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Connectivity index in neutrosophic trees and the algorithm to find its maximum spanning tree

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Abstract: In this paper, we first define the Neutrosophic tree using the concept of the strong cycle. We then define a strong spanning Neutrosophic tree. In the following, we propose an algorithm for detecting the maximum spanning tree in Neutrosophic graphs. Next, we discuss the Connectivity index and related theorems for Neutrosophic trees.

Keywords: Neutrosophic trees; totally and partial Connectivity indices; maximum spanning tree; strong spanning tree; strong cycle; strong edge

1. Introduction

In recent years, neutrosophic graphs as one of the new branches of graph theory has been welcomed by many researchers and a lot of work has been done on the features and applications of this particular type of graph [1, 2, 4-6, 17-25]. One of these is finding the spanning tree in neutrosophic graphs. In an article by S.Broumi et al. [7], an algorithm for finding the minimum spanning tree is presented. Using the score function, they calculated a rank for each edge, then constructed a minimum spanning tree based on the lowest score. Other people, including I.Kandasamy [13], also provided algorithms for the minimum spanning tree in the Double-Valued neutrosophic graph.

What we present here is an algorithm for finding the maximum spanning tree in neutrosophic graphs. Our proposed algorithm is similar in appearance to the algorithm presented in [7] but differs from it. First, the algorithm is presented for graphs that have weighted edges, while our algorithm includes the general state of the neutrosophic graphs. The second difference is in how you choose to build the tree. In [7], the score function is used and we use the strength function. The strength function has the advantage of having a more realistic view of indeterminacy-membership (I). In fact, in this function, we have improved the effect of effect indeterminacy-membership (I). In [7, 16], the effect of falsity-membership (F) and indeterminacy-membership (I) was the same, which does not seem very appropriate due to the different nature of falsity-membership (F) and indeterminacy-membership (I).

The definition of a neutrosophic tree used in this paper is similar in structure to the definition given in [12]. The difference between the two definitions stems from the difference in the definition of the strength of connectivity between the two vertices.

2. Preliminaries

In this section, some of the important and basic concepts required are given by mentioning the source.

Definition 1. [3] A single-valued neutrosophic graph on a nonempty V is a pair $G=(N,M)$. Where N is single-valued neutrosophic set in V and M single-valued neutrosophic relation on V such that

$$T_M(uv) \leq \min\{T_N(u), T_N(v)\},$$

$$I_M(uv) \leq \min\{I_N(u), I_N(v)\},$$
$$F_M(uv) \leq \max\{F_N(u), F_N(v)\},$$

For all $u, v \in V$. N is called single-valued neutrosophic vertex set of G and, M is called single-valued neutrosophic edge set of G, respectively.

Definition 2. [12] A connected SVN-graph $G = (N, M)$ is said to be a SVN-tree if it has a SVN spanning subgraph $H = (N, B)$ which is a tree, where for all edges uv not in H satisfying

$$T_M(uv) < T_B^{\infty}(uv), \qquad I_M(uv) > I_B^{\infty}(uv), \qquad F_M(uv) > F_B^{\infty}(uv).$$

3. Neutrosophic tree

In this section, the types of edges are first classified and defined in terms of edge strength. Then we will provide some other definitions depending on the type of edges. Based on the strength of connectivity between the end vertices of an edge, edges of neutrosophic graphs can be divided into two categories as given below.

Definition 3. An edge uv in a neutrosophic graph $G = (N, M)$ is called

a. A ***weak*** edge if $CONN_{(G-uv)}(u,v) = CONN_G(u,v)$ *and* $CONN_G(u,v) \neq M(uv)$,

b. A ***neutral*** edge if $CONN_{(G-uv)}(u,v) = CONN_G(u,v)$ *and* $CONN_G(u,v) = M(uv)$,

c. A $\mathbf{I}-$***strong edge*** if $CONN_{(G-uv)}(u,v) < CONN_G(u,v)$ *and,*
$$CONN_G(u,v) = \big(T_M(uv), I_M(uv), F_M(uv)\big) = M(uv),$$

d. A $\mathbf{II}-$***strong edge*** if $CONN_{(G-uv)}(u,v) < CONN_G(u,v)$ *and*, $CONN_G(u,v) \neq M(uv)$.

Example 1. Consider the neutrosophic graph $G = (N, M)$ on $V = \{a, b, c, d, e, f\}$ as shown in figure 1.

