Journal de la Société Française de Statistique
Vol. 152 No. 1 (2011)

# Sensitivity analysis of interference optical systems by numerical designs

**Titre:** Analyse de sensibilité de systèmes interférentiels par des plans d'expériences numériques

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**Abstract:** Optical interference corresponds to the interaction of two or more coherent lightwaves yielding a resultant irradiance that deviates from the sum of the component irradiances. Furthermore, in an optical system involving interferences, the complexity of the output increases tremendously with the number of input parameters. Subsequently, the output of an interference system is very sensitive to any variation of input parameters. We performed sensitivity analyses to qualify two examples of such systems: coherent laser beam combining setup and multidielectric interference filters.

Coherent laser beam combining setup is based on amplitude splitting, where a primary light source is divided in an array of laser amplifiers overlapping in free space, so that the emitted lightwaves constructively interfere resulting in various intensity pattern. An interference optical filter is a stack of multiple thin layers from different dielectric materials with spectraly transmittance properties. The wavelength selection results from constructive interferences that take place between incident and reflected waves.

Because of the high number of parameters (around 20 in both cases) and in order to identify the most critical interactions, different space filling designs (low discrepancy sequences, latin hypercubes ...) are used. At first, the intrinsic quality of these designs is determined using two slightly different methods: the radar and the Minimum Spanning Tree (MST). While the radar method measure the uniformity of a design based on the analysis of all the 2-dimensional projections of this design, the MST method characterizes the distribution of the points in the original space. In a second part, we compare the intrinsic quality of these different designs with the extrinsic quality determined by the sensitivity analysis study of coherent beam combining and interference filters. Finally, in this paper, we are able to conclude on the best designs to perform sensitivity analyses on interference optical systems, which are systems with high interactions. Moreover, this study gives clues on the design to be used to analyze sytems with more than 100 parameters.

**Résumé :** Les interférences optiques correspondent à des interactions entre deux ou plus ondes lumineuses cohérentes, dont la résultante est une intensité différente de la somme des intensités de chaque onde. De plus, dans un système optique utilisant des interférences, la complexité de la sortie d'un tel système croît énormément avec le nombre de paramètres d'entrées. Il en résulte que la sortie d'un sytème interférentiel est très sensible à toute variation des valeurs des paramètres d'entrées. Nous réalisons une analyse de sensibilité pour qualifier deux types de tels sytèmes : la combinaison cohérente de faiceaux lasers et les filtres interférentiels.

La combinaison cohérente de faisceaux est un système basé sur la division en amplitude où la lumière d'une source lumineuse est divisée dans une matrice d'amplificateurs et recombinée en espace libre, de telle sorte que les faisceaux issus de chaque amplificateur interfèrent constructivement pour obtenir l'intensité souhaitée. Un filtre optique interférentiel est un empilement de couches minces de différents matériaux diélectriques afin d'obtenir les propriétés

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de transmissions spectrales désirées. La sélection d'une longueur d'onde particulière résulte des interférences entre les ondes transmises et réfléchies.

En raison du grand nombre de paramètres (autour de 20 dans chaque cas) et pour évaluer les interactions les plus critiques, différents plans d'expériences (suite à faible discrépance, hypercube latin ...) sont utilisés. Dans un premier temps, la qualité intrinsèque de ces plans est déterminée en utilisant deux méthodes : le radar et l'arbre de longueur minimal (ALM). Tandis que la méthode du radar mesure l'uniformité d'un plan en se basant sur l'analyse des projections des points du plan dans tous les sous-espaces de dimension 2, la méthode de l'ALM charactérise la distribution des points du plan dans son espace d'origine. Dans un deuxième temps, nous comparons la qualité intrinsèque des plans avec leur qualité extrinsèque par l'étude de l'analyse de sensibilité de la combinaison cohérente et d'un filtre interférentiel. Finalement, cette étude permet de conclure sur le meilleur plan pour réaliser une analyse de sensibilité pour l'étude des systèmes interférentiels, qui sont des systèmes comportant de hautes interactions entre les paramètres. De plus, cette étude donne des indications sur les plans à utiliser sur des systèmes comportant plus de 100 paramètres.

***Keywords:*** numerical designs, sensitivity analysis, sensitivity analysis, optics
***Mots-clés :*** plans d'expériences numériques, analyse de sensibilité, optique
***AMS 2000 subject classifications:*** 49Q12, 62K99, 78A10

## 1. Introduction

In the field of computer experiments, simulations of complex phenomena are getting more and more realistic. Although computer power has highly increased over the last years, simulations including numerous parameters are still very time consuming. The use of numerical designs is an effective method to build metamodels, to explore codes with an high dimensional space of parameters and to build sensitivity analyses. In the most complex interference optical systems, there is no direct analytic formula to express the relationship between the outputs and the inputs of the computer code. In these optical systems, the intensity of a sum lightwaves is different from the sum of individual lightwaves intensities. These systems are also highly sensitive too any variation of an input parameter, especially because of the interactions with all the other parameters. So the impact of uncertainties of input parameter values and the possible interactions between these input parameters on the output must be assessed to identify the most critical parameters and interactions. Then, it is possible to specify the manufacturing characteristics to obtain the desired level of performance of the optical interference system.

