PLENARY PANEL: MATHEMATICS IN DIFFERENT SETTINGS

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When we think about the title "Mathematics in different settings", a number of questions arise. For example:

- *How many mathematics are there – one or many? Is there a mathematics that is "prior to", or independent of, any setting?*
- *What (who) is it that makes settings "different"? And how does this relate to social differences among people?*
- *What is an appropriate typology of different settings – for research or for curriculum design purposes? Relatedly, we might ask: who decides what is "important"?*
- *What is the nature of relations among policy arrangements, research and educational institutional settings?*
- *How are different settings represented in mathematics teaching and assessment?*
- *What is the relationship of mathematics education researchers to any setting?*

This plenary panel will explore a range of these questions.

MATHEMATICS: ONE OR MANY?

We can try to answer these questions using several principles and illustrations. First, we acknowledge that our understanding of "mathematics" depends on the historical and cultural context. Thus, mathematics has changed substantially since the end of World War II, for example because of the availability of new technologies. And in mathematics education, we have become familiar with the ideas that the different mathematics done in different cultural settings may exhibit radical differences in appearance. For example, it has been suggested that a mathematics based in a language, such as Maori, which is different to languages such as Portuguese or English may be understood as a different mathematics (Barton, 2008).

In response to this, on the one hand, it is argued that despite the differences in appearance, "street mathematics" and "school mathematics", for example, still share universal underlying principles (Nunes, Schliemann & Carraher, 1993; Noss, Hoyles, Kent & Bakker, 2010). On the other hand, some in the mathematics education community have taken on board ideas from social theory that different (in some sense) versions of mathematics are created in different sites through different social practices (e.g. Lave, 1988; Walkerdine, 1988). The differences in position among different authors, concerning the degree of construction versus representation (of "reality") that one attributes to (different) mathematics, are important, and this debate will go on.

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Further examples come from the types of mathematical / statistical modelling routinely done in industrial (e.g. quality control) and in commercial (e.g. risk assessment) settings, , which appear to be undertaken in very different ways from the ways that mathematics is produced in universities. This raises questions about seeing various types of mathematics as research mathematics which is then simply "transferred" to (or transformed) in applied and educational settings.

What makes settings "different"?

Thus, if we take the view that there are many different mathematics – or that they are based in many different settings –the question moves on to how they are different. One answer often given is simply to name the different settings in a commonsense way, but this does not allow us to analyse what may be key similarities and differences among settings. Other answers emphasise that the different settings are characterised by different types of situated cognition or situated learning (Lave & Wenger, 1991; Watson & Winbourne, 2008), or are constituted in different activities (e.g. Williams & Wake, 2007; Jaworski & Potari, 2009), or by different social practices in different discursive contexts which produce different meanings (e.g. Morgan, 2009; Evans, 2000; Hall, 1992). Or that the different settings are lived in by different people. Here the awareness of social differences can lead to raising questions of social justice (e.g. Burton, 2003). One aspect of this is the structured differences in the distribution of mathematics knowledge, which has been argued as being reproduced through differentiation within the educational system, in which mathematics plays a key role (Bernstein, 1990).

A TYPOLOGY OF SETTINGS: FIELDS OF ACTIVITY

Bourdieu's (1998) concept of fields of activity is useful in delineating a first list of settings that we wish to consider in the Plenary Panel. He includes:

- the Political and the Bureaucratic field: policy decisions and their implementation
- the Scholastic field: research
- the Cultural field: including education ["symbolic control"]
- the Economic field: production

And in the intersection of these, we could consider:

- Civil society: citizenship.

Each of these settings can provide the context for one or more different mathematics. Thus the economic field of production and distribution would provide the basis for several mathematics, including "workplace mathematics", "financial mathematics", and some "street mathematics". Civil society would provide the context for (some) "street mathematics", "everyday mathematics", and "mathematics for citizenship".

Research, institutional and policy settings

At the same time, there are complex relations across fields – policy, educational institutions, research, and production. These relations involve the exercise of power, and concern the control of resources and the marshalling of these to promote certain outcomes, e.g. the way that certain mathematics is legitimised for educational purposes. For one early study of different stakeholders' influence in national curriculum development, see Ernest (1991) – but researchers in every country can contribute accounts which combine elements of particularity and generality relevant to policy formulation (Noyes, in press). For example, much of the curriculum taught in classrooms depends on the relationship at a given time between policy and research settings, in mathematics education, and also in mathematics (Lerman & Tsatsaroni, 2009; Adler & Huillet, 2008; Young, 2006; Bernstein, 1990).

REPRESENTATION OF DIFFERENT SETTINGS IN SCHOOL MATHEMATICS TEACHING

Returning to the mathematics classroom, much has been written in recent years about the desirability of attempting to represent different settings in mathematics teaching – to provide motivation for students, "authenticity", preparation for the world of work, and so on. A number of problems have also been outlined, and the differential consequences for students from different social classes outlined (e.g. Cooper & Dunne, 2000; Forman & Steen, 2000).

These questions point to the problem of "harnessing" outside settings for use in the mathematics classroom – as illustration, motivator, or as a context for drawing out and generalising the reasoning skills of a competent adult (e.g. Schliemann, 1995) – and also to the perennially perplexing problem of "learning transfer" (e.g. Williams & Wake, 2007; Lobato, 2009; Evans, 2000). These issues will be taken up by several of the speakers.

ROLE OF THE MATHEMATICS EDUCATION RESEARCHER

There is clearly scope for a range of positions for the mathematics education researcher to take vis-a-vis the setting. These might include the following:

- objective *reporter* of what is "really" going on
- *producer of 'accounts'* from those engaged in the activities of the setting
- *advocate* for social or educational change
- *activist*, working alongside those engaged in trying to bring about changes

These latter roles are generally based on ideas of social justice. Of course we must distinguish a researcher's ideological self-positioning from the contradictory positionings that may arise within the research and teaching practices that the individual may be involved in.

We expect to learn more about different possible roles, and about the way that they are taken up, in the National Presentation at PME-34 of Brazilian research / action with disadvantaged adults.

