Pacioli and humanism: pitching the text in Summa Arithmetica
Patricia McCarthy, Alan Sangster and Greg Stoner
Accounting History 2008; 13; 183
DOI: 10.1177/1032373207088178

The online version of this article can be found at:
http://ach.sagepub.com/cgi/content/abstract/13/2/183

Published by:
$\bigcirc$SAGE Publications
http://www.sagepublications.com

On behalf of:

The Accounting History Special Interest Group of the Accounting and Finance Association of
Australia and New Zealand

Additional services and information for *Accounting History* can be found at:

Email Alerts: http://ach.sagepub.com/cgi/alerts

Subscriptions: http://ach.sagepub.com/subscriptions

Reprints: http://www.sagepub.com/journalsReprints.nav

Permissions: http://www.sagepub.com/journalsPermissions.nav

Pacioli and humanism: pitching the text in *Summa Arithmetica*

Patricia McCarthy
Open University
Alan Sangster
Middlesex University
Greg Stoner
Glasgow University

Abstract

Despite the wide cross-disciplinary influence of Fra' Luca Pacioli's Summa de Arithmetica, Geometria, Proportioni et Proportionalita (Summa), *it has been criticized as being both difficult to read and written in a mixture of bad Italian and bad Latin; but, paradoxically, intellectuals of Pacioli's day praised the style of writing in* Summa. *Can both viewpoints be correct? The answer to this question is sought by identifying what may have inspired Pacioli to write* Summa *in the manner he did. In doing so, the article considers the times in which he lived and, in particular, the impact that Renaissance Humanism and Humanist Education may have had upon his writing style. The article finds both views were correct in their own timeframes and contexts and that Pacioli's writing style was both an appropriate one with which to address a contemporary merchant society and one which would impress and gain the approval of his fellow humanist educators and patrons.*

Introduction

Pacioli's *Summa* (1494) contained the first known printed exposition of double entry bookkeeping, which has been identified as the foundation of modern accounting (Fogo, 1905). It is also acknowledged as having given rise to advances in mathematics during the sixteenth century (Rose, 1976); to have provided the catalyst for the development of statistics in the seventeenth century (Strathern, 2001); and, to have assisted the development of perspective in architecture and art (Ciocci, 2003).

Consideration by accounting historians of Pacioli's bookkeeping treatise,[1] and the style of writing he adopted have typically focused on the accounting content (for example Geijsbeek, 1914; Yamey, 1994, 2004; Nobes, 1995). A similarly narrow focus has also been adopted in other disciplines, such as mathematics, when considering the content of *Summa* relevant to their fields. Little has been written concerning the motivation behind his writing in his chosen style, a style which, it could be argued, was instrumental in bringing what he wrote to the attention of the world.

Pacioli had a range of choices open to him concerning the language and writing style to use when he wrote *Summa*. As a pre-university teacher of abbaco (see for example Grendler, 1989; Ciocci, 2003; Camerota, 2006), he had many years experience of teaching in the vernacular. As a university teacher, he would have lectured in Latin and would have known how to use it effectively. He would also, as a Franciscan preacher, have been used to addressing a common crowd and catching their attention so that they listened to his message.[2] He would have known precisely how to pitch his text for his intended audience.

The style of writing was praised at the time of its publication by, among others, the highly educated bibliophile, Guidobaldo, Duke of Urbino (Taylor, 1942, p.196). Yet, from approximately 50 years after its publication up to the present day, *Summa* has been criticized as having been difficult to read and poorly written in a mixture of bad Italian and bad Latin.[3]

This article investigates whether two such contradictory viewpoints may be "correct" by attempting to identify what may have inspired Pacioli to use the style of instructional writing that he adopted, incorporating switches of language, classical referents and popular sayings to convey ideas, concepts, advice, images and moral values.

When investigating issues of this type that are rooted in a different time and culture from today, the context of the period during which it occurred must be considered or we risk misunderstanding the approach taken (Pin, 1993, p.166). Pacioli wrote *Summa* when the humanist movement was gathering pace in Renaissance Italy and when its influence was widespread through all branches of society, particularly in education and the arts. To this end, the next two sections of this article look at the nature of Renaissance Humanism and at Humanist Education and the influence of his humanist mentors, and Humanism in general, upon Pacioli.

In order that Pacioli's writing style may be set in context, overviews are then presented in the context of Renaissance Humanism, of both *Summa* and the humanist-influenced decoration of the *Sala dell'Udienza* (the *Sala*) in the *Collegio del Cambio* (Moneychangers Guild) in Perugia, the city in which Pacioli held his first university-administered appointment and where he spent more than eight years teaching during the 1470s and 1480s. These are followed by a comparison of the use of language and other devices in Pacioli's *Summa* with the frescos of the *Sala*. Finally, the analysis is discussed and conclusions drawn.

Renaissance Humanism

The Renaissance began in Italy in the late fourteenth century and spread throughout much of Western Europe in the period up to around 1620. Fuelled by the rediscovery of classical Greek and Roman texts, it was brought to life and incorporated into Italian culture by Dante (1265–1321/1993) and, in particular, Petrarch (1304–74). By 1400, it had emerged as a recognizable intellectual movement (Grendler, 2006, p.3). Its influence accelerated with the immigration of refugees bringing large numbers of ancient Greek and Roman texts following the fall of Constantinople in 1453 (Hooker, 1999), an event that virtually coincided with the invention of the moveable-type printing press, which, in turn, enabled such ancient texts to become widely available.

A massive expansion of trade and the growth of the merchant class in Renaissance Italy, both in size and in terms of individual wealth, provided a basis for the development of Humanism, a philosophical ideology that justified activities that had for a long time been discouraged – consideration of self and personal gain – and encouraged the pursuance of personal pleasure in life rather than conducting life in servitude or in deference to others (Kreis, 2004).

Humanism developed from a desire for human expression, for the rediscovery of old values, including the importance of civic virtue and of doing good rather than simply knowing what is good, and the replacement of dogma with truth or, more precisely, convincing argument. It was "a cultural and educational program which emphasized and developed an important but limited area of studies" (Kristeller, 1961, p.10): grammar, rhetoric, poetry, history and moral philosophy – the *studia humanitatis* (the humanities) (Grendler, 2006, p.5).

Adherents to Humanism used the standards of classical Greece and Rome to measure their efforts, learn how to challenge received wisdom, and to seek and embrace change. Those described by the term "*umanista*" ("humanist") were generally scholars, students and teachers of the humanities in the humanist Latin (or "Grammar") schools and in the universities (Kristeller 1961, p.9; Grendler, 1967).

Upon leaving the humanist education system, students worked in many of the governments within Renaissance Italy, as advisers, secretaries, civil servants, lawyers

and judges; and they encouraged people to develop their full potential, to be less narrow in their focus and, to develop their talents in whatever field they lay, so leading to the many polymaths of the Renaissance – the multi-talented 'Renaissance Men' (Grendler, 2006).