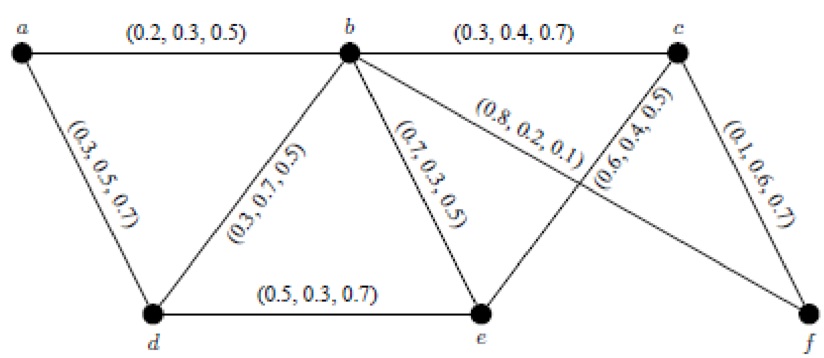


Figure 1. A neutrosophic graph

Table 1. The strength of connectedness between each pair of vertices u and v.

	$CONN_G(u,v)$	$CONN_{G-uv}(u,v)$	$M(uv)$
a,b	(0.3, 0.3, 0.5)	(0.3, 0.5, 0.7)	(0.2, 0.3, 0.5)
a,d	(0.3, 0.3, 0.5)	(0.2, 0.3, 0.5)	(0.3, 0.5, 0.7)
b,c	(0.6, 0.4, 0.5)	(0.6, 0.4, 0.5)	(0.3, 0.4, 0.7)
b,d	(0.5, 0.3, 0.5)	(0.5, 0.3, 0.7)	(0.3, 0.7, 0.5)
b,e	(0.7, 0.3, 0.5)	(0.3, 0.4, 0.7)	(0.7, 0.3, 0.5)
b,f	(0.8, 0.2, 0.1)	(0.1, 0.6, 0.7)	(0.8, 0.2, 0.1)
c,e	(0.6, 0.4, 0.5)	(0.3, 0.4, 0.7)	(0.6, 0.4, 0.5)
c,f	(0.6, 0.4, 0.5)	(0.6, 0.4, 0.5)	(0.1, 0.6, 0.7)
d,e	(0.5, 0.3, 0.5)	(0.3, 0.5, 0.5)	(0.5, 0.3, 0.7)

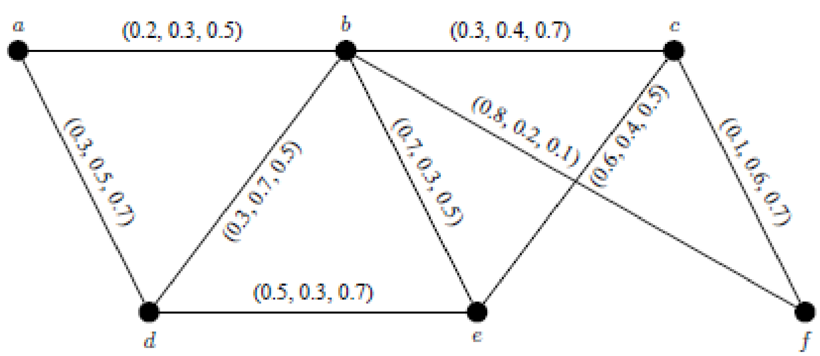
As can be seen in Table 1, edge bc and cf are weak, be, bf and ce are I $-$ $strong\ edges$, and ac, ad, bd and de are II $-$ $strong\ edge$.

Definition 4. A path in a neutrosophic graph is called a I $-$ $strong\ path$ if all its edges are I $-$ $strong$ and called a II $-$ $strong\ path$ if all its edges are II $-$ $strong$. Also is said to be a $strong\ path$ if all its edges are either I $-$ $strong\ edge$ or II $-$ $strong\ edge$.

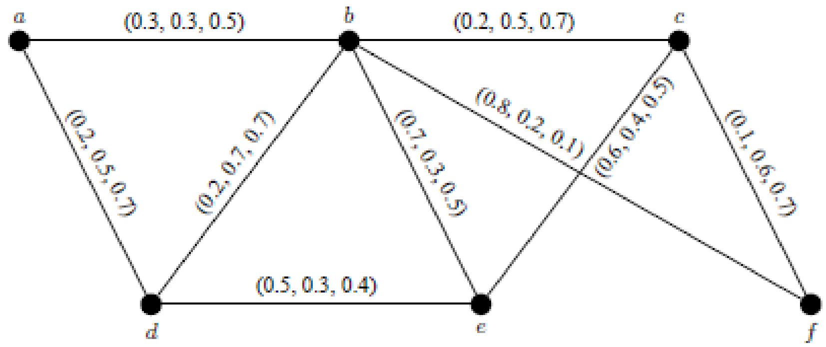
Definition 5. Let $G = (N, M)$ be a neutrosophic graph and C be a cycle in G. C called strong cycle if all its edges are either I $-$ $strong\ edge$ or II $-$ $strong\ edge$.