Organisation of the Presentation

The introduction above suggests a number of topics for discussion in this Plenary Panel:

(1) Mathematics in school, college, university, teacher development settings
(1a) Representation of different "outside" settings in mathematics teaching
(2) Mathematics in civil society, everyday life, citizenship
(3) Mathematics in workplaces
(4) Application of school or college mathematics learning to out-of-school settings
(5) Social difference, mathematics and social justice
(6) Complex relations among educational institutions, research, policy

The four participants will together address these issues in the following ways.

Silvia Alatorre considers mathematics in several settings – school, everyday life, citizenship and teacher development – and the challenges for social justice in each, in the context of 21st century Mexico, where issues of social inequality are particularly evident. She emphasises teacher development as one of the necessary ways to challenge the "vicious circles" that she experiences currently in the settings with which she is familiar.

Henk van der Kooij reminds us of three commonly-accepted goals for (mathematics) education: to prepare for citizenship, for work and for further learning. He argues that there is only one mathematics – but with several approaches to it, and different goals for learning. He thereby distinguishes between mathematics for general education in school/college settings, and mathematics for vocational education and training (VET) in work settings. Henk's emphasis on the idea of *situated abstraction* raises the question of how we can use insights from research to construct different pathways to different curricular goals – in a way that might be able to avoid the traditional hierarchical orderings between different types of school or college mathematics.

Despina Potari considers in particular University teaching and workplace settings. She presents one vignette from each setting, and uses activity theory to focus on the way that the tools mediate the action of "making connections", within and between such settings. The analysis of the two cases indicates that in both settings there is a network of connections between the tools that frame the invariant mathematical objects which are situated in the actual practice of the two communities.

Andy Noyes describes trends in school mathematics in contemporary England, and considers whether future settings relevant for mathematics learned by today's students can be the basis for the curriculum. He explores the value of ideas of general education, citizenship, and critical pedagogies, as a basis for an education in mathematics that might be socially empowering. He counter-poses the idea of using the "immediate lived realities" of students as the key setting for mathematics education, on the basis that, for these students, the challenges of citizenship start now.

Inevitably the contribution of each colleague is grounded in a consideration of goals for mathematics education, and for education generally. It will be clear that each of these colleagues is writing from her/his own national setting, but is aiming to propose to us ideas that will stimulate thinking about the settings that all of us inhabit and work in.

In all these contributions, we can see arguments for the importance of being clear about what is the setting / context of each episode of activity described. It is not that mathematical activities in different settings are "just different". We must acknowledge *and describe* differences in mathematics in different contexts, as they are structured and placed in hierarchies, based on relations of power, in ways that tend to be reproduced and amplified, as Silvia illustrates, by educational institutions. The challenge is how we can avoid reproducing such dichotomies or hierarchies that function to privilege one "type of mathematics" over another – usually the academic or the school-based, over the practical / vocational. One way is for researchers to aim to uncover elements in each setting that are usually unacknowledged, and which are necessary and efficient for the successful completion of the activity at hand. For example, Despina points out that "the elaborated formula used by the technician is a contextual transformation of a common mathematical formula" for the resistance – which presumably has advantages such as efficiency of use, in the setting in which the technician works. And Silvia calls for an alternative social model which, rather than considering someone as an underachiever in terms of a deficit in dominant (e.g. school mathematical) practices, instead accepts social difference and multiple practices, and seeks to represent and to build upon informal numeracy practices and social "funds of knowledge".

Thus, Henk's paper tries to balance policy trends and (mathematics education) research findings, and to propose that there is sufficient common ground between the two sets of interests to consider the needs of two different types of mathematics education (curriculum) – that is, for different mathematics to be offered to "academic" and "vocational settings", but without either being seen as "more" or "less". Andy, in the current relative absence of similar opportunities in his national setting, aims to develop the potentials in the ideas of general education and citizenship to construct settings where mathematics education might be "reset".

Working in ways such as these, the mathematics education community might be able to exploit ideas discussed here, such as *situated abstraction* – based on analyses of "practical" (e.g. workplace) settings – so as to be able to construct discrete programmes of study that serve different educational goals, while valuing the different learners and the different settings in which they live and work.

WHAT IS THE RELEVANCE OF MATHEMATICS IN ISSUES OF SOCIAL JUSTICE?

Silvia Alatorre

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Four possible settings are considered for a possible answer to the question at hand: school, everyday life, citizenship, and teacher development. Also considered is the vulnerability of different groups in these different settings.

When we seek to comprehend a complex system of many interrelated factors, we must necessarily choose those factors (and their interconnections) that we consider at the very heart of the whole. Such is the case when we try to answer the question posed in the title. For that, I will consider four interrelated settings where mathematics plays a crucial role: school, everyday life, citizenship, and teacher development.

In trying to understand the maze of interrelated vicious circles that these four settings produce, my starting point is of course the situation in Mexico, where I come from. Like in many other non-first-world countries high levels of inequity prevail; in education, for instance, two indicators may give a hint about this:

- Of people aged over 15, 10% never attended school, 19% have only completed 6 years of schooling, and only 11% have more than 12 years of schooling (INEGI, 2005);
- Private schools are generally better off than public schools, but they only serve the more privileged 8% of students (*ibid*). For instance, recently published research about the mathematics knowledge of the university students found that 24% of those who had previously attended only private schools had marks above M+SD (i.e. more than 1 standard deviation above the mean), while that ratio was 10% among those who had previously attended only public schools (González, 2009).

First setting: school

Although school is envisioned by many (particularly politicians) as the social equalizer *par excellence*, it can become the basis of a vicious circle where social injustice can progressively be amplified. In school children who belong to underprivileged milieux are usually dragged into a chain of increasing disadvantage. Deprived conditions – such as the physical setting of the school, overcrowded classrooms, unfavourable health, nutritional and emotional conditions, and, not least, teachers with an inadequate training – lead to deprived learning, such as insufficient comprehension of concepts and methods, misconceptions, inability to apply the knowledge in everyday situations, etc.

