thesis on which this work is based on: the information paradox cannot be solved if we work in the classical Einstein theory, unless ad hoc hypothesis is introduced or fundamental principles of physics are abandoned, as for example the limit of speed of light (in the instantaneous evaporation of the second escape route we presented) or the unitarity principle of quantum mechanics.
We argue that quantum field theory in curved spaces is a useful theory that describes correctly the Hawking evaporation, but it is unable to describe the full picture (in particular the real full dynamics of the collapsing star) and without a quantum theory of gravity the paradox that arise in this approach cannot be reasonably solved.
Just as classical mechanics cannot keep in account quantum effects like the quantum tunneling of a particle through a barrier, the Einstein theory of gravity is not able to describe correctly the full dynamics of a collapsing star, as well as the dynamics of matter that reaches the Planck scale in energy density. This is the real source of the information loss paradox.
From this perspective, the information loss paradox is an apparent paradox, in the sense that it doesn't regard the real physics of black holes, but only models and theories that don't keep in account all the fundamental features of gravity, like its quantum behaviour

at the Planck scale.

We will come back to this point in the next section, in which we'll present a different effective LQG-based model that also avoids the information loss paradox without renouncing to some fundamental principle of physics.

We conclude this section with a last consideration about the shock wave.

If we assume that the shock wave is not physical, motivated by heuristic arguments like the fact that it is a feature that is not present in other models, we can assume at the speculative level that the bounce is actually time-symmetric, and the star in the post-bounce phase expands in a time-reverse way with respect to the contracting phase.

Computing the life-time of the process is extremely easy, and requires simply doubling the time required for the contracting phase. What one obtains is therefore

$$T \propto M \tag{3.4}$$

and even though this result would avoid the information paradox in the same way we described before, it would be in contrast with observations.

In fact, if the life-time is given by 3.4, stars of only few solar masses that become black holes in the very early stage of the Universe should already be seen emerging as white holes, but no tracks of them have been found in astrophysical observations.

Finally, we want to make a consideration about a feature of this model.

The Hawking radiation appears to be a subleading effect of the dynamical process star-black hole-shock wave; this is coherent with thermodynamic evaporation of macroscopic systems: in classical physics there are no large-mass systems that radiate all (or almost all) their energy through thermal radiation.

Instead, thermal radiation is usually an almost irrelevant effect happening during the dynamics of macroscopic objects, which are characterized by a temperature kept almost constant.

A simple example of this is the early evolution of the stars: many physical processes happen during the life of a star before collapsing in a black hole (if the star becomes a black hole), and even if such stars radiate, thermal emission doesn't affect significantly their evolution.

In the next section we'll present a second effective LQG-model proposed in literature, that is substantially different from the former, and in a completely different way avoids the information loss paradox.

3.2 Avoidance in black-to white hole transition

The way in which this model avoids the information loss paradox has been anticipated in the first chapter: is the escape route associated with remnants. In this paragraph we will describe it in more detail and we will give different arguments that motivate such escape. Firstly we have to point out that the remnant escape is ruled out by the Page formulation of the paradox, since we recall that the paradox for Page arises when the star evaporated for half of its original mass, after a time called Page time.

Once such point is reached and surpassed, there are not enough particles in the interior that can be entangled with the particles forming the Hawking radiation, and the whole state of the system cannot be described anymore as a pure state, but becomes mixed.

This means on one side loss of information during the dynamics, on the other side violation of unitarity of the time evolution of the system, since if we evolve backward in time a final mixed state we cannot reach the initial pure state through an operator that is the hermitian conjugate of the time evolution operator.

It seems at first sight that this quantum gravity model, as well as many others that predict remnants, are ruled out by the Page argument.

There is however a large part of the quantum gravity community (both from String theory, Loop Quantum Gravity, and other approaches) which believe that the Page formulation of the paradox is wrong.

Before going into detail, for sake of completeness we also mention that there is also a large group convinced that both the Page formulation and the remnant solution are correct, and are trying to avoid the paradox introducing hypothesis more or less motivated by quantum gravity considerations.

We won't focus on this line of research, instead we'll describe the considerations behind the first line.

