



Neutrosophic bg-closed Sets and its Continuity

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Abstract: Smarandache introduced and developed the new concept of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. Aim of this paper is we introduce and study the concepts Neutrosophic b generalized closed sets and Neutrosophic b generalized continuity in Neutrosophic topological spaces and its Properties are discussed details.

Keywords: Neutrosophic bg closed sets, Neutrosophic bg open sets, Neutrosophic bg continuity, Neutrosophic bg maps.

1. Introduction

Neutrosophic system plays important role in the fields of Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, Mechanics, decision making, Medicine, Management Science, and Electrical & Electronic, etc.. Topology is a classical subject, as a generalization topological spaces many type of topological spaces introduced over the year. T Truth, F -Falsehood, I- Indeterminacy are three component of Neutrosophic sets. Neutrosophic topological spaces(N-T-S) introduced by Salama [22,23]etal., R.Dhavaseelan[10], Saied Jafari are introduced Neutrosophic generalized closed sets. Neutrosophic b closed sets are introduced by C.Maheswari[17] et al.Aim of this paper is we introduce and study about Neutrosophic b generalized closed sets and Neutrosophic b generalized continuity in Neutrosophic topological spaces and its properties and Characterization are discussed details.

2. Preliminaries

In this section, we recall needed basic definition and operation of Neutrosophic sets and its fundamental Results

Definition 2.1 [13] Let \mathfrak{X} be a non-empty fixed set. A Neutrosophic set \mathcal{J}_1^* is a object having the form

$$\mathcal{J}_1^* = \{ \langle x, \mu_{\mathcal{J}_1^*}(x), \sigma_{\mathcal{J}_1^*}(x), \gamma_{\mathcal{J}_1^*}(x) \rangle : x \in \mathfrak{X} \},$$

$\mu_{\mathcal{J}_1^*}(x)$ -represents the degree of membership function

$\sigma_{\mathcal{J}_1^*}(x)$ -represents degree indeterminacy and then

$\gamma_{\mathcal{J}_1^*}(x)$ -represents the degree of non-membership function

Definition 2.2 [13].Neutrosophic set $J_1^* = \{ \langle x, \mu_{J_1^*}(x), \sigma_{J_1^*}(x), \gamma_{J_1^*}(x) \rangle : x \in \mathfrak{X} \}$, on \mathfrak{X} and $\forall x \in \mathfrak{X}$

then complement of J_1^* is $J_1^{*C} = \{ \langle x, \gamma_{J_1^*}(x), 1 - \sigma_{J_1^*}(x), \mu_{J_1^*}(x) \rangle : x \in \mathfrak{X} \}$

Definition 2.3 [13]. Let J_1^* and J_2^* are two Neutrosophic sets, $\forall x \in \mathfrak{X}$

$$J_1^* = \{ \langle x, \mu_{J_1^*}(x), \sigma_{J_1^*}(x), \gamma_{J_1^*}(x) \rangle : x \in \mathfrak{X} \}$$

$$J_2^* = \{ \langle x, \mu_{J_2^*}(x), \sigma_{J_2^*}(x), \gamma_{J_2^*}(x) \rangle : x \in \mathfrak{X} \}$$

Then $J_1^* \subseteq J_2^* \Leftrightarrow \mu_{J_1^*}(x) \leq \mu_{J_2^*}(x), \sigma_{J_1^*}(x) \leq \sigma_{J_2^*}(x) \& \gamma_{J_1^*}(x) \geq \gamma_{J_2^*}(x)$

Definition 2.4 [13]. Let \mathfrak{X} be a non-empty set, and Let J_1^* and J_2^* be two Neutrosophic sets are

$J_1^* = \{ \langle x, \mu_{J_1^*}(x), \sigma_{J_1^*}(x), \gamma_{J_1^*}(x) \rangle : x \in \mathfrak{X} \}$, $J_2^* = \{ \langle x, \mu_{J_2^*}(x), \sigma_{J_2^*}(x), \gamma_{J_2^*}(x) \rangle : x \in \mathfrak{X} \}$ Then

1. $J_1^* \cap J_2^* = \{ \langle x, \mu_{J_1^*}(x) \cap \mu_{J_2^*}(x), \sigma_{J_1^*}(x) \cap \sigma_{J_2^*}(x), \gamma_{J_1^*}(x) \cup \gamma_{J_2^*}(x) \rangle : x \in \mathfrak{X} \}$
2. $J_1^* \cup J_2^* = \{ \langle x, \mu_{J_1^*}(x) \cup \mu_{J_2^*}(x), \sigma_{J_1^*}(x) \cup \sigma_{J_2^*}(x), \gamma_{J_1^*}(x) \cap \gamma_{J_2^*}(x) \rangle : x \in \mathfrak{X} \}$

Definition 2.5 [23].Let \mathfrak{X} be non-empty set and τ_N be the collection of Neutrosophic subsets of \mathfrak{X} satisfying the following properties:

1. $0_N, 1_N \in \tau_N$
2. $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$
3. $\cup T_i \in \tau_N$ for every $\{T_i : i \in j\} \subseteq \tau_N$

Then the space (\mathfrak{X}, τ_N) is called a Neutrosophic topological space(N-T-S).

The element of τ_N are called Ne.OS (Neutrosophic open set)

and its complement is Ne.CS(Neutrosophic closed set)

Example 2.6.Let $\mathfrak{X} = \{x\}$ and $\forall x \in \mathfrak{X}$

$$A_1 = \langle x, \frac{6}{10}, \frac{6}{10}, \frac{5}{10} \rangle, A_2 = \langle x, \frac{5}{10}, \frac{7}{10}, \frac{9}{10} \rangle$$

$$A_3 = \langle x, \frac{6}{10}, \frac{7}{10}, \frac{5}{10} \rangle, A_4 = \langle x, \frac{5}{10}, \frac{6}{10}, \frac{9}{10} \rangle$$

Then the collection $\tau_N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$ is called a N-T-S on \mathfrak{X} .