Humanist education

During the Middle Ages, education was built around the seven liberal arts,[4] but Humanism changed this, first in Renaissance Italy, then in the rest of Europe. Petrarch led the way by rejecting the Aristotlean-based Scholastic education of the Middle Ages as being too abstract and disembodied, too dry and too scientific (Grendler, 2006, pp.1–2). Vergerio (1368–1444), in the first humanist pedagogical treatise (1402–3) took forward Petrarch's ideas and proposed a more appropriate focus for pre-university study. He advocated the retention of the liberal arts but suggested a very different foundation for education – the study of the humanities.

The humanities were studied in the humanist elementary schools and then further developed in the humanist secondary schools. Two of the subjects taught, grammar and rhetoric, were inherited from the liberal arts *Trivium*. However, the third subject in the *Trivium*, logic, was replaced by poetry, history and moral philosophy.

With its emphasis upon the humanities as a preparation for civil life, humanist education "sought to foster good character and learning in youth and included a strong emphasis on history, moral philosophy and eloquence" (Grendler 1987, pp.341–2) – the art of using language to convince others. To that end, facility of expression, elegance of expression and harmony, imitating the rhetorical skills in Latin recommended by the Roman, Cicero (106–43 BC), were key skills.

Instruction in how to use Latin in the style of the classic authors (that is "good" Latin)[5] was a key element of this education (Grendler, 2006, p.3). It was viewed as training for citizenship and was fostered and flourished in the courts of powerful princes, nobles, monarchs, and in the city and papal states, whose patronage was vital to fund humanist studies and its expression in architecture, art, and printing.[6]

Mathematics was not taught in the Latin schools (Black, 2007). It was taught in the vernacular (that is, spoken language of the day) abbaco schools, which focused on business and its practicalities and, in particular, mathematics relevant to business (Van Egmond, 1981; Radford, 2003). In universities, there was some teaching of mathematics with students studying three of the liberal arts *Quadrivium* - geometry, arithmetic, and astronomy - but not the fourth, music (Kristeller, 1965, p.147; Grendler, 2002). However, there were very few teachers of these subjects in the universities (Grendler, 2002; Black, 2007). Most university students studied one of the three specialist subjects: theology, medicine, or law.

In order to reach a wider audience, adherents to Humanism such as Pacioli's mentor, Leon Battista Alberti (1404–72), were prepared to mediate their humanist

attachment to Latin elegance by using the vernacular (Taylor, 1942, p.110), as Pacioli himself did in *Summa*. They still, however, maintained attachment to Latin as an instructional device, particularly in universities where all instruction and all texts were in Latin (Grendler, 2004, p.3).

Humanist education did not become the norm in northern and central Italian Latin schools until around 1450 (Grendler, 1989, p.139). Pacioli's Latin has been described as "bad", that is Scholastic Latin (see for example Ciocci, 2003, p.24), which suggests he may have received a Scholastic Latin school education. However, this is unlikely. His first job was as tutor to the sons of a Venetian merchant in 1464 (Antoni, 1995, p.266)[7] when he was at most 19, and possibly as young as 16. To have been able to take on this role, it is likely that he attended a vernacular abbaco school, not a Latin school (see Rankin, 1992; Ciocci, 2003, p.16).

The abbaco schools were an alternative to the Latin schools rather than a part of the Latin school educational process. The abbaco school curriculum was taught in the vernacular and Latin was not taught beyond a basic level. The only exception to this was in the city of Florence (Grendler, 1989, 1995).[8]

Three of the greatest influences upon the young Pacioli during the 1460s and early 1470s – Piero della Francesca (1416–92); Federigo, Duke of Urbino (1422–82); and Leon Battista Alberti – were all leaders in their fields, multi-talented, and they all exhibited humanist ideals. Della Francesca was an artist and mathematician (Vassari, 1550; Cossali, 1857; Mancini, 1916; Camerota, 2006); Federigo was a powerful patron and one of the most impressive scholars of his day (Burckhardt, 2002); Alberti was a polymath,[9] and one of the leaders of the humanist education movement (Santayana, 1930).

An example of the influence they had upon Pacioli can be seen in the choice he made when he became a Franciscan friar at some point between 1472 and 1475 (Cavazzoni, 1995). Of the two branches of the order, he elected to join the less restricted Conventuals. They had relative freedom to move where they pleased and could own property. Mirroring the humanist ideal concerning the importance of the individual, the Conventuals believed that some individual freedom and the receipt of rewards was necessary to enable true scholarship to take place and so underpin their work as preachers (Moorman, 1968).

As will be shown in the next section, an examination of Pacioli's *Summa* reveals how he embraced the ideals of Humanism and humanist devices and harnessed them where appropriate to support his writing and to ensure that, in the spirit of Humanism, his work would have widespread contemporary appeal.

Summa

Pacioli's seminal work is a compendium in two volumes. The first volume contains Arithmetic, Algebra, and a variety of subjects of a commercial nature, including

bookkeeping. The second volume contains geometry and trigonometry. The book is reputed to have been purchased by merchants from all over Europe (Favier, 1998), which supports a view that the primary audience for *Summa* was neither mathematicians nor university students of mathematics – vernacular literature was not written for the university-educated or those educated in the Latin schools (Kristeller, 1959/1992, p.24) – but was the abbaco-school-educated Italian merchant class (Strathern, 2001), of which the merchants would have used it as a reference text, which may explain the survival of a large number of copies to the present day (see Sangster, 2007).[10]

The contents of *Summa* reflect two central themes of Renaissance Humanism:

1. It mirrors the humanist ideal of rediscovering classic texts and making them available to the masses. For example, in *Summa*, Pacioli summarized and translated into the vernacular some of the algebra and geometry of Euclid (c.300 BC) and presented it for the first time in a vernacular printed text.

2. It embraced the humanist educational principle of bringing important work, old or new, within the reach of as wide an audience as possible. Through *Summa*, Pacioli sought to disseminate advice and instruction in matters not previously readily available, such as operating a business, and also to give the merchant class an intellectually wider education than that which a simple instructional manual on business would have provided. His success at doing so can be seen in the plaudits the book has received over the last 500 years. For example, it is recognized that:

*   Pacioli was first person to write that the coordination of the rules and accounts of a business not only had to be done, but that it was fundamental and necessary for good governance (Bariola, 1897, pp.369–70);

*   accounting today can be traced directly back to *Summa* (Fogo, 1905; Geisjbeek, 1914);

*   *Summa* laid out the programme for Renaissance mathematics (Rose, 1976);

*   mathematical problems within it led, some 150 years later, to the development of the theory of probability (Strathern, 2001); and,

*   it was important to and assisted in the development of perspective in architecture and art (Ciocci, 2003, p.19).

