ACTIVITIES PLANNING AND RESOURCES ASSIGNMENT ON DISTINCT PLACES: A MATHEMATICAL MODEL

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Abstract. In France, the Hospital Community of Territory has been defined since the settlement of the pricing by activity (T2A) in 2004, and the new hospital governance. This new community allows the pooling of the hospital's human and material resources of any place in the same territory. It aims at increasing the continuity of health care. A Hospital Community of Territory is made up of several distinct places, material and human resources. A medical exam needs one human resource and one material resource, both of them compatible with the exam. The objective is to create a decision aid tool which will plan the exams with the assignment of the human resources and the material resources. In this paper, we propose a mathematical model which is tested with randomly generated instances. It proposes exams planning taking into account the assignment of material and human resources in the community over a given horizon planning.

Keywords. Hospital Community of Territory, exams planning, resource assignment, resource pooling, Integer Programming Modeling.

Mathematics Subject Classification. 90B50.

1. Introduction

Nowadays, the main goal of a hospital system is to improve its efficiency. The two main objectives are, on one hand, to improve the hospital's financial asset, and on the other hand to maximize the quality of service.

There are inequalities between several places through a same district. Indeed, some hospitals can have patients waiting a few months or more, to obtain an appointment. Other hospitals have almost no waiting time. These two kinds of hospitals can be very close to one other geographically speaking. The idea is that by moving or pooling some resources, the load of these two kinds of hospitals could be balanced. The resource pooling approach seems to be one of the best solutions to increase hospital efficiency.

In France, some rules and laws are often created in order to improve hospital organization. One of the recent and very important ones was the settlement of the pricing by activity (T2A). The result is the creation of the Hospital Community of Territory (HCT) in 2008. The HCT is a group of several hospitals which reach an agreement to achieve common goals. A new organization in the health care sector is the guarantee of efficient budget management and of good health care quality preserving the proximity support.

One of the objectives of this HCT is proximity, saving the patient from having to undertake frequent trips to be treated or examined.

A lot of HCT have already been created in France. In 2010, there were twenty-nine HCT. Some administrative departments can have up to four HCT. There are one hundred and one departments in France. We can therefore assume that there are at least one hundred HCT.

These communities are new. They need a decision aid tool to anticipate and foresee their establishment and running. In this paper, we will discuss the field of medical imaging, where patients are examined for example by an MRI, an X-ray, or a scanner. All the hospitals do not have the same material resources so a patient may have to go to another hospital, one that he may not be used to. Nowadays, more and more medical exams are prescribed, so we need to think about the organization of the HCT applied to the medical imaging field.

In this article, we propose a mathematical model which deals with this issue. The objective is to build exams planning with a material and human resources assignment through the HCT. The first part describes the problem by defining the physical, logical and decisional subsystems. The second part is the state of the art about operational research for managing health care and theoretical models to solve problems about manufacturing system. With our model, we can tackle several problems which are described in the third part with the mathematical model. Section four presents our experiments and results. The conclusion and further research are given in section five.

2. Problem description

The ASDI methodology (Analysis, Specification, Design, Implementation) has been developed for the design and implementation of simulation environments, that can be applied to both a system that already exists and to one that still has to be conceived [18]. To analyze a complex system, ASDI advises to separate the

system into three subsystems: the physical subsystem, the logical subsystem and the decisional subsystem. Each subsystem interacts with the two others [7].

The physical subsystem is composed of the physical entities used to realize the set of activities: the resources, their geographical distribution and their interconnections. The logical subsystem consists of the flows the system should process, all activities concerning the processing of these flows, and the entities in the system which are related. The decisional subsystem is the management center, which contains the rules about management, resource assignment and system running.

2.1. Physical subsystem

The HCT consists of several places. There is a known distance between these places. In each place, there is one or many material resources. A material resource is from one type (MRI, X-ray, scanner). Each material resource has a planning with its available hours per period. For example, we can assume that an MRI engine is only available five hours on Monday because it needs a maintenance operation or because an external physician reserved it.

Human resources compose a medical crew. The composition of this crew depends on the considered exam. This crew can have a specific number of technicians, radiologists, stretcher bearers, specialists depending on the examined body part, *etc.* In the following, we assume that the medical crew is composed of only one human resource. The human resources belong to a given hospital, the place where they are employed, but they can work at the other places within the same HCT. Thus, they can be asked to move between the different places. For instance, a human resource can work at place A on Monday morning, then at place B on Monday afternoon, at place B again on Thursday morning and then again at place A on Thursday afternoon. A time is given to the human resources to travel from one place to another. The human resources can use one type of material resource or several depending on their skills. Each human resource has a schedule defining his regulatory work time, along with breaks and vacations.

2.2. Logical subsystem

By definition, an exam needs a human resource and a material resource. Both of them have to match. The material resource has to be from the needed type to perform the assigned exam and the human resource must own the skills to use this needed type of material resource. An exam should be completed before a due date. Each exam has a reference place where the exam has to be done, if possible. Each exam has a known process time. This process time could depend on the assigned human resource and the used material resource. In our case, the process time is supposed to be constant, independent of the chosen human and material resources. Each exam starts in one period and finishes in the same period.