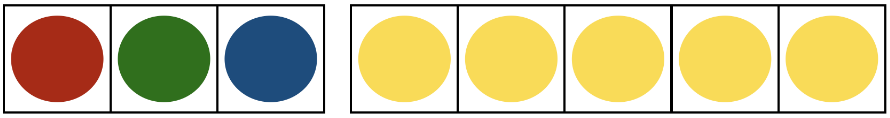
To describe the escape we have to recall the main steps of the information loss paradox in the Page formulation. In the Page analysis we introduced the von Neumann entropy and we stated firstly that such entropy has to be the same for the internal system (the star not evaporated yet) and the external system (the whole set of particles that form the Hawking radiation).

Secondly, we assumed that the von Neumann entropy for the interior (and thus for the exterior) cannot be larger than the thermodynamical entropy given by the Bekenstein-Hawking expression 1.19.

The calculation that brings to this inequality is based on an assumption that we didn't

stress during the presentation of the Page formulation, but here is crucial.
The assumption is that the amount of information in the interior has to be equal (or eventually smaller) to the information lying on the horizon. This assumption realizes the well known holographic principle, for which the whole information inside a certain volume (that in our case is the black hole volume) can be completely recovered by a theory constructed on its boundaries.
It is the reason why the von Neumann entropy (which describes the possible configurations of the interior) has to be equal to or less then the thermodynamical entropy (which is given exclusively by the area of the horizon of the black hole).
To have a further insight, we mention that the Bekenstein-Hawking entropy formula has been recovered both in Loop Quantum Gravity and String theory from the counting of quantum gravity states on the horizon.
In the loop theory in particular the number of possible spin network states compatible with the topology of the horizon have been calculated, and the calculation brought to the Bekenstein-Hawking entropy if one fixes the γ factor (the Immirzi parameter, a free parameter of the theory) to ~ 1 (see 42, 43).
To summarize, following the Page argument the entropy of matter inside the horizon has to be less or at most equal (in the case that the interior is in a maximally entangled state) to the entropy of quantum gravity states of the horizon. However, if in the interior there are more states available than $e^{S_{BH}}$, the Page argument fails.
In fact, if the von Neumann entropy is allowed to be larger than the thermodynamical entropy, the entanglement between the interior and the exterior can grow even if the external evaporated mass becomes larger than the internal mass (so after the Page time). This would mean that the whole state can remain pure even after the Page time and if the interior matter is allowed to escape (through a white hole in the quantum gravity model we analyzed), the whole information can be recovered at very late times.
Two main questions arise naturally at this point: how can the information increase in the interior even if the amount of energy decreases due to evaporation?
The intuitive answer to this question is what brought Page to construct the second half of the Page curve, for which the entanglement entropy starts to decrease after the Page time: in the Page perspective, if the von Neumann entropy is maximized by $ln(N)$, where N is the dimension of the internal Hilbert space, since the area of the horizon shrinks, and thus its mass, such limiting quantity has to decrease in time (since the number of particles decrease), and once the entanglement entropy reaches such value from below,

the same decreasing has to happen to S_{vN}.
The key idea here is that the interior contains much more states than the ones assumed by Page. The entropy in fact is not linked only with the number of particles in a system, but also with the possible states that each particle can have according to a macroscopic configuration.
This is true also in classical statistics:



The system on the left has a larger entropy than the system on the right even if contains less objects, since we can create more configurations with the same constitutive ingredients. The system on the right has actually zero entropy, since it can achieve only one distinguishable configuration.
This means that a small number of particles which have access to an increasing range of quantum states have an increasing entropy.
Therefore, even if the number of particles in the interior decreases and becomes less than the external one, the entanglement entropy of the interior can in principle increase.
This can be mathematically proved, and the proof is based on the fact that the interior of a black hole with a small shrinking horizon can have an enormous increasing volume, that reflects into an increasing number of possible internal states for the system.
This could seem counter-intuitive at first sight, but recall that the geometry of the black hole is not euclidean: an euclidean geometry requires a relation between a spherical area and its volume of kind $V \propto A^{3/2}$ (if one increases, the other has to increase). Non-euclidean geometries allow an inverse-like relation between the area and the internal volume, and this is what happens.
Notice that we are not speaking here about the volume of the star inside the hole, that goes like r^3, but the volume of the whole black hole; the quantum state of the particles inside the star depends on the size of the black hole, not only on the size of the star, that clearly decreases during the evaporation.Even if we don't show the calculations behind this idea, we'll given an argument that enforces this position; more details can be found in [44].
Let us thus consider the collapsing star, when the star is still in the internal classical region ($L(t) >> r_{in}$) for sake of simplicity; the consideration can be easily extended at each time of the collapse. Let us take two different time slicing, call them Σ_1 and Σ_2 (fig.

