ABSTRACT

Mathematics held an important place in the first twelve of years of the *Educational Times* (1847-1923), and in November 1848 a department of mathematical questions and solutions was launched. In 1864 this department was reprinted in a daughter journal: *Mathematical Questions with Their solutions from The Educational Times* (*MQ*). This thesis concentrates on the development of this department from its inception until 1862, when William John Clarke Miller became its editor; and is considered in terms of the editors, contributors and mathematics. To facilitate this research, a source-oriented database using κλειω (kleio) software was constructed. It contains data taken from the questions and solutions and also miscellaneous items from the journal. Database analysis was used in conjunction with traditional, archival sources; for example, the respective, previously unknown correspondence of two of the main contributors, Thomas Turner Wilkinson and Miller.

The development of the department fell into two main periods: the early 1850s when it was edited by Richard Wilson then James Wharton and had an educational bias; and the late 1850s when it was dominated by Miller and Stephen Watson who contributed moderately complex problems of a reasonably high standard on conic sections, probability and number theory.

In 1850 Miller started contributing with a group of pupils and masters, including Robert Harley, from the Dissenters' College, Taunton. Another group of contributors which emerged was one of northern geometers, with whom Wilkinson was connected. He collaborated with Thomas Stephens Davies on geometry and this influenced his contributions to the department.

Miller edited the department from 1862 to 1897 and *MQ* from 1863 to 1897 and made *MQ* an international journal of renown for its original research. It contained contributions from some of the most eminent national and international mathematicians, including Cayley, Sylvester, Hirst and Clifford. The start of this new phase is briefly introduced and reviewed.

THE DEVELOPMENT OF THE MATHEMATICAL DEPARTMENT OF THE *EDUCATIONAL TIMES* FROM 1847 TO 1862

CONTENTS

2 THE ORIGINS OF THE *ET*

3 THE LAUNCHING OF THE MATHEMATICAL DEPARTMENT AND ITS EARLY EDITORS

4 THE *ET* DATABASE

5 OVERALL QUANTITATIVE ASSESSMENT OF THE DEPARTMENT

6 CONTRIBUTORS TO THE DEPARTMENT

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APPENDIX A

APPENDIX B - ON DISK

This appendix is included as a computer file. This file, **appendb.doc**, a Word 6.0 file, can be found on the disk in a pocket inside the back cover of this thesis.

THE DATABASE FILES - ON DISK

The computer file containing the *ET* database is called **ET11.dat** and is a text file. It can be found on the disk in a pocket inside the back cover of this thesis.

PUBLISHED WORKS

Burt, J. and James, T.B. 'Source-Oriented Data Processing. The triumph of the micro over the macro?', delivered at the International Conference of the Association for History and Computing, Moscow, August 1996 and published in *History and Computing*, **8.3** As this paper was jointly delivered, it is preceded by a letter from Dr James outlining our respective involvement in the paper.

0 INTRODUCTION

0.1 Scope of the thesis

The starting point for this thesis was 'A Note on *The Educational Times* and *Mathematical Questions*' by I. Grattan-Guinness in *Historia Mathematica* which draws attention 'to a unique pair of journals' [Grattan-Guinness 1992 76]. The note contains a short description of the history and contents of the two journals and lists some of the eminent mathematical contributors from Britain and abroad. The final paragraph contains the statement, 'More to the point, however, is that nobody knows what these volumes contain (or what questions were left untackled).' The desire to begin to analyse the mathematical activity in these journals formed the impetus for this project.

Victorian mathematics is an area which has not been very well-served in the literature, as will be demonstrated in the literature review (see 0.2). Unfortunately this is even more true for mathematical journals. In the recent *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, E. Neuenschwander remarks at the beginning of his chapter on mathematical journals, 'The importance of professional mathematical journals for scientific interchange in the present-day mathematical community is unquestioned. Nevertheless, there has been no comprehensive study of the historical development of mathematical journals. The special studies thus far available on this theme are practically unknown, and are mainly confined to a single journal. This article can, therefore, provide only a preliminary overview of the subject, and compile the literature for further research.' [Neuenschwander 1994 1533-9]. The research in this thesis on the mathematical problems in an educational journal thus includes much pioneering work.

The *ET* ran from October 1847 until December 1923. A department of numbered mathematical questions started in August 1849. This soon became a regular feature and comprised new questions and some solutions to previous ones. The department continued, under different editors, until 1917. The total number of questions published was approximately 19,000. These questions and solutions lend themselves to database analysis but due to the amount of material involved a decision had to be taken as to which years should be analysed. Should the data be sampled at regular intervals, for example, every five years, and cover the entire lifespan of the department, or be taken from a particular period? It was decided to concentrate on the first years of the department as this would shed light upon how and why it came into being. A glance at the department from the mid 1860s onwards revealed contributions from many eminent mathematicians; but prior to this the contributors were not immediately recognisable. In addition the editor of this department was unknown before the mid 1860s. These gaps in knowledge formed the basis for the specific questions which the thesis addresses:

1) Why and when was the mathematical department started in the first place?
Why was there a mathematical department - did mathematics hold a special place in the *ET*? What needs did the department aim to satisfy? Who was it aimed at? When precisely did it start and what were the first questions?

2) Who were the editors of the mathematical department?
Did the editors have any specific aims and/or objectives? Were these actually carried out? Did the editors contribute any questions/solutions themselves?

3) What was the profile of the mathematical department itself?
How many questions were posed and how many of these were solved? How long did it take to solve the problems and did this vary over time? Were there any special sub-sections of questions and solutions? Did these last throughout the whole period? How many questions were unsolved and was there any particular reason for this? How many solutions were there per question? How many questions were solved by their proposer and did this vary over time? Were there any trends in the department? Which contributors were implicated in the different phases/sections of the department, and what were their associated mathematical contributions?

4) What was the constitution of the community of mathematicians actively contributing to the department?

Were they major or minor mathematicians? Were there any regional or institutional networks of mathematicians? What were the main periods and relative time spans of activity of the contributors? Were any textbooks mentioned in the questions/solutions which pointed to particular mathematical communities? Was there evidence of a community of mathematicians contributing to several journals? Briefly, was there any correlation between the contributors to the department and those involved in the setting up of the London Mathematical Society (LMS), (founded 1865) or the Association for the Improvement of Geometrical Teaching (AIGT), (founded 1871)?

5) What kind of mathematics was contained in the department?

What was the level of mathematics in the questions and solutions and how did it change over time? How does the mathematics in the questions/solutions reflect (or not) the mathematical activity in Britain? What broad mathematical subject areas did the questions and solutions fall into? Did any questions specify solution by a particular method? If so, was this method actually used in the solution? What was the balance of pure and applied mathematics? Were any new areas of mathematics developed in the mathematical department?

0.2 Literature review

In 'Notes on some minor English mathematical serials' by R. C. Archibald published in 1929 in *The Mathematical Gazette* [Archibald 1929 379-400], he observes that many prominent mathematicians contributed questions/solutions to mathematical journals. He mentions that although these questions were generally not of the greatest difficulty, they were still reasonably taxing and there were often well-constructed solutions which were instructive to the reader. He also notes that contribution to these journals was widespread compared with other countries. He lists many minor mathematical journals, giving an outline of their contents, details of their editors and on occasions highlighting major contributors. He also indicated further sources of information. In particular, he highlighted as a main source works by and about

Thomas Turner Wilkinson (1815-1874). Wilkinson's articles in the *Mechanics Magazine*, volumes 54-9 1848-1853 form an expansive study of mathematical journals, including both well-known and rare journals.

Other works on mathematical journals include shorter summary articles; 'Mathematical Journals' by J.W.L. Glaisher [1880] and 'Notice sur le journalisme Mathématique en Angleterre' by J.S. Mackay [1893]. The only study of a single Victorian mathematical journal appears to be T. Perl's study of '*The Ladies' Diary* or *Woman's Almanack*, 1704-1841' [Perl 1979 36-53]. This includes an analysis of the mathematical categories of the questions and also a survey of how many were contributed by women.

Victorian mathematics is not served much better. Studies of Cambridge mathematics predominate as Cambridge was the mathematical centre of Victorian Britain [Grattan-Guinness 1994a 1484]. *A History of the Study of Mathematics at Cambridge* by W.W. Rouse Ball in 1889 was and still is regarded as a seminal work on Cambridge mathematics. H. Becher [1980 1-48] discusses William Whewell, the applied mathematician and philosopher, and Cambridge, where he shaped the mathematical curriculum. Grattan-Guinness [1985 84-111] also discusses mathematics and mathematical physics at Cambridge from 1815 to 1840 with reference to French influences.

A more general text on Victorian mathematicians is A. Macfarlane's *Lectures on Ten British Mathematicians of the Nineteenth Century*, [1916], which provides a snapshot of the characters and mathematics of ten eminent Victorian mathematicians, mainly academics. There are a few biographies, including several of the logician George Boole, including [MacHale 1985], several of William Rowan Hamilton, the developer of quaternions, including [O'Donnell 1983], and [Hankins 1980], and one on Augustus De Morgan by his wife [De Morgan 1882].

E.G.R. Taylor's work on *The Mathematical Practitioners of Hanoverian England, 1714-1840* [1966] surveys mathematical practitioners - navigators, instrument makers and the like, from the early eighteenth century to almost the mid nineteenth century. The only major period study of a particular mathematical subject is J. Richards' book *Mathematical Visions: The Pursuit of geometry in Victorian England* [1988]. This covers both high level geometry by major mathematicians, for example by Arthur Cayley, and also Euclidean geometry in schools.

Mathematics education in England is covered by *A History of Mathematics Education in England* by A.G. Howson [1982]. This is in effect a history of nine mathematical educators, but it also includes valuable information on individuals and movements of interest in the Victorian era. W. H. Brock and M. H. Price [1980] discuss changes in mathematical education in Victorian Britain in 'Squared paper in the nineteenth century: instrument of science and engineering, and symbol of reform in mathematical education'. Brock's paper on 'Geometry and the Universities: Euclid and his Modern Rivals 1860-1901' [1975 21-35] and Price's 'Mathematics in English education 1860-1914: Some questions and explanations in curriculum history' [1983 271-84] cover the attempt by the Association for the Improvement of Geometrical Teaching (AIGT) to remove the Euclidean stranglehold on geometrical teaching. The AIGT became the Mathematical Association, and the history of both associations is charted by Price in his work *Mathematics for the Multitude?* [Price 1994].

V. Chapman's book *Professional Roots: The College of Preceptors in British Society* [1985] appears to be the only comprehensive account of the College from its inception to recent times. It contains many references to the *ET* but hardly mentions mathematics. 'The Contribution of the College of Preceptors' was the subject of an MPhil dissertation by R. Willis at Leeds University in 1994.

0.3 Outline of the thesis

0.3.1 Methodology

There are several different historiographical approaches which could be employed for studying the material in the mathematical department of the *ET*. Possibilities include the study of a particular mathematical subject, for example geometry, throughout the life of the *ET*, or the contributions of a particular mathematician throughout the life of the journal. These methods were not chosen here as they address only a part of the mathematical life of the *ET*. Instead the mathematical department is studied in a wide variety of aspects over a defined period of time, August 1849-April 1862.

The archival methods involved in this thesis will now be summarised. It was initially extremely difficult to obtain photocopies of the relevant volumes of the *ET* as there were only two known complete sets of the *ET* in the UK. One was at the British Library Newspaper Library at Colindale, London, and although it was possible to view four volumes at a time by prior arrangement, it was prohibitively expensive to photocopy the material. The other set originally belonged to the College of Preceptors, and had been stored in an attic on their premises. A more suitable repository was required for such a unique collection of journals, and the College allowed its removal to the Institute of Education Archives, in London. For various reasons it was many months before the volumes were available for inspection by the general public. This factor introduced considerable delay and inconvenience to the work undertaken here. When the volumes were eventually accessed, it was possible to photocopy the mathematical department from each month, and also any other items of mathematical interest.

0.3.2 Thesis contents

Chapter one provides the mathematical backdrop for this thesis, commencing with a survey of French mathematics after the revolution of 1789 and then looking at the influence of French mathematics on the development of the calculus in Britain resulting in the emergence of the Analytical Society. The teaching of mathematics at the major British institutions from the 1810s to the 1840s is delineated. Learned societies and mathematical journals are examined during the same time span. The major British mathematical developments from the 1840s to the 1860s are reviewed. Finally, significant mathematical developments from elsewhere are noted. Chapter two gives the corresponding background information to the inauguration of the College of Preceptors and the founding of the *ET*. The place of mathematics generally in the *ET* is then surveyed.

Chapter three investigates the inception of the mathematical department and discusses the early mathematical editors. Chapter four introduces the construction and analysis of the database from which results are used in subsequent chapters. Chapters five to seven are carried out using a top-down approach, starting from the profile of the department, proceeding to a study of the contributors and lastly the mathematics. Chapter five takes as its starting point the profile of the mathematical department and identifies major sections and trends within it. The corresponding contributors and their mathematics are then discussed . Chapter six starts with the contributors and scrutinises the community of mathematicians connected with the department and in tandem considers their mathematics. Chapter seven takes as its starting point the mathematics in the questions and solutions themselves and considers the range of subjects covered. Chapter eight summarises the mathematical developments in the department post 1862, when the daughter journal was launched, and also in the wider mathematical community with the foundations of the LMS and AIGT. Reflections on the department observed nearer the end of the nineteenth century are then given. The status and contents of the department in the latter years is contrasted with that in the early years. There follows a critique of the database methodology and finally conclusions are drawn.

0.4 Acknowledgements

I would like to acknowledge the help and encouragement I have received from many different quarters. I am indebted to my director of studies, Professor Ivor Grattan-Guinness for his unstinting guidance, direction and inspiration, and to my second supervisor Dr Tony Crilly for his invaluable comments and advice. Especial thanks are due to Professor Eduardo Ortiz for his help at the beginning of this project and to Dr Tom James for reading through the thesis. Thanks also to Professor Jon Press for his comments on the relational model in chapter four.

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ET - *Educational Times*

MQ - *Mathematical Questions with Their Solutions from the Educational Times*

DNB - the Dictionary of National Biography

MC - Miller Correspondence

DC - Davies Correspondence

LMS - London Mathematical Society

AIGT - Association for the Improvement of Geometrical Teaching

BSHM - British Society for the History of Mathematics

AHC - Association for History and Computing

I BRITISH MATHEMATICS, 1840 TO 1865

1.0 Chapter overview

This chapter provides a backdrop against which to analyse the mathematical material in the *ET* and serves to set the work in this thesis in context. The mathematical problems in the *ET* need to be evaluated in the context of mathematical development in Britain from the 1840s to the mid 1860s. Before this can be delineated, the state of British mathematics at the end of the eighteenth century will be outlined with its stunted development being seen to be strongly influenced by Newtonian calculus. This is contrasted with the situation in France where Lagrange's approach to the calculus used the differential and integral calculus of Leibniz and Euler and stimulated mathematics generally with important work emerging in analysis, especially in applications. Following the Revolution of 1789 there appeared several highly influential institutions in France, particularly the *Ecole Polytechnique* (1794) which was to be the breeding ground for many fine pure and applied mathematicians. These institutions were vital for the training of soldiers and as a consequence the teaching of science and mathematics was deemed important. Many of them, such as the *Ecole des Ponts et Chaussées* were engineering schools. Teaching at these institutions acquired a professional status and research also abounded. Significant journals appeared that covered both research and educational mathematics. The major works of most of the mathematicians of this era are referenced in the *ET*.

Continental mathematics, particularly Leibnizian and Lagrange's versions of the calculus, were forcibly brought to the attention of British mathematicians by the Analytical Society at Cambridge University in 1815. In the light of this, British mathematics in institutions, journals and education is then surveyed and is compared with the French situation. The low level of research activity and the predominance of Euclidean geometry in Britain when compared with France [Grattan-Guinness 1985 84-111] is particularly noted. Major areas of mathematical development from 1840 to the 1860s are then set out to see whether there is any reference to them in the problems, and whether the problems contain material of a similar level.

This chapter does not contain original material and the main sources used are Grattan-Guinness (1980) for French mathematics and Howson (1982) and Becher (1980) for British mathematics. General sources consulted include Kline (1972), Eves (1990) and Grattan-Guinness (1994).

1.1 Prehistory of British and Continental mathematics

This section begins by studying the independent development of versions of the calculus by both the Briton Isaac Newton (1642-1727) and the German Willhelm Gottfried Leibniz (1646-1716) in the late seventeenth century. Their work set up the calculus as a new and powerful method which could be applied to different areas of mathematics and used to address a large range of problems. Each man devised his own new algebraic notation and techniques which were more powerful than just using geometrical methods and led to the solution of various physical and geometrical problems. Area, volume and similar problems had previously been approached via summation methods but both men formed the vital link between these problems and integration, the reverse process of differentiation. The theories were different however, and it is this difference which played a large part in the development of mathematics in Britain in the eighteenth century and had ramifications in the early nineteenth century.

In 1666, Newton introduced the new concepts of fluent and fluxion, both based on ideas of motion. A fluent x was a quantity which flowed in time with the fluxion $\dot{x}$ as its velocity. He extended these ideas to the realm of geometry in his great work *Philosophiae naturalis principia mathematica* (1687) where he used the idea of geometrical quantities in motion. In this scheme, a line is seen as being generated by a moving point, a plane by a moving line etc. The problem he was trying to solve was how to discover the rate of change of a particular quantity, given the motion. Similarly, the converse problem also presented itself to him - how to discover the motion given the rate of change. In *Tractatus de quadratura curvarum* (1704) Newton defined fluxions in terms of the ratio of finite increments. The 'limits of prime and ultimate ratios' as he termed them were rooted in a geometrical context and were reminiscent of the limiting processes found in the method of exhaustion of Eudoxus and Archimedes [Guicciardini 1994 310]. Newton also used series, especially power series, to represent functions. To summarise, Newton's invention of the calculus was based on the limiting values of ratios of geometrical quantities in motion and used the notation of x and $\dot{x}$.

Leibniz's discoveries regarding the calculus were made in autumn 1675 [Grattan-Guinness 1997 244] approach to the calculus was carried out using fundamentally different notions. His approach to the integral calculus was to take a Cartesian curve and divide the area below the curve into a series of rectangular strips. He then calculated the area below the curve as the sum of the rectangles ydx where x is the abcissa, dx is an infinitesimally small section of the abcissa to the right of x and y is the ordinate. dx was termed the differential of x, the infinitesimal case of a forward difference. The slope of the tangent to the curve was given by dy/dx, literally the ratio of the infinitesimal ordinate dy to the infinitesimal abscissa dx. Leibniz preferred not to rely upon series and tried to obtain results in finite terms. Thus Leibniz's method was purely geometrical with no suggestion of motion.

A bitter dispute was instigated in the early eighteenth century by Newton as to whether Leibniz had appropriated Newton's work. With hindsight it transpires that Newton carried out much of his work before Leibniz while Leibniz did invent the calculus independently. At the time these facts were not clear and there was a great deal of hostility regarding priority of invention. Mathematicians sided with Newton or Leibniz, with the overall result that mainly continental mathematicians, particularly the Swiss brothers James and John Bernouilli, supported Leibniz and mainly British mathematicians supported Newton.

Partly due to this dispute, the reaction of the mathematical world to these differing methods of using the calculus was marked. Newton's approach was seen as being logical but with an unwieldy notation whereas Leibniz's method and notation could be readily used. It was possible however to use Leibniz's method and notation and relate this to Newton's method and notation. Most eighteenth century mathematicians adopted Leibniz's method and notation, especially after the 1740s when Newton's tradition atrophied after MacLaurin, and Leibniz's influence as a result was huge. Both approaches were very powerful when applied to problems in mechanics.

Newton's work suffered further attack when Bishop George Berkeley (1685-1753) criticised Newton's terminology and his methods of proof in a pamphlet entitled *The Analyst* in 1734. The Scot Colin Maclaurin (1698-1746) was the chief defender of Newton's methods in his book *A Treatise of Fluxions, in Two Books* (1742), which was highly influential in British mathematics. Maclaurin was one of the best mathematicians of the eighteenth century and published his first major work, *Geometria Organica* at the age of twenty one. This work was dedicated to Newton although it did not make use of fluxions. At the beginning of the eighteenth century few mathematicians were familiar with fluxions whereas following Maclaurin's work many other treatises on fluxions appeared. According to Guicciardini 'It was about the middle of the century that the world of 'philomaths' and, perhaps, some students in the military academies and universities began to practise with Newton's dots.' [Guicciardini 1989 55]. Thus the British settled down with their fluxional notation in the second half of the eighteenth century and the mathematical developments that did occur were mainly geometrical.

Meanwhile, continental mathematicians built on the analytical methods introduced by Leibniz. There had been a cessation in the exchange of ideas between Britain and the Continent due to the priority controversy, and this was to prove devastating to the development of English mathematics. The result was the stultification of English mathematics whilst the continental mathematicians forged ahead developing Leonhard Euler's (1707-83) and Leibniz's analytical methods. 'The isolation of English mathematicians from their continental contemporaries is the distinctive feature of the history of the latter half of the eighteenth century [Rouse Ball 1889 117]. In addition, to replace both Newton's and Leibniz's approaches, Joseph Louis Lagrange (1736-1813) introduced the concept of functions and their derivatives using Taylor's series as a means to link a function to its derivative. His approach was mistaken in its assumption that such a series always existed, but his approach was an important development mathematically and influenced French mathematics after the Revolution of 1789.

1.2 French mathematics after the Revolution of 1789

There was a golden era of French mathematics after the Revolution of 1789 which was to filter through to Britain in the early nineteenth century. Due to the Revolution, new educational and professional institutions arose in France and these were to become extremely influential in terms of the development of mathematics. Journals connected with these institutions appeared and also some others which arose independently [Grattan-Guinness 1988a 208].

1.2.1 The *Ecole Polytechnique*

The *Ecole Polytechnique* was a new institution founded 1794 shortly after the Revolution. It was conceived as an institution for the training of engineers in a broad sense, including the military and naval as well as civil and mechanical engineers and initially bore the name *Ecole Centrale des Travaux Publics* (Central School for Public Works). This broad ideal was practically unrealisable as Paris was unsuitable for training the naval engineers so the function of the *Ecole Polytechnique* was changed to one of equipping the engineers with the multifarious techniques they would need, hence the new name of the *Ecole Polytechnique*. Mathematics was a core subject, others being physics, chemistry and engineering.

Of particular interest are the staff of this institution which employed admissions examiners, also examiners who were responsible for passing or failing students at the end of each year and lastly teachers. Most importantly, a *professeur* (teacher) could not simultaneously be an *examinateur* (examiner) or vice versa. A table of the key *professeurs* and *examinateurs* is now given, adapted from Grattan-Guinness [1988b 201].

Fig. 1.1 Mathematicians of interest on the staff of the *Ecole Polytechnique*

Mathematician	**Function**	**Subject**	**Time Period**
Lagrange	*Professeur*	Analysis/Mechanics	1794-1799
Lacroix	*Professeur*	Analysis/Mechanics	1799-1808
"	*Examinateur*	Analysis/Mechanics	1808-1815
Laplace	*Examinateur*	Analysis/Mechanics	1794-1799
Legendre	*Examinateur*	Analysis/Mechanics	1799-1816
Monge	*Professeur*	Descriptive Geometry	1794-1810
Poisson*	*Professeur*	Analysis/Mechanics	1808-1815
"	*Examinateur*	Analysis/Mechanics	1815-1840
de Prony	*Professeur*	Analysis/Mechanics	1794-1814
"	*Examinateur*	Analysis/Mechanics	1816-1838
Arago*	*Professeur*	Descriptive Geometry	1810-1816
"	*Professeur*	Machines	1818-1831
Cauchy*	*Professeur*	Analysis/Mechanics	1816-1830
Hachette	*Professeur*	Descriptive Geometry	1799-1815
"	*Professeur*	Machines	1806-1816
Liouville*	*Professeur*	Analysis/Mechanics	1839-1850
Sturm	*Professeur*	Analysis/Mechanics	1840-1855

* represents a graduate of the *Ecole Polytechnique*, or *polytechnicien*

Many of the men listed above were highly influential in the development of mathematics in the early nineteenth century and will now be considered individually together with their mathematical contributions these being: Lagrange, Lacroix, Laplace, Legendre, Monge, Arago, Poisson, Fourier and Cauchy. Carnot did not teach or examine at the *Ecole Polytechnique* but will be considered here alongside Monge.

1.2.2 Major mathematicians connected with the *Ecole Polytechnique*

As well as his work on the calculus (see below), Lagrange completed two other great works - *Traité de resolution des équations numériques de tous degrés* (1767), which used continued fractions to approximate the real roots of an equation and his impressive *Méchanique analitique* (1788), which presented statics and dynamics on terms of solely algebraic principles such as that of least action. He also wrote important papers on number theory, and his work on the theory of equations inspired Evariste Galois (1811-1832) to develop group theory.

Lagrange's position as a founding professor of the *Ecole Polytechnique* gave him a platform to teach the calculus using his own algebraic methods. He described them fully in his *Théorie des fonctions analytiques* (1797) and *Leçons sur le calcul des fonctions* (1806). Lagrange contributed enormously to the high mathematical standard at the *Ecole Polytechnique.* He was succeeded as *professeur* by Sylvestre François Lacroix (1792-1867). Lacroix developed a love of mathematics which was encouraged by Monge. He was also deeply influenced by the philosophers the Marquis de Condorcet (1743-94) and the Abbé de Condillac (1714-60). He was an outstanding textbook writer but did not produce any research level mathematics of note. Lacroix wrote an influential textbook on the calculus which compared all three methods: *Traite du calcul differentiel et du calcul integral* (three volumes 1797-1800 and second edition 1810-1819), the differentials of Leibniz being prominent. In the early nineteenth century this book was to have profound effects on a group of men at Cambridge - the Analytical Society (see 1.1.2 below).

Pierre Simon Laplace (1749-1827) was a contemporary of Lagrange and published two considerable works - *Traité de mécanique céleste* (5 volumes 1799-1825) on theoretical astronomy and *Théorie analytique de probabilités* (1812 and later editions) on probability. Other work included geodesy and differential equations.

Adrien-Marie Legendre (1752-1833) produced important work in both elementary and higher mathematics. His *Eléments de Géometrie* (1794) was an attempt to resolve ambiguities with Euclid's parallel postulate (the fifth) and contained Euclid's propositions in a clearer and more understandable form. The seminal English translation was made by John Farrar of Harvard University and a further one was made by the Scot Thomas Carlyle which was later revised and used extensively in America. In higher mathematics, Legendre published the seminal *Essai sur la théorie* *des nombres* (1797-8) on number theory. Between 1811 and 1819 he produced *Exercices du calcul intégral* in three volumes on the integral calculus. He extended it in the three volume *Traité des fonctions elliptiques et des intégrals Eulériennes* (1825-32) which covered elliptic functions and beta and gamma functions. Together with Lagrange, Lacroix and Laplace, Legendre formed part of the theoretical mathematical backbone of the *Ecole Polytechnique*, the first two as *professeurs* of Mechanics and the second two as *examinateurs* of Mechanics, Lacroix being *professeur* then *examinateur*.

The founding *professeur* of descriptive geometry was the geometer Gaspard Monge (1746-1818). Monge became a physics instructor at the Collège in Lyon when only sixteen. Monge's geometrical methods were initially classified as secret by the military and withheld from the public but were later disseminated as descriptive geometry and were to have a significant effect on the development of geometry in the nineteenth century. During the restoration of 1816 he was removed from the *Institut de France* in retribution for his close alliance with Napoléon.

At the *Ecole Polytechnique* Monge proved himself an exceptional teacher and he inspired a generation of talented geometers including Jean Victor Poncelet (1788-1867). In addition to his *Traité de Géométrie Descriptive* (1799) he also produced a foundational work in differential geometry - *Application de l'Analyse a la Géometrie* (1795) which contained the concept of lines of curvature of a surface. Monge also contributed enormously to the field of solid analytic geometry as a result of his lectures.

Lazare Nicolas Marguerite Carnot (1753-1823) studied at Mézieres under Monge. He was not a *professeur* or *examinateur* at the *Ecole Polytechnique* but is considered here as his work complements that of Monge. He composed his *Géometrie de position* and *Essai sur la théorie des transversales* in exile. His son Hippolyte became minister of public instruction in 1848.

Siméon-Denis Poisson (1781-1840) was appointed as *professeur* of Mechanics after Lacroix. He entered the *Ecole Polytechnique* in 1798 where his ability was noticed by Lagrange and Laplace. After graduating he obtained a post as a *professeur*. Poisson's work was mainly on differential equations and functions and elasticity, electricity and magnetism, probability theory and physical astronomy. His main books were *Traité de mécanique* (1811 and 1833) *Théorie nouvelle de l'action capillaire* (1831), *Théorie mathématique de la chaleur* (1835) and *Recherches sur la probabilité* (1837).

Jean-Baptiste Joseph Fourier (1768-1830) was also an applied mathematician. He attended the *Ecole Normale* and made such a good impression that he was offered an assistant teaching post at the *Ecole Polytechnique*. He was a close friend of Monge and taught for a short while at the *Ecole Polytechnique*. He produced new work in the area of heat in 1807 which was eventually published in 1822 as *Traité analytique de la chaleur*. This work was later to inspire the great British applied mathematicians William Thomson (later Lord Kelvin) (1824-1907) and James Clerk Maxwell (1831-79). He also introduced 'Fourier' series in the course of the *Traité analytique de la chaleur*. These series have proved invaluable in a vast range of applied mathematical problems.

Gaspard Clair François Marie Riche de Prony (1755-1839) studied at the *Ecole des Ponts et Chaussées* and in an active engineering career directed a monumental project to produce new decimal tables (*Tables du Cadastre)* for use with the new metric system. De Prony was an engineer of distinction and a gifted teacher. He cultivated contacts with foreign engineers but did not appear to visit Britain. Charles Babbage (see 1.1.2 below) would have heard of De Prony's enormous task when he visited Paris in 1819; this was to have a profound effect on Babbage [Campbell-Kelly and Aspray 1996 11].

Dominique Francois Jean Arago (1786-1853) entered the *Ecole Polytechnique* in 1803 and became a *professeur* there in 1810, holding the office of *secrétaire* to the *Bureau des Longitudes* in 1805. His chief interests were in optics, astronomy and geophysics, aided in the latter by his friend Alexander von Humboldt (1769-1859). He was a capable mathematician but made his mark with his entrepreneurial work.

Augustin Louis Cauchy (1789-1857) entered the *Ecole Polytechnique* in 1805 and then moved on to study at the *Ecole des Ponts et Chausées* after which he was an active engineer for a while. He studied the monumental works of Laplace and Legendre. Cauchy was made a *professeur* of analysis and mechanics at the *Ecole Polytechnique* in 1816. He brought great rigour to pure mathematics and he particularly tightened up the area of analysis with his work on the divergence and convergence of infinite series. He also contributed to real and complex function theory, determinants, probability, mathematical physics and differential equations. Following the restoration of 1816 he was appointed to the re-formed *Academie des Sciences* without having to undergo the uncertain process of election. After the revolution of 1830 he had to leave the *Ecole Polytechnique* but he took up his professorship again in 1848, the intervening years having been spent in exile in Turin and Prague.

This concludes the brief survey of major mathematicians involved with the *Ecole Polytechnique*, although it must be stressed that these are only a few of a large contingent. Mathematicians at the *Ecole Polytechnique* will now be considered who were involved with journals - Jean Nicolas Pierre Hachette (1769-1834), Joseph Liouville (1809-1882) and Jacques Charles Francois Sturm (1803-1855).

1.2.3 Journals connected with the *Ecole Polytechnique* or its staff

The *Journal Polytechnique* was launched in 1795 and swiftly bore the title *Journal de l'Ecole Polytechnique*. It contained the lecture notes of the *Ecole* but rapidly became effectively a research journal in pure and applied mathematics serving the high-powered mathematicians working there. The situation was redressed by Hachette in 1804 who started the *Correspondance sur l'Ecole Imperiale Polytechnique*. This included general news of the *Ecole* and articles appropriate for the students to read, some being submitted by current or recently-graduated students. The journal continued until 1816 when Hachette was retired. According to [Grattan-Guinness

1990 93] 'Both the *Journal* and the *Correspondance* seem to have been quite widely circulated.'

Joseph Diaz Gergonne (1771-1859) commenced his *Annales de mathématiques pures et appliquées* in 1810, to address the same lacks as Hachette's *Correspondance* above. Gergonne's *Annales* paid attention to 'pedagogical concerns as well as to research interests' [Grattan-Guinness 1990 194]. This journal ran until 1832 and contained many articles by Gergonne who used his own name or the pseudonym - 'a subscriber'.

Gergonne's *Annales* were to inspire Liouville who read them at the age of 15 and was sufficiently able mathematically to write his own comments about their contents. Liouville graduated from the *Ecole Polytechnique* in 1825 and then moved on to the *Ecole des Ponts et Chausées.* In 1836 he decided to launch a journal *Journal des Mathématiques pures et appliquées.* He wanted this to be devoted entirely to mathematics and from the beginning it was a high level journal aimed at mathematical research and not pedagogy or elementary mathematics. 'Liouville's journal, as it was known, turned out to be like Crelle's in its level, and from the start it emulated its German predecessor in international importance and renown' [Grattan-Guinness 1990 1227]. The German August Leopold Crelle (1780-1855) had launched the *Journal für die reine und angewandte Mathematik* (Journal for pure and applied mathematics) in 1826. He hoped to attract a broad range of mathematical articles but soon Crelle's journal, as it was informally known, became filled with pure mathematical articles.

Liouville's journal was preceded by another French journal when Audebart, Baron de Ferrusac began his *Bulletin général et universal des annonces et des nouvelles scientifiques* in 1823. It was re-shaped in 1824 as the *Bulletin universel des sciences et de l'industrie.* This normally appeared monthly and contained abstracts, reviews and original articles on all aspects of science, technology and medicine, including mathematics. Most importantly it covered material from other countries. The Swiss Sturm succeeded Ferrusac as editor for four volumes of the series *Sciences mathématiques, astronomiques, physiques et chimiques.*

In 1831 the *Journal officiel de l'instruction publique* appeared, containing details of life at the Université plus wider educational news from France and abroad. There were also book reviews, articles and reports of meetings at the Académie. As it was not an official publication, it underwent a name change to the *Journal général de l'instruction publique* in 1834.

The *Nouvelles annales de mathématiques* appeared in 1842 and was edited by Olry Terquem and Camille Gerono. This journal was pitched at the same mathematical level as Hachette's *Correspondance* and Gergonne's *Annales*, but its sub-title revealed that it was specifically for the candidates to the *Ecole Polytechnique* and *Normale*. The *Archiv der Mathematik und Physik* had appeared in Germany the previous year with similar aims.

In 1835 the *Académie des Sciences* launched the *Comptes rendus de l'Académie des Sciences* which contained the reports of its weekly meetings plus details of members' and non-members' papers.

A summary of these journals in chronological order is given in fig. 1.2

Fig. 1.2

Journal	**Launch**	**End**	**Editor**	**Orientation**
Journal de l'Ecole Polytechnique	1795			initially student then research.
Corresponance sur l'Ecole Imperiale Polytechnique	1804	1816	Hachette	student
Annales de mathématiques pures et appliquées	1810	1832	Gergonne	student and research
Bulletin général et universal...	1823		Ferrusac then Sturm	diverse scientific material

Journal für die reine und angewante Mathematik	1826		Crelle	research
Journal officiel de l'instruction publique	1831			student
Comptes rendus de l'Académie des Sciences	1835			research
Journal de mathématiques pures et appliquées	1836		Liouville	research
Archiv der Mathematik und Physik	1841			student
Nouvelles annales de mathématiques	1842		Olry Terquem & Camillo Gerona	student

1.2.4 Other institutions

After the Revolution, the *Académie des Sciences* was replaced by the *Institut de France*, which contained several classes, that of the mathematical and physical sciences being the principal. The *Académie* was restored in 1818 after the fall of Napoléon.

In 1810 Napoléon founded the '*Université Impériale de France*' to replace the old system of *universités* that disappeared in 1793. Its remit included training for teachers. It included the *Ecole Normale*, an élite institution and reincarnation of an institution of the same name which had started in 1795 but ran for only four months. By mid century it began to gain significance for science and mathematics.

After they had graduated from the *Ecole Polytechnique*, students attended the '*écoles d'application*'. There were two major *écoles d'application* which are particularly noteworthy here - the *Ecole des Ponts et Chaussées*, founded in 1747, and the *Ecole des Mines*, both of which continued relatively unchanged throughout the Revolution. The *Conservatoire des Arts et Métiers* was founded in 1793 to cater for teaching at an elementary level and also to house equipment. This low-key activity continued until 1819 when it was reformed by Arago who introduced three new chairs, one being in geometry and one in mechanics. The *polytechnicien* Dupin occupied this chair and propounded many current theories in mechanics. Poncelet became its *Directeur* in 1849. The three *écoles* above served the area of engineering, but did not address commerce or industry. This was rectified in 1829 by the founding of the *Ecole Centrale des Arts et Manufactures*. Mechanics and engineering were taught here, in addition to chemistry and chemical technology, Liouville being one of the mathematics *professeurs*.

The above survey shows the vibrancy of French mathematics after the Revolution. There were numerous mathematicians of a very high calibre and the new institutions encouraged research in higher mathematics, new courses to be developed in mathematics and new journals for the dissemination of this material. It was a halcyon period. The confluence of military and mathematical education in the institutions above resulted in 'the growth of the professionalization of science' [Grattan-Guinness 1988b 210]. Moving back to England, a very different scenario was apparent.

1.3 The Analytical Society and other reforms

Returning to the issue of the calculus in Britain, the influence of Robert Woodhouse (1773-1827), who obtained his M.A. at Cambridge in 1798, is now outlined. He became a fellow of Gonville and Caius College, was the Moderator of the Tripos for some years and later concentrated on astronomy. He was greatly influenced by Lagrange's approach to the calculus and agreed with him on many points His views on the calculus appeared in his book *The Principles of Analytical Calculation* (1803) and '(t)hrough him the British came into contact with the Lagrangian algebraical school' [Guicciardini 1989 129]. This had a reforming effect on British mathematics which unfortunately did not percolate down to students, even though Woodhouse was interested in teaching. Disgruntled students at Cambridge felt forced to take the initiative themselves and the Analytical Society was formed at Cambridge in 1812 with Charles Babbage at the helm.

Charles Babbage (1791-1871) was the son of a banker and received his education, unfortunately not of a high standard, at Totnes grammar school. Due to his wide reading of mathematics textbooks he became a fine self-taught mathematician. He had heard of Lacroix's great treatise on the calculus, the *Traite du calcul differentiel et du calcul intégral* and on his way up to Cambridge University in 1810 he decided to go to London and purchase it. Due to the Napoleonic wars, French books were rare and expensive in England and this book was much more costly than Babbage had imagined. However, he purchased it and read it avidly upon arriving in Cambridge. At Cambridge Babbage had hoped to receive guidance from the mathematical tutors on any mathematical difficulties he came across during his reading. He was deeply disappointed to discover that not only could his tutors not answer his questions, but also that some of them appeared not to be familiar with the textbooks or the mathematics they contained and merely directed Babbage to turn instead to the elementary mathematics that would appear in the Cambridge exams.

One source of consolation to Babbage was that he now had access to papers from the academies of Berlin, Paris and Petersburg via the Cambridge libraries. In October 1811 he purchased eight volumes of the *Journal de l'Ecole Polytechnique*. Babbage noticed the continental notation in Woodhouse's book on analytical calculation and also in Lagrange's *Théorie des fonctions analytiques* (1797). He then vigorously took up this notation and helped form the 'Analytical Society' in 1812 which advocated the 'd-ism' of Leibniz as opposed to the 'Dot-age' of Newton' [Hyman 1982 24] which was enshrined in Cambridge teaching. Two other notable members of this society were George Peacock (1791-1858) and John Herschel (1792-1871). This society wished to influence the teachers at Cambridge to accept continental versions of the calculus, and they translated a treatise of Lacroix' textbook on the calculus (a much shorter version of his *Traité*). The society folded in 1814, having provoked outrage at Cambridge, but the reforms they wished to occur did come about but in a more measured and circumspect manner [Hyman 1982 20-7]. Babbage went on to mathematical research in algebras before turning to calculators, Herschel also researched in these areas before he carried on his father's astronomical work, and Peacock remained at Cambridge working on the foundations of algebra which were influenced by Woodhouse's work on the calculus. According to Guicciardini [1989 137], '(the) most important contribution from the Analytical Society's members was that they initiated a trend of research which characterized much of British mathematics up to Arthur Cayley (1821-1895) and George Boole (1815-1864). The *Memoirs of the Analytical Society* were centred on the calculus of operators and on functional equations.'

Cambridge was not the only centre of reform; the continental influence had also reached Scotland and Ireland. In Scotland, despite a geometrical tradition, John Playfair (1748-1819) advocated the study of analytical mathematics and used Laplace's techniques in his astronomy lectures in Edinburgh. His work on the history of mathematics disparaged the fluxional tradition, and this made an impression on James Ivory (1765-1842) who went on to work on elliptic integrals and especially potential theory.

Reformers at Dublin university were influenced by the works of Laplace and this brought about a change in their applied mathematics teaching. John Brinkley (1763-1835) was connected with both the Cambridge and Dublin reforms as he was educated at Cambridge then played a key role in the Dublin reforms as the holder of the Andrews chair of Astronomy at Trinity College. A key mathematician of this school was William Rowan Hamilton (1805-1865), who already produced important work in his teens (see 1.6.3).