Definition 2.7.Let (\mathfrak{X}, τ_N) be a N-T-S and $J_1^* = \{ \langle x, \mu_{J_1^*}(x), \sigma_{J_1^*}(x), \gamma_{J_1^*}(x) \rangle : x \in \mathfrak{X} \}$ be a

Neutrosophic set in \mathfrak{X} . Then J_1^* is said to be

1. Neutrosophic b closed set [17] (Ne.bCS) if $Ne.cl(Ne.int(J_1^*)) \cap Ne.int(Ne.cl(J_1^*)) \subseteq J_1^*$,
2. Neutrosophic α -closed set [7] (Ne. α CS) if $Ne.cl(Ne.int(Ne.cl(J_1^*))) \subseteq J_1^*$,
3. Neutrosophic pre-closed set [25] (Ne.Pre-CS) if $Ne.cl(Ne.int(J_1^*)) \subseteq J_1^*$,
4. Neutrosophic regular closed set [7] (Ne.RCS) if $Ne.cl(Ne.int(J_1^*)) = J_1^*$,
5. Neutrosophic semi closed set [7] (Ne.SCS) if $Ne.int(Ne.cl(J_1^*)) \subseteq J_1^*$,
6. Neutrosophic generalized closed set [10] (Ne.GCS) if $Ne.cl(J_1^*) \subseteq H$ whenever $J_1^* \subseteq H$ and H

is aNe.OS,

7. Neutrosophic generalized pre closed set [17] (Ne.GPCS in short) if $Ne.Pcl(J_1^*) \subseteq H$ whenever $J_1^* \subseteq H$ and H is a Ne.OS,
8. Neutrosophic α generalized closed set [15] (Ne. α GCS in short) if $Neu \alpha-cl(J_1^*) \subseteq H$ whenever $J_1^* \subseteq H$ and H is a Ne.OS,
9. Neutrosophic generalized semi closed set [24](Ne.GSCS in short) if $Ne.Scl(J_1^*) \subseteq H$ whenever $J_1^* \subseteq H$ and H is a Ne.OS.
10. Neutrosophic generalized α closed set [11] (Ne. G α CS in short) if $Neu \alpha-cl(J_1^*) \subseteq H$ whenever $J_1^* \subseteq H$ and H is a Ne. α OS.
11. Neutrosophic semi generalized closed set [24](Ne.SGCS in short) if $Ne.Scl(J_1^*) \subseteq H$ whenever $J_1^* \subseteq H$ and H is a Ne.SOS.

Definition 2.8.[9] (\mathfrak{X}, τ_N) be a N-T-S and $J_1^* = \{ \langle x, \mu_{J_1^*}(x), \sigma_{J_1^*}(x), \gamma_{J_1^*}(x) \rangle : x \in \mathfrak{X} \}$ be a

Neutrosophic set in \mathfrak{X} . Then

Neutrosophic closure of J_1^* is $Ne.Cl(J_1^*) = \cap \{ H : H \text{ is a Ne.CS in } \mathfrak{X} \text{ and } J_1^* \subseteq H \}$

Neutrosophic interior of J_1^* is $Ne.Int(J_1^*) = \cup \{ M : M \text{ is a Ne.OS in } \mathfrak{X} \text{ and } M \subseteq J_1^* \}$.

Definition 2.9. Let (\mathfrak{X}, τ_N) be a N-T-S and $J_1^* = \{ \langle x, \mu_{J_1^*}(x), \sigma_{J_1^*}(x), \gamma_{J_1^*}(x) \rangle : x \in X \}$ be a

Neutrosophic set in \mathfrak{X} . Then the Neutrosophic b closure of J_1^* (Ne.bcl(J_1^*) in short) and

Neutrosophic b interior of J_1^* (Ne.bint(J_1^*) in short) are defined as

$Ne.bint(J_1^*) = \cup \{ G / G \text{ is a Ne.bOS in } \mathfrak{X} \text{ and } G \subseteq J_1^* \}$,

$Ne.bcl(J_1^*) = \cap \{ K / K \text{ is a Ne.bCS in } \mathfrak{X} \text{ and } J_1^* \subseteq K \}$.

Proposition 2.10. Let $(\mathfrak{X}, \mathcal{N}_\tau)$ be any N-T-S. Let J_1^* and J_2^* be any two Neutrosophic sets in (\mathfrak{X}, τ_N) . Then the Neutrosophic generalized b closure operator satisfy the following properties.

1. $Ne.bcl(0_N) = 0_N$ and $Ne.bcl(1_N) = 1_N$,
2. $J_1^* \subseteq Ne.bcl(J_1^*)$,
3. $Ne.bint(J_1^*) \subseteq J_1^*$,
4. If J_1^* is a Ne.bCS then $J_1^* = Ne.bcl(Ne.bcl(J_1^*))$,
5. $J_1^* \subseteq J_2^* \Rightarrow Ne.bcl(J_1^*) \subseteq Ne.bcl(J_2^*)$,
6. $J_1^* \subseteq J_2^* \Rightarrow Ne.bint(J_1^*) \subseteq Ne.bint(J_2^*)$.

NEUTROSOPHIC b GENERALIZED CLOSED SETS

In this part we introduce neutrosophic bG closed sets its properties are discussed.

Definition 3.1.

A Ne. set J_1^* in an NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is called Neutrosophic b generalized CS (briefly Ne.(bG)CS) iff $Ne.bCl(J_1^*) \subseteq J_2^*$, whenever $J_1^* \subseteq J_2^*$ and J_2^* is Ne. (b)OS in \mathfrak{X} .

Example 3.2.

Let $\mathfrak{X} = \{j_1, j_2\}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X} where $J_1^* = \langle x, (\frac{2}{10}, \frac{5}{10}, \frac{8}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}) \rangle$.

Then the Neutrosophic set $J_2^* = \langle x, (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}) \rangle$ is a Ne.bGCS in \mathfrak{X} .

Remark 3.3.

A Ne. set J_1^* in a NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is called Ne.(b)generalized open (briefly Ne.(bG)OS) if its compliment J_1^{*c} is Ne.(bG)CS.

Theorem 3.4.

Every Ne.-CS in $(\mathfrak{X}, \mathcal{N}_\tau)$ is Ne.(bG)CS.

Proof.

Let J_1^* be a Ne.CS in NSTS \mathfrak{X} . Let $J_1^* \subseteq J_2^*$, where J_2^* is Ne.(b)OS in \mathfrak{X} . Since J_1^* is Ne.CS it is Ne.(b)CS and so $\text{NeuCl}(J_1^*) = \text{Ne.bCl}(J_1^*) = J_1^* \subseteq J_2^*$. Thus $\text{Ne.bCl}(J_1^*) \subseteq J_2^*$. Hence J_1^* is Ne.(bG)CS.

Example 3.5

Let $\mathfrak{X} = \{j_1, j_2\}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X} where $J_1^* = \langle x, (\frac{3}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}) \rangle$. Then the

Neutrosophic set $J_2^* = \langle x, (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$ is a Ne.bGCS but not a Ne.CS in \mathfrak{X}

Theorem 3.6.