3.1):

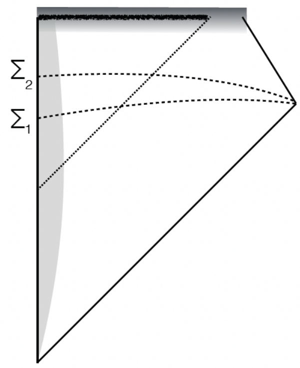


Figure 3.1: *Classical Penrose diagram of a collapsing star, with two space-like* t = *constant hyper-surfaces. (Credits: 44)*

Here we consider the classical Penrose diagram, not the quantum gravity corrected one, but the consideration can be immediately extended to the quantum model.
Since we are outside the deep quantum region of the space-time, considerations about ordinary quantum field theory are assumed to hold (as stated before, the reasoning keeps holding also in the deep quantum region); let us consider the portions of these two hypersurfaces internal to the horizon, and call them Σ_1^{in}, Σ_2^{in}.
Let us consider the Hilbert spaces living on such internal hypersurfaces. We notice now that the quantum state on Σ_1^{in} is completely determined once the state on Σ_2^{in} is known, because Σ_1^{in} is entirely contained in the past of Σ_2^{in}. Notice also that the opposite is not true, since particles can reach Σ_2^{in} even without passing through Σ_1^{in}, if they pass across the horizon after $t(\Sigma_1)$. This implies that distinguishable states on Σ_1^{in} can be distinguished by local observables on Σ_2^{in} (after they are time evolved), but not the opposite. Therefore this brings to conclude that the number of distinguishable states on Σ_2^{in} is larger than the ones on Σ_1^{in}. The entropy in the internal region increases in time.
It is clear from this simple consideration that this entropy cannot be captured entirely by the Bekenstein-Hawking formula. In fact, once the horizon shrinks the thermodynamical entropy decreases, while the interior entropy has to increase. This motivates the position that the thermodynamical entropy cannot take in account the whole amount of internal entropy of the system.
It's worth to mention here that there are several arguments supporting this position, and maybe as many in the opposite direction.
We won't go into detail in the discussion, that is an open problem, and is outside the

aims of this work.
We conclude the paragraph by stressing that this way to avoid the paradox is completely different from the one proposed in the first model, even if the two models are built from the same quantum gravity theory.
This is possible because in the construction of the models the authors made different assumptions motivated by different arguments, and the final outputs turn out to be different. One of the main common points is that both the models avoid the information loss paradox, and in particular the fact that quantum gravity plays a crucial role to make this happen.
In the last section we will make a more direct comparison between the two models, stressing the common points and the features that make them strongly different.

3.3 Comparison between the two models and conclusions

In the previous sections we analyzed two competing models supposed to describe the gravitational star collapse based on Loop Quantum Gravity. Let us study more in detail their main differences and similarities.
Firstly, we have that both the models consider the star as a pressureless, homogeneous and isotropic fluid. Thus for both the models the interior of the star is governed by the effective Friedmann equations 2.14 and 2.15 for the pre-bounce phase. The post-bounce phase as predicted by the models however is completely different: the shock-wave model admits the production of a shock-wave of matter which propagates outward, containing almost the whole mass of the initial star (the whole mass minus the small amount of mass evaporated during the collapsing phase through the Hawking process); the white hole remnant model admits instead a post-bounce dynamics governed by the Friedmann equations, so describing an expanding star, with a mass of order M_P.
In the second model the Hawking process is expected to extract almost all the mass of the original star during the pre-bounce phase. This is due to the construction of the exterior: to have an external trapped region that bounces in an anti-trapped region, the mass of the interior has to be of order M_P to have a tunneling probability of order ~ 1.
On the other hand, the white hole shock-wave model has a static exterior, and no anti-trapped regions form during the dynamics: the exterior of the star keeps to be a trapped region, and the outgoing motion of the shock is allowed by the existence of an inner

horizon r_{in} that is dynamical.