Because of the high dimensional space (more than 20 in general), the computation of a particular set of the design parameters is still time-consuming. We use numerical space-filling desings [1], as proposed in [2], to identify the most critical interactions and to build a metamodel, which approximates the relationship between the inputs and outputs of the code by an explicit formula, and limit the number of computer runs. Space-filling designs are the only designs which provide information about all portions of the experimental region, due to their evenly distributed points [3, 4]. The chosen metamodel is a polynomial function, therefore, the important factors will be determined by the significant coefficients values. To assess the potential efficiency of numerical design to exhibit the interactions in high dimensional spaces, we study and use different types of designs. This methodology is used to perform a sensitivity analysis [5] on two different systems using optical interferences: coherent beam combining setups [6, 7] and multidielectric interference filters [8]. Subsequently, we are able to determine the influence of the errors in each system: phase errors of each lightwave in coherent combining and coating layer refractive index errors in interference filters.

At first, we analyse the quality of different numerical designs, by using two methods [9, 10]. These methods are directly analyzing the designs by exploring their point distribution, thus exploring their intrinsic quality and exhibiting the best design with respect to this quality. In a second part we describe the physical properties of the systems. Then, we perform sensitivity analyses on our interference optical systems using all the designs studied previously, thus exploring the extrinsic quality of the designs and enabling us to point out the best types of numerical designs to be applied to these systems. Finally, by comparing the intrinsic and extrinsic quality of these designs, we conclude on the best design for studying systems with a high number of input parameters and a high level of interactions.

## 2. The numerical designs

When the relationship between inputs and outputs of code is not explicitly known, space filling designs are more interesting, because they spread the computer runs evenly throughout the studied space. Thus, to insure the quality of the designs one should use methods to analyze the distribution of the points. In this section we will describe the methods used to analyze the designs, where we will not use classical approaches like discrepancy to analyse our designs, because, as mentioned in Franco et. al. [10], those classical criteria are insufficient to conclude about the uniformity of a point distribution.

The different designs we use, are :

- low discrepancy sequences (deterministic designs): Sobol [11] and Faure [12]
- latin hypercubes designs [13]
- random designs, based on a uniform probability distribution
- WSP [14]

The goal is to find out, which designs are suitable in our cases: 18 dimensions and 29 dimensions. In the case of non deterministic designs, the distribution of the points can be slightly different each time we generate a design. We compare the statistics given by both methods on 5 latin hypercubes, 2 or 3 WSP, 5 random designs.

### *2.1. Radar criterion*

The quality of a design is determined by the radar method [9] using the DICE packages in R-software developed in the framework of the DICE consortium [15, 16]. This method is based on the analysis of all the 2-dimensional projections of the design. In each of this 2-dimensional projection, the points are projected on a rotated line. For each rotation step of the line, the Greenwood statistic [17, 18] is calculated on the projections of the all points to qualify the uniformity of the projected points on this line. The Greenwood statistic is a measure of the uniformity of a point distribution on a line. It is calculated from a sample of $N$ values in the interval $[0;1]$ as follows: let $x_1,\ldots,x_N$ be the sample values in ascending order. We define $d_i = x_i - x_{i-1}$, for $i = 2,\ldots,N$, and $d_1 = x_1$ and $d_{N+1} = 1 - x_N$. The Greenwood statistic is then: $G = \sum_{k=1}^{N+1} d_i^2$. It has been here adapted to the different sizes taken by the line during its rotation.

So the radar scanning statistic is a method based on the analysis of the spacing between the projected points on the rotated line. At the end of the analysis the radar points out the worst

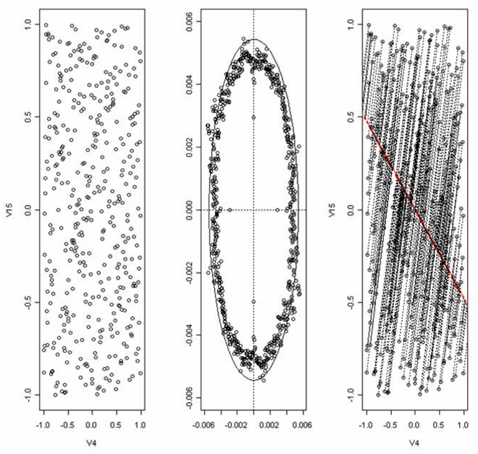



FIGURE 1. *Representation of the worst pair of dimensions (4 and 15) of one of the 400 points, 18 dimensional latin hypercube design. Radar value: 0.0062.*