One manifestation of deprived learning can be seen in a phenomenon that we have called "school-engendered errors" or SEE: errors that are made more often as the subjects' schooling increases, at least within a certain range. For instance, in a survey conducted in Mexico City with adults in the street (Alatorre et al., 2002), subjects were given three measures of shelving (1.5m, 1.30m, 1.465m) and were asked which shelf was longer. The two most frequent errors are what Stacey & Steinle call the "L" and the "S" mistakes (see e.g. Stacey, 2005), which are incorrect ideas about how the string of decimals should be ("long" or "short") to make a decimal number larger. The "L" mistake generally originates in reading the decimal part as an integer (thus saying that 1.465 is the largest and 1.5 the smallest), and in the sample it was made more frequently by subjects with low levels of schooling. The "S" mistake may originate in thinking that the longer the string, the smaller the fraction it refers to (1.5 is the largest because it is in tenths, whereas 1.465 is the smallest because it is in thousandths), and in the sample the *higher* the subjects' schooling (up to high school), the more often they made it. The "S" mistake seems to be either promoted by or at least not effectively corrected by the school system. Such school-engendered errors are also more probable within socioeconomically disadvantaged milieux.

Second setting: everyday life

Of course, school is not the only setting in which mathematics learning takes place, and everyday life is repeatedly a source of more grounded knowledge. Often, it is evident that schooling is neither a necessary condition nor a sufficient one for the ability to think mathematically, or to apply mathematical tools correctly. This was seen, for example, among adults to whom we posed problems of proportional reasoning (Alatorre & Figueras, 2005): Illiterate people usually gave correct and informal answers to the problems with which they were more familiar, and the more schooled subjects often gave incorrect answers, even when responding about familiar contexts.

Everyday life is both a source of mathematical knowledge and a field where the mathematical knowledge is applied. Regardless of *how* and *where* we learn it, we must be able to know how much change is due from a $200 (200 peso) banknote after a purchase of $115.20, to predict how much we will pay in a purchase of 76 items worth $12.30 each, to figure out the amount of fabric needed for a tablecloth, to calculate the price of a travel ticket sold by a machine, to understand that a bargain of 2 for $279.90 instead of the regular price of $126.40 is no bargain at all, to know that a 20% discount calculated after a 50% discount is not a 70% discount, to comprehend how much is being charged in the electricity, water or phone bill, to foresee how much we will be paying for credit, etc.

Regardless of how and where we learn it, yes, but the truth is that these important pieces of knowledge are held all the more often by people who had at least some (good) schooling. Everyday life can be part of a vicious circle where less favoured mathematical enculturation triggers unfavourable conditions for everyday life and in turn is triggered by them (Bishop, 1991).

In Mexico (as in many other countries) there are groups of people who are especially vulnerable to this vicious circle: indigenous people. Their difficulties include the language, the fact that both the educational system and the "modern" life ignore their traditional culture and the mathematics imbedded in it, the divorce between the traditional measuring units and the metric system, etc. The understanding of this phenomenon lies within the field of ethnomathematics (see e.g. D'Ambrosio, 1985), and one of its components is the linguistic conditions in which learning and life take place. A framework for this has been proposed by Barwell (2005), but it does not include a situation that has been increasingly common in Mexico: indigenous people migrate looking for work to the United States, where they are treated as "Hispanic", thus fostering a double or triple alienation (Bengoechea, 2010).

Mathematics in real life can vary between being a relatively theoretical exercise to being a material necessity for survival (Walkerdine, 1990). Consider for instance these situations concerning an adult buying cheese: (a) she chooses the best quality regardless of the price; (b) she compares the different prices and presentations and chooses the best value; (c) she chooses the cheapest package; (d) she offers a couple of small coins and takes whatever amount of cheese she is given. Only in situations (b) and (c) is there actually some maths involved; buyer (a) does not need the mathematics although she could probably picture herself in situations (b) or (c); and for (d) the amount of cheese received will be insufficient anyway (she is even beyond using mathematics as a "material necessity for survival").

The vicious circle of daily life as an amplifier rather than a buffer for social differences need not be so. It has an ideological foundation; schooled numeracy has a much higher status than home or street practices, as Walkerdine (1990) and Baker & Street (2000) have pointed out. The latter have argued that instead of considering that somebody is an underachiever in terms of being deficient in dominant practices, there could be an alternative social model accepting social notions of difference and multiple practices and seeking to represent and build upon informal numeracy practices and "funds of knowledge".

Third setting: citizenship

The setting of daily life is highly interrelated with that of citizenship. More and more our ability to grasp the issues relevant to our participation in society and to have a bearing on them depends on our ability to read, process and interact with information and with gadgets that are mathematical in their origin and/or in their presentation.

Globalisation has done this for us: almost everybody in the world needs to handle information that has mathematical form and/or content. Almost everybody needs to be able to decipher a graph in the newspaper without relying on the journalist for its interpretation, to critically participate in major civic decisions such as the changing of fuel for ecologically safe energy sources, to understand the rules for electing representatives, to participate in discussions about the distribution of the national budget, to relate “big” numbers associated to the national affairs with their “smaller scale” regional or micro-regional counterparts, to understand the effects of the rise and fall of interest rates, to comprehend that an average income does not mean that everybody receives it, and so on. The numeracy issues related to citizenship are numerous.

The fact that this is also a vicious circle is not surprising but needs to be said: people with reduced opportunities for mathematical enculturation, either from school or from everyday life, also are deprived of opportunities to participate in the possible change, through active citizenship, of precisely those conditions. Of course people who are able to handle information that has mathematical form and/or content are not protected against social injustice, but certainly those who are not able to do so are even less protected.

Fourth setting: Teacher development

I would finally like to turn to the fourth setting, which includes another vulnerable group: schoolteachers, particularly those at the elementary level. This group is positioned at a turning point in the educational cycle. In Mexico, teachers are trained in special schools called Escuelas Normales, which they attend after high school. The curriculum for the Escuela Normal does not include courses in mathematics (nor, for that matter, in Spanish or history or biology), except for one or two courses in the teaching of the subject matter. Thus, the future teacher does not have the opportunity to reconceptualise whatever misconceptions or incomplete learning s/he starts with, and from there the problems can only grow. Again, the cumulative effects are greater and produced more quickly in underprivileged surroundings, those where the Escuelas Normales receive students with less adequate knowledge of the content matter, and have fewer resources (human, economic, etc.) to make up the deficiencies.

