Optimising Cash Flows in APVIOBPCS Using Control Theory

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Abstract

This paper describes the investigation of the effects on profitability of production for an Automatic Pipeline, Variable Inventory and Order Based Production Control System (APVIOBPCS) production system using as an example electronic RAM production. The investigation includes varying the order up to inventory and WIP using PID control to illustrate the profit gain from using control analysis. The results show that greater PID gains for a controlled production system will increase profitability. The value of desired inventory level increases profitability with a defined optimum value.

Keywords: *Optimisation, APVIOBPCS, Production, Control Theory, Inventory.*

Introduction

The current economic climate, after a recession, has made the economic consequences of inventory control even more important than usual. Lalonde and Pohlen [1] identify a critical need for supply chain management to deal with costs on a regular basis. Benita Beamon [2] in her important review of 1997 discusses criteria used by researchers to categorise supply chain design methods. Many of these include cost as an essential criterion. She categorised supply chain performance measures involving economics as:

- Cost minimisation
- Sales Maximisation
- Profit maximisation
- Inventory investment minimisation
- Return on investment maximisation

These have been investigated by a number of authors, but they have not proposed a single definitive solution.

Costs are a significant issue in the design and manufacturing process which have been revolutionised in recent years by the availability of the DFM-C software systems [3]. However, these systems do depend critically on the rate of production and knowledge of the interrelationship between design/production costs and supply chain costs. It is only in the last few years that analysts have paid any attention to marketing trends in the sales problem [4].

In order to make any realistic offers to customers they would need to know the implications of their choices on their inventory strategy, delivery and profitability. The purpose of this paper is to explore the development of a model that can be used in real-time to compute profit and costs of various inventory ordering policies. Disney and Grubbström [5] have examined the APVIOBPCS model, which is used here, to obtain analytic expression for the economic performance of a generalised order-up-to policy for random demands. They show that an optimal costs solution is no longer the same as that for inventory alone. Several authors have used Net Present Value (NPV) Analysis to investigate e-commerce [6], disassembly processes [7], MRP systems [8]; EOQ models [9] and make –to-order and make-to-stock systems [10]. Most business operators do not

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understand NPV and it may not be as relevant in the current almost low interest economic climate. This work is an extension of earlier work by White and Censlive [11] that showed that the profitability was better with PID control. This paper will show how the profitability can be controlled with the PID gains.

APVIOBPCS model

The problems of production and inventory have posed significant problems throughout the history of manufactured goods. Many different analytical techniques, such operational research methods, have been used with no one technique being wholly adequate in providing all the necessary solutions to planning and optimisation. However the consideration of the information flows, the effects of batch sizes and scheduling has now reached a position where the economic penalties of excess stock and the western poor practices are thrown into sharp relief by lean production regimes, such as that practiced by Toyota in the global market place. Disney & Towill [12] at Cardiff University, White and Censlive [13] and others have analysed Supply Chains with Automatic Pipeline, Inventory and Order Based Production Control Systems (APIOBPCS) and more latterly APVIOBPCS, extensively, to determine system stability and optimisation [14]. In these systems, the actual inventory at the distributor is compared with the agreed re-order point. The reorder decision is arrived at by balancing consideration of Customer Service Levels (CSL) whilst not building up excessive stocks.

In this APVIOBPCS model of a factory and sales system we see that the distributor produces virtual sales orders assuming a typical pattern of behaviour. These orders are further modified by the factory using human experience of the learning curve over time (this has the effect of producing an exponential delay). This is added to a fraction of the inventory error, plus a fraction of the Work In Progress (WIP) error. This comprises the order rate, which then will after a delay, cause production to be completed. From this completion rate the virtual sales rate is subtracted and the excess accumulation of these products leads to the inventory.