2.3. Decisional subsystem

The considered problem in this paper is to design a model which, from a set of exams, builds a schedule matching the triplet {period, human resource, material resource} to an exam. A planning is the assignment of one human resource, one material resource and one period to each exam. We are in a predictive approach, all the exams can be processed from the beginning of the planning horizon. The aim of this model is not to make a precise schedule of the exams but to attribute a period to each exam. This planning has to respect some constraints and optimize criteria. In the following, the decision levels are defined. Then the criteria to optimize, the constraints to respect and the considered problems are defined.

2.3.1. *Levels*

To identify the different problems that can be met in manufacturing systems, three decision levels have been defined [1, 16]. The three levels depend on the planning horizon used to apply the decisions. The considered model described in this paper can be used within these three levels. The objectives for each level are not the same.

- In the long term, the used level is the strategic one. The main objective is to size the system, that is to say: is it useful to create an HCT with the concerned places, does the concerned hospital need to join the HCT, is it justified to buy other material resources, etc.
- In the mid-term, the second level is the tactical one. The objective is the sizing of the resources, that is to say to determine the number of resources needed in each place, and the type of the material resources or the skills of the human resources.
- In the short term, we are considering the operational level. It concerns the assignment and/or planning problems. The objectives are to plan the exams and assign them to one human resource, one material resource and one period, and to determine the planning of each resource.

The three levels differ from each other by the length of the planning horizon (respectively several years, several months and one week) and the size of the period (respectively one month, one week and one half-day).

2.3.2. *Criteria*

The criteria may not be the same depending on the used level. Generally, the criteria which are the most relevant are the following.

Concerning the patient's comfort, the criterion is the number of exams carried out at their reference place. If an exam could not be assigned at its reference place, another criterion could be the distance between the reference place and the effective one for each exam.

Concerning the hospital criteria, the number of exams done before their due date has to be maximized. In another case, we would have taken the just-in-time

as a criterion. In our problem, we prefer to assign the exams before their due date so if the considered patient needs other exams or surgery, the next act could be done on time.

Regarding the economic aspect, the criteria are about the costs. One of the criteria is the number of exams planned during the planning horizon. If an exam is not planned during the current planning horizon, it will be reconsidered during the next one. Another one is the occupancy rate of each place, each material resource, and each human resource.

All the above mentioned criteria can be optimized in each level. At the operational level, two additional criteria will be studied: the number of moves of the human resources within the HCT and the number of overtime hours of the human resources.

2.3.3. Constraints

Constraints can be divided into two categories: hard constraints and soft constraints. Hard constraints have to be respected while soft constraints can be broken.

The hard constraints are:

- The open time of the material resources: an exam can only be assigned to a material resource while this resource is available.
- The human resource skills and the material resource compatibility: the material resource has to be able to process the considered exam, and the human resource has to be able to work on this material resource.
- When an exam is planned on a material resource with a human resource during a period, the considered human resource has to be assigned during this period to the place where the material resource is located.
- If a human resource is allowed to move during the planning horizon, his moves are constrained: a human resource can only work at one place over one period.

The following constraints are soft constraints:

- The exams should be assigned before their due dates.
- The exams should be planned over their reference places.
- Human resources should work during their predefined schedule. Otherwise, additional time will be considered as overtime hours. Overtime hours are limited in time.

2.4. Considered problems

From this description, the main problem has been broken down into five problems of increasing difficulty. The objective for each of them is to write the mathematical formalization, and to solve it. Each problem takes into account some of the above described points.

- Problem 1 is the most basic one. We suppose that the human resources are always available. Only the material resources are considered.

TABLE 1. Problem characteristics.

Problem	1	2	3	4	5
Period and material resource assignment to the exams	To determine	To determine	To determine	To determine	To determine
Human resources assignment to the places		Imposed	Imposed	To determine	To determine
Overtime hours of the human resources			To determine		To determine
Level	S&T	S&T	O	S&O	O

- Human resources are considered in problem 2 and in the following problems. In problem 2, they are able to work on one or several types of material resources. The human resources cannot move, they are always working at the same place, and they cannot work overtime.
- In problem 3, the human resources can work overtime. But they can only work at the place which employs them.
- In problem 4, human resources cannot work overtime, but they can work at several places. They can move from one place to another.
- In problem 5, human resources can work overtime and they can work at several places.

Table 1 summarizes the five problems and the characteristics of the HCT taken into account. An empty box means that the case is not studied in the considered problem. The last line of the table defines in which level the considered problem can be solved (S: Strategic, T: Tactical, O: Operational).

3. OPERATIONAL RESEARCH FOR MANAGING HEALTH CARE

Over the last ten years, many operational research methods have been used to solve health care problems. As examples, we can cite [14] which deals with the management of an elective surgery waiting system by using a simulation model. Mixed Integer Programming is solved in an exact way in [23] to treat the surgery case applied to a British Columbia health authority and in [15] to address the problem of planning the patient flow in hospitals subject to scarce medical resources. [12] solves an MIP thanks to heuristics to tackle the optimization of surgery sequencing and scheduling decisions under uncertainty. [6,8] use the MIP in an exact way and with heuristics to solve respectively the operating theater planning problem and the capacity planning decisions that allocate surgical specialties to operating-room days. [21] uses a Monte-Carlo simulation with a MIP and metaheuristics to solve a stochastic surgery planning problem.