The shock-wave carries the inner horizon in its motion, and this allows its trajectory to be time-like as measured through the internal metric (space-like trajectories are prohibited in General Relativity, since require superluminal velocities); however, the shock-wave trajectory is space-like if measured by the external metric.

This difference between the interior and exterior comes from the fact that the space-time is discontinuous across the shock-wave; the fact that the trajectory is space-like if measured by the external Schwarzschild metric is not a conceptual issue, since no external observer is able to observe the outgoing motion of the shock, since is hidden behind the outer horizon until the shock reaches r_{out}: from that point on the trajectory becomes time-like both for the internal and external metric, and the shock becomes visible from the exterior.

Therefore both of the models describe in a consistent (but different) way the emergence of the remaining matter from the outer horizon.

Another important difference is the life-time predicted by the two models: the shock-wave model predicts a life-time (from the production of the outer horizon in the collapsing phase to the disappearance of the horizon in the post-bounce phase) of order $\sim M^2$, while the black-to-white hole model predicts a life-time $> M^3$. As we pointed out in the first chapter, the value of the life-time is crucial for the information loss paradox, as stated by Page. In fact, the paradox arises once the mass is evaporated by an amount of $M/2$, that means a time (the Page time) of $T_{Page} \propto \left(\frac{M}{2}\right)^3$. Therefore, on one side $M^2 < T_{Page}$ for macroscopic black holes, which means that the shock-wave model avoids the paradox in an extremely simple way.

On the other side, the black-to-white hole model is affected by the paradox in the Page formulation, but there are several arguments that suggest that the Page formulation doesn't keep in account all the possible physical states of the system (the internal ones), and if one allows the von Neumann entropy to increase after the Page time, due to the increasing of such internal states, the paradox is avoided as well. This means that both models avoid the information loss paradox, which affects the Hawking model based on QFT in curved space-times, although in two complete different ways. Now we look at their main common point, that is at the basis of the philosophical position at the core of this work.

We notice that the crucial ingredient that allows to avoid the paradox in each model is the quantum gravity effect of repulsion that allows the star to bounce. If we neglect it, the star in both models collapses in a physical singularity, and the information in its interior would be completely destroyed by the infinite curvature that characterizes the singularity.

The dynamics would end there, and no shock-wave neither an expanding star can exist. The information loss paradox would enter inevitably in the game, and no process would allow to avoid it. The only possibility then would be to sacrifice some fundamental principle of physics, like superluminal behaviour in GR or loss of unitarity in quantum evolution, as described in the escapes of the first chapter. We notice instead that once quantum gravity effects are kept in account, models can be constructed that avoid the information paradox without forcing to renounce to fundamental principles of physics.

These two models (but also other that we didn't consider in this work) constitute our philosophical position about the information loss paradox: such paradox arises *only* if we don't keep in account quantum gravity. Within models based on a fundamental quantum theory of gravity like Loop Quantum Gravity this paradox disappears naturally, without the need of introducing ad hoc processes which force us to abandon physical principle that are supposed to hold also in black hole physics. Our perspective is based on the fact that the information paradox is not a paradox of black hole physics, but of black hole models constructed from the classical Einstein theory of gravity. During the collapse the star reaches a regime in which Einstein theory looses its validity and predictability, and if this is not kept seriously in account one is forced to reach physically absurd conclusions like violation of unitarity of quantum mechanical processes.

We want to underline that both the model proposed here are not complete, and are far away from being the realistic description of a star collapse. In fact, pressureless matter (dust) is a strong approximation for the composition of a star, and once pressure is kept in account significant physics can emerge determining corrections to the pressureless dynamics.

Moreover, if the first model allows the star to have a tail (a spread instead of a well defined boundary), the second model is based on the Oppenheimer-Sneyder assumption (a well defined boundary). This could be a too restrictive assumption, limiting the model ability to capture the physical dynamics of the star. Also both of the model are based on assumptions that are not yet completely proved: the shock-wave model is based on some choices, technically gauge fixing conditions before quantization and the polymerization of the field variables, that could determine a dynamics different from the real one.