1.4 British mathematics and higher education

> 'When we turn to England, we find no single institution which offers points of comparison with the *Ecole Polytechnique* in every particular. If we consider the latter as a school of engineers, and especially of military engineers, we can only find the Academy at Woolwich with which to compare it. But if we view the French school as an institution in which the higher mathematics are so taught as to produce teachers and investigators, we must consider it as occupying the position which in our country is filled by the University of Cambridge.'
> [Augustus De Morgan 1835a 334]

1.4.1 Cambridge

Following the account of the entry of Continental mathematics into Britain, a brief general survey of mathematics education in Britain will now be given. Cambridge was by far the most influential institution in the eighteenth century and the state of mathematics in Cambridge reflected the general standard in Britain with no outstanding developments. The teaching of mathematics at Cambridge was also not of a high standard and passing the examinations was given undue stature. The examination was entitled the 'Tripos' after the three-legged stool of an erstwhile examiner. In the early part of the nineteenth century there were three classes of graduate - 'wranglers', 'senior optimes' and 'junior optimes'. 'Poll men' were those holding a non-honours poll degree. The wranglers took extra exams of a higher level. Encouragement for mathematics was provided by the institution of a prize for attainment in mathematics by Robert Smith in 1770. However, as can be seen from Babbage's experience, even with such an incentive, the level of mathematics was not always high.

In the 1820s the innovations brought in by the Analytical Society had an effect on new generations of undergraduates, particularly via Peacock's teaching. George Biddell Airy (1801-1892), Augustus De Morgan (1806-1871) and William Whewell (1794-1866) were among the first students to be influenced whilst Arthur Cayley, James Joseph Sylvester (1814-1897), George Green (1793-1841), William Thomson and

George Gabriel Stokes (1819-1903) were major students in the 1830s and 1840s. Rouse Ball [1889 128] stated 'The character of the instruction in mathematics at the university has at all times largely depended on the text-books then in use'. New textbooks arrived outlining these developments, and Whewell's books on mechanics were especially influential as they applied new philosophical thinking and mathematical techniques to Newtonian mechanics. Whewell was very influential within Cambridge in the 1840s as Master of Trinity College. Research papers were published with the Royal Society or in the *Proceedings of the Cambridge Philosophical Society*, which was started in 1819; the *Philosophical Magazine* took shorter papers. These were effectively the Cambridge mathematical journals, the *Cambridge (and Dublin) Mathematical Journal* (1839-54) appearing later (see 1.5.1).

Whewell's strong influence on the development of both the pure and applied mathematics curricula at Cambridge will now be considered in detail. Returning to the development of the calculus, it was seen above that the French mathematicians had developed analytical methods for the calculus as opposed to their British counterparts who still adhered to synthetic-geometric ones. In particular, the applied mathematicians Lagrange and Laplace applied analysis to mechanics. The analysts of the French school mistrusted geometrical derivations because the intuitive approach they represented produced inconsistencies within analysis and so had restricted use. To counter these difficulties, analysts turned instead to algebraic foundations [Becher 1980 1-2]. This wariness of geometry by the French analysts contrasts strongly with the British love of geometry, particularly Euclidean, and shows the lines of demarcation which to a certain extent polarised mathematics in the two countries.

Analysis may have been a preferable method for the French to use, but it was not without its difficulties. For a student to use analysis, it had to be taught concertedly - not quickly or superficially. This posed no problem in France where the propounders of analysis were also ardent teachers - for example, Cauchy. The French *professeurs* did lecture and some also wrote text books and course material. Their courses equipped students to take up a career in mathematics. The situation was entirely different at Cambridge where the emphasis was on a liberal education with mathematics at it centre. The chief function of mathematics was felt to be for training the mind, and geometry was the principal tool for achieving this end. After graduation, a Cambridge graduate may take up one of a number of professions, so there was no notion of training up mathematicians, as there was in France. Thus the mathematics that was taught at Cambridge had to be simple to understand, appeal to the intuition and address the traditional problems in mixed mathematics [Becher 1980 10].

After the reforms brought about by the Analytical Society, it became increasingly difficult for the curriculum to cover both the new analysis and the old eighteenth century geometry. The result was a de-stabilising effect on the mathematical Tripos. Whewell was the prime mover in restoring the mathematical equilibrium. Whewell studied at Trinity College, was second wrangler in 1816 and became a Fellow of Trinity in 1817. He was a college tutor from 1818 to 1839, professor of Moral Philosophy from 1837 to 1855 and Master of Trinity from 1841 to 1866. He was responsible for the introduction of Continental mechanics at Cambridge. He pressed for the continued expansion of applied mathematics in the curriculum in the 1830s, but he felt that a similar expansion of analysis, especially pure analysis would destroy the foundations of a liberal education. He then became a fierce opponent of the analysis that had come to dominate the curriculum in his eagerness to defend a 'liberal education'. To this end he wrote textbooks and tracts and sought to influence University institutions. His ideas were generally realised [Becher 1980 3].

At the beginning of the nineteenth century passing the hardest Tripos exams required the ability to answer taxing mathematical problems at speed. The ablest students would have private tuition to get through this hurdle and the student who performed best earned the prestigious title of 'senior wrangler'. (Even though education at Cambridge was intended to be 'liberal', classics did not form a part of the Tripos until 1827 when the Classical Tripos began. A Moral Sciences Tripos and a Natural Sciences Tripos were inaugurated in 1851 but before any of these three Triposes could be attempted it was necessary to pass the elementary mathematical exam that poll-men took.) The curriculum at the beginning of the nineteenth century contained algebra, trigonometry, arithmetic, geometry, fluxions, hydrostatics, astronomy, optics and mechanics based on the geometric approach of Newton's *Principia*. Analysis, beyond elementary trigonometry and algebra, comprised fluxions, based on and applied to geometry and mixed (applied) mathematics, and the indiscriminate use of infinite series. There was no trace of abstract algebraic analysis of the continental type, pure or applied, in the curriculum [Becher 1980 7]. At this time, the pure mathematics text books in use were Euclid, Samuel Vince's *Trigonometry and Conic sections*, William Dealtry's *The Principles of Fluxions: Designed for the Use of Students in the University*, the beginning of Newton's *Principia* and lastly James Wood's *Algebra* which contained a section on analytic geometry. For applied mathematics they used Wood's *Optics and Mechanics*, Vince's *Hydrostatics* and *Astronomy* and Newton's *Principia.* These textbooks made use of geometrical diagrams instead of algebraic formulae as these were felt to be clear, obvious and easily understood by the student. This obsession with the geometrical, however, stopped British mathematicians being able to tackle continental analysis.

Robert Woodhouse, however, did see the benefit of the Lagrangian algebraic approach to the calculus, as outlined in 1.1.2, but his book *The Principles*, above, did not become a standard text at Cambridge. However, following the efforts of the Analytical Society (see 1.1.2), in 1817 George Peacock used continental notation in the Tripos and the Analytical Society's translation of Lacroix' work replaced the fluxional texts as the standard work. Whewell used continental methods in his *Elementary Treatise on Mechanics* (1819) which then became the standard text and also in his *Treatise on dynamics* (1823). Whewell's emphasis, however, was to keep analysis as a tool for applied mathematics and to accentuate the practical aspect of applied mathematics via geometrical diagrams. Pure mathematics was to be subservient to applied during Whewell's time at Cambridge.

Textbooks on analytic geometry began to be used at Cambridge, beginning with Henry Parr Hamilton's in 1826. This was not initially a success as it included diagrams in the appendix. A version in 1828 contained conic sections derived using a focus and a directrix, with equations of the second degree dealt with at the end of the book. John

Hymer's work on analytic geometry published in 1837 became the standard text. He used general transformation techniques and presented equations of the second degree in a more general manner than Hamilton had done. Hymers pointed the student towards analytic methods and away from synthetic. Peacock's *Treatise on Algebra* (1830) became standard but he did not pursue rigour in analysis as Cauchy did in France by rejecting diverging infinite series. Hymer's book *Treatise on differential Equations and on the Calculus of Finite Difference* [1839] similarly became standard but it lacked rigour. Matthew O'Brien's book on calculus of 1842 was the most rigorous to be used at Cambridge. O'Brien moved from Cambridge to the Royal Military Academy at Woolwich (see 1.4.3). In the late 1830s and early 1840s, Cambridge students had excellent texts in analytic geometry, algebra, and calculus, which had continental techniques and procedures but not continental foundations. Newtonian mathematics was not studied, and only about one in seven questions required synthetic-geometric solutions. Students spent their time instead on studying algebraic operations [Becher 1980 23]. This curriculum produced the excellent mathematicians of the 1830s and 1840s mentioned above.

However, Whewell was uneasy that pure mathematics was dominating the curriculum. He turned against analysis seeing it as artificial when compared to Euclidean geometry and inclined instead to the geometrical mathematics of the eighteenth century. He effected a reform of the Tripos in 1848 whereby students studied elementary mathematics comprising Euclid, arithmetic, elementary trigonometry and algebra, conic sections treated geometrically, dynamics, elementary statics, hydrostatics without calculus, elementary optics and the beginning of Newton's *Principia* for two years. This material was covered in a textbook by Harvey Goodwin in 1846. More able students would study algebra, differential and integral calculus, analytic geometry, hydrostatics, mechanics, advanced astronomy and optics. Students would use compilations of earlier works when studying this material. The ablest students would read whole works - Newton's *Principia*, Euler's *Mechania*, Lagrange's *Théorie des fonctions* and his *Mécanique analytique*, Monge's *Application de l'algèbre à la géométrie* and Laplace's *Mécanique céleste*. Although a sound collection, this list included no current work on analytic geometry or algebra. Whewell did not wish them to study the latest developments by mathematicians such as Poisson and Carl Friedrich Gauss (1777-1855) in journals and memoirs but rather concentrate on solid texts. This contrasted sharply with continental practice where research material was included in lectures. In the 1840s in Cambridge, students were so occupied with their private tutors that they did not even attend lectures.

The reforms mentioned above were scrutinized by a Parliamentary Commission in 1850 which was concerned with administrative statutes and the curriculum of the universities of both Oxford and Cambridge. 'When interviewed by the Commissioners, most Cambridge mathematicians agreed that (1) the scope of the Tripos had become too extensive before the restrictions imposed by the mid-century reforms, (2) the peak of the analytic revolution had been reached approximately ten years previously, (3) the examinations had become too analytic to the detriment of fixing physical principles, and (4) the regulations of 1848 had made mathematical studies more geometric and less analytic. In general those who had contributed little to mathematical research voiced uniform approval of the reforms. The attention of students was no longer unduly fixed upon the dextrous use of symbols, to the neglect of natural relations in the application of mathematics to physical subjects.' [Becher 1980 40]. In their report of 1852 the commissioners supported Whewell's stance in upholding geometrical methods and using analytical methods merely as a tool for applied mathematics. By 1854, the Tripos questions contained a majority of applied mathematics and problems requiring algebraic solutions were outnumbered by those requiring synthetic-geometric solutions, including Newtonian ones which made up more than forty percent of the examination. Geometry was emphasised over algebra, intuition over abstract rigour, problem-solving over mathematical processes, Newton over Lagrange and Whewell over O'Brien [Becher 1980 42]. Thus by the 1850s mathematics at Cambridge lagged behind the continent again in the area of pure mathematics although Whewell's approach did produce many fine applied mathematicians including Thomson, Stokes and his student J. C. Maxwell.

Mathematics at Oxford, where it had been neglected, contrasted sharply with that at Cambridge [Howson 1982 126]. 'Even in 1850, Responsions, an examination held between the third and seventh terms of the undergraduate's residence [at Oxford], demanded only Euclid, Books 1 and 2, or algebra to simple equations; whilst Moderations, an intermediate examination taken between Responsions and the final degree examination, asked for the first three books of Euclid and 'the first part' of algebra or, as an alternative to mathematics, logic. 'Pure mathematics' was only for those few who read for mathematical honours' [Howson 1982 112]. Such a syllabus was not designed to engender mathematical expertise in the Oxford students.

1.4.2 The London colleges

The universities of Oxford and Cambridge were joined in 1826 by the London University. This had secular foundations which were radically different to those of Oxford and Cambridge and as a result it was viewed with caution by the Establishment and refused a Royal Charter. A rival institution, King's College, was set up in London in 1829 and this was pro-Establishment. Conformity to the thirty nine articles of the Church of England was a necessary pre-requisite to membership of King's College, as it was for membership of Oxford and Cambridge. However, unlike Oxford and Cambridge, neither the London University nor King's College London could confer their own degrees. This situation changed in 1836 with the setting up of the University of London to examine students from the two London colleges and elsewhere. The London University changed its name to University College London to avoid confusion [Rice 1996 377].

Augustus De Morgan became the first mathematics professor at both London University in 1828 and University College London in 1837, having resigned his position in at the London University in 1831 over a matter of principle. He remained at University College London until 1866 when he again resigned on a matter of principle. Students on his mathematics course were split into four groups, two junior and two senior, each having a lower and higher division. The junior classes covered simple arithmetic and algebra, the first six books of Euclid, solid geometry and plane trigonometry. The senior classes covered spherical trigonometry, conic sections, applications of algebra to geometry, higher algebra, differential and integral calculus and their applications, differential equations, the calculus of variations and some probability theory. His pupils also referred to lessons in number theory, theory of equations, and double algebra [Rice 1996 380]. De Morgan had very strong views regarding the teaching of mathematics and was against his students 'cramming' to pass examinations. Although educated at Trinity College Cambridge himself, being fourth wrangler in 1827, he deplored the Cambridge Tripos with its emphasis on solving difficult problems and preferred the Oxford system where the student was gauged on his knowledge of the subject rather than his performance compared to his peers.

University College was thus dominated in its early years by De Morgan and he was responsible for teaching some pupils who were later to become distinguished, including the politician and writer Walter Bagehot (1826-1877) and the logician William Stanley Jevons (1835-1882).

> 'Other notable De Morgan pupils include the prolific textbook author, Isaac Todhunter (1820-1884); Cambridge Tripos coach, E.J. Routh (1831-1907); and originator of the Four Colour Problem, Francis Guthrie (1831-1899).....Among De Morgan's last students were the astronomer Arthur Cowper Ranyard (1845-1894), and the Professor's own son, George Campbell De Morgan (1841-1867). These two men were jointly responsible for the founding of the University College Mathematical Society in 1864. This quickly evolved into the London Mathematical Society and continued to meet at University College with professor De Morgan as its first president throughout its first two years.
>
> The Society's second president, by coincidence, had also been one of De Morgan's first students, albeit briefly, when the college first opened for lectures in October 1828. The Jewish mathematician, James Joseph Sylvester attended De Morgan's lectures at the new London University for its first five months at the age of only 14. Yet even during this short period of time he made a substantial impression on its professor. He was withdrawn by his family in February 1829 on the grounds of his immaturity and inability to control his volatile temper when taunted by fellow students, presumably for his youth and religious background. He transferred to the school attached to Liverpool's

> Royal Institution before entering St John's College, Cambridge in 1831. In January 1837, he was second wrangler in the Tripos, although the University's religious tests prevented him from taking his degree and staying on at Cambridge.' [Rice 1996 382-3]

In addition to the mathematics professorship which De Morgan dominated for so many years, there was also the professorship in natural philosophy. Sylvester held this professorship from 1837-1841. He did not find teaching this subject easy, especially the practical side, and students who attended Sylvester's lectures found natural philosophy treated as applied mathematics as taught at Cambridge. Topics covered included dynamics, statics, hydrostatics, elliptic motion, gravitation, optics, and astronomy, with little or no reference to heat, electricity, or magnetism. Students needed a knowledge of algebraic notation, proportion, and trigonometric functions to enter the first year. To get into the second year, they needed to be familiar with conic sections, quadratic equations, and spherical trigonometry and for the third year, analytical geometry and the differential and integral calculus were required [Rice 1996 384].

Sylvester, being unhappy teaching natural philosophy, yearned to teach pure mathematics and took up a chair of mathematics at the University of Virginia in 1841. He was succeeded as professor of natural philosophy at University College London by Richard Potter (1799-1886) who was keen on the experimental side, producing papers on optics [Rice 1996 383-4].

At King's College the Reverend Thomas Grainger Hall (1803-81) occupied the mathematics chair from 1830-1869 but did not make any great impression on the development of mathematics. He was fifth wrangler in 1824 and fellow of Magdalen College Cambridge. He wrote textbooks based on his courses - *A Treatise on Differential and Integral Calculus* (1834), *A Treatise on Plane and Spherical Geometry* (1836), *Elements of Algebra* (1840) and *Elements of Descriptive Geometry* (1841). The professor of natural philosophy from 1831-1844, the Reverend Henry Moseley (1801-1872), had studied at St. John's College Cambridge and become seventh wrangler in 1826. He produced distinguished results in hydrodynamics. He was succeeded by the Reverend Matthew O'Brien (1814-55) who held the chair until 1854. O'Brien studied at Trinity College Dublin and Caius College Cambridge and was third wrangler in 1838. He went on to hold a professorship in mathematics at Woolwich (see 1.4.3). Thomas Minchin Goodeve (1821-1902) then held the chair in natural philosophy at King's College until 1860. He was ninth wrangler in 1843 and also went onto to lecture in astronomy at Woolwich (see below). The mathematical physicist, James Clerk Maxwell then occupied the chair until 1865 [Rice 1996 390-3].

1.4.3 The military establishments

De Morgan above mentioned the military schools and their impact upon mathematics. There were three main military schools, the Royal Military Academy at Woolwich, the Royal Military College at Sandhurst and the Royal Naval Academy at Portsmouth. The Royal Military Academy was founded in 1741 to give a mathematical grounding to inexperienced military personnel. In contrast to Cambridge, continental mathematics was known at this institution as the engineers needed to keep abreast of scientific developments. Charles Hutton (1737-1823), John Bonnycastle (1750-1821) and Olinthus Gregory (1774-1841) were important teachers in the period prior to 1840, Hutton being the professor of mathematics, succeeded in this post by Gregory. Samuel Hunter Christie (1784-1865) succeeded Gregory as professor of mathematics in 1838 and wrote a new text book to replace Hutton's *Course* which included a small amount on the differential and integral calculus. Although these teachers did not make notable mathematical contributions themselves based on Continental mathematics, they did draw attention to French work. This was done by writing textbooks, encyclopaedias and dictionaries which served as a channel for continental mathematics.

Hutton wrote *A Course of Mathematics* (1798, 1801) based on many years of teaching, and it was to become a standard text until the 1830s. This covered arithmetic, logarithms, geometry, algebra, trigonometry, mensuration, conic sections, mechanics, fluxions, hydrostatics, hydraulics, pneumatics, resistance of fluids and gunnery. In the

1830s the differential and integral calculus were introduced by Samuel Christie. Hutton was the editor of the *Ladies' Diary* from 1773/4 to 1817, abridged the *Philosophical Transactions* (1665 - 1800) and published the *Mathematical and Philosophical Dictionary* (1795/9). 'Hutton's *Dictionary* reveals a deep and extensive knowledge of continental works and is still used by historians as a valuable source.' [Guicciardini 1989 112]. His research interests lay in the areas of engineering, geodesy and the convergence of series. He approved of following Greek geometrical tradition to initially shape mathematical thinking, but then advocated 'modern analysis' to extend it. He did not however use it in his own research or teaching, he merely drew attention to it .

'Bonnycastle was a prolific writer of elementary textbooks which often ran into many editions' [Taylor 1966 280-1]. Both Bonnycastle and Gregory contributed to the *Ladies' Diary* and Gregory came to the editor, Hutton's attention via this journal. Gregory was the editor of the *Ladies' Diary* from 1818 to 1840, having edited the *Gentleman's Diary* from 1804-1819. Gregory's courses were upon astronomy, mechanics and elementary physics and he drew upon works by Lagrange and Carnot's *Géométrie de Position.*

Another journal associated with this establishment was *The Mathematician*, established and issued at Woolwich between 1845 and 1850 by Professor Thomas Stephen Davies (1795-1851), Dr William Rutherford (1798?-1871) and Stephen Fenwick [Wilkinson 1851a 126] and [Abram 1876 86]. Davies was a frequent contributor to mathematical journals, the main ones being the *Gentleman's and Lady's Diary*, the *Philosophical Magazine* and the *Mechanics' Magazine*. He was interested in geometry from two different standpoints, the historical and the current. He became a Fellow of the Royal Society of Edinburgh in 1831 and was elected a Fellow of the Society of Antiquaries in 1840. He was appointed as a mathematical master at Woolwich in 1834. Major areas of his research included the properties of the trapezium, Pascal's hexagramme mystique, Brianchon's theorem, symmetrical properties of plane triangles and the geometry of three dimensions. He also devised a new system of spherical geometry. He produced a book of solutions to Hutton's

Course of Mathematics which contained 4000 solutions of problems from a range of different mathematical areas and of varied degrees of difficulty. He also produced a twelfth edition of Hutton's *Course of Mathematics*, in two volumes, in 1841-3 [*DNB*].

Rutherford taught at a school at Woodburn from 1822-5 and was a master at Corporation Academy, Berwick from 1832-7. In 1838 he became a mathematical master at Woolwich where he was appreciated by his pupils for his lucidity. He was a council member of the Royal Astronomical Society from 1844-7. He contributed many questions, solutions and papers to *The Lady's Diary* from 1822-69 and contributed also to *The Gentleman's Diary*. 'He always delighted in a 'pretty problem', although his mathematical studies were quite of the north-country type.' He also produced several editions of Hutton's *Course of Mathematics* (1841, 6, 54, 60) and also Bonnycastle's *Algebra* (1848). His work included papers on Pascal's theorem, theorems in co-ordinate geometry and numerical equations [*DNB]*. Information from Mr W.J. Miller is given as a source for this *DNB* article, and the citation in the article regarding Rutherford's north-country type mathematics is perhaps from him.

Sylvester has been mentioned in connection with his teaching at University College London which he left in 1841 for the University of Virginia (see 1.4.2). This was not a success and he returned to London and took up actuarial work while still continuing his mathematical research. Upon Christie's retirement in 1854, Sylvester applied for the post of mathematics professor at Woolwich. He was not appointed and Rev. O' Brien (see 1.4.1) held the position for just a year until his death in 1855. Sylvester was then successful in obtaining the professorship which he retained until 1870. The level of mathematics teaching at Woolwich was not high and did not stimulate Sylvester [Rice 1996 401-2].

The Royal Military College at Sandhurst was founded in 1799 and had a Junior and Senior department. The material studied in the Junior department was more elementary than at Woolwich and covered arithmetic, geometry, mensuration, algebra, conic sections, mechanics, hydrostatics, hydraulics and pneumatics. On leaving the

College officers would be thoroughly conversant with the first six books of Euclid. Charles Law, a publisher of the *ET*, wrote a guide to Sandhurst in 1849. In this guide, an officer claimed to have studied the differential and integral calculus, statics, dynamics and practical astronomy [Anonymous 1849 55]. Wallace and Ivory were both masters here from the early 1800s until 1819. Thomas Leybourn, editor of the *Mathematical Repository* was a master from 1802 to 1839.

This concludes the survey of British educational institutions. Elementary education will be covered in chapter two when nineteenth century education is considered. British journals and mathematical societies will now be discussed.

1.5 British journals and mathematical societies

1.5.1 Overview of the development of mathematical journals

The lack of a comprehensive study of the historical development of mathematical journals is lamented by Neuenschwander [1994 1533-9] (see 0.1). He adds that where studies into mathematical journals are undertaken, they are subsequently little-known, and frequently confined to a single journal. With the above provisos, Neuenschwander gives an overview of the general development of mathematical journals, whilst indicating literature for further research.

The first scientific journals were published by academics and other learned societies in the seventeenth century [Meadows 1979 and Houghton 1975]. Mathematical articles did not appear in their own section, but along with articles about other subjects. This tradition continued into the eighteenth century, where other journals appeared of a more popular nature. An important example of such a journal is the *Ladies' Diary* (1704-1840). The professionalisation of science following the Napoleonic wars brought with it an attendant increase in the number of subject-specific journals. These journals did not enjoy longevity and were not of a high standard.

The first journal to break significantly this mould was Joseph Diaz Gergonne's *Annales de Mathématiques Pures et Appliquées* (1810-31) (see 1.2.3). This journal was not allied with any academic institution, and its success was such that it was imitated in other countries. One such imitation was the *Journal für die reine und angewandte Mathematik* (Journal for pure and applied mathematics) informally entitled 'Crelle's journal' (1826-), published in Germany (see 1.2.3) but at a higher mathematical level than Gergonne's journal. Another was the *Journal des Mathématiques pures et appliquées*, informally known as 'Liouville's Journal' (1836-) published in France (see 1.2.3). Their counterpart in Great Britain was *The Cambridge (and Dublin) Mathematical Journal* (1839-1854) which preceded *The Quarterly Journal of Pure and Applied Mathematics* (1855-1927), founded by J.J. Sylvester. The initial contributors to *The Cambridge (and Dublin) Mathematical Journal* were mainly Cambridge men, including De Morgan, Cayley, Sylvester and Stokes. The first editor was Duncan F. Gregory (1813-1844), a graduate of Trinity College Cambridge.

Journals connected with an institution were the *Journal de l'Ecole Polytechnique* (1795-) (see 1.2.3) in France and in Britain *The Proceedings of the Cambridge Philosophical Society* (1819-) (see 1.4.1). Another important scientific institution was the British Association for the Advancement of Science (BAAS) which was founded in 1831 and commissioned *Reports* on various branches of science, including mathematics. The BAAS held annual meetings in various cities throughout Britain and played an important role in the development of science throughout western Europe in the nineteenth century [MacHale 1985 66]. The Royal Society published reports in its *Philosophical Transactions* (1665-) and in France the *Académie des Sciences* in the *Comptes rendus de l'Académie des Sciences* (1835-).

A new breed of mathematical journals appeared in the second half of the nineteenth century to accompany various new mathematical societies. These journals contained contributions from society members, and also the proceedings of the society. An important example for the purpose of this thesis is the *Proceedings of the London Mathematical Society* (1865-) (see 8.2.1). Similar journals appeared world-wide during the latter half of the nineteenth century.

The above journals contain specifically mathematical material. Neuenschwander also mentions journals covering material of a more peripheral mathematical nature, such as the teaching of mathematics. Such journals were aimed at mathematics students, mathematics teachers and lay mathematicians. His examples from Great Britain are the *Ladies' Diary* (1704-1804), the *Gentleman's Diary* (1741-1840) and the *Mathematical Gazette* (1894-) (see 8.1).

1.5.2 Archibald's work on British mathematical journals

A major work on British mathematical journals is by Raymond Clare Archibald [1929 379-400]. He lists two general sources of information on mathematical journals, Thomas Turner Wilkinson's material in the *Mechanics Magazine* from 1848-1853 (see 3.4.2 below) and John Sturgeon Mackay's article in the *Compte Rendu de l'Association Française pour l'Avancement des Sciences* [1893 303-8]. Mackay's work is a short summary whilst Wilkinson's work is comprehensive and voluminous.

Archibald covers 43 serials and also includes associated material connected with them. For each serial, he gives the size of the pages in centimetres, the locations of any institutions holding a set, with details of missing numbers, details of the editors, and sometimes the date the first mathematical question appeared and the total numbers of questions and their solutions published. Sometimes important results were outlined together with details of the attendant mathematicians Only later serials of interest here with their dates of publication as given by Archibald will now be listed: the *Lady's Diary* (1704 - 1840), the *Gentleman's Diary* (1741-1840), *or* the *Mathematical Repository*, the *Lady's and Gentleman's Diary* (There are two separate journals of this name. One is the amalgamation of the two serials above and ran from 1841-1871. The other ran from 1747-1792 and was often referred to as *Burrow's Diary* or *Carnan's Diary* after editors of these names.), the *Scientific Receptacle* (there are two separate journals of this name, one from 1791-1819 and the other in 1825), the *Mathematical Repository* (1770-1835), the *Gentleman's mathematical Companion* (1798-1827), the *Enquirer* (1811-3), *Quarterly Visitor* (1813-4), the *Leeds Correspondent* (1814-23), the *Students' Companion* (1822-3), the *Liverpool Apollonius* (1823-4), *Enigmatical Entertainer and Mathematical Associate* (1827), the

Scientific Mirror (1829-30), the *Private Tutor and Cambridge Mathematical Repository* (1830-1), the *York Courant* (1829-46), the *Northumbrian Mirror* (1837-41), the *ET* (1847-1915) and lastly the *Association for the Improvement of Geometrical Teaching Reports* (1871-93). The majority of the 43 serials were short-lived and appeared in the latter half of the eighteenth century or early part of the nineteenth. It is of interest that although several of these serials are aimed at both sexes, two are aimed specifically at ladies. Also, three concern students and six come from northern cities or counties. The only journals of this kind in existence at the time the *ET* was launched were *The Lady's and Gentleman's Diary, The Cambridge (and Dublin) Mathematical Journal* (1839-1854), and *The Mathematician.*

A study of the *Ladies' Diary* was carried out by Teri Perl [1979 36-53]. (This journal was frequently referred to as the *Diary* or the *Lady's Diary.*) She charted the rise of this popular annual magazine from its early days with an almanac format and containing articles of general interest, to a journal format containing a selection of enigmas, queries and mathematical questions. These mathematical items were sometimes couched in poetical language, or indeed sometimes verse. The *Ladies' Diary* achieved a longevity which was not characteristic of its contemporaries, showing that the combination of stimulating mathematical problems and an interesting format was attractive to its readership. A main focus of Perl's was to investigate how many and which women contributed to the *Ladies' Diary.* She gave a table of the number of women who answered questions from each mathematical category. She summarizes her paper saying 'The beginning of the *Ladies Diary* coincides with the popularization of mathematics and the growth of mathematical literacy. However, as mathematical literacy spread in response to developing technology's requirements for more mathematically sophisticated workers, women, not part of this need, were left behind. This effect is reflected in the decline in the number of women contributors over the life of the publication.' [Perl 1979 36].

Charles Hutton was editor from 1773/4 - 1817 and was also professor of mathematics at the Royal Military Academy, Woolwich from 1773-1807, a fellow of the Royal Society and writer of textbooks (see 1.4.3). Olinthus Gregory was the final editor from 1818-40 and was also professor of mathematics at the Royal military Academy, Woolwich, from 1807-38.

The *Lady's and Gentleman's Diary* (1841-1871) was published annually and was edited by W.S.B. Woolhouse (1809-93). It contained mathematical papers and problems, some of a difficult nature. Prize questions were posed by the editor and the prize question for 1844 was a difficult combinatorial problem, solved for one case by Thomas Penyngton Kirkman (1806-95) and later worked on by Cayley and Sylvester.

Although not in existence when the *ET* was founded, *The Mathematical Repository*, or *Leybourn's Repository* as it was informally entitled, deserves mention here as it only ceased publication in 1835 and held much valuable material. It contained mathematical questions, Cambridge problems, original essays and mathematical memoirs taken from important works. Leybourn was helped to edit this journal by friends such as Hutton. Many of the best English mathematicians contributed to this journal including Babbage, Peacock, Herschel and Hutton [Archibald 1929 390-1].

1.5.3 Wilkinson's work on mathematical periodicals

Also appearing in Neuenschwander's bibliography is the proliferation of articles on mathematical periodicals by Thomas Turner Wilkinson (1815-1875) which appeared in the *Mechanics Magazine* from 1848 to 1853. Wilkinson, the self-taught mathematician from Burnley, Lancashire, is discussed in depth in 3.3.1 and 3.4 below as having possible involvement with the early mathematical editor of the mathematical department. In the biography of Wilkinson by Abram [1876 77-94], Wilkinson's fascination with mathematical periodicals is traced:

> In 1846 I contributed some mathematical exercises, &c, to the Mechanics' Magazine. These brought me into correspondence with Professor Davies, Professor De Morgan, Sir James Cockle and others. I purchased my first mathematical periodical, the Lady's Diary, in 1835, and the contents of this so interested me that I began to collect all the English periodicals which contained mathematics. In process of time (sic) I had in my possession the most extensive series of these works out of London. I made these an object of special study,

> and at the suggestion of Professor Davies I commenced a series of articles on our English Mathematical Periodicals in the Mechanics' Magazine during 1848, which extended through various volumes of that work up to 1854.

This whereabouts of this large collection of periodicals is currently unknown. Wilkinson left his correspondence to the Chetham Society, Manchester, but his other papers were auctioned but it appears that the collection of periodicals is not traceable in the auction catalogue. However, Archibald [1929 394] states that the copy of the *Enigmatical Entertainer and Mathematical Associate* in the Brown University library, U.S.A. was Wilkinson's. It is possible that David Eugene Smith (1860-1944), a historian of mathematics, purchased part of Wilkinson's collection, (in 3.5.2 and 8.4, Smith purchasing William John Clarke Miller's (1832-1903) correspondence and collection of photographs will be noted).

Even if the physical remains of Wilkinson's periodical collection are currently missing, the influence that he brought about in the sphere of mathematical periodicals during his time is tangible and undeniable. His work on the mathematical periodicals in the *Mechanics Magazine* is detailed and comprehensive. The introduction to these articles appeared in volume 48, 1848 and reads as follows:

MATHEMATICAL PERIODICALS

> Sir,-Allow me to contribute my mite towards furnishing the information respecting mathematical periodicals required by 'Mathematicus' at p. 488 of your last volume. After I have furnished the serials in my own possession, perhaps some others of your correspondents will supply the descriptions of those I have not been fortunate enough to obtain. I am, Sir, yours, &c,
>
> THOMAS WILKINSON
>
> Burnley, Lancashire, Dec. 29, 1847.

Wilkinson described each journal in turn using the following subsections: Origin, Editors, Contents, Questions, Contributors and Publication. Most of these are self-explanatory, but Questions contained the total number of questions proposed and answered, and also details of any prize questions. Contributors held the names of principal contributors. Where journals held original papers, he listed these and sometimes he went into detail about individual problems. For some journals, he gave a list of mathematical categories, together with some of the questions which fell into them.

The above gives an outline of the studies carried out so far on mathematical journals, but what of their influence? Archibald refers to Playfair [1808 282] as stating,

> A certain degree of mathematical science, and indeed no inconsiderable degree, is perhaps more widely diffused in England, than in any other country in the world. *The Ladies' Diary*, with several other periodicals and popular publications of the same kind, are the best proofs of this assertion. In these, many curious problems, not of the highest order indeed, but still having a considerable degree of difficulty, and far beyond the mere elements of science, are often to be met with; and the great number of ingenious men who take a share in proposing and answering these questions, whom one has never heard of anywhere else, is not a little surprising. Nothing of the same kind, we believe is to be found in any other country....The geometrical part.....has always been conducted in a superior style; the problems proposed have tended to awaken curiosity, and the solutions to convey instruction, in a much better manner than is to be found in more splendid publications.

The participation in the majority of these journals by philomaths and the way the solutions were useful for teaching purposes was thus a hallmark of English mathematical journals at the beginning of the nineteenth century. This concludes the survey of mathematical journals up to the mid nineteenth century. A survey of English and French scientific periodicals in the mid-nineteenth century will now be summarized as its contribution is relevant to the study of mathematical journals.

1.5.4 English and French scientific periodicals

The contrast between popular science periodicals in London and Paris from 1820-75 has been analysed by Sheets-Pyenson [1985 549-72]. She asserts that social reformers, zealous publishers and voluntary associations created popular literature to promulgate useful knowledge which appeared in the urban centres of Paris and

London during the middle decades of the nineteenth century. Popular scientific literature was seen as being sufficiently cheap to attract the attention of the working classes and keep them from spending their time reading what was perceived as potentially politically subversive literature. Scientific periodicals were then a safe way of stimulating the increasingly literate working class. Periodical editors in London and Paris shared these aims, demonstrated by continuous mirroring of each other's periodicals. However, the scientific cultures in the two countries as revealed by the scientific periodicals were very different, being 'shaped by the dissimilar characteristics of their audiences, editors and high scientific communities' [Sheets-Pyenson 1985 549].

In England, the distribution of scientific periodicals on a large scale was facilitated by the technological advances in the printing trade and the use of the railways and the postal system for transporting them. Mechanized printing began to increase the output of the publishing industry in the 1820s and at this point popular scientific periodicals started to appear, their production reaching a maximum in the 1860s. They could be classified as general science, natural history and mechanics' magazines, the latter being read by self-improving artisans. Periodicals in each category would have a similar format and fall in a similar price range, being specifically targeted at a particular social group. Readers would influence these periodicals by sending letters, articles, notes and queries, sometimes anonymously. Editors filtered these contributions and may have even fabricated some of them to fill up their pages. In so doing, they actively tried to influence their readers [Sheets-Pyenson 1985 552]. 'The ideologies of science that appeared in popular science periodicals, then, mirrored above all editors' concerns, aspirations, and prejudices' [Sheets-Pyenson 1985 553].

By the nineteenth century, the production and distribution of periodicals and books was centred in London. Most of the natural history, general science and mechanics' magazines produced here were in compact and portable octavo format. Woodcuts and line engravings were used to illustrate them. They were written in comprehensible terminology as opposed to scientific jargon to ensure they were accessible to the working classes. There was some recycling of material amongst these periodicals, but each also carried its own reviews of other periodicals and books [Sheets-Pyenson 1985 552-3].

The readership of these periodicals is summarised as follows.

> Viewed in its totality, the audience for popular science periodicals in England resembled a pyramid. At the highest level were the hundreds of upper and middle class subscribers to natural history periodicals who could afford to pay a shilling per monthly issue...At a level beneath them were thousands of middle class readers of general science periodicals. The moderate price (four pence) and emphasis on the applications of science in these periodicals, though not to the exclusion of pure scientific topics, made them particularly appropriate for the leisure hours and even the libraries of the scientifically-inclined subscribers. Finally, at the lowest level, inexpensive (at several pence an issue) and technologically-oriented mechanics' magazines provided tens of thousands of urban workers with advice on how to advance their social position through scientific pursuits [Sheets-Pyenson 1985 555].

In the 1860s, a new image was advanced for scientific activity, turning away from the previous notion of the self-made scientific entrepreneur to the professional ideal of the scientific expert. Mechanics magazines contributed to this change by their interest in the achievements of scientists and engineers, but they were nonetheless disinclined to relinquish their hallmarks of amateur participation [Sheets-Pyenson 1985 555].

French periodicals fell into three categories, the first of which was 'useful knowledge journals' for the working classes. These were superseded in the 1850s and 60s by periodicals in the other two categories, general science weeklies and general science annuals. These covered pure science and its applications and in them high scientific material was made intelligible to the layman. All these periodicals highlighted the utilitarian aspects of science but discussion of pure science *per se* was the domain of the high scientific community. The French periodicals were in an encyclopaedic format and did not specialize like the English journals. Little was divulged by the editors as to their philosophical aims, except the encyclopaedic format and they did not use the periodicals as organs for scientific societies [Sheets-Pyenson 1985 555-9]. She contrasts French and English editors thus,

> French popular science periodical editors were part of a tightly knit, self-conscious community of professional *vulgarisateurs* who launched their careers by serving as science columnists for Parisian newspapers. English editors, by comparison, were striking in their heterogeneous occupations and concerns. They were united only by their activity as amateur scientists and amateur science writers. Unlike their French counterparts, English editors were little aware of each other's efforts to popularize science [Sheets-Pyenson 1985 559].

Some of the French editors held degrees and some had carried out research. This ceased when they became editors as research and popularising science were held to be mutually exclusive. In contrast, the English periodical editors seldom held a degree, and their broad aim was to promote science among their fellow amateur scientists [Sheets-Pyenson 1985 559-61].

A brief mention will now be made as to how commercial forces affected scientific journals in Victorian Britain, the subject of a study by Brock [1980 95-122]. Stamp duty stood at 4d per sheet in 1816 and was payable on any journal containing news of a general nature. This stricture did not apply to the *Philosophical Magazine*, but did to the *Mechanics Magazine*. Stamped paper needed to be purchased in advance from a Stamp Office and the easiest places to do this from then distribute the journals was London, Dublin or Edinburgh. Advertisements in journals were also taxed but this was rescinded in 1853, and stamp duty was no longer levied after August 1853 either. The third tax on journals was paper duty and this forced cheap periodicals like the *Mechanics Magazine* to use inferior paper and restrict the journal to 16 pages. Paper became cheaper in 1857 and paper tax was abolished in 1861. Some journals continued to pay for stamping as this gave them free postage to provincial readers.

There were special postal arrangements for educational material and a book rate which allowed cheap bulk postage of periodicals to provincial centres [Brock 1980 98-101]. 'By the 1860s, too, the complicated duties on the import of foreign literature had been abolished and the international mails flowed sufficiently freely to make the précis, translation and dissemination of foreign scientific news more efficient, so lessening the insularity of British periodicals' [Brock 1980 105]. Direct readers subscriptions were relatively few in the nineteenth century but had grown substantially by the twentieth century [Brock 1980 107]. Editors were frequently well-respected scientists who contributed original material to their journals [Brock 1980 111]. A mark of a journal's success was indicated by it containing details of government examinations [Brock 1980 112].

This detailed survey of French and English scientific periodical will provide a useful backdrop when considering the *ET*. The survey of literature on journals is now concluded and mathematical societies will now be examined.

1.5.5 Mathematical movements

Mathematical societies had been formed on similar principles at Spitalfields, London and in Manchester in 1717 and 1718 respectively. The subject of most interest to the members of the Manchester Mathematical Society was geometry and in 1794 a Mathematical Society was set up in Oldham [Rice, Wilson and Gardner 1995 403]. Several of its members contributed frequently to some of the mathematical journals mentioned above and Jeremiah Ainsworth (1743-1784), James Wolfenden (1754-1841) and John Butterworth (1774-1845) became known as distinguished geometers. The London Mathematical Society, founded in 1865, will be discussed in 8.2.1.