So, planning and scheduling procedures are necessary to improve this field. A literature review has recently been done [5] in the field of operating rooms. [3]

makes out a literature review about scheduling of human resources. Many theoretical models have been developed to solve problems about manufacturing system (FS, FSH, RCPSP, FMS, *etc.*). Frequently, these models are used as a base to treat the health care problems.

In medical imaging, an exam is processed by human resources and a material resource. A hospital needs to plan the exams which will occur at its place by assigning to them the concerned resources. The HCT needs the same kind of planning as one hospital, but to improve the organization of this HCT, pooling and sharing the resources between these hospitals or moving the patients are envisaged. The assignment of the exams to the places is implicit because one material resource is assigned to one exam and a material resource belongs to a specific place. A period is also assigned to each exam.

Our problem is compared to a Resource Constrained Project Scheduling Problem (RCPSP) where a set of resources is assigned to activities. A state of the art about RCPSP can be found in [20, 25]. In our case, exams can be seen as activities and human and material resources as two sets of resources. RCPSP is NP-hard in the strong sense [4]. Today, RCPSP and its extensions are still studied [26]. RCPSP is usually studied in one place problem.

Multi-mode RCPSP considers activities which can be assigned to resources with different modes [24]. An activity can be made by different combinations of resources. Depending on this combination, the process time to execute this activity will be different. In the hospital field, an exam does not need the same process time depending on the human resource's experience or his specialty. If a human resource is working on one specific material resource, we can imagine that he will work better on it than a human resource who can work on several material resources. In the same way, an exam does not need the same process time if it is made by a new material resource or an old one. To be more specific, our problem can be seen as a multi-skills RCPSP, and not only a multi-mode RCPSP [2]. An activity can be made depending on different modes, these modes depending on the skills of the resources.

In [10, 11], authors consider that an activity needs several resources. Each resource can be chosen in a set of resources. In the hospital field, two big sets can be made if we just separate the human resources from the material ones. More sets can be defined if the resources are grouped by their capacity to work with other resources, for instance a human resource who could work on an MRI. These groups are not necessarily disjointed. If one group defines the capacity of the human resources to work on one type of material resources, a human resource can be part of two of these groups if he can work on two types of material resources.

Another aspect which could be considered is the individualization of the resources [17]. For instance, the resources are differentiated by their work time for the human ones, or by their open time for the material ones. Moreover, if a human resource does not want to work on a specific material resource, some incompatibilities can be defined. In a more general way, we could define in this way the capacity of each human resource to work on one or more material resources.

In the following, we propose a formalization of our problem. This problem is new and difficult. It is new because it is based on the RCPSP but with the combination of other characteristics: multi-places and multi-skills with resources' individualization. It is difficult because it is based on the RCPSP which is NP-hard. Moreover, the size of the real problem is significant. For instance, for an HCT made by two places composed of ten material resources during one month, five thousands exams have to be planned.

4. Mathematical model

The considered data, variables, constraints and criteria are described in this section.

4.1. Data

The data used in each problem are the following. Let be:

N the set of exams to plan in the HCT during the considered planning horizon.

K the set of distinct places in the HCT.

L the set of material resources available in the HCT.

A the set of types of material resource.

M the set of human resources.

T the set of periods.

t_i the process time of exam $i \in N$.

tt the time for a human resource to go from one place to another.

d_i the due date of exam $i \in N$.

$$r_{i,k} = \begin{cases} 1 \text{ if place } k \in K \text{ is the reference place of exam } i \in N, \\ 0 \text{ otherwise.} \end{cases}$$

$$b_{m,a} = \begin{cases} 1 \text{ if human resource } m \in M \text{ can work on type } a \in A \text{ of material} \\ \quad \text{resource,} \\ 0 \text{ otherwise.} \end{cases}$$

$$c_{i,a} = \begin{cases} 1 \text{ if exam } i \in N \text{ needs type } a \in A \text{ of material resource,} \\ 0 \text{ otherwise.} \end{cases}$$

$$c_{i,l} = \begin{cases} 1 \text{ if exam } i \in N \text{ can be handled by material resource } l \in L, \\ 0 \text{ otherwise.} \end{cases}$$

$$d_{l,a} = \begin{cases} 1 \text{ if material resource } l \in L \text{ is from type } a \in A, \\ 0 \text{ otherwise.} \end{cases}$$

$$s_{k,l} = \begin{cases} 1 \text{ if material resource } l \in L \text{ is located on place } k \in K, \\ 0 \text{ otherwise.} \end{cases}$$

$$v_{m,k,s} = \begin{cases} 1 \text{ if resource } m \in M \text{ is working at place } k \in K \text{ during period } s \in T, \\ 0 \text{ otherwise.} \end{cases}$$

$$v_{m,k,0} = \begin{cases} 1 \text{ if human resource } m \in M \text{ is employed by place } k \in K, \\ 0 \text{ otherwise.} \end{cases}$$

$p_{l,s}$ the open time of material resource $l \in L$ during period $s \in T$.

$q_{m,s}$ the work time of human resource $m \in M$ during period $s \in T$.

H_{max} the total maximum time a human resource can work over a period.