Every Ne.(b)CS in $(\mathfrak{X}, \mathcal{N}_\tau)$ is Ne.(bG)CS.

Proof.

Let J_1^* be a Ne.(b)CS in NSTS \mathfrak{X} . Let $J_1^* \subseteq J_2^*$, where J_2^* is Ne.(b)OS in \mathfrak{X} . Since J_1^* is Ne.(b)CS, $\text{Ne.bCl}(J_1^*) = J_1^* \subseteq J_2^*$. Thus $\text{Ne.bCl}(J_1^*) \subseteq J_2^*$. Hence J_1^* is Ne.(bG)CS.

Example 3.7. Let $\mathfrak{X} = \{j_1, j_2\}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X}

where $J_1^* = \langle x, (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{2}{10}) \rangle$. Then the Neutrosophic set $J_2^* = \langle x, (\frac{8}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{9}{10}, \frac{5}{10}, \frac{1}{10}) \rangle$ is a Ne.bGCS but not a Ne.bCS in \mathfrak{X} .

Remark 3.8.

- (i). Every Ne. (bG)CS is Ne.(Gb)CS.
- (ii). Every Ne.(sG)CS is Ne.(bG)CS.
- (iii) Every Ne.(G α)CS is Ne.(bG)CS.

Example 3.9.

Let $\mathfrak{X} = \{j_1, j_2\}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X}

where $J_1^* = \langle x, (\frac{3}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{2}{10}) \rangle$. Then the Neutrosophic set $J_2^* = \langle x, (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$

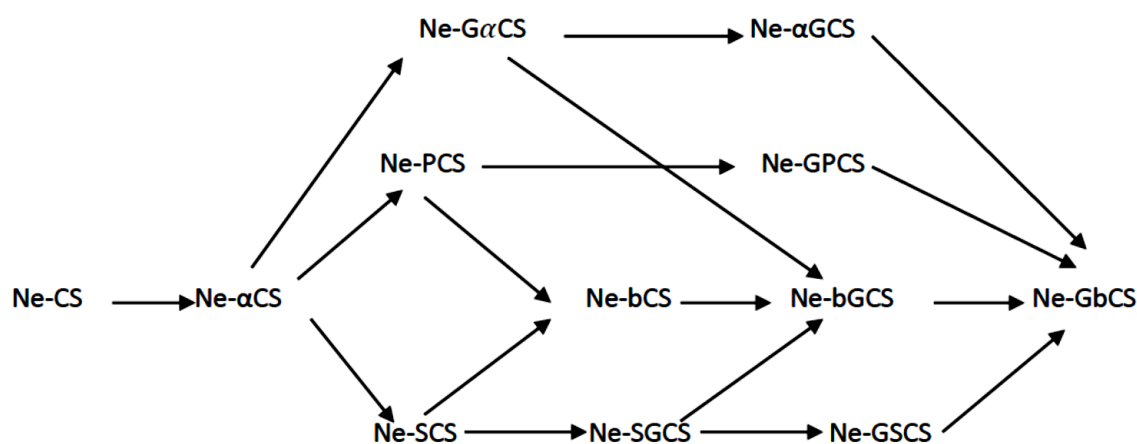
is a Ne.GbCS but not Ne.(bG)CS in \mathfrak{X}

Example 3.10.

Let $\mathfrak{X} = \{j_1, j_2\}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X} where $J_1^* = \langle x, (\frac{2}{10}, \frac{5}{10}, \frac{8}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}) \rangle$.

Then the Neutrosophic set $J_2^* = \langle x, (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$ is a Ne.bGCS in \mathfrak{X} is not Ne.(sG)-CS

Diagram:1



Theorem 3.11.

A Ne. set J_1^* of a NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is called Ne.(bG)OS iff $J_2^* \subseteq \text{Ne.bInt}(J_1^*)$, whenever J_2^* is Ne.(b)CS and $J_2^* \subseteq J_1^*$.

Proof.

Suppose J_1^* is Ne.(bG)OS in \mathfrak{X} . Then J_1^{*c} is Ne.(bG)CS in \mathfrak{X} . Let J_2^* be a Ne.(b)CS in \mathfrak{X} such that $J_2^* \subseteq J_1^*$. Then $J_1^{*c} \subseteq J_2^{*c}$, J_2^{*c} is Ne.(b)OS in \mathfrak{X} . Since J_1^{*c} is Ne.(bG)CS, $\text{Ne.bCl}(J_1^{*c}) \subseteq J_2^{*c}$, which implies $(\text{Ne.blnt}(J_1^*))^c \subseteq J_2^{*c}$. Thus $J_2^* \subseteq \text{Ne.blnt}(J_1^*)$.

Conversely, assume that $J_2^* \subseteq \text{Ne.bInt}(J_1^*)$, whenever $J_2^* \subseteq J_1^*$ and J_2^* is Ne.(b)CS in \mathfrak{X} . Then $(\text{Ne.blnt}(J_1^*))^c \subseteq J_2^{*c} \subseteq J_3^*$, where J_3^* is Ne.(b)OS in \mathfrak{X} . Hence $\text{Ne.bCl}(J_1^{*c}) \subseteq J_3^*$, which implies J_1^{*c} is Ne.(bG)CS. Therefore J_1^* is Ne.(bG)OS.

Theorem 3.12.

If J_1^* is Ne.(bG)CS in $(\mathfrak{X}, \mathcal{N}_\tau)$ and $J_1^* \subseteq J_2^* \subseteq \text{Ne.bCl}(J_1^*)$, then J_2^* is Ne.(bG)CS in $(\mathfrak{X}, \mathcal{N}_\tau)$.

Proof.

Let J_3^* be Ne.(b)-OS in \mathfrak{X} such that $J_2^* \subseteq J_3^*$, then $J_1^* \subseteq J_3^*$. Since J_1^* is a Ne.(bG)CS in \mathfrak{X} , it follows that $\text{Ne.bCl}(J_1^*) \subseteq J_3^*$. Now $J_2^* \subseteq \text{Ne.bCl}(J_1^*)$ implies $\text{Ne.bCl}(J_2^*) \subseteq \text{Ne.bCl}(\text{Ne.bCl}(J_1^*)) = \text{Ne.bCl}(J_1^*)$. Thus $\text{Ne.bCl}(J_2^*) \subseteq J_3^*$. Hence J_2^* is Ne.(bG)CS in \mathfrak{X} .

Theorem 3.13.

If J_1^* is Ne.(bG)OS in $(\mathfrak{X}, \mathcal{N}_\tau)$ and $\text{Ne.blnt}(J_1^*) \subseteq J_2^* \subseteq J_1^*$ then J_2^* is Ne.(bG)-OS in $(\mathfrak{X}, \mathcal{N}_\tau)$.