Although not a mathematical society, The Mechanics Institutes movement will now be examined briefly as some mathematicians, for example, George Boole, were connected with them. The first Mechanics' Institution appeared in Glasgow in 1823 and grew out of a series of scientific lectures given by George Birkbeck (1776-1841) to working men. He then went on to found the Birkbeck Mechanics' Institution in London in 1824 and this eventually became Birkbeck College. Similar institutions then appeared throughout England providing working men with classes, lectures and libraries. They were opposed by some members of the Church of England but supported by dissenters. By the 1830s the movement in many places had lost its popularity which then belonged to lectures on literary topics. There now follows a survey of the major mathematical subjects developed in Britain from around 1840 to the mid 1860s.

1.6 A survey of British mathematics from around 1840 to the mid 1860s by subject area

1.6.1 Algebra

Negative and imaginary numbers were not well accepted in Britain in the eighteenth century and indeed were rejected by William Frend (1757-1841) and Francis Maseres (1731-1824) at its end. The British placed particular emphasis on clear foundations in geometry (see 1.4.1) and required the same sound basis for algebra [Pycior 1994 798-800]. George Peacock, Cambridge tutor and later Dean of Ely (see 1.3 and 1.4.1) expounded the foundations of algebra seriously in his *Treatise on Algebra* (1830), although Babbage, Herschel and Woodhouse had all contributed towards this subject. Peacock divided algebra into two types, arithmetical algebra and symbolical algebra. In his view, in symbolical algebra, these signs and symbols did not represent specific quantities or operations on them, but could take on various interpretations. Thus manipulation must come before interpretation. Negative numbers were thus admissible in symbolical algebra where they were deemed unfit for arithmetical algebra. Algebra was thus extended to a more abstract phase where the rules governing manipulation of the symbols was more important than the meaning of the symbols. Peacock used as his laws for symbolical algebra the same laws that applied for arithmetical algebra, explaining his actions by what he called 'the principle of the permanence of equivalent forms.' [Peacock 1830 71].

Peacock's work on algebra was taken up by Augustus De Morgan (see 1.4.1 and 1.4.2). In 1835, De Morgan [1835b] endorsed Peacock's work and tried hard from then onwards to establish algebra as a basic component of liberal education [Panteki 1991 191]. He realised that Whewell's emphasis on geometry was detrimental to the development of pure mathematics generally and specifically to algebra as well as analysis (see 1.4.1). His work in algebra was encapsulated in his *Trigonometry and*

Double Algebra [1849] in which he laid down 14 laws of symbolic algebra. 'Single' algebra concerned algebra without the use of angles and 'double' algebra with them, that is when considering complex numbers. He was concerned that the symbolical forms were satisfactorily interpreted rather than emphasis being placed on how these forms were manipulated. Symbolical algebra was thus established by the British mathematician Peacock and developed further by De Morgan in the mid-nineteenth century.

1.6.2 Symbolic logic

De Morgan also contributed to the field of symbolic logic in his *Formal Logic; or, The Calculus of inference, necessary and probable* [1847]. He contributed the 'generalized copula' which he represented as transitive, commutative and reflexive. He also introduced various non-syllogistic forms of inference, developed the theory of the quantified predicate, formed the so-called 'De Morgan's laws' which isolated logical relations between compounds and aggregates and introduced the concept of a universe of discourse [De Morgan 1860 331-58].

George Boole was born in 1815 in Lincoln where his father, John Boole was a cobbler by trade but studied mathematics and astronomy in his spare time. John Boole was involved in the Mechanics Institute in Lincoln (see 1.5.5), the movement which encouraged self-improvement in working men. George Boole was also self-taught and mastered mathematics and languages. As a young man he ran a school for some years near and in Lincoln, but in 1849 was appointed to the newly founded Queen's College at Cork in Ireland where he died 15 years later [Grattan-Guinness 1982 34]. He was a passionate teacher and was also interested in other social issues. As Boole had not received a Cambridge education, he had an independence of mind that was to enable him to take new steps in mathematics which he may not have taken had he been educated in the Cambridge tradition.

Boole conceived of a comprehensive logic which would subsume syllogistic logic and be modelled on algebra and his ideas were given in his books *The Mathematical Analysis of Logic* (1847) and *An Investigation of the Laws of Thought, on which are* *founded the Mathematical Theories of Logic and Probabilities* (1854). The operator methods which he used in solving differential equations (see 1.6.6) deeply influenced his approach to his work on logic. Boole's emphasised the logic of classes, denoted by x, y, z and individual numbers were represented by X, Y, Z. The null class was represented by 0 and the universal class by 1. An operation called election produced xy and this denoted the intersection of two classes. Boole built up a unique system of symbolic logic which did not fit easily into any previous mould. The most striking example of this was his assertion that $x^2=x$. This is at first very strange, but becomes more comprehensible when portrayed as $x(1-x)=0$ where $1-x$ is the complement of x. Intuitively this states that the intersection of the class x and the class not x is the null class. 'Both De Morgan and Boole can be regarded as reformers of Aristotelian logic and the initiators of an algebra of logic. The effect of their work was to build a science of logic which henceforth was detached from philosophy and attached to mathematics' [Kline 1972 1191].

1.6.3 Quaternions

Algebra was developed by Peacock and De Morgan in terms of its foundations, but it was also extended in a different way by the introduction of quaternions by the Irishman William Rowan Hamilton (1805-1865). Hamilton was a gifted linguist and applied mathematician. He entered Trinity College Dublin in 1823 and produced ground-breaking work on geometrical optics in 1827. In 1827 he was appointed Professor of Astronomy at Trinity College Dublin and also Royal Astronomer of Ireland. Hamilton introduced the idea of complex numbers as real number pairs (a, b) instead of a + ib, where $i^2 = -1$. He wished to extend the use of complex numbers from two to three dimensions. After many years of cogitation, he inaugurated number quadruples (a,b,c,d), or quaternions which represent a + bi + cj +dk where $i^2 = -1$ and $j^2 = -1$ and $k^2 = -1$, i, j, k representing the qualitative units directed along the three axes respectively of a Cartesian diagram. Hamilton's discoveries, announced in 1843 at a meeting of the Royal Irish Academy, were published in his *Treatise on Quaternions* [1853] and *Elements of Quaternions* [1866], the latter posthumous. Hamilton believed his quaternions would be as useful to mathematics as the calculus and he was enthusiastically supported by Peter Guthrie Tait (1831-1901), professor of

Mathematics at Queen's College Dublin. However, applied mathematicians did not generally use quaternions but they did lead to new methods in vectors which were adopted by physicists. Vectors were developed by James Clerk Maxwell (see 1.6.7) in his book *A Treatise on Electricity and* Magnetism (1873).

1.6.4 Determinants and matrices

Determinants evolved from the solution of systems of linear equations. Work on them was carried out by Leibniz (see 1.1), Euler, Colin Maclaurin, Laplace, Cramer (1704-52), Lagrange and Gauss. To develop the inklings of determinant theory found in Gauss's *Disquisitones arithmeticae* (1801), Cauchy laid the foundations for the systematic development of the new theory of determinants, including rules for multiplying them. Cauchy's work lay mostly unnoticed for the next 30 years until Carl Gustav Jacob Jacobi (1804-51) published three definitive works on determinants in 1841. He gave a definitive definition of a determinant but his approach was abstract. Sylvester worked on determinants over a long period of time, producing many new results, including an improved method of eliminating x from an *n*th degree and an *m*th degree polynomial [Kline 1972 800-3]. Cayley also contributed to the theory of determinants.

Any determinant contains an array of elements. Sylvester wished to study this array so he called it a 'matrix' in 1850. The laws of matrix algebra were studied in their own right by Cayley in his paper 'Memoir in the theory of matrices' (1858). Cayley was interested in the connection between the theory of functions and matrix theory and formulated the Cayley-Hamilton theorem, that every square matrix satisfies its own characteristic equation. Cayley stated he had proved this for square matrices of size two and three. Cayley's work on matrices was not taken up and developed by other mathematicians in general [Grattan-Guinness and Ledermann 1994 778-80].

Biographical details of Sylvester's career at University College London were set out in 1.4.2. He was a professor at the University of Virginia from 1841 to 1845, and became an actuary and a lawyer in London from 1845 to 1855. He then became a professor of mathematics at the Royal Military Academy, Woolwich (see 1.4.3) until

1871. He was professor of mathematics at Johns Hopkins University from 1876 until 1884 and during this time he set up the *American Journal of Mathematics*. He returned to England to take up the Savilian chair of geometry at Oxford University, a post he held until his death [Kline 1972 797].

Cayley was senior wrangler and Smith's prizeman at Cambridge in 1842. Although elected a fellow of Trinity College, Cambridge, he only worked there for three years as assistant tutor as he did not wish to take holy orders (see 2.1). He then turned to the legal profession, where he remained until 1863, when he accepted the newly created Sadlerian professorship at Cambridge.

1.6.5 Invariant Theory

Invariance is the study of entities which remain constant despite changing conditions. Invariant theory is a branch of algebra developed in the mid-nineteenth century 'to seek out and catalogue invariant algebraic forms'. The simplest type of algebraic expression which was considered in this subject was the binary form:

$$ax^2 + 2bxy + cy^2$$

Forms of any number of variables were also treated, but in the nineteenth century the binary form was the focal point of research [Crilly 1994 787-8].

Boole created the subject of invariant theory in his paper on linear transformations [1842 1-20, 106-19]. Boole also found the absolute invariants under the transformation in this situation: those expressions which remain precisely the same before and after the transformation [MacHale 1985 55]. He did not develop invariant theory, the task falling instead to Cayley, Sylvester and the Irishman George Salmon (1819-1904).

Invariant theory was associated with several branches of mathematics including the study of polynomials, number theory, the study of determinants and algebraic geometry which centred around the study of curves and surfaces described by algebraic forms of three and four variables. Cayley worked on this from the 1840s, making particular contributions in the 1850s and early 1860s [Crilly 1994 789].

Invariant theory was a major topic of research during this period in the United Kingdom.

1.6.6 Operator theory, differentiation and integration

Boole together with Gregory played a crucial part in the development of operator theory which he then applied to many different areas of the differential and integral calculus and to invariant theory. In his paper 'On a General Method in Analysis' [1844] Boole defined operators such as D, the operation of differentiation where

$$D: = d/dx$$

This operator was then used by Boole in his version of Taylor's series [Grattan-Guinness 1982 34-5]. This abstract work in operator theory introduced by Boole thus added to that already achieved by Peacock, Hamilton and De Morgan in the area of algebra.

The question of the definition of the limit was a still a vexed question in the calculus for British mathematicians during this period. 'The closest Cambridge came to a text with continental rigour was Matthew O'Brien's calculus book of 1842' [Becher 1980 21]. This was soon replaced by a text book by William Walton published in 1846 which reverted to Lagrange's method of using derivatives and used many of the concepts the Analytical Society (see 1.3) had striven to eliminate. De Morgan and Boole both contributed works on the differential calculus but it was Isaac Todhunter's textbooks on the calculus which became very popular, reaching many editions, even though the treatment of the calculus still lagged behind that on the continent.

1.6.7 Applied mathematics

On the Continent, partial differential equations found extensive use in applied mathematics in the nineteenth century. In France, Fourier and Poisson used one to express heat flow. Poisson applied such equations to the areas of gravitation, electricity and mechanics. The English mathematician George Green (1793-1841) built on Poisson's work on potential theory and produced original work in hydrodynamics and electricity and magnetism. Green's work was neglected until Sir William Thomson arranged for it to be published posthumously in the early 1850s.

Green was the first great English mathematician to continue the work done on the Continent after analysis was introduced to England. His work inspired the distinguished school of mathematical physicists at Cambridge which included Thomson, Stokes, and Clerk Maxwell [Kline 1972 684].

Other applied subjects treated by British mathematicians at this time were astronomy, optics, dynamics and mechanics. An article on optics by Herschel and one on waves and tides by Airy appeared in the *Encyclopaedia Metropolitana* (completed 1845) whilst Peter Barlow (1776-1862), professor at the Royal Academy, Woolwich, produced articles on mechanics, hydrodynamics, optics, astronomy, electromagnetism and pneumatics [Grattan-Guinness 1985 87]. Airy, Herschel, Babbage and De Morgan were all members of the Astronomical Society, a society in the forefront of scientific development in England, founded in 1820 and receiving its Royal Charter in 1831 [Rice, Wilson and Gardner 1995 402]. Airy produced many works on astronomy, his first work in the early 1820s covering lunar theory, the shape of the earth, precession and nutation and he also worked on the preparation of solar tables . In optics, he worked on refraction, reflection, interference and diffraction. Whewell's works on dynamics covered the rotation of bodies and the laws of motion. His work on mechanics was based on the lever principle, not on kinematics as in France [Grattan-Guinness 1985 101-2].

1.6.8 Geometry

Euclidean geometry has already been seen to have occupied a peculiar position in British mathematics in the eighteenth and nineteenth centuries. It was perceived as being clear to see and understand and its study was felt to develop the mind generally (see 1.4.1). In all the subjects covered above, British mathematicians made major and fundamental contributions, but in geometry this was not the case as Britain was firmly rooted in an Euclidean geometrical past. Analytic geometry was tentatively treated in a chapter of Wood's *Algebra* at the beginning of the nineteenth century (see 1.4.1). Henry Hamilton's work on analytic geometry of 1826 lacked diagrams in the main body of the text and was not popular. Hymer's work on analytic geometry (1837) became the standard text. This contained general transformation techniques and gave a general treatment of equations of the second degree. Hymers favoured analytic methods over synthetic.

Projective geometry was a branch of geometry which was developed extensively in the nineteenth century, initiated by Desargues (1593-1662), Monge and Carnot. Poncelet made it into an independent field of study. In his treatise on projective geometry of 1822, he 'generalized Monge's projection along parallel rays to projection along a pencil of intersecting rays, and thus rediscovered projective transformations. He also saw that in this way he could unify the study of the conics, and so obtain for geometry a level of generality that he felt had hitherto been the province of algebra (geometry being tied too closely to specific figures).' [Gray 1994a 902-3]. His work was tremendously influential, stimulating Gergonne, Brianchon (1783-1864) and Chasles (1793-1880), amongst others to further extend the subject. Brianchon developed the principle of duality, commenced by Pascal.

Projective geometry had two branches, synthetic and analytic. Jacob Steiner (1796-1863) was the principal architect of synthetic projective geometry, his contributions including work on the principle of duality, homothetic ranges and pencils, harmonic division, the projective geometry of conic sections and the theory of curves and surfaces. Analytic projective geometry was developed by Julius Plücker who defined line coordinates and a method of abridged notation. He defined ordered triples (x, y, t) such that for a point with Cartesian coordinates (X,Y), $x=X/t$
$y=Y/t$. When $t=0$, the triple (x, y, 0) represents the ideal points at infinity described by Kepler, Desargues and Poncelet. He also described cubic and quartic curves.

Michel Chasles (1793-1880) produced fine work in both synthetic and analytical projective geometry. His history of geometrical methods *Aperçu historique sur l'origine et le développement des methodes en géométrie* (1837) sought to promote geometrical methods which he felt were waning in France due to the popularity of analytical methods. Carnot similarly wished to see geometry freed of analytic notation. Chasles promoted the fact that geometrical methods are simple and intuitive as opposed to analytical methods which were not rigorously defined. His treatises on geometry (1852) and conic sections (1865) built on Steiner's and Poncelet's work but also repeated some results already been produced by Plücker (Chasles did not read German). Chasles named the cross ratio the anharmonic ratio [Kline 1972 835, 849].

The work begun on the continent on projective analytic geometry was built on by the British mathematician, Cayley, who also produced the equations of the 27 lines on every cubic surface in 1849 [Kline 1972 859], Salmon having demonstrated their existence. Cayley in his work on quantics of 1859, he considered the connection between invariant theory and projective geometry and worked to describe projective geometry in such a way that it contained metrical geometry. Sylvester was also a strong advocate of projective geometry, as was Thomas Archer Hirst (1830-1892), a graduate of Marburg University who taught at UCL from 1865-1870 and wrote extensive diaries cataloguing his visits to continental mathematical colleagues [Richards 1988 130, 136, Rice 1996 385]. Cayley, Sylvester and William Kingdon Clifford (1845-1879) also played a prominent part in developing n-dimensional geometry, the first seeds of this subject being sown by George Green in 1833. Clifford graduated from King's College London, and was second wrangler and Smith's prizeman at Cambridge in 1867. He became professor of applied mathematics at University College London from 1871-9 [Rice 1996 387].

Whilst projective geometry was consolidated and expanded, work was still progressing in synthetic Euclidean geometry with new results published on the inscribed and escribed triangles of triangles [Kline 1972 837].

1.6.9 Probability

This subject is covered by Todhunter's *History of the Theory of Probability from the time of Pascal to that of Laplace* (1865). Probability theory was developed by Laplace, De Moivre and Euler. De Morgan wrote *An Essay on Probabilities and on their application to Life Contingencies and Insurance Offices* in 1838 in the Cabinet Cyclopaedia series. This treatise sought to render Laplace's general methods in probability theory accessible to those without considerable mathematical knowledge and then apply the principles to practical situations - life contingency and insurance.

Boole was another British mathematician who contributed to probability theory. His work *An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities* included an interpretation of the probabilities of compound events in terms of Boolean combinations of simple ones. This concludes the survey of British mathematics from 1840 to 1865.

1.7 Summary

This chapter has surveyed French mathematics after the Revolution and shown the high level of pure and applied mathematics that emerged as a result, encouraged by the educational system which actively produced mathematicians and scientists. In contrast, British mathematics was somewhat stagnant following the priority controversy over the discovery of the calculus between Newton and Leibniz which then prevented mathematical interchange between the British mathematicians and their Continental counterparts who sided with Leibniz. This situation ended when a group of disgruntled Cambridge students formed the Analytical Society in 1812 in order to bring Continental practices regarding the notation used for the calculus to England.

The discussion of higher education in England revealed the dominance of Cambridge regarding the teaching of mathematics. However, the end product of the Cambridge education system at that time was not mathematicians *per se* as in France, but rather gentlemen ready for a career in the Church or the military, for example. To this end, the rationale at Cambridge was to provide the students with a 'liberal' education, and this objective was actively pursued by William Whewell, who was highly influential at Cambridge from the 1840s to the mid 1860s. Whewell favoured Euclidean geometry as he felt it developed clarity of thought and mathematical reasoning, and introduced Continental applied mathematics to Cambridge. He was sceptical about introducing Continental analysis into Cambridge, however.

The London colleges emerged in the 1820s, University College (originally the London University) catering for those with dissenting backgrounds who were unable to graduate from Oxford or Cambridge and King's College being a pro-establishment reaction to University College. As a mathematics teacher, De Morgan was hugely influential at University College over a considerable period of time and he was joined later by another brilliant mathematician, Sylvester, who was not as capable a teacher. At the military establishments, Continental mathematics was taught and here the mathematics teacher Hutton developed his *Course of Mathematics* which was a valuable text, continually updated by different mathematicians. In the late eighteenth and early nineteenth centuries Hutton was editor of a very important journal, the *Ladies' Diary*, and Rutherford, Davies and Fenwick edited the *Mathematician* from 1845 to 1850.

The *ET* was launched during a period of decline of mathematical journals, these having been chronicled by Wilkinson. The main journals at this time were the long-standing *Ladies' Diary*, containing varied contributions from reasonably eminent mathematicians and also philomaths, the *Cambridge (and Dublin) Mathematical Journal* which had De Morgan, Cayley, Sylvester and Stokes as early contributors and lastly the *Mathematician* emanating from the Royal Military Academy at Woolwich. Eminent mathematicians published their papers in the *Proceedings of the Cambridge Philosophical Society*, *Philosophical Transactions of the Royal Society*, and the *Philosophical Magazine*.

General science periodicals were read by many middle class people and mechanics magazines were read by the working classes wanting to improve their social position. The notion of the self-made scientist gave way to that of the professional in the 1860s. The opinions and the aspirations of the editors, who were often respected scientists, had a definite impact on the content of scientific journals. Commercial considerations also shaped science journals, particularly as they were subject to stamp duty, paper tax and advertisement tax. There was very real pressure on editors to ensure their journals covered at least the cost of printing and publishing. In the 1860s conditions improved to facilitate the interchange of journals between Britain and the Continent. Very little work has been carried out on mathematical journals to date, and that which has is mainly confined to the study of an aspect of a particular journal. Archibald's survey of 1929 remains the seminal work in this area.

Finally the chapter contained a survey of the main developments of British mathematics which are relevant to this thesis. Peacock laid radically new foundations in algebra in a treatise published in 1830, and his work was built on by De Morgan. A further extension to algebra, quaternions, was conceived of by Hamilton. These were complex numbers represented as real number pairs which were non-commutative. Boole devised an entirely new system of logic in works published in 1847 and 1854, this work being influenced by operator methods which he used to solve differential equations. De Morgan further developed Boole's work in this area. Cayley and Sylvester developed matrices, determinants and invariant theory, Salmon contributing to the latter area. De Morgan and Boole contributed to the differential calculus and Todhunter wrote the definitive text book on this subject. In applied mathematics Green used Continental methods to develop electricity and magnetism and hydrodynamics and his work inspired Thomson, Stokes and Clerk Maxwell at Cambridge. Whewell worked on mechanics, Airy on astronomy and waves and tides and Herschel on astronomy and optics. Developments in geometry in the first half of the nineteenth century were effected by the French mathematicians Monge, Poncelet, Carnot, Gergonne, Brianchon and Chasles and the German mathematicians Steiner and Plücker. Cayley, Sylvester, Hirst and Clifford then built on their work from the mid to late century. Lastly De Morgan and Boole worked on practical problems in probability.

2 THE ORIGINS OF THE *ET*

2.0 Chapter overview

Chapter two covers events in approximately chronological order and starts with a brief overview of British elementary education at the beginning of the 19th century, highlighting the lack of uniformity in the quality of teaching. The founding of the College of Preceptors is then charted, a body whose aim was to standardise the teaching profession by examining the teachers. Members of this College started a journal, the *Educational Times (ET)* in 1847. The aims and objectives stated in the *ET* are discussed and the place of mathematics in general in it is analysed. Rev. Dr Richard Wilson and James Wharton, both mathematicians and members of the College Council, were instrumental in initiating the examination of school pupils and the rationale for this innovation was publicised in the *ET*. The three important commissions in the middle of the century are outlined and the state of mathematics in elementary schools is set out.

2.1 British elementary education

Education had been provided mostly by a system of endowed grammar schools up until the mid 18th century. However, by the beginning of 19th century, there were three broad categories of education, provided by a) the endowed grammar schools, each accommodating a small number of pupils, b) various national voluntary organisations which gave children of the 'labouring classes' some elementary education of a basic nature and c) an expanding collection of fee-charging, privately-owned schools [Birchenough 1930 1]. Education in this period was often closely associated with matters of religion and class.

The Church of England had held a strong grip on education since the Reformation. The 1662 Act of Uniformity required all clergymen and every public schoolmaster keeping any public or private school and every person instructing or teaching any youth in a private family to conform to the liturgy of the Church of England. In practice, this meant subscribing to the Thirty-nine Articles. The remit of the Act also encompassed masters, fellows, tutors and chaplains in any college or university. This Act remained in force until 1869 and caused an insuperable problem for Dissenters.

'For Roman Catholics and most kinds of Protestant dissenters the legacy of the eighteenth century was a set of legal disabilities.....In the main they were still formally excluded from ministerial or administrative office, from commissions in the armed services, and from the universities:.....' [Thomson 1991 59]. Young [1977 74] alludes to the bitterness which arose between Dissenters and the Church of England over education - 'The Dissenters, who had wrecked Graham's Bill of 1843 for educating factory children in Church schools with a conscience clause, took the field against the Committee of Council......In effect,....the Government grant would be a subsidy in aid of the education of little Churchmen, or the conversion of little Dissenters.' To avoid having to subscribe to the Church of England in educational matters, Dissenters created their own educational establishments [Howson 1985 45], the University of London (later University College London) being the first university to admit Jews, Dissenters and women (see 1.1.3).

Turning now to the issue of class, Dr Andrew Bell and Joseph Lancaster, a Dissenter, were prominent reformers of education for the working class during the early 19th century. These two innovators independently reintroduced and popularized the monitorial system, which comes under category b) above. Lancaster's system was formalised with the aid of trustees into the Royal Lancasterian Institution in 1810. The monitorial system was one in which children taught each other and concentrated on the basic subjects - the 3 Rs. Bell published his *Sketch of a National Institution* for training the children of the poor in 1808 and in 1811 'The National Society for Promoting the Education of the Poor in the Principles of the Established Church throughout England and Wales' was founded [Birchenough 1930 41-52]. The large monitorial schools were favoured in 1838 by J. Kay Shuttleworth (1804-1877), who was appointed as secretary of the Committee of Council of Education in 1839 and who is regarded as the founding father of the modern English educational system [Stewart 1972 90]. Kay Shuttleworth and his friend E. Carlton Tufnell used their own resources to set up a training college for teachers at Battersea in 1839-40. This was a success and was built upon by the government. 'Trained teachers, public inspections, the pupil-teacher system, the combination of religious with secular instruction with liberty of conscience, and the union of local and public contributions were all provided for or foreseen by him.' [*DNB*].

However, even within the monitorial system, the educational pioneers Bell and Lancaster, had rival methods of education and each organisation had its own system of training its teachers. Given the three different educational categories above with their attendant various religious influences, there was little conformity in the teaching profession as a whole in Britain. The desire to standardise the teaching profession in Britain brought about the formation of the College of Preceptors which was founded in 1846.

2.2 The inauguration of the College of Preceptors.

The College of Preceptors is believed to be the oldest surviving association of teachers in the United Kingdom and had a clear sense of mission from the beginning. This was set out in resolutions reached by its members and confirmed by Royal Charter on 28 March 1849, namely to ensure that prospective teachers would be adequately qualified, both academically and professionally, being validated by a panel of teachers provided by the College of Preceptors [Chapman 1985 v]. The College itself was founded as a result of frustration felt by a group of teachers, mostly head masters, towards the lack of uniformity in the teaching profession. The members of this founding group included Richard Stokes, a head master in Chipping Ongar, Essex, John Parker, a recently graduated assistant master at the same school, Henry Stein Turrell, a head master in Brighton specializing in modern languages, especially French, Rev. Dr Richard Wilson, a classicist and head master in London, Dr William Ballantyne Hodgson, a head master in Manchester, Mr Stephen C. Freeman, a head master in Enfield, Middlesex, Joseph Payne a head master in Camberwell whose area of interest was philology and James Wharton from Kinver in Staffordshire [Chapman 1985 11-4].

It was John Parker who originally had the idea of a College of Schoolmasters, a notion that was encouraged by Richard Stokes. He outlined his thoughts at a meeting of the Literary and Scientific Society in Brighton in 1845, where he read a paper on the subject. He indicated that his desire to standardise the teaching profession was stimulated by the standardization of the medical profession in the 1820s. He asserted that the duties of a school teacher were equally as important as those of doctors, lawyers or the clergy. He pointed out that many children of the middles classes were educated at private schools where there was no way of ascertaining the teachers' ability. He made it clear that his objective was to form a body capable of assessing teachers' competency independently of religious issues. His suggestions were warmly received and he was sufficiently encouraged to try them out on a wider audience. He then promoted his proposals via an advertisement in a national newspaper. Circumstances were ripe for rapid communication at this time, the great age of the railways having begun in the 1830s, and the penny post being introduced by Sir Rowland Hill in 1840 [Thomson 1991 13, 81]. Parker received many replies in support of his schemes from English school masters [Chapman 1985 10, 16].

Resulting from Parker's initiatives, a series of meetings took place early in 1846 culminating in the creation of an interim committee to explore in detail this idea of a *College of Preceptors,* this name having been chosen so as not to preclude the large numbers of women employed in the teaching profession. Turrell headed this interim committee and at a meeting in London on 20 June 1846 proposed resolutions encapsulating the founding aims on which this professional association was to be constructed. The membership of this newly-established College grew rapidly, 60 masters having enrolled at the above meeting in London, and the number having swelled to 1000 before June 1847. The character of these members was commented on by Boole (see 1.6.2, 1.6.6 and 1.6.9) in a letter to Thomson (later Lord Kelvin - see 1.4.1 and 1.6.7) in September 1846 [MacHale 1985 284] - 'The Royal College of Preceptors (I don't much like the name), a body of more than three hundred schoolmasters including many of the most respectable masters of private boarding schools in the South....' The College was directed by a Council with Turrell as President and James Eccleston B.A., D.B. Reid F.R.S. and William Ballantyne

Hodgson LL.D as vice-presidents. The Dean of the Council was the Rev. Dr Richard Wilson (1798 - 1879), a man who features prominently in this thesis. There was concern that the College should be seen to be financially above board and hence a firm of bankers, a solicitor and three auditors were associated with the College. Lastly, the roles of Secretary and Treasurer of the Council were filled by John Parker.

The setting-up of the examinations, held half-yearly, was led by Wilson and Payne who made bible history and the theory and practice of education mandatory subjects. The first candidates were able to choose from tests at a lower or higher level in mathematics, classics, commerce and foreign languages. Appropriate qualifications for those passing examinations were consolidated during the first two years of the College's life. To qualify for a certificate, a successful candidate had to pass the third class of either the mathematical or classical test, otherwise they would receive a diploma. A candidate on gaining a second class pass became an Associate of the College of Preceptors, A.C.P. and a first class pass, a Licentiate of the College of Preceptors, L.C.P. Work of distinction in education, science, literature or the fine arts was needed to become a Fellow of the College of Preceptors, F.C.P. Gowns were chosen to correspond to each award which were meant to approximately equate to traditional university awards, A.C.P. to a university degree B.A., L.C.P. to an M.A. and F.C.P. to a doctorate [Chapman 1985 28-31].

Such a burgeoning group of teachers recognized the need for support from influential members of society and they managed to enlist the President of the Royal Society, the Marquis of Northampton, as their Chief Patron. Support also came from other eminent Royal Society members including Sir John Lubbock, the vice president of the Royal Society, James Joseph Sylvester, Sir Richard Westmacott and J.W. Gilbart who became vice patrons of the College. Of the 32 vice patrons, just over a dozen were Members of Parliament and amongst the others were two Aldermen and the Recorder of Birmingham. This College further strengthened its constitutional position by the procurement of the Royal Charter on 28 March 1849, which gave it stability [Chapman 1985 45]. The College made use of a newly-founded journal, the *ET* (see 2.3), to advertise its efforts to raise money to gain the Royal Charter [*ET* Apr. 1848

137] and also its eventual success [*ET* Dec. 1850 49]. The College now had a firm foundation and built on this base to influence the teaching profession in both the nineteenth and twentieth centuries [Chapman 1985].

It would appear that the innovations introduced by the College have not been attributed to them and Chapman observes that 'One can only conclude that the College had committed the unpardonable sin of the twentieth century: a lack of communication.' [Chapman 1985 xi]. He then states even more emphatically: 'Reference has been made to the College in a number of books dealing with the expansion and changes in British education but they often amount to no more than the odd line or a foot-note. If there can be such scanty references to a group of men and women who have initiated and influenced such changes in British society, then the picture of the past hundred years or more must be incomplete and perhaps even inaccurate' [Chapman 1985 xi].

A possible explanation can be found in [Silver 1983 29-30] who states that:

> Discussions of English education in the middle of the twentieth century have been overwhelmingly about *the system*, about the structure of the system, about access to the system, about the organization of secondary education, about the organization of universities and teacher education and technical education and the binary system, about numbers and ages of transfer and sizes of school, about percentages and finances, about the policies of political parties, about the structure of examinations, about urban aid and priority areas. Involved in all of this are implications about and interests in the role of the state, the balance of national and local government, the roles of professional experts and the lay public, pressure groups and interest groups and decision making, the power of the exchequer, manpower forecasting and economic efficiency. Also involved are concepts such as those of rights (of children or of parents), democracy and equality (in relation to educational and social mobility), and freedom (to buy private education). That there are widely differing views about all of these does not hide the fact that this kind of political and ideological discussion has been dominant in education, certainly since the 1930s. In looking for historical

> explanations of this system and all it entails historians have accepted a set of definitions or terms of reference that have produced most, though certainly not all, historical work on Victorian education. The interests of the twentieth century help to explain the silences about the nineteenth. There are, indeed, historians who have asked questions about the educational content or impact of social movements and the press, architecture and publishing, industry and towns. Overall, however, the historian's perspectives have tended to focus his attention on the growth of the system, people who have demonstrably made contributions to the system, processes which are still discernible in the system, social and economic changes easily (if mechanically) applicable to a discussion of the system.

It may be that the College has not been seen to contribute to the development of the system and thus the advances it made have been overlooked.

2.3 The launch of the *ET*

In 1847, individual members of the College were instrumental in founding *the ET*, which had as its subtitle on the first issue published on 2 October 1847; 'a Monthly, Stamped Journal of Education, Science and Literature' [*ET* Oct. 1847 1] (see 1.5.4 for discussion of stamped journals). This journal ran from 1847 to 1923, when it changed title, it has now become *Education Today*. The *ET* featured many articles concerned with achieving standards of excellence in the fields of education, science and literature and the energy and inspiration of those who brought it into being are greatly in evidence on its pages. It would appear that it reached a large audience comprising many different sections of society. It had a substantial, and in some areas, influential readership, and yet paradoxically, it is currently little-known. This is because its contribution to the development of the fields of education and mathematics, to name but two, has been overlooked, and as a result, it has been neglected.

The first issue of the *ET*, - Volume 1, number 1, appeared on 2 October 1847, priced 6d. This puts it at the cheaper end of the range of journals discussed in 1.5.4., but it was more expensive than the *Mechanics Magazine*. It was published by Thomas Taylor, 31, Nicholas Lane, London, but the editor of this auspicious journal was conspicuous by his anonymity - a situation that would remain unchanged for many years. Each issue contained 16 pages (see 1.5.4) and a notice at the end of this first issue stated: 'The *ET* will appear on the first of every month, stamped 6d; unstamped 5d. Parties paying 6s in advance will be supplied with the journal for 12 months' [*ET* Oct. 1847 16]. The stamped version carried the red penny post on the front page. A notable feature of the first issue of this journal that is very striking is the assurance with which it was written. There was a long letter [*ET* Oct. 1847 6] from Isaac Reeve commenting on the purpose of the College of Preceptors. This letter referred to previous discussions concerning the College and gave a strong sense of an already established entity instead of a virgin enterprise.

The first page [*ET* Oct. 1847 1] contained a miscellany of irregularly laid out, boxed-in advertisements for general items (for example envelopes, beauty products, photos - aimed at nobility, gentry, managers, merchants, engineers and the like.) Amongst these general advertisements were those offering school places and also the services of tutors. The very first advertisement stated:

> Tuition - London -- A Gentleman, A Member of the College of Preceptors, who passed the Higher Classical examinations last June, having the afternoon of Wednesday and Saturday at his disposal, is anxious to enter into an Engagement with any Gentleman requiring the assistance of a CLASSICAL and GENERAL MASTER, either in Scholastic or Private Teaching. The Advertiser can refer to his present Principal, The Proprietor of a school of the first respectability. - Old Kent Rd.

The College of Preceptors was thus underway in promoting one of the first teachers to pass one of its examinations.

There were several prominent and noteworthy articles in this first issue, first the selection from the examination papers for June 1847 [*ET* Oct. 1847 3]. These delineated the expansive knowledge and broad thinking of those who posed the questions and the resulting desire for comprehensive and thoughtful answers from the examination candidates. Some questions were posed in foreign languages and reference was made to leading continental educationalists, for example, the Swiss Pestalozzi (1746-1827), and also to German methods of education. All the questions in this selection were thought-provoking, searching and called for answers displaying powers of critical analysis concerning the methods, objectives and content of the educational system. Some of the fundamental notions of the founders of the College can be seen in these questions. One question went thus : 'Trace the analogy between the profession of teaching and that of medicine.' highlighting John Parker's inspiration for bringing the College into being. Another question stated: 'What subjects should constitute the curriculum of studies in schools for the higher and middle classes in this country?', clearly outlining the educational target area of the College.

The next significant item was a letter [*ET* Oct. 1847 5] from Rev. Dr Wilson. Wilson was born in Westmoreland [*ET* Nov. 1879 313-4 and Venn 1940-54 526]. He was educated mainly at Kendal Grammar school, and went up to Cambridge with an Exhibition in 1820. At St John's College he obtained his B.A. in 1825, being 17th wrangler on the Tripos list. Shortly afterwards he became a Fellow of the College, and gained his M.A. in 1827. The Bishop of Ely ordained him Deacon in 1828 and Priest in 1829. He took private pupils for many years at Cambridge, one such being James Joseph Sylvester (see 1.4.2, 1.6.4 and 1.6.5). He was awarded the degree of D.D. in 1839. He was then head master of St Peter's Collegiate school, Eaton Square, London. Wilson was one of the earliest Fellows of the College of Preceptors, and was a member of the College Council until 1877, holding the office of Dean from 1848 to 1859. He was an examiner for the College for a range of subjects, including classics and Hebrew. He was known for his kind and courteous character, his wide-ranging knowledge and his constant interest in education. He continued to live mainly in London until his death. Wilson will be discussed in 3.3 as a possible editor of the mathematical section.

The above-mentioned letter included a transcript of Rev. Dr Wilson's address prior to the distribution of certificates on 26 June 1847. In it, he enunciated the rationale of the College, and commenced by stating: 'Alluding to the office of schoolmaster among the middle classes of society, for to this portion of society does the College of Preceptors address itself...'. The College were thus extremely clear as to whom they were appealing - the middle classes. Kay-Shuttleworth's schemes were being put into place to train teachers for the labouring classes (see above) but the middle classes had not been catered for. A few of the Council members went to see Lord John Russell (1792-1878) to seek government support. They put it to his Lordship that "whilst the higher and lower classes were not allowed to be educated by unexamined teachers, yet the education of the middle classes was left to uncertified, incompetent and irresponsible persons."' Lord Russell had been involved in recent educational controversy (see 2.1) and had no wish to support measures that might cause further trouble [Chapman 1985 24].

Returning to Rev. Dr Wilson's address, he stated that a College comprising school teachers was best placed to examine school teachers, as opposed to tutors from Oxford, Cambridge, University College London or Kings College London and that the public concurred with this view. He declared it was now necessary to separate the functions of teacher and divine office, traditionally held by the clergy, the professional school teacher now being a distinct and independent entity. He concluded by mentioning his hope that every school, whether for females or males, 'would become a training college in itself' and that 'many respectable heads of schools will have two or three articled pupils'.

Another item of consequence was the announcement of the Ladies Department [*ET* Oct. 1847 5]. Joseph Payne and Stephen Freeman both advocated educating women and the inclusion of women in the College's strategy from the outset reflected their interests. For their era, these ideas are atypical and portray the forward thinking of the members of the College. A collateral College for women was envisaged at first and ladies took their own exam in June 1848. However, these plans were soon altered as women began taking examinations alongside men. From 1849 onwards, women were to feature in the life of the College and in the pages of the *ET* [Chapman 1985 26].

Lastly, this first issue contained a short list of the objectives of the journal [*ET* Oct. 1847 8] followed by a longer and more detailed prospectus [*ET* Oct. 1847 9-10]. 'Our objects in establishing this journal are :-

First to act as the organ of educators in their communications with the public or amongst themselves;

Secondly to furnish to those who intend to enter the profession, and to join its junior members, that instruction in the principles of the art of teaching, and in their practical application, which is indispensably necessary to enable them to perform their duties successfully;

Thirdly to point out the true bearing of all questions connected with education, and in particular to advocate the interests of the educator by showing that they are inseparably bound up with the true interests of the community; and

Fourthly, to bring together information of every kind relating to education; and in this department special attention will be paid to educational works, old as well as new, which will be analysed and criticised in such a way as to enable teachers to judge accurately of their merits, the want of trustworthy information of this kind being a desideratum which has long been seriously felt.'

A more detailed prospectus was then given, as follows:

> The proposed contents of the *ET* will be:
>
> 1. Reports of the proceedings of the College of Preceptors; communications to the council from local boards in connection with it; original correspondence on matters of education, science and literature in general, letters from assistants to the Dean of the College seeking for information and aid in the prosecution of their studies preparatory to the College exams, - the answers to which, when considered to be generally useful and interesting will be published in the Journal, authentic copies of the examination papers of the College will be given at the close of each exam, and properly-arranged lists of those who have passed them and received the College certificates.
>
> 2. Papers on the science and art of education in which it will be endeavoured to elucidate and render practically applicable the general truths discovered and

taught by the profound metaphysicians of various countries, but which are still unknown to the great mass of teachers, and have hitherto exerted but little influence on practical education. explanations of the various systems of education which have been pursued at different periods and in various countries. This department will present to the reader a digest of all that has been done by the great minds which have been directed to education. Each discoverer will be allowed to expound the merits of his system, substantially in his own words; and a resumé of the advantages and disadvantages of each plan will be given, founded on reasons which, being fully expressed, may be judged of by every reader. Papers on female education, and on the duties and provinces of the female teacher; showing their vast importance, especially in a moral point of view, not only upon young women but upon the whole population.

3. Educational politics, exemplifying the influence of the state on education in various countries; statistics of crime as connected with those of education; manners and customs viewed as the results of early discipline; the position of the educator and its effect upon the state of education, and through that upon the condition, moral and intellectual, of the whole community; normal schools and centralisation.

4. Papers for the purpose of conveying exact and scientific information on the subjects of the greatest utility and interest to the enlightened educator. Among the subjects in this department will be concise lectures on physics and the natural sciences, intended to give a general view of those branches of knowledge, and to direct the student to the best works upon them. Special attention will be paid to the points most likely to perplex the unaided student; and the best made of presenting them to the youthful pupil, so as to be intelligible to him, will be explained. In this division will be included papers on physical culture, and the intimate connexion between the condition of the body and that of the mind. Lastly, full and faithful reviews of books, especially of those intended to be used in schools, or likely to be serviceable to the instructor, including those which are already well known, but respecting which we may

entertain opinions different from those that are prevalent; digests of the transactions of scientific bodies; the proceedings of universities, colleges, chartered and endowed schools, preferments, vacancies etc.