In the 1960's Forrester [15] applied the methods of Industrial Dynamics simulation to the problem of production systems. Forrester devised the method of System Dynamics to describe problems that were not amenable by other means, primarily those, which included human decisions. The principle arguments were that ALL such processes could be described by the feedback loops inherent in the information flows and the delays represented in those flows. Since he recognised the inherent non-linearity of all the processes being described he was only able to use a basic continuous simulation package DYNAMO to obtain numerical results [15]. He used exponential transfer functions to represent the delays, as he believed that they were a better representation of physical reality for many real situations.

Recent work using analytical methods by Towill's group at Cardiff University is the basis of the work presented here. Different forms of IOBPCS models are described in Ferris and Towill [16].

The main criticism raised by Riddalls et al. [17] of Forrester's simulation methods is that they do not give general management decision support Towill's control-theoretic models allow an analytical approach to the inventory problem, while retaining the dynamics of the situation, unlike traditional operational research (OR) methods described by statistical or quasi-static methods. The disadvantage of the System Dynamics models with their extensive ad hoc non-linearity is that few or any general analytical predictions can be made. In principle they capture the main features of the systems while allowing a greater depth of analysis. These problems can also be represented as state-space equations, either as continuous or discrete models.

In the early descriptions of System Dynamics, delays were assumed to be exponential in form, in control terms exponential lags. Hence the delays due to the actual production processes, for example, are described by a simple single time constant T_p. In the models used here the differential equations are modelled using Laplace transforms assuming that a large number of items are being handled. These techniques are limited to linear models but have great utility in allowing both general expressions to be obtained.

The key to the dynamic behaviour of the inventory system is the rate of ordering. The main structural system that has been investigated in this present work is that of the Automatic Pipeline, Variable Inventory and Order Based Production Control System (APVIOBPCS). The inventory error (EINV) is represented by the difference between a desired level of inventory (DINV) and the actual inventory (AINV). A smoothing function is used to obtain the average sales consumption (AVCON) as a function of the virtual consumption rate VCON. In his earlier work, Towill [18] showed that the averaging techniques used in industry to determine long term sales trends could be modelled by an exponential lag in a continuous model. This value is then used to obtain the order rate (ORATE) given to the production facility, wherever it is. There is generally a production delay inherent in the manufacturing processes.

The Order rate (ORATE) is obtained from the sum of a fraction of the exponentially smoothed virtual sales plus a fraction of the error in inventory plus a fraction of the perceived error in Work In Progress. The error in inventory is supplied by a variable demand. Payment is often requested at this point. (The term “error” is used in the control engineering sense for the fed back difference signal).

This model, implemented in SIMULINK, is shown in Figures 1 & 2 with the cost functions included.

The basic structure of these models as implemented by Towill’s group uses a proportional controller in the loops carrying the inventory and WIP error signals. Other authors [19] and [20] have proposed PID controllers. These were shown by Censlive [21] to give lower inventory stock outs than the best proportional control. A second model was therefore implemented with PID controllers to see whether they were more effective in the cost based system models.

Case study

The example used here uses the costs and throughput data of an automated PCB assembly plant described in Deif [22]. The product in this case is a RAM module. Deif used data from a real case, with the following cost functions which allow for:

- Holding cost C_H, *calculated from the quantity of unsold RAM/week,* Q_H*, and then converted to holding cost using the following equations:*

$$Q_H = production - demand \tag{1}$$

$$C_H = Q_H \, iP_r \tag{2}$$

- The backlog cost, C_B, *is calculated from backlog quantity* Q_B *and then multiplying by the backlog penalty* P_B *and the cost of loss of goodwill* C_{LGW} *as shown below:*

$$Q_B = demand - (production + Q_H) \tag{3}$$

$$CB = QB\,(Pb + C_{LGW})\,P_S \tag{4}$$

The values quoted by Deif [22] are:

P_r=*$30*
P_s=*$100*
P_B=*0.01% of the selling price*
C_{LGW}=*0.01% of the selling price*
i=0.2%

Since there is a high volume flow rate of product we have used a continuous model in this case.