Proof.

Let J_1^* be Ne.(bG)OS and J_2^* be any Ne. set in \mathfrak{X} such that $\text{Ne.blnt}(J_1^*) \subseteq J_2^* \subseteq J_1^*$. Then J_1^{*c} is Ne.(bG)CS and $J_1^{*c} \subseteq J_2^{*c} \subseteq \text{Ne.bCl}(J_1^{*c})$. Then J_2^{*c} is Ne.(bG)CS. Hence J_2^* is Ne.(bG)OS of \mathfrak{X} .

Theorem 3.14.

Finite intersection of Ne.(bG)CSs is a Ne.(bG)CS.

Proof.

Let J_1^* and J_2^* be Ne.(bG)CSs in \mathfrak{X} . Let $\mathfrak{F} \subseteq J_1^* \cap J_2^*$, where \mathfrak{F} is Ne.(b)CS in \mathfrak{X} . Then $\mathfrak{F} \subseteq J_1^*$ and $\mathfrak{F} \subseteq J_2^*$. Since J_1^* and J_2^* are Ne.(bG)CSs, $\mathfrak{F} \subseteq J_1^* = \text{Ne.blnt}(J_1^*)$ and $\mathfrak{F} \subseteq J_2^* = \text{Ne.bInt}(J_2^*)$, which implies $\mathfrak{F} \subseteq (\text{Ne.blnt}(J_1^*) \cap (\text{Ne.blnt}(J_2^*)))$. Hence $\mathfrak{F} \subseteq \text{Ne.bInt}(J_1^* \cap J_2^*)$. Therefore $J_1^* \cap J_2^*$ is Ne.(bG)CS in \mathfrak{X} .

Theorem 3.15.

A finite union of Ne.(bG)OS is a Ne.(bG)OS.

Proof.

Let J_1^* and J_2^* be Ne.(bG)OS in \mathfrak{X} . Let $J_1^* \cup J_2^* \subseteq \mathfrak{F}$, where \mathfrak{F} is Ne.(b)OS in \mathfrak{X} . Then $J_1^* \subseteq \mathfrak{F}$ or $J_2^* \subseteq \mathfrak{F}$. Since J_1^* and J_2^* are Ne.(bG)OS, $\text{Ne.bCl}(J_1^*) = J_1^* \subseteq \mathfrak{F}$ or $\text{Ne.bCl}(J_2^*) = J_2^* \subseteq \mathfrak{F}$, which implies $\text{Ne.bCl}(J_1^*) \cup \text{Ne.bCl}(J_2^*) \subseteq \mathfrak{F}$. Hence $\text{Ne.bCl}(J_1^* \cup J_2^*) \subseteq \mathfrak{F}$. Therefore $J_1^* \cup J_2^*$ is Ne.(bG)OS in \mathfrak{X} . However, union of two Ne.(bG)CSs need not be a Ne.(bG)CS as shown in the following example.

Example 3.16.

Let $\mathfrak{X} = \{j_1, j_2\}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T. on \mathfrak{X}

Where $J_1^* = \langle x, (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{2}{10}) \rangle$.

Then Neutrosophic set $J_1^* = \langle x, (\frac{1}{10}, \frac{5}{10}, \frac{9}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{2}{10}) \rangle$,

$J_2^* = \langle x, (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$ is a Ne.bGCSs but $J_1^* \cup J_2^*$ is not a Ne.bGCS in \mathfrak{X} ,

Theorem 3.17.

If J_1^* is Ne.(b)OS in $(\mathfrak{X}, \mathcal{N}_\tau)$ and Ne.(bG)CS, then J_1^* is Ne.(b)CS in $(\mathfrak{X}, \mathcal{N}_\tau)$.

Proof.

Let J_1^* be Ne.(b)OS and Ne.(bG)CS in \mathfrak{X} . For $J_1^* \subseteq J_1^*$, by definition $\text{Ne.bCl}(J_1^*) \subseteq J_1^*$.

But $J_1^* \subseteq \text{Ne.bCl}(J_1^*)$, which implies $J_1^* = \text{Ne.bCl}(J_1^*)$. Hence J_1^* is Ne.(b)CS in \mathfrak{X} .

Definition 3.18.

A NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is called a Neutrosophic bT_{1/2} space (in short Ne.(b)T^{*}_{1/2} space) if every Ne.(bG)CS in \mathfrak{X} is Ne.-CS.

Definition 3.19.

A NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is called a Neutrosophic T_{1/2} space (in short Ne.T_{1/2}space) if every Ne.(bG)CS in \mathfrak{X} is Ne.(b)CS.

Theorem 3.20.

A NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is Ne.(b)T_{1/2} space iff every Ne. set in $(\mathfrak{X}, \mathcal{N}_\tau)$ is both Ne.(b)OS and Ne.(bG)OS.

Proof.

Let \mathfrak{X} be Ne.(b)T_{1/2}space and let J_1^* be Ne.(bG)OS in \mathfrak{X} . Then J_1^{*c} is Ne.(bG)CS in \mathfrak{X} . By definition all Ne.(bG)CS in \mathfrak{X} is Ne.(b)CS, so J_1^{*c} is Ne.(b)CS and hence J_1^* is Ne.(b)OS in \mathfrak{X} .

Conversely, let J_1^* be Ne.(bG)CS. Then J_1^{*c} is Ne.(bG)OS which implies J_1^{*c} is Ne.(b)OS. Hence J_1^* is Ne.(b)CS. Every Ne.(bG)CS in \mathfrak{X} is Ne.(b)CS. Therefore \mathfrak{X} is Ne.(b)T_{1/2} space.

Theorem 3.21.

A NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is Ne.(b)T_{1/2} space iff every Ne. set in $(\mathfrak{X}, \mathcal{N}_\tau)$ is both Ne.OS and Ne.(bG)OS.

Remark 3.22.

A NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$ is

- (i) Ne.(b)T_{1/2}space if every Ne.(bG)OS in \mathfrak{X} is Ne.(b)OS.
- (ii) Ne.(b)T^{*}_{1/2}space if \forall Ne.(Gb)OS in \mathfrak{X} is Ne-open.

Remark 3.23.