In Table 2, data are related to the considered problems. An empty box means that the data is not used.

4.2. VARIABLES

In problem 1, the considered variable will be the assignment of one exam to one material resource during one period.

$$x_{i,l,s} = \begin{cases} 1 \text{ if exam } i \in N \text{ is assigned to material resource } l \in L \text{ during} \\ \quad\ \text{period } s \in T, \\ 0 \text{ otherwise.} \end{cases}$$

In problem 2, the variable will be the assignment of one material resource and one human resource to one exam during one period.

$$x_{i,l,s,m} = \begin{cases} 1 \text{ if exam } i \in N \text{ is assigned to material resource } l \in L \\ \quad\ \text{and human resource } m \in M \text{ during period } s \in T, \\ 0 \text{ otherwise.} \end{cases}$$

The variable which counts overtime hours per human resource and per period will be used in problems 3 and 5 where overtime hours are allowed.

$z_{m,s}$ computes the number of overtime hours of human resource $m \in M$ over period $s \in T$.

In problems 4 and 5, where human resources can move from one place to another, a variable assigns one human resource to one place during one period.

$$y_{m,k,s} = \begin{cases} 1 \text{ if human resource } m \in M \text{ is working at place } k \in K \text{ over} \\ \quad\ \text{period } s \in T, \\ 0 \text{ otherwise.} \end{cases}$$

At the beginning of the planning horizon, the human resource is starting from the employer place: $y_{m,k,0} = v_{m,k,0}$.

Another variable is the move of the human resource at the beginning of the considered period.

$$w_{m,s} = \begin{cases} 1 \text{ if human resource } m \in M \text{ is moving at the beginning of} \\ \quad\ \text{period } s \in T, \\ 0 \text{ otherwise.} \end{cases}$$

Table 3 summarizes the used variables for each problem.

TABLE 2. Considered data in each problem.

Problem	1	2	3	4	5
Exams to be planned	N	N	N	N	N
Distinct places	K	K	K	K	K
Material resources	L	L	L	L	L
Types of material resource		A	A	A	A
Human resources		M	M	M	M
Planning horizon	T	T	T	T	T
Process time of the exams	t_i	t_i	t_i	t_i	t_i
Given time to the human resource to go from one place to another		tt	tt	tt	tt
Due date of the exams	d_i	d_i	d_i	d_i	d_i
Reference place of the exams	$r_{i,k}$	$r_{i,k}$	$r_{i,k}$	$r_{i,k}$	$r_{i,k}$
On which material type a human resource is able to work		$b_{m,a}$	$b_{m,a}$	$b_{m,a}$	$b_{m,a}$
By which type of material resource the exam can be processed		$c_{i,a}$	$c_{i,a}$	$c_{i,a}$	$c_{i,a}$
By which material resource the exam can be processed	$c_{i,l}$				
Type of each material resource		$d_{l,a}$	$d_{l,a}$	$d_{l,a}$	$d_{l,a}$
Place of each material resource	$s_{k,l}$	$s_{k,l}$	$s_{k,l}$	$s_{k,l}$	$s_{k,l}$
Place where the human resource is working during a period		$v_{m,k,s}$	$v_{m,k,s}$		
Place where the human resource is employed		$v_{m,k,0}$	$v_{m,k,0}$	$v_{m,k,0}$	$v_{m,k,0}$
Material resource open time	$p_{l,s}$	$p_{l,s}$	$p_{l,s}$	$p_{l,s}$	$p_{l,s}$
Human resource work time		$q_{m,s}$	$q_{m,s}$	$q_{m,s}$	$q_{m,s}$
Total maximum work time			$H_{\max}$		$H_{\max}$

TABLE 3. Considered variables in each problem.

Problem	1	2	3	4	5
Assignment of an exam to a material resource during a period	$x_{i,l,s}$				
Assignment of an exam to a material resource and a human resource during a period		$x_{i,l,s,m}$	$x_{i,l,s,m}$	$x_{i,l,s,m}$	$x_{i,l,s,m}$
Number of overtime hours of each human resource during a period			$z_{m,s}$		$z_{m,s}$
Assignment of a human resource to a place during a period				$y_{m,k,s}$	$y_{m,k,s}$
Move of a human resource at the beginning of a period				$w_{m,s}$	$w_{m,s}$

TABLE 4. Possible criteria to minimize depending on the considered problem.

Problem	1	2	3	4	5
Criterion 1 or 3	x	x	x	x	x
Criterion 2	x	x	x	x	x
Criterion 4			x		x
Criterion 5				x	x

4.3. CRITERIA

In this paper, we consider the following criteria:

- Criterion 1: number of exams done after their due date.
- Criterion 2: number of exams not performed at their reference place.
- Criterion 3: number of exams not done during the planning horizon.
- Criterion 4: number of overtime hours of the human resources.
- Criterion 5: number of moves of the human resources.