Definition 6. Let $G = (N, M)$ be a neutrosophic graph. G called a neutrosophic tree if it has no strong cycle.

Example 1. Consider a neutrosophic graph $G = (N, M)$ and $H = (A, B)$ as shown in figure 2.



a. G is not a neutrosophic tree



b. H is a neutrosophic tree

Figure 2. a. G is not a neutrosophic tree and b. H is a neutrosophic tree

It is clear from fig 1 that G is not a neutrosophic tree. Since G contains strong neutrosophic cycles. Cycles such as $abda$, $abeda$, $abceda$, ect. are strong neutrosophic cycles in G. But H is a neutrosophic tree, H has no strong neutrosophic cycle.

Definition 7. Let $G = (N, M)$ be a connected neutrosophic graph and T, is a neutrosophic spanning subgraph of G that T spanned by the vertex set of G and T^* is a tree. If the edges of T are selected from G such that for each edge uv of T, uv is either $\text{I} - strong\ edge$ or $\text{II} - strong\ edge$. Then T called a strong spanning tree and denoted by (SST).

Definition 8. Let $G = (N, M)$ be a connected neutrosophic graph with at least one strong spanning tree. Then the strength of strong spanning tree in G is defined and denoted by

$$S(T) = \sum_{uv \in T} S(uv) = \sum_{uv \in T} \frac{4 + 2T_M(uv) - 2F_M(uv) - I_M(uv)}{6}.$$

Also, F called maximum spanning tree if $S(F) \geq S(T)$ for any strong spanning tree T.

Theorem 1. Let $G = (N, M)$ be a connected neutrosophic graph. Then G is a neutrosophic tree if and only if the following conditions are equivalent for any $u, v \in V$.

a. uv is a $\text{I} - strong\ edge$
b. $\left(CONN_{TG}(u, v), CONN_{IG}(u, v), CONN_{FG}(u, v)\right) = (T_M(uv), I_M(uv), F_M(uv))$.

Proof. This theorem can be easily proved by defining a strong edge.
□

Definition 9. Let $G = (N, M)$ be the Neutrosophic Graph. The ***partial connectivity index*** of G is defined as

$$PCI_T(G) = \sum_{u,v \in N} T_N(u) T_N(v) CONN_{T_G}(u, v),$$
$$PCI_I(G) = \sum_{u,v \in N} I_N(u) I_N(v) CONN_{I_G}(u, v),$$
$$PCI_F(G) = \sum_{u,v \in N} F_N(u) F_N(v) CONN_{F_G}(u, v),$$

Where $CONN_{T_G}(u,v)$ is the strength of truth, $CONN_{I_G}(u,v)$ is the strength of indeterminacy and $CONN_{F_G}(u,v)$ is the strength of falsity between two vertices u and v. we have

$$CONN_{T_G}(u,v) = \max\{\min T_M(e) \mid e \in P \text{ and } P \text{ is a path between } u \text{ and } v\},$$
$$CONN_{I_G}(u,v) = \min\{\max I_M(e) \mid e \in P \text{ and } P \text{ is a path between } u \text{ and } v\},$$
$$CONN_{F_G}(u,v) = \min\{\max F_M(e) \mid e \in P \text{ and } P \text{ is a path between } u \text{ and } v\},$$

Also, the ***totally connectivity index*** of G is defined as

$$TCI(G) = \frac{4 + 2PCI_T(G) - 2PCI_F(G) - PCI_I(G)}{6}.$$

3.1. Maximum spanning tree

In this section, a version of the maximum spanning tree discussed on a graph by strength of edges. In the following, we propose a neutrosophic maximum spanning tree algorithm, whose computing steps are described below. Note that the strength function $S(uv) = \frac{4+2T_M(uv)-2F_M(uv)-I_M(uv)}{6}$ is used to label here.

The algorithm for finding the maximum spanning tree (MST)

Here, the input is adjacency matrix $M = \left[\left(T_M(u_iu_j), I_M(u_iu_j), F_M(u_iu_j)\right)\right]_{n\times n}$ of the neutrosophic graph $G = (N, M)$, and output is a tree F with weighted edges.