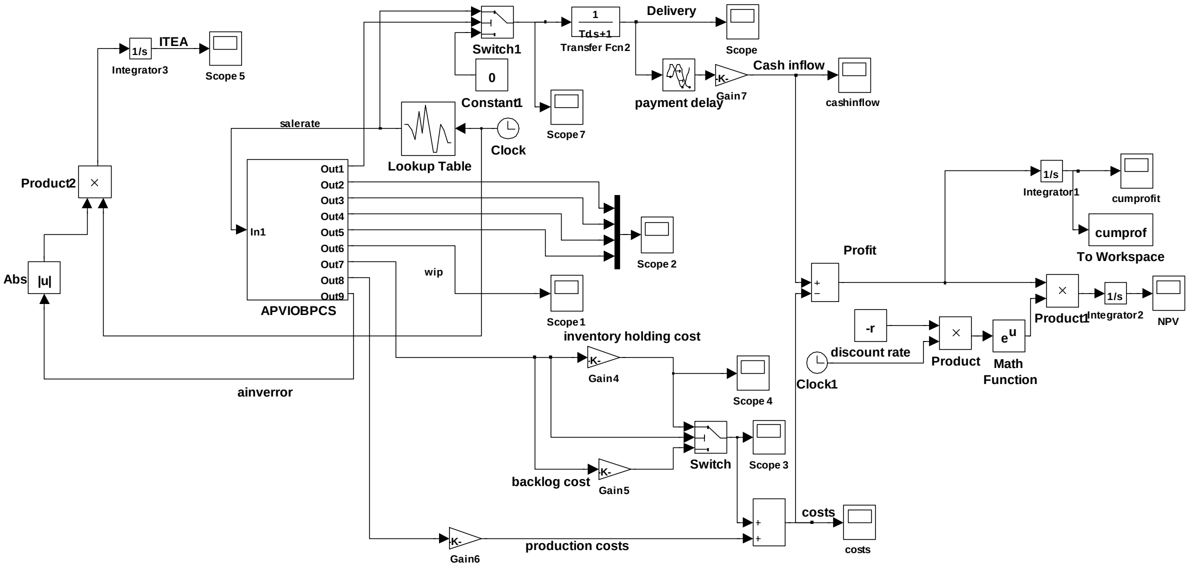

Figure 1. Schematic of the APVIOBPCS with cost functions implemented in SIMULINK.