| number of points of the design | random | sobol | latin hypercube | WSP | faure |
|---|---|---|---|---|---|
| 200 | 0.0145 | 0.0830 | 0.0148 | 0.0196 | 0.161 |
| | 0.0145 | | 0.0132 | 0.0269 | |
| | 0.0147 | | 0.0145 | | |
| | 0.0144 | | 0.0145 | | |
| | 0.0153 | | 0.0169 | | |
| 400 | 0.0064 | 0.042 | 0.0062 | 0.0124 | 0.0692 |
| | 0.0068 | | 0.0063 | 0.0256 | |
| | 0.0065 | | 0.0063 | | |
| | 0.0065 | | 0.0064 | | |
| | 0.0066 | | 0.0062 | | |

TABLE 1. *Radar values on 18 dimensional designs*

2-dimensional projection and the worst direction in this projection and the higher the value is, the less uniform the distribution is.

As an example figure 1 and 2 show the output of the radar function for 2 different designs. In the case of a latin hypercube (fig. 1), there is no significant alignment of points. But, we can see in figure 2, which is a Faure design, that the projection in the plane 4-6 contains point alignments. Thus, due to the loss of information, the design is less efficient, if the code is a function of a linear combination of the parameters 4 and 6.

Finally, the radar enables it to find in which direction the design will be less effective. Because the designs will be used on systems on which we have no information concerning favoured directions, we want the points to be distributed as evenly as possible. The statistics of our designs are given in tables 1 and 2.

But in higher dimension, the comparison of different designs by this criterion is more difficult.

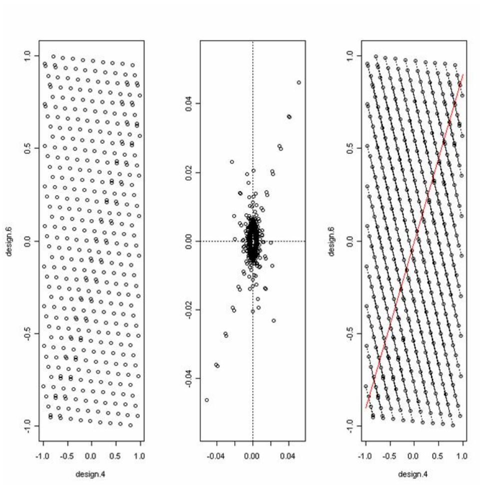
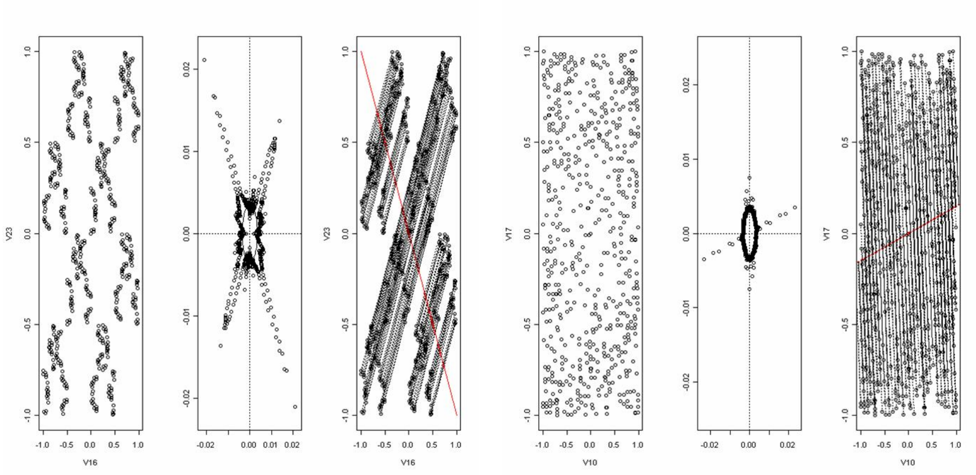



FIGURE 2. *Representation of the worst pair of dimensions (4 and 6) of the 400 points, 18 dimensional Faure design. Radar value: 0.069.*

| number of points of the design | random | sobol | latin hypercube | WSP | faure |
|---|---|---|---|---|---|
| | 0.0041 | 0.0298 | 0.0040 | 0.0088 | 0.085 |
| | 0.0039 | | 0.0040 | 0.0236 | |
| 614 | 0.0040 | | 0.0040 | 0.0044 | |
| 598 for WSP1 and WSP2 | 0.0040 | | 0.0040 | | |
| | 0.0042 | | 0.0041 | | |

TABLE 2. *Radar values on 29 dimensional designs*



(a) (b)

FIGURE 3. *Radar on the Sobol and WSP 29-dimensional designs. (a): Sobol, radar value: 0.0298 (b): WSP, radar value: 0.0236.*

For example, the values in 29 dimensions for the Sobol and the second WSP design are almost the same. But figure 3 shows a great difference between these two designs. The use of the single radar value is not always sufficient to accurately assess the quality of a design and a graphic representation of the worst pair of dimensions can be needed.