In a NSTS $(\mathfrak{X}, \mathcal{N}_\tau)$

- (i) Every Ne.T_{1/2} space is Ne.(b)T_{1/2}
- (ii) Every Ne.(b)T_{1/2} space is Ne.(Gb)T_{1/2}
- (iii) Every Ne.(b)T_{1/2} space is Ne.(Gb)T_{1/2}

4. Ne.(bG)-Continuous and Ne.(Gb)-closed mappings

In this section, Neutrosophic bg-CTS maps, Neutrosophic bg-irresolute maps, and Neutrosophic bg-homeomorphism in Neutrosophic topological spaces are introduced and studied.

Definition 4.1.

A mapping $q: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is said to be Neutrosophic b generalized Continuous (Ne.(bG)-CTS), if $q^{-1}(J_1^*)$ is Ne.(bG)CS in \mathfrak{X} , for every Neutrosophic-CS J_1^* in \mathfrak{Y} .

Theorem 4.2.

$q: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(bG)-CTS iff the inverse image of each NSOS in \mathfrak{Y} is Ne.(bG)OS in \mathfrak{X} .

Proof.

Let J_2^* be a Ne.(bG)OS in \mathfrak{Y} . Then J_2^{*c} is Ne.(bG)CS in \mathfrak{Y} . Since q is Ne.(bG)-CTS $q^{-1}(J_2^{*c}) = (q^{-1}(J_2^*))^c$ is Ne.(bG)CS in \mathfrak{X} . Thus $q^{-1}(J_2^*)$ is Ne.(bG)OS in \mathfrak{X} .

Converse, is obvious.

Theorem 4.3.

Every Ne.-CTS map is Ne.(bG)-CTS.

Proof.

Let $q: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ be Ne.-CTS function. Let J_1^* be a Ne. OS in \mathfrak{Y} . Since q is Ne.-CTS, q^{-1} Ne. OS in \mathfrak{X} . Mean while each Ne.OS is Ne.(bG)OS, q^{-1} is Ne.(bG)OS in \mathfrak{X} . Therefore q is Ne.(bG)-CTS.

Example 4.4.

Let $\mathfrak{X} = \{j_1, j_2\} = \mathfrak{Y}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X} $\mathcal{N}_\sigma = \{0, J_2^*, 1\}$ on \mathfrak{Y} , then Then the Neutrosophic sets

$$J_1^* = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right) \rangle.$$

$$J_2^* = \langle x, \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right) \rangle \text{ is a Ne.bGCS in } \mathfrak{X}.$$

Identity mapping $q: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ q is Ne.(Gb)-CTS but not Ne.-CTS

Definition 4.5

A mapping $q: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is said to be Neutrosophic b-generalized irresolute (briefly Ne.(bG)-irresolute), if $q^{-1}(J_1^*)$ is Ne.(bG)CS set in \mathfrak{X} , for each Ne.(bG) CS J_1^* in \mathfrak{Y} .

Theorem 4.6.

Every Ne.(bG)-irresolute map is Ne.(bG)-CTS.

Proof.

Let $q: \mathfrak{X} \rightarrow \mathfrak{Y}$ be Ne.(bG)-irresolute and let J_1^* be Ne.-CS in \mathfrak{Y} . Since every Ne.-CS is Also Ne.(bG)CS, J_1^* is Ne.(bG)CS in \mathfrak{Y} . Since $q: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)-irresolute, $q^{-1}(J_1^*)$ is Ne.(bG)CS. Thus inverse image of every Ne.CS in \mathfrak{Y} is Ne.(bG)CS in \mathfrak{X} . Therefore the function $q: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)-CTS. The converse is not true.

Example 4.7.

Let $\mathfrak{X} = \{j_1, j_2\} = \mathfrak{Y}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X} $\mathcal{N}_\sigma = \{0, J_2^*, 1\}$ on \mathfrak{Y} , then

Then the Neutrosophic sets

$$J_1^* = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{6}{10}\right) \rangle, \text{ and } J_2^* = \langle x, \left(\frac{8}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{4}{10}, \frac{6}{10}, \frac{7}{10}\right) \rangle .$$

Then Identity mapping $q: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$

We have $J_3^* = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$ is a Ne.(bG)-CTS maps but not Ne.(bG)-irresolute maps.

Theorem 4.8.

Every Ne.(bG)-CTS map is Ne.(Gb)-CTS.

Proof.

Clear from the fact that Ne.(bG)CS is Ne.(Gb)CS.

Theorem 4.9.

Let $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$, $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ be two mappings. Then

- (i) $\zeta \circ \varrho$ is Ne.(bG)-CTS, if ϱ is Ne.(bG)-CTS and ζ is Ne.-CTS.
- (ii) $\zeta \circ \varrho$ is Ne.(bG)-irresolute, if ϱ and ζ are Ne.(bG)-irresolute.
- (iii) $\zeta \circ \varrho$ is Ne.(bG)-CTS if ϱ is Ne.(bG)-irresolute and ζ is Ne.(bG)-CTS.

Proof.

(i) Let J_2^* be Ne.CS in \mathfrak{Z} . Since $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ is Neutrosophic CTS, by definition $\zeta^{-1}(J_2^*)$ is Ne.CS of \mathfrak{Y} . Now $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)-CTS so $\varrho^{-1}(\zeta^{-1}(J_2^*)) = (\zeta \circ \varrho)^{-1}(J_2^*)$ is Ne.(bG)CS in \mathfrak{X} . Hence $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)-CTS.

(ii) Let $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ be Ne.(bG)-irresolute and let J_2^* be Ne.(bG)CS subset in \mathfrak{Z} . Since ζ is Ne.(bG)-irresolute by definition, $\zeta^{-1}(J_2^*)$ is Ne.(bG)CS in \mathfrak{Y} . Also $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)-irresolute, so $\varrho^{-1}(\zeta^{-1}(J_2^*)) = (\zeta \circ \varrho)^{-1}(J_2^*)$ is Ne.(bG)CS. Thus $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)-irresolute.

(iii) Let J_2^* be Ne.(b)-CS in \mathfrak{Z} . Since $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ is Ne.(bG)-CTS, $\zeta^{-1}(J_2^*)$ is Ne.(bG)CS in \mathfrak{Y} . Also $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)-irresolute, so every Ne.(bG)CS in \mathfrak{Y} is Ne.(bG)CS in \mathfrak{X} . Hence $\varrho^{-1}(\zeta^{-1}(J_2^*)) = (\zeta \circ \varrho)^{-1}(J_2^*)$ is Ne.(bG)CS in \mathfrak{X} . Thus $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)-irresolute.

Theorem 4.11.

If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(b)*-CTS and $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ is Ne.(bG)-CTS then $\zeta \circ \varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ is Ne.(bG)CTS if \mathfrak{Y} is Ne.(b) $T_{1/2}$ -space.

Proof.