Step 1. Input matrix M;

Step 2. Using the strength function $S(u_iu_j) = \frac{4+2T_M(u_iu_j)-2F_M(u_iu_j)-I_M(u_iu_j)}{6}$, convert the neutrosophic matrix into a strength matrix $S = \left[S(u_iu_j)\right]_{n\times n}$;

Step 3. Iterate steps 4 and 5 until all $n-1$ elements of S are either labeled to 0 or all the nonzero elements of the matrix are labeled;

Step 4. Find the M either column or row to compute the unlabeled maximum element $S(u_iu_j)$, which is the value of the corresponding are $e(u_iu_j) \in M$;

Step 5. If the corresponding edge $e(u_iu_j) \in M$ of chosen S produce a cycle whit the previous labeled entries of the strength matrix S than set $S(u_iu_j) = 0$ else label $S(u_iu_j)$;

Step 6. Design the tree F including only the labeled elements from the S which will be computed MST of G;

Step 6. Stop (end algorithm).

Example 3. Consider a neutrosophic graph $G = (N, M)$ on $V = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ as shown in Figure 3.

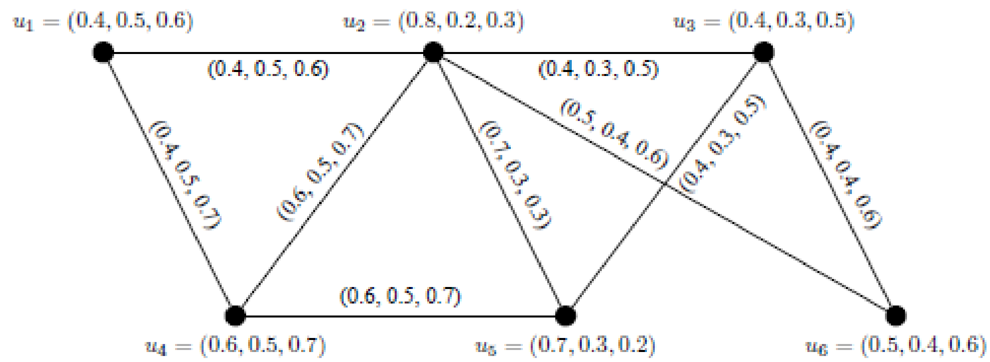


Figure 3. a neutrosophic graph G on $V = \{u_1, u_2, u_3, u_4, u_5, u_6\}$

And

$$M = \begin{bmatrix} 0 & (0.4, 0.5, 0.6) & 0 & (0.4, 0.5, 0.7) & 0 & 0 \\ (0.4, 0.5, 0.6) & 0 & (0.4, 0.3, 0.5) & (0.6,0.5, 0.7) & (0.7,0.3, 0.3) & (0.5, 0.4, 0.6) \\ 0 & (0.4, 0.3, 0.5) & 0 & 0 & (0.4, 0.3, 0.5) & (0.4, 0.4, 0.6) \\ (0.4, 0.5, 0.7) & (0.6,0.5, 0.7) & 0 & 0 & (0.7, 0.3, 0.2) & 0 \\ 0 & (0.7, 0.3, 0.3) & (04, 0.3, 0.5) & (0.6, 0.5, 0.7) & 0 & 0 \\ 0 & (0.5, 0.4, 0.6) & (0.4, 0.4, 0.6) & 0 & 0 & 0 \end{bmatrix}.$$

Using the strength function $S(u_i u_j) = \frac{4+2T_M(u_i u_j)-2F_M(u_i u_j)-I_M(u_i u_j)}{6}$ we have

$$S(u_i u_j) = \begin{bmatrix} 0 & 0.517 & 0 & 0.483 & 0 & 0 \\ 0.517 & 0 & 0.583 & 0.550 & 0.750 & 0.567 \\ 0 & 0.583 & 0 & 0 & 0.583 & 0.533 \\ 0.483 & 0.550 & 0 & 0 & 0.550 & 0 \\ 0 & 0.750 & 0.583 & 0.550 & 0 & 0 \\ 0 & 0.567 & 0.533 & 0 & 0 & 0 \end{bmatrix},$$