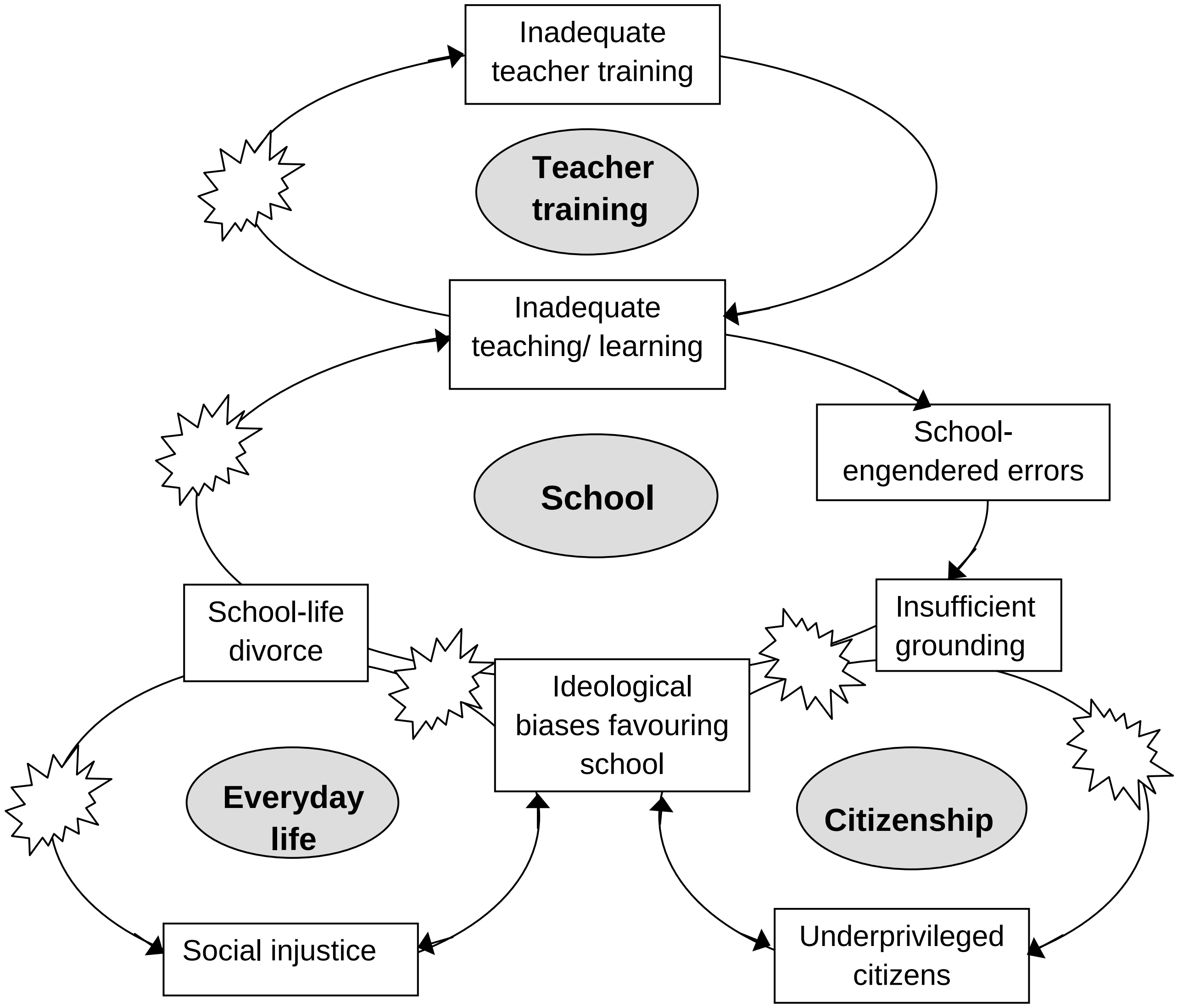
In recent years, some members of this community have proposed that the Mathematical Content Knowledge (MCK) proposed by Shulman (1986) may not be *that* important after all, and I agree that if one is thinking of a second-grade teacher there may be many pieces of knowledge (whether related to the teaching of mathematics or not) that are far more important than, say, the density of rational numbers. I also agree with Bishop (1991) that teachers, as the mathematical enculturators, should be selected according to criteria regarding their ability to communicate and their commitment to the mathematical enculturation process, and that the principles of their education should contemplate an understanding of Mathematics as a cultural phenomenon, its values, and its technical level, as well as the development of a strong metaconcept of the Mathematical enculturation process generally.

Nevertheless, I would argue that elementary teachers need to master the MCK of the mathematics that they are supposed to teach; for instance, speaking of rational numbers, no teacher should think that the sequence 0.60, 0.70, 0.80, 0.90 is followed by 0.100. In Mexico, some elementary teachers do. Of course, such gaps in crucial MCK may well be evident in other countries, too.

Thus, it can be said that the vicious circle of teacher development is the cornerstone of the whole structure of problems. In Mexico, the social conditions for breaking this vicious circle involve many difficulties, going from the scarcity of well-qualified teacher educators to the corruption of the teachers' union.

OVERVIEW

In a very simplified diagram, all these complex interactions may look like this, where the vulnerability of underprivileged people is represented by a flash icon:



Evidently, breaking into these vicious circles is not an easy job. It involves a multiplicity of actors and a social decision to consider which entry point is more likely to lead to worthwhile and lasting social change. Maybe we should start with teacher development and include in it an effort towards what Andy Noyes calls in his contribution to this panel "Mathematics for social empowerment"?

MATHEMATICS AT WORK

Henk van der Kooij

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Official documents in many countries state that the goals of mathematics education are threefold: to prepare for citizenship*, for* work *and for* further learning*. But in many educational practices, the main emphasis is still on procedural fluency, especially in algebra, needed for further learning. This paper describes how mathematics for and in work differs from mathematics learned at school and tries to seek for a better balanced mathematics curriculum in secondary education.*

ONE MATHEMATICS, DIFFERENT SETTINGS

There are many research reports, based on empirical studies and other ones on more theoretical considerations, that discuss how mathematics embedded in work settings is different from the way mathematics is learned and practised in the typical traditional school settings (Hoyles & Noss, 1996; Forman & Steen, 2000). Different names, like school maths, work maths, street maths and others, seem to point to the fact that there are many different types of mathematics. However, I prefer to think of just one mathematics, with different approaches to it, depending on the goals for learning in a given educational setting.