Suppose J_1^* is Ne.(b)-CS subset of \mathfrak{Z} . Since $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ is Ne.(bG)CTS $\zeta^{-1}(J_1^*)$ is Ne.(bG)CS subset of \mathfrak{Y} . Now since \mathfrak{Y} is Ne.(b) $T_{1/2}$ -space, $\zeta^{-1}(J_1^*)$ is Ne.(b)-CS subset of \mathfrak{Y} . Also since $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(b)*-CTS $\varrho^{-1}(\zeta^{-1}(J_1^*)) = (\zeta \circ \varrho)^{-1}(J_1^*)$ is Ne.(b)-CS. Thus $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)-CTS.

Theorem 4.12.

Let $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ be Ne.(bG)-CTS. Then ϱ is Ne.(b)-CTS if \mathfrak{X} is Ne.(b) $T_{1/2}$ space.

Proof.

Let J_2^* be Ne.-CS in \mathfrak{Y} . Since $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)CTS, $\varrho^{-1}(J_2^*)$ is Ne.(bG)CS subset in \mathfrak{X} . Since \mathfrak{X} is Ne.(b) $T_{1/2}$ space, by hypothesis every Ne.(bG)CS is Ne.(b)-CS. Hence $\varrho^{-1}(J_2^*)$ is Ne.(b)CS subset in \mathfrak{X} . Therefore $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(b)-CTS.

Theorem 4.13.

Let $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ be onto Ne.(bG)-irresolute and Ne. b*CS. If \mathfrak{X} is Ne.(b) $T_{1/2}$ -space, then $(\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(b) $T_{1/2}$ -space.

Proof.

Let J_1^* be a Ne.(bG)CS in \mathfrak{Y} . Since $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)irresolute, $\varrho^{-1}(J_1^*)$ is Ne.(bG)CS in \mathfrak{X} . As \mathfrak{X} is Ne.(b) $T_{1/2}$ -space, $\varrho^{-1}(J_1^*)$ is Ne.(b)CS in \mathfrak{X} . Also $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne. b*CS, $\varrho(\varrho^{-1}(J_1^*))$ is Ne.(b)CS in \mathfrak{Y} . Since $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is onto, $\varrho(\varrho^{-1}(J_1^*)) = J_1^*$. Thus J_1^* is Ne.(b)CS in \mathfrak{Y} . Hence $(\mathfrak{Y}, \mathcal{N}_\sigma)$ is also Ne.(b) $T_{1/2}$ -space.

Theorem 4.14.

Let $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ be Ne.(bG)-CTS and $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ be Ne.g-CTS. Then $\zeta \circ \varrho$ is Ne.(bG)-CTS if \mathfrak{Y} is Ne. $T_{1/2}$ space.

Proof.

Let J_1^* be Ne.-CS in \mathfrak{X} . Since ζ is Ne.g-CTS, $\zeta^{-1}(J_1^*)$ is Ne.g-CS in \mathfrak{Y} . But \mathfrak{Y} is Ne. $T_{1/2}$ space and so $\zeta^{-1}(J_1^*)$ is Ne.-CS in \mathfrak{Y} . Since ϱ is Ne.(bG)-CTS $\varrho^{-1}(\zeta^{-1}(J_1^*)) = (\zeta \circ \varrho)^{-1}(J_1^*)$ is Ne.(bG)CS in \mathfrak{X} . Hence $\zeta \circ \varrho$ Ne.(bG)-CTS.

Theorem 4.15.

If the bijective map $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(b)*-open and Ne.(b)-irresolute, then $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(bG)-irresolute.

Proof.

Let J_1^* be a Ne.(bG)CS in \mathfrak{Y} and let $\varrho^{-1}(J_1^*) \subseteq J_2^*$ where J_2^* is a Ne.(b)OS in \mathfrak{X} . Clearly, $J_1^* \subseteq \varrho(J_2^*)$. Since $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(b)*-open map, $\varrho(J_2^*)$ is Ne.(b)-open in \mathfrak{Y} and J_1^* is Ne.(bG)CS in \mathfrak{Y} . Then $\text{Ne.bCl}(J_1^*) \subseteq \varrho(J_2^*)$, and thus $\varrho^{-1}(\text{Ne.bCl}(J_1^*)) \subseteq J_2^*$. Also $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ irresolute and $\text{Ne.bCl}(J_1^*)$ is a Ne.(b)-CS in \mathfrak{Y} , then $\varrho^{-1}(\text{Ne.bCl}(J_1^*))$ is Ne.(b)CS in \mathfrak{X} . Thus $\text{Ne.bCl}(\varrho^{-1}(J_1^*)) \subseteq \text{Ne.bCl}(\varrho^{-1}(\text{Ne.bCl}(J_1^*))) \subseteq J_2^*$. So $\varrho^{-1}(J_1^*)$ is Ne.(bG)CS in \mathfrak{X} . Hence $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)-irresolute map.

Definition 4.16.

A mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is said to be Neutrosophic bg-open (briefly Ne.(bG)OS) if the image of every Ne.-OS in \mathfrak{X} , is Ne.(bG)OS in \mathfrak{Y} .

Definition 4.17.

A mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is said to be Neutrosophic bg-CS (briefly Ne.(bG)CS) if the image of every Ne.CS in \mathfrak{X} is Ne.(bG)CS in \mathfrak{Y} .

Definition 4.18.

A mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is said to be Neutrosophic bg*-open (briefly Ne.(bG)*-OS) if the image of every Ne.(bG)OS in \mathfrak{X} is Ne.(bG)OS in \mathfrak{Y} .

Definition 4.19.

A mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is said to be Neutrosophic bg-CS (briefly Ne.(bG)*-CS) if the image of every Ne.(bG)CS in \mathfrak{X} is Ne.(bG)CS in \mathfrak{Y} .

Remark 4.20.

- (i) Every Ne.(bG)*-CS mapping is Ne.(bG)CS.
- (ii) Every Ne.(bG)*-CS mapping is Ne.(Gb)*-CS.

Theorem 4.23.

If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.CS and $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ is Ne.(bG)CS, then $\zeta \circ \varrho$ is Ne.(bG)CS.

Proof.

Let J_1^* be a Ne.CS in \mathfrak{X} . Then $\varrho(J_1^*)$ is Ne.CS in \mathfrak{Y} . Since $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ is Ne.(bG)CS, $\zeta(\varrho(J_1^*)) = (\zeta \circ \varrho)(J_1^*)$ is Ne.(bG)CS in \mathfrak{Z} . Therefore $\zeta \circ \varrho$ is Ne.(bG)CS.