Finally, we can conclude, that the low discrepancy sequences have the worst intrinsic quality. One should be very cautious using the Faure design, due to its alignments, which could lead to inconsistent results.

### *2.2. MST criterion*

The second method used to qualify computer experiments, is based on a tree constructed from the set of points of the design. This tree, which is called Minimum Spanning Tree (MST), contains all the points, has no cycle and the sum of the length of its edges is minimal. The MST built on a space filling design of n points is calculated by algorithms [19, 20], where the effort for computation time evolves as O($n^2$). In these algorithms, the MST is grown from a single node (the points of the design) by adding the closest node to current tree at each stage along with the edge corresponding to that closest distance. Depending on the starting point there may be more than one MST for a given set of points, but all of the MST's have the same length-edge histogram [21, 22]. The statistical information used from the histogram are the average ($m$) and the standard deviation ($\sigma$) of the edge lengths. In the ($m$, $\sigma$) plane, all distribution of points can be plotted and easily compared with well-characterized distributions (for example, perfectly ordered or random ones) as shows the Figure 4.

The design is then associated by the mean $m$ and the standard deviation $\sigma$ of the edge lengths, which characterize the type of distribution of the points (cluster, ordered . . . ) [10, 21]. In the ($m$, $\sigma$) plane, the best space filling designs are characterized by a high average length of MST branches to

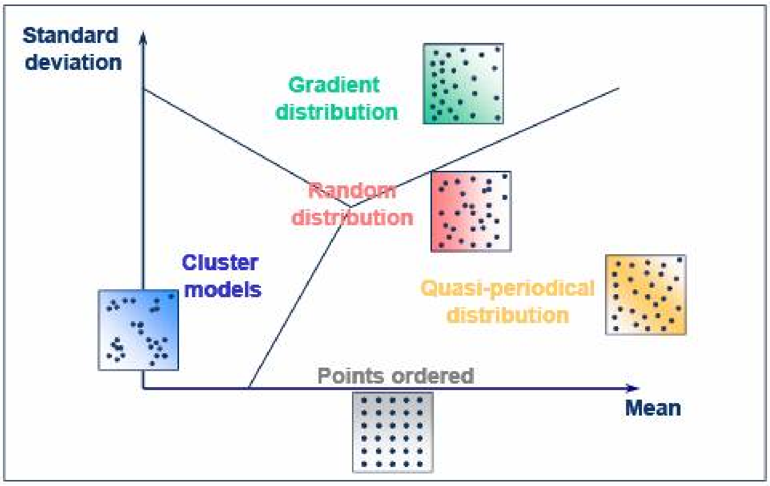



FIGURE 4. *Representation of different types of distributions in the (m, σ) plane [10]*

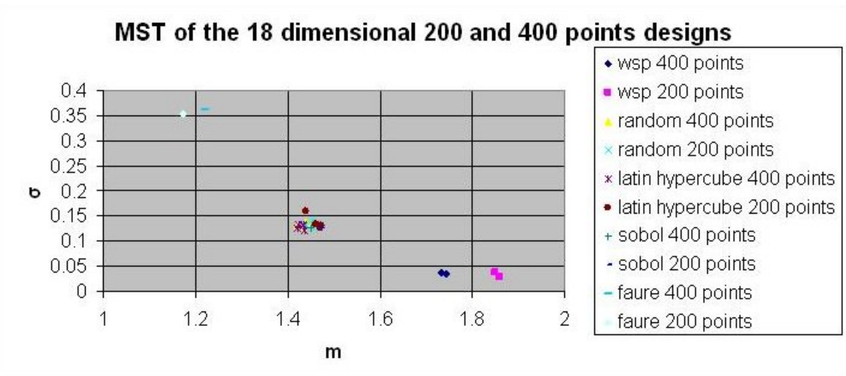



FIGURE 5. *Representation of standard deviation vs average of the edges length for the 18 dimensional designs.*

fill the space and a small standard deviation to obtain a sufficient regularity. Moreover, with ordered structure ($\sigma$=0), the points are not evenly spread across the projection of the experimental space onto all subspaces. The results are presented in figures 5 (18 dimensions) and 6 (29 dimensions).

We can see that the WSP designs seem to be the best as they have the smallest standard deviation and the largest mean. Even outranked by the WSP, the other designs are very similar in terms of quality, except the Faure design, which is clearly the worst.