As discussed in the PISA framework (OECD, 2002), a modern view on mathematics as a discipline emphasizes 'the study of pattern' and 'dealing with data' (Steen, 1990; Devlin, 1994). This is not a traditional content-driven look at a possible mathematics curriculum for school, but rather a way to describe the field of interest. The choice in the PISA study to describe the scope of mathematics education by four overarching ideas: (*patterns in*) *quantity*, (*patterns in*) *shape and space,* (*patterns in*) *change and relationships* and (*dealing with*) *uncertainty* opens up a novel way of thinking about a better balanced curriculum for (secondary) education that truly takes into account the threefold goal (society, work and further learning).

CHARACTERISTICS OF MATHEMATICS AT WORK

> Mathematics in the workplace makes sophisticated use of elementary mathematics rather than, as in the classroom, elementary use of sophisticated mathematics. (Steen, 2003)

This phrase summarizes briefly the findings of many research studies on mathematics at work and other out-of-school settings. Where mathematics learned at school is embedded in a well-defined formal structure, the mathematics used in the workplace is embedded in the context of work. Practitioners at work do use *situated abstraction* in which local mathematical models and ideas are used that are only partly valid in a different context because they are connected to anchors within the context of the problem itself (Noss and Hoyles, 1996; Hoyles, Noss, Kent & Bakker, 2010).

The idea of situated abstraction is very helpful for understanding what happened in a reform project for senior high school vocational education (engineering) in the Netherlands (van der Kooij, 2001). The principle was that mathematics should become supportive for the vocational courses and therefore the mathematical subjects were presented in the context of engineering. Although the students were considered to be low achievers in formal algebra, it was found that they could do algebra as long as the variables were real quantities with a meaning (time, length, speed, mass) – but that most of them got lost as soon as variables became abstract (*x* and *y*). An example: most students were able to do calculations on the pendulum equation $T = 2\pi\sqrt{\frac{l}{g}}$ with T (in s) the period, l (in m) the length of the pendulum and g (in m/s^2), but many of them had no idea how to deal with a more general (and more simple, but meaningless) equation like $y = 2\sqrt{x}$. For that reason, instead of going for full abstraction and generalization, we emphasized transfer from one context to another: how is a strategy or procedure similar in a different context and what is different. Most of the time, this transfer is not complete – in the sense that every context gives rise to its own modification of the method that is 'applied' in that context (Evans, 1999).

Some important general aspects of mathematics in context (of work) are (Bakker et al, 2008; Steen, 2001; Hoyles et al, 2002):

- reading and interpreting tables, charts and graphs,
- use of IT (like spreadsheets),
- dealing with numbers, often not precise and with units of measurement,
- proportional reasoning,
- statistical process control activities,
- representing and analysing data,
- multi-step problem solving.

Strangely enough, most of these aspects are not found in mathematics curricula in secondary education.

COMPETENCES AND SKILLS FOR THE WORKPLACE: THE ECONOMIC SETTING

Policymakers in the United States (SCANS, 1991) and Europe (European Communities, 2007) have described the competencies that are needed for future workers in a world in which "the globalization of commerce and industry and the explosive growth of technology on the job" (SCANS report) asks for skills that are different from the traditional ones learned at school. Competencies for work are described in the SCANS report (Steen, 2003) as the ability of people to use:

- resources (allocating time, money, material, and human resources)

- information (acquiring, evaluating, organizing, maintaining, interpreting, communicating, and processing)
- systems (understanding, monitoring, improving, and designing)
- technology (selecting, applying, maintaining, and troubleshooting).

Skills needed for such competences are split up as follows:

- "basic" skills: arithmetic, estimation, reading graphs and charts, logical thinking, understanding chance
- "thinking" skills: evaluating alternatives, making decisions, solving problems, reasoning, organizing, planning
- personal qualities: responsibility, self-esteem.

The European Union describes seven key competencies for life long learning needed for personal life and for work. Key competence 3 (EU, 2007):

> Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts).

Yet I would expect to find that neither the described aspects of mathematics in work nor the competences and skills described by the policymakers are found to any great extent in mathematics curricula for primary and secondary education in most countries.

Nevertheless, the intended change to competency based education (CBE, advocated in the European Union as the driving force for Life Long Learning) offers opportunities to reconsider the kind of mathematics education that truly aims at goals related to the three settings: society, work and further learning.

RECONSIDERING MATHEMATICS AT SCHOOL

If the threefold goals of mathematics education (citizenship, work and further learning) are taken seriously, the claims above should be considered when rethinking a balanced mathematics curriculum to achieve these goals. For vocational training this seems more straightforward than for general education. In vocational education the mathematical content should be defined in terms of applicability for the workplace setting. For engineering, some important key aspects are: direct and inverse proportionality, absolute and relative numbers, relationships between more than two quantities with dimensions and units of measurement included, reading and interpreting complex graphs, logarithmic scaling, tolerances and significance, and a basic idea of chance in the context of uncertainty.

For general education, the PISA framework has the potential in helping to define a balanced curriculum. The four overarching ideas (Quantity, Space and shape, Change and relationships, Uncertainty) and the eight competencies (Thinking and reasoning, Argumentation, Communication, Modelling, Problem posing and solving, Representation, Using symbolic, formal and technical language and operations, Use of aids and tools) can be used to structure and design a balanced curriculum in which all three goals are met.

One example of a balanced curriculum in general education can be found in the Netherlands: Mathematics A (de Lange, 1987), designed for a special group of high school students: those preparing for a study in social sciences. It was a well-balanced whole of statistics and chance, discrete mathematics and applied calculus with many applications to and starting points within the context of several disciplines. Problem posing and solving activities, mathematizing and developing one's own strategies were seen as at least as important as training for procedural fluency.

DISCUSSION

> We believe, after examining the findings of cognitive science, that the most effective way of learning skills is "in context," placing learning objectives within a real environment rather than insisting that students first learn in the abstract what they will be expected to apply. (SCANS, 1991, viii)

Mathematics education that starts in (real life) contexts can have, if designed well, a natural flow from concrete to general/abstract. As stated before, a sense-making learning line does not need to end in the formal, abstract world of pure mathematics for all students. For most students, especially those who don't aim at college level study in natural sciences, situated abstraction with transfer from one context to another is motivating and enough to get prepared for work and for living as a "constructive, concerned and reflective citizen" (OECD, 2002).

So, why are curricula in general not balanced? Maybe because of the deeply rooted belief that mathematics has to be learned in a linear way - from arithmetic, via algebra to functions (linear, quadratic, polynomials, exponential and periodic) and calculus, with some geometry and maybe some statistics alongside. This is how most curricula are designed and it does not reflect the modern view on the nature of mathematics (Steen, 1990; Devlin, 1994).