5. Biographies of celebrated teachers, bibliographical accounts of scholastic literature, and the history of educational establishments, both at home and abroad.

6. Advertisements connected with every department of scholastic business.

In this short list of objectives, the editorial of the *ET* highlighted its wish to communicate issues of current educational interest (incidentally, not only those representing the views of the College of Preceptors.) They then described the didactic role envisaged for the journal placed in its appropriate social context. Lastly, they referred to their aspirations as critical reviewers of a wide range of educational works.

In the longer prospectus, six specific objectives of the journal were stated. The first contained a desire to be an organ of official communication for the College of Preceptors both generally and specifically relating to publication of material relating to examinations (although the *ET* was not the official journal of the College until a new series was launched in April 1861 when it carried the subtitle *'and Journal of the College of Preceptors'*). In the second, an aspiration was articulated to promulgate in England little known continental educational methods. Also in point two, the wish was expressed to publish material focussed specifically on women, both as learners and teachers, with emphasis placed on their high value. This was an extraordinary statement for the period when equal opportunities in education did not exist for men and women. Point three embodied the political aims of the journal and point six was concerned with the practical matter of advertising.

Point four is of especial interest as it contained direct reference to scientific material. In it, the editorial board delineated the desire to transmit in a clear manner scientific information for use by educators. It also highlighted the plight of the unaided students, and expressed a wish to help them. (This focus is one that was expanded upon by the well known mathematician, writer of mathematical text books, Cambridge don and examiner- Isaac Todhunter in his *the Conflict of Studies and Other Essays* [1873]. Todhunter devoted an entire chapter of his book to 'Private Study of Mathematics' and dwelt upon the difficulties of the self-taught student when faced with examinations.) The editorial board of the *ET* also signalled their intent to provide instructive book reviews. News of scientific bodies was to be given, accompanied by information concerning schools, universities and other educational establishments. Point five highlighted the aim to produce specific accounts of the influence of educators, educational establishments and educational literature. Many examples could be cited to exemplify the fulfilment of these objectives in the early issues of the *ET*. For example, the selection of examination papers set in June 1847 [*ET* Oct. 1847 3] contained the following questions:

> 2) The Germans make a distinction between Pädagogik, the science of Education; Methodik, the science of Methods and Didaktik, the art of teaching. How would you define their respective provinces?
>
> 4) State the leading features of Pestalozzi's, Jacotot's and De Fellenberg's systems of Education, and point out their several excellencies and defects.
>
> 9) Discuss the following educational dogmas
>
> 1. The faults of a school are to be sought for in the master.
> 2. The object of education is to stimulate the pupil to educate himself.
> 3. The pupil should never be allowed to leave a subject until he thoroughly understands it.
> 4. On ne s'instruit pas en s'amusant. (Enjoying oneself is not instructive.)
> 5. Non multa sed multum. (Not many but much)
> 6. Festina lente. (Make haste slowly.)

These examination questions clearly delineate the aspiration to promote continental methods of education as expressed in the first, concise list of objectives, mentioned above. Many of the French journals summarised in fig. 1.2 (see 1.2.3) were similar in some aspects to the *ET*. For example, the *Journal officiel de l'instruction publique* contained university news and wider national and international educational items and

Ferrusac's *Bulletin universel des sciences et de l'industrie* published reviews and abstracts of scientific material. However, there is no evidence that the editors of the *ET* were directly influenced by these journals.

The many extracts from the first issue of the *ET* thus accentuate the clarity of the aims and objectives of the College it emanated from, and also the high standards sought from those wishing to acquire the qualifications awarded by the College. The nature of the educational material discussed in this journal reflects the fact that some of the founding members of the College mentioned above were certainly polymaths, certain of them being particularly interested in mathematics and science. The aims and objectives outlined above were adhered to and examples of their fulfilment in relation to point four specifically form the body of the next section.

2.4 The place of mathematics in the *ET*.

2.4.1 Mathematics in the early years of the *ET*

Prominence was given to the field of mathematics throughout the life of this journal and examples from issues from its first few years amply supply illustrations of this point.

On the first page of the first issue can be found an advertisement for the winter lectures (1847-8) of the 'South Islington Commercial and Mathematical School', a minor example, maybe, but its positioning and timing make it noteworthy. Referring again to the selection of examination papers from June 1847, given in the first issue, there are several questions pertaining specifically to mathematics, which together with languages and science were termed in those days as 'modern studies' [Chapman 1985 49]. Out of 21 questions, 11 were concerned with educational issues of a general nature and the other 10 addressed topics focussing on specific subject areas. Of these 10, five related directly to mathematics and are shown below.

> 10) Explain and contrast the analytical and synthetical (or constructive) methods of teaching. Illustrate the applications of each to the teaching of Arithmetic or Latin.

14) What is the nature and range of the mental discipline effected by the study of Mathematics?

15) What are the main deficiencies in our common methods of teaching Mathematics? How would you propose to remedy them? As an illustration, explain how you would teach Arithmetic to an elementary class.

17) Which do you consider from personal experience, the best books for teaching the following subjects:- Greek, Latin, French, German, Algebra, Trigonometry, Arithmetic, Geography, General History and English Grammar

18) Give an outline of the plan you would pursue in teaching Greek, Euclid or History.

Again, the incisiveness of these questions demonstrates the desire to stimulate the examinees to pursue excellence in their teaching of mathematics and not merely to lead their pupils to 'cram' for exams (see 1.4.2). There was obviously dissatisfaction in the examiners' minds concerning the current teaching of mathematics (question 15) and this contrasted with their lofty opinion of mathematics suggested by question 14 - a most penetrating question. The mathematics examiners for June 1847 [*ET* Jun. 1847 1] were John Hind, Wharton, and Boole (see 16.2, 1.6.6 and 1.6.9). James Wharton (d. 1862) received his B.A. from St. John's College, Cambridge in 1834 and was the author of mathematical textbooks on algebra and arithmetic. He was a member of the College Council for many years and also of the College Board of Examiners [*ET* Apr. 1862 21 and Venn 1940-54 419] (also see below). John Hind (1796-1866) graduated from St John's College, Cambridge and was 2nd wrangler and 2nd Smith's prizeman in 1818. He taught in Sydney then was ordained a deacon in Cambridge [Venn 1940-54 419 and *DNB*]. He was the author of mathematical works on algebra and the comets and was a member of the Royal Astronomical Society. By June 1849, the examiners were John Hind, Boole and Sylvester [*ET* Jun. 1849 243]. In December 1850, Thomas Stephen Davies (see 1.4.3) was added to this trio [*ET* Dec. 1850 49]. The mathematical ability of the majority of these examiners then was exceptionally high, again demonstrating the important position it held in the College, although it was noteworthy that the examiners changed from year to year. The positions held by the examiners was also noteworthy, Sylvester and Davies being members of the Royal Society, Davies teaching at Woolwich, where Sylvester was to teach from 1855 to 1870 and Hind being a member of the Royal Astronomical Society (see 1.6.7). No minutes of any examination committees for these early years appear to have survived.

The subjects covered by the mathematical tests were published in the first issue of the *ET* [Oct. 1847 5]. The higher mathematical test covered 'the higher departments of mathematics and its applications, especially in algebra, conic sections, the differential and integral calculus, statics and dynamics.' The lower mathematical test covered 'arithmetic, four books of Euclid with deductions and problems, the elements of algebra and the elements of plane trigonometry.' The higher test had many subjects in common with the material studied by the more able students at Cambridge under Whewell's reforms of 1848 (see 1.4.1), except that the Cambridge syllabus also contained analytic geometry, hydrostatics, mechanics, advanced astronomy and optics and the College syllabus also contained conic sections. The lower test is similar to the elementary mathematics studied at Cambridge, except that it lacked the applied mathematics. Similarly, it resembled the course studied at Woolwich, except that the Woolwich course also contained applied mathematics and fluxions. Details of the examinations carried out at Woolwich were published in the [*ET* Nov. 1859 257]. Mathematics was one of nine subjects studied, and pure mathematics was worth 2,000 marks and mixed mathematics 1,500. It is possible that the College mathematical examiners wanted to promote pure mathematics in reaction to Whewell's emphasis on applied mathematics (see 1.4.1). Hind was a member of the Royal Astronomical Society and would have surely had the expertise to examine in these subjects.

Excellent examiners, however, did not entice excellent candidates as there were none for the first mathematical higher test. Six men did gain the lower test in mathematics in June 1847 as did two men in June 1848. In April 1848, Rev. Dr Wilson referred to the introduction of mathematical prizes in the following extract 'The College has already, by its Dean, declared that it considers mathematical studies among the most important, yet at present most neglected branches of education; and an illustrious mathematical scholar has shown his opinion on the subject by offering a prize for the greatest proficients in mathematics at the College examinations.' By August 1849. there was to be found adjacent the names of those having passed the mathematical examinations, the names of R. Jones M.C.P. (Member of the College of Preceptors) who was designated the Sir John Lubbock's Mathematical Prizeman, and A. Smith M.C.P, Mr Wire's Mathematical Prizeman. This distinction was a high honour considering Sir John Lubbock was the vice president of the Royal Society and vice-patron of the College (see 2.2). David William Wire, of Lewisham, was a vice-patron of the College. The introduction of mathematical prizes was perhaps stimulated by a desire to emulate the Smith's prize awarded for mathematical prowess at Cambridge (see 1.1.3).

Great emphasis was given throughout the *ET* to thorough understanding of each subject promulgated between its pages. It was to this end that model answers to the mathematics exam questions were published. In November 1847, the arithmetic test and the solutions to the questions on algebra from January's exam, 1847, were published [*ET* Nov. 1847 21-2]. The examination paper published in [*ET* Mar. 1849 138] is reproduced in fig. 2.1 Appendix A. There was an examination paper in 'Arithmetic' and one in 'Arithmetic and Algebra'. The questions were searching, asking for definitions of arithmetical and algebraic terms and supporting examples. On the 'Arithmetic' paper, which contained 21 questions, question one asked for a definition of least common measure and greatest common measure, then requested examples. Question four read 'Shew, by arithmetical example, that multiplying a given quantity by a proper fraction decreases its value, and dividing it increases its value. Why is this?' There were then several questions involving a practical application of arithmetic. Question seven was a practical problem which involved ratios, but at the end of the problem, the candidate was required to state the reason for the method of solution. These questions are very similar in character to those set in 1847 at Battersea Training School by Tate as cited in [Howson 1982 217] (see Tate below).

The 21 arithmetic questions included four separate questions on 'Artificers' and Work' comprising building problems. The 'Arithmetic and Algebra' examination again required definitions, explanation and investigation of methods not just arithmetical or algebraic manipulation. Questions requiring arithmetical and algebraic manipulation were again set in a practical context - the number of bricks in a wall, the cost of cloth and the weight of gold being examples. This section contained 17 questions, question 16 requesting an explanation of the signs and symbols used in arithmetical algebra.

In April 1850, a selection of the examination papers from January 1850 'Euclid with Deductions' was published [*ET* April 1850 164] and is shown in fig 2.2 Appendix A. This paper also required definitions and explanations. None of Euclid's propositions were referred to by number and the exercise did not comprise regurgitation of any part of Euclid. The first question asked for information on the axioms, including the parallel axiom and asked for a substitute definition. The majority of the questions involved straight lines, triangles, circles and squares, an example being question four 'To draw a straight line through a given point parallel to a straight line. Describe a circle which shall touch two given parallel lines, and pass through a given point between them.' A detailed knowledge of the axioms and propositions in the first books of Euclid would be needed to answer such questions.

In [*ET* Apr. 1848 149-51] there was a long item on education by 'A Collegian, London' containing a large section on the importance of mathematics in both schools and commercial establishments and a desire expressed that the College of Preceptors should lead in demonstrating *how* to teach mathematics in a lively and appropriate manner. The following citation summarises these mathematical aims. 'The College has already by its Dean, declared that it considers mathematical studies among the most important, yet at present most neglected, branches of education; and an illustrious mathematical scholar has shown his opinion on the subject by offering a prize for the greatest proficients in mathematics at the College examinations.[7]

> At Cambridge, Dr Whewell and others hail the proceeding of the College in this direction as an earnest of improvements in primary instruction. Already many of our commercial schools are devoting their attention to a sound and diligent course of mathematics; and the supplementary educational establishments which

have arisen under the appellations of athenaeums and mechanics' institutions, at which thousands of youths are labouring to compensate the defects of instruction by a diligent application to mathematical as well as classical and other studies, are sufficient proofs of the necessity of some change in our methods of school education. Let it not be supposed that we would wish mathematics only to be studied, but we are aware of opinion that they ought to form the staple in commercial schools, and that in classical schools they ought to engross at least one-half of the hours of study.

The desirability of concentrating on a few subject areas - mathematics, classics and chemistry, was then expressed, with lectures given and published in the form of model lessons [*ET* Apr. 1848 150].

In the early issues, there was a profusion of articles on the teaching of arithmetic. [*ET* Oct. 1848 5] contained a lengthy article entitled 'A Model Lesson in Arithmetic', attributed to 'B.A', possibly in response to the article mentioned directly above. It was in the format of a teacher/pupil dialogue, where the teacher asked a question of a class of about thirty boys and pupils were selected at random to answer. The answers given were ideal and showed a full understanding of why the rules of arithmetic were effective. The subject of the lesson was decimal fractions and there was considerable detail concerning all the different situations in which they occurred. Terms were delineated by the pupil and explanations and examples provided of certain results. It is hard to imagine a pupil being able to give such lucid answers which outlined not only certain arithmetical rules, but also why they held.

The Dean's (Rev. Dr Wilson's) report was published in a somewhat later volume of *ET* [*ET* Jul. 1854 224] and the Dean reiterated the desire 'that schoolmasters will now universally instruct their pupils through principles rather than rules'. One of the examiners commented on the examinands poor performance in the last set of examinations compared with the high standards achieved in previous ones. 'Notes on Teaching Arithmetic' by J.B. was printed in [*ET* May 1857 99] and contained hints for the successful organisation of an arithmetic class. It recommended mental arithmetic, as well as 'sound and thorough Arithmetic' but was chiefly concerned about the mechanics of the lesson itself. It stated that arithmetic lessons used the power of the mind rather than the memory and could be therefore be carried out in the afternoon when the memory was flagging. A variety of activities was suggested for use during the lesson, which was preferably held in a separate room with a blackboard. The old practice of leaving the student to work on problems unassisted day after day was deplored. A warning was given against the antithesis of this practice - explaining everything to the pupil. Instead the teacher should observe the progress of the pupils and be ready to guide them in their own learning so that 'whilst the ideas may be the master's, yet let them come from the pupil's mind and lips'.

Another article by J.B. giving teaching hints appeared in [*ET* June 1857 123] and was entitled 'Hints on teaching Algebra'. The importance of a firm foundation in arithmetic was stressed before attempting algebra. A rationale for teaching algebra was given

> The reasons why Algebra should form a branch of study are numerous. It is now taking a place in not only a liberal, but a moderate education; its aid in Arithmetic is invaluable, being the royal road by which many complicated and high-sounding rules are resolved; it stands on the threshold of all mathematical studies; it quickens the intellect and develops latent genius. And, again, the pursuit of this branch is recognized as a feature of the educational movement of the present age. The Committee of Council foster its cultivation, and the Royal College of Preceptors must deem its importance great, since in their scholastic examinations, it is required not only for First and second Classes, but even for the Third.

Teaching algebra at an early age was then recommended, and the teacher exhorted to keep his pupils' interest. The simple equation was stipulated to no longer be a boundary, nor the quadratic 'regarded as invincible'. Formulae and progressions were stated to be 'more enticing than horse-shoe problems', quite the opposite approach to that undertaken in the arithmetic and algebra examination which contained several practical problems. Less time spent on practical problems would result in most of this 'almost boundless science' being taught universally in most schools. Three hours a week was given as a possible time to spend on algebra and it was strongly recommended that pupils study independently, so their genius would be 'uncrammed'. The article concluded with the hope that a wide range of pupils would start to study algebra. This article is longer than its counterpart on arithmetic, and is less focussed. The desire to see pupils motivated by the love of algebra *per se*, gleaned without the use of practical examples, does seem somewhat unrealistic and out of character with the questions in the earlier examinations. A third article appeared in [*ET* Jul. 1857 146] on the 'Use of Geometry Theoretical and Practical'. This discussed the best time of day for a geometry lesson and the best age to start teaching it then progressed to practical applications of geometry. Again, the article lacked focus and was couched in somewhat poetic language. All three articles show the importance of these basic mathematical topics to the editors of the *ET*.

A letter was printed in [*ET* Dec. 1847 55] asking the editor which books were recommended for the mathematics exam, and Rev. Dr Wilson replied. He advocated Hind's *Algebra* or Lund's edition of Wood's *Algebra* and referred to a list given earlier in the same volume. He also mentioned that Euclid would be studied via *viva voce* where students would be asked to write out propositions. The list from [*ET* Dec. 1847 41] is as follows -

Algebra	Tate's *Algebra Made Easy*
	Colenso's *Algebra* especially 5th edition
	Wood's *Algebra* edited by Lund
Geometry	Pease's *Practical Geometry*
	Pott's *Euclid*
	Lardner's *Euclid*
Trigonometry	Hall's
	Snowball's
	Hymer's
Arithmetic	Tate's *First Principles of Arithmetic*
	Macleod *Mental Arithmetic* 2 parts

Hopkin's *Manual of Mental Arithmetic*
Crossley's *Arithmetic*
De Morgan's Colenso's Hind's Wharton's

Below this list there appeared a note mentioning a publication received. The book concerned was *the Principles and Practice of Arithmetic and Mensuration with the Use of Logarithms* by J. Wharton M.A. with the sanction of the College of Preceptors. Wharton's involvement with the College had resulted in them publicly endorsing his work, although it appears in the above list almost as an afterthought. The list is not attributed to anyone, but since Rev. Dr Wilson referred to it, it must be assumed he approved of it. Wilson does not mention his own book on trigonometry in this list, published in 1831 and being lucid with clear explanations. In the preface, Wilson acknowledged help from colleagues at Cambridge, and inspiration from Poinsot's *Sections Angulaires* and Airy's article on Trigonometry in the *Encyclopaedia Metropolitana*. Hind's book on arithmetic was very good with clear explanations. His *Elements of Algebra* [1839] required a working knowledge of arithmetic and progressed from elementary rules and theorems in algebra, in the light of arithmetic, to symbolic algebra, the basis for analytical investigations [Hind 1839 preface]. It contained appendices with examples. Again, this was a good text book.

Thomas Tate (1807-88) figures prominently in the list. He was the editor of the mathematical column of the *York Courant* from around 1834 to 1846 when it ceased and was responsible for the introduction of a 'Juvenile' section [Howson 1982 98-9]. He was one of the country's first teacher trainers, working at Kay-Shuttleworth's (see 2.1) Battersea Training School. Tate was a gifted teacher who used examples of practical problems in his teaching. He set down his teaching principles in his *Philosophy of Education* (1854, then 1857) in which he states that 'arithmetic cultivates the reasoning powers and induces habits of exactness and order' [Tate 1857 248 cited in Howson 1982 114]. The syllabus at Battersea included algebra, arithmetic, mechanics and mental arithmetic and Tate's many textbooks were universally used.

Bishop Colenso's work also features prominently in the above list. Colenso (1814-83) was mathematics master at Harrow from 1838-42. His textbooks were widely used in Victorian schools, particularly his *Arithmetic* [Howson 1982 259]. Wood's *Algebra* had a geometrical bias and was used at Cambridge (see 1.4.1). Hymer's works on analytic geometry and differential equations were standard at Cambridge (see 1.4.1). To summarise the list, it features Tate's innovative books and emphasises arithmetic.

Good mathematical textbooks, both new and old were reviewed in detail in the *ET* and those considered highly beneficial and instructive for personal study were warmly recommended. A review in February 1848 of *the Young Ladies' Arithmetic* by Samuel Goodwin [*ET* Feb. 1848 107] highlighted sentiments very much in line with those of the editorial as can be seen from these citations '..[this] work adds another to the list of treatises which proceed upon the now happily disappearing system of teaching arithmetic by mere rule.' and 'We can see no reason why girls as well as boys should not be taught the principles of arithmetic; we believe their minds to be equally capable of development and that it is quite as important that their powers be educed.' The main criticism of the work is that the rationale is not given for the mechanical methods of performing arithmetic.

The Rev. J. Steen's *Treatise on Mental Arithmetic* was approved by its reviewer in [*ET* February 1848 108], with the proviso that little time should be spent in teaching mental arithmetic in schools as this would detract from teaching the principles of arithmetic. Steen, of the Belfast Royal Academical Institution, responded to this review in [*ET* Mar. 1848 125] by stating that mental arithmetic, if taught correctly, could help develop the powers of the mind. He outlined his teaching methods which accentuated the underlying principles and hoped the reviewer would reconsider his opinion as Steen felt his disapprobation would turn teachers away from this relatively new discipline of mental arithmetic. The response of the reviewer was published below this letter and stated that it was more important to put arithmetic generally on a sound footing by expounding the underlying principles and that concentrating on mental arithmetic, even if carried out well as Steen advocated, would detract from this. He asserted the College of Preceptors was determined to remedy these defects in the teaching of arithmetic. The syllabus at Battersea training College included mental arithmetic as well as arithmetic and Tate used Pestalozzi's method of teaching arithmetic to instruct pupils regarding underlying principles, and the right approach for different cases in arithmetic as well as using rules [Kay-Shuttleworth 1862 341-5, cited in Howson 1984 108,110].

The distinction given to mathematics in this journal was too much for one reader who wrote in February 1850: 'I see in every number of your valuable periodical very interesting displays of mathematical talent, creditable in the highest degree, and instructive, but no classical question...!'

The personal interest and involvement of many of the strategic members of the College in mathematics were displayed in this journal. John Parker had penned three mathematics books which were advertised in [*ET* Nov. 1847 31]. A 'Paper read by Mr Wharton at the Conversazione held by the College of Preceptors, 26 June 1848', was reported in [*ET* Aug. 1848 242-3]. Wharton was one of the mathematical examiners for the College examinations and in this article he pointed out the necessity of mathematical study to give 'accuracy of reasoning and power of sustained thought'. He then went on to say 'Cambridge has of late expressed its determination to encourage only accurate mathematical learning, and the College will do its utmost with that University for the attainment of so desirable an object'. Wharton's interest were not confined to mathematics as he was also a classics examiner in June 1850.

2.4.2 Articles and books by Wharton

In [*ET* Jan. 1849 82] there was an outline of a lecture on the study of mathematics given at a College meeting by Wharton. The outline started with a review of the history of mathematics then contained a discussion of the lack of mathematical study at the public schools, at Oxford, for the armed forces and for those in Parliament. Teaching comprising 'a never-ending succession of rules' and lacking a logical method was then attacked. Cambridge wranglers who had 'crammed' for exams were pronounced unfit to teach. The Cambridge system was then denounced with its '*grinders*' who prepared pupils for the examination by making them copy out entire mathematics books without the examples.

Wharton advocated a strict matriculation examination to improve standards of students before they entered Cambridge and also the provision of good textbooks at higher than elementary level, instead of merely translating French textbooks. He stated 'Cambridge at present holds a very doubtful position in regard to the French schools and her transatlantic competitors have already taken the lead in many studies.' He went on to recommend that Cambridge should clearly state the level of mathematical attainment it required from students upon entrance, 'the three books of Euclid and Simple Equations according to Bridges' *Algebra*' being popularly claimed as being all that Cambridge tutors required. Putting principles in the form of rules was censured and teachers were encouraged to cultivate their pupils' reasoning, even if this produced a profusion of questions. He suggested that arithmetic would 'fill the void so much felt in female education' and 'teach them to think accurately and consecutively'. The article concluded with a citation of an eminent Cambridge man who stated that it was useless trying to teach higher mathematics at Cambridge when the basics had not been grasped and also strongly recommended 'a large and judicious collection of Problems'.

Wharton published many books, some of which ran to several editions and many containing examples from examination papers, for example the Civil Service. Wharton's *Elements of Plane Trigonometry* [1849] was aimed at school children and treated angles as ratios instead of the older practice of considering them as lines [Wharton 1849 2]. It also had an introduction to algebraic geometry which he felt had been neglected. He recommended Wilson's, Hind's, Snowball's and Hymer's trigonometry books for further reference. The material in this book covered the standard definitions and formulae in plane trigonometry and had examination papers from Emanuel College Oxford, the College of Preceptors and St. John's College Cambridge.

Wharton's *Logical Arithmetic* [5th edition, 1859] was for school classes 'to render youth familiar with the use of numbers, to make them independent of rules in their calculations, and to give them method in writing out at examinations' [Wharton 1859 preface]. The book commenced with definitions, then considered fractions, decimals, logarithms, proportions, interest and contained some Civil Service papers, College of Preceptor papers and Oxford middle Class papers. The definitions were over-simplistic, declaring there to be three kinds of number, odd, even and fractional. The 1861 edition started with a history of the development of number systems and had additional material on mental arithmetic. The books were small and slim and the explanations and definitions were not of a high standard, they were not of the same calibre as Wilson's work on trigonometry.

Wharton's *Examples in Algebra: being a collection of more than 2000 examples...* [1848] came mostly from Cambridge examination papers and was intended to progress from elementary to more complex problems. It included fractions, surds, permutations and combinations, the binomial theorem, series, simple equations, quadratic equations, simultaneous equations and indeterminate equations. His *Complete Solutions of Every Class of Examples in Algebra....* was published posthumously in 1863 and contained an anonymous preface from Cambridge, which is most revealing.

> This Publication, entitled "Solutions of Examples in Algebra," is the first part of a work designed as a help to Students of that science. The first part, complete in itself, was nearly finished when the Author was, after a painful illness, removed by death. The book will be a useful companion, more especially to those Students who have not access to a living Instructor to explain their difficulties.
>
> The late James Wharton, B.A., M.C.P., was a Schoolmaster, and the author of several Elementary Books. He was educated at St. John's College, Cambridge, and his name appeared fourteenth in the list of Senior Optimes, in the year 1834. He was one of the originators of the Royal College of Preceptors and a most active and zealous promoters of its success. He was also, for sometime, one of

> its Mathematical Examiners, and he continued to labour for the interests of the College as long as his health enabled him to do so.
>
> The Second Part of the work is intended to complete, if the First Part should be found to accomplish the design of the Author.

This preface stated Wharton's achievements but the statement that he was fourteenth senior optime for his year showed him to be in a much lower league than De Morgan , and Wilson who were wranglers (see 1.4.1).

Logical Arithmetic was unfavourably reviewed in the *ET*. Wharton answered his reviewers in [*ET* Feb. 1848 99-100]. His reply, entitled 'On Arithmetical Instruction', was somewhat arrogant and discourteous, immediately claiming that the reviewer had not read the work carefully and had also made errors. He denounced the reviewer as being afraid of new methods and wishing to retain the old rules. He declared the reviewer 'wants to have a large mess of principles, something like Professor De Morgan's book, which, at his recommendation I have inspected; and my opinion of which is that, with all its excellence, there is too much explanation'. Given the emphasis placed on explanation evinced in the examinations of 1847, this seems a strange line for Wharton to have taken. It certainly seems at odds with the emphasis on principles given by Wilson in a later issue (see above). Wharton included a section of a Cambridge book which showed that

$$am/bm=a/b$$

and attacked this as being unnecessary, as such a statement should be regarded as an axiom, 'or at least as data'. He asserted that his statement that $1/2=2/4$ needed no further explanation and that arithmetic could be taught satisfactorily from such a basis. He suggested that 'arithmetic be brought as nearly as possible to common sense, and let it exercise common reason, and be made the ladder to higher subjects.' He emphasised that boys should be made to reason, not understand. He felt the College examinations had already 'done very great good' but that lengthy textbooks were detrimental, even they did attempt to propound the principles of arithmetic. He claimed that boys he had taught for three months were capable of tackling difficult examples from other books, such boys being the antidote to Dr Whewell's problem that 'young men come to Cambridge entirely unprepared to proceed in the University course, because they are often entirely devoid of any previous mental cultivation'.

Wharton's reliance on his students' acceptance of his axioms compares poorly with De Morgan's desire to set out a firm foundation for his arithmetic. Rice discusses De Morgan's views in his doctoral dissertation which focuses on De Morgan as educator,

> De Morgan complained that, in their elementary study of the subject, the majority of students were not imbued with any notion of sound reasoning; instead he said "all is rule and work" [De Morgan 1831 271]. Moreover, he criticised the lack of rigour in properly defining arithmetical terms and concepts, such as fractions: "He [the pupil] has been accustomed to the consideration of several things of the same kind, but rarely to that of the division of one of these objects into *equal* parts. His *half* has, most probably, been merely a division into any two parts whatsoever, and can accordingly, with perfect consistency, talk of the larger and the smaller half" [De Morgan 1833 210]. This point was of no small consequence since he had observed from experience that "the want of a familiar acquaintance with common and decimal fractions is the source of nine out of ten of the difficulties which are commonly found in the study of algebra" [De Morgan 1833 221-2]. The first step, therefore, in training sound mathematicians was to ensure that their notion of all the concepts they were required to employ was rigorous and exact [Rice 1997 112].

Although both De Morgan and Wharton both deplored teaching arithmetic by rules, De Morgan was prepared to tackle what he saw as the root of the problem while Wharton simply stated his axioms (or 'data') without clearly defining his basic concepts. De Morgan wrote many mathematical treatises for the Society for the Diffusion of Useful Knowledge, a society which sought to spread scientific knowledge via cheap, clear treatises. This society published some of De Morgan's most popular books and also many of his articles in its *Quarterly Journal of Education* and *Penny Cyclopaedia* [Howson 1982 84-6]. De Morgan was known as an excellent teacher and his arguments are certainly more convincing than Wharton's, which are weak.

Wharton's defence of his article was followed by a reply from the reviewer, who was at a loss to discover his so-called errors or understand why he was deemed to approve of arithmetic taught by rule, which he said he deplored. He criticised Wharton's practice of using axioms for arithmetical properties that should be proved, declaring Wharton's reasoning to be 'somewhat loose'. He also criticised Wharton's error of going from the particular to the general, but did agree with him about the 'defective state of mathematical instruction in most of our public and private schools'. The reviewer's response is lucidly written and cogently argued, showing Wharton to be clearly inferior in his grasp of teaching arithmetic. The editor of the *ET*, in publishing the review and the response to it, was certainly at pains to be impartial regarding this matter.

Wharton wrote to Whewell on 3 May 1850 saying

> Dear Sir, My name may perhaps be unknown to you, or rather forgotten amidst the multitude of your important occupations. I now however write to you that having been for years the constant corrector both by Pamphlets and by means of the *Educational Times*, of an improved method of study and teaching both in the University and in the Public Schools, and also having introduced considerable improvement in Private Schools, I should be particularly glad of an opportunity of taking a place as Mathematical Assistant in a Public School, and then to have an opportunity of trying effectually, what can be effective in that way by diligence and authority in our Public Schools. I address you because your recent pamphlet proves your earnestness in the cause, and also because you may be likely to be consulted in the subject, and from Heads to whom I am unknown might wish to make use of my experience. Trusting you will excuse the liberty

The outcome of this letter is not known, but it shows Wharton's desire to be seen as a champion of mathematics education

2.4.3 Articles concerning De Morgan

De Morgan had other articles published in the *ET*. He wrote an article on 'Decimal Coinage' in the 'Companion to the Almanac for the year 1848' which formed the basis for discussion for an item in [*ET* Feb. 1848 102]. His lecture in October 1848 at the commencement of the Session of the Faculty of Arts, in University College London 'On the Effects of Competitory Examinations, Employed as Instruments in Education' (see 1.4.2) was reported in full in [*ET* Dec. 1848 56-9]. This article accentuated the need for depth of study in a particular area, not a breadth of study as in the liberal education at Cambridge. In it, De Morgan advocated private study, with the main use of lectures being to pick up useful hints.

In [*ET* Sep. 1854 273-4], M.C. of Elizabeth College, Guernsey (probably Mortimer Collins, see 5.5.2), wrote an article on 'Double Algebra' which featured De Morgan's work of that title. He denounced learning arithmetic by rule and stated that De Morgan was 'the only Englishman who has attempted to raise the standard of Elementary Algebra'. He especially recommended De Morgan's *Trigonometry and Double Algebra* and works by the German Martin Ohm, pronouncing these to be admirable school books. He pointed out that De Morgan fell short of establishing algebra 'as a significant and symbolic calculus'. He praised Colenso's book as being the best school book on the subject in England, but commented that it failed to fulfil 'the logical requirements of the science'. He then called upon the College to go beyond Cambridge and 'demand for its First Class, a thoroughly philosophic knowledge of Algebra as a science'. This should include a) definitions of symbols, b) the use of infinite series, c) the notion of limits, d) the notation of functions, e) different scales of origin and f) equations of the first and second degree. He praised De Morgan for producing problems for such equations on topics like levers and specific gravities - mechanical problems, instead of examples involving 'old ladies dealing in eggs'. The tenor of this article is on a much higher plane than the previous ones on arithmetic by Wharton (see above) and indicates the writer had a broader and deeper grasp of mathematics than him.

Wharton and Wilson thus provide examples of men intimately connected with the College who published articles on mathematical issues. Another such was Alexander Kennedy Isbister who joined the College Council in 1857. For his first paper delivered to the College, he chose as his subject the teaching of Euclid [Chapman 1985 55 and *ET* Apr. 1861 3-5]]

2.4.4 Articles concerning Whewell

Whewell (see 1.1.3) has already been mentioned in this chapter, in connection with his approbation of 'the proceedings of the College in this direction as an earnest of improvements in primary instruction [*ET* Apr. 1848 150] and also, according to Wharton, his desire that boys should arrive at Cambridge with satisfactory mathematical grounding [*ET* Feb. 1848 99-100]. John Parker gave a review of Whewell's *A Liberal Education in general, and with especial reference to the University of Cambridge, Part II* in [*ET* Apr. 1850 158-9]. He emphasised Whewell's important position at Cambridge and took pleasure in the fact that Whewell advocated the same aims as those stated in the *ET*, quoting Whewell's assertion that the faculty of reason is cultivated by the study of mathematics. There is then a discussion of the Classical Tripos, arguing that it could concentrate on logical reasoning and systematic arrangement rather than mere translation, and that this would put it on the same footing as the Mathematical Tripos.

Parker discussed Whewell's opinion of the need for improvement of the Cambridge system, but did not share Whewell's criticism of Goodwin's Course (see 1.4.1), merely echoing the need for improvement of the system. He agreed with Whewell that Cambridge should produce sound textbooks on every aspect of elementary mathematics, 'but as regards the definition of Trigonometrical functions, the questions raised may be questioned'. Parker applauded Whewell's boldness in criticising the teaching in the public schools and quoted him as saying that improvement in education would only occur if there was concomitant improvement in the schools and the universities. Parker reproduced a long quotation highlighting the lack of sound mathematical teaching at several of the great schools, emphasising the utility of arithmetic for practical purposes, and stating 'Arithmetic is in itself a good discipline of attention and application of mind'. Whewell then discussed the necessity for the mathematical preparation of school pupils before going to Cambridge and Parker drew attention to the need for a matriculation examination (see 2.4 below). Parker concluded the review by recommending Whewell's work to anyone interested in education, particularly of the middle classes.

This review sparked off some correspondence and in [*ET* Jul. 1850 223-4], F.C., a Cambridge alumnus had a letter published which praised Whewell for balancing the claims of both the classics and mathematics for a place in a liberal education. He then went on to say that boys should be classified at school according to their mathematical ability, not their classical ability, and taught at an appropriate level. He agreed with Whewell that the Grammar schools should provide a satisfactory grounding for the Universities.

On a lighter note, each month the *ET* contained a column featuring the names of famous people from the past who had been born, or had died in the same month. Many famous mathematicians and scientists were listed here, including Newton, Leibniz, Kepler, d'Alembert and Boyle. News was given of other educational institutions - a section entitled 'University Intelligence' brought news from Oxford, Cambridge, Dublin and the London Universities. This completes the analysis of the place of mathematics generally in the early years of the *ET*. Educational reforms involving secondary schools will now be addressed.

2.5 Later educational reforms

In Parker's review of Whewell's work on a liberal education (see 2.3 above), Whewell was quoted in [*ET* Apr. 1850 159] as declaring 'It may be said that the University and the Colleges ought to compel the schools to teach the Elements of Mathematics, by requiring a certain quantity of Mathematics of all their students. This may be said, and it is true, and I hope will be acted upon. I hope the Colleges or the University will require of all the students a real and practical acquaintance with Arithmetic and Geometry.' Parker reinforced these statements and added a warning 'The difficult point respecting a matriculation examination is now fully shown; and it is hoped that the University will be induced to make themselves equal to the times in which they live; and if not, either some violent change will be imposed, or the University will gradually become disused and neglected, except by those who fatten on its income.' The theme of school matriculation had already been taken up by Wharton in [*ET* Jan. 1849 82] (see 2.3 above) and was developed in a lecture reported in [*ET* Feb. 1850 102]. This report focuses on the significance of sound arithmetical teaching stating '(t)his subject is especially important to many Members of the College, because it happens to be the only part of mathematics which they teach.' The report concludes with a call to Cambridge - 'let Cambridge fit and enact a certain standard by means of a matriculation examination, and their requirements are sure to be met. It is their neglect and indifference which keeps down the standard- they demand accurate Arithmetic and a thorough knowledge of equations and Algebra, why then do they not exact that or its equivalent from everyone' He then referred to books used in schools - 'people are content to use such books as those of Walkinghame, Bridge, and Bonnycastle, or some such like, of which they never comprehend one half, and their pupils comprehend nothing.'

According to Chapman [1985 38-42], it was a founder member of the College, Mr Hall, who had originally written to the College Council advocating school matriculation examinations. His ideas became reality in August 1850, when a committee set up to look into matter produced a full scheme for their adoption. Their plans for school examinations were very similar to those in place currently. The school principal or his nominated assistant would conduct the examination while an examiner appointed by the College would supervise the entire proceedings. Additionally, the supervisor would add his own exam questions to ensure no 'cramming' of pupils. In November 1850, a circular was sent to all members of the College who were head masters, asking them if they wished to participate. Only William Goodacre, a Dissenter, of the Standard Hill Academy, Nottingham, replied. Rev. Dr Wilson and John Parker paid their first visit to this school in December 1850, then examined the boys on 23 and 24 December. This visit attracted publicity and by 1853, twenty more schools had been added to the one in Nottingham. The work grew and was joined in 1856 by the Society of Arts but was eventually taken over by Oxford and Cambridge. This action aggrieved members of the College who saw the examination of school pupils as their territory. The front page of [*ET* Jun. 1857 121] carried an article on 'Oxford Examination of Middle Class Schools' claiming the College and its efforts in the area of school examinations had been ignored by Oxford as the College ran on unsectarian principles. It was pointed out most forcibly that University tutors were not equipped to examine school pupils as they had never taught in such an institution. Such a clash with Oxford and Cambridge threw the College into a crisis, and both Wharton and Wilson were casualties [Chapman 1985 49, 53].

Mathematics in the *ET* and the reforms concerning school examinations have been discussed above. Finally the royal commissions centred on the 1860s will be surveyed as they yield information regarding the state of mathematics in the period under consideration. This survey is taken from [Price 1994 16-8]. The Clarendon Commission considered the nine public schools which had ancient foundations and included Eton, Harrow and Winchester. The Taunton Commission considered a variety of schools between these public schools and the elementary schools, which had been investigated by the Newcastle Commission (1858-61). As a result of the Clarendon Commission (1861-4), mathematics teaching in the public schools gained a firmer footing with the appointment of specific mathematics masters. However, most boys only studied four books of Euclid, some algebra and some arithmetic. The same topics emerged from the Taunton report, arithmetic being dominant, together with some dissatisfaction with the range of mathematics taught. Pupil's performance in geometry was observed to be below that in arithmetic. Although school teachers admitted mathematics was useful and valuable, there was no enthusiasm to extend mathematics teaching to any depth.

2.6 Summary

This chapter gives an overview of English education, showing how the upper and lower classes were catered for to some degree, but the middle classes were received education of a varying quality. The desire to standardise teaching by examination was behind the emergence of the College of Preceptors. The launch of the *ET* is then discussed, a journal intimately connected with the College of Preceptors. The pervasion of mathematics in this journal right from its inception is then set out.

Mathematics was treated with respect and much thought and care was given to its dissemination. The teaching of mathematics, particularly arithmetic, was particularly focussed upon, in both articles and examination questions, and the needs of the self-taught student were singled out. The state of mathematics teaching in secondary schools , as reported in three royal commissions in the 1860s, is summarised.

In the *ET*, contributions from eminent mathematicians were in evidence, especially De Morgan, and the mathematical examiners for the College of Preceptors were mathematicians of the highest calibre. Emphasis was placed on the unsectarian nature of the College, but education at Cambridge was frequently referred to. Two prominent members of the College, Rev. Dr Wilson, the Dean, and Mr Wharton, a mathematical examiner, wrote prominent articles accentuating the importance of good mathematical teaching based upon principles rather than rules, although Wharton's work was of a much lower standard than De Morgan's. Wilson and Wharton were also instrumental in starting up matriculation examinations in schools, an activity which was annexed by Oxford and Cambridge in the late 1850s. Although, the loss of their endeavour was a set-back for them, all the specifically mathematical activity just mentioned, occurring mostly at the beginning of the life of the *ET*, was extremely vibrant and encouraging. Such a high level of attention to mathematics was a very auspicious beginning for the subject in the *ET*. The development of the mathematical section against this fertile mathematical backdrop will now be set out.