### *2.3. Conclusion*

Finally, by analysing the intrinsic quality of the different designs, the conclusions of the two methods are slightly different. In fact, these methods are analysing two different aspects of a design and are complementary: the radar seeks for points alignments in 2-dimensional subspaces while the MST analyses directly the design and enables to classify the type of points distribution.

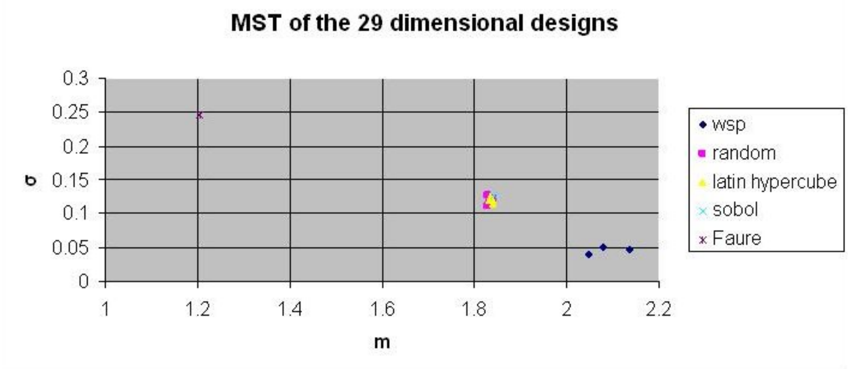



FIGURE 6. *Representation of standard deviation vs average of the edges length for the 29 dimensional designs.*

Using simultaneously both criteria we can conclude that the Sobol and Faure sequences are the designs with the worst quality. The WSP design appears to the best design: it is the best with respect to the MST method and is acceptable with respect to the radar method. In the next section, we apply our designs to perform sensitivity analysis to our optical systems.

## 3. Sensitivity analysis of interference optical systems

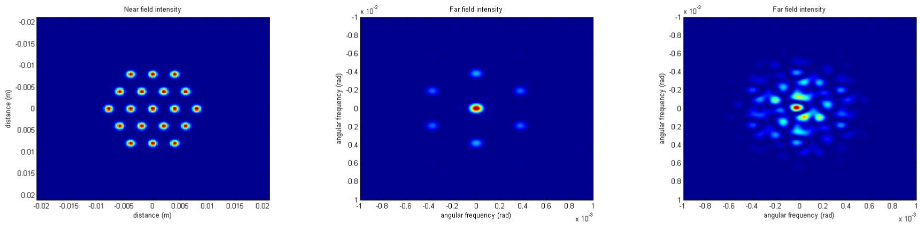
The interference optical systems presented in this paper are based on amplitude splitting, where a primary wave is divided into two ore more segments, which travel different paths before overlapping and interfering. Therefore, variation of path length differences and amplitude differences for one of the waves modify the resulting intensity pattern. However, especially when dealing with many interfering waves, it is not obvious which wave parameter or interaction between parameters are the most critical in terms of intensity pattern distortion. The metamodel that we will use to assess these interactions is a second order polynomial function:

$$f(X_1,\ldots,X_n) = a_0 + \sum_{k=1}^{n} a_k \cdot X_k + \sum_{k=1}^{n} b_k \cdot X_k^2 + \sum_{0<i<j\leq n} c_{ij} \cdot X_i \cdot X_j$$

We chose a second order polynomial function instead of a first order because we know that the response in both cases has an extremum in the center of the domain. The coefficients are estimated using a least square approach. The R-square coefficient will guarantee the quality of this polynomial regression. In both cases we look at the interaction coefficients $c_{ij}$ and at the value $a_0$ to insure of the extrinsic quality of our design ($a_0$ is the value at the extremum).

### *3.1. Coherent laser beam combining setup*

High power density is required to increase range and sensitivity in many applications. However the ultimate power density available in a single laser amplifier chain is limited by nonlinear effects and damage threshold in the amplifying medium [23]. Coherent combining of laser beams is an alternative way for power scaling while maintaining low beam divergence [7, 24]. In coherent




(a) (b) (c)

FIGURE 7. *The intensity is given in an arbitrary unit. (a): near field. (b): far field with equal phases. (c): far field with non-equal phases.*

combining, several lasers are packaged together into an array, all the array elements operate with the same wavelength, and the relative optical phases of the elements are controlled. In fact, without an active control of the phases, destructive interferences between waves will reduce the average power density in the intensity pattern. Therefore, it is crucial to know, the sensitivity of the system towards residual phase differences between laser sources.

As an example, we study a hexagonal array of 19 lasers (fig. 7). This configuration guarantees a high fill factor resulting in very efficient coherent beam combining with more than 70% power concentrated in the center of the diffraction pattern.

When phase differences remain (fig. 7 (c)), the combining efficiency decreases resulting in spreading of the power in large areas. Different criteria are used to qualify this efficiency. In this case, we present results with the Mask Encircled Power (MEP) criterion which is very convenient for many applications. It is the fraction of power contained in a fixed angular aperture, here a 1 mrad centered circular aperture.