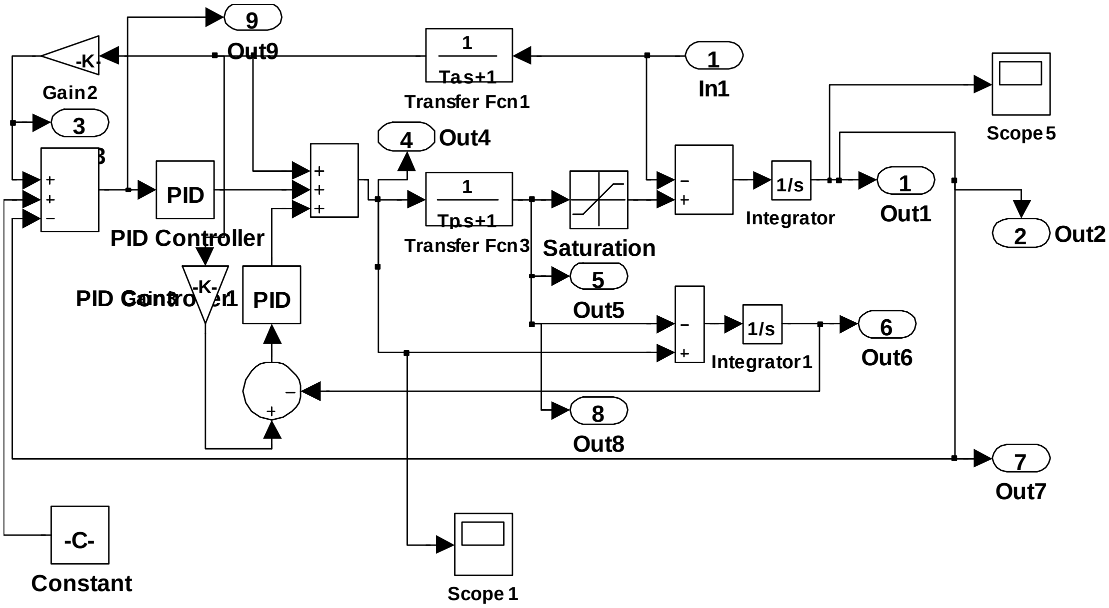

Figure 2. APVIOBPCS model.

Results

The model (see Figure 3) simulates 52 weeks of activities and the results for the cumulative profit are shown in Figure 4 for the proportional control using Disney and Towills' gain values [12] and for the better PID gains found by Censlive. These are plotted with data using the optimum PID gains found by Censlive using an ITSE criterion.

The plots show profit is initially negative for all three situations but becomes positive after 7 weeks for the ITSE optimised PID and 10 and 14 weeks for the other two cases. The cumulative profit is shown in Figure 4 illustrating the economic superiority of the PID controlled system.

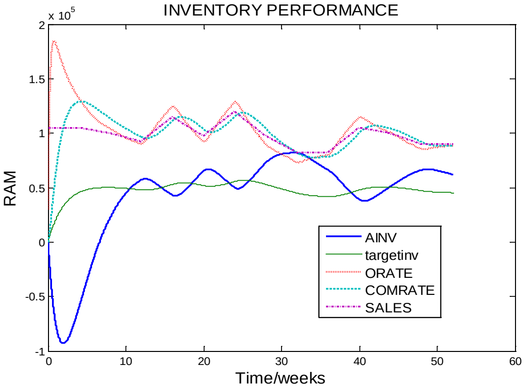

Figure 3. Inventory performance of the APVIOBPCS with proportional control using sales data from Deif (2007).

In the paper by Deif the sales rate varies from month to month and this effect is shown in Figure 3 using Dief's data. The inventory follows the sales rate after the initial stock out at the start of the simulation. Notice that the variable "desired inventory level" is a smoothed version of the sales rate. After the initial peak both the order rate and the completion rate follow the sales demand.

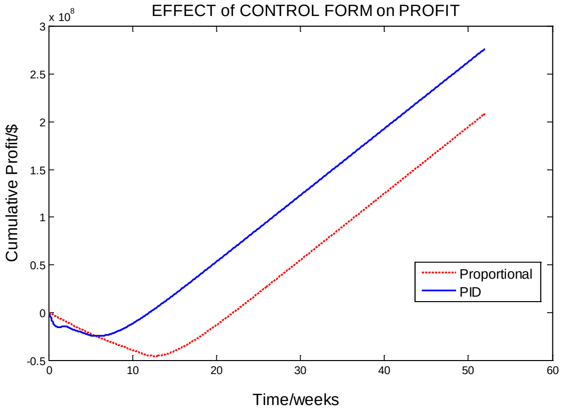

Figure 4. Cumulative Profit for the APVIOBPCS with comparison of P and PID control for a step sales input.

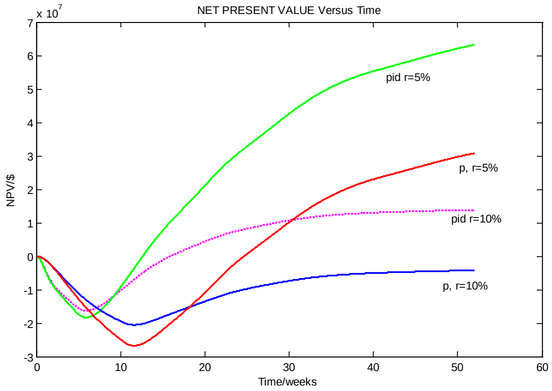

Figure 5. NPV value comparisons.