3 THE LAUNCHING OF THE MATHEMATICAL DEPARTMENT AND ITS EARLY EDITORS

3.0 Chapter overview

Chapter three commences by tracing the beginnings of the mathematical department from the first mention of a question in March 1848 to the launching of the department in August 1849. The period from August 1849 to April 1862 is surveyed as a whole. Comments by the mathematical editors are analysed to attempt to ascertain the rationale, specific and general, behind the department. A major policy is set out which concerned providing problems for school pupils and resulted in the appearance of the junior department from 1851 to 1854. Important factors affecting the department are discussed, namely space considerations. Several important components of the department are visited, including prize questions, letters to correspondents and the request for original problems. The evidence regarding the mathematical editors from the inception of the mathematical department to April 1862 is then set out and arguments put forward as to their identity. These figures include Wilson, Wharton, Wilkinson and Miller. Wilson and Wharton have already been discussed in 2.4. Biographical details are given for Wilkinson and Miller.

3.1 The inception of the mathematical department

3.1.1 A survey of the first questions posed

Mathematical questions became a regular and well known part of this journal, but there is confusion in the literature as to how and when they started. Archibald [1929 396] stated 'Numbered mathematical questions seem to have begun with the issue for August 1849, although questions, solutions and mathematical papers appeared occasionally before this.' The first 'generally mathematical' question came from a preceptress in Manchester, dated 10 November 1847. It read as follows [*ET* Dec. 1847 48]:-

THE MOON'S ORBIT

> Mr. Editor,- I am happy to find that your columns are not to be confined to the wants and wishes of the educators of one sex, but that ladies also may state their experience and their difficulties, and request your aid.
>
> I, in common with many other governesses, make constant use of *Keith on the Globes*, and many teachers, to whom I have pointed out the following passage, are as much in the dark as to its meaning as I am. 'While the moon revolves round the earth in an elliptical orbit, she likewise accompanies the earth in its elliptical orbit around the sun; by this compound motion her path is everywhere CONCAVE to the sun.' I confess I cannot understand this. It seems evident that, owing to its compound motion, the moon must cross over or under the line of the earth's orbit twice every month. Now, this being the case, it appears to me that her orbit must be serpentine, and that therefore during one-half of the month her orbit will be concave, and during the other half, convex, towards the sun. If, you sir, or any of your correspondents, would give such a solution of this difficulty as ladies could understand, accompanied by a diagram, showing that the moon's orbit is everywhere concave towards the sun, or point out any error there may be in this passage I have quoted, it would confer a great obligation upon many teachers: and upon, sir, your obedient servant.

This plea was answered in detail and three trochoids were printed to illustrate the reply. Unfortunately the figures were stated to have been incorrectly drawn, but they still served their purpose. Two solutions by correspondents Mr Reeve and Mr Tabor were given to this problem the following month. They were referred to in [*ET* Feb. 1848 103] when the editors replied to Mr Reeve's complaints that his diagram had not been printed, and the terms used in his solution did not correspond to Mr Tabor's diagram. The first solution in December 1847 was unattributed. If it was by an editor, then this would be a case of an editor *and* correspondents providing solutions, more than was asked for by the preceptress. This astronomical/mathematical problem was strategic because it embodied so much of the *raison d'être* of the journal's founders (see 2.4.1 and 2.4.2) - here was a teacher asking for mathematical clarification of a problem to aid her in her teaching from the editor, or from any correspondents to the journal.

In [*ET* Mar. 1848 183] came the first reference to a separate section of mathematical questions. In the 'Notices To, And Queries From, Correspondents' section appeared a contribution from G.I. as follows:

> G.I. suggests the advantage of devoting half a page in each number of the *Educational Times* to original geometrical, algebraical, and arithmetical questions for solution. We have already inserted several communications of this kind, and shall always be happy to do so when the questions proposed seem calculated to be useful and interesting.

Several issues are striking when analysing this short note. First, the subjects chosen accord with those already accentuated in *the ET*, geometry, algebra and arithmetic (see 2.4.1). Next, G.I. advocates original questions, perhaps as opposed to examples from text books. The suggestion received qualified approval from the editors, who stated that problems of this kind had already been inserted, and would be again if they were 'useful and interesting'. Clearly, there was no intention to pursue mathematics for its own sake.

Presumably in response to this, a question appeared in [*ET* May 1848 183]. This was not in a separate section, but again in the 'Notices To Correspondents' SECTION. This was posed by A.B. who requested the solution of a cubic equation:

$$y^3 + 9y + 6 = 0$$

A general solution of a cubic equation was then stated, and then this was used to produce the specific solution to the cubic equation given. It is not clear here whether A.B. had supplied this solution, or the editors. Clarification of this issue was provided in [*ET* Jun. 1848 205] when a letter appeared 'On The Solution Of Cubic Equations'. This stated:

> Sir,-The cubic equation which you solved in your last number, was sent, not for the sake of a solution, but with a request that it might be inserted in *the Educational Times*, as a problem for your readers, and *requiring the three surd roots;* will you then, allow me, notwithstanding the mistake that has arisen, to observe that it is not the mere solution, but the manner of it, that I think of importance in your paper. We all know how common the practice has been at Cambridge , for men to get up for the examination dozens of formulae, of the

deduction of which they never understood anything; and at the same time I would rather see an example solved by depression, whenever it is possible, than by any formula. For the depressing process, the example in question is a most unapt selection, but nevertheless, I would propose the following as a solution suited to the examinations...(There followed a solution using depressed equations, involving the substitution: $y = x + z$, then the production of a factor of the cubic in terms of y, leaving a quadratic equation in y to be solved: the depressed equation.)

May 16, 1848 A COLLEGIAN.

The main purpose behind the above problem was then didactic, and related specifically to teaching at Cambridge. It is noticeable that the two communications mentioned above were pseudonymous. However, the use of the 'A.B.' recalls Augustus De Morgan using that pseudonym in 1863 , on the flyleaf of the book *From Matter to Spirit* which he co-authored anonymously together with his wife. 'A Collegian' also wrote a long article on education generally in April 1848, with special emphasis on mathematics (see 2.4.1). The sentiments expressed by 'A Collegian' are concomitant with De Morgan's educational principles, especially his loathing of the practice of cramming for examinations. It could be then, that De Morgan was instrumental in shaping the mathematical department at its foundation.

There were no other communications of a specifically mathematical nature until [*ET* Nov. 1848 36-7] when an article 'On the theory of parallels' appeared. It was submitted by Thomas J. Leeming of Broad Oak, Gloucestershire and directed to 'the notice of your professional readers', an educational focus again. It contained an attempt to simplify 'the doctrine of Parallels, as laid down in Euclid. Here it was suggested that the cumbersome '12th (so-called) axiom' be discarded and the following substitution made for the 35th definition:- 'Parallel right lines are such as are in the same plane and are perpendicular to the same right line'. Other Euclidean propositions were then re-formulated using this definition and the resulting set of definitions which then emerged was stated by Mr Leeming to be, 'A great simplification of the matter for the purposes of instruction...'. He then wished to know of any reasonable objection to its more general adoption.

That same month, a specifically mathematical question did appear in its own small section entitled 'MATHEMATICAL QUESTION' and read as follows,
'Required, the solution of the following equation (sic), first by a *quadratic*, and secondly by a *cubic*:

$$x - y = 8$$
$$x^4 - y^4 = 14\,560$$

P.Y.D.W. CLERICUS Storrington, Sussex'

This question received a response in [*ET* Dec. 1848 62] from two different solvers:- Thos. Morley in Bromley, Kent, and Peebles. Morley solved the equations as requested using firstly quadratics, then Cardan's method. Peebles' solution was prefaced by a line, presumably inserted by the editors, stating the solution to be substantially the same as the second of Morley's, 'but more fully and, perhaps, more clearly stated'. This was certainly a portent of what was to come for many years in the mathematical department, several different solutions to a problem, each bringing its own clarity concerning a certain aspect of the problem in hand.

Below these two solutions came two new questions numbered (1), (2), under the heading 'Questions In The Diophantine Analysis' proposed by T. Morley, M.C.P., Bromley, Kent. The initials M.C.P. represent 'Member of the College of Preceptors' (see 2.2), one of the College's own graduates. The first question required a general integral solution to be found for the sides of a right angled-triangle to be a complete nth power, and also the specific case when $n = 5$ to be discussed. The second question concerned a specific problem with a plane triangle. Sides in arithmetical progression were required which satisfied a trigonometric expression involving the sides and angles of the triangle on the left hand side, and a square integer number on the right hand side. These questions were of moderate difficulty.

In [*ET* Dec. 1848 60-1], there was also a response to Mr Leeming's article on the theory of parallels. This was in a section entitled 'Original Correspondence' and came from Thomas Kimber, King's College London. Kimber alluded to a lecture on the study of logic delivered to members of the College of Preceptors by Dr Latham, and went on to analyse Leeming's assertion by reducing it to a logical form and comparing it to Euclid's 12th axiom also reduced to a logical form. He mentioned how difficulties related to Euclid's 12th axiom were overcome in two leading colleges of the University of London by using the following as a substitute for Euclid's 12th axiom - 'two lines which intersect cannot *both* be parallel to a third line'. He concluded that Leeming had added nothing original in his previous article, but that a M. Bertrand of Switzerland had made a worthy and so far neglected contribution by considering infinite spaces between the parallel lines, which he had incorporated into his teaching on the subject.

In [*ET* May 1849 82] an article appeared on 'Divisibility Ad Infinitum' by R.A.A. in which he discussed whether a line containing an infinite number of points could be used to produce an infinite line and make it easier to imagine. His article caused a good deal of correspondence, supporting the existence of the infinite line. Both Leeming's and Reeve's articles dealt with the difficulties of Euclid's parallel postulate and tried to resolve them. These difficulties had been addressed independently by Janos Bolyai and Nicolai Lobachevsky in the 1830s and resulted in the formation of a new, radical branch of geometry, non-Euclidean geometry [Gray 1994b 880]. Non-Euclidean geometry did not become popular in Britain until the late 1860s [Richards 1988 74].

3.1.2 The initial structure of the mathematical department

A steady stream of numbered mathematical questions and solutions then followed. In [*ET* Jan. 1849 8-7] a new question was posed by T. Morley under the heading 'Mathematical Question', and two solutions published under the heading 'Answers To The "Questions" In Last Month's "Times"'. The first solution was by Morley, and the second by an esteemed correspondent, J.W. Brighton. This was probably James Wharton (see 2.4.1 and 2.4.2), whose reply to an unfavourable review on his work on arithmetical instruction in February 1848 was signed 'J. Wharton, Brighton'. Two solutions and two new questions were given in [*ET* Feb. 1849 108] under the heading 'Mathematical Questions', one of the questions being posed by a pseudonymous 'Enquirer', with reference to a similar question in Wood's *Algebra*.

The heading 'Mathematical Questions And Solutions' first appeared in [*ET* Mar. 1849 133] and comprised two solutions and two new questions. It was stated that two other solutions had been received, but that they were undesirable as they needed diagrams to explain them. Later solutions did include diagrams, so there must have been a change in policy at some stage. In a separate section, 'Answers To Correspondents', immediately below, there was a note saying that three questions proposed by a Staffordshire correspondent were 'too easy of solution to be suited to our columns.' This indicates that the mathematical editors had a lower limit in mind for the standard of the mathematical questions, even though they were thinking of problems suitable for use in the school room. In May, June and July 1849 there appeared the section 'Mathematical Questions And Solutions', containing just a few questions and solutions each month. The principal contributors during this early period were Thomas Morley, Wharton, Leeming and 'Geometricus'. The total number of questions posed in this time was 25, two of them being numbered. Thus in the short time from the first germ of an idea for a mathematical department in March 1848 to the introduction of the MATHEMATICS QUESTIONS AND SOLUTIONS department in March 1849, the mould had been cast.

3.2 Survey of editorial policy

In the above review of the initial mathematical questions, instances can be seen of the editors printing a small note to explain their choice of a particular question or solution. These snippets were to be found in either the 'Mathematical department' itself, preceding a problem, or as an endnote, or in the 'Answers To Correspondents' section. This trend was to continue throughout the life of the journal and many of the editors' aims and objectives were divulged in this way. Some of these comments occurring throughout the period under consideration will now be analysed. Following this will be a study of the mathematical editors themselves.

3.2.1 Space considerations

Throughout the period under consideration, there was constant complaint from the mathematical editors that there was insufficient space for their mathematical department. It may be that commercial considerations played a pivotal role in determining the length of non-paying contributions to the *ET* (see 1.5.4). In [*ET* Sep. 1849 273], a note to mathematical correspondents lamented the lack of space to print the several communications which had been received, giving an indication of the good response to the newly-launched mathematical department. In this particular issue, a whole page was devoted to model solutions for the last examination of the commercial class, and just under half a page to the 'Mathematical department' itself. The mathematical editors were thus making a policy decision as to how to use their allocated space and model solutions obviously took a high priority, as was seen in 2.4. Their choice will be returned to in 3.2.2.

In [*ET* May 1850 132], some contributors were informed that 'their solutions of the questions were much more voluminous than required' and they were encouraged to be as succinct as possible and to study, for example, Wharton's *Algebra* for good model solutions. The lack of space was mentioned again in [*ET* Apr. 1852 166], and in [*ET* Jul. 1852 238] it was reported that there were 'deferrals due to great pressure of matter'. A cause of the pressure was specified in [*ET* Oct. 1852 16] when it was stated that 'From the sudden pressure of Advertisements, several solutions are unavoidably deferred till our next number'. This was blamed again as the cause of abridged space for mathematics between February 1853 and August 1861 [1]. Want of space (cause unspecified) was declared between January 1853 and May 1861 [2]. In [*ET* Jul. 1854 236] the mathematical editors stated 'In consequence of the important results arising out of the recent examinations of the College, we are obliged to make the Mathematical Department a blank for this month'.

[1] [*ET* Feb. 1853 111; Feb. 1855 41; Aug. 1861 114]
[2] [*ET* Jan. 1853 87; Nov. 1853 42; Jun. 1854 214; Apr. 1856 95; May 1861 16]

Attempts were made to mollify mathematical contributors. In [*ET* Mar. 1858 70] it was stated that 'we hope to pay special attention to our mathematical correspondents next month'. In [*ET* Aug. 1856 188], contributors were told that 'We have in reserve some Solutions which shall appear as soon as possible'. A mild remonstration was issued to contributors in [*ET* June 1853 212], however, with 'We always endeavour to insert the questions which are sent to us; but if a correspondent send us a dozen at once, and several others do the same, as is often the case, it is very evident that we can find places for only a few out of the whole'.

3.2.2 Junior policy

It was seen in 3.2.1 that model solutions were given high priority in September 1849. Producing such solutions exemplified the desire to provide material for students and their teachers alike to learn from. This desire, stated explicitly, was clearly a main plank of the rationale of the mathematical editors. In [*ET* Feb. 1850 107], mathematical contributors were thanked for some very able solutions, and it was stated that 'We have also been invited to admit some questions of less difficulty, which may tempt less able veterans in Geometrical Science'. The use of the term 'veterans' clearly precluded juniors but did set the trend for questions of an easier nature. This contrasts with the editorial comment regarding the questions of too easy a nature rejected in March 1849. In these early stages of the mathematical department, it would appear that the editors were trying to find the right level to encourage contributors to pitch their questions. A specific objective regarding mathematical teaching was articulated in [*ET* Aug. 1850 254],

> We shall pay attention to the suggestions of some of our distinguished contributors from Ireland and Yorkshire; but at the same time, we beg to assure them, that our object has hitherto been, rather to introduce amongst teachers sound methods of mathematical demonstration, than to lead a *few* to display the powers of their extraordinary mathematical genius; and our design was begun and carried out with these views, amongst a great number of other duties. We feel highly grateful for the great support that has been acceded to our aspirations; and we trust that the advocates of mathematical education will not fail to exercise their powers in promoting a study which is on all sides

> acknowledged to be the greatest developer of the powers of human mind, and the best promotive of a sound and unbiassed judgement in human affairs.

In [*ET* Apr. 1851 160], there appeared the following,

> A correspondent offers the following suggestion:
>
> > The columns of your journal devoted to Mathematical Questions, &c, are highly beneficial to Assistants and Teachers in general; but owing to the advanced character of the majority of these Questions, the benefit is confined exclusively to the above parties. There are many pupils in our schools who would willingly test their ability by attempting some of the solutions, were they not so much beyond their powers.
> >
> > I would therefore suggest, that a column be devoted each month to a few *original* questions of an elementary nature, adapted to the capacities of boys moderately advanced in the study of Mathematics.
>
> In reply to the above suggestion, we beg leave to state, that it has always been intended to give a certain number of easy questions, which appeared calculated for the purpose above mentioned. In the present Number, we have arranged some questions under the head of 'Junior Mathematics' and we shall endeavour to adapt that department to the purpose proposed.'

This marked the inception of the 'Junior' Mathematical department which ran until December 1854. Original problems were proposed for the juniors, reiterating the initial suggestion made by G.I. in March 1848 to have original problems.

In [*ET* Dec. 1854 362] an arithmetic examination paper was inserted in the columns 'for the use of Masters of Schools who may wish to examine their pupils according to what is here suggested'. Later in [*ET* May 1859 118], it was stated that it was 'advantageous to mathematical students to offer different methods of solution given by distinguished correspondents.' The aspiration to reach mathematical students was maintained, for in [*ET* Dec. 1859 286] appeared the following in the *Answers to Correspondents* section: '...we wish to have somewhat easy questions now and then for the sake of our juveniles'.

3.2.3 General rationale

In [*ET* Jan. 1850 85] the following appeared, 'Correspondents are informed that Solutions by others, are preferred, if equally deserving, to those sent by the proposer of any question.' Scrutiny of those questions solved by the proposer in 5.6 will determine how closely this objective was carried out. One tactic employed by a voluminous and strategic correspondent, Thomas Turner Wilkinson (see 3.4.1), was to pose questions using his own name, then solve them using a pseudonym, for example Wilkin Thomasson, or 'Hibernicus'.

The following comments shed light upon the editor's criteria for selecting questions and solutions for publication from the plethora they evidently received. In [*ET* May 1850 182], 'It is difficult to select the *best* out of a number of solutions, which are often very similar in nature, and possess various points of excellence.' In [*ET* Apr. 1854 166], reference to external sources was made, as follows:

> 'We would respectfully suggest to some of our contributors, that although it may be desirable to point to certain authorities, we should prefer having every solution entire of itself, but, of course, without encumbering it with mere algebraical reductions, so that every solution should be a model of excellence and neatness. We are afraid that these points have not been kept sufficiently in view, in connection with our object of improving and extending mathematical learning.'

In [*ET* Mar. 1857 68] was found 'Mr Levy.-We thank you for your solution, and for the information that Question 939 is the same as Question 50; a good thing will bear repetition. Problems in Mathematics necessarily recur.' In [*ET* Apr. 1857 92] appeared the comment on '..the great difficulty in selecting those solutions which are likely to be of most service to others, as it regards method.' The editors were therefore looking for excellent, self-contained solutions, preferably not by the proposer and which demonstrated a clear method, reinforcing the didactic and utilitarian emphasis.

3.2.4 Original problems

In 3.2.2, a tension can be seen between printing questions suitable for juniors and printing those of a higher level for more advanced mathematicians. One contributor in particular who advocated original questions was William John Clarke Miller (1832-1903), who will be of considerable importance later (see 3.5). A note addressed to Mr Miller was printed in [*ET* Jun. 1857 141] went as follows:

> We appreciate highly your remarks. We should indeed like to propose more *new* Questions than we do, but we have acted on the notion that our scientific friends *must see* the Solutions in our pages. We are indeed most grieved that the beautiful problems and most valuable matter cannot at present be presented by us to the public eye.

Miller was to become a major figure in the life of the mathematical department of this journal, and it is revealing to see his aims being discussed at this stage of its life. Indeed in [*ET* Jan. 1857 20], his plan for a separate mathematical reprint was discussed in the 'Answers To Correspondents' section in a most condescending manner:

> Mr Miller.-We wish you were at this moment at our elbow. We are quite sure you would be pleased with us, because we are so *pleased with you*. You suggest the propriety of our publishing a Mathematical Supplement. We are most ready to take your hint and act upon it; but then starts up the consideration that the Supplement must be *paid* for; and two or three hundred sixpences would go but a very little way to effect so necessary a business. You must therefore be like ourselves. We try to do what is right, and to please as many persons as possible; and we measure our actions by practicability, and not by *liking*. Perhaps folks at Eltham can *do as they like;* we cannot do so in London. There is, however, one grand consolation-viz.; that it is not good for any one to have it in his power to do as he likes; young boys and girls of fortune frequently act so: but only see what a mess they soon make of it!....

The reference to two or three hundred sixpences may be an indication of the print run of the *ET*, or the number of subscribers to the mathematical department. Stamp duties and advertisement tax were no longer payable in 1857, and paper was cheaper (see 1.5.4), but it was still necessary for a journal to cover its costs.

However, in [*ET* Jul. 1857 162] there was evidently a change in attitude towards Miller's suggestion concerning new problems as the following letter was published:-

> Mr Miller, ...We propose acting upon your hint; i.e. to propose more new problems for the sake of our mathematical readers, it being taken for granted that their solutions will not always be published.

As Miller had only made the suggestion in June, this was really a quick turn-around and inspires conjecture that the editors received a large response in Miller's favour.

In [*ET* Dec. 1859 286] came the comments '.....One is tempted to say there is nothing new under the sun, and yet THERE IS. Real mathematicians are discovering new properties and new relations daily.' These comments mark a growing awareness of the presence and power of original mathematical work which was to have so much influence on the mathematical development of the journal after Miller became its editor (see chapter 8).

3.2.5 Prize questions

In [*ET* Oct. 1851 17], the following appeared:

PRIZE QUESTION

> Sir, I wish you to propose, in your next Number, the following Question, to be answered in the Number for December, and I will give for the best solution, copies of your Journal for twelve months, beginning with the October Number.
> Theon.

The question taken from Matthew Stewart's *General theorems* published in 1746, concerned a given circle with two right lines given by position. Given any point on its circumference, two lines were required to be drawn from this point and perpendicular to the first two right lines. The sum of the cubes of these two lines then had to be shown to be equal to a solid whose base was the sum of the squares of two other lines. The question, moderately difficult, was in the areas of number theory and geometry. Later, in [*ET* Sep. 1856 216], an obituary notice appeared for Mr Henry Buckley, who had died aged 49. It stated that he contributed to the *ET* under the pseudonyms of 'Theon' and Geometricus, and that he was a pupil of the Lancashire mathematician John Butterworth. Henry Buckley could then be the poser of the first prize question in the *ET* and one of the earliest contributors to the mathematical department (see 3.1.2).

Four more prize questions appeared in [*ET* May 1852 189], this time accompanied by offers of free copies of *ET* for two years to the winner. Three of these questions were algebraic, and the last geometric. These were solved by W. Grayson, who was awarded the prize, but similar solutions were received from Stephen Watson and Isaac Hind of Keigthley. 'Pappus' inserted a geometrical prize question in [*ET* Nov. 1852 39], with a year's free *ET* as the prize. T.T Wilkinson solved this in [*ET* Mar. 1853 140] and he referred to winning this prize in his autobiography (see 3.4.1).

In [*ET* Oct. 1853 17], Hibernicus set a prize question which ran as follows: 'By the aid of lines not exceeding the second order, inscribe a quadrilateral in a circle given in magnitude and position, so that two of its opposite sides shall pass through given points within the circle, and such, that the rectangle contained by the straight lines joining the given points with the intersection of the diagonals shall be a given space or a minimum.' Note that lines not exceeding the second order were required, corresponding to the general emphasis of British geometry at the time (see 1.2.8). This was then amended slightly in [*ET* Jan. 1854 90] and the 'corrected enunciation' of the prize question printed. This was answered by the proposer in [*ET* Aug. 1854 258]. The solution given by Hibernicus was substantial, two columns long, concerned lines of the second order and cited the *Mathematical Companion* and also the work of half a dozen Lancashire geometers (see 3.4.4).

3.2.6 Letters

From [*ET* Oct. 1849 14], there appeared details of which solutions had been received from whom. This was to continue through much of the period under consideration. Frequently, along with these details there would be a personal comment to a contributor. These letters covered a wide variety of issues ranging from stating that a communication had been lost, for example to a detailed discussion of a particular branch of mathematics touched upon in a question. In [*ET* Apr. 1850 156] the following was published 'Garçon, Ecclesall,-too trifling.' The unfortunate Garçon was never heard of again. Occasionally letters were used to point out deficiencies on the part of the contributor and sometimes included corrective reading. Another example is taken from [*ET* Aug. 1851 259] where 'I.J. is recommended to obtain Pott's Euclid, 8vo. edition, and in conjunction with the text to work out the deductions belonging to each book; and, if possible, also to obtain some assistance, if in the first place he meet with any difficulties. T.M. will find a solution of the question in the article by Agrestis; and if he intend to come up to the College examinations, he ought to acquire method and system even in working arithmetic.' The didactic aim of the editors of the mathematical department is again seen clearly in this letter.

In 1857, these letters took on a new dimension, becoming more detailed and the comments more personal; the following from [*ET* Apr. 1860 92] is an extreme example:

> Mr Stephen Watson. Surely you are somewhat hard on us; but we wish our friends to let us have a bit of their minds. When a man is in a passion, the best thing for him to do is to go to the next milestone, or the nearest guide-post, and walk round and round it as quick as possible, at the same time uttering aloud all his complaints, just as if it were quite impossible that any one should hear what is so indignantly and furiously uttered. Sheridan used to say that when a man was in a towering passion, he should sit down and write a letter full of the most angry and insulting expressions; and when this was done, he recommended that a comfortable sleep should be had recourse to before the epistle was despatched. Of course Mr Sheridan knew the missive would be sent into the fire. You say that the Editor of the *Lady's Diary* conducts his editorial labours much better than we do; and that therefore many Diarians complain of us. We have plenty to say in defence. We could edit the Mathematical portion of this most excellent annual with the utmost ease. The Editor has only once in the year to make up his account; and it is surely very easy to select the best solutions, whilst the rest is merely noticed with politeness: now we have to repeat the same operation twelve times a year, and that, too, with great hindrance from want of space. You

> complain that more than one solution of the same question is frequently given: we hold that it is highly instructive to see the same conclusion arrived at by different processes. You allege against us that all our questions are not answered, and that some are even *unanswerable*: we do not believe that the facts of the case sustain this opinion. You ridicule the First Book of Euclid *mania*: we reply, it is not a *mania* at all, it is purely an accident, which can easily be avoided. In Cambridge nothing is more common than to limit the nature of the solution; but we quite agree with you that the farmer, who takes it into his head to wheel home his harvest in a wheel-barrow, must be regarded as a fool for his pains. We, however, thank you for your remarks, and admire very highly your mathematical investigations; and we humbly hope that you will favour us with your lucubrations from time to time. We will try to behave better for the future that is, if we can.

To publish such a letter shows a want of feeling, and will be referred to when attempting to identify the mathematical editors. In [*ET* Jul. 1860 165], the editors sought to ameliorate the situation by printing the following, ' We did not say you were in a 'towering passion.' We merely made a hypothesis, and ventured a wise course of action in relation to the hypothesis, and not to you. Again, we say, we admire your mathematical skill exceedingly.' Whilst the personal comments in the April 1860 letter are distasteful, the policy statements contained illuminate the number of solutions per question, and also the subject matter (First Book of Euclid mania). It is also interesting to note the comments regarding 'unanswerable questions'. This implies an expectation on behalf of Watson that all questions should be both answerable, and answered in the *ET*. The mathematical editor appears to wish to be seen to concur with this view.

3.2.7 Other items

There was a period from [*ET* Dec. 1850 60] to [*ET* Oct. 1851 14] when extracts of the journal the *Liverpool Apollonius* was reprinted. This was prefaced by the note: 'Having been requested by some of the first Geometricians of the day, to publish a selection from the Geometrical Solutions by the late Mr J.H. Swale of Liverpool, it

has been thought desirable also to give, in successive Numbers of this Paper, reprints of the principal part of the Problems, &c. published in the *Liverpool Apollonius*, which was edited by Mr Swale, and has now become exceedingly scarce.-Ed.'
This publication occupied quite a large proportion of the space allocated to the Mathematical department. A policy regarding rare problems was given explicitly in [*ET* Jan. 1851 85] which stated that the object was to supply new questions in mathematics, and also to reproduce old ones which were becoming scarce. In [*ET* Aug. 1851 259], it was asserted that 'The Swale papers and the *Apollonius* will not be published, except in the *Educational Times*.'

Disputes broke out between mathematical correspondents and these were alluded to and sometimes published in this section of the *ET*. Claims over priority could continue for quite some time, with long, detailed, heated letters from both parties. On occasion, the editor would favour one correspondent, other times the debate would be drawn to a close.

Mathematical articles were also published in the mathematical department itself, or near it. Sometimes, an article about mathematics would appear elsewhere in the journal as a general article. For example, in [*ET* Jun. 1850 198-201], a general article was published about mathematics as a branch of general education. Specifically mathematical examples of articles are: 'On Sundry Spirals' by S.M. Drach [*ET* Dec. 1853 64-5], 'Spherical Geometry' by the Rev. T. Gaskin [*ET* Apr. 1854 166-7] and an article on 'Double Algebra' by M.C. Collins, Elizabeth College, Guernsey [*ET* Sep. 1854 273-4].

Last, the 'Answers To Correspondents' section contained practical information concerning printing. Details were given regarding circulation, the preparation of diagrams and the preferred date for sending in mathematical correspondence. In [*ET* Sep. 1852 282], an increase in circulation was reported thus:-

> D.D. (Oxford).-The great increase in our current circulation during the progress of the current volume, has encouraged us to make extraordinary exertions for the improvement of the next. Of the October number, the commencement of the

> new volume, it will be necessary to print a very large impression, to meet the demand we have ascertained will be made for it. Not only is it rapidly gaining ground in the Universities, and getting recognised in our scholastic establishments, from Colleges to Sunday-schools, as the proper and efficient organ of education, but it is winning the favour of the Clergy, for the earnestness of its exertions in the advocacy of religious instruction as the foundation of all knowledge. Numerous friends of education among the laity have expressed themselves equally gratified with the manner in which both science and learning have been laid before the general reader; and from every quarter we have met with hearty encouragement and equally hearty commendation. The vast improvement which the next volume will display, will prove, we hope, that we are not unworthy of the patronage we have received.

It has not been possible to trace the actual print runs made and thus check whether the increase in circulation actually took place, and if so, its size. The perceived extent of the journal's influence and acceptance given above and its purported success in its treatment of science and learning are extremely noteworthy in terms of its overall aims. In [*ET* Aug. 1856 188] appeared 'It is not improbable that we may soon have an eight-page wrapper, especially if our circulation continue to increase so rapidly.' In [*ET* Aug. 1851 259], a request was made for diagrams to be made more distinct and in [*ET* Nov. 1852 39] that mathematical correspondence should be sent in the first week of the month.

To summarise editorial policy, the didactic aim of the editors of the mathematical department is very evident, with references given to suitable books and even individual guidance offered to contributors regarding their mathematical method. The instigation of the junior section was a natural extension of this policy, with questions specifically provided for junior members to solve. The original suggestion for starting the mathematical department contained a recommendation for original geometric, algebraic and arithmetic questions. As these subjects were the ones highlighted in the *ET* (see 2.4.1), it is not surprising to see Wharton involved in posing early questions, as he was so prominent in discussing mathematics in of general articles in the *ET*. In the first few years, the mathematical editors tried to indicate the required level for the questions sent in by contributors, rejecting some questions as too easy and stating that others were too hard, indicating perhaps that they had in mind a certain level for a specific target audience.

In contrast to this strong emphasis on teaching, a lone opinion was expressed that original problems, perhaps unsolvable, should be introduced into this mathematical department. This suggestion received an initial snub, but was then accepted in theory. In response to a contributor in 1860, it was revealed that the general direction of the department still appeared to be to provide solutions for the questions that had been posed, and not leave them unanswered.

3.3 The succession of editors

3.3.1 Identifying the editors

The *ET* was not the work or property of any individual or body. In [*ET* Jun. 1851 206] there appeared '*the Educational Times* does not belong to the College of Preceptors, nor is it in any way under the control of the College, and we shall therefore have no objection to publish, whenever we can find space, a series of the Cambridge Examination Papers.' Further, no nominal details were published whatever concerning the editors of the *ET*. Such anonymity affords no clue as to the general editors of the *ET*, and also veils the identity of the editors of the mathematical department, particularly in the early years. Evidence from the *ET* itself regarding the mathematical editors will now be set out, then supporting material from secondary sources. Biographical details will then be supplied regarding any characters involved with the inception or running of the mathematical department and all the evidence will be analysed against the material in the mathematical department itself.

In Miller's (see 3.5) obituary, it was said that 'He succeeded Mr Warton (sic) as Mathematical Editor in 1861' [*ET* Mar. 1903 138]. When the mathematical reprint was launched in 1864, it had Miller's name on the flyleaf as editor, so it is certain that he was the mathematical editor of the reprint, and consequently of the mathematical department in 1864. Wharton died on 25 March 1862 after a long illness and his obituary appeared in [*ET* Apr. 1862 21]. Wharton corresponded with Miller and his letter of 12 March 1860 (MC) said 'My dear sir, I received your corrections for my examples in Algebra; accept my most grateful thanks to you through Mr Hopkirk, but I think they are also especially due to you from myself and that I might take the liberty of offering them to you on being a most valuable correspondent to the Educational Times for ten years during which I was the mathematical editor.' He wrote to Miller again on 24 January 1861 (MC) and commented that 'I thought you had been aware that Dr Wilson had been Mathematical Editor of the E.T. for a short period about four years or less and that in consequence my asking him to take it during my absence from London for a few months.' His letter to Miller of 20 May 1861 (MC) stated that 'that department is again placed in my hands', referring to the mathematical department of the *ET*.

The obituary announcement for Wharton said nothing about his mathematical editorship. In [*ET* May 1862 43-4] came the following under the heading: 'Notices To Mathematical Correspondents';

> We shall be glad to receive solutions of the following questions...The solutions already sent have been unfortunately lost in consequence of the sudden death of a gentleman connected with this Journal, to whom they had been entrusted for revision for the press. We shall therefore feel obliged if our correspondents will send us fresh copies of their solutions of these questions...
>
> We have made special arrangements for extending the space devoted to Mathematics, and we hope by increased attention to the selection of elegant and original solutions, and by a careful revision of the press, to render this department of the journal still more worthy of the attention of the many distinguished mathematicians, both at home and abroad, who contribute to our columns.

The implication from this notice is that Wharton was the gentleman who had suddenly died and that he had been mathematical editor right up until his death. In that case, he would have carried on as mathematical editor from May 1861, as stated in his letter to

Miller, until his death in March 1862. The new mathematical editor of May 1862 who took over after Wharton's death, undoubtedly Miller, expressed his long-held desire to see original problems posed and solved in the *ET*. He also made his target audience very clear - distinguished mathematicians. His aims were certainly not purely didactic. His development of the mathematical department will be studied in 3.5.1 and also 8.1 and 8.3.

There is another source of evidence which corroborates Wharton's editorship. Davies' letters to Wilkinson are discussed fully in 6.2.4, but some will be discussed now as they touch upon Wharton. In his letter of 1 October 1850 (DC), he said,

> I don't know how far the power of Wharton at the "Ed^l Times" is absolute, but the two last nos. (which he sent me) inspire me with a favourable opinion of the work. I lent him the *Apollonius*, and he promised to give the "inscribed polygon" problem from it this month - which you see he has not done. As he has now some of the MSS. I think this had better not be done but those entered upon at once. I will tell him so when he calls, which I expect he will tomorrow. He is now only 2 miles off, at Blackheath.

He continued upon this theme in his letter of 12 October 1850 (DC),

> Wharton was here last night. We talked over several matters - amongst them the publication of Swale's papers. He proposes to give a portion of the papers on tangencies from the *Apollonius* next month in the Ed^l Times and to follow up with the remainder; and then proceed to the MSS. Your collections from which you will by that time be able to have made and arranged. The plan appears to be a good one. He asked me in the name of the Council of the College of Preceptors to examine for them next Xtmas. My only difficulty is my state of health: but I have consented subject to that proviso.

Hence it would appear that Wharton was editor for 10 years from 1850 to 1860, and that some time prior to 24 January 1861, Wharton handed the editorship back to Wilson, possibly due to the long illness referred to in his obituary. He regained the editorship in 20 May 1861 and he held it then until his death. Presumably Wilson was editor from 1849 to 1853, sharing this duty with Wharton for three years.

Information about the mathematical editorship also came from Thomas Turner Wilkinson, who will be seen in 6.2 to be a major contributor to the mathematical department during the period under consideration. The memorial written by W.A. Abram [1876 77-94] contained Wilkinson's autobiography as passed on to Abram a few months before his death [Abram 1876 81]. In his autobiography, Wilkinson stated that 'When the *Educational Times* was established, I was one of its earliest promoters, and had the principal share in introducing the mathematical department into that journal. This portion was at first edited by Richard Wilson, D.D., author of an excellent treatise on Trigonometry, and it is now (1873) under the care of W.J.C. Miller, B.A., of Huddersfield College.' Wilkinson's assertion concerning his own role in initiating the mathematical department is not confirmed by supporting material from another source, but it is quoted by [Archibald 1929 397].

Archibald also added the statement [Archibald 1929 397]

> As far back as 1858, at least, it would seem as if this editorship had passed into the hands of Wm. John Clarke Miller (1832 - 1903), who, in 1861, conceived the idea of devising some plan whereby the contributions to the mathematical columns might be preserved, apart from other matter, in a more convenient form. After ascertaining the views and desires of contributors and obtaining the necessary promises of support, there was published in July 1864 the first volume of *Mathematical Questions with their solutions. From the 'Educational Times'* (92 pp. 15.2 x 24.5 cm), the contents being mainly a reprint of what had appeared previously, July 1863 to June 1864.

Archibald's dates do not accord with the information contained in Wharton's letters to Miller.

Details in Miller's obituaries and biography regarding his editorship after Wharton are unclear. In Miller's biography [Finkel 1896 158-63], it is stated 'It was in 1861 that he conceived the idea of devising some plan whereby the contributions to the mathematical columns of the *ET*, which had been for some years under his Editorship,....'. Miller's obituary [Anonymous 1903; 525, 606] is even less clear, it said 'Mr Miller received his early education at the West of England College, Taunton, and was afterwards mathematical master there, subsequently he was appointed Professor of Mathematics, and finally Vice Principal of Huddersfield College. At this time he became connected with the *ET*, and was its Mathematical Editor for nearly 40 years.' Miller actually submitted his first contribution to the *ET* in September 1850 when he was a school boy at the Dissenters' College, Taunton (see 6.3). In September 1861 came the first instance of a solution of Miller's bearing his position as mathematical master at Huddersfield. Hence, the statement in the obituary can be seen to be incorrect. Similar inaccuracies can be found in the obituary to Miller in the *ET* [Mar. 1903 138]. It would seem that Miller actually held the mathematical editorship from May 1862 to 1897, a period of 35 years, when he resigned due to ill-health [Anonymous 1903 525, 606]. Hence data is entered into the database from August 1849 to April 1862.

3.3.2 Summary of the editors

Biographical material about Wilson, Miller and Wharton will now be set out to help identify the editors. Wilson was referred to in his obituary [*ET* Nov. 1879 313-4] as having 'kindly and courteous manners' and being of 'upright and benevolent character'. Wharton's personal characteristics were not mentioned in his obituary in [*ET* Apr. 1862 21] which read as follows

> 'We regret to announce the death of Mr James Wharton, M.A. of St. John's College, Cambridge, who departed this life on the 25th March after a long and painful illness. Mr Wharton was for many years a member of the Council and of the Board of Examiners of the College of Preceptors, which owes to him a deep debt of gratitude for his exertions in establishing and promoting it. His sufferings before his death were very great, but they were borne with Christian fortitude and resignation.'

There is no mention here of Wharton's personality, apart from his stalwart bearing of his suffering.

Turning now to Miller, in the *Richmond and Twickenham Times* for 17 August 1889, there was reference to 'the pleasant, gracefully worded and erudite little speeches of Mr W.J.C. Miller' and also 'remarks, which are always appropriate, often profound, invariably couched in the happiest words.' These citations were related to Miller's contributions to a literary society, the Richmond Athenaeum, which he founded in 1887 [Finkel 1896 161].

The factual, polite letter in [*ET* Aug. 1851 259] would seem consistent with Wilson's character. The letter to Watson in [*ET* Apr. 1860 92] was presumably by Wharton. After Wharton's death, the personal letters from the mathematical editors to the correspondents disappeared completely from the end of the mathematical department and did not appear at all during Miller's editorship.

From the available evidence, the following assumptions can be drawn concerning the mathematical editors:

- 1849 Wilson editor, perhaps helped by Wilkinson,
- In 1850s, Wharton was an editor, perhaps with Wilson,
- From the style of the letters to correspondents, Wharton would appear to be the sole editor in 1857,
- Around January 1861 Wharton asked Wilson to take over the editorship until May 1861,
- Wharton regains editorship until April 1862,
- April 1862 Wharton died and Miller took over from May 1862 until his retirement in 1897.

It is worth noting here that an overall editor of the *ET* was identified in Stephen Watson's letter of 17 May 1861 to Miller (MC). Watson said 'I hope change in the editorship of the E.T. will be an improvement, it can hardly be a deterioration. I hope Mr Isbister will try to bring the mathematical part, at least, into a satisfactory and systematic order.' There are also letters from Isbister to Miller (MC) in which the running of the mathematical department is touched upon.