Table 4 summarizes how the constraints are instantiated for each problem. The mathematical formulation of these criteria is the following. In problem 1, $x_{i,l,s,m}$ is replaced by $x_{i,l,s}$.

$$C_1 = |N| - \sum_{i \in N} \sum_{l \in L} \sum_{\substack{s \in T \\ s \le d_i}} \sum_{m \in M} x_{i,l,s,m} \tag{4.1}$$

$$C_2 = |N| - \sum_{i \in N} \sum_{k \in K} \sum_{l \in L} \sum_{s \in T} \sum_{m \in M} x_{i,l,s,m} \cdot s_{k,l} \cdot r_{i,k} \tag{4.2}$$

$$C_3 = |N| - \sum_{i \in N} \sum_{l \in L} \sum_{s \in T} \sum_{m \in M} x_{i,l,s,m} \tag{4.3}$$

$$C_4 = \sum_{m \in M} \sum_{s \in T} z_{m,s} \tag{4.4}$$

$$C_5 = \sum_{m \in M} \sum_{s \in T} w_{m,s}. \tag{4.5}$$

The method of weighting criteria [9] is adapted in this paper. The objective function is obtained by summing the different criteria with a coefficient ω_c. Usually, the problem is written as $\min \sum_{c=1}^{C} \omega_c \cdot C_c$, with C_c a criterion and ω_c the relative coefficient. The coefficients are chosen such as $\sum_{c=1}^{C} \omega_c = 1$. We could also use the epsilon-constraint method [19] by changing some criteria into constraints.

Moreover, lexicographic order allows to take into account a preference between the objectives [13]. Thus, the user − that is to say the planner who could use this model to do the HCT assignment − can choose the criteria which are the most relevant according to him, by changing the value of the different coefficients.

In this paper, we use a lexicographic order and we propose to choose the coefficients such as the criteria are easily readable. Moreover, criterion 1 and criterion 3

are dependent, they cannot be taken into account together. Either C_1 or C_3 is optimized. Thus, the used objective function is (4.6) or (4.7).

$$\min C_1.10^{\omega_1} + C_2.10^{\omega_2} + C_4.10^{\omega_4} + C_5.10^{\omega_5} \tag{4.6}$$

$$\min C_2.10^{\omega_2} + C_3.10^{\omega_3} + C_4.10^{\omega_4} + C_5.10^{\omega_5} \tag{4.7}$$

For instance, if the objective function is $\min C_1 + C_2 10^{\omega_2}$, ω_2 is chosen such as $10^{\omega_2} > C_1$. If the used objective function is (4.6) and the result is 120 345 678, lets assume that $\omega_1 = 7, \omega_2 = 4, \omega_4 = 2$ and $\omega_5 = 0$. Thus, the values of the criteria are $C_1 = 12, C_2 = 34, C_4 = 56$ and $C_5 = 78$.

4.4. Constraints

For these problems, some constraints have to be respected.

- Constraint 1 ((4.8) or (4.9)): material resources open times have to be respected because an exam cannot be done apart from these material resources open times.
- Constraint 2 (4.10): human resources work times have to be respected because an exam cannot be done apart from these human resources work times. Travel time is included in work time.
- Constraint 3 (4.11): for each period, for each human resource, overtime hours of the human resource are taken into account only when it is necessary, if the human resource works out of his defined work time.
- Constraint 4 (4.12): an exam is assigned to one material resource and one period at best. An exam may not be assigned.
- Constraint 5 (4.13): an exam is assigned to one material resource, one human resource and one period at best because it may not be assigned.
- Constraint 6 (4.14): an exam is assigned to a compatible material resource.
- Constraint 7 (4.15): an exam is assigned to a compatible material resource and to a human resource who is able to work on this material resource.
- Constraint 8 ((4.16) or (4.17)): the human resource who is assigned to the considered exam has to work at the place where the used material resource is located during the considered period. In (4.16), assignment of the human resource on the place is given, whereas in (4.17), this assignment is determined by the model.
- Constraint 9 (4.18): per each human resource and period, overtime hours are limited in time.
- Constraint 10 (4.19): a human resource cannot move twice during one period.
- Constraint 11 (4.20): moves of human resources are taken into account only if they are not working at the same place as during the previous period.
- Constraint 12 ((4.21) or ((4.22) and (4.23))): integrity constraints.
- Constraint 13 ((4.24) or (4.25) or ((4.24) and (4.25))): non negativity constraints.

Table 5 summarizes the constraints which are considered for each problem. The constraints are written as follows:

$$\sum_{i \in N} t_i.x_{i,l,s} \leq p_{l,s}, \forall l \in L, \forall s \in T \tag{4.8}$$

$$\sum_{m \in M} \sum_{i \in N} t_i.x_{i,l,s,m} \leq p_{l,s}, \forall l \in L, \forall s \in T \tag{4.9}$$

$$\sum_{l \in L} \sum_{i \in N} t_i.x_{i,l,s,m} + w_{m,s}.tt \leq q_{m,s}, \forall m \in M, \forall s \in T \tag{4.10}$$

$$\sum_{i \in N} \sum_{l \in L} t_i.x_{i,l,s,m} - q_{m,s} + w_{m,s}.tt \leq z_{m,s}, \forall m \in M, \forall s \in T \tag{4.11}$$

$$\sum_{l \in L} \sum_{s \in T} x_{i,l,s} \leq 1, \forall i \in N \tag{4.12}$$

$$\sum_{l \in L} \sum_{s \in T} \sum_{m \in M} x_{i,l,s,m} \leq 1, \forall i \in N \tag{4.13}$$