Finally the black-to-white hole model is completely based on the fact that one can perform

classical coordinate transformations on the effective metric (following what one classically does in General Relativity), that allow to obtain the maximal extension of the external Schwarzschild metric, but in the shock-wave model is extremely clear that such transformations are not allowed, and deformations of the classical transformation rule for the metric tensor have to be kept in account if one deals with an effective quantum metric.
They are also far away from being completely understood, since for example is not completely clear if the shock-wave is a coordinate artifact coming from the particular coordinates used to describe the dynamics.
On the other side the maximally extended external metric described in the black-to-white hole model doesn't come from any equations of motion, and the only prescription used for its construction is physical intuition and the requirement that the metric is continuous everywhere. In general relativity there are many metrics that one can construct which are continuous but are absolutely non-physical, or require unphysical matter to be generated.
Despite this, already at this stage the information paradox is considered avoided in both of them (it is proved mathematically in the first model, while in the second there are calculations, although not conclusive, that support such statement). Therefore, even if we are probably far away from having an exact and definitive model unanimously accepted that describes the physics behind the collapse of a star, the models that we have at hand in this stage already avoid the information loss paradox, and as stated before the avoidance is based crucially on the quantum behaviour of gravity at the Planck regime, a feature kept in account by any reliable quantum theory of gravity, which is completely neglected by Einstein theory, and consequently by the Hawking model based on the former.
Our idea is that since quantum gravity theories like Loops or Strings are mature enough to find applications in astrophysical models like black holes and star collapses, is worth to study the eventual paradox *within* such theories, that keep in account physics in the planckian regime, abandoning the idea that within classical general relativity the paradox can be avoided in a credible way.
This because the existence of the paradox from our perspective is due to a theory (general relativity) that is pushed too far beyond its domain of validity, thus lost for such physical process its predictive power and reliability.
The information loss paradox therefore has to be interpreted as a paradox coming from a theory that doesn't capture the fundamental processes behind the gravitational collapse, and disappears once quantum gravity is kept in account (in its regime of validity). We

can make a comparison between the information paradox and other paradoxes coming from classical physics.

When Bohr at first modellized the atomic structure as electrons moving classically around the nucleus in circular orbits, suddenly a paradox turned out: how can an hydrogen atom (or an heavier atom) remain stable if electrons are negatively charged, and protons have opposite sign?

This brings out a paradox that physicists tried to solve (within the classical framework) making a parallelism with Newtonian gravity: electric attraction between electrons and protons allows the orbit to remain circular as well as gravitational attraction allows the almost circular motion of the moon around the earth. However they realized that this explanation couldn't be reliable, since it was known that electric charges under acceleration radiate (emitting light with certain wavelengths), loosing progressively energy.

With the discover of quantum mechanics, physicists realized that such apparent paradox was a paradox in classical physics, not in quantum mechanics. Bohr at first realized that to avoid this issue the only possibility was that electrons could have only precise quantized values of energy, and that they couldn't radiate in a continuous manner. Then, when the theoretical building of quantum mechanics was mature enough and was enough understood to be applied to physical models, calculations have been performed to confirm Bohr intuition: electrons cannot collapse toward atomic nuclei. A paradox that was apparently unsolvable within the context of classical physics, was elegantly and naturally avoided by quantum mechanics.

On the same line, we can briefly discuss about another important paradox concerning classical mechanics: when physicists started to formalize mathematically the black body, an apparent unsolvable problem comes out: from precise classical calculations using Maxwell equations the black body should emits photons with infinite power (the well known ultraviolet catastrophe). This prediction was supported by classical physics, but gave rise to a wrong prediction: no infinite power was observed during black body radiation.

The solution of the problem has been found by Planck (1900), which assumed that photons inside the box could exchange energy only in discrete pockets. This weird assumption, that is considered the real born of quantum mechanics, allowed to solve completely the apparent paradox arising from classical calculations, and brought to the famous (and experimentally verified) Planck law.

This is another clear example that paradox arises when one tries to push a theory (in these cases classical electromagnetism and classical mechanics) beyond its regime of validity.

Probably some physicists facing these kind of problems with classical tools have thought to abandon some well-established principles of classical physics to solve the paradoxes, but the correct answer was different: a new theory was needed to describe such kind of natural phenomena.

These lessons from the past teach us a lot: paradoxes are crucial in physics, because they force us to think and create new research lines, that can eventually end up in new physical theories. On the other side, it is worth to keep in mind that maybe the avoiding paradoxes effort using the same theory that produced them can be useless: history taught us that paradoxes can be a signal that a new more fundamental theory is needed, and our thesis lies exactly in this direction: the information paradox is a paradox for the gravitational collapse within quantum field theory in curved space-time, and needs a more fundamental theory to be avoided.