Theorem 4.24.

If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is a Ne.(bG)CS map and \mathfrak{Y} is Ne.(b) $T_{1/2}$ space, then ϱ is a Ne.-CS.

Proof.

Let J_1^* be a Ne.CS in \mathfrak{X} . Then $\varrho(J_1^*)$ is Ne.(Gb)-CS in \mathfrak{Y} , since ϱ is Ne.(Gb)CS. Again since \mathfrak{Y} is Ne.(b) $T_{1/2}$ space, $\varrho(J_1^*)$ is Ne.-CS in \mathfrak{Y} . Hence $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is a ϱ -CS.

Theorem 4.25.

If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is a Ne.(bG)CS map and \mathfrak{Y} is Ne.(b) $T_{1/2}$ space, then ϱ is a Ne.(b)-CS map.

Theorem 4.26.

A mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(bG)CS iff for each Ne. set J_1^* in \mathfrak{Y} and Ne.OS J_2^* such that $\varrho^{-1}(J_1^*) \subseteq J_2^*$, there is a Ne.(bG)OS J_3^* of \mathfrak{Y} such that $J_1^* \subseteq J_3^*$ and $\varrho^{-1}(J_3^*) \subseteq J_2^*$.

Proof.

Suppose ϱ is Ne.(bG)CS map. Let J_1^* be a Ne. set of \mathfrak{X} , and J_2^* be an Ne.OS of \mathfrak{X} , such that $\varrho^{-1}(J_1^*) \subseteq J_2^*$. Then $J_3^* = (J_2^{*c})^c$ is a Ne.(bG)OS in \mathfrak{X} such that $J_1^* \subseteq J_3^*$ and $\varrho^{-1}(J_3^*) \subseteq J_2^*$.

Conversely, suppose that \mathfrak{X} is a Ne.CS of \mathfrak{X} . Then $\varrho^{-1}((\varrho(\mathfrak{X}))^c) \subseteq \mathfrak{X}^c$, and \mathfrak{X}^c is Ne.OS. By hypothesis, there is a Ne.(bG)OS J_3^* of \mathfrak{Y} such that $(\varrho(\mathfrak{X}))^c \subseteq J_3^*$ and $\varrho^{-1}(J_3^*) \subseteq \mathfrak{X}^c$. Therefore

$$\mathfrak{X} \subseteq \left(\varrho^{-1}(J_3^*) \right)^c \text{ Hence } J_3^{*c} \subseteq \varrho(J_3^*) \subseteq \varrho \left(\varrho^{-1}(J_3^*) \right)^c \subseteq J_3^{*c}, \text{ which implies } \varrho(\mathfrak{X}) = J_3^{*c}.$$

Since J_3^{*c} is Ne.(bG)CS, $\varrho(\mathfrak{X})$ is Ne.(bG)CS and thus ϱ is a Ne.(bG)CS map.

Theorem 4.27.

If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ and $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ are Ne.(bG)CS maps and \mathfrak{Y} is Ne.(b)T_{1/2} space, then $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)CS.

Proof. Let J_1^* be a Ne.-CS in \mathfrak{X} . Since $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(bG)CS, $\varrho(J_1^*)$ is Ne.(bG)CS in \mathfrak{Y} . Now \mathfrak{Y} is Ne.(b)T_{1/2}space, so $\varrho(J_1^*)$ is Ne.-CS in \mathfrak{Y} . Also $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ is Ne.(bG)CS, $\zeta(\varrho(J_1^*)) = (\zeta \circ \varrho)(J_1^*)$ is Ne.(bG)CS in \mathfrak{Z} . Therefore $\zeta \circ \varrho$ is Ne.(bG)CS.

Theorem 4.28. If J_1^* is Ne.(bG)CS in \mathfrak{X} and $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is bijective, Ne.(b)-irresolute and Ne.(bG)CS, then $\varrho(J_1^*)$ is Ne.(bG)CS in \mathfrak{Y} .

Proof.

Let $\varrho(J_1^*) \subseteq J_2^*$ where J_2^* is Ne.(b)OS in \mathfrak{Y} . Since ϱ is Ne.(b)irresolute, $\varrho^{-1}(J_2^*)$ is Ne.(b)OS containing J_1^* . Hence $\text{Ne.bCl}(\varrho(J_1^*)) \subseteq \varrho^{-1}(J_2^*)$ as J_1^* is Ne.(bG)CS. Since ϱ is Ne.(bG)CS, $\varrho(\text{Ne.bCl}(\varrho(J_1^*)))$ is Ne.(bG)CS contained in the Ne.(b)OS J_2^* , which implies $\text{Ne.bCl}(\varrho(\text{Ne.bCl}(\varrho(J_1^*)))) \subseteq J_2^*$ and hence $\text{Ne.bCl}(\varrho(J_1^*)) \subseteq J_2^*$. So $\varrho(J_1^*)$ is Ne.(bG)CS in \mathfrak{Y} .

Theorem 4.29.

If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(bG)CS and $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ is Ne.(bG)*-CS, then $\zeta \circ \varrho$ is Ne.(bG)-CS.

Proof. Let J_1^* be Ne.CS in \mathfrak{X} . Then $\varrho(J_1^*)$ is Ne.(bG)CS in \mathfrak{Y} . Since $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ is Ne.(bG)*-CS. Thus $\zeta(\varrho(J_1^*)) = (\zeta \circ \varrho)(J_1^*)$ is Ne.(bG)CS in \mathfrak{Z} . Therefore $\zeta \circ \varrho$ is Ne.(bG)CS. If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ and $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ are Ne.(bG)*CS maps, then $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)*CS.

Theorem 4.30.

Let $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$, $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ be two maps such that $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)CS.

- (i) If ϱ is Ne.-CTS and surjective, then ζ is Ne.(bG)CS.
- (ii) If ζ is Ne.(bG)-irresolute and injective, then ϱ is Ne.(bG)CS.

Proof.

(i). Let \mathfrak{X} be Ne.CS in \mathfrak{Y} . Then $\varrho^{-1}(\mathfrak{X})$ is Ne.CS in \mathfrak{X} , as ϱ is Ne.-CTS. Since $\zeta \circ \varrho$ is Ne.(bG)CS map and ϱ is surjective, $(\zeta \circ \varrho)(\varrho^{-1}(\mathfrak{X})) = \zeta(\mathfrak{X})$ is Ne.(bG)CS in \mathfrak{Z} . Hence $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ is Ne.(bG)CS.