One of the benefits of studying mathematics in the workplace is that we can see that the mathematics used in different workplace settings and in daily life is far from "standard" – and it is learned in these settings in different ways, many of which are far from "linear".

MAKING CONNECTIONS IN TWO DIFFERENT SETTINGS: THE ROLE OF TOOLS

Despina Potari

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This paper addresses the issue of making connections in two different settings, the university mathematics teaching and the workplace. Through the analysis of two vignettes from each setting by using Leontev's triadic model (activity – actions-operations), it attempts to indicate similarities and differences concerning this particular action in the two settings, by focusing on the tools and their interrelations.

INTRODUCTION

Making connections is at the heart of mathematical activity. In the practice of research mathematicians, making connections provides the wide mathematical picture either by relating the mathematical productions to the real world or by connecting different research areas and results (Burton, 1999). Boaler (2003) argues that we need to shift our attention from the knowledge categories to mathematical actions recognizing the action of making connections as central both for the development of mathematics and for the development of students' mathematical meaning. Bishop (1988) also focuses on the construction of mathematical meaning and considers making connections as a way that an individual attributes meaning to a mathematical object. In the practice of mathematics teaching and mathematics teacher education Rowland, Huckstep and Thwaites (2005) in the knowledge quartet recognise "connection" as a category of prospective elementary teachers' knowledge. They argue for the importance of this knowledge for teaching and they extend it beyond the structural connections within mathematics itself to the awareness of the relative cognitive demands of different topics and tasks. In the context of the workplace, making connections focuses on the translation of different representational means. Williams and Wake (2006) claim that this translation is important in the workplace and it can be facilitated especially for an outsider by making connections with more concrete, relatively "universal" cultural resources such as the use of metaphors and models. Roth and Bowen (2001) focusing on the graph related practices claim that professionals attribute meaning to graphs by making connections between the phenomena to which the graph pertains and its structure.

In an activity theory perspective (Leont'ev, 1978) a three-tiered explanation of Activity is recognized focusing on its different components: activity, actions and operations. Human *activity* is always energized by a motive, *actions* translate activity-motive into reality and *operations* accomplish the actions instrumentally. Under this framework, making connections can be considered as actions undertaken to perform an activity while operations accomplish the actions that are mediated through a number of tools. These tools frame the construction of mathematical meanings as a number of studies, particularly in the workplace, have demonstrated (eg. Pozzi, Noss & Hoyles, 1998). The study of Triantafillou and Potari (in press) also indicates that the persistent involvement in mathematical practices through the interrelation of a wide range of mathematical and non- mathematical tools leads to the generation of invariant mathematical concepts and processes.

In this paper, I will try to exemplify how the actions of "making connections" are related to the tools employed by the participants in two different settings, the university mathematics teaching and the workplace. In particular, I will focus on similarities and differences among the tools and the way they interrelate and shape the mathematical practices based on the action of making connections.

TWO EXAMPLES OF MAKING CONNECTIONS FROM TWO DIFFERENT SETTINGS

I present two vignettes below. The first comes from a study in which I have recently been involved with a colleague, Theodossios Zachariades, and a postgraduate student, Georgia Petropoulou, in studying university mathematics teaching in the mathematics department where I have been working for the last two years. The second comes from the PhD work of Chrissavgi Triantafillou that I have supervised which is based on the study of mathematical practices in a Greek telecommunication organization.

Vignette 1: From University teaching

The lecturer who is both a research mathematician and a mathematics educator is teaching the Bolzano theorem in a Mathematical Analysis course to first year mathematics undergraduates: "Let, for two real a and b, $a < b$, a function f be continuous on a closed interval [a, b] such that f(a) and f(b) are of opposite signs. Then there exists a number $x_0 \in [a, b]$ with $f(x_0) = 0$." He sketches a graph to support the meaning of the theorem by pointing out to the students the continuity of the function and the fact that it will cross the x- axis: "We see that the graph is a continuous line. This means that it will cut the interval [a, b]. We need to prove it. It is not enough that we see that it crosses the x-axis". Then he poses the following problem to the students: "Suppose we have $\sqrt{2}$ and we want to approach it by a sequence of rational numbers which converges to $\sqrt{2}$". He draws a number line on the board and represents the numbers 1, 2 and $\sqrt{2}$. He says "$\sqrt{2}$ is between 1 and 2. What if we want to go closer to $\sqrt{2}$?" One student suggests the middle of the interval and the teacher uses this idea to construct a sequence of intervals: [1, 2] [1. 1.5] [1.25, 1.5] … by comparing the square of the most recently calculated "middle number" with 2 so that $\sqrt{2}$ is included in each of these intervals. This example is leading the students through a number of questions to the idea of nested intervals that represents them in the number line. By using the theorem of nested intervals he concludes that the intersection of the intervals $[a_o, b_o]$ $[a_1, b_1]$ … $[a_n, b_n]$ is $\sqrt{2}$. Then he uses the idea of the construction of the intervals as the key idea of the proof of the Bolzano theorem. He carries on performing the formal proof by focusing on the construction of the nested intervals and by relating it to the graph of the function.

Distance (km)	Diameter (mm)
Up to 3	Φ0.4
From 3 – 6	Φ0.6
From 6 – 9	Φ0.8
From 9 - 10	Φ0.9

Fig. 1: The table representation.

Vignette 2: From the workplace

The vignette comes from an informal discussion between the researcher and the technician. The issue discussed is about the difficulties that apprentices face to make sense of an everyday action, that of locating the exact position of the fault in the local underground copper-wiring network. The underlying mathematics in this action is the proportional relation among three quantities the resistance, diameter and length of the copper wires which is expressed through the connection of various communicating tools such as tables, graphs, formulas. The technician offered an explanation of the phenomenon that could help the apprentices to understand it. He used the elaborated formula $L= R\cdot 45\cdot d^2$ to reason about the relation described in Fig. 1 between the diameter d of the wires and the corresponding distance L between the subscriber and the network headquarters.