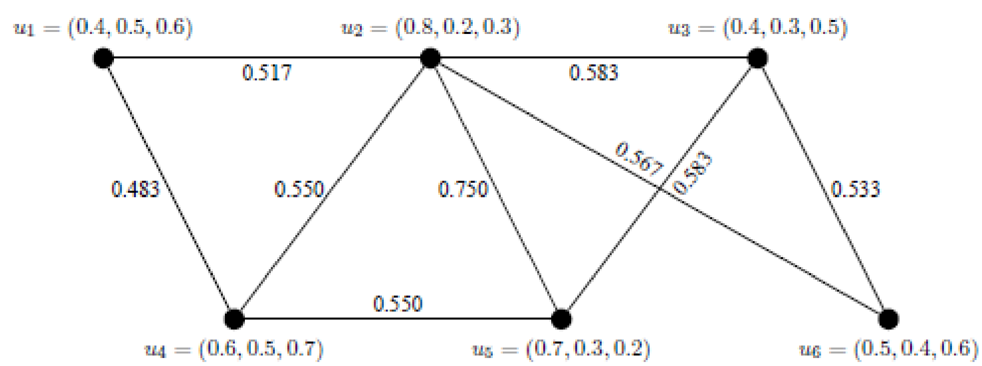


Figure 4. A neutrosophic graph G whit strength of edges

Now search the matrix S to find the maximum value and select the edge corresponding to the row and column of that element. The following figure edge u_2u_5 is highlighted.

$$S(u_iu_j) = \begin{bmatrix} 0 & 0.517 & 0 & 0.483 & 0 & 0 \\ 0.517 & 0 & 0.583 & 0.550 & 0.750 & 0.567 \\ 0 & 0.583 & 0 & 0 & 0.583 & 0.533 \\ 0.483 & 0.550 & 0 & 0 & 0.550 & 0 \\ 0 & 0.750 & 0.583 & 0.550 & 0 & 0 \\ 0 & 0.567 & 0.533 & 0 & 0 & 0 \end{bmatrix},$$

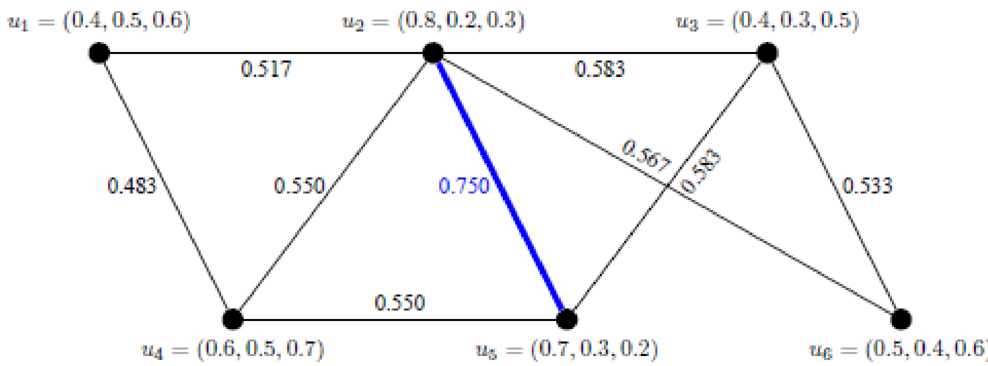


Figure 5. An edge u_2u_5 is highlighted

The next maximum element 0.583 is marked and corresponding edges u_2u_3 and u_3u_5, but the simultaneous selection of these two edges causes the formation of a cycle, so we choose one of these two edges arbitrarily and ignore the other.

$$S(u_iu_j) = \begin{bmatrix} 0 & 0.517 & 0 & 0.483 & 0 & 0 \\ 0.517 & 0 & 0.583 & 0.550 & 0.750 & 0.567 \\ 0 & 0.583 & 0 & 0 & 0.583 & 0.533 \\ 0.483 & 0.550 & 0 & 0 & 0.550 & 0 \\ 0 & 0.750 & 0.583 & 0.550 & 0 & 0 \\ 0 & 0.567 & 0.533 & 0 & 0 & 0 \end{bmatrix},$$

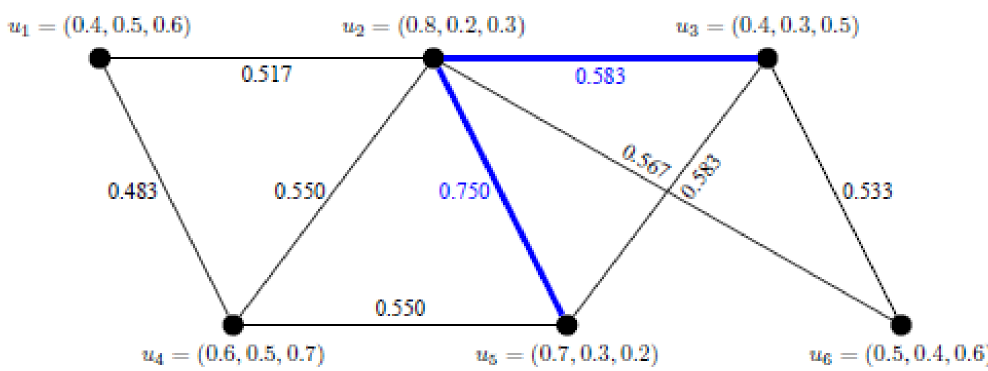


Figure 6. An edge u_2u_3 is highlighted

Continuing this process, edges u_2u_6, u_2u_4, and u_2u_1 are selected, respectively. The maximum spanning tree is obtained as figure 8.