Before leaving Wilson and Wharton, Davies' opinion of them will be given as stated in his letter of 13 October 1849 (DC),

> My own view of the College of Preceptors is unchanged, and you know what they were before it was charted. You will feel what mopes (?) it is composed of presently. The Turrells, and the Parkers and the Whartons and the Wilsons are still much in it, and indeed, they are likely to do so for years to come. It is indeed possible that more able men might step in - and more unselfish men too. In that case it might become a useful institution though it seldom happens that a body is reformed from within, and especially a chartered body, which like the wind has been described as a chartered libertine.

Wilkinson and Miller will now be discussed in detail.

3.4 Thomas Turner Wilkinson

3.4.1 Biography

A long memorial of Wilkinson was compiled by William Alexander Abram [1876 77-94] and appeared in the *Transactions of the Historical Society of Lancashire and Cheshire*. Wilkinson wrote his autobiography a few months before his death on 6 February 1875 and this was included in Abram's memorial. An obituary for 'The Late Alderman Wilkinson F.R.A.S.' appeared in *the Preston Guardian* on 10 February 1875. In December 1914, there was a short biography of 'The Late Alderman Wilkinson F.R.A.S.' under the title 'Lancashire Workers of the Past' [Anonymous 1914-5]. He clearly had made an impact outside the sphere of mathematics.

Thomas Turner Wilkinson was born on the 17 March 1815 at Abbot House, Mellor, near Blackburn. His name is a concatenation of his parents' - Mary Turner, and William Wilkinson. His mother died when he was nearly two and his father cared little for his two sons or their education. After the death of his mother, he and his brother remained with his grandparents, under the special care of his aunt Ann. When his father remarried, Wilkinson returned to his father's house, but his brother was adopted by his aunt.

He received some early education from the age of seven to 12 at his cousin's school, where his ability in reading and arithmetic were noted. He returned to his father's farm and neglected his studies for three or four years. He was once visited by his brother who amazed him with his ability to perform difficult arithmetical calculations. This goaded Wilkinson into action and he determined to surpass his brother in this area. He was encouraged to progress with his mathematics by his uncle, John Wilkinson, who possessed a good library of books. He was freely able to avail himself of these, and thus his general knowledge improved dramatically. He obtained Pinnock's *Catechism of Algebra*, and discovered he could understand it without a teacher. He then mastered Bonnycastle's *Algebra*, then his *Mensuration*. Next, his uncle obtained Malton's *Royal Road to Geometry* for him, and this became an established favourite.

Around 1830 he became involved in the instigation of a Mutual Improvement Society in the village of Mellor. His autobiography does not specifically mention his connection with the Burnley Mechanics' Institute, but both the memorials dwell on his long involvement (over 30 years) with it. He held several different offices within this institution, including teacher of book-keeping and the higher branches of mathematics, chairman of the directors, auditor, secretary and vice-chairman of the directors. Honorary membership was conferred upon him in January 1858. His contributions to the society were not only mathematical, he also taught geography, geology, history, local history, literature, folklore and astronomy.

He formed acquaintances with local groups of mathematicians and attended their occasional meetings. Here he met George Aspen who gave him private tuition in trigonometry. Another member described the mechanics needed for an orrery and this kindled Wilkinson's interest in astronomy, which was later to be fuelled by the appearance of Halley's comet. He obtained further tuition in general matters from Rev. William Hartley of Balderstone. He then procured a book on hydrostatics plus several treatises of the Society for the Diffusion of Useful Knowledge.

Wilkinson appeared set to apply his knowledge to a trade when he entered the counting-house of a cotton spinners, W. & J. Statter's at Mellor Brook, as a book-keeper. Here he was introduced to cotton spinning calculations, and he was soon called upon by other firms in time of need. However, the firm became bankrupt, and this particular avenue of opportunity folded. His career then commenced as a teacher, his first job being at Burnley Grammar school. He was to work here for two short stretches, followed by a long term of office of 30 years as assistant (or second) master. Interspersed with his teaching at Burnley were three occasions when he commenced a school, the National School at Crawshawbooth, the Parish Schools at Habergham Eaves and the private adventure school - Mount Pleasant School. Whilst under his direction, the Parish School was the first in the district to be inspected by the government. He also had a pawnbroking business in Cheapside, Burnley.

Wilkinson was well-versed in the classics, wrote poetry, hymns and important books on local geology and local history. His books on Burnley Parish Church and Burnley Grammar School are still well regarded. He also wrote on Lancashire folklore and ballads. He was a member of the following societies, to all of which he contributed articles regularly and prolifically, the Historic Society of Lancashire and Cheshire which met at Liverpool, the Chetham society, the Literary and Philosophical Society, the Geological Society, and the Literary Club which met at Manchester. He was the vice-president of the Burnley Literary and Scientific Club. The Burnley Literary and Philosophical Society was inaugurated in 1861 and the educational pioneer Sir James Kay-Shuttleworth (see 2.1) became president and Wilkinson vice-president. Wilkinson held many positions in local office, member of the Corporation of Burnley, councillor, chairman of the finance committee, alderman and Overseer of Habergham Eaves.

Wilkinson was married twice, firstly to Agnes Ward, of Preston in 1837, and then to Angelina Harrison of Burnley. No mention is given by him as to why the first marriage was terminated. He does give details of his second wife's lineage, stating a connection with the first Lord Ribblesdale [Abram 1876 92]. He had five children, two of whom died, leaving one son and two daughters. He was devoted to his second wife and never recovered following her sudden death on 13 April 1874. He was smitten with a serious illness soon after, and died nine months later on 6 February 1875.

3.4.2 Mathematical periodicals

Wilkinson was very involved with the local mathematical community, but the discovery of mathematical periodicals allowed him access to a much wider mathematical sphere. He purchased his first one, the *Lady's Diary*, (see 1.5.2) in 1835, and '..the contents so interested me that I began to collect all the English periodicals which contained mathematics' [Abram 1876 88]. He sent his first contribution to the *Lady's Diary* in 1837, and was awarded the mathematical prize in 1852. He contributed many solutions and mathematical essays to that journal (editor W.S.B. Woolhouse). Around the late 1830s, he discovered the *York Courant* (editors Thomas Tate, then William Tomlinson), and he and several of his pupils contributed to its mathematical and philosophical section. He also contributed to the *Northumbrian Mirror* and the *Preston Chronicle* (editor Septimus Tebay).

Regarding his collection of mathematical periodicals, Wilkinson claimed to have the 'most extensive series of these works out of London' [Abram 1876 88]. He stated:

> I made these an object of special study, and at the suggestion of Professor Davies I commenced a series of articles on our English Mathematical Periodicals in the *Mechanics' Magazine* during 1848, which extended through various volumes of that work up to 1854. I think about 28 of these works are there summarized and described. I afterwards followed them up to another series, entitled *Notae Mathematicae*, which extended to about twelve papers.

There were actually 29 articles and they form a substantial corpus of material (see 1.5.3). His treatment of the subject was thorough, meticulous and comprehensive. He also contributed many other short mathematical essays to this journal. These papers were much valued, and led to his being elected a Fellow of the Royal Astronomical Society, in December, 1850. His interest in mathematical periodicals, their beginnings and history, was to be one of Wilkinson's hallmarks.

His encounters with mathematical periodicals brought him into contact with a wider mathematical audience; but they also were the means of introducing him to distinguished mathematicians. He acquainted himself with T. S. Davies (see 1.4.3), one of the editors of the *Mathematician*, which was set up at Woolwich between 1845 and 1850. Wilkinson and Davies maintained considerable correspondence, now held at Chetham's library, and the tone of the letters suggests they became firm friends. Davies clearly had a high opinion of Wilkinson, on the subject of the cultivation of geometry in Lancashire, he stated that

> If our Queries on this subject be productive of no other result than that of eliciting the able and judicious analysis subsequently given by Mr Wilkinson ..., they will have been of no ordinary utility....Mr Wilkinson has shown himself to possess so many of the qualities essential to the historian of mathematical science, that we trust he will continue his valuable mathematical researches in this direction still further. It cannot be doubted that Mr Wilkinson has traced with singular acumen the manner in which the spirit of geometrical research was diffused amongst the operative classes, and the classes immediately above them - the excisemen and the country schoolmaster [Pen-and-Ink 1850 436-8].

Wilkinson wrote a long obituary of Professor Davies in the [*ET* Mar. 1851 125-7] in which he said that 'Pen-and-Ink' was a pseudonym regularly used by Davies.

3.4.3 Wilkinson and the *ET*

Following his links with the *Mathematician*, Wilkinson claimed to have been involved with the *ET* from its inception. His assertions were mentioned in 3.3.1, but a fuller citation from his autobiography will now be given [Abram 1876 87],

> When the *ET* was established, I was one of its earliest promoters, and had the principle share in introducing the mathematical department into that journal. This portion was at first edited by Richard Wilson, D.D., author of an excellent treatise on Trigonometry, and it is now (1873) under the care of W.J.C. Miller, B.A., of Huddersfield college. This work contains very many mathematical questions and solutions, which I have contributed at different times. I also gained the two mathematical prizes which were offered by the proprietors for the best solutions to certain questions. Already more than 4000 questions have

> been proposed and answered in this work. A lengthy paper on Porisms, from me, will be found in this journal.

Wilkinson also became involved with a Blackburn group of mathematicians who formed a bi-yearly journal, the *Student's Companion* which started in 1822 but only lasted two issues. Wilkinson described this group in great detail, commenting upon the occupations and mathematical abilities of each member. Many of the above group contributed to the *Lady's Diary*. Many were schoolmasters, others had occupations such as weaver, blacksmith, land surveyor, cotton manufacturer and farmer. He also mentioned the pseudonyms used by these people, a very pertinent point as these were in such common use in the time period under consideration here. One later member of the group was John Garstang, erstwhile assistant master of Blackburn Grammar School, teacher of the Rev. Robert Harley and contributor to the *ET*. Wilkinson deemed Garstang to be a good mathematician who late became an actuary for the Blackburn Savings' Bank.

Wilkinson also mentioned Septimus Tebay (1820-1897) who occasionally frequented this group. 'Septimus Tebay, B.A., then of Preston, but latterly head master of Rivington School, occasionally joined us at Blackburn, when we met at Mr Garstang's; he is one of the ablest of our Lancashire mathematicians, and has gained many prizes in the *Diary*. He is also the author of an excellent treatise on Mensuration' [Abram 1876 88]. Tebay entered St John's College Cambridge in 1852 and was 27th wrangler in 1856. He had been a labourer in Preston gas-works who had taught himself mathematics and then been sent to Cambridge by a gentleman in Preston. After retiring from Rivington, he ran a public house in Bolton [Venn 1940-54 134]. Tebay also contributed to the *ET* and there was a dispute between Tebay and Wilkinson conducted on the pages of the *ET* and alluded to by [Brierley 1878 29] as follows,

> At pages...........there is a painful correspondence between him and Mr Septimus Tebay, of Rivington, in which Mr Tebay (truthfully to my own knowledge) accuses him of exchanging solutions of mathematical problems with other correspondents in order to increase the number of each, a kind of plagiarism

> much too common amongst the last generation of mathematicians. In one of his letters, Mr Tebay well observes, 'The system of exchange I believe to be practised to a considerable extent among the present degenerate race of Lancashire mathematicians of the glory of which so much has been said. Such is the general wreck, that with one or two exceptions there does not remain a single spark of that sterling genius which characterises the labours of Butterworth, Smith and Wolfenden.' There is an article on porisms in one of the numbers of the *Educational Times* by Mr Wilkinson, almost entirely the work of another man, who in turn borrowed most of his work from the papers of Mr Butterworth.

This jaundiced view of Wilkinson is not echoed in any other biographical material. Indeed, the memorial in the *Guardian* states that 'It is often recorded of many worthy and eminent men that they had bitter enemies, but in the case of Alderman Wilkinson, he seemed to be held in universal respect, and his death seems to arouse a feeling of reverence and esteem towards his name by those who were acquainted with him which is only seldom witnessed.' It also contained a reference to Wilkinson, 'who, in a quiet and unostentatious way, did great service to the generation by his zealous labour in the field of literature, and who served the town with equal zeal in the capacity of an instructor of its youth and of a representative man.' Abram talked of himself as a junior historical researcher, and referred to Wilkinson as a kind friend and mentor, willing to share his expertise in that field with any who were interested. It would appear, then, that this scorn felt by Brierley towards Wilkinson was an isolated example of bad feeling towards Wilkinson.

3.4.4 The Lancashire Geometers

In the above quotation from Brierley [1878 29] was mentioned 'The present degenerate race of Lancashire mathematicians, of the glory of which so much has been said.' This referred to another of Wilkinson's strong mathematical interests besides the periodicals; his fascination with 'the Lancashire Geometers'. Wilkinson gave a paper on 'The Lancashire Geometers and their Writings' to the Literary and Philosophical Society of Manchester on 28 December 1852. This appeared in their transactions in 1854 and was a substantial 34 pages long [Wilkinson 1854 123-57]. His article addressed the challenge put forward at the York meeting for the British Association for the Advancement of Science in 1831 by Mr Harvey who had said,

> There had long existed a devoted band of men in the North of England, resolutely bound to the pure and ancient forms of geometry, who in the midst of the tumults of steam-engines, cultivated it with unyielding ardour, preserving the sacred fire under circumstances which would seem from their nature most calculated to extinguish it. In many modern publications, and occasionally in the Senate House Problems proposed to the candidates for honours at Cambridge, questions are to be met with derived from this humble but honourable source. The true cause of this remarkable phenomenon I have not been able clearly to trace.

Wilkinson traced the mathematical lives of many Lancashire mathematicians and highlighted the fact that some of the Lancashire Geometers were hand-loom weavers, who suspended diagrams from their looms and studied them as they span [Wilkinson 1854 130].

3.5 William John Clarke Miller

3.5.1 Biography

William John Clarke Miller, mathematical teacher and editor, medical administrator and mathematician, was born on 31 August 1832 at Beer, South Devon. During his childhood he developed a love of books and of nature and was educated at the village school. Miller's parents were Dissenters and would not allow him to be educated in a conformist establishment, thus he entered the newly-created West of England Dissenters' Proprietary School, Taunton, in August 1847, aged 15. Here, mathematics and classics were held in equal esteem and Miller became proficient in both. However, mathematics became his passion and he was to be remembered years later by a fellow pupil with 'book in hand, probably Euclid's *Elements*' [Record 1948 39].

In the early 1850s, a group of several students and three school masters (one being Robert Harley) from the above Dissenters' school, started sending questions and solutions to the mathematical department of the *ET* (see 6.3). Miller's first solution to a mathematical problem appeared in [*ET* Sep. 1850 277]. He became a mathematical master at this school which had fortuitously formed a connection with the University of London in 1848 to enable it to award University degrees. He graduated with mathematical honours in 1854. He then taught at several institutions before eventually becoming vice principal at Huddersfield College in Yorkshire and bearing the title professor. There was a lull in his contributions to the *ET* in the early 1850s, but he renewed them in 1855. Miller's wish to study at Cambridge, had been thwarted because of his parents' religious stance. Thus his only access to high-level mathematics was vicariously through the problem pages of the *ET*.

In 1865 he applied for the post of mathematical professor at Owen's College, Manchester, the precursor to Manchester University. On 31 May 1865 he wrote to Cayley (MC) asking him to be his referee for the post. Cayley had already nominated Harley and thus was unable to oblige, but he did express his high opinion of Miller's mathematical abilities. Miller did not obtain the post and never broke into the higher echelons of mathematical tutors.

All his mathematical editorship was a hobby, by profession he was also General Secretary and Registrar of the General Medical Council from 1876 to 1897, a post in which his administrative abilities were greatly appreciated. The British Medical Council gave him a gratuity of £1000 in appreciation of his services, when ill-health forced him to resign, and his obituary appeared in the British Medical Journal [Anonymous 1903 525].

In 1896 he was struck with aphasia, due to pressure of work. He suffered from this until his death on 11 February 1903 from influenza and bronchitis.

3.5.2 The Miller correspondence

Miller kept up an enormous correspondence which was mainly connected with his editorship of the *ET*. This correspondence has been located recently in the David Eugene Smith collection at Columbia University [Grattan-Guinness 1994c 204-5].

This entire collection contains a wealth of fascinating material, much of it previously unknown. It includes material from approximately 240 correspondents, including Wilkinson, Watson, Rutherford, Cayley and Clifford. Many other well-known mathematicians are also included in the list. Much of the correspondence took place outside the time period covered by this thesis. From this correspondence it can be seen that Miller clearly played a pivotal role in the mathematical networks of his day. However, as he did not rise to public prominence, it would appear none of his many contributions to late nineteenth century mathematics has been recognised or remembered.

David Eugene Smith purchased Miller's editorial material and Miller discussed his terms for selling his collection to Smith in a letter dated 11 October 1894, (MC). In this letter Miller also revealed that he had a unique collection of photographs of mathematicians that he had built up over 30 years. Many of these are familiar now and some have been appearing recently on the inside cover of the back page of the London Mathematical Society Newsletter, citing the David Eugene Smith collection as their source of origin. It would appear to be unknown that Miller was the initial owner of these photographs.

3.6 Summary

The general mathematical and scientific aims described in the *ET* were realised in part by the inception of a mathematical department in November 1848. The fact that it remained only a department in an educational journal until its separate reprint in 1864 means that it can't be treated as a stand-alone mathematical journal. This part of the journal soon possessed an established format which was to remain effectively unchanged for many years.

The mathematical editors from the beginning of the department to April 1862 had a very clear rationale. They wanted it to contain solutions demonstrating sound mathematical method to aid students and teachers alike. Sometimes several different methods of solution were given for a single question, in order to demonstrate their different merits. The didactic rationale found a particular focus in the junior section, which contained questions for school pupils to solve and ran from April 1851 until December 1854. The desire to use the mathematical department for didactic purposes was an extension of the general rationale regarding mathematics in the *ET*, propounded by Wharton and Wilson, as seen in 2.4.1. Wharton's didactic aims were linked to Whewell's educational tenets regarding teaching at Cambridge (see 1.4.1). Hence the influences on the mathematical department in the early years came from influential members of the College of Preceptors, and also an important member of Cambridge University. Wilson shared the same desire to use the mathematical department to encourage good practice in teaching. Several of the French mathematical journals in 1.2.3, fig. 1.2 were oriented towards students but there is no evidence that these influenced Wilson and Wharton.

Miller wrote to the mathematical editors in 1857 evincing a desire to see original questions set, with the proviso that the solutions would not always necessarily appear. He was immediately and sharply rebuffed, but not long afterwards his ideas were accepted in principle by the editors of the mathematical department. He took over as mathematical editor in May 1862, and immediately announced his aims to attract contributions from 'distinguished mathematicians, both at home and abroad.' He went on to introduce a separate reprint of the mathematical department which fulfilled these aims (see 8.1.1).

Wilkinson claimed to be involved in introducing the mathematical department to the *ET* but this is not verified by any evidence from another source. Wilson posed six questions which were solved by Wilkinson, four in 1854, two concerning astronomy and two in 1860 concerning problems from the first book of Euclid. In [*ET* Sep. 1854 267], Wharton's books on arithmetic, algebra and trigonometry were advertised and recommendations were printed underneath each advertisement. In particular, underneath the advertisement for trigonometry ran 'This work is the most suitable and the best work I have seen for schools' *T.T. Wilkinson, Esq., F.R.A.S., Dr Wilson, London &c. &c.*' Hence there may have been collaboration between Wilson and Wilkinson. Wilkinson corresponded with Miller from August 1865 to September

1874, but nowhere did he mention his early involvement with the mathematical department. Wilkinson is thus categorized as an important supporter of the department.

The general mathematical aims of the *ET*, then the creation of the mathematical department and the rationale behind it have been summarised. How the data from the mathematical department was incorporated into a database will now be examined.

4 BUILDING THE *ET* DATABASE

4.0 Chapter overview

Chapter four charts the search for a suitable database for the data in the mathematical department. The use of databases in the field of the history of mathematics is surveyed, with reference to the British Society for the History of Mathematics (BSHM) and this is then related to the history of this project. Database use in the field of history is then briefly discussed, with reference to the Association for History and Computing (AHC). Two suitable methodologies are then set out, the relational and the source-oriented. The source-oriented method was chosen for this project and is delineated in detail.

4.1 Databases related to the history of mathematics

When this project began in October 1992, there was very little experience of databases within the field of the history of mathematics. Research in this field is supported by the BSHM, an international society, which is very active in disseminating information concerning research at all levels. Few databases containing material concerning the history of mathematics were known to the BSHM at that time. There was a database containing data about potential theory, constructed at Melbourne University. Here, information concerning potential theory was taken from a card index system to a simple database comprising three cross-referenced stacks in Hypercard$_{TM}$, (an Apple Mackintosh$_{TM}$ application). A simple (flatfile) database of women mathematicians was created at the Open University using the personal computer (PC) application FoxmakerPro$_{TM}$. These flatfile databases were the only known examples at the start of this project. Later on in the project life cycle, studies on French journals were reported in *Science et Techniques en Perspective Regards sur la Science: Le journal scientifique* [1994]. Dody [1994 24-179] discussed the '*Correspondance sur l'Ecole Polytechnique*', and Duvina [1994 179-218] '*Le journal de Mathématiques pures et appliquées sous la férule de J. Liouville*' both presenting results in the forms of charts and tables, with no mention of database analysis, implying the use of a spreadsheet. Experienced colleagues from computer science strongly recommended that neither a spreadsheet nor a flatfile database was suitable for the *ET* data, and a relational system was suggested.

A distinction needs to be drawn between a particular methodology and a computer application needed to implement this methodology. The relational methodology will be summarised below and it is certainly suitable for this project. (The relational methodology is not the only alternative. The object-oriented database methodology could also be considered [Date 1994 630-704 and Harvey and Press 1996 203-7].) At the start of this project, the computer application (software) that was actually available to implement the relational methodology was Borland's dBaseIV$_{TM}$ for DOS, a particularly difficult ('user-unfriendly') package for an inexperienced user. Harvey and Press [1996 36] comment that 'dBaseII's 'dot prompt' - a blank screen with a single full-stop to 'prompt' the user - was justly notorious'. The same blank screen was present in dBaseIV$_{TM}$ and this application was found to be problematic. It is always necessary to design a database then translate this into a database model. With dBaseIV$_{TM}$, it was not easy to change a database once created. Advances in the development of database software have brought about applications that are much more flexible in this respect; for example, Microsoft Access$_{TM}$, where changes to database design are easy to implement. The latter was just available at the beginning of the project but it was too expensive and also required more memory than was present on the computing equipment available. It was then that contact was made with members of the Association for History and Computing, also an international society, who had much wider experience of the use of computing in history, particularly databases in history.

The AHC 'has acted as a forum for the exchange of ideas and information about courseware and research tools such as databases and bibliographies' [Speck 1994 28]. Much information about all aspects of history and computing have been disseminated in *History and Computing I, II and III*, three books containing contributions from AHC annual conferences. The AHC has its own international journal, *History and Computing* (1989-) which includes papers on current research in history and computing.

The seminal work on databases in the field of history is *Databases in Historical Research* by Harvey and Press [1996]. Database theory is expounded in detail and case studies are set out which exemplify methodological points. A range of methodologies is covered and accompanying computer software is discussed. A typical example of a historical database is one containing data from a census [Mawdsley and Munck 1993 47-81] and [Harvey and Press 1996 98-102]. There are several other books to guide the historian as to which computer system to use. Mawdsley and Munck [1993] give descriptions of the creation of both simple and relatively complex historical databases, the latter focussing on a database for data concerning the French Parliamentary Convention of 1792-5. Greenstein [1993] covers similar database areas and Burnard [1989] gives a description of (database) relational theory .

A large project on the Samuel Hartlib papers resulted in the creation of a mammoth database stored on CD-ROM [*The Hartlib Papers on CD-ROM* 1995]. The database comprised scanned copies of the original papers plus a full transcription thereof. A database management system (DBMS) was also housed on the CD-ROM to facilitate full analysis of both types of material, original and transcribed.

Resulting from the databases created, there is now an appreciable quantity of datasets (particular sets of data stored in a particular DBMS or other format). These are available to anyone for research or educational purposes from the History Data Unit at Essex University, and have been compiled into a list [Schürer, Anderson and Duncan 1992].

The Internet attracts interest from ever-enlarging sections of society, particularly within the academic and business sections. The quantity of historical information available via this facility is growing [Southall 1993 110-20] and [Gibson 1995 81-9]. An example of this is the 1801 census of Norway which can be accessed and analysed via the Internet, using powerful database software CensSys reported at the international conference of the AHC in Moscow, 1996 [Kolle, Oldervill, Solli 1996 118].

Discussion with members of the AHC led to the choice of a source-oriented DBMS for this project, and in particular, the use of the software κλειω. The source-oriented methodology was suitable, as will be argued below. κλειω was available, affordable and flexible and could be used on the computing equipment in use at the time. A large project at the Open University on British Mathematics, 1860-1940 and sponsored by the Leverhulme Trust also used the source-oriented methodology and κλειω software. The Britmath database thus formed was reported in [Barrow-Green 1996 137-47 and Burt and James 160-9].

The wealth of expertise in the AHC concerning databases contrasted sharply with that in the BSHM, so a conference on 'Databases in the History of Mathematics, Science and Technology' was held in October 1994 in order to inform those in the mathematical sciences of current advances in database methodology and practice. A variety of database projects was set out and the relational and source-oriented methodologies discussed.

The decisions and influences leading to the selection of a database for the *ET* material will now be expanded.

4.2 The selection of a Database Management System (DBMS)

Any methodology must provide a suitable theoretical framework for storing the data from the mathematical department, then analysing the data in order to answer the appropriate questions posed in chapter 0. Once a theoretical model has been chosen, before a physical database can be created, appropriate computer software must be selected to perform this task. Such software will allow the creation of a structured pool of data. If the database is to be shared between several users, the software will allow multiple access whilst only allowing editing of the material by, for example, a database administrator. The Database Management System (DBMS) will also control query processing and security and integrity constraints.

Traditional relational databases allow the imposition of constraints on the data entered, so that, say, sections of this data may consist of a 3 digit number only, whilst the data in another section may contain up to 20 alphabetic characters. Such data would then be termed 'structured'. Such a system can be awkward for those with unstructured data. Another problem can emerge if data arises infrequently, as care must be taken to ensure the database does not waste computer facilities by allocating space to a section of data which will be mostly blank. Harvey and Press state that '..it remains generally true to say that the essential characteristic of RDBMSs is that they handle structured data. They do not offer an ideal solution for projects which involve loosely structured data or free text. In such cases, text or document management systems may be more appropriate to the historian's needs.' [1996 55]. For the historian with large amounts of unstructured data, a textual information management system (TIMS) would be suitable. TIMSs have database management features in common with DBMSs, for example data integrity and security. In contrast to DBMSs, however, they allow for the storage and searching of large documents [Harvey and Press 1996 56]. It must be noted, though, that RDBMSs are continually evolving and that the distinction between TIMSs and RDBMSs is not clear cut. Microsoft Access$_{TM}$ is an example as it has structured fields but also supports searchable memo fields which can accommodate large quantities of text.

Alternatively, there is software which will facilitate the creation of a database which imposes few restraints when entering data. Such a system allows the database creator to fashion the database around the irregularities of the source and is known as 'source-oriented.' Software for either of the above-mentioned systems comes under the category of a DBMS, but also incorporates TIMSs features as it will handle the storage and searching of large documents.

Within the historical community there is debate at present concerning the most appropriate type of (DBMS) for historical projects. This mainly concerns those with complex material and centres on the relative merits of traditional relational database systems versus source-oriented database systems such as κλειω [Denley 1994 33-43]. These are the two systems that were considered for this work, so a description of the rationale and method of functioning of each system now follows.

4.2.1 The relational model

A relational database consists of relations, or tables [Harvey and Press 1996 119]. Each relation consists of attributes and contains many records, or tuples. Each record of data occupies a row of the table. All data values are atomic (or scalar), meaning that at every row and column position in each table, there is only one data value, never several [Date 1994 55-6]. In order to guarantee that these records are unique, attributes are distinguished which have unique values for each record. It is possible to include composite attributes within this definition, that is a combination of attributes which together make each row of the table unique. In addition to uniqueness, these attributes must have the quality of minimality, that is, no attribute can be discarded from a composite attribute without losing the uniqueness. Any attribute or composite attribute possessing the qualities of uniqueness and minimality is then labelled a 'candidate key'. Should there be more than one candidate key, the simplest one (i.e. the one containing the least attributes) is chosen as the primary key, and the others dubbed 'alternate' keys. These keys form the basis for moving from relation to relation as they provide the link between tables. An attribute which is a primary key in one relation but which occurs in a different relation to provide the bridge between the two is called a 'foreign key' [Date 1994 112-6 and Harvey and Press 1996 120].

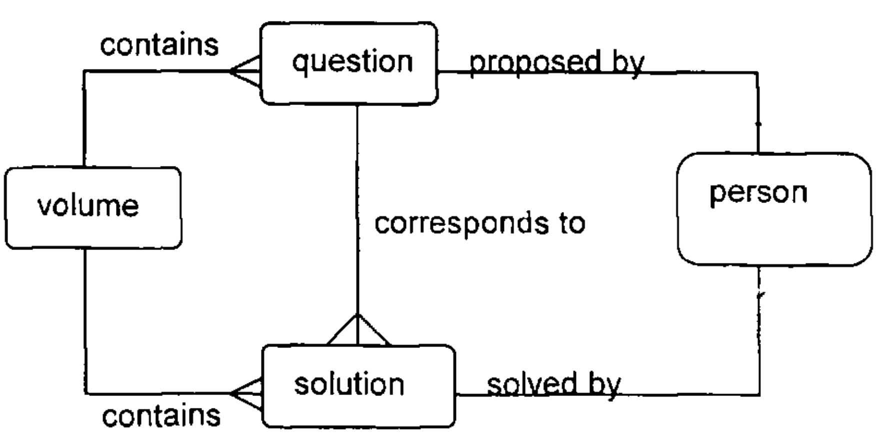
Before a relational (and indeed any) database can be built, data modelling (or logical design) must take place [Date 1994 269-346 and Harvey and Press 1996 119-30]. Normalization is a bottom-up method of modelling data and it is the method advocated for relational database design by Date. An alternative method of designing database systems to that of normalization will be mentioned briefly. Entity-relationship modelling (ERM) is a top-down approach. It initially involves a thorough scrutiny of the data to distinguish suitable objects to act as the basic building blocks.

These are entitled 'entities', and it is vital to choose them correctly. This selection is not always obvious or unambiguous. (It is to be noted though, that database construction is an area where there may never be a single, ideal database constructed from a particular collection of data as different people may want to answer different queries from this data and thus may choose different entities.) Following the choice of entities comes the choice of 'attributes', pieces of information which describe each entity. The relationship between the entities is then formulated and the first modelling stage, ERM is then complete and can be illustrated using a E-R diagram [Harvey and Press 1996 103 and Date 1994 347-66].

Normalization and ERM are different data modelling methods, with different approaches and outcomes. Normalization was developed by Codd [1972] and ERM by Chen [1976]. Despite their differences, both Date [1994 364] and Harvey and Press [1996 138] acknowledge that some aspects of ERM modelling can be very useful prior to carrying out the normalization process in the normalization method. Date advocates the use of the E-R diagramming techniques and Harvey and Press state that 'ERM is a useful approach to database projects irrespective of whether RDA (Relational Data Analysis) is ultimately carried out. It is an excellent way of mapping out a situation, clarifying thought and providing a provisional data model that can be understood by non-specialists who may ultimately use the database.' In this context, as a modelling aid prior to normalization, an E-R diagram of the *ET* data is now given.

4.2.1.1 The *ET* data

Fig. 4.1 Initial E-R diagram of *ET* data



In ERM, a straight line between drawn entities means the relationship is 1:1, so each question is proposed by one person. A straight line ending in a 'crows foot' means that the relationship is one to many. So for one volume there may be many questions.

These entities are given below (in upper case), with their respective attributes in parentheses. The primary key is underlined.

VOLUME(<u>volume-date</u>, volume-number, part-number, volume-format, editorial-comment (found mainly in the letters to correspondents section), book-information (information about mathematical books outside the mathematical department), articles (concerning mathematics or directly related subjects), mathematicians, institutions (relevant to the thesis), examinations (relevant to the thesis), volume-miscellaneous (miscellaneous material of interest.)
volume-date is the primary key for this entity, and the composite key, volume-number and part-number is an alternate key.

QUESTION(<u>question number</u>, status (for example showing whether question was junior), categories (mathematical categories the question falls into), methods (methods requested to solve the question), question-miscellaneous, question-book-id (unique identifier for a book in a question), question-book-title, question-book-date, author-surname, author-firstname, author-title, question-book-miscellaneous, question-journal-id (unique identifier for a journal in a question), question-journal-title, question-journal-editor, question-journal-volume, question-journal-year, question-journal-miscellaneous

SOLUTION(<u>solution-number</u>, status, categories, methods (methods actually used to answer the question), solution-miscellaneous, solution-book-id, book-title, book-date, author-surname, author-firstname, author-title, book-miscellaneous, solution-journal-id, journal-title, journal-editor, journal-volume, journal-year, journal-miscellaneous

PERSON(<u>person-id</u>, type (proposer or solver), status (male, female or pseudonym), surname, firstnames, title, qualifications, town, county, country, institution, occupation, person-miscellaneous)

The data in these initial tables is then submitted to the strict rules of normalization. Before this can take place, the concept of functional dependency must be outlined as it is a key concept in the normalization process. Date [1994 272] supplies a formal definition of functional dependency.

> Let R be a relation, and let X and Y be arbitrary subsets of the set of attributes of R. Then we say that Y is **functionally dependent** on X - in symbols
> $X \rightarrow Y$

(read 'X functionally **determines** Y' or simply 'X arrow Y') - if and only if each X-value in R has associated with it precisely one Y-value in R. X is then known as a **determinant.**

If in a given relation, a given attribute C is functionally dependent on an attribute B, and the attribute B is functionally dependent on an attribute A, then attribute C is **transitively dependent** on attribute A.

4.2.1.2 Functional dependencies and normalization

The functional dependencies in each table are now identified.

In VOLUME

volume-date **determines** each of editorial-comment, book-information, articles, mathematicians, institutions, examinations, volume-miscellaneous

part number and volume number together **determine** each of editorial comment, book information, articles, mathematicians, institutions, examinations, volume miscellaneous

institution **determines** examination

In QUESTION

question number **determines** each of status, categories, methods, question-miscellaneous, question-book-id, book-title, book-date, author-surname, author-firstname, author-title, book-miscellaneous, question-journal-id, journal-title, journal-editor, journal-volume, journal-year, journal-miscellaneous

question-book-id **determines** book-title, book-date, author-surname, author-firstname, author-title, book-miscellaneous

question-journal-id **determines** journal-title, journal-editor, journal-volume, journal-year, journal-miscellaneous

Similarly for SOLUTION

In PERSON

person-id **determines** each of surname, firstnames, title, qualifications, proposer miscellaneous

town **determines** both county and country institution **determines** occupation

The normalization process will now be set out. This is a formal process with well-defined rules [Harvey and Press 1994 119]. The overall aim of normalization is now set out,

> A truly normalized collection of tables (relations) is one that is structured so that they cannot contain redundant data. The normalization process is often described in terms of stages known as first, second, third, fourth and fifth normal forms (1NF-5NF). The general idea is that at each successive stage of normalization certain undesirable features are eliminated from the initial unnormalized table. First normal form is obtained by eliminating repeating groups, second normal form is obtained by eliminating non-primary-key attributes which are not functionally dependent on the whole of the primary key, and third normal form is obtained by eliminating functional dependency between non-primary-key attributes. (Third normal form is similar to Boyce-Codd normal form, but does not deal satisfactorily with overlapping candidate keys.) Fourth normal form deals with multi-valued determinancies, and fifth normal form deals with a rather unusual situation known as join dependency which is of little practical significance [Howe 1989 87].

Definitions are now given of the various normal forms.

First Normal Form (1NF)

A relation is in 1NF if and only if all the underlying domains contain scalar values only [Date 1994 296].

Second Normal Form

A relation is in second normal form (2NF) if and only if it is in 1NF and every non-primary-key attribute is **fully** functionally dependent on the primary key [Robinson 1993 29].

Third Normal Form

A relation is in third normal form (3NF) if and only if it is in 2NF and every non-primary-key attribute is non-transitively dependent on the primary key [Robinson 1993 35].

BCNF

When a relation is in Boyce-Codd normal form (BCNF), every attribute which is not part of the (primary) key is a fact about the key, the whole of the key and nothing but the key. More formally, a relation is in BCNF if and only if every determinant is a candidate key [Robinson 1993 24-5].

To convert the four relations VOLUME, QUESTION, SOLUTION and PERSON into 1NF, all repeating groups must be identified, removed and placed in separate relations. For example, in VOLUME, there may be several articles corresponding to any one volume date, so articles is not a scalar attribute, it is a repeating group. Other attributes falling into this category are:

In VOLUME
editorial-comment, book-information, articles, mathematicians, institutions, examinations, volume-miscellaneous,

In QUESTION
question-categories, question-methods, question-miscellaneous
question-book-miscellaneous,
question-journal-miscellaneous

Similarly for SOLUTION

In PERSON
title, qualifications, person-miscellaneous

For the relations to be in 1NF, these repeating groups must be removed from the initial groups and placed in their own relations, with an appropriate primary key. The new relations are given below, with the foreign keys given in italics,

From VOLUME
EDITORIAL-COMMENT(editorial-comment-id, editorial-comment-details, *volume-date*)
BOOK-INFORMATION(book-information-id, book-name, book-details, *volume-date*)
ARTICLES(article-id, article-name, article-details, *volume-date*)
MATHEMATICIANS(mathematician-id, mathematician-name, mathematician-details, *volume-date*)
INSTITUTIONS(institution-id, institution-name, institution-details, *volume-date*)
EXAMINATIONS(examination-id, examination-details, *volume-date*, *institution-id*)
VOLUME-MISCELLANEOUS(volume-date, volume-misc1, volume-misc2,..)

From QUESTION
QUESTION-CATEGORIES(question-number, category1, category2, category3,...)
QUESTION-METHOD(question-number, method1, method2,...)
QUESTION-MISCELLANEOUS(question-number, question-misc1, question-misc2,.)
QUESTION-BOOK-MISCELLANEOUS(question-book-id, question-book-misc1, question-book-misc2,..)
QUESTION-JOURNAL-MISCELLANEOUS(question-journal-id, question-journal-misc1, question-journal-misc2,...)

Similarly for SOLUTION

From PERSON
PERSON-TITLE(person-id, person-title1, person-title2,..)
PERSON-QUALIFICATIONS(person-id, person-qualfn1, person-qualfn2,..)
PERSON-MISCELLANEOUS(person-id, person-misc1, person-misc2,..)

After repeating groups have been removed, the remaining attributes, arranged in relations with appropriate primary keys, are,

VOLUME(volume-date, volume-number, part-number, volume-format)
QUESTION1(question-number, status, question-book-id, book-title, book-date, author-surname, author-firstname, author-title, question-journal-id, journal-title, journal-editor, journal-volume, journal-year)
and SOLUTION1 has corresponding attributes to QUESTION1.

PERSON(person-id, type, status, surname, firstnames, town, county, country, institution, occupation)

These 24 relations now satisfy the conditions for 1NF, and must be inspected to see if they satisfy those for 2NF. In all of these tables, there are no composite primary keys. Hence the attributes in each table depend upon the whole of the primary key, so each table is in 2NF.

Each relation must now be checked to see that it is in 3NF. Hence there must be no transitive dependencies. That is, no attribute must depend on the primary key indirectly, via another attribute that is not an alternate key.

The relations which contain transitive dependencies are, PERSON, QUESTION1 and SOLUTION1. In PERSON, town **determines** county and country, and institution **determines** occupation. Both sets of attributes must be removed to their respective tables, leaving:

PERSON(person-id, type, surname, firstname1, firstname2, firstname3, *town-id*, *institution-id*)
PERSON-TOWN(town-id, county, country)
PERSON-INSTITUTION(institution-id, institution-details, occupation)

In QUESTION1, question-book-id **determines** book-title, book-date, author-surname, author-firstname, author-title and journal-book-id **determines** journal-title, journal-editor, journal-volume, journal-year. Similarly for SOLUTION1. Again, both sets of attributes must be removed to their respective tables, leaving

QUESTION2(question-number, status)
BOOK-IN-QUESTION(question-book-id, book-title, book-date, author-surname, author-firstname, author-title, *question-number*)
JOURNAL-IN-QUESTION(journal-book-id, journal-title, journal-editor, journal-volume, journal-year, *question-number*)

Similarly for SOLUTION2, BOOK-IN-SOLUTION and JOURNAL-IN-SOLUTION.

All the relations are now in 3NF. To be in BCNF, every determinant must be a candidate key. The primary key is the only determinant in all the relations except

VOLUME, where volume-number and part-number form a composite alternate key. Hence the relations are in BCNF.