"If we want our resistance to be up to 350 ohms [he had explained that this is roughly the maximum resistance that a network can bear to have and is half the resistance in the loop] in the case of a wire phi 0.4 the telephone works up to two kilometres and five hundred meters [he calculates $350x45x0.4^2= 2520$ m]. Now, if we want to go further, we change the diameter to zero point eight and now we can go up to ten kilometres. [He calculates $350x45x0.8^2 = 10080$m]". Then he makes a sketch to demonstrate the local lines for both cases.

THE ANALYSIS OF THE TWO VIGNETTES

Vignette 1

Activity-motive	Teaching Bolzano's theorem	To promote students' understanding of the proof.
Actions-goals	Connecting a graph of a function to the conditions of the theorem.	To help students develop a sense of the theorem.
	Providing a familiar context (the √2).	To help students identify the key idea of the proof of the theorem (the nested intervals).
	Making connections between algebraic and graphical notations (use of number line and the graph of a function).	To create a visual representation to make explicit to the students the steps of the formal proof.
Operations -conditions	Use of different communicative tools (mathematical symbols, metaphors, graphs); thinking tools (theorems, concepts, processes, familiar proof).	Blending informal and typical mathematical tools.

Table 1: Teacher activity related to Leont'ev's three levels in vignette 1

Vignette 2

Activity-motive	Explain a mathematical relation used in his every day practice.	To help the researcher and the apprentices to make sense of the phenomenon.
Actions-goals	Connecting the table, the formula and the actual working situation.	To help the researcher and the apprentices develop an understanding of the proportional relation in the working context.
Operations -conditions	Use of different communicative tools (mathematical formulas, metaphors, diagrams – drawing, table); thinking tools (performing calculations, using algebraic relations).	Blending the real situation to the representations (formula, table).

Table 2: Technician's activity related to Leont'ev's three levels in vignette 2

Similarities and differences between the actions of making connections in the two settings

In both cases, making connections seemed to be a central action aiming to help newcomers (the students in the case of university teaching and the researcher and apprentices in the workplace case) to make sense of aspects of that practice. At the university, making connections is crucial to the production of mathematics and to its teaching, while in the case of the workplace, it is important for the technician completing a job efficiently with the existing resources. This is realised through a variety of communicative and thinking mathematical tools in the sense that Mellin-Olsen (1987) characterises them. The communicative tools include mathematical symbols, metaphors, graphs and diagrams while the thinking tools are concepts and processes in both cases. In the mathematics teaching context of the university the thinking tools are more complex than in the workplace context as they refer to a number of mathematical objects and their relations. They are cultural products of the community of mathematicians over a long period of time and carry certain beliefs and conventions existed in this community. On the other hand, in the workplace thinking tools are mathematically less elaborate and do not have the decontextualised and general character that the mathematical tools in university mathematics have. Moreover, they are not recognised by some technicians as mathematical tools although they use them (see for example Hoyles, Noss & Pozzi, 2001). The communicative tools used in both cases differ in terms of the conventions that have been established in the historicity of the two activity systems; the one refers to the practice of mathematicians and the other to the particular workplace practice. In both practices the communicative tools are embedded in the particular activity: in the first case they have an impersonal character, while in the second they are idiosyncratic. So, in the case of the workplace activity the formulas, the tables have different linguistic features from the ones used in formal mathematics. For example, the elaborated formula used by the technician is a contextual transformation of the common formula ($R = \rho \cdot L/s$, where ρ represents the resistivity of the material that the wire is made of and s the cross-sectional area of the wire); as in the work context all the wires are made of the same material, the resistivity takes a particular value and the wires are classified according to their diameter.

By focusing on the way that the tools are interrelated in the process of making connections we see two different paths. In the first case, the teacher uses less formal communicative tools (the sketches of graphs and of a number line) to attribute meaning to the more formal mathematical symbols. He also uses the structure of a familiar concrete case to help students understand the structure of the proof of Bolzano's theorem and consider the semantic character of proof (Weber, 2005). In the second case, the technician bases his reasoning on the three communicative tools (the table, the elaborated formula and his personal drawing) by starting from concrete cases in the table, by applying the formula and finally by linking to the actual phenomenon through his drawing. In both cases the ultimate goal is to construct or use appropriate thinking tools in order to see the invariant mathematical object, the proof of the Bolzano theorem in the first case and the proportional relation in the second. The question that still remains is how transparent the tools employed for the action of making connections are for an outsider in both of the two activity systems. This requires an understanding of the meaning of the tools but also of other situational factors such as the practice of the community, its traditions and rules and the division of labor (Engestrom, 1999).

CONCLUDING REMARKS

Making connections is a central and common action in two different practices, the university mathematics teaching and the workplace. These practices although very different in motives and goals use a variety of tools that are different in a way but also share some common characteristics. In both cases there is a network of connections between the tools that frame the invariant mathematical object (see Nunes, Schliemann & Carraher (1993) for a discussion about the need to "understand the mathematical invariants as well as the particulars of the situations" (p. 139) in order to function well in cultural contexts). Although in the case of university mathematics teaching, the issues of abstraction and generalization are crucial, the invariant mathematical objects are situated in the actual practice of the two communities in the sense that they have meaning in relation to the rules, beliefs and traditions of each community. An outsider needs to become a participant of this practice to get a sense of the invariant mathematical object. So, I would agree with the position of Henk van der Kooij in this plenary panel that we could talk about "just one mathematics with different approaches to it, depending on the goals for learning in a given educational setting". Moreover, by exploiting theoretical frames and research tools that have been used in recent decades in workplace mathematics, we can possibly illuminate important issues relating to mathematics teaching and learning in the formal education system at the university level.

RESETTING SCHOOL MATHEMATICS

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The aims, curricula, and pedagogies of school mathematics vary across space/place - nations, states, districts, schools and classrooms - and over time. Such distinctions are the objectification of historic struggles for the soul of school mathematics, played out at various scales of activity. School mathematics is not the same as academic mathematics and needs to fulfil, as part of a state education system, certain social, political and economic functions. Particular future settings for young people strongly frame their experiences of school mathematics. Here I will briefly consider dominant and alternative settings for school mathematics.

INTRODUCTION

The gate keeping role of mathematics is well documented (Skovsmose, 1998; Volmink, 1994). In the highly performative English education system, the threshold for progression to advanced level study at age 16, and the many life opportunities that arise from that, is five or more subjects passed at grade A*-C, including English and mathematics. This is the point where students are broadly divided into academic and vocational tracks. The most common difficulty faced by school leavers is achieving this grade in mathematics. It is noteworthy that students from the poorest fifth of households are half as likely to *make the grade* in mathematics as the wealthiest fifth and for the top two grades (A*/A) the ratio increases to over 5:1 (Noyes, 2009a).