The central laser beam in the array will be our reference and the 18 other lasers will have the following phase variation: $\Delta\phi = \phi_{laser} - \phi_{center}$ with $\phi_{center} = 0$.

We use the previous designs in dimension 18 to perform a sensitivity analyses. The laser in the center will stay with a constant phase equal to zero, while the others have a phase varying between $-\frac{\pi}{10}$ and $+\frac{\pi}{10}$ meaning that the domain of definition of the MEP is . When all the phases are equal to 0, the MEP is at its maximum. Therefore, theoretically, the $a_0$ coefficient should be equal to this maximum MEP value. The error on $a_0$ is used to asses the quality of the design. For almost all designs, the metamodel has a good quality with an R-square greater than 0.95. After analysis of the metamodel, all designs have the same trend: a laser interacts 10 times more with its closest neighbours than with the others and up to 50 times more than with a diametrically opposite laser.

For the 200 points designs (around 10 points per dimensions), the error on the $a_0$ coefficient is lower or around 1%, only the Sobol design has an error of roughly 10%.

For the 400 points designs (around 20 points per dimension), all the designs have an error on $a_0$ less than 1%.

The low discrepancy sequences with around 10 points per dimension do not deliver accurate results and a greater number of points is preferable. Therefore for the interference filters, we will keep 20 points per dimension but with a higher number of dimensions.

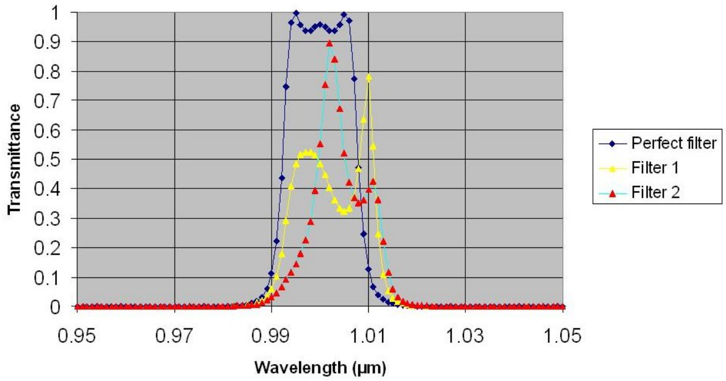



FIGURE 8. *Spectral transmittance of the perfect filter and of two filters (Filter 1 and Filter 2) with errors on refractive index values.*

### 3.2. Multidielectric interference filters

In optics, bandpass filters are used to select a specific spectral range. Optical thin films coatings are efficient components to select a narrow wavelength bandwidth from an optical signal. Refractive index errors or thickness errors during the manufacturing of these layers can dramatically impair the desired optical properties. Because of the spectral selectivity accuracy that is needed for a bandpass filter, thickness $t$ and the refractive index $n$ of each layer have to be controlled very precisely during the coating manufacturing. We point out the most critical layer parameter and quantify the interactions between those parameter errors by the study of the optical filter transmittance.

This study is done by the 29 dimensional designs described in table 2 on the following three-cavity bandpass filter composed of 29 layers:

Substate/HLHL4HLHLH L HLHL4HLHLH L HLHL4HLHLH/air

where H and L are quarter-wave layers ($n \times t = \lambda_0/4$ with $\lambda_0$ = 1 µm) of high (H) and low (L) refractive index values respectively.

This coating is an assembly of three basic bandpass filter: HLHL4HLHLH.

Due to the knowledge of the optical properties of this filter, the most critical layers or layer interactions belong to the blocks L4HL, which are the fundamental basis of each optical cavity, and between the 4H-layers of these blocks, whose characteristics correspond to the center of the bandpass of each optical cavity.

Furtherance, we limit the study to errors on refractive index as both optical thickness $n \times t$ and reflectivity of a layer are driven by refractive index value. The maximum refractive index error value is 2.5%. Two examples of the errors of the refractive index error values on the optical properties of the filter are presented on fig. 8: Filter 1 and Filter 2 curves show out the changes in the spectral transmittance of the filter due to errors on refractive index values of each layer.