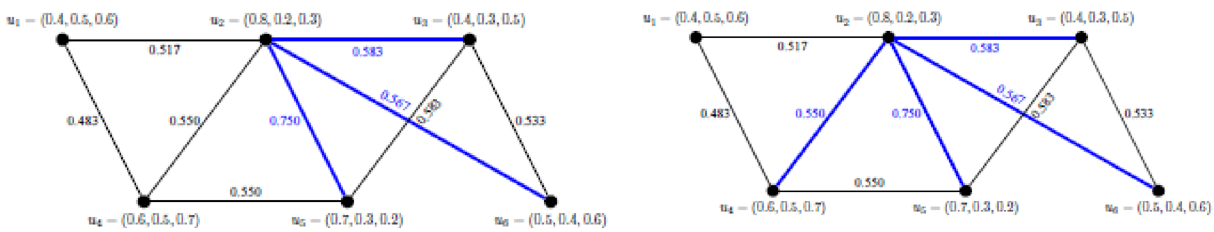


Figure 7. The edges u_2u_6 and u_2u_4 are highlighted

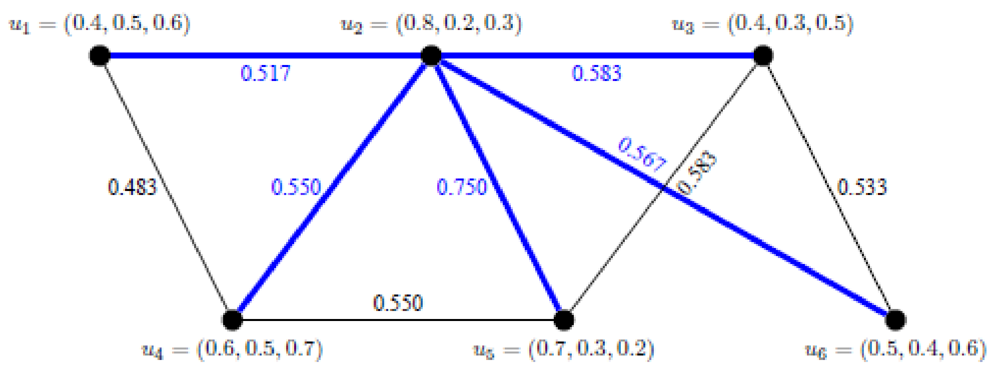


Figure 8. Maximum spanning tree (MST)

As it was observed, the selection of the maximum spanning tree was not unique, so neutrosophic graph $G = (N, M)$ is not a neutrosophic tree, also G contains a strong neutrosophic cycle.

Note. Obviously, if $G = (N, M)$ has a unique strong spanning tree, it will also have a unique maximum spanning tree, but the conversely is not necessarily true.

3.2. Partial connectivity index in the neutrosophic tree

In this section, the results of examining the Partial connectivity index and totally connectivity index on the neutrosophic trees are presented and proved.

Theorem 2. Let $G = (N, M)$ be a neutrosophic graph. Then $TCI(G - uv) = TCI(G)$ if and only if either uv is a weak edge or neutral edge.

Proof. The proof of this theorem is clear using definition 8.
□

Corollary 1. Let $G = (N, M)$ be a neutrosophic graph and, uv is an edge in G, uv is a bridge if and only if uv is either $\mathrm{I} - strong\ edge$ or $\mathrm{II} - strong\ edge$.

Corollary 2. Let $G = (N, M)$ be a neutrosophic graph. Then for any uv, $TCI(G - uv) \neq TCI(G)$ if G^* is a tree.

Theorem 3. Let $G = (N, M)$ be a connected neutrosophic graph whit strong spanning tree (SST) T. for any $uv \in M$, where uv is an edge of T, then either

$$PCI_T(G - uv) < PCI_T(G)$$
$$or$$
$$[(PCI_I(G - uv) > PCI_I(G)) \vee (PCI_F(G - uv) > PCI_F(G))]$$

Hence we have $TCI(G - uv) < TCI(G)$.