$$\sum_{l \in L} \sum_{s \in T} x_{i,l,s}.(1 - c_{i,l}) = 0, \forall i \in N \tag{4.14}$$

$$\sum_{l \in L} \sum_{s \in T} \sum_{m \in M} x_{i,l,s,m}.(1 - d_{l,a}.b_{m,a}) = 0, \forall i \in N, \forall a \in A | c_{i,a} = 1 \tag{4.15}$$

$$x_{i,l,s,m} - \sum_{k \in K} s_{k,l}.v_{m,k,s} \leq 0, \forall i \in N, \forall l \in L, \forall m \in M, \forall s \in T \tag{4.16}$$

$$x_{i,l,s,m} - \sum_{k \in K} s_{k,l}.y_{m,k,s} \leq 0, \forall i \in N, \forall l \in L, \forall m \in M, \forall s \in T \tag{4.17}$$

$$q_{m,s} + z_{m,s} \leq H_{max}, \forall m \in M, \forall s \in T \tag{4.18}$$

$$\sum_{k \in K} y_{k,m,s} = 1, \forall m \in M, \forall s \in T \tag{4.19}$$

$$y_{m,k,s} - y_{m,k,(s-1)} \leq w_{m,s}, \forall k \in K, \forall m \in M, \forall s \in T \tag{4.20}$$

$$x_{i,l,s} \in \{0,1\}, \forall i \in N, \forall l \in L, \forall s \in T \tag{4.21}$$

$$x_{i,l,s,m} \in \{0,1\}, \forall i \in N, \forall l \in L, \forall m \in M, \forall s \in T \tag{4.22}$$

$$y_{m,k,s} \in \{0,1\}, \forall k \in K, \forall m \in M, \forall s \in T \tag{4.23}$$

$$z_{m,s} \geq 0, \forall m \in M, \forall s \in T \tag{4.24}$$

$$w_{m,s} \geq 0, \forall m \in M, \forall s \in T \tag{4.25}$$

4.5. COMMENTS

These models allow us to take into account more particular cases than described here. For instance, it can be assumed that the reference place of exams is imposed.

TABLE 5. Used constraints in each problem.

Problem	1	2	3	4	5
Constraint 1	(4.8)	(4.9)	(4.9)	(4.9)	(4.9)
Constraint 2		(4.10)		(4.10)	
Constraint 3			(4.11)		(4.11)
Constraint 4	(4.12)				
Constraint 5		(4.13)	(4.13)	(4.13)	(4.13)
Constraint 6	(4.14)				
Constraint 7		(4.15)	(4.15)	(4.15)	(4.15)
Constraint 8		(4.16)	(4.16)	(4.17)	(4.17)
Constraint 9			(4.18)		(4.18)
Constraint 10				(4.19)	(4.19)
Constraint 11				(4.20)	(4.20)
Constraint 12	(4.21)	(4.22), (4.23)	(4.22), (4.23)	(4.22), (4.23)	(4.22), (4.23)
Constraint 13			(4.24)	(4.25)	(4.24), (4.25)

It was a soft constraint and it becomes a hard constraint. If the criterion "number of exams done at their reference place" becomes a constraint, all the exams which are done during the planning horizon will be assigned to a material resource which is on their reference place. Equation (4.2) becomes:

$$\sum_{s \in T} \sum_{m \in M} x_{i,l,s,m} \leq \sum_{k \in K} s_{k,l}.r_{i,k}, \ \ \forall i \in N, \forall l \in L. \tag{4.26}$$

A human resource can be imposed to process an exam. If human resource number 2 has to be used for exam i, a new constraint can be:

$$\sum_{i \in N} \sum_{l \in L} \sum_{s \in T} \sum_{\substack{m \in M \\ m \neq 2}} x_{i,l,s,m} = 0. \tag{4.27}$$

In the same way, a material resource, a place or a period can be imposed to one or more exams. The material resource l_1 can be imposed to the human resource m_1. The variable $x_{i,l,s,m}$ will be equal to zero for the couples (l,m) different from (l_1, m_1).

In the same way as overtime hours of the human resources can be allowed, overtime hours of the material resources could be allowed too.

Another point which could be studied is emergency. An emergency rate could be defined, if the exam has to be done before one period, some periods, or if there is no emergency. Depending on this emergency rate, the model could allow or not some delay compared to the due date. Some exams could be done after their due date meanwhile other exams will have to be done before their due date.

4.6. SIZE OF THE PROBLEMS

The size of the problem is depending on the considered problem (data and constraints) and the dataset. Table 6 summarizes the number of variables and