The sensitivity analysis assessment is performed using the following response R (merit func-

| Design used and number of computer runs | Score | $a_0$ |
|---|---|---|
| Random 1, 614 runs | 6/9 | 0.71 |
| Random 2, 614 runs | 4/9 | 0.75 |
| Random 3, 614 runs | 7/9 | 0.82 |
| Random 4, 614 runs | 7/9 | 0.88 |
| Random 5, 614 runs | 8/9 | 0.81 |
| Sobol, 614 runs | 6/9 | 0.68 |
| Latin Hypercube 1, 614 runs | 6/9 | 0.88 |
| Latin Hypercube 2, 614 runs | 5/9 | 0.90 |
| Latin Hypercube 3, 614 runs | 7/9 | 0.87 |
| Latin Hypercube 4, 614 runs | 5/9 | 0.81 |
| Latin Hypercube 5, 614 runs | 6/9 | 0.89 |
| WSP 1, 598 runs | 7/9 | 0.30 |
| WSP 2, 598 runs | 6/9 | 0.02 |
| WSP 3, 614 runs | 6/9 | 0.53 |
| Faure, 614 runs | 1/9 | 1.43 |

TABLE 3. *Score on the identification of the major interaction coefficients and $a_0$ value for the 29-layers filter*

tion):

$$R = \sqrt{(T(\lambda_i) - T_p(\lambda_i))^2}$$

where T($\lambda_i$) is the transmittance of the filter at the wavelength $\lambda_i$ in the case of the computer experiment (with error on refractive index values), $T_p$ ($\lambda_i$) is the transmittance of the perfect filter, and $\lambda_0$=1 µm is the central designed wavelength of the perfect filter. The refractive indices vary in [2.325; 2.375] for the H layers and [1.275; 1.325] for the L layers.

So, the merit function R evaluates the transmittance influence of the refractive index error values on the spectral domain [0.9 µm, 1.1 µm].

The most critical layer interactions will be identified by checking the values of the coefficients from the polynomial regression. The quality of the different designs is assessed using the following criteria:

- Value of the R-square obtained with the polynomial regression.

- Identification of the major interaction coefficients within the 9 major interaction coefficients (between 4H-layers and within L4HL blocks) $c_{ij}$ : $c_{4-5}$ $c_{5-6}$ $c_{5-15}$ $c_{5-25}$ $c_{14-15}$ $c_{15-16}$ $c_{15-25}$ $c_{24-25}$ $c_{25-26}$. The score highlights the number of major interactions identified by the design, compared to the list. For example, a score of 6/9 means that 6 of the highest interactions identified by the metamodel are within the 9 highest interactions.

- Deviation of $a_0$ value from 0. In the case of no refractive index error, the value of R is null so the exact value of $a_0$ is 0.

The range of values obtained for the response R with all the designs is [0 ; 4.30]. The mean of the response values is 1.81 and the mean square error is 0.63. For almost all the designs, the metamodel obtained has an R-square greater than 0.96.

The results obtained for the most critical interactions of layers are shown in the "score"column of table 3.

Faure designs performs very badly with 1/9 score and a very strong deviation of $a_0$.

The random designs obtain variable score from 4/9 to 8/9 as the latin hypercube and WSP designs have comparable scores.

The results obtained with the value of $a_0$ are presented in the 3rd column of table 3. In terms of $a_0$ deviation, the lowest are obtained by WSP designs. The $a_0$ values of random designs and LHS designs are comparable but we can notice that the $a_0$ values of random designs are a little lower than those obtained with LHS designs. In the case of deterministic designs build with discrepancy sequences, Sobol design is between WSP and random designs.

Finally, this study exhibits the low quality of designs built by low discrepancy sequences and unstable quality of random designs. It appears that the WSP design has the best extrinsic quality.

### *3.3. Conclusion*

The study of two interference optical systems, which are characterized by a lots of parameter interaction, by various numerical designs, points out that 20 points per dimension are necessary to determine interactions and obtain a satisfactory metamodel. The low discrepancy designs should not be used due to points alignments. The latin hypercubes gave results nearly similar as random designs. The quality of the metamodel assessed by the $a_0$ coefficient, R-square value and the interactions in the case of the optical filter points out the WSP design quality.

Even though variations are found in the score and the $a_0$ coefficient, because using a space-filling design to analyze a system leads to approximation, results are still consistent. Therefore, the WSP designs seem to be the most promising to explore spaces with more than 30 dimensions.

## 4. Conclusion

The intrinsic quality of different designs was determined by the MST and radar criteria. The use of the radar criterion requires a graphical representation of the worst pair of point projections. These criteria are complementary to exclude designs and in our study we never obtained inconsistent results. Only the MST criterion points out higher quality of WSP designs.

The results of sensitivity analysis of interference optical systems exhibit the WSP design quality, because of the quality of the metamodel, as well as the ability of assessing interactions.

The intrinsic and the extrinsic quality show good agreement, which lead to the conclusion that the study of systems with a high level of interactions and a large number of parameters should be done using designs with points distributed as evenly as possible. The sensitivity analyses are indeed well connected to the mathematical properties of the designs (point alignments or quasi-periodical point distribution).

Further work will analyze maximin or improved latin hypercubes, which could also be used as the construction of such design attempts to optimize the sample with respect to an optimum euclidean distance between design points, which should lead to quasi-periodical distribution. Moreover, the study of these designs will be conducted with a larger number of parameters.