Proof. Suppose $G = (N, M)$ be a connected neutrosophic graph whit strong spanning tree (SST) T. Since T is SST then any edge of T is either I $-$ $strong\ edge$ or II $-$ $strong\ edge$. By Corollary 1, for each $uv \in M$, uv is a bridge. Then $PCI_T(G - uv) < PCI_T(G)\ or\ [(PCI_I(G - uv) > PCI_I(G)) \vee (PCI_F(G - uv) > PCI_F(G))]$.

□

Theorem 4. Let $G = (N, M)$ be a connected neutrosophic tree and G^* is not a tree. Then there exists at least one edge $uv \in M^*$ such that $TCI(G - uv) = TCI(G)$.

Proof. Let $G = (N, M)$ be a neutrosophic tree and G^* is not a tree. Hence there is at least one cycle in G^*. As respects a tree is a connected forest, there exist $uv \in M^*$ so that at least one of the following

$$T_M(uv) < CONN_{T(G-uv)}(u, v),$$
$$I_M(uv) > CONN_{I(G-uv)}(u, v), \qquad F_M(uv) > CONN_{F(G-uv)}(u, v))$$

Then

$$PCI_T(G - uv) = PCI_T(G)\ \ and\ \ PCI_I(G - uv) = PCI_I(G)\ \ and\ \ PCI_F(G - uv) = PCI_F(G)$$

Therefore, $TCI(G - uv) = TCI(G)$.

□

Theorem 5. Let $G = (N, M)$ be a connected neutrosophic graph then G is a neutrosophic tree if and only if G has a unique strong spanning tree.

Proof. Suppose $G = (N, M)$ is a connected neutrosophic graph with only one strong spanning tree T. Then G has no strong edges except the edges of T. hence G has no strong cycle. Therefore by definition 6, G is a neutrosophic tree. Conversely, assume that G is a neutrosophic tree. Again according to definition 6, G lacks a strong circle. Therefore, there is only one strong path between the two arbitrary vertices of G. then the strong spanning tree of G is unique.

□

Theorem 6. Let $G = (N, M)$ be a connected neutrosophic graph and T the corresponding SST of G. Then $TCI(T) = TCI(G)$ if and only if T is the unique strong spanning tree of G.

Proof. Suppose $G = (N, M)$ is a connected neutrosophic graph and T the corresponding SST of G. And $TCI(T) = TCI(G)$. Now, shown that T is a unique strong spanning tree of G. Proof of this is easily possible using Theorem 5. Conversely, assume that T is the unique strong spanning tree of G. It is clear that to obtain the connectivity index of G, only the strong paths will be the same paths of T. then $TCI(T) = TCI(G)$

□

Corollary 3. Let $G = (N, M)$ be a neutrosophic tree with the unique strong spanning tree (T) and the unique maximum spanning tree (F). Then $TCI(T) = TCI(G) = TCI(F)$.

Theorem 7. Let $G = (N, M)$ be a connected neutrosophic graph and $uv \in M^*$. Then $TCI(G - uv) < TCI(G)$ for any uv and $\left(CONN_{TG}(u,v), CONN_{IG}(u,v), CONN_{FG}(u,v)\right) = (T_M(uv), I_M(uv), F_M(uv))$ if and only if G^* is a tree.

Proof. Suppose $G = (N, M)$ is a connected neutrosophic graph and G^* is a tree. It is clear $TCI(G - uv) < TCI(G)$. Since G^* is a tree, for any $uv \in M^*$, $G - uv$ is not connected. Also for any $uv \in G$ we have $\left(CONN_{TG}(u,v), CONN_{IG}(u,v), CONN_{FG}(u,v)\right) = (T_M(uv), I_M(uv), F_M(uv))$. Conversely assume that for each uv, $TCI(G - uv) < TCI(G)$ and $\left(CONN_{TG}(u,v), CONN_{IG}(u,v), CONN_{FG}(u,v)\right) = (T_M(uv), I_M(uv), F_M(uv))$, then both uv is a neutrosophic bridge and a $\mathrm{I}-strong\ edge$. By theorem 1, G is a tree. Since, for each uv, $TCI(G - uv) < TCI(G)$, G^* is a tree.