Pacioli's instructional style exhibits a clear humanist influence through the inclusion of epigrams,[11] laudatory verses,[12] and dedicatory letters to his patron in the Introduction to the book; using language to suit his audience, switching from the vernacular to Latin where appropriate – he even did this with the two rhetorical verses

in the Introduction, one of which is in the vernacular and the other in Latin; and he includes features of humanist writing in the form of autobiographical information and personalized examples in his text.[13]

The next section describes the humanist decoration of the *Sala dell'Udienza* in the *Collegio del Cambio* in Perugia. Parallels are then drawn between it and Pacioli's humanist-influenced instructional style.

The *Sala dell'Udienza* in the *Collegio del Cambio,* Perugia

At the end of the fifteenth century, Perugia was an important trading city in central Italy (Banker, 1997), approximately two-thirds of the way from Venice to Rome; and was developing as an important centre for art and culture (Blanshei, 1979). Various trade and craft guilds had become essential institutions in the life and politics of the city, and membership of one of the guilds was a requirement for political office. Due to the needs of merchants to exchange the various currencies in use, and to facilitate the creation and hiding of loans,[14] few of the guilds were as important to the city as the Moneychangers (or Bankers) Guild.

Construction of the *Collegio del Cambio* building in Perugia on behalf of the Moneychangers Guild was completed in 1457. The programme for the decoration of the entrance hall, where merchants and moneychangers met and discussed their business – the *Sala* – was drawn up by the humanist, Francesco Maturanzio, and the painting of the frescos on the walls and ceiling were commissioned by the guild in 1496 and undertaken by another humanist, Perugino (Blanshei, 1979).[15] The frescos were completed between 1498 and 1500.

The iconography of the *Sala* carries an overwhelmingly moral tone. It is a sophisticated mix of referents with wide appeal that demonstrated the learning, culture and civic status aspired to or attained by those engaged in commerce, by whom and for whom it was built and decorated (Banker, 1997; Fusetti & Virilli, 2003). It was highly fashionable in its day and was the source of influence for other major works of art of the period, including the Sistine Chapel in Rome (Michelangelo) and the Papal apartments in the Vatican (Perugino and Raphael).

The fresco decoration of the *Sala* covers most of the walls above the intarsia panels, and the ceiling. In true humanist style, all the figures and imagery are from classical sources or depict classical figures. For example, Cato the Elder (234–149 BC),[16] is depicted standing guard on the short wall to the right of the entrance, representing civic virtue. Upon entry, the wall on the left depicts the four cardinal virtues in two lunettes: Prudence and Justice in the first (see Figure 1) and Fortitude and Temperance in the second. Between them is a self-portrait of the artist, Perugino. The far-end wall depicts the Transfiguration and the Nativity of Christ as representations, respectively, of the theological virtues of Faith and

Figure 1: *Sala dell'Udienza,* **long wall to left of entrance door, first Cardinal Virtue lunette (of two): Prudence and Justice with Six Antique Wise men[17]**

Charity (Love of God). The main decoration on the remaining wall is a fresco of the third theological virtue, The Almighty appearing before angels, prophets and sibyls (prophetesses), signifying Hope (Redemption).[18]

The ceiling decoration depicts astrological gods chosen to represent the first seven Orders of Angels as described by Dante (1308–21/1993).[19] All are riding in triumphal chariots, a feature inspired by the triumphs of Roman emperors and the *Triumphs* of Petrarch,[20] an iconography that was very much in vogue in this period and featured, for example, on Pierro della Francesca's diptych[21] paintings of Federigo, Duke of Urbino and his wife, Battista Sforza, painted in 1465.

This programme of decoration was elegant, beautiful and, as required for a humanist audience, informed by ancient sources. In keeping with the humanist ideal that pictures should have a dual purpose (to please and to instruct – Fara, 2007) it also included a strong moral philosophical message and, relevant to any comparison with *Summa*, uses Latin in two different ways: as an eloquent appeal and for instructional emphasis. More detailed analysis of the frescos is included in the next section when the devices incorporated within them are compared with the writing style adopted by Pacioli.[22]

Figure 2: *Sala dell'Udienza,* ornamentation of the ceiling, detail of Apollo (centre)

Comparison of the use of language and other devices in Pacioli's *Summa* and in the frescos of the *Sala dell'Udienza*

Summa was written in the vernacular with occasional phrases in Latin; and the same mix of languages was used throughout the book (Antinori, 1995, p.29).[23] The humanist influence on Pacioli's writing in *Summa* is clear to see (Belloni, 1994, p.43) and from that perspective Pacioli's use of the vernacular is not surprising – Dante had done so when he wrote his *Divine Comedy* so as to make it available to the unlearned (Boccaccio, 1355–9/1987, p.263) – and the greatest humanist educators of the mid and late fifteenth century, including Pacioli's mentor, Leon Battista Alberti, promoted the use of the vernacular in order to reach the widest possible audience.

Had Pacioli written *Summa* in Latin, he would have had a major problem: the dead language simply did not have the vocabulary to present and explain mathematical theories without artificially extending it (Taylor, 1942, p.144). More importantly in this context, the merchant class, for whom Pacioli wrote *Summa* (Sangster *et al.*, 2008), was not generally educated in either the scholastic or

humanist schools and so would not typically have had sufficient knowledge of Latin to understand a book written in that language. For a book such as *Summa*, the vernacular was the language to use. However, use of the vernacular was not without its problems. There were a number of regional dialects spoken in what was to become Italy and no clear indicator of which would ultimately become the dominant language.

Some writers (for example Lee, 1989) have suggested that it was printed in Tuscan (Pacioli's native dialect) but with some (mainly) Venetian variants. An alternative and more compelling view of the nature of the vernacular used by Pacioli was offered some 10 years ago by reference to his manuscript book, *De Viribus Quantitatis* (written 1496–1508). In the introduction to its 1997 translation into Italian, Marinoni assesses the quality of Pacioli's language and the style of writing. He concludes (Marinoni, 1997, p.x) that Pacioli's vernacular was neither pure Tuscan, nor Tuscan mixed with Venetian, but a hybrid mixture of dialects from the markets of northern Italy. Belloni (1994) used Pacioli's handwritten Perugian abbaco manuscript of 1478 as an exemplar of the manner in which Pacioli wrote in the vernacular. His conclusion concerning the language used by Pacioli was the same as Marinoni's.

It seems likely, therefore, that the vernacular in *Summa* was typical of the language used by merchants across Northern Italy at that time. As Pacioli's primary goal in writing *Summa* was to educate the merchant class and to do so in the most effective way, a way that set down the foundations upon which future generations of merchants could build, this choice of vernacular made complete sense.