## 5. acknowledgments

The authors wish to thank the members of the DICE consortium for the fruitful discussions.

## References


[1] G. E. P. Box, W. G. Hunter, J. S. Hunter, *Statistics for experimenters*, Wiley and Sons, New-York, 1978

[2] O. Vasseur, M. Claeys-Bruno, M. Cathelinaud, M. Sergent, "High-dimensional sensitivity analysis of complex optronic systems by experimental design: Applications to the case of the designs and the robustness of optical coatings", *Chinese Optics Letters* **8** (s1), pp. 21 - 24, 2010

[3] P. C. Young, "Data-based mechanistic modelling, generalised sensitivity and dominant mode analysis", *Comput. Phys. Commun.* **117**, pp 113 - 129, 1999

[4] A. A. Giunta, S. F.Wojtkiewicz, and M. S. Eldred, *Overview of modern design of experiments methods for computational simulations*, In Proceedings of the 41st AIAA Aerospace Sciences Meeting and Exhibit, AIAA-2003-0649, 2003

[5] A. Saltelli, K. Chan, E.M. Scott, *Sensitivity analysis*, Wiley, New-York, 2000

[6] T. M. Shay, V. Benham, Lt. J. Spring, Capt. B. Ward, F. Ghebremichael, M. A. Culpepper, A. D. Sanchez, J. T. Baker, Sgt. D. Pilkington, R. Berdine, *Self-Referenced Locking of Optical Coherence by Single-detector Electronic-Frequency Tagging*, In Proceedings of SPIE, Vol. 6102, 61020V, 2006

[7] V. Jolivet, P. Bourdon, B. Bennai, L. Lombart, G. Goular, E. Pourtal, G. Canat, Y. Jaouen, B. Moreau, O. Vasseur, "Beam shaping of single-mode and multimode fiber amplifier arrays for propagation through atmospheric turbulence", *IEEE. J. Sel. Topics of Quantum Electron.* **15**, pp. 257 - 268, 2009

[8] H. A. Macleod,, *Thin film optical filters*, Adam Hilger Ltd, Bristol, 1986

[9] O. Roustant, J. Franco, L. Carraro, A. Jourdan, *A radial scanning statistic for selecting space-filling designs in computer experiments*, In proceedings of MODA-9, Bertinoro, Italy, 2010

[10] J. Franco, O. Vasseur, B. Corre, M. Sergent, "Minimum Spanning Tree: A new approach to assess the quality of the design of computer experiments", *Chemometrics and intelligent laboratory systems* **97**, pp. 164 - 169, 2009

[11] I. M. Sobol, "Uniformly distributed sequences with an additional uniform property", *USSR Computational Math. and Math. Phys.* **16** (5), pp. 236 - 242, 1976

[12] H. Faure, "Discr?pance de suites associées à un système de numération (en dimension s)", *Acta Arith.* **41**, pp. 337 - 351, 1982

[13] M. Stein, "Large Sample Properties of Simulations Using Latin Hypercube Sampling", *Technometrics* **29**, pp. 143 - 151, 1987

[14] M. Sergent, R. Phan-Tan-Luu, J. Elguero, *Statistical analysis of solvent scales*, Part 1 An. Quim. Int. Ed 93, 1997

[15] http://cran.r-project.org/

[16] http://www.dice-consortium.fr/

[17] J. S. Rao, M. Kuo, "Asymptotic results on the greenwood statistic and some of its generalizations", *J. R. Statist. Soc. B.*, **46**, pp. 228 - 237, 1984

[18] M. Greenwood, "The statistical study of infectious diseases", *J. R. Statist. Soc.*, **109**, pp. 85-110, 1946

[19] J. B. Kruskal, *On the shortest spanning subtree of a graph and travelling salesman problem*, In proceedings of the American Mathematical Society **7**, pp. 48 - 50, 1956

[20] R. C. Prim, "Shortest connection networks and some generalizations", *Bell system technical journal* **36**, pp. 1389 - 1401, 1957

[21] C. Dussert, G. Rasigni, M. Rasigni, J. Palmari, "Minimal spanning tree: A new approach for studying order and disorder", *Physical Review B.* **34**(5), pp. 3528 - 3531, 1986

[22] C. Zhan, "Graph-theoretical methods for detecting and describing gestalt clusters", *IEEE Trans. Comput.*, **C-20**, pp. 68 - 86, 1971

[23] Y. Jaouen, G. Canat, S. Grot, S. Bordais, "Power limitation induced by nonlinear effects in pulsed high-power fiber amplifiers", *C. R. Physique.* **7**(2), pp. 163 - 169, 2006

[24] T. Y. Fan, "Laser beam combining for high-power, high-radiance sources", *IEEE J. Sel. Topics of Quantum Electron.* **11**(3), pp. 567 - 577, 2005