□

Theorem 8. Let $G = (N, M)$ be a connected neutrosophic graph such that G^* is a star graph. If v_1 is the center vertex and for any $uv \in M^*$,

$$T_M(uv) = \min\{T_N(u), T_N(v)\},\ \ I_M(uv) = \min\{I_N(u), I_N(v)\},\ F_M(uv) = \max\{F_N(u), F_N(v)\}.$$

Also $\forall\, j \geq 2$, $t_1 \leq t_j$, $i_1 \leq i_j$ and $f_1 \geq f_j$ where $t_j = T_N(v_j)$, $i_j = I_N(v_j)$ and $f_j = F_N(v_j)$ for $j = 1, 2, \ldots, n$. Then

$$PCI_T(G) = t_1 \sum_{j=1}^{n-1} t_j \sum_{k=j+1}^{n} t_k,$$
$$PCI_I(G) = i_1 \sum_{j=1}^{n-1} i_j \sum_{k=j+1}^{n} i_k,$$
$$PCI_F(G) = f_1 \sum_{j=1}^{n-1} f_j \sum_{k=j+1}^{n} f_k.$$

Proof. Let $G = (N, M)$ be a neutrosophic graph such that G^* is a star graph and v_1 is the center vertex. Therefore for any vertex v_j, we have

$$CONN_{TG}(v_1, v_j) = T_M(v_1 v_j) = \min\{T_N(v_1), T_N(v_j)\} = T_N(v_1),$$
$$CONN_{IG}(v_1, v_j) = I_M(v_1 v_j) = \min\{I_N(v_1), I_N(v_j)\} = I_N(v_1),$$
$$CONN_{FG}(v_1, v_j) = F_M(v_1 v_j) = \max\{F_N(v_1), F_N(v_j)\} = F_N(v_1).$$

Then

$$\sum_{k=2}^{n} T_N(v_1) T_N(v_k) CONN_{TG}(v_1, v_k) = (T_N(v_1))^2 \sum_{k=2}^{n} T_N(v_k) = t_1^2 \sum_{k=2}^{n} t_k,$$

Too for any $j, k \neq 1$, we have $CONN_{TG}(v_j, v_k) = T_N(v_1) = t_1$. Hence

$$
\begin{aligned}
PCI_T(G) = \sum_{u,v\in N} & T_N(u)T_N(v)CONN_{T_G}(u,v) \\
&= \sum_{k=2}^{n} T_N(v_1)T_N(v_k)CONN_{TG}(v_1,v_k) + \sum_{k=3}^{n} T_N(v_2)T_N(v_k)CONN_{TG}(v_2,v_k) + \cdots \\
&+ T_N(v_{n-1})T_N(v_n)CONN_{TG}(v_{n-1},v_n) \\
&= \left(T_N(v_1)\right)^2 \sum_{k=2}^{n} T_N(v_k) + T_N(v_1)\sum_{k=3}^{n} T_N(v_2)T_N(v_k) + \cdots + T_N(v_1)T_N(v_{n-1})T_N(v_n) \\
&= \left(T_N(v_1)\right)^2 \sum_{k=2}^{n} T_N(v_k) + T_N(v_1)\sum_{j=n}^{n-1} T_N(v_j) \sum_{k=j+1}^{n} T_N(v_k) = t_1 \sum_{j=1}^{n-1} t_j \sum_{k=j+1}^{n} t_k.
\end{aligned}
$$

Using a similar proof, we can show that $PCI_I(G) = i_1 \sum_{j=1}^{n-1} i_j \sum_{k=j+1}^{n} i_k$ and $PCI_F(G) = f_1 \sum_{j=1}^{n-1} f_j \sum_{k=j+1}^{n} f_k$.
□

Theorem 9. Let $G = (N, M)$ be a connected neutrosophic graph such that $G^* = C_n$. Then the following are equivalent.

a. $TCI(G - uv) = TCI(G)$ for any uv.
b. M is a constant function.
c. G has n strong spanning tree whit $S(T) = \gamma$ that γ is a constant value.

Proof. Suppose $G = (N, M)$ be a neutrosophic graph with $G^* = C_n$.

a → b Assume that $TCI(G - uv) = TCI(G)$ for any uv. This means that deleting each edge will not change the value of the connectivity index. Therefore, the membership function will be the same for all edges.

b → c Assume that M is a constant function. Hence all the edges of G are $I - strong\ edge$. Since removing each edge from the cycle will result a new tree of G. then the number of strong spanning trees of G will be n and strength of any strong spanning tree is a constant value.

c → a Assume that G has n strong spanning tree whit $S(T) = \gamma$ that γ is a constant value. It is clear for each edge of G we have $TCI(G - uv) = TCI(G)$.
□

4. Conclusion

In the paper, deals with a maximum spanning tree (MST) and a strong spanning tree (SST) problem under the neutrosophic graphs. Also, the Partial connectivity index and totally connectivity index in neutrosophic trees was presented here and some results obtained from the study of this index in trees were presented and proved. It should be noted that the results obtained in this article can be generalized to directed neutrosophic graphs, bipolar neutrosophic graphs and interval-valued neutrosophic graph, in general.

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