As evidenced by the printing of a second edition of *Summa* in 1523, this proved an appropriate form of the language for a book intended mainly for the merchant class. Further support for the appropriateness of the form of the vernacular adopted by Pacioli in *Summa* is given by its having been highly praised by his patron, the humanist, Guidobaldo, Duke of Urbino (Taylor, 1942, p.196) who was neither Venetian nor Tuscan. But *Summa's* hybrid vernacular plus occasional Latin later led to some commentators echoing Caxton,[24] and describing the language used as barbaric (see for example Franci & Rigatelli, 1985, p.62; Yamey, 1994, p.18).

Pacioli did, nevertheless, include some Latin in *Summa*. His motivation to do so can be explained through consideration of its similar use in the *Sala* frescos.

Use of Latin in the *Sala* and by Pacioli

The vernacular is not used in the humanist-designed and decorated *Sala*. Latin is the only language in evidence and, in all cases, the Latin in the *Sala* is either of an instructional nature or statements of significant prophetic truths. It is employed in two ways,

each of which was commonly in use in paintings during the Renaissance. The first is in rhetorical verse, in the form of either laudatory verse or epigram, within or alongside the images. These styles of rhetoric were very popular in humanist circles, and much admired. One example in the *Sala* is a laudatory verse below the self-portrait of Perugino (translated from Guerrini, 2004, p.419):

> If the art of the painter had been lost by now,
> He has reclaimed it.
> But if it had not been invented up until now,
> He has created it.

Another is below the portrait of Cato (Figure 3) reflecting his moral stance. Fusetti and Virilli (2003) translate it as:

> Whoever you are, whether you stand up to pronounce a discourse with
> solemn words
> Or you hasten to render justice to the people,
> Set aside your own affections.
> He whose heart is troubled by love or hatred
> Cannot follow the straight path

Figure 3: *Sala dell'Udienza,* Cato standing above an epigram in Latin

Laudatory verses and epigrams were intended to please the ear and the senses and to impress the courts of Humanism. They were used to draw attention to the importance of civic virtue and responsible citizenship, to the importance of using rather than ignoring the learning of ancient times, and to encourage the adoption of humanist views of reason (science).

The text in the laudatory verses below Perugino and Cato were written for the *Sala* but in the classical style. The second use of Latin in the *Sala* is in the form of quotations from ancient classical authors.

Mirroring this form of use of Latin in the *Sala*, Pacioli uses Latin in *Summa* to draw attention to things of importance, invariably exaltations to good practice or moral behaviour. In doing so, he is, in effect, applying the art of oratory to instructional writing and, in so doing, supporting and promoting the moral philosophy facet of Renaissance Humanism.[25]

In an obvious attempt to add a stamp of courtly style and as a marker of the humanist movement, on the reverse of the first folio of the Introduction to *Summa* (the front of the page being a list of contents) and after a dedicatory letter to the Duke of Urbino, Pacioli included two rhetorical verses. One of these verses is in Latin: "The Epigram of Fra Pompilius to the Reader". It praises the book and Pacioli (Lucas) to its readers. Taylor (1942, pp.188–90) presents an abbreviated translation:

> The things which have been wasting away in the midst of their hiding places
> Lucas oh friend has restored to you
> Specks of everything in the world.
> Whatever you want to do and where you live this book will help you.
> Most books do not. Consequently this book is unique.

Both this verse and Perugino's verse in the *Sala* are designed to make claims of authority through rhetoric and reference to recovered ancient authority.

There is, nevertheless, a significant difference between the use of Latin in *Summa* and in the *Sala*. Pacioli used Latin in *Summa* not to make a separate point, but for emphasis. He enabled his readers to always understand what was being said in Latin by either following the Latin immediately with a translation into the vernacular or by making the same point in the surrounding vernacular text. The Latin drew the attention of readers to the surrounding text as they searched for its meaning or, if they understood the Latin, as they noted the repetition of what was being said in Latin, so emphasizing the point being made.

For example, in Chapter 4 of the bookkeeping treatise in *Summa* there is a quotation from St Mathew's Gospel in Latin, "*Primum quaerite regulum dei, et haec omnia adjicietur vobis*" followed immediately by its translation in the vernacular. The original text in *Summa* is shown in Figure 4.

Figure 4: Extract from *Summa* folios 199 verso (back) and 200 recto (front)
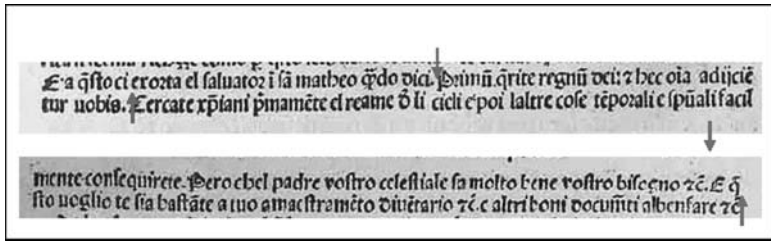
The quote in Latin is between the first two arrows, Pacioli's translation of the phrase into the vernacular is between the second and the third arrows, and the final sentence of the chapter (in the vernacular) is between the third and fourth arrows.

Other examples of the use of Latin to emphasize a point, all taken from the bookkeeping treatise, include:

- In Chapter 1, "*Ubi non est ordo, ibi est confusio*" ("where there is no order there is confusion") followed by the translation into the vernacular.

- In Chapter 4, a quotation from municipal law, first in Latin "*vigilantibus et non dormientibus Jura subveniunt*" ("the law helps those that are awake, not those that sleep") followed by the vernacular translation.

- In Chapter 4, Pacioli reinforces his advice for merchants to keep God in their mind and to observe mass daily and assures them that they will not lose their riches by this means with a quotation from Latin verse: "*Nec caritas opes, nec missa minuit iter etc.*", this time with no translation but the surrounding text conveys a similar message to the one given in Latin.

In contrast, the imagery of the frescos provides a second message rather than an explanation of the Latin phrases. Consequently, anyone who did not understand the Latin in the *Sala*, would have needed someone to provide an explanation before they could understand the message being conveyed in Latin.

Nevertheless, just as it was used in *Summa*, Latin was used in the *Sala* to make a point. It was also designed to impress. It represented quality and old and good values and, irrespective of whether they understood Latin or not, it would have instilled in those within the *Sala* the perception that this was a "good" place to be, a place of virtue, and a place of trust – a place of high morals and ethics; and a place where humanist ideals were to be found and followed.

The use of Latin in *Summa* and in the *Sala* is consistent with the humanist goal to engage and persuade the intended audiences. In both cases, use of Latin carries strong moral philosophical messages as well as portraying eloquent rhetorical style. As in the *Sala*, all the Latin phrases used by Pacioli are of an instructional nature or are statements of significant prophetic truths. In both *Summa* and the *Sala*, the intended audience was the same – merchant society, and its noble patrons. The humanist symbolism discussed in the following section served a similar purpose.