Changing the desired inventory constant, T_V, only affects the point at which cash inflow starts, hardly changing the costs. This is confirmed in the weekly profit figures (see Figure 7). The deficit is nearly the same but the date at which profit starts coming in is earlier for the higher T_V constant.

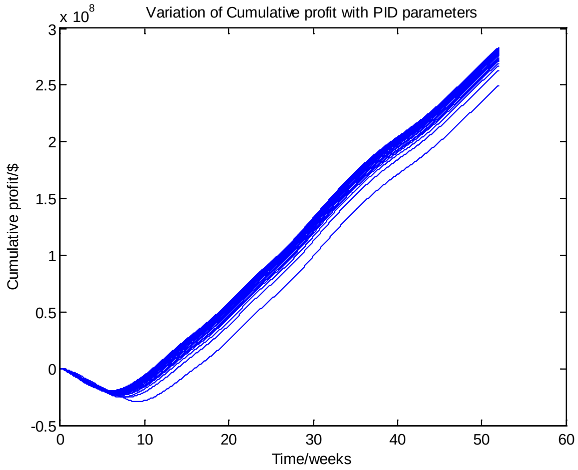

Figure 6.Cumulative profit in the APVIOBPCS with PID control using data from Deif (2007) for varying PID gain.

The model we have used here is a standard linear representation of APVIOBPCS but with non-linear features to compute the cash flows. These equations can be modified to include different formulations of costing or additional terms. It is clear that the performance measures described by Beamon can be evaluated relatively easily using this model. Figure 6 shows clearly that greater PID gain increases the profitability. This is because the period of undershoot is less with greater gain. It is also clear from our model that in this case of high volume order rates the level of inventory does not have the penalty effect commonly assumed.

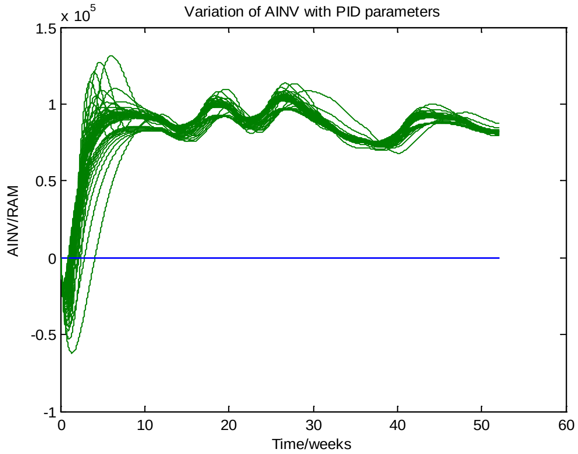

Figure7. AINV for the APVIOBPCS with PID control using sales data from Deif (2007) for varying values of PID gain.

Figure 7 shows how the inventory level varies with differing PID gains. The peak overshoot and peak undershoot vary considerably but a number of the differing gains give a very similar output for the inventory level. The Order rate (see Figure 8) illustrates an operational issue with the order rates generated by differing PID gains. The initial spike in orders gets larger with PI, nearly doubling in value. Although, as can be seen the effects of the gain give improving profits with greater gain.

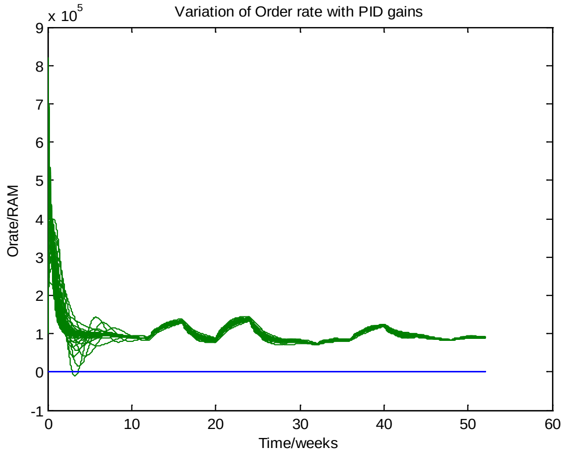

Figure 8. ORATE for APVIOBPCS with PID control using sales data from Deif (2007) for varying values of PID gain.