TABLE 6. Size of the problems.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
$\|N\|.\|L\|.\|T\|$	$\|N\|.\|L\|.\|T\|.\|M\|$	$\|N\|.\|L\|.\|T\|.\|M\|$ $+\|M\|.\|T\|$	$\|N\|.\|L\|.\|T\|.\|M\|$ $+\|M\|.\|T\|$ $+\|M\|.\|K\|.\|T\|$	$\|N\|.\|L\|.\|T\|.\|M\|$ $+2.\|M\|.\|T\|$ $+\|M\|.\|K\|.\|T\|$
$\|L\|.\|T\|$ $+2.\|N\|$ $+\|N\|.\|L\|.\|T\|$	$2.\|N\|.\|L\|.\|M\|.\|T\|$ $+\|M\|.\|T\|$ $+\|M\|.\|K\|.\|T\|$ $+\|L\|.\|T\|$ $+\|N\|.\|A\| + \|N\|$	$2.\|N\|.\|L\|.\|M\|.\|T\|$ $+3.\|M\|.\|T\|$ $+\|M\|.\|K\|.\|T\|$ $+\|L\|.\|T\|$ $+\|N\|.\|A\| + \|N\|$	$2.\|N\|.\|L\|.\|M\|.\|T\|$ $+3.\|M\|.\|T\|$ $+2.\|M\|.\|K\|.\|T\|$ $+\|L\|.\|T\|$ $+\|N\|.\|A\| + \|N\|$	$2.\|N\|.\|L\|.\|M\|.\|T\|$ $+5.\|M\|.\|T\|$ $+2.\|M\|.\|K\|.\|T\|$ $+\|L\|.\|T\|$ $+\|N\|.\|A\| + \|N\|$

constraints as a function of data. The second line is about the number of variables in each problem, and the third one is about the number of constraints.

In the worst case of the studied instances in this paper, the number of variables is 175.250 and the number of constraints is 352.670.

5. DATASETS AND RESULTS

The proposed models summarized in Table 5 have been tested using the version 12.4.0 of CPLEX. The host machine is powered by an Intel Xeon X5687 quad-core CPU running at 3.6 GHz.

The obtained results for problem 2 and problem 4 are compared in Section 5.1. We will show how a pooling of resources can improve the number of processed exams and why it is better to allow the human resources to work at several places. We will show the limit of the exact method in Section 5.2.

5.1. RESOURCES POOLING

In the chosen dataset, the HCT is composed of two different places. Four material resources are located on these two places. Each material resource is from one type. Three types exist: MRI, X-ray and scanner. Three human resources are working at these places. On the first place, there is one material resource from MRI type and one from scanner type, and two human resources. Both of these human resources can work on both of these types (MRI and scanner). On the second place, there are two material resources, one from X-ray type and the other from scanner type. Only one human resource works at this place. Figure 1 describes the HCT. Tables 7 and 8 summarize the data.

A list of exams has to be planned in this HCT. Each exam has a process time, a reference place and a due date. The objective is to maximize the number of exams done during the planning horizon. One hundred exams have to be planned during three periods.
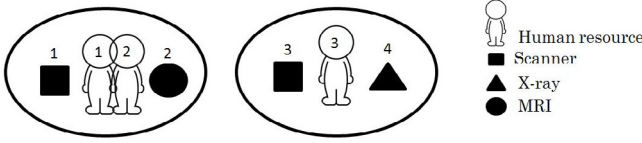
FIGURE 1. HCT composition.

TABLE 7. HCT data.

Exams	N	100
Places	K	2
Material resources	L	4
Material resource types	A	3
Human resources	M	3
Periods	T	3

TABLE 8. Ability of the human resources on the material resource types.

	MRI	Scanner	X-ray
Human resource 1	Yes	Yes	
Human resource 2	Yes	Yes	
Human resource 3		Yes	Yes

In problem 2, the assumption is that human resources always work at the same place. In problem 4, the human resources can work either at one place, or at the other. In the created instance, only 88 exams out of 100 are done if the human resources cannot move, and all the exams are done if the human resources can move during the planning horizon.

This is easily explained: suppose that there are less X-ray exams to do than the other types. All of the human resources are occupied at the beginning. Because there are less X-ray exams to do, the human resource who was working on the X-ray resource is now free. He cannot work on the scanner because the second human resource is working on it. On the second place, the human resource is either working on the scanner, or on the MRI. If the human resources are allowed to move, the human resource who is free on the first place can go to work on the scanner at place two, while the other human resource on place two is working on the MRI. If they cannot move, more time is needed to execute all of the exams.

It is also interesting to study the occupancy rate of each resource in both of the considered problems. The occupancy rate is the sum of the process times of the exams the resource is processing over its open time. The occupancy rate of material resource l_1 is given by Equation (5.1).

$$\tau_{l_1} = \frac{\sum_{i \in N} \sum_{s \in T} \sum_{m \in M} x_{i,l_1,s,m} . t_i}{\sum_{s \in T} p_{l_1,s}}. \tag{5.1}$$
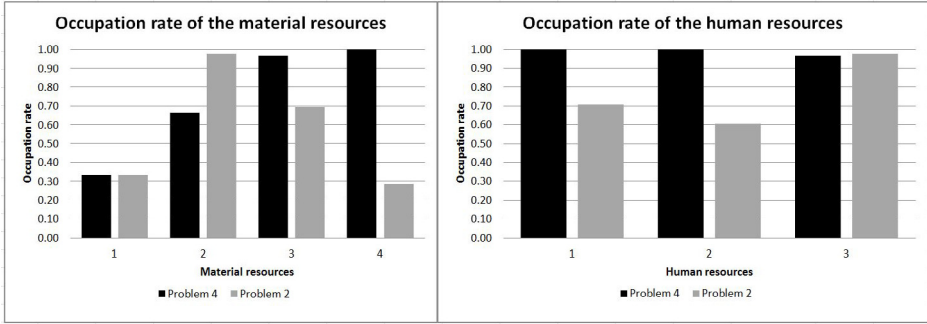
FIGURE 2. Occupancy rate of material and human resources.