As QUESTION2 contains only one attribute, it could be combined with QUESTION-MISCELLANEOUS, giving:

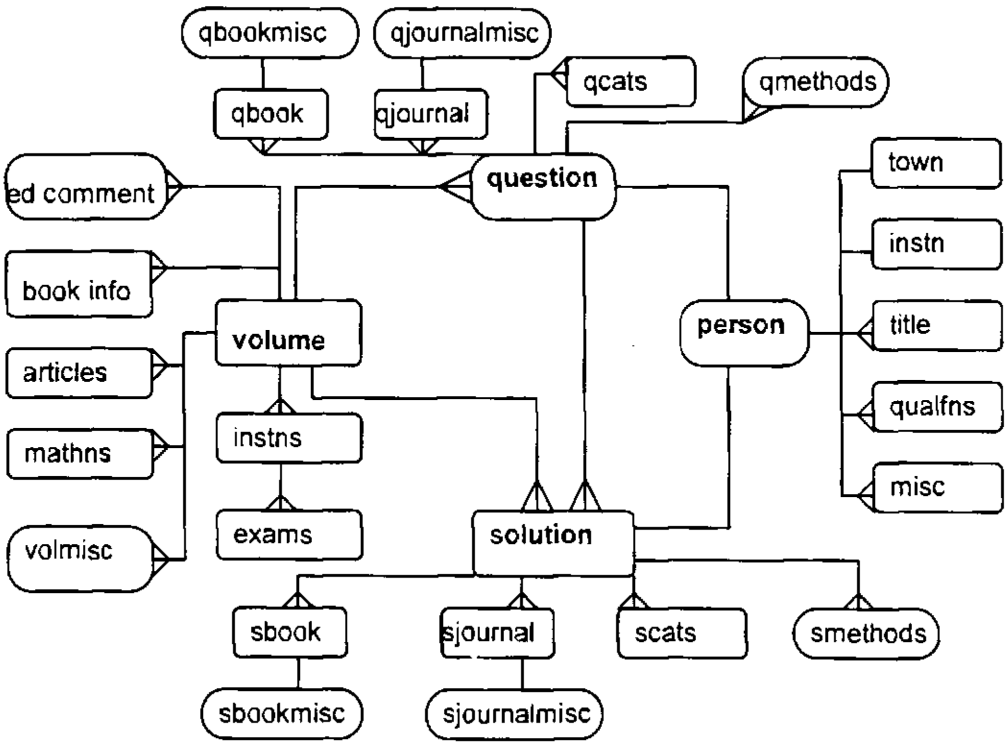
QUESTION3(question-number, status, question-misc1, question-misc2,.) which is in BCNF. Similarly for SOLUTION3. These are both still in BCNF.

The final list of 28 relations in BCNF is,

VOLUME(volume-date, volume-number, part-number, volume-format)
EDITORIAL-COMMENT(editorial-comment-id, editorial-comment-details, *volume-date*)
BOOK-INFORMATION(book-information-id, book-name, book-details, *volume-date,*)
ARTICLES(article-id, article-name, article-details, *volume-date*)
MATHEMATICIANS(mathematician-id, mathematician-name, mathematician-details, *volume-date*)
INSTITUTIONS(institution-id, institution-name, institution-details, *volume-date*)
EXAMINATIONS(examination-id, examination-details, *volume-date*, *institution-id*)
VOLUME MISCELLANEOUS(volume-date, volume-misc1, volume-misc2,..)
QUESTION3(question-number, status, question-misc1, question-misc2, *volume-date*)
QUESTION-CATEGORIES(question-number, category1, category2, category3,...)
QUESTION-METHOD(question-number, method1, method2,...)
BOOK-IN-QUESTION(question book id, book-title, book-date, author-surname, author-firstname, author-title, *question-number*)
QUESTION-BOOK-MISCELLANEOUS(question-book-id, question-book-misc1, question-book-misc2,..*question-number*)
JOURNAL-IN-QUESTION(journal-book-id, journal-title, journal-editor, journal-volume, journal-year, *question-number*)
QUESTION-JOURNAL-MISCELLANEOUS(question-journal-id, question-journal-misc1, question-journal-misc2,...*question-number*)
SOLUTION3(solution-number, status, solution-misc1, solution-misc2, *volume-date*)
SOLUTION CATEGORIES(solution-number, category1, category2, category3,...)
SOLUTION METHOD(solution-number, method1, method2,...)
BOOK-IN-SOLUTION(solution-book-id, book-title, book-date, author-surname, author-firstname, author-title, *solution-number*)

SOLUTION-BOOK-MISCELLANEOUS(solution-book-id, solution-book-misc1, solution-book-misc2,..*solution-number*)
JOURNAL-IN-SOLUTION(journal-book-id, journal-title, journal-editor, journal-volume, journal-year, *solution-number*)
SOLUTION-JOURNAL-MISCELLANEOUS(solution-journal-id, solution-journal-misc1, solution-journal-misc2,...*solution-number*)
PERSON(person-id, type, surname, firstname1, firstname2, firstname3, *town-id, institution-id, question-number*)
PERSON-TOWN(town-id, county, country)
PERSON-INSTITUTION(institution-id, institution-details, occupation)
PERSON-TITLE(person-id, person-title1, person-title2,..)
PERSON-QUALIFICATIONS(person-id, person-qualfn1, person-qualfn2,..)
PERSON-MISCELLANEOUS(person-id, person-misc1, person-misc2,..)

Fig. 4.2 Final E-R diagram of the normalised *ET* database



(The entity names in figure 4.2 are slight modifications of the entities given above to facilitate a more compact presentation.)

This model provides a reliable and well-founded system for entering and subsequently analysing all the *ET* data. The 28 relations above would have been needed to enter the required data into a relational database. There are now many suitable Windows$_{TM}$ computer programs capable of executing such a task, such as Microsoft Access$_{TM}$, dBase for Windows$_{TM}$, Paradox$_{TM}$ (Wright [1994 116-24] reviews four databases for Windows$_{TM}$). These are very powerful and reasonably user-friendly programs. They each have the facility to analyse the database using a version of the purpose-built programming language for querying databases, Structured Query Language (SQL). With Access$_{TM}$, it is possible to store material in a 'memo' field, which is 'well-suited to storing information which varies in size, like descriptions, comments,...' [Harvey and Press 1996 55]. Using memo fields, it would be possible to incorporate the relations comprising miscellaneous details into another relation, for example,

PERSON(person-id, type, surname, firstname1, firstname2, firstname3, person-miscellaneous, *town-id*, *institution-id*) where person-miscellaneous is stored in a memo field.

There are many advantages to normalization as a modelling technique. Technical advantages are that data redundancy is eliminated thus reducing the amount of processor time needed to perform database queries and simplifying database operations like updating and deleting. More importantly, it enforces data integrity, ensuring that there are no ambiguities in the data. Academic advantages are that it identifies the entities and corresponding relationships between them which are then used to model the data and also that it is suitable for use with record linkage [Harvey and Press 1996 128]. These advantages are considerable and the relational model of the *ET* data outlined above is suitable and practical.

The relational model itself is frequently described in mathematical terms using concepts and notation from set theory and relational algebra [Date 1994 139-211]. The reliability and portabilty of the relational system make it very desirable and it is much favoured by many. It is suitable, both theoretically and practically for much historical research. A historian may prefer not to use it as normally it prevents the storage of the data in a form similar to its original.

Data have to be segregated to some extent to fit them into a relational model. The very act of splitting the data into discrete blocks which slot into a system where rigid rules apply is not necessarily a simple procedure where historical data is concerned. With mathematical or scientific data, it is possible that there will be little ambiguity when deciding how to construct a model for that data (still realising however that data modelling with scientific material can still be very complex). Data relating to the business world may also be modelled successfully using the relational model. The use of the relational model for historians is explored in depth by [Bradley 1994 75] who states,

> It is now apparent why the relational database theorists concentrate upon the business world. It is because the relational model is most suited to structuring information about enclosed universes where uncertainties and ambiguities are limited. Furthermore, the relational model performs best when it presents a static rather than dynamic picture of the universe it is representing.

With historical data collected from a given time span, there may be difficulties arising from the fact that the meaning or categories of the data may change with time. Alternatively, other external parameters may change, for example country name or political status. These points are expanded upon in [Denley 1994 33-43].

Conceptually, then, it can be difficult to place historical data into a relational model because it can contain irregularities that make it almost impossible to fit into a strict and inflexible structure. It can be imagined these difficulties have been overcome and that the data has been cast into a relational model. The process of entering this data into the database entails inserting the various details concerning each of the separate entities into separate database tables. With say Microsoft Access$_{TM}$, it is possible to have several tables open at once, but certainly not the 28 that are needed for a relational model of the *ET* database. And while it would now be possible, using Microsoft Access$_{TM}$ 2.0 or 7.0 to create data entry forms to alleviate this somewhat, this was not possible at the beginning of this project as Microsoft Access$_{TM}$ was not a viable option. This practical aspect was a deciding factor when choosing a DBMS. The *ET* database contains details of several thousand mathematical problems and a high standard of data entry was imperative for the resulting integrity and quality of the project. The act of entering data pertaining to each question into 28 different relations appeared unsatisfactory and likely to lead to error in data entry when changing from relation to relation. (It must be noted, however, that Microsoft Access$_{TM}$, say, has very powerful facilities to enforce data integrity.) Also, it would not be an instantaneous matter to retrieve all the data relating to one question as the relevant data from each relation would have to be located. It must be emphasised that this is a matter of personal preference. The relational methodology is suitable for this project which could then be successfully implemented with Microsoft Access$_{TM}$. Indeed, the κλειω database has been converted into a Microsoft Access$_{TM}$ database (although it is not yet fully normalised), prior to being made available to other researchers via the History Data Unit, Essex University.

In software engineering, when data are transferred from a paper document to a computer system, it is preferable to make the layout of the user-interface, which may be a form, as similar as possible to the layout of the original paper form [Curtis 1994 473]. This point is made in connection with database data entry by Breure [1995 30-49] who states that:

> The historian draws information from documents and he has to accept that data from a coherent text fragment are split over different tables. When we use standard interface facilities, this means that data that belong together are not displayed together (loss of *data context*)...In contrast with things in our modern world, historical entities often lack reliable identifiers. Either the historian himself or the system has to provide these identifiers.

Again, this could be overcome by the use of a TIMS.

To summarize, the relational model is well-established theoretically and well-supported now by user-friendly software. It is suited for this research but was not chosen at the beginning of this project as it involved the splitting of the data for insertion into multiple database tables causing an increased probability of error in data entry. Also, flexible and user-friendly software which is now standard was not made available for this project, instead inflexible and non-user-friendly software was proffered. The relational model does have advantages when it comes to querying the data, as RDBMSs are fast, powerful and the results can be exported easily to a spreadsheet for statistical analysis. It is also possible to create database reports in most RDBMSs.

4.2.2 Source-oriented data modelling and the κλειω DBMS

κλειω is a source-oriented DBMS, an emphasis which makes it different from the relational model in intent and application. With historical material, any artificial alterations undertaken to fit the data into a database may result in the loss of crucial data and/or meaning, thus obviating the whole exercise. For example, there may be instances of a person's occupation which do not fit neatly into a relational database, so a decision may be taken to discard or alter awkward cases. This may be undesirable for the historical researcher. κλειω has as its paradigm the creation of a DBMS which allows enormous flexibility in the design of databases. In essence, the historical data shapes the database, rather than the DBMS shaping the data entered into it.

The development of κλειω began in the late 1970s by Dr Manfred Thaller of the Max-Planck Institut für Geschichte in Göttingen. Queries in the first version were Latin-based, thus emphasising the fact that it was aimed at historians, not computer scientists nor those in business. An English version was launched in 1993, overseen by an advisory committee from a variety of institutions including Queen Mary and Westfield College, London, King Alfred's College, Winchester and Southampton University. Version 5.1.1 comprises two levels, 'standard' and 'high tec', the latter involving sophisticated image-handling capabilities. Standard κλειω was the version used for the *ET* database and will thus be delineated.

With κλειω, it is possible to enter data in a form as close to the original as is desired, with the proviso that 'The objective of reproducing all the information in an original source may not, of course, always be appropriate to a historical research project' [Harvey and Press 1996 196]. It is then feasible to designate codes for specified sections of data, whilst leaving the data itself untouched. This is achieved via the creation of a 'codebook', a file which acts as a translation table, to convert sets of character strings into numerical variables. This facility of codebooks lying over the data, as it were, allows codes to be refined with hindsight and the retention of the original data. There are no decisions imposed on the length of fields before data is entered, as there is in a relational system. Instead, one is free to insert data almost limitlessly.

A κλειω database is constructed in a hierarchical manner. Within this framework, however, there is enormous freedom. It is permissible to have repeating groups (see 4.2.1.3), a situation not admissible in the relational system. Similarly, many items of data can be entered in a space allowed for one item in the relational system. 'Fuzzy' data, that is, data which contains an inherent degree of uncertainty as to its origin/authenticity, can be admitted. Comments can be added which are then noted as being distinct from the raw data and are not searchable with the original data. These facilities are best described by the authors of the English κλειω *Tutorial*,

> κλειω has the ability to accept all forms of historical source material in a format that relates to the source. Historical sources can present information in forms that are very hard to reconcile with the conventions of a traditional database. On the simplest level, a census return may contain an entry containing two distinct occupations. This can be described as a multi-value variable. In κλειω these **entries** can be combined in such a way that they remain in context but can be made logically equivalent for processing. **Elements** can contain any

> number of entries, which in turn can have different **aspects** (e.g. the original spelling, or the editor's comments, can be stored alongside the main version to be processed), **views** (e.g. Latin and vernacular equivalents of the same data could be stored as alternative views) and **visibility** (a quantitative estimate of the value or reliability of the information). These features allow fuzzy data (for example, a surname which might also represent an 'occupation' - such as Fletcher) to be defined as such, the user then having to choose at the information retrieval stage how to interpret such data. Elements are contained in **groups** of information (not unlike records in relational database design) which are related to each other in a hierarchical fashion, i.e. are logically subordinate or superordinate to each other....The size of database is limited only by the capacity of the machine being used; the complexity of a database is virtually unlimited (elements can contain up to two million characters; there can be up to 32,000 different element names, and up 32,000 different groups). [Woollard and Denley 1993 xiv-xv].

All these concepts add up to κλειω's data model which has been described as 'a semantic network tempered by hierarchical considerations' [Thaller 1991 155]. Its flexible nature gives κλειω a 'rubber band data structures' facility [Denley 1994 37]. The fluid nature of creating a database with κλειω marks it out as an 'organic' DBMS.

Atypical settings of historical data is a matter addressed by the κλειω DBMS by means of its various data types, and its logical environment. κλειω supports seven different data types, text, date, number, category, relation, location and image. The data type category allows the use of 'value-added' data, relation allows entries to be linked (incidentally it is also possible to link several κλειω databases), location is used in connection with map production and image with image processing. The logical environment is a collection of 'logical objects' which are κλειω and/or user-defined instructions which determine how sections of the data is interpreted. This environment is stored within κλειω and can be altered. These data facilities lend themselves to the processing of information with atypical date systems, for example, weights and measures systems. Very complex logical environments can be created to cater for, say, special festivals. Equally, a simple but effective logical environment can be used for classifying data. Using the category data type, it has been possible to produce a set of characters which show whether the problem proposers were male, female or used a pseudonym, whether the question was solved by the proposer, whether it was repeated. Within this logical environment, κλειω enforces exclusion within these categories, so a proposer is *only* one of male, female or pseudonym.

Once the κλειω database has been formed, permanent catalogues can be created. A catalogue contains contents of elements, plus a pointer for each term showing where it is located in the database. This selection can then be marked-up and analysed with greater rapidity than would be the case if the selection were made afresh each time from the whole database.

A theoretical possibility for the future would be to use the 'high tec' version of κλειω and scan in the actual copies of the problems from *ET*. The relevant facsimile of the problem could then be allied to the data stored in the database about that particular problem. This would then be very close to 'an edition of the *ET*' and would be a further example of 'the Database as Edition' [Thaller 1991 156-9]. Unfortunately, there is a practical impediment to this, as the quality of the *ET* printing is very poor, with the reverse side frequently showing through.

The above analysis conveys the advantages of the κλειω DBMS. It is not without its disadvantages, however. κλειω functions via the out-moded system of batch processing. Copious errors may appear following compilation, which may require many iterations to rectify. Also, it is necessary to delete certain system files following a faulty compilation, and this can be time-consuming. More seriously, data must normally be entered in the same order as a user-defined κλειω structure file. If major changes are made in the structure file, normally the order of the corresponding data file must similarly be altered and the potential for this to be very time consuming is large indeed. Querying can be long-winded, and changes have to be made to the queries before they can be exported to a spreadsheet package for analysis.

4.3 The κλειω *ET* database

The construction of the κλειω *ET* database has been undertaken with much care and deliberation. Working within the restrictions of κλειω has enforced a disciplined and rigorous approach, to avoid future pitfalls. A course in κλειω was completed and a attendant project undertaken, this being a pilot study (or prototype) for the *ET* database. This small initial study allowed the creation of version 1 of the *ET* database which was populated with data from the first year of *ET*. Samples of data throughout the life of *ET* were also taken and analysed to ensure uniformity in data structure during all periods of the journal's life. The structure diagram for version 1 of the *ET* database is shown in appendix A in figure A4.1.

The *ET* database has been through many refinements before reaching its final form. During the pilot study, the database was queried and tested to ensure it could fulfil all the tasks that would be required of it. Alterations in the structure of the database were then made in response to the above tests and this process was continued until there was confidence that the database was appropriately constructed for the assignment in hand.

The final version of the *ET* database is now described in full.

With κλειω it is possible to input the data into a data file first, then create a structure file to fit it. This procedure was not followed in this instance, as a κλειω structure file was fashioned first, with the data being placed subsequently into a corresponding κλειω data file.

The *ET* database was designed to be as close to the original data in *ET* as possible. Each month *ET* had a mathematical department named 'Mathematical Questions and Solutions'. The format of this section varied during the life of the journal, but during the initial stages it commenced with a subsection of 'Solutions' of previously posed questions. The question to be solved was repeated above each solution, with all the details of the proposer. The solution was printed together with the details of the solver. If the solution were by the proposer, then the statement 'by the proposer' would replace the solver details. Following the 'Solutions' section came a 'New Questions' section with several mathematical questions each having details of its proposer. Last of all was a miscellaneous section with snippets of editorial comment directed at particular mathematical correspondents or stating editorial policy. This latter section has proved invaluable for tracing the changes in policy throughout the life of the *ET*. This comprises all the data of interest in the mathematical department itself. Other sections of the journal may also contain items of interest. Details of mathematical examinations, examiners, other institutions or mathematicians may appear in disparate parts of any month's issue of the *ET*.

To translate the above data into a κλειω database, a κλειω structure file must be created. This must contain **groups** of information (see 4.2.2), each group consisting of a number of **elements**. Before describing the structure file itself, a structure diagram will be given delineating the groups (shown in boxes) and their respective elements (to the side of or below each group).

The κλειω model produced is source-oriented in that it all the data pertaining to a particular month is encapsulated in a form similar in order and context to that in the *ET*. The only exception to this is that it was not possible to incorporate all the mathematics from the problems themselves into the basic version of κλειω, and scanning them in and then marking them up was also not a possibility. The information that was required from the mathematical data, for this research and for subsequent research, was the mathematical categories they belong to. The work in progress by Dody [1994 24-179] and Duvina [1994 179-218] also uses similar categories for the mathematical papers in the respective journals under investigation. Other researchers using the *ET* data would want to find problems posed or solved by a particular mathematician, or problems in a particular category, for example, geometry or number theory.

Each group will now be described in detail in relation to the relevant monthly edition of the *ET*.

REFERENCE is the first group and is an identifier for each separate volume of the *ET*. It is a restriction of κλειω that this first group must contain only one element which must be alphanumeric in character and up to 12 characters long. The corresponding element, **refnum** comprises the concatenation of the string 'et', the volume number then the part number. The volume number was given in roman numerals and corresponded to the year of publication. Volume I related to October 1847 to September 1850, volume II, October 1850 to September 1851. Each issue of the journal had a number also, so October 1847 was labelled Vol. I No. 1. Similarly, October 1848 carried Vol II No. 13. In this database, the journal number is referred to as part number to avoid ambiguity.

VOLUME contains the following elements:-

> **voldate**, - the date given on the front page of each month's issue in the format '1 Sep 1847'.
> **volnum**, - the journal volume number, as described above.
> **partnum** - the journal part number as described above.
> **volformat** - the section and subsection headings used in the 'Mathematical Questions and Solutions' section.

Each volume contained general information which may be of particular interest. This is stored in the group VOLINF. Next there are many instances of solutions, represented by the group S. Similarly, there are many new questions, represented by the group Q. To be precisely true to the original source, it would be necessary to repeat the details of each question when entering the solution details. This would have entailed a waste of space and effort and was not carried out as the details given regarding the question of a particular solution are precisely the same as the originally proposed question.

VOLINF contains the following elements,

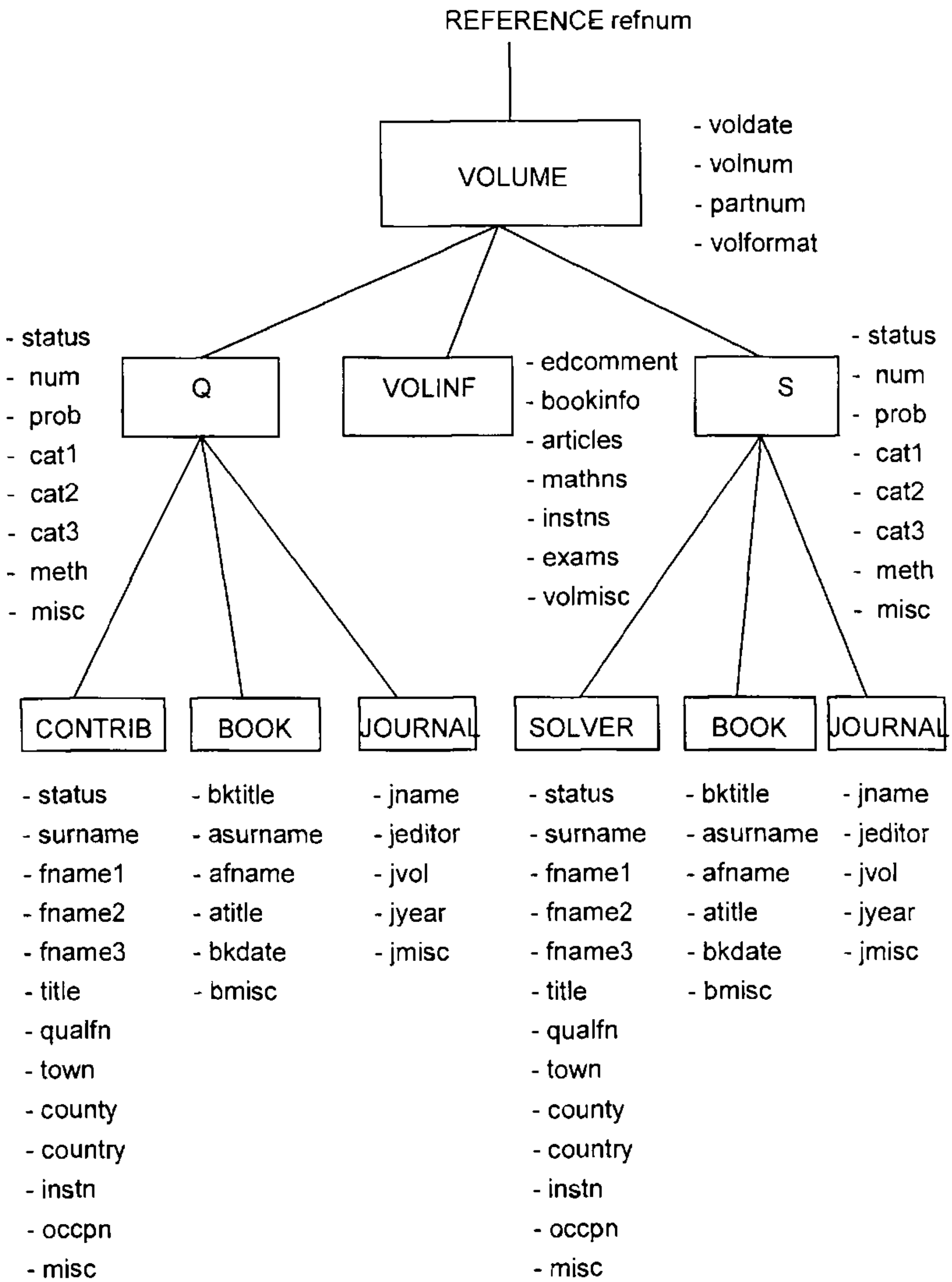
edcomment - editorial comment with particular reference to mathematics.
bookinfo - book review or other mention of mathematical book.
articles - particular article/s written by or about mathematicians.
mathns - specific mathematician/s.
instns - specific institution/s of direct or indirect mathematical interest.
exams - mathematical exams and the corresponding examiners.
volmisc - any item of mathematical interest not fitting into one of the above categories.

Q contains the following elements,

status - descriptor/s selected from the ensuing list, reproposed, junior mathematics, proposed and contains diagram. ('Proposed' is included here to facilitate a quick count of proposed questions.)
num - the question number given in hindu-arabic numerals throughout, and not in roman.
prob - this element is a repeat of the question number and is used specifically for relating a given problem to a given solution via the data type relation.
cat1, cat2 and cat3 - are all categories relating to the mathematics contained in the question. Details of these categories are given in 4.4.1.
meth - indicates the stated preferred method of solution requested for a given question.
misc - any question details not pertinent to the above elements e.g. reference to a particular theorem.

4.3.1 The *ET* structure diagram

Fig. 4.3



S is a group containing the elements,

status - descriptor/s selected from the ensuing list, solved by proposer, contains diagram and junior mathematics.

num, prob, cat1, cat2, cat3 and misc are mirror images of **num, prob, cat1, cat2, cat3 and misc. meth** contains the method **actually used** to solve the question under consideration.

VOLINF has no groups subordinate to it, but both Q and S do have subordinate groups. Q has 3 groups subordinate to it, CONTRIB, BOOK and JOURNAL and these may appear in any order.

CONTRIB is the group containing all details pertaining to the proposer of the question. Its elements are,

status - a descriptor from the range 'male', 'female' or 'pseudonym'.

surname, fname1, fname2, fname3 - nominal details.

title - for example Mr, Mrs, Miss, Dr, Professor.

qualfn - for example, M.A., D.D.

town - town or city stated. (This may be town of residence, or of workplace.)

county - county stated. (ditto for county)

country - country stated.' (ditto for country)

instn - institution stated.

occpn - occupation stated.

cmisc - any miscellaneous detail concerning the proposer that does not fit into any other category.

BOOK contains the following elements,

bktitle - the title of the book.

asurname - the author's surname.

afname - the author's first name.

bkdate - any date given in relation to the book.

bmisc - any miscellaneous detail concerning the book that does not fit into any other category.

JOURNAL contains the following elements,

jname - the name of the journal.

jeditor - the editor of the journal.

jvol - the particular volume of the journal.

jyear - the year of publication of the journal.

jmisc - any miscellaneous detail concerning the journal that does not fit into any other category.

S has three groups subordinate to it, SOLVER, BOOK and JOURNAL. These may appear in any order.

SOLVER contains exactly the same elements as CONTRIB.

BOOK and JOURNAL are precisely the same for CONTRIB and SOLVER.

4.3.2 The *ET* κλειω structure file

The κλειω structure file, *ET*.MOD, given in appendix A table A4.2 contains the necessary κλειω commands to implement the model shown above. Comments are inserted into κλειω files via the κλειω command 'note'. This structure file falls into two main sections, the logical environment and the group definitions.
At the beginning of the logical environment is a logical object to define the dating system used. Following this is a logical object to define the categories used in the various status elements. Incorporated within this are exclusion mechanisms to ensure that a proposer/solver cannot simultaneously be male and female. Lastly there is a logical object making use of the relation data type. Its function is to relate given questions to their solutions. This is achieved by the dedicated elements **q:prob** and **s:prob**. Thus if question 150 is proposed in June 1850 and solved in August 1850, κλειω is able to link these two groups across the different volumes via **q:prob** and **s:prob** to facilitate querying of the contents of both the question 150 and the solution to 150. This logical object needs activating by a command following the compilation of the *ET* data file. This command will be shown in appendix A4.3.

The above outlines the design and structure of the *ET* database. The entry of data into this database will now be discussed, commencing with the general details in VOLUME and working down the structure diagram towards a detailed analysis of one question and its related solution. Before the latter can be undertaken, the mathematical categories used for the questions and solutions will be elucidated.

Fig. 4.4

THE EDUCATIONAL TIMES.

A Monthly Stamped Journal of Education, Science, and Literature.

Vol. III., No. 33.] JUNE, 1850. [Stamped—Price 6*d*.

CONTENTS.

Note.—*Subscription, 6s. per annum, payable in advance to Mr. Charles Henry Law,* 131 *Fleet Street, London; to whom all Books for Review, and Communications for the Editor, are to be sent.*

and the apostles ... or heaven. ... used ... instead of the name of God. ... of ... to heaven ... never introduced it as we Christians do; but always paraphrased it either by Adonay or ... in Heaven's Christ warned the Jews not to swear by heaven, because of its being God's throne; only because that the Jews began to ... that ... substituted for ... (4.) All the prophets from Moses to Malachi ... to Israel, as "words of God;" which, in the language of the apostles is ... oracles of God."

(5.) Voice or קוֹל and word or דָּבָר are often used the one for the other in the use of words which is common in almost every language, e. g., "Non intelligo quid sonet haec vox voluptas." (Cic.) "Vox populi, vox dei." In these cases vox is used instead of sermo or verbum. The Greek φωνή means sometimes voice and sometimes word or language. ... קוֹל or, voice, is often used instead of דָּבָר word.

The Talmud speaks of a certain voice from heaven, which is called בַּת קוֹל, filia vocis. "The Rabbis called revelations of God to pious men, after ancient prophecy ceased in Israel," the voice from heaven. But according to the crooked way of the Rabbinic school, it was customary to apply any accidental utterance of a scriptural passage to the then existing circumstances. So we read in the Talmud (tract. Sabbath) that Rabbi Simon, son of ... and R. Jonathan, were about setting out on a journey, for the purpose of calling on their friend Rabbi Samuel, then master of a high-school in Babylon. They wished to be directed by a voice from heaven. They passed a school just when the scholars were reading from the Bible, "and Samuel died" (1 Sam. xxv. 1). They took it as an intimation that their friend had actually died, and ... and they found that he really was dead.

Tosephoth ad Sanhedrin, vol. II. Col. I. declares that בַּת קוֹל, filia vocis, was not a voice heard from heaven, but an echo; whence בַּת קוֹל.

I am, Sir, Yours truly,

T. A.

Bristol, 3rd March, 1850.

Sir,—I have found the inclosed "Hints" very useful in this School; and it has occurred to me that, if published in the *Educational Times*, they might be of service at this season by inducing other teachers to adopt the plan of distributing something similar. I submit the matter, therefore, to your judgment.—I am, Sir, yours most obediently,

W. B. Hodgson.

HINTS FOR THE HOLIDAYS.

"Valde iptur, mi fili, ... persuade, esse te quidem mihi carissimum, sed multo fore cariorum, si talibus monumentis praeceptisque laetabere." (*Cicero De Officiis.*)

1.—Let no day pass over without some stated and regular work. Two or three hours so spent will give zest to your amusements, and save life from running to waste.

2.—Study chiefly those branches in which you feel that you are deficient rather than those that you like most. We dislike what we find difficult and do not understand;—overcome the difficulties, learn to understand, and liking will surely follow. ... at the time what puzzles you; after a few days' progress, the puzzle will often disappear.

3.—Keep a journal, however brief, of your day's occupations and experiences, and every night consider what and how far the past day has been of service to you:—

"What profitable labours is left unbroken;
What kind word to thy playmates spoken;
Whom hast thou pitied, and whom forgiven;
How with thy faults hast duty striven."

4.—Do wholly whatever you do. Work when you do work, play when you do play. Avoid listlessness and dawdling. Let the bow be either well bent, or unstrung. Between the two states there is no room for useful effort, or refreshing rest. The secret of health, bodily or mental, lies in the alternation of, first—one labour with another, then—of labour with rest.

5.—If in the country, besides enjoying the fresh air and fine scenery, take opportunity to study some branch of natural history, for which there are but little time and means in school, and for which the taste must be formed in early life if at all. Whether you prefer Botany (plants), or Entomology (insects), or Ornithology (birds), or Conchology (shells), in any of these you will learn to observe carefully, to arrange judiciously, and to trace admiringly, the wisdom, unity, and beauty of design which run through all nature.

6.—Go to rest early and rise early,—were it only to enjoy as much as possible of the sunlight and sunshine. Work rather in the morning than in the evening.

7.—Take as much active exercise as you can in the open air. Your main business at this season is to form the body, and with it, and through it, the mind, with which the labours of your coming life must be accomplished. See that the body, "the mind's temple," be well built, compact, pure, and graceful.

8.—Remember, above all things, that in LIFE there is no vacation. Divide or designate time how we may, for its every moment you are accountable to a higher than teacher or parent. All time not spent in your own improvement (whether in study, or worship, or society, or exercise, or needful rest) or in doing good to others, (and that, too, is your own improvement,) is worse than lost. It leaves its sad traces in weakened powers, disordered desires, defeated hopes, and self-reproach, deepening as years advance.

9.—If, finally, you neglect these hints, (as the majority of you probably will,) be candid enough to tell, when next we meet, whether the neglect of them has increased your enjoyment, either at the time or in retrospect. Even so, the seed will not be wholly lost; and the seed scattered vainly for this season, may bear fruit the next.

W. B. H.

Chorlton High School, Dec., 1849.

MATHEMATICAL QUESTIONS AND SOLUTIONS.

SOLUTIONS.

LIV. (*Proposed by Mr. C. Tunnicliffe.*)—Required the amount of any capital at the end of a year at compound interest, supposing the interest payable at every instant.

Solution by Mr. T. Elliott, Bracknell.

Suppose the interest to be paid n times a year, and let r = rate per £1. per annum. Let also P = principal, and $A_1, A_2, \ldots A_n$ the successive amounts. Then—

$$A_1 = P + P\frac{r}{n} = P\left(1 + \frac{r}{n}\right)$$

$$A_2 = A_1 + A_1\frac{r}{n} = P\left(1 + \frac{r}{n}\right)^2$$

&c.

$$A_n = P\left(1 + \frac{r}{n}\right)^n$$

$$= P\left(1 + n\frac{r}{n} + \frac{n(n-1)}{2}\frac{r^2}{n^2} + \ldots\right)$$

$$= P\left(1 + r + \frac{1 - \frac{1}{n}}{2}r^2 + \ldots\right)$$

which, when n is infinite,

$$= P\left(1 + r + \frac{r^2}{1.2} + \frac{r^3}{1.2.3} + \ldots\right).$$

$= P\,e^r$, e being the base of Naperian Logarithms.

Solution by Dunman, Glasgow.

Let an unit of capital have become x during the time t by the continuously capitalised interest, at the annual rate r per unit; then the increment of capital dx, during the instant dt, is $dx = xrdt$ $\therefore \frac{dx}{x} = rt$; and, integrating, $x = e^{rt}$, which between the limits $t = 0$ and $t = 1$, is e^r = amount of one unit of capital at the end of one year. If the capital at the beginning of the year be a, its amount at the end of the year will $\therefore$ be ae^r; and, generally, at the end of any time (expressed in n years) $= ae^{rn}$.

LV. (*Proposed by Mr. W. T. Bush.*)

$$\left.\begin{array}{ll} xy = x^2 - y^2 \ldots\ldots\ldots\ldots\ldots & (1) \\ 2x^2(x-y) = x^3 + y^3 \ldots\ldots\ldots & (2) \end{array}\right\} \text{to find}$$

x and y without quadratics.

Solution by W. C., Jersey.

$4x^4(x-y)^2 = (x^3 + y^3)^2 \ldots\ldots (2)^2$

$x^2y^2 = (x^2 - y^2)^2 \ldots\ldots (1)^2$

$4x^4(x-y)^2 = 5x^2y^2 \ldots\ldots (2)^2 - (1)$

$4x^2(x-y)^2 = 5y^2 \ldots\ldots$ div. by x^2

$x(x-y)^2 = \frac{1}{2}\,.\,y^2$

$x(x-y) = \frac{1}{2}\sqrt{5}\,.\,y \ldots\ldots (3)$

$x(x-y) = y^2$ By (1)

$y^2 = \pm\frac{1}{2}\sqrt{5}\,.\,y$

$y = \pm\frac{1}{2}\sqrt{5}$ or 0

$2x^2(x-y) + xy = 2x^2 \ldots\ldots (1) + (2)$

$2x(x-y) + y = 2x$

$\pm\sqrt{5}\,.\,y + y = 2x$. By sub. from (3)

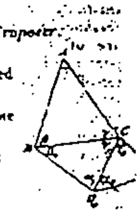
$x = \frac{(1 \pm \sqrt{5})y}{2}$

$x = \frac{1}{4}(5 \pm \sqrt{5})$, or 0.

LVI. (*Proposed by Mr. T. Elliott.*)—ABC is a triangle. Any two of its sides, AB and AC, are produced, and the centre O, of the circle which touches BC and the other two sides produced, is found. The triangle BO,C is formed, and any two of its sides are produced, and the centre O, of the circle which touches the unproduced side and two produced sides, is found. The triangle O,O, ... found; and so on. It is required to express the angles of the nth triangle formed in this manner in terms of the angles of the original triangle; and to show what form that nth triangle assumes when n is infinite.

Solution by the Proposer.

Let the angles be designated as in the diagram, and assume P = 2 right angles. Then:



$$a_1 = \frac{1}{2}(P - a)$$

$$a_2 = \frac{1}{2}(P - a_1) = \frac{1}{2}\left(P - \frac{P}{2} + \frac{a}{2}\right)$$

$$a_3 = \frac{1}{2}(P - a_2) = \frac{1}{2}\left(P - \frac{P}{2} + \frac{P}{4} - \frac{a}{4}\right)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_n = \frac{1}{2}\left(P - \frac{P}{2} + \ldots \mp \frac{P}{2^{n-1}} \pm \frac{a}{2^{n-1}}\right)$$

$$= \frac{P}{2}\left(1 - \frac{1}{2} + \frac{1}{4} - \ldots \mp \frac{1}{2^{n-1}}\right)$$

$$\frac{P}{2} \cdot \frac{2}{3}\left(1 \mp \frac{1}{2^n}\right) \pm \frac{4}{2^n} = \frac{P}{3}\left(1 \mp \frac{1}{2^n}\right)$$

$$\pm \frac{4}{2^n}$$

$= \frac{P}{3}$ when n is infinite.

The expressions for β^n and γ^n are precisely similar, and therefore the triangle is ultimately equilateral.

LVII. (*Proposed by Mr. R. Harley, Mathematical Tutor, Dissenters' College, Taunton.*)—Show how to find $2n+1$ numbers, *i.e.* an odd series of numbers, such that the sum of their squares and that of their cubes shall be expressed by the same cube number.

Solution by the Proposer.

First $(3x)^3 + (4x)^3 + (5x)^3 + (8x)^3 + (10x)^3 + (16x)^3 + (20x)^3 +$ &c.....to $(2n+1)$ terms $=$ $(3x)^3 + x^3(4^3 + 5^3)(1 + 2^3 + 4^3 +$ &c. ..to n terms$) =$ $(3x)^3 + (3x)^3(2^{3n} - 1) = (3 \cdot 2^n x)^3 =$ a cube number.

Again, similarly we find

$(3x)^2 + (4x)^2 + (5x)^2 + (8x)^2 + (10x)^2 + (16x)^2 + (20x)^2 +$ &c.....to $(2n+1)$ terms $=$ $\frac{41 \cdot 2^{2n} - 14}{3}$, which being equated with the cube $(3 \cdot 2^n x)^3$, we get

$$x = \frac{41 \cdot 2^{2n} - 14}{3^4 \cdot 2^{3n}}$$

Hence the mode of determining numbers to answer the condition specified is obvious. For example, —let it be required to find *five* such numbers. In this case $n = 2$.

$$\therefore x = \frac{41 \cdot 2^4 - 14}{3^4 \cdot 2^6} = \frac{107}{864}; \text{ and the}$$

numbers sought are $\frac{107}{288}, \frac{107}{216}, \frac{535}{864}, \frac{107}{108}, \frac{535}{432}$;

the sum of their squares and the sum of their cubes being each expressed by the cube $\left(\frac{107}{32}\right)^3$.

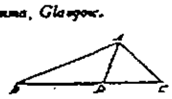
Note.—The former part of the preceding solution suggests an easy method of resolving a given cube number into any odd number of cubes. For, since $(3x)^3 + (4x)^3 + (5x)^3 + (8x)^3 + (10x)^3 +$ &c. to $(2n+1)$ terms $= (3 \cdot 2^n x)^3$; if a^3 be the cube which it is required to divide into $(2n+1)$ cubes, put $(3 \cdot 2^n x)^3 = a^3$, then $x = \frac{a}{3 \cdot 2^n}$; and the cubes sought are

$$\left(\frac{a}{2^n}\right)^3, \left(\frac{a}{3 \cdot 2^{n-2}}\right)^3, \left(\frac{5a}{3 \cdot 2^n}\right)^3, \left(\frac{a}{3 \cdot 2^{n-3}}\right)^3, \left(\frac{5a}{3 \cdot 2^{n-1}}\right)^3,$$

&c.......to $\left(\frac{5a}{3 \cdot 2}\right)^3$.

LIX. (*Proposed by Mr. George Swift, Norwich.*)—Given the base, the sum of the angles at the base, and the difference of the sides, to construct the triangle.

Solution by Digamma, Glasgow.



In a straight line BC take BD = the given difference of the sides, and from D draw DA making the $\angle$ ADC = half the given sum of the angles at the base. From centre B, and with radius = the given base, describe an arc cutting DA in A. Join BA, and draw AC making with AD $\angle$DAC = $\angle$ ADC; ABC is the required triangle. For $\because$ $\angle$ ADC = DAC $\therefore$ AC = CD $\therefore$ BC — AC = BD = the given difference of the sides. And $\because$ $\angle$ B + $\angle$BAD = $\angle$ADC, adding $\angle$ DAC to both sides, $\angle$ B + $\angle$BAC = $\angle$ ADC + $\angle$DAC = (by construction) the given sum of the angles at the base; also AB is (by construction) = the given base. $\therefore$ ABC is the required triangle.