Use of other humanist symbolism by Pacioli and in the *Sala*

Similarly to Pacioli's use of Latin being comparable to its use in the *Sala*, there are symbolic similarities between *Summa* and the frescos in the *Sala*, including references to the Cardinal Virtues, Theological Virtues, Dante and Cato.

Virtues and sayings

The virtues have a dominating presence in the *Sala*. The four Cardinal Virtues (Fortitude, Temperance, Justice and Prudence) are shown in the frescos along the long left wall with Latin inscriptions on waxed tablets or, more probably, slateboards; and the three Theological Virtues (Faith, Hope and Charity) appear in the other major frescos. In the fresco of the third theological virtue (God appearing before angels, prophets and sibyls), the prophets and sibyls each carry a "phylactery" – a sash or a scarf – bearing a Latin inscription which emphasizes their part in the prophecies of the Redemption, quoted in each case from "The Divine Institutions" by Lactantius (c.240–320 AD). For example, the text on Isaiah's phylactery translates from the Latin as "*behold the Virgin, she will conceive*".

The frescos abound with such Classical referents of Humanism. Each of the Cardinal Virtues is supported by three prominent figures from Roman and Greek antiquity, identified by their inscriptions. The source for the inscriptions was "*Factorum ac dictorum memorabilium libri IX*" [Nine books of memorable deeds and sayings] by Valerius Maximus (c.20BC–50AD), in which morals and civic virtues were represented by exemplary figures from ancient history. The book was deemed of such importance to the humanist movement that it was one of the first books to be printed in Venice after the first press was established in the city in 1469, and was widely used in humanist circles during the Renaissance (Fusetti & Virilli, 2003, p.16).

Pacioli did not adhere to the strictly humanist approach adopted in the *Sala* of using classical ancients as the sources of authority. Rather, he adopted a practice common among Franciscan preachers of taking material from whatever source it can be found (Moorman, 1968) to present wisdom, moral ideals and good

196

habits. He used Cato, an ancient authority, and Dante, an early humanist authority, and references to scripture; and he took wisdom from wherever it could be found in everyday parlance, common sayings, municipal law, and from commonly known poets. His wider use of sources clearly presented him with many more examples that could be used than would have been the case had he adopted a strictly humanist ancient-classical-based approach.

In the bookkeeping treatise, Pacioli uses these sayings mainly in Chapters 1 and 4, where he is laying out the approach he wants his readers to adopt; and in the concluding chapters, 34 and 35, when they act as reminders. He is particularly keen on emphasizing watchfulness and hard work. Three examples from Chapter 4 illustrate the manner in which he did this. (All translations are from Crivelli, 1924, p.17):

A common proverb:

> A merchant rightly resembles a cock which, among other things, is the most watchful bird that exists. In winter or in summer it makes its nocturnal vigils, at no time resting.

On the same theme but quoting from municipal law:

> Help comes to him who is watchful and not to him that sleeps.

And then another common proverb:

> A merchant's head is also compared to one that has a hundred eyes; yet these are not enough for him, either in words or in actions.

Pacioli utilizes sayings and expounds good deeds in line with, and in a manner that mimics, those of Valerius Maximus. Furthermore, and in keeping with both his role as a friar and the humanist education principle of education for a good Christian life, he opens and closes his instructions with exhortations that each page of a merchant's books should start with the Sign of the Cross and that God's name should always be kept in mind. For example, in the opening part of Chapter 3 of the bookkeeping treatise, he gives an example of how to head up an inventory and states that you should start with the following form of words in the vernacular: "*Al nome de dio. 1493. a di. 8. novembre in venegia*" ("In the name of God, this 8th day of November, 1493, in Venice").

Dante and Cato: the order of the world and civic virtue

The ceiling of the *Sala* shows the ruling of human destiny by the astrological signs and seven of the Orders of Angels (as defined by Dante). It clearly situates the moneychangers and merchants (who undertook their business in the *Sala*) beneath the established order of the heavens. Pacioli also uses Dante, in Chapter 4 of the bookkeeping treatise:

> Alas my son, it is necessary that you shake yourself, for one does not attain to fame in fine feathers and under quilts. He who wastes his life under these

leaves only a trace similar to that left by smoke in the air or by foam on the water. (Crivelli, 1924, pp.17–18)

Similarly to the use of the planetary gods by Dante within the *Sala* to represent moral order, they are featured by Pacioli in another reference to Dante concerning Mars, this time mirroring the victory represented by the chariots on the ceiling of the *Sala*.

Work should not seem strange to you, for Mars never granted victory to those who resting fed themselves. (Crivelli, 1924, p.18)

The guardianship of Cato at the entrance to the *Sala*, with his associations with defence of civil order, suggests lofty aspirations that both demand an exemplary standard of public morals and flatter Perugia by implying that the city values can be equated with those of the great days of the Roman Republic. Pacioli uses Cato in a similar manner in *Summa*, advising merchants to learn to be good mathematicians and to be careful to learn practices from teachers of good reputation, as translated by Taylor (1942):

… keep in mind the precept of Cato the moralist: "Learn from the wise and teach the ignorant yourself, that is, don't learn from ignoramuses who have more leaves than grapes." (Taylor, 1942, p.63)

Discussion and Conclusion

This article has found that Pacioli's choice of language and use of classical referents and sayings to convey ideas, concepts, advice, images and moral values in *Summa* appears to have been deliberate; it was calculated and was consistent with the humanist age in which Pacioli wrote his text. Analysis of the *Sala* frescos and of the text of *Summa* suggests that what Perugino did with his palette, Pacioli did with his pen.

The humanist programme of the decoration in the *Sala dell'Udienza* is a sophisticated mix of referents with wide appeal that demonstrated the learning, culture and civic status aspired to or attained by those engaged in commerce by whom and for whom it was built. As the decoration of the *Sala* claims a status for merchants and moneychangers, so Pacioli claims a status for merchants in *Summa*.

Pacioli's *Summa* was an innovation in its day. He called it a "compendium". It contained the mathematics that was taught in the abbaco schools (Franci & Rigatelli, 1985), thus inferring that it was primarily intended for the merchant class and their abbaco teachers and that it was designed to help them in the conduct of commercial affairs (Sangster *et al.*, 2008). Supporting this view, Pacioli is considered to have been one of the foremost teachers of abbaco (Camerota, 2006, p.327) and as much a teacher of abbaco as he was a master of theology (Ciocci, 2003). *Summa* is considered to be the finest example, and the most comprehensive of all the abbaco texts (Jayawardene, 1971; Rowland, 1995), and to have represented

a bridge between the practical knowledge of technicians and merchants and the theoretical teaching of mathematics in university (Ciocci, 2003, p.23).

The examples presented in this article illustrate the moral authorities Pacioli appealed to in his writing and also his teaching (*Summa* is written in the style of a didactic lecture); and show that his approach has major humanist similarities with the moralist programme used in the decoration of the *Sala dell'Udienza*.