(ii). Let \mathfrak{X} be a Ne. CS in \mathfrak{X} . Then $(\zeta \circ \varrho)(\mathfrak{X})$ is Ne.(bG)CS in \mathfrak{Z} . Since ζ is Ne.(bG)-irresolute and injective $\zeta^{-1}(\zeta \circ \varrho)(\mathfrak{X}) = \varrho(\mathfrak{X})$ is Ne.(bG)CS in \mathfrak{Y} . Hence ϱ is a Ne.(bG)CS.

Theorem 4.31.

Let $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$, $\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ be two maps such that $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)*CS map.

- (i) If ϱ is Ne.(bG)-CTS and surjective, then ζ is Ne.(bG)CS.
- (ii) If ζ is Ne.(bG)-irresolute and injective, then ϱ is Ne.(bG)*-CS.

Theorem 4.32. Let $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ then the following statements are equivalent

- (i) ϱ is Ne.(bG)-irresolute.
- (ii) for every Ne.(bG)CS J_1^* in \mathfrak{Y} , $\varrho^{-1}(J_1^*)$ is Ne.(bG)CS in \mathfrak{X} .

Proof.(i)⇒ (ii)Obvious.

(ii)⇒(i) Let J_1^* is a Ne.(bG)CS in \mathfrak{Y} which implies J_1^{*c} , is Ne.(bG)OS in \mathfrak{Y} . $\varrho^{-1}(J_1^{*c})$ is Ne.(bG)-open in \mathfrak{X} implies $\varrho^{-1}(J_1^*)$ is Ne.(bG)CS in \mathfrak{X} . Hence ϱ is Ne.(bG)-irresolute.

Neutrosophic bg- homeomorphism and Neutrosophic bg*-homeomorphism are defined as follows.

Definition 4.33.

A mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is called Neutrosophic bg-homeomorphism (briefly Ne.(b)-homeomorphism) if ϱ and ϱ^{-1} are Ne.(bG)CTS.

Definition 4.34.

A mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is called Neutrosophic bg*-homeomorphism (briefly Ne.(bG)*-homeomorphism) if ϱ and ϱ^{-1} are Ne.(bG)irresolute.

Theorem 4.35.

Every Ne.-homeomorphism is Ne.(bG)-homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

Example 4.36.

Let $\mathfrak{X} = \{j_1, j_2\} = \mathfrak{Y}$, $\mathcal{N}_\tau = \{0, J_1^*, 1\}$, is a N.T.on \mathfrak{X} $\mathcal{N}_\sigma = \{0, J_2^*, 1\}$ on \mathfrak{Y} ,

Then Neutrosophic sets

$$J_1^* = \langle x, \left(\frac{10}{10}, \frac{5}{10}, \frac{0}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right) \rangle,$$

$$J_2^* = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{6}{10}, \frac{6}{10}, \frac{4}{10}\right) \rangle \text{ and } J_3^* = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right) \rangle$$

Define mapping $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ by $\varrho(j_1) = j_1$ and $\varrho(j_2) = j_2$

Then ϱ is Ne.(bG)-homeomorphism but not Ne.-homeomorphism

Theorem 4.37.

Every Ne.(bG)*-homeomorphism is Ne.(bG)- homeomorphism.

Proof.

Let $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ be Ne.(Gb)*-homeomorphism. Then ϱ and ϱ^{-1} are Ne.(bG)-irresolute mappings. By theorem 4.7 ϱ and ϱ^{-1} are Ne.(bG)-CTS. Hence $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(bG)-homeomorphism.

Theorem 4.38.

If $\varrho: (\mathfrak{X}, \mathcal{N}_\tau) \rightarrow (\mathfrak{Y}, \mathcal{N}_\sigma)$ is Ne.(bG)-homeomorphism and

$\zeta: (\mathfrak{Y}, \mathcal{N}_\sigma) \rightarrow (\mathfrak{Z}, \mathcal{N}_\delta)$ is Ne.(bG)-homeomorphism and \mathfrak{Y} is Ne.(b)T_{1/2} space,

then $\zeta \circ \varrho: \mathfrak{X} \rightarrow \mathfrak{Z}$ is Ne.(bG)-homeomorphism.

Proof.

To show that $\zeta \circ \varrho$ and $(\zeta \circ \varrho)^{-1}$ are Ne.(bG)- CTS. Let J_1^* be a Ne.OS in \mathfrak{Z} . Since $\zeta: \mathfrak{Y} \rightarrow \mathfrak{Z}$ is Ne.(bG)- CTS, $\zeta^{-1}(J_1^*)$ is Ne.(bG)open in \mathfrak{Y} . Then $\zeta^{-1}(J_1^*)$ is a Ne.-open in \mathfrak{Y} as \mathfrak{Y} is Ne.(b)T_{1/2} space. Also since $\varrho: \mathfrak{X} \rightarrow \mathfrak{Y}$ is Ne.(bG)- CTS, $\varrho^{-1}(\zeta^{-1}(J_1^*)) = (\zeta \circ \varrho)^{-1}(J_1^*)$ is Ne.(bG)-open in \mathfrak{X} . Therefore $\zeta \circ \varrho$ is Ne.(bG) CTS. Again, let J_1^* be a Ne.OS in \mathfrak{X} . Since $\varrho^{-1}: \mathfrak{Y} \rightarrow \mathfrak{X}$ is Ne.(bG)- CTS, $(\varrho^{-1})^{-1}(J_1^*) = \varrho(J_1^*)$ is Ne.(bG)OS in \mathfrak{Y} . And so $\varrho(J_1^*)$ is Ne.-open in \mathfrak{Y} since \mathfrak{Y} is Ne.(b)T_{1/2} space. Also since $\zeta^{-1}: \mathfrak{Z} \rightarrow \mathfrak{Y}$ is Ne.(bG)-CTS, $(\zeta^{-1})^{-1}(\varrho(J_1^*)) = \zeta(\varrho(J_1^*)) = (\zeta \circ \varrho)(J_1^*)$ is Ne.(bG)-open in \mathfrak{Z} . Therefore $((\zeta \circ \varrho)^{-1})^{-1}(J_1^*) = (\zeta \circ \varrho)(J_1^*)$ is Ne.(bG)OS in \mathfrak{Z} . Hence $(\zeta \circ \varrho)^{-1}$ is Ne.(bG) - CTS. Thus $\zeta \circ \varrho$ is Ne.(bG) - homeomorphism.

Funding: This research received no external funding.

Acknowledgments: The authors are highly grateful to the Referees for their constructive suggestions.

Conflicts of Interest: The authors declare no conflict of interest

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Received: May 6, 2020. Accepted: September 20, 2020