Figure 9 illustrates what happens if the desired level of inventory is changed. Here we can see an optimum value of T_V of around 1.3 to 1.5 depending on the interest rate. This alters the cost of holding stock. It may be that the formulation used here is a little simple and may be greater since warehouse costs are not included. However it does show an optimum peak value for profit.

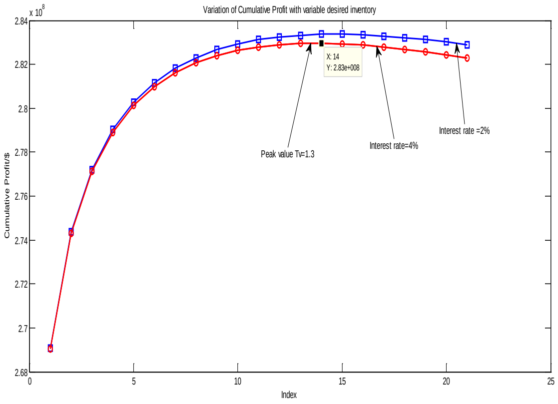

Figure 9. Variation of profit with desired inventory level.

It can also be seen that the effect of production time delay is small but reduces the profit with increasing production delay. If the lowest ITAE PID gains are used then the difference between the value of cumulative profit over 52 weeks from this set of gains is less than 1% less than the best value obtained.

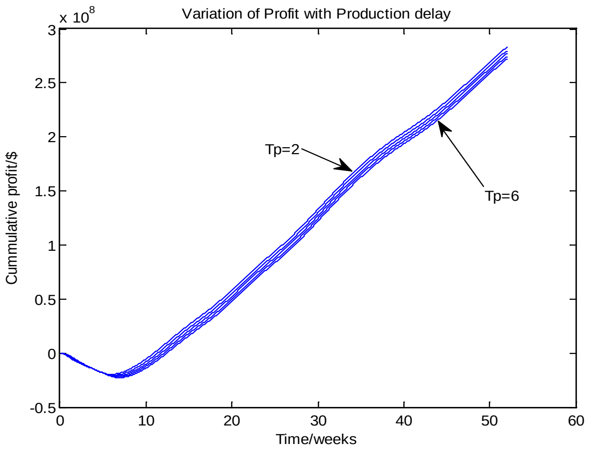

Figure 10. Variation of profit with production delay.

Conclusions

- PID control allows profit to be generated earlier and at a larger rate than for proportional control.
- Variable desired inventory allows greater profitability with an optimum value T_V for this problem of around 1.3 to 1.5.
- Increasing production delays cause lower profitability.
- Cumulative profit is increased by increased PID gains but excessive order rates are produced for the highest values.
- The difference in values of profitability close to the optimum are quite small, especially if the lowest ITAE inventory value gains are chosen.

Symbols

AINV	Current Inventory level
AVCON	Average sales rate
CONS	Sales consumption or market demand
COMRATE	Rate of production
CSL	Customer service levels
C_{LGW}	estimated cost for lack of goodwill
EINV	Error in inventory level
EPOS	Electronic point of sale
EWIP	Error in WIP
DINV	Desired inventory
i	Interest rate
ORATE	Outstanding level of supplier orders
P_S	Selling Price
P_r	Production costs
TINV	Target inventory
T_a,	Smoothing time constant
T_i	Inventory order constant time.
T_w	WIP order constant time.
T_P	Production delay time
T_{Pbar}	Multiplier from smoothed sales to WIP
T_V	Constant multiplier to AVCON

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