In this thesis work we presented two tentative models within loop quantum gravity (a tentative theory at this stage), and showed that also at this level the paradox is avoided. Let's conclude with a consideration about the credibility of the models that we presented, with a comparison between the previous and the Hawking model.

One can correctly argue that the approximations on which these models are based can be compared with the approximation of a classical space-time on which the Hawking calculation is based on.

There is however a crucial difference between these kind of approximations: in the previous models the assumptions have to be seen as approximations of a fundamental quantum theory of gravity, that at this stage cannot be used in its full generality since the computational complexity of the task.

However, such approximations lie on a model which contain the fundamental features of the full quantum theory, and even if they can reasonably be wrong (bringing to wrong models) the models are built on the main lessons of the full theory, and the paradox avoidance comes from such lessons.

To be more explicit, the existence of the quantum bounce of matter, due to a switch between an attractive and repulsive behaviour of space-time is a generic feature of each LQG-based models, both in black hole physics and cosmology.

The avoidance of the singularity, that allows the information not to be completely destroyed, (by hitting the singularity) is a crucial element to avoid the information paradox. Such common feature of the models is independent on the choices made by the particular models, and comes from a fundamental feature of the full theory: matter cannot reach an energy density larger than the planckian one, since a planckian mass cannot be compressed in a space smaller than the minimum volume allowed in loop quantum gravity, that is the minimum eigenvalue of the quantum volume operator.

In the same way, a mass M cannot be compressed in a volume smaller than $\sim \Delta R_S$ (the volume of the star at the bounce in the shock-wave model). This to emphasize that even if approximations are made, all the LQG-based models are based on fundamental features of the full theory, completely absent in GR-based models like the Hawking one. The assumptions behind the two effective models can be seen in some way as the ones performed by Bohr in his atomic model: the model follows the principles of quantum mechanics, but without exact calculations from the full theory has to be seen as an approximated description of the atomic physics. Here we are in a similar situation, in which we have approximate models that have to be seen as attempts to encode the fundamental features of the quantum theory in the astrophysical dynamics of a collapsing star.

A very last remark: the whole thesis is based on the expectation that loop quantum gravity is the correct quantum extension of general relativity.

About this is worth to mention that the full theory is not based on ad hoc assumptions or approximations, but it is a rigorous quantization of general relativity developed using techniques that come from the quantization of other field theories.

This means that there is no theoretical reason to believe that the theory is wrong, since is based on principle belonging both to general relativity and quantum mechanics, applied rigorously and using techniques that are considered to be valid in the quantization of other kind of field theories. It is ultimately possible that the theory is wrong, but to prove this there are only two possible ways: criticize mathematically the construction, finding errors or internal inconsistencies, or perform experiments to invalidate its predictions.

While the first way is under investigation (and so far didn't produce any significant result), the second possibility is outside the domain of our actual experimental power.

An important thing to notice here however is that almost all the actual lines of research in quantum gravity (for example string theory) are based on the assumption that exist objects with minimal length ($\sim l_p$), and energy cannot be compressed more than that. This means that the concept of singularity is avoided in almost all the main approaches to quantum gravity currently under investigation, and we expect that models built from such approaches would eventually solve the information paradox as well.

However, a crucial feature that distinguishes LQG from such approaches is that the planckian size is not assumed as an external input, but is a mathematical result of the loop quantization of Einstein theory of gravity.

If by a side one can argue that the assumptions of planckian strings can be arbitrary and not reflect a physical phenomenon, the quanta of space of loop quantum gravity are not assumed a priori but come from calculations in the theory, as well as the energy levels in the hydrogen atom are mathematically derived from ordinary quantum mechanics.

The principles on which LQG is based are the principles of Einstein theory and the ones of quantum mechanics, nothing more.

In conclusion, we underline that the fundamental features of both the effective model presented here (which allow to avoid the information paradox) come from a theory which merges the principles of quantum mechanics and Einstein gravity, and the crucial difference with the Hawking model is that even if the quantum field describing the radiation is obviously based on quantum mechanics, the space-time is treated *classically*; therefore the semi-classical model in our perspective misses the intrinsic quantum nature of space-time at the Planck scale, and this is the deep reason which makes the paradox arising.

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