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SPECTRUM OF TWO-DIMENSIONAL DYNAMICAL SYSTEMS WITH COMPLETELY POSITIVE ENTROPY

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One of interesting classes of two - dimensional dynamical systems are the systems with completely positive entropy. Some properties of these systems have been discovered by J.P. Conze in [1]. One of the most interesting properties of these systems is their spectrum.

Conze has shown that if (X,μ, G) is a system in which G is generated by a pair of commuting automorphisms being K - system then (X,μ, G) has a countable infinite Lebesgue spectrum.

Recently we have shown in [2] that any system (X,μ, G) with completely positive entropy has a countable infinite Lebesgue spectrum.

In order to obtain our result we shall make use of some results appearing in [1] and [2] .

Let $\{T, S\}$ be an ordered pair of generators of G.

Lemma 1 [1] . There exists a measurable partition ζ of X such that

(1) $T^k S^l \zeta \leq \zeta$, where $(k, l) \prec (0, 0)$,

(2) $\bigvee_{(k,l)\in Z^2} T^k S^l \zeta = \varepsilon$,

(3) $H(\zeta \mid \zeta^-_{T,S}) = H(\zeta \mid S^{-1}\zeta) = h(G)$.

Let σ be a measurable T - invariant partition of X and let $\bar{\sigma}$ be a Pinsker closure of σ .

Lemma 2 [2] . There exists a measurable partition ζ of X such that

(4) $\sigma \leq T^{-1}\zeta \leq \zeta$,

(5) $\bigvee_{n=0}^{\infty} T^n \zeta = \varepsilon$,

(6) $\bigwedge_{n=0}^{\infty} T^{-n}\zeta = \bar{\sigma}$.

Using these lemmas and the separability of $L^2(X,\mu)$ one can construct a countable transfinite sequence $\{\sigma_\alpha\}_{\alpha<\alpha_0}$ of measurable partitions of X with properties :

(7) $\sigma_0 = \varepsilon$,

(8) $\sigma_{\alpha+1} \lneq \sigma_\alpha$,

(9) $T\sigma_\alpha = S\sigma_\alpha = \sigma_\alpha$,

(10) if α is a limit ordinal number then $\sigma_\alpha = \bigwedge_{\gamma<\alpha} \sigma_\gamma$,

(11) G has countable Lebesgue spectrum in $L^2(\sigma_\alpha) \ominus L^2(\sigma_{\alpha+1})$, $\alpha<\alpha_0$

(12) $L^2(X,\mu) \ominus \mathbb{1} = \bigoplus_{\alpha<\alpha_0} \left(L^2(\sigma_\alpha) \ominus L^2(\sigma_{\alpha+1}) \right)$.

Two last properties imply the desired result.

Now, let us suppose $h(G) < \infty$.

Conze and independently Katznelson and Weiss have proved

<u>Lemma 3</u> ([1] , [3]) . If G is aperiodic and $h(G) < \infty$ then there exists a measurable partition ξ of X such that $H(\xi) < \infty$ and $\bigvee_{(k,l)\in\mathbb{Z}^2} T^k S^l \xi = \varepsilon$.

Using this result one can prove that for G having completely positive and finite entropy we have $\sigma_1 = \nu$

The last property means that an arbitrary ordered pair $\{T, S\}$ of generators of such groups is a K - system in the sense of Conze.

It would be very interesting to explain if the assumption $h(G) < \infty$ is essential .

References

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