In *Summa*, Pacioli embraces humanist education ideals. His choice of language would have been careful and deliberate. In his introductory letters and dedications in *Summa*, as well as through the inclusion of two rhetorical verses, he demonstrated that he understood how to appeal to his fellow humanists, admirers and patrons. He also used Latin as an instructional device in a humanist style that had significant similarities with its use in the *Sala*; and he did so in a manner that ensured that his message was conveyed as widely as possible – an ideal of humanist education.

As mentioned in the Introduction to this article, Pacioli had a range of choices open to him concerning the language and writing style to use when he wrote *Summa*. He would have known precisely how to pitch his text for his intended audience and he clearly sought to do so both through his choice of language and in his incorporation of humanist referents within the text.

His writing in *Summa* sought wide and public appeal among the merchant class in a way not dissimilar to that sought by the designers of the decoration in the *Sala*, and so educated and impressed its merchant class audience. *Summa* was a serious attempt inspired, at least in part, by humanist thinking in education, to raise the level of business education for those running and those aspiring to run businesses. His use of the vernacular in *Summa* was appropriate for such a text and his use of Latin and instructional imagery in a humanist style was consistent with and would have appealed to those for whom the book was intended.

In answer to the question posed at the start of this article, the views of both the critics of Pacioli's writing style and of his supporters were justified within their own contexts and time. However, as the "Italian language" was developing rapidly in the late fifteenth century, Pacioli's critics ought, perhaps, to have paid more heed to that situation and, possibly, to the mores of printing in 1494 before lambasting him in the manner they did. In 1525, Tuscan was identified as the preferred source dialect for a common Italian literary language. It is unsurprising, therefore, that 50 years after publication *Summa*'s hybrid version of the vernacular was considered outdated by, for example, the mathematician Federico Commandino (1509–75)[26] and in need of updating to the Italian of the mid-sixteenth century.

Pacioli's Latin in *Summa* was clearly not the "good" Latin of the classical period aspired to by the humanists. However, it does not matter which form of Latin it was. What mattered was whether Pacioli's Latin was a correct representation of the form of Latin he was using (Grendler, 2006). Given the praise his writing received at the time of its publication, it probably was.

Despite the linguistic criticisms it received, for 50 years from 1494, Pacioli's *Summa* was the most widely read book on mathematics in Italy (Olschki, 1918) and the fitness-for-purpose of Pacioli's use of language and style in late fifteenth-century Italy is indisputable. Further, Pacioli's humanist-inspired literary style was undoubtedly a significant step in the search for an appropriate style of educational writing in the humanist tradition and represented a major step in the development of humanist approaches to education – approaches which are still important in education today.

Pacioli was a man of his time. His primary goal was to educate those around him, to help them in their daily lives. He was writing for men of his world, not the world of 50 or 500 years into the future. That he did both is a testament to the quality of the contents of *Summa* and to his choice of style of composition and writing, irrespective of how they were presented linguistically.

Had Pacioli not adopted a humanist style in writing *Summa*, it is doubtful that it would have gained either the audience or the dissemination it did, doubtful if mathematics would have progressed at the pace it achieved in the sixteenth century, and questionable whether bookkeeping would have developed along the lines he set down. Indeed, his bookkeeping treatise would possibly have remained as unknown and uninfluential as the five pages on bookkeeping written in Naples in 1458 by the Ragusan, Benedikt Kotruljević (also known as Benedetto Cotrugli), but not published until 1573 (Tucci, 1990), by which time the world had moved on, a fate that certainly did not befall *Summa* when it was published in 1494.

Notes

1. *Particularis de Computis et Scripturis*, the bookkeeping treatise contained within Pacioli's *Summa*.
2. See, for example, Moorman (1968) and Lesnick (1989).
3. See, for example, Yamey (1994). See Belloni (1994) for a discussion of the variability of the Italian in *Summa*.
4. For more details on the background to and nature of the seven liberal arts of the *Trivium* and the *Quadrivium*, see, for example, Grant (1999a,b). For a summary of what constituted the seven liberal arts, see http://members.aol.com/oldenwilde/members/diu/quadriv.html (available on 6 May 2007). Black (2007) stated that teaching of *all* seven liberal arts had ceased in Italian schools, possibly by the eleventh century and certainly by the thirteenth century.
5. Petrarch divided history into eras defined by their culture. The Roman or Classical or Ancient era, which ended in the fourth century, was typified by the use of "good" Latin. In the Middle Ages that followed it, people learnt and wrote "bad" Latin (Grendler, 2006, pp.3–4).
6. For further information on humanist education, see, for example, Santayana (1930) and, for the development of humanism in general, Kristeller (1965) and Burkhardt (2002).

7. Researchers disagree over the actual date, some suggesting 1465.
8. In Florence, abbaco was taught for a couple of years part-way through school education.
9. Alberti was a philosopher, architect, musician, painter, sculptor, poet, cryptographer, polyglot, and writer, as well as being one of the leaders of the development of humanist education.
10. Notwithstanding the primary market for *Summa* not being university mathematic students, it was considered such an important book on mathematics that Bernardino Baldi (1553–1617), the renowned humanist of the Court of Urbino (and first biographer of Pacioli), who wrote the first European history of mathematics between 1587 and 1595, *Vite de' Matematici*, promoted the reading of *Summa* to friends and colleagues.
11. A short, witty poem expressing a single thought or observation.
12. Verses expressing or conferring praise.
13. See Kristeller (1992) for further details on the manner in which texts were written in Renaissance Italy
14. Loans on which interest was paid, though essential to commerce, were officially regarded by the Church as usurious and un-Christian. Moneychangers, therefore, trod on lucrative and essential but tricky ground – see, for example, De Roover (1974).
15. Pietro Vanucci who, according to Taylor (1942), was a good friend of Pacioli during his time in Perugia.
16. A Roman politician and the first Censor of the Senate (charged with guarding public moral). He also wrote poetry and moral aphorisms.
17. Figures 1 to 3 are reproduced with permission from Fusetti, S. and Virilli, P., (2003). The Collegio del Cambio in Perugia – Comments on the restoration, Assisi: Editrice Minerva.
18. This interpretation of these three religious scenes is taken from Fusetti and Virilli (2003).
19. The acknowledged source of the planetary deities in the *Sala* is a set of prints first published in Florence in 1464 (Fusetti & Virilli, 2003).
20. " … Love triumphs over Man; Chastity triumphs over Love; Death triumphs over both; Fame triumphs over Death; Time triumphs over Fame; and Eternity triumphs over Time". *The Sonnets, Triumphs, and Other Poems of Petrarch. By Various Hands, with a Life of The Poet by Thomas Campbell,* p.CXL (http://www.gutenberg.org/dirs/1/7/6/5/17650/17650-h/17650-h.htm).
21. A work consisting of two painted or carved panels that are hinged together. This example had the portraits of Federigo and his wife hinged together and, on the back, paintings of triumphal chariots.
22. For further examples of the decoration within the *Sala*, see http://www.wga.hu/index1.html and http://pietro-perugino.gemaelde-webkatalog.de/

23. "Inoltre, il Trattato XI risulta perfettamente integrato negli Trattati: stessa lingua usata, stesse espressioni idiomatiche, stessi proverbi, stesse citazioni bibliche che si trouvano più volte anche negli altri trattati."