My ongoing research with 16 year olds suggests that for many of the most able students the motivation to succeed in mathematics is due, in part at least, to its exchange value. Moreover, at a national level, international comparisons such as TIMSS and PISA fuel government anxieties about educational outcomes and the implications for future economic prosperity in a changing global order. The dominant economic rationalisation for school mathematics (Gutstein, 2009; Noyes, 2009b) tends to reinforce particular forms of curriculum and assessment, pedagogy and learner experience.

The contribution that mathematics makes to social differentiation is related to the economic setting that shapes curriculum and assessment. Despite the increasing entanglement of the mathematical/scientific and the social in modern life there remains far too little attention to this in English mathematics classrooms. Embedding modelling and problem-solving into the curriculum has proved difficult here and there is negligible critical pedagogy. Student experience of mathematics is largely procedural and utilitarian (skills, functionality).

A new version of our national mathematics qualification for 16-year olds is about to be launched in September 2010 and embodies a tension between two future settings:

training skilled workers for technical and trade work ('functional mathematics') and preparing another group of young people for academic courses of further and higher study (advanced level mathematics). In trying to satisfy these quite different purposes the curriculum is, arguably, unfit to meet either goal. Moreover, these two broad settings constrain school mathematics.

Ernest (2004, p. 316) suggests six aims for the curriculum:

- utilitarian knowledge
- practical, work-related knowledge
- advanced specialist knowledge
- appreciation of mathematics
- mathematical confidence
- social empowerment through mathematics

The political/economic settings can be seen in the first three points: *maths for work* and *maths for the academy*. This reflects the age-old tension between vocational and academic education, represented by Ernest's 'industrial trainers' (points 1/2) and 'old humanists' (point 3), both groups of which have considerable influence upon policymaking. I want to argue that the final setting should become a priority for school mathematics: *mathematics for social empowerment.*

RESETTING SCHOOL MATHEMATICS

If we reject the purely economic rationale then how might we rethink school mathematics? I want to outline three framing ideas that have different philosophical and historical origins but prioritize the social over the economic. Here the emphasis is on the learner of mathematics as well as learning of mathematics. [I have considered these in more depth elsewhere (Noyes, 2007)]

- education for citizenship
- the principle of general education
- critical pedagogies (of access and dissent)

I start from the first aim of the National Curriculum for mathematics, which is that the school curriculum should

> aim to promote pupils' spiritual, moral, social and cultural development and prepare all pupils for the opportunities, responsibilities and experiences of adult life.

I do not want to explore the extent to which mathematics classrooms satisfy this aim but suffice to say that there is plenty of room for improvement. Such preparedness for life, which must be broader than the economic productivity of work, can be thought of as citizenship education in its broadest sense.

But democratic citizenship is necessarily critical. So a critical school mathematics (Skovsmose, 1994) would prioritize the equipping of citizens with the capabilities to understand and engage with the mathematically-formatted, data-rich world around them. As Gutstein argues, drawing upon the work of Paolo Freire, school mathematics should encourage learners to "read and write the world with mathematics" (Gutstein, 2006). Importantly, the setting for Gutstein is the immediate lived realities of the students with whom he works, not future work or political engagement.

The German notion of *allgemeinbildung*, which is a general, liberal education befitting a modern, technological, 'risk society' setting, is also instructive here. It develops "competence for self-determination, constructive participation in society, and solidarity towards persons limited in the competence of self-determination and participation" (Elmose & Roth, 2005, p. 21). Mathematics must empower students to function in this modern setting, one in which data and mathematical technologies are increasingly woven into the fabric of modern living.

Heymann (2003) details his vision for (German) school mathematics as framed by general education. His thesis details examples of how mathematics can be highly relevant to each of the six goals below. They are similar in spirit to the National Curriculum aim cited above:

- preparing for later life
- promoting cultural competence
- developing an understanding of the world
- promoting critical thinking
- developing a sense of responsibility
- practicing communication and cooperation

These different ideas are espoused by scholars for whom the *social* setting of modern risk society is as important as *economic* or *knowledge* settings for school mathematics. Although motivated by views of a better, more socially just, society, their political positions are more or less radical and/or liberal. Heymann argues that his mathematics curriculum for general education cannot be realised in a top-down fashion but would better be enacted in small steps by individuals for whom this makes sense. This principle is echoed by Povey (2003, p. 56) when she argues that "to harness mathematics learning for social justice involves rethinking and reframing mathematics classrooms so that both the relationship between participants and the relationship of participants to mathematics (as well as the mathematics itself) is changed". Such approaches are made more difficult in the current climate of educational managerialism, accountability measures and cultures of performativity.

Emphasising citizenship, general education and/or critical pedagogy does not mean that learners shouldn't engage with challenging mathematics for its own sake or future use. Indeed, for those who are currently marginalised by school (mathematics), it is acutely important that a 'pedagogy of access', i.e. the highest quality teaching to achieve the best possible outcomes in high stakes tests, complements any critical pedagogic approach or 'pedagogy of dissent' (Morrell, in Gutstein, 2006). It would be counterproductive to develop more socially just, critical, '*allgemeinbildung*' approaches to school mathematics if the socially differentiated attainment in traditional high stakes mathematics assessments continues to be reproduced.

So, in sum, I have argued that the current framing of school mathematics, in England at least, is predominantly economic (in work and academic settings). A *mathematics for social empowerment* needs to become far more prominent in both designed and enacted curriculum in order to equip 21st century citizens with the (mathematical) means of self-determination in a technological, risk-society setting. The three ideas discussed above – citizenship, general education, and critical education – offer useful existing directions of travel. Rebalancing the framing settings for school mathematics is critically important for young people and for future society, nationally and internationally. However, it is hard to see how such a frame for mathematics education policy and curriculum might gain high-level influence in a neo-liberal, market-driven education system where economics provides many of the generative metaphors. Therefore, following Heymann, this paper is an appeal to those for whom this argument makes sense; to start acting, or to continue to act, in ways that develop mathematics education as a setting for social empowerment.

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