These rates per resource are shown in Figure 2. The occupancy rates are better with problem 4 than with problem 2. When human resources can move, resources are used more efficiently and more exams are done.

Pooling resources is not always useful, it depends on the system. If there is the same number of human resources as the number of material resources, or if each human resource can only work on one material resource, pooling resources may not be useful. Pooling seems to be really efficient on the system when there are less human resources than the material ones, and if the human resources are able to work on several material resources, as in the previous example.

5.2. COMPUTATIONAL EXPERIMENTS

The models described in this paper are all solved in an exact way. All the problems are studied in this part. The datasets were randomly generated but represent real data. In all the datasets, the HCT is composed of three places, seven material resources, three types of material resources, five human resources. Five sizes of problem have been treated: 100, 200, 300, 400 and 500 exams have to be assigned. The planning horizon depends on the number of exams to assign. In the largest studied problem, the planning horizon is composed of ten periods, which represents at our scale, five working days, so one week. Instances are composed of 2.100 up to 175.250 variables and 4.400 up to 352.670 constraints depending on the considered problem and size of dataset. For more details, refer to Table 6 which details the number of variables and constraints per problem.

Results are summarized in Table 9. The solver CPLEX has been used to solve the problems by using the Mixed Integer Linear Programming. For each size of dataset, for problems 1 to 4, every combinations of criteria have been tested. For problem 5, because the number of possible combinations of criteria is large, we chose twenty-nine of them, the ones which are the most significant. The time limit of the solver is set to thirty minutes. An empty box $(-)$ means that the solver does not find the optimal solution in less than thirty minutes. In the other cases, the running time can be from some seconds until less than thirty minutes. The

TABLE 9. Number of iterations to solve the considered problems.

Exams	100	200	300	400	500
			Problem 1		
1 criterion	374	1148	1239	1580	39 974
	21	22	8	0	2103
2 criteria	41 167	19 314	35 072	57 080	–
	3684	3553	5280	6856	–
			Problem 2		
1 criterion	236	453	821	1570	1948
	0	1	5	34	14
2 criteria	17 857	15 771	–	–	–
	4861	1642	–	–	–
			Problem 3		
1 criterion	222	453	669	39 064	1713
	0	0	2	2422	2
2 criteria	5389	9 486	–	–	–
	1244	1273	–	–	–
3 criteria	–	–	–	–	–
	–	–	–	–	–
			Problem 4		
1 criterion	1397	513 507	79 036	1 008 101	1 350 326
	11	4841	1188	6482	1905
2 criteria	1213	–	–	–	–
	2	–	–	–	–
3 criteria	1302	–	–	–	–
	1	–	–	–	–
			Problem 5		
1 criterion	1041	403 543	367 914	1 930 366	–
	0	4246	4444	4601	–
2 criteria	1069	–	–	–	–
	0	–	–	–	–
3 criteria	1461	–	–	–	–
	26	–	–	–	–
4 criteria	71 236	–	–	–	–
	2733	–	–	–	–

number of iterations needed by the solver to find the optimal solution is written in
Table 9. The number of nodes in the CPLEX branch-and-bound tree is also given.
In each box, the first result is the number of iterations, and the second one is the
number of nodes.

The larger the number of exams to plan in the HCT is, the larger the number
of iterations to compute the objective value is. In the same way, the number of
iterations increases with the number of criteria. The limit of the model is quickly

reached. If two criteria are studied, the solver does not find any solution in less than thirty minutes for more than 200 exams for problems 2 and 3. If three criteria or more are optimized, the limit is 100 exams for problems 4 and 5.

Thanks to these models, we can study the operational level if we only want to optimize one criterion. These models could also help us to study the tactical and strategic levels.

6. Conclusion and further research

The Hospital Community of Territory (HCT) is a new hospital organization that aims at a better efficiency of the health care sector. One idea can be the resources pooling. Pooling the resources can help the efficiency of the system, so exams could be done sooner and resources could be used more efficiently. In this paper, an HCT has been modeled. We have proposed five integer programming models for planning medical imaging in this context. In this article, a mathematical model is proposed and solved by a MILP solver. Exact resolution has been experimented to our models to validate them.

Optimization is done successfully for instances which deal with small problems, over less than one week. However if we want our model to be more realistic, we should plan exams over one month. With exact methods, constraints programming might be envisaged. Alternatively, other refinement (Bender's decomposition, additional cuts, *etc.*) should be considered. Because of the size of the problems, we plan to propose heuristics and metaheuristics with an epsilon-constraint method [22] to provide quickly an approximate solution to all of these problems. Thanks to approach methods, we could increase the size of the planning horizon and of the periods to study the tactical and the strategic levels. Sizing the number of human and material resources could be considered, as the study of the entry of a new place in the HCT.

In future work, other criteria can be studied. For instance, distance between the different places should be taken into account. Moreover, one patient can have several exams. Simultaneity between these exams should be prohibited. An exam could need not only one human resource but several in a crew. Rules govern the composition of these crews. Stochastic aspect should be integrated. Human resources could be late or missing, material resources could have failures.

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