24. Concerning the standard of English in a book of the fourteenth century, in 1490 the pioneering English printer William Caxton (1490/1963) wrote: "[I] took an old book and read therein, and certainly the English was so rude and broad that I could not well understand it. And also my lord Abbot of Westminster had shown to me recently certain evidences written in old English for to translate it into our English now used. And certainly it was written in such a manner that it was more like Dutch than English. I could not translate it nor bring it to be understood".

25. It was suggested by one referee that this occasional use of Latin adopted by Pacioli was typical of vernacular printed books of the fifteenth century. We draw attention to it because his use of Latin appears premeditated and for a specific purpose, rather than simply because it was the "normal thing to do".

26. Commandino was considered the seminal translator of Euclid from Greek into Latin (1562) and then into Italian (1575). He also translated works by Archimedes, Ptolemy, Aristarchus, Pappus, Apollonius, Eutocius, Heron and Serenus. He founded the Urbino School of Mathematics, thought highly of the content of *Summa* and introduced the work to other important mathematicians of his day (Rose, 1976).

References

Antoni, T. (1995), "Luca Pacioli e lo Studio Generale di Pisa", in *Convegno Internazionale Straordinario per Celebrare Fra' Luca Pacioli*, Venezia, Aprile 1994, Corsico (MI): Ipsoa Scuola D'Impressa, pp.265–9.

Antinori, A. (1995), "Luca Pacioli e il trattato XI", in *Convegno internazionale Straordinario per Celebrare Fra' Luca Pacioli*, Venezia, Aprile 1994, Corsico (MI): Ipsoa Scuola D'Impressa, pp.27–37.

Banker, J.R. (1997), "The Social History of Perugia in the Time of Perugino", in Becherer, J.A. (ed.), *Master of the Italian Renaissance: Pietro Perugino*, New York: Rizzi International, pp.37–52.

Bariola, P. (1897), *Storia della Ragioneria Italiana*, Milano: Presso L'Autore. Facsimile reprint (1998) Amaduzzi, A. (ed.), Bari: Cacucci Editore.

Belloni, G. (1994), "Nota Filologico-Linguistica", in *Luca Pacioli Trattato di Partita Doppia Venezia 1494*, Venice: Albrizzi Editore, pp.35–44.

Black, R. (2007), personal communication, May.

Blanshei, S.R. (1979), "Population, Wealth, and Patronage in Medieval and Renaissance Perugia", *Journal of Interdisciplinary History*, Vol.9, No.4, pp.597–619.

Boccaccio, G. (1355–9/1987), "Trattatello in laude di Dante", in Grendler, P.F. (ed.), *An Italian Renaissance Reader*, Toronto: Canadian Scholars Press, pp.212–73.

Burckhardt, J. (2002), *The Civilisation of the Renaissance in Italy*, New York: The Modern Library, Random House.

Camerota, F. (2006), "Teaching Euclid in a Practical Context: Linear Perspective and Practical Geometry", *Science & Education*, Vol.15, No.2–4, March, pp.323–34.

Cavazzoni, G. (1995),"Tractatus Mathematicus ad Discipulos Perusinos (Parte Prima)", in *Convegno Internazionale Straordinario per Celebrare Fra' Luca Pacioli*, Venezia, April 1994, Corsico (MI): Ipsoa Scuola D'Impressa, pp.39–48.

Caxton, W. (1490/1963), "Preface from the Preface to Enydos", in Culley, W.T. and Furnivall, F.J. (eds.), *Caxton Eneydos*, London: Early English Text Society. (http://www.courses.fas.harvard.edu/~chaucer/language.html)

Ciocci, A., (2003), *Luca Pacioli e la Matematizzazione del Sapere nel Rinascimento*, Bari: Cacucci Editore.

Cossali, P.D.P. (1857), *Elogio di Fra Luca Pacioli*, Roma: Tipografia Delle Belle Arti.

Crivelli, P. (1924), *Double-Entry Book-Keeping, Fra Luca Pacioli*: an original translation, London: Institute of Book-keepers.

Dante, A. (1308–21/1993), *The Divine Comedy*, in trans. Sisson, C.H., intro. Higgins, D.H., Oxford: Oxford University Press.

De Roover, R. (1974), "Scholastic Economics: Survival and Lasting Influence from the Sixteenth Century to Adam Smith", in Kirshner, J. (ed.), *Business, Banking and Economic Thought in Late Medieval and Early Modern Europe*, London: University of Chicago Press, pp.306–35.

Fara, P. (2007), *Introduction to Scientific and Medical Portraits*, London: Roger Gaskell. (http://www.rogergaskell.com/pdfs/gaskell%20portraits.pdf)

Favier, J. (1998), *Gold and Spices: The Rise of Commerce in the Middle Ages*, trans. Higgitt, C., New York: Holmes and Meier.

Fogo, J.R. (1905), "History of Book-Keeping", in Brown, R. (ed.), *A History of Accounting and Accountants*, Edinburgh: Augustus M. Kelly Publishers/Institute of Chartered Accountants of Scotland. [2004 reprint (New York: Cosimo Books Inc.), pp.93–170]

Franci, R. and Rigatelli, L.T. (1985), "Towards a History of Algebra from Leonardo of Pisa to Luca Pacioli", *Janus*, Vol.72, No.1–3, pp.17–82.

Fusetti, S. and Virilli, P. (2003), *The Collegio del Cambio in Perugia: Comments on the Restoration*, Assisi: Editrice Minerva.

Geijsbeek, J.B. (1914), *Ancient Double-Entry Bookkeeping*, Denver, CO: Geisjbeek.

Grant, H. (1999a), "Mathematics and the Liberal Arts", *The College Mathematics Journal*, Vol.30, No.2, pp.96–105.

Grant, H. (1999b), "Mathematics and the Liberal Arts II", *The College Mathematics Journal*, Vol.30, No.3, pp.197–203.

Grendler, P.F. (1967), "Five Italian occurences of Umanista, 1540–1574", *Renaissance Quarterly*, Vol.20, No.3, pp.317–25.

Grendler, P.F. (1987), *An Italian Renaissance Reader*, Toronto: Canadian Scholars Press.

Grendler, P.F. (1989), *Schooling in Renaissance Italy: Literacy and Learning*, *1300–1600*. Baltimore, MD: The Johns Hopkins University Press.

Grendler, P.F. (1995), "What Piero Learned in School: Fifteenth-century Vernacular Education", in Lavin, M.A. (ed.), *Piero della Francesca and his Legacy, Studies in the History of Art 48*, Washington, DC: National Gallery of Art, pp.161–74.

Grendler, P.F. (2002), *The Universities of the Italian Renaissance*. Baltimore, MD: The Johns Hopkins University Press.

Grendler, P.F. (2004), "The Universities of the Renaissance and the Reformation", *Renaissance Quarterly*, Vol.57, Spring, pp.1–42.

Grendler, P.F. (2006), *The European Renaissance in American Life*. Westport, CT: Praeger.

Guerrini, R. (2004), "Tituli e Tradizione Classica in Perugino", in *Atti del Convegno Internazionale di Studio*, 25–8 Ottobre 2000, Perugia: Volumnia, pp.415–22.

Hooker, R. (1999). *Humanism*. (http://www.wsu.edu/~dee/REN/HUMANISM.HTM)

Jayawardene, S.A. (1971), 'Luca Pacioli', in Gillispie, C.C. (ed.), *Dictionary of Scientific Biography*, New York: Charles Scribners Sons, pp.274–77. (http://www.chlt.org/sandbox/lhl/dsb/page.269.php)

Kreis, S. (2004), *Renaissance Humanism*. (http://www.historyguide.org/intellect/humanism.html)

Kristeller, P.O. (1959/1992), "The Scholar and his Public in the Late Middle Ages and the Renaissance", in Mahoney, E.P. (ed.), *Medieval Aspects of Renaissance Learning*, New York: Columbia University Press.

Kristeller, P.O. (1961), *Renaissance Thought, the Classic, Scholastic, and Humanist Strains*, New York: Harper Torchbooks.

Kristeller, P.O. (1965), *Renaissance Thought II, Papers on Humanism and the Arts*, New York: Harper Torchbooks.

Kristeller, P.O. (1992), "Medieval Aspects of Renaissance Learning", Mahoney, E.P. (ed.), *Medieval Aspects of Renaissance Learning*, New York: Columbia University Press, pp.3–25.

Lee, G.A. (1989), "Manuscript Additions to the Edinburgh University Copy of Luca Pacioli's *Summa de Arithmetica*", *Abacus*, Vol.25, No.2, pp.125–34.

Lesnick, D.R. (1989), *Preaching in Medieval Florence: The Social World of Franciscan and Dominican Spirituality*, Athens and London: The University of Georgia Press.

Mancini, G. (1916), "L'opera 'De coribus regolaribus' di Piero Franceschi detta Della Francesca, usurpata da Frà Luca Pacioli", Memorie della R. Academia dei Lincei VXIV, VIIB, pp.441–580.

Marinoni, A. (1997), *Introduction to Luca Pacioli's De Viribus Quantitatis*, Milan: Ente Raccolta Vinciana.

Moorman, J. (1968), *History of the Franciscan Order: From its Beginnings to 1517*, Oxford: Clarendon Press.

Nobes, C. (1995), "Pacioli's Tractatus: A Note on a Mystery and a Review of Some Commentaries", *Accounting, Business and Financial History*, Vol.5, No.3, pp.379–84.

Olschki, L. (1918), *Geschichte der Neusprachlichen Wissenschaftlichen Literatur* [History of the New Linguistic Scientific Literature], Heidelberg: Carl Winter.

Pacioli, L. (1494), *Summa de Arithmetica, Geometria, Proportioni et Proportionalita*, Venice: Paganino de' Paganini.

Pacioli, L. (1496–1508), *De Viribus Quantitatis*, Manuscript Codice 250 held by the Biblioteca Universitaria di Bologna.

Pin, A. (1993), "The Contribution of Luca Pacioli to the Development of Business accounting", *Economic Notes by Monte dei Paschi di Siena*, Vol.22, No.2, pp.161–77.

Radford L. (2003), "On the Epistemological Limits of Language: Mathematical Knowledge and Social Practice during the Renaissance", *Educational Studies in Mathematics*, Vol.52, No.2, pp.123–50.

Rankin, F.K.C. (1992), "The Arithmetic and Algebra of Luca Pacioli (c.1445–1517)", PhD thesis, Warburg Institute, University of London.

Rose, P.L. (1976), *The Italian Renaissance of Mathematics*, Genève: Librairie Droz.

Rowland, I.D. (1995), "Abacus and Humanism", *Renaissance Quarterly*, Vol.48, No.4, pp.695–727.

Sangster, A. (2007), "The Printing of Pacioli's *Summa* in 1494: How many Copies were Printed?", *Accounting Historians Journal*, Vol.34, No.1, pp.125–46.

Sangster, A., Stoner, G. and McCarthy, P.A. (2008). "The Market for Luca Pacioli's Summa Arithmetica", *Accounting Historians Journal*, Vol.35, No.1, pp. 111–34.

Santayana, S.G. (1930), Renaissance Educators, Boston, MA: Warburg Institute, Meador Publishing.

Strathern, P. (2001), *Dr Strangelove's Game: A Brief History of Economic Genius*, London: Hamish Hamilton.

Taylor, R.E. (1942), *No Royal Road: Luca Pacioli and his Times*, Chapel Hill: North Carolina University Press.

Tucci, U. (1990), *Benedetto Cotrugli, Raguseo, Il Libro dell'Arte di Mercatura*, Venezia: Aresenale Editrice.

Van Egmond, W. (1981), *Practical Mathematics in the Italian Renaissance: A Catalog of Italian Abbacus Manuscripts and Printed Books to 1600*, Firenze: Editrice Giunti Barbèra.

Vassari, G. (1550), *Le Vite de' piú Eccellenti Architetti, Pittori, et Scultori Italiani, da Cimabue Insino a' Tempi Nostril*, Firenz: Lorenzo Torrentino.

Vergerio, P.P. (1402–3), *De Ingenius Moribus et Liberalibus Studiis Adulescentiae* [On Noble Customs and Liberal Studies of Adolescents], in Grendler, P.F. (ed.) *An Italian Renaissance Reader*, Toronto: Canadian Scholars' Press, pp.341–61.

Yamey, B.S. (1994), "Luca Pacioli, the Summa and De Scriptus" in Gebsattel, A. and Yamey, B.S. (eds.), *Luca Pacioli: Exposition of Double Entry Bookkeeping, Venice 1494,* Venice: Albrizzi Editore, 11–33.

Yamey, B.S. (2004), "Pacioli's De Scripturis in the Context of the Spread of Double Entry Bookkeeping", *De Computis: Revista Española de Historia de la Contabilidad*, No.1, Diciembre, pp.142–54.

Address for correspondence: Alan Sangster, Middlesex University Business School, The Burroughs, Hendon, London NW4 4BT, UK. E-mail: a.j.a.sangster@btinternet.com