LX. (*Proposed by Mr. T. Wilkinson, Burnley.*)—Find the sum of n terms of the series, $\tan. \phi + \frac{1}{2} \tan. \frac{1}{2}\phi + \frac{1}{4} \tan. \frac{1}{4}\phi + \frac{1}{8} \tan. \frac{1}{8}\phi +$ &c.

Solution by Mr. J. Alsop, Dissenters' College, Taunton.

Let S denote the sum of the given series.

$$\text{Since } \cot. \phi = \frac{\cot.^2 \frac{\phi}{2} - 1}{2 \cot. \frac{\phi}{2}};$$

$$\therefore \tan. \frac{\phi}{2} = \cot. \frac{\phi}{2} - 2 \cot. \phi.$$

$$\text{Therefore } \frac{1}{2} \tan. \frac{\phi}{2} = \frac{1}{2} \cot. \frac{\phi}{2} - \cot. \phi;$$

$$\frac{1}{4} \tan. \frac{\phi}{4} = \frac{1}{4} \cot. \frac{\phi}{4} - \frac{1}{2} \cot. \frac{\phi}{2};$$

$$\frac{1}{8} \tan. \frac{\phi}{8} = \frac{1}{8} \cot. \frac{\phi}{8} - \frac{1}{4} \cot. \frac{\phi}{4}, \text{ \&c.}$$

$$\therefore \frac{1}{2^{n-1}} \tan. \frac{\phi}{2^{n-1}} = \frac{1}{2^{n-1}} \cot. \frac{\phi}{2^{n-1}} - \frac{1}{2^{n-2}} \cot. \frac{\phi}{2^{n-2}}.$$

Adding these terms,

$$S - \tan. \phi = \frac{1}{2^{n-1}} \cot. \frac{\phi}{2^{n-1}} - \cot. \phi.$$

$$\text{Therefore } S = \frac{1}{2^{n-1}} \cot. \frac{\phi}{2^{n-1}} - \cot. \phi + \tan.$$

$$= \frac{1}{2^{n-1}} \cot. \frac{\phi}{2^{n-1}} - \frac{\cot.^2 \phi - 1}{\cot. \phi}.$$

$$= \frac{1}{2^{n-1}} \cot. \frac{\phi}{2^{n-1}} - 2 \cot. 2\phi.$$

LXI. (*Proposed by Mr. T. Wilkinson, Burnley.*) Required the sum of the squares of the reciprocals of the *sixth* order of figurate numbers, $\frac{1}{1^2} + \frac{1}{8^2} + \frac{1}{36^2} + \frac{1}{120^2} + \frac{1}{330^2} + \frac{1}{792^2} +$ &c. *ad infinitum*; it being understood that 1, 3, 6, 10, 15, &c., is reckoned the *first* order.

Solution by the Proposer.

By Dr. Rutherford's method of summation (*Art.* 1. *Ladies' Diary*, 1837.) putting $S = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} +$ &c. *ad infinitum* $= \frac{\pi}{6}$, and A, B, C, &c., the several series there given, we have $n = 6$; and $(1-1)^6 = 1 - 6 + 15 - 20 + 15 - 6 + 1$. $\therefore \frac{1}{1^2} + \frac{1}{8^2} + \frac{1}{36^2} + \frac{1}{120^2} +$ &c., *ad infinitum* $=$

$$7^2 \left\{ \begin{array}{l} 924S - 1584A + 990B - 440C \\ \quad + 132D - 24E + F \\ - 6^2S_1 + 180A_1 - 240B_1 + 180C_1 \\ \quad - 72D_1 + 12E_1 \\ - 15^2S_2 + 600A_2 - 450B_2 + 180C_2 \\ \quad - 30D_2 \\ - 20^2S_3 + 600A_3 - 240B_3 + 40C_3 \\ - 15^2S_4 + 180A_4 - 40B_4 \\ - 6^2S_5 + 12A_5 \\ - \quad S \quad \ldots \end{array} \right.$$

$= 7^2\left(154\pi^2 - \frac{114023}{73}\right)$; by substitution and reduction:—the sum of the series required.

Q. E. I.

Solution by Digamma, Glasgow.

The summation of such series by the operose methods of the Calculus of Limited Differences and of the Integral Calculus has been superseded by the simple and easy processes published by Messrs. Beecroft[1], Rawson[2], Rutherford[3], Woolhouse[4], and Young[5]; whose ingenious investigations must be consulted by those who are desirous of becoming acquainted with this subject.

The following results are derived from Mr. Beecroft's formula (q) p. 95[1].

The general term of a series of the squares of the reciprocals of the figurate numbers of the sixth order (on the supposition here made,) is

$\frac{1^2 \cdot 2^2 \cdot 3^2 \cdot 4^2 \cdot 5^2 \cdot 6^2 \cdot 7^2}{n^2(n+1)^2(n+2)^2 \ldots (n+6)^2}$. Let the numerator $= a^2$, and the series to be summed is

$$a^2 \left\{ \frac{1}{(1.2.3.4.5.6.7)^2} + \frac{1}{(2.3.4.5.6.7.8)^2} + \frac{1}{(3.4.5.6.7.8.9)^2} + \text{\&c. in infinitum.} \right\}$$

The sum is found, as above, to be

$$\frac{77a^2}{43200}\left\{\frac{\pi^2}{6} - \frac{130277}{79200}\right\} = 45276\left\{\frac{\pi^2}{6} - \frac{130277}{79200}\right\}$$

$$= 7546\left\{\pi^2 - \frac{130277}{13200}\right\}.$$

(1) Lady's and Gentleman's Diary for 1848. (2) The Mathematician, vols. i. and ii. (3) Lady's and Gentleman's Diary for 1832. (4) Ib. for 1836. (5) Philosophical Magazine, N.S., vols. x. and xi., and in his Mathematical Dissertations (1841), p. 120 *et seq.*

LXII. (*Proposed by Nemo.*)

Shew that $\frac{1}{1 + x^{m-n} + x^{m-p}} + \frac{1}{1 + x^{n-m} + x^{n-p}} + \frac{1}{1 + x^{p-m} + x^{p-n}} = 1$.

Solution by the Proposer.

Correct the error in the print in the second fraction, and then multiplying the numerator and denominator of the third fraction by x^{m-p}, it becomes $\frac{x^{m-p}}{1 + x^{m-p} + x^{m-n}}$.

$$\therefore \frac{1}{1 + x^{m-n} + x^{m-p}} + \frac{1}{1 + x^{n-m} + x^{n-p}} + \frac{1}{1 + x^{p-m} + x^{p-n}}$$

$$= \frac{1 + x^{m-p}}{1 + x^{m-n} + x^{m-p}} + \frac{x^{m-n}}{1 + x^{m-n} + x^{m-p}}$$

$$= \frac{1 + x^{m-n} + x^{m-p}}{1 + x^{m-n} + x^{m-p}} = 1.$$

LXIII. (*Proposed by Nemo.*) Given $x^4 + y^4 = xy(x+y)^2$ and $y^2\sqrt{x} = \sqrt{(x+y)^3}$ to find x and y.

Solution by Mr. James Tomlinson, Ashton-under-Willows, Lancashire.

From 1. dividing by $x + y$,

$x^4 - x^3y + x^2y^2 - xy^3 + y^4 = x^2y + 2x^2y^2 + xy^3$

Transposing $x^4 - 2x^3y - x^2y^2 - 2xy^3 + y^4 = 0$

Div. by x^2y^2, $\frac{x^2}{y^2} - 2 \cdot \frac{x}{y} - 1 - 2\frac{y}{x} + \frac{y^2}{x^2} = 0$

$$\frac{x^2}{y^2} + \frac{y^2}{x^2} - 2\left(\frac{x}{y} + \frac{y}{x}\right) = 1$$

Completing the square and extracting the root,

$\frac{x}{y} + \frac{y}{x} = 3$ or -1 ...

Taking the 1st value, &c. $\left(\frac{x}{y}\right)^2 + 1 = 3 \cdot \frac{x}{y}$

Transposing and completing the square, &c.

$\frac{x}{y} = \frac{3 \pm \sqrt{5}}{2}$ ...

From $2\,xy^2 = (x+y)^2$;

$\frac{3 \pm \sqrt{5}}{2} \cdot y^2 = \left(\frac{x}{y} + 1\right)^2 \cdot y^2 =$

$\left(\frac{5 \pm \sqrt{5}}{2}\right)^2 \cdot y^2$

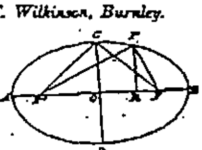
$\therefore\ y^2 = \frac{(5 \pm \sqrt{5})^2}{4 \cdot (3 \pm \sqrt{5})} = \frac{50 \pm 10 \cdot \sqrt{5}}{4}$,

and $y = \frac{5}{2} \cdot \sqrt{2 + \frac{2}{5}\sqrt{5}}$,

and $x = \frac{\sqrt{50 \pm 20\sqrt{5}}}{3 + \sqrt{5}} = \frac{5}{2}\sqrt{10 \pm \frac{22}{5}\sqrt{5}}$.

LXIV. (*Proposed by Nemo.*)—At what point of an ellipse is the angle formed by the focal distances the greatest?

Solution by Mr. T. Wilkinson, Burnley.



Let AB, CD be the axes of the ellipse. Put OB $= t$; OC $= c$; OF $= e$; OR $= x$....

Then by Conics FP $=$

$\frac{t^2 + ex}{t}$; F'P $= \frac{t^2 - ex}{t}$; F'P $= 2e$

$\therefore$ cos. F'PP $= \frac{t^4 - 2e^2 t^2 + e^2 x^2}{t^4 - e^2 x^2}$.

But when the angle is a *maximum*, its cosine is a *minimum*.

$\therefore\ \frac{4t^2 e^2 x\,(t^2 - e^2 t^2)\,dx}{(t^4 - e^2 x^2)^2} = 0.$ Whence $x = 0$;

and consequently the angle F'CF at the extremity of the conjugate axis CD is the greatest possible.

Q. E. I.

Solution by Diyamma, Glasgow.

Let F and F' be the foci, P the required point of the ellipse whose focal distances are r and r'. FF' $= 2f$. By a property of the curve $r + r'$ $= 2a =$ axis major .... (1)

From triangle FPF' and...... (1)

$\cos.\frac{P}{2} = \frac{a^2 - f^2}{r\,(2a - r)}$ ...... (2)

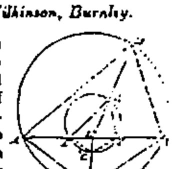
Now, angle P will be the greatest when cos. $\frac{P}{2}$ is least, which it will be when the denominator of (2) is greatest, the numerator being constant.

Let $u = r\,(2a - r) = 2ar - r^2 - a^2 + a^2 = a^2 - (a - r)^2$; and u will have its greatest value for $(a - r)^2 = 0$, whence $r = a = r'$, by (1)

$\therefore$ P is at either extremity of the axis minor.

LXV. (*Proposed by Nemo.*)—If on the given chord of a circle as a base triangles are inscribed in the circle, it is required to find the locus of their centres of gravity.

Solution by Mr. T. Wilkinson, Burnley.



Let AC be the given chord, ADC; ABC any two positions of the inscribed triangles; H the middle of the base, G and G' the centres of gravity of the two triangles. Then since in *all* positions of the triangle ADC, HG $= \frac{1}{3}$ HD, in the extreme cases when HD coincides with AG, we have HE $= \frac{1}{3}$ HA and HF $= \frac{1}{3}$ HC. $\therefore$ E and F are points in the locus and EF $= \frac{1}{3}$ AC is constant. Consequently EG, FG are parallel respectively to AD, DC, and $\angle$ EGF $= \angle$ ADC $=$ given vertical angle, and $\therefore$ the locus of G is a circle by Euclid iii. 21. Similarly the $\angle$ EG'F $= \angle$ ABC, and the locus of G' is a circle. Hence since $\angle$ADC $+ \angle$ABC $= \angle$ EGF $+ \angle$ EG'F $=$ two right angles, the locus of the centres of gravity G and G' is the circle EGFG'E.

Q. E. D.

LXVI. (*Proposed by Nemo.*)—If $\phi - a$, ϕ, $\phi + a$, be three angles whose cosines are in harmonical progression, show that

$$\cos.\ \phi = \sqrt{2}.\ \cos.\ \frac{a}{2}.$$

Solution by Mr. T. Wilkinson, Burnley.

Since the cosines are in *harmonical*, their reciprocals will be in *arithmetical*, progression.

$$\therefore\ \frac{1}{\cos.\,(\phi + a)} + \frac{1}{\cos.\,(\phi - a)} = \frac{2}{\cos.\,\phi}.$$

$$\therefore\ \frac{2\cos.\,\phi\ \cos.\,a}{\cos.\,(\phi + a)\cos.\,(\phi - a)} = \frac{2}{\cos.\,\phi}.$$

$$\text{Whence}\ \frac{\cos.\,\phi\ \cos.\,a}{\cos.^2\,\phi - \sin.^2\,a} = \frac{1}{\cos.\,\phi}.$$

$$\therefore\ \cos.^2\,\phi = \frac{\sin.^2\,a}{1 - \cos.\,a} = \frac{1 - \cos.^2\,a}{1 - \cos.\,a}$$

$$= 1 + \cos.\,a. = 2\cos.^2\,\frac{a}{2}.$$

$$\therefore\ \cos.\,\phi = \sqrt{2}.\ \cos.\,\frac{a}{2}.$$

Q. E. I.

LXVII. (*Proposed by Nemo.*)

Given $x^3 - 12x^2 + 57x - 94 = 0$, to find x.

Solution by Diyamma, Glasgow.

Take away the second term by substituting $y + 4$ for x...

$\therefore y^3 + 9y + 6 = 0$, which, solved by Cardan's method, gives $y = 3^{\frac{1}{3}} - 9^{\frac{1}{3}}$

$\therefore x = 4 + 3^{\frac{1}{3}} - 9^{\frac{1}{3}} = 3.3621659....$

The equation divided by

$x - 4 - 3^{\frac{1}{3}} + 9^{\frac{1}{3}}$ gives

$x^2 - \left(8 - 9^{\frac{1}{3}} - 3^{\frac{1}{3}}\right)x + 19 + 5 \times 9^{\frac{1}{3}} - 3^{\frac{1}{3}} = 0$

whence the other two values of x, which are imaginary, are

$x = 8 + 9^{\frac{1}{3}} - 3^{\frac{1}{3}} \pm \left\{-3\,(6 + 9^{\frac{1}{3}} + 3^{\frac{1}{3}})\right\}^{\frac{1}{2}}$.

NEW QUESTIONS.

LXVIII. (*Proposed by Mr. W. H. Hoare, Lower Clapton.*)—The rent of a farm is £n, and a fine of £b. is demanded for a lease of n years: what ought the rent to be, if the fine be remitted?

LXIX. (*Proposed by Mr. E. Maxwell Dillon, A. B.*)—

$$\frac{2y^2 - 6\sqrt{x}}{\sqrt{x}} + (4y^2 - 16x^{\frac{1}{2}})^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}\ (1)$$

$$x^{\frac{1}{2}} + \sqrt{8\,(y - x^{\frac{1}{2}}) - 4} = y + 1 ......\ (2)$$

to find x and y.

LXX. (*Proposed by Mr. W. O. Slinthin, Leyburn.*)—If a^1, b^1, be the segments of the hypotenuse made by a line bisecting the right angle, the

$\frac{a^1\,b^1}{a\,b} = \frac{a^2 + b^2}{(a + b)^2}$; required proof.

LXXI. (*Proposed by Mr. W. O. Slinthin, Leyburn.*)—Three right lines are given in position: it is required to construct a triangle which shall have its vertices on the given lines, and its perimeter a *minimum*.

LXXII. (*Proposed by Mr. W. O. Slinthin, Leyburn.*)—Given $x^2 + y^2 = 34$,

$xy + y = 15$; to find x and y by quadratics, if possible.

LXXIII. (*Proposed by Mr. W. O. Slinthin, Leyburn.*)—If 5 be taken from the sum of the squares of two consecutive numbers each prime to 3, the remainder will be divisible by 36; required proof.

LXXIV. (*Proposed by Mr. T. Elliott.*)—If straight lines be drawn from the angular points of a triangle, through any point within it, to meet the opposite sides, and the points of section be joined, the former lines will be cut harmonically.

LXXV. (*Proposed by Mr. E. M. Collins.*)—Inscribe a square in the segment of a circle.

LXXVI. (*Proposed by B. H.*)—Take any point D in a triangle, and through D draw a straight line which shall bisect the triangle (by the first books of Euclid.)

LXXVII. (*Proposed by Mr. James Lavin, Swinford, County Mayo, Ireland.*)—Through a given point within a circle given in magnitude and position, draw a chord; such that the rectangle of the ratio of straight lines joining its extremities with another given point, shall be given; and shew the analysis.

LXXVIII. (*Proposed by Mr. James Lavin, Swinford, County Mayo, Ireland.*)—AB is one of the principal axes of an ellipse given every way, CD an ordinate to it given in position. Required the point P in the Periphery, so that by drawing AP and PB, the figure CmPB shall be a given or a maximum (m being the intersection of AP with the ordinate CD); and give the analysis.

LXXIX. (*Proposed by Hibernicus.*)—Find the relation of a log. from the differential equation

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{y^2 - x^2}{y^2 x^2}.$$

LXXX. (*Proposed by Hibernicus.*)—Divide a given right line into two parts, such that the equilateral triangle described on one part shall be n times that on the other, within the limits of the first Book of Euclid.

LXXXI. (*Proposed by Hibernicus.*)—Find (by aid of lines not exceeding the second order) a point in a straight line given in position, such that the rectangle contained by tangents drawn from the required point to two circles given in magnitude and position, shall be equal to a given space; and shew the analysis.

LXXXII. (*Proposed by Hibernicus.*)—Prove by the first Book of Euclid that the inscribed equilateral triangle, which has one of its sides parallel to the least side of any plane triangle, is greater than either of the equilaterals similarly inscribed in it with respect to the other two sides.

LXXXIII. (*Proposed by Hibernicus.*)—

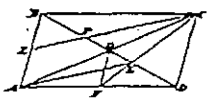
circle given in magnitude and position, place a chord of a given length, such that the rectangle contained by straight lines, joining its extremities with two given points, shall be equal to a given space or a maximum, and shew the analysis.

LXXXIV. (*Proposed by Hibernicus.*)—In the right line joining the centres of two given spheres, find geometrically a point, such that to an eye placed in it, the sum of the visible surfaces shall be a maximum, and shew the analysis.

LXXXV. (*Proposed by Hibernicus.*) — Given $\frac{4}{x}+\frac{5x}{4}=x^{\frac{3}{2}}+\frac{x^{2}}{4}+6$. Find x by quadratics.

LXXXVI. (*Proposed by Hibernicus.*) — Divide a given circular segment into two parts having a given ratio by a right line parallel to its chord.

LXXXVII. (*Proposed by Mr. E. M. Dillon, Taunton.*)—

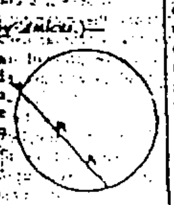


To prove within the limits of first Book of the Elements that "If right lines CF, CE, be drawn from one angle of a parallelogram ABCD to the points of bisection of the opposite sides AB, AD, they trisect the transverse diagonal BD."

LXXXVIII. (*Proposed by Amicus.*)—

Shew that $(a+b)^{n}=(n+1)a^{n}\Big\{1+$ [illegible] $\left(\frac{b^{2}}{a^{2}}-\frac{b}{a}+1\right)$ $+\frac{n}{2}$ [illegible] $\frac{n-2}{4}\left(\frac{b^{3}}{a^{3}}-\frac{b^{2}}{a^{2}}+\frac{b}{a}-1\right)$ $+\frac{n}{2}\cdot\frac{n-1}{3}\cdot\frac{n-2}{4}\cdot\frac{n-3}{5}\left(\frac{b^{4}}{a^{4}}-\frac{b^{3}}{a^{3}}+\right.$ [illegible] $\left.\right)+\text{\&c.}\Big\}$.

LXXXIX. (*Proposed by Amicus.*)—



Let PA be drawn through a given point, B and terminated by the circumference of a given circle, be cut in B so that the rectangle PA . PB = a given space, it is required to determine the locus of the point B.

To Mathematical Correspondents.—A contributor complains of the short time allowed for the solution of Mathematical problems; but if the time were extended, we feel fully assured that we should be altogether unable to examine and acknowledge the greatly increased productions of contributors.—Correspondents should place their names or their signatures on each of their papers.

Solutions also received—

56, 58, 59, 60, 61, 62, 63, 64, 65, 66, by Digamma, Glasgow.

54, 64, by Mr. T. Elliott.

54, 56, 58, 64, by Mr. T. J. Leeming, Woolwich Common.

54, 60, 62, 65, 66, 67, by Mr. Thomas Cranston, East Keswick, Yorkshire.

54, 55, 59, 60, 67, by Mr. John Giblin, Glasgow.

54, 59, by Mr. C. Tooncliffe, Marton.

55, by Mr. Robert Davidson, Glasgow.

55, 59, 60, by W. O., Jersey.

65, 66, 67, by Mr. J. Brookes, Manchester.

58, 60, 61, 64, 65, 66, 67, by Mr. T. Wilkinson, [illegible]

59, 67, by Mr. G. Swift, Norwich.

59, 60, 64, 65, 66, by Mr. J. Alsop, Taunton.

59, 60, 63, 64, by J. M., Biggleswade.

63, 67, by Mr. J. Tomlinson, Ashton-le-Willows, Lancashire.

64, 65, by G. H. S.

67, by Mr. Wm. Lundy, Rastrick.

One new question sent is illegible.

Want of space prevents the insertion of several Articles and Problems this month; but they shall appear in the following Number.

ANSWERS TO CORRESPONDENTS.

Communications intended for insertion in our next should reach us before the 16th inst.

We cannot undertake to return rejected Articles.

Press of matter obliges us to keep back several valuable contributions; others under consideration.

R. W.—Many Numbers of the *Educational Times* are out of print. An early application to the publisher is advisable.

J. J. T. W.—"Principals of Schools, and Teachers, established prior to the date of the Charter, shall be recommended to the Council by a written certificate, signed by two or more Members, or other persons of known respectability, which certificate shall specify the Christian and Surname of the candidate, the date at which he established his school or became a Teacher, and his place of residence," &c. (*Extract from the Bye Laws of the College of Preceptors.*) Mr. Parker, Secretary of the College, will supply further particulars.

REVIEWS.

Parental Wisdom; or, The Philosophy and Social Bearings of Education, with historical illustrations of its power, its political importance, &c. By the Author of "The Wrongs of Poland." Second Edition. Saunders and Otley.

This book is certainly the production of an earnest Educator, but, as certainly, it cannot be regarded as the work of a clever book-maker.

The Author, led away by the heartfelt interest he takes in the subject, and relying on the extent and kind of his own experience, treats with little deference the intelligence and good will which his readers bring to bear on it. In hortation and discussion he is therefore tediously laborious; taking great pains to explain points which all his readers must understand, and urging them to duties which few will be disposed to shirk. This prolixity has the additional unhappiness of being intentional. It is not the babble of an ancient Educator muttering his experience with the garrulity of age, but the precise and pompous spouting of a school-room rhetorician, bent on uttering "words, words, words."

Hear the grave profession to this effect.

"To plead the more effectually with the parental heart—and to win the acquiescence of that most interesting and influential class, 'the fair mothers of our isle,' (on whom as to earlier and abiding impressions so much depends,) as well as to give all the attraction in his power to a much hackneyed but all-important theme, he has given the rein to contemplative thought, and freely decked his composition with such graces of figurative language as might consistently harmonize with, and serve to illustrate, a subject so lofty in its character, and so extensive in its application. In this liberty he has but followed the counsel of Longinus, the reputed father and oracle of criticism, who maintained that good thoughts could not be too well dressed, and that there was as much difference in an idea presented to the mind, through the medium of dignified and of common-place language, as in beholding an object by the light of the sun, or the glimmering of a taper."

Notwithstanding this flourish, those who read the book through will, we fear, join with us in thinking that the counsel of Longinus has been imprudently disregarded in this instance, and that the good thoughts of the book (and it contains many), are much too well dressed, or excessively over-dressed.

As this is the only quarrel we have with the Author, we get over it at once, in order that our notice of his well-meant effort to promote the cause of Education may not be unpleasantly qualified at the close. It is a good and wise habit to find out the faults of a new acquaintance as readily as may be, provided there is an honest intention, when the worst is ascertained, of parting friends with him, if he will allow it.

There are six Essays in this volume on the following subjects: I. The Importance of Education; II. Parental Responsibility; III. A Philosophical Theory of Education; IV. On the Prevalent Levity of Youth; V. Testimony derived from Ancient and Modern History; VI. Practical Hints to Parents.

These titles prove that the Author is as skilful a logician as he is an inapt disciple of Longinus. The dependence of these Essays on each other, and their collective connexion with the triple title affixed to the work, cannot very easily be seen. The Essays themselves answer better to their respective headings—old sermons to texts—presenting no novelty in matter or style, not even departing, for the sake of illustration, from the beaten track, which abounds with many hackneyed examples of the subject. Of the promised adornments with which these common-places are relieved, the following is a specimen,—obstructions the reader will have to overleap in nearly every page.

"On the boundless sea of metaphysical discussion the author of these pages has no desire to embark; philosophical theories he deems little worth, as they bear on matter of fact. Amid the troubled waters of speculative controversy he would not venture his bark, nor willingly encounter the rough waves of that fathomless abyss, where the farther we launch, the more fearful are the storms to be weathered; he would rather steer near the shore far from its hidden rocks, its quicksands and its breakers, which to many a gallant bark has proved fatal."

If, however, the Author has not [illegible] theory or practice; added to the stock of [illegible] tional experiences, nor even reproduced [illegible] the better things, he has entered with [illegible] fulness into the conflicting "sentiments" [illegible] respect to the treatment of childhood, and [illegible] and, weighing the evidence fairly, he [illegible] pronounces a discreet and moderate judgment, from time to time condescending to present with a good thought in its natural state, [illegible] the usual prodigality in dress. Witness this passage on the education of infants.

"In the most approved plans [illegible] simplicity, very properly, forms the leading [illegible] and is, perhaps, the most [illegible] part of the system. In whatever relates to the training of the infant mind, every departure from simplicity is a manifest defect; when, therefore, [illegible] for infantile education speak of infallible modes of teaching children to reflect ere they can give distinct utterance to their thoughts, [illegible] nature incline us to question the wisdom [illegible] of the attempt; supposing it were even possible, it is very doubtful whether it would be advisable, and those who profess [illegible] infants, are far more likely to injure than [illegible] them. Such means may, [illegible] produce a precocious and brittle wit, but this is opposed to a solid and rational [illegible] ensuing might thus be superinduced on that which, with an ignorant few, [illegible] for superior intelligence; but the [illegible] would reject the absurdity, [illegible] reason, that the right direction of [illegible] far exceeded any [illegible]

"It is more than probable that [illegible] nobler ideas and more [illegible]

4.4 Data entry in the *ET* database

Facsimiles of the original material referred to here can be found in fig. 4.4, June 1850, which contains a broad selection of data of interest, will be used as an example to illustrate how data is selected for entry into the database. It is to be emphasised that data entered into the database is kept in a form as close to the original as possible. Each group will now be examined in turn.

VOLUME

voldate JUNE 1850
volnum 3
partnum 33
volformat MATHEMATICAL QUESTIONS AND SOLUTIONS; SOLUTIONS; NEW QUESTIONS.

The first three are taken from the front page and the last one from page 205, inside the journal.

VOLINF

edcomment greatly increased productions of contributors

This is taken from the end of the mathematical department and points to the editorial comment regarding the sharp increase in contributions and their inability to cope with them.

bookinfo A Short and Easy course of Algebra. By THOMAS LUND, B.D.

This is located at the end of the mathematical department, in the 'REVIEWS' section.

articles long article 'On Mathematics as a branch of General Education' by J.Kendall Esq. - engineers, actuaries, astronomers.

The title of the article appears on the front page. 'Engineers...' is a selection of the text from this article giving a hint of its scope.

mathns [empty]

exams Examiners:- Professor Boole, J.J. Sylvester Esq., M.A., F.R.S. &c. Rev J. Hind M.A., F.C.P.S., F.R.A.S. &c.

This information is obtained from the front page.

instns [empty]

volmisc Mathematical Master wanted - Dissenter greatly preferred.

This is taken from the advertisements on the front page.

VOLUME and VOLINF have been investigated, but Q and S cannot be until the mathematical categories have been defined.

4.4.1 The codification of the questions/solutions into mathematical categories.
Three mathematical categories, **cat1**, **cat2**, **cat3** were deemed sufficient to describe the areas of mathematics contained in the questions. There is a separate element **meth** describing the style/method of solution actually requested in the text of the question.

Similarly, any solution can have up to three categories, **cat1**, **cat2**, **cat3** describing the areas of mathematics contained in the solutions. There is a separate element **meth** describing the style/method/s actually used in each solution.

The following is a table of the abbreviations used in the database for the styles/methods used for the questions and solutions.

Table 4.5

geom	Euclidean geometry, approach unspecified
geom-syn	solved using Euclidean geometry via synthetic approach
geom-an	solved using Euclidean geometry via analytic approach
alg	algebraic
ind	inductive
raa	*reductio ad absurdum*
mp	*modus ponens*
an	analytic *per pro* Descartes as opposed to Analysis of Cauchy, Weierstrass etc.
lim	limits etc using tools of Analysis (limits, convergence, infinitesimals...)
co-ord	analytic methods using rectilinear co-ordinates

disc	discursive
trig	trigonometrical

The mathematical categories are now given. They are organised in appropriate sections. The codes used in the database are also given, plus extra explanation where appropriate.

Table 4.6

ARITHMETIC (ARITH)	
arith	Arithmetic
num	Theory of Numbers:
ALGEBRA (ALG)	
appeqns	approximating roots of eqns
algmisc	miscellaneous algebra
algexp	use of algebraic expressions
seqns	solving equations
seqnsq	solving equations by quadratics
redeqns	reducing equations
reseqns	resolving equations
exp	expansions (binomial etc.)
gps	theory of groups
log	logic
TRIGONOMETRY (TRIG)	
ptrig	planar trigonometry
strig	spherical trigonometry
trigf	trigonometric Functions
gon	goniometric
GEOMETRY (GEOM)	
geom	elementary geometry - general planar
geom-sol	elementary geometry - general solid
rgeom	rectilinear geometry - lines, rectangles, triangles etc.
cirtri	circles and triangles in elementary geometry
cgeom	circular circles only, in elementary geometry
curv	other curves other curves, spirals etc.
conic	conic sections

sangeom	solid analytic geometry
pangeom	plane analytic geometry - general
pangeomc	plane analytic geometry - circular
pangeomr	plane analytic geometry - rectilinear
pangeomcr	plane analytic geometry - both circular and rectilinear
proj	projective geometry
desc	descriptive geometry
DIFFERENTIATION & INTEGRATION (DIFF)	
diffc	differential calculus
de's	differential equations
intc	integral calculus
ints	integrals
diffeqn	difference equations
APPLIED MATHS (APP)	
static	statics
dyn	dynamics
eng	engineering
ast	astronomy
hyd	hydrostatics/dynamics
optic	optics
ANALYSIS (AN)	
anlim	analysis using limits
func	functions
fseries	finite series
iseries	infinite series
PROBABILITY, STATS (PROB)	
comb	combinatorics
prob	probability
mstats	mathematical statistics
MISC (MISC)	
misc	misc

The above list will now be repeated with a fuller description of each category.

ARITHMETIC (ARITH)

arith (arithmetic)

Problems involving numerical calculations only, without algebra. These include financial problems.

num (theory of numbers)

This includes diophantine analysis, as well as equations where, for example, the right hand side is given as a small square, representing a square number.

ALGEBRA (ALG)

appeqns (approximating roots of eqns)

Here approximating methods, such as Horner's, are used to estimate roots of equations.

algexp (use of algebraic expressions)

This is where normally a solution contains a substantial use of algebraic manipulation without solving, reducing or approximating equations. This category would be used where a geometrical problem calls for the production of a particular algebraic expression.

seqns (solving equations)

This covers all polynomials.

seqnsq (solving equations by quadratics)

This is used in both questions and solutions where solution via quadratics is named.

redeqns (reducing equations)

Used when equations are required to be reduced.

reseqns (resolving equations)

Used when equations are resolved into factors.

algmisc (miscellaneous algebra)

This is for problems where algebra is used, but doesn't fit into any of the above categories.

exp (expansions - binomial etc.)
Expansions such as Taylor's, binomial come under this category as opposed to infinite series.

gps (theory of groups)
This includes any early work in this area.

log (logic)
Specifically involving logic

TRIGONOMETRY (TRIG)

ptrig (planar trigonometry)
Plane trigonometry using plane trig expressions.

strig (spherical trigonometry)
Spherical trigonometry using expressions in spherical coordinates.

trigf (trigonometric functions)
Used where substantial use of trigonometric functions is used in e.g. an algebraic context as opposed to in plane trigonometry.

gon (goniometric)
Where purely angles are considered.

GEOMETRY (GEOM)

geom (elementary geometry - general planar)
General planar elementary geometry. This category is used very rarely, as it is very broad. It has only been used in the posing of a broad question about Euclidean geometry, where it is impossible to narrow it further into rectilinear or curvilinear.

geom-sol (elementary geometry - general solid)
Euclidean solid geometry

rgeom (rectilinear geometry)
Lines, rectangles, triangles etc in Euclidean geometry.

cirtri (circles and triangles in elementary Euclidean geometry)

Circular cgeom circles only, in elementary Euclidean geometry

sangeom (solid analytic geometry)

Solid analytic geometry

pangeom (plane analytic geometry)

This category is to be used with **curv** and **conic** below to show they are being used in the context of plane analytic geometry and not Euclidean.

pangeomc (plane analytic geometry - circular)

Plane analytic geometry with circles.

pangeomr (plane analytic geometry - rectilinear)

Plane analytic geometry with lines, rectangles, triangles and so on.

pangeomcr (plane analytic geometry - circular)

Plane analytic geometry with both circles and lines, rectangles, triangles and so on.

proj (projective geometry)

To satisfy description of projective geometry as it stands today, and not including the modern geometry of the time unless these conditions were satisfied.

desc (descriptive geometry)

As per the work and influence of Gaspard Monge.

curv (other curves)

Other curves, spirals etc. This is found with pangeom for plane analytic geometry, or rgeom or cgeom for Euclidean geometry.

conic (conic sections)

This is found with pangeom for plane analytic geometry, or rgeom or cgeom for Euclidean geometry.

DIFFERENTIATION & INTEGRATION (DIFF)

diffc (differential calculus)

Use of the differential calculus.

de's (differential equations)
Use of differential equations.

diffeqn (difference equations)
Use of difference equations.

intc (integral calculus)
Use of integral calculus.

ints (integrals)
Use of integrals in, for example, area under curve calculations.

APPLIED MATHS (APP)

static (statics)
Mechanics without motion.

dyn (dynamics)
mechanics with motion.

eng (engineering)
Problems involving engineering - trains, canals etc.

ast (astronomy)
Astronomical problems excluding curvature of earth (see geodesic below).

hyd (hydrostatics/dynamics)
Problems involving fluids.

optic (optics)
Optical problems - including refraction.

geo (geodesic)
Problems involving e.g. curvature of earth.

ANALYSIS (AN)

anlim (analysis)
Analysis using limits, that is, using very similar methods to the way analysis is used today.

func (functions)
Using functions of a one or more variables.

fseries (finite series)
Finite series, but excluding binomial expansions.

iseries (infinite series)
Infinite series, but excluding binomial expansions.

PROBABILITY, STATS (PROB)

comb (combinatorics)
Combinations and permutations.

prob (probability)
Probability and chance.

mstats (mathematical statistics)
Mathematical statistics.

MISC (MISC)

misc (miscellaneous)
Anything that has an unusual aspect merits this final category.

The above categories were arrived upon after studying the mathematics in the questions/solutions, devising categories, reinspecting the mathematics then revising the categories. This process went through many iterations before the final product above was decided upon and will be described in full in 4.5.2. The Royal Society Catalogue of Scientific Papers was consulted, even though this was collated at a much later date [1908]. The mathematical classifications were checked for compatibility with categories found in the *ET* itself, for example, in examination papers.

Now the mathematical categories have been defined, the decisions taken when entering data from a question and its related solution can now be inspected.

4.4.2 A detailed analysis of data entry into κλειω of one question.

Question 20, posed in December 1849 will be analysed in detail. The question runs as follows, 'XX. Required a simple method of finding the equation to the Lemniscate of Bernouilli.'

The entry into the database for **Q** is as follows

q$o/20/020/pangeom/curv///Lemniscata of Bernouilli

q$ denotes the group **Q**, the elements are separated by a slash / sign and are as follows:

status = o [as the question is proposed for the first time, not reproposed.]
num = 20
prob = 020
cat1 = pangeom (as the curve entitled 'The Lemniscate of Bernouilli' falls into the category of plane analytic geometry.)
cat2 = curv as the Lemniscate of Bernouilli falls into the category of other curves. **cat3** is blank.
No particular method of solution is requested in the question, so **qmeth** is blank.
misc contains the string 'Lemniscate of Bernouilli'.

There are no references to books or journals in this question, so **BOOK** and **JOURNAL** are empty, that is, all their elements are blank.

CONTRIB is not blank as details are given about the proposer. The database entry is:

contrib$m/Tebay/Septimus/////Preston

Here **contrib$** denotes the group **CONTRIB** with elements:

status = m [as Septimus is a recognised male forename].
surname = Tebay
fname1 = Septimus
No details are given about his title, qualifications, county, country, institution or occupation. Hence **fname2, fname3, title, qualfn, county, country, instn, occpn** are all blank. However, the town is given, so
town = Preston

Note that there are four blank elements between **fname1** and **town** (**fname2, fname3, title** and **qualfn**). These are denoted by ///// five slashes containing four blank spaces. The blank elements **county, country, instn, occpn and cmisc** do not have to be denoted in that manner as they fall after the last entered non-blank element.
The corresponding 'solution' is given in January 1850 and is as follows:

s$si/20/020/pangeom/curv/rgeom/co-ord
solver$m/Wilkinson/Thomas/////Burnley
s$s/20/020/pangeom/curv/conic/co-ord
solver$m/Wilkinson/Thomas/////Burnley
s$d/20/020/pangeom/cgeom/trigf/an/polar equation
solver$m/Tebay/Septimus/////Preston

There are three solutions to this question.

The first solution

s$si/20/020/pangeom/curv/rgeom/co-ord
solver$m/Wilkinson/Thomas/////Burnley

s$ denotes the group **S**

status = si	(means that the question has been solved and that a diagram is given).
num = 20	
prob = 020	
cat1 = pangeom	(the equation of the curve is given).
cat2 = curv	(the Lemniscate of Bernouilli belongs to the other
curves	category).
cat3 = rgeom	(rectangular coordinates are used).
meth = co-ord	(coordinate geometry is the method used to solve this question).

solver$ denotes the group **SOLVER** with elements:

status = m	(male)
surname = Wilkinson	
fname1 = Thomas	
town = Burnley	

All the other elements are blank.
No books or journals are mentioned, so **BOOK** and **JOURNAL** simply do not appear in the data file. κλειω is a very economical system and it is only necessary to enter what is there.

<u>The second solution</u>

s$s/20/020/pangeom/curv/conic/co-ord
solver$m/Wilkinson/Thomas/////Burnley

This is very similar to the first solution. The elements are identical, with the exception of

status = s	(no diagram is given for the second solution)
cat3 = conic	(as the equations for the hyperbola and ellipse are used).

<u>The third solution</u>

s$d/20/020/pangeom/cgeom/trigf/an/polar equation
solver$m/Tebay/Septimus/////Preston

The elements are the same as in solutions 1 and 2 except for

status = d	(solved by the proposer, Septimus Tebay).
cat2 = cgeom	(a circle is used as the basis of the solution).
cat3 = trigf	(trig. functions play an important part in this solution).
meth = an	(the solution is solved via analysis as per Descartes).
misc = polar equation	

4.4.3 Selected data entry examples

Examples will now be given of questions and solutions containing information relating to books and/or journals. Other elements of interest will also be highlighted.

Example 1

s$si/1/001/geom///geom-an/Euclid 33.3
solver$m/Wilkinson/Thomas/////Burnley/Lancashire
book$Horae Geometry/Davies////Prop 7

misc contains a reference to Euclid book 33 propostion 3.
book$ represents the group **BOOK**
bktitle is Horae Geometry
asurname is Davies

bmisc contains Prop 7

Example 2

s$di/21/021/rgeom/alg//geom-an;alg/Analysis; imagine the thing done;construction

solver$p/Geometricus

book$Geometry/Simpson

solmeth contains two entries:-'geom-an' and 'alg' as both were used in the solution. Both can be queried equally in the database and have equal value.
solmisc contains 'Analysis; imagine the thing done; construction'. This highlights the fact that an analytic approach to geometry was accentuated.
status is 'p' indicating that the solver used a pseudonym.
surname is 'Geometricus', the pseudonym.

Example 3

q$o/29/029/cirtri////first book Euclid

contrib$m/Wilson/R///Rev/D.D.

This question demonstrates the use of the elements **title** and **qualfn**.
title is Rev
qualfn is D.D. [Doctor of Divinity]

Example 4

s$si/17/017/cirtri///geom-an/Analysis;pure geometry

solver$m/Wilkinson/Thomas/////Burnley

journal$Mathematical Companion//10//page 107

journal$ represents the group **JOURNAL**
jname is Mathematical Companion
jvol is 10
jmisc is page 107

Example 5

s$d/32/032/cgeom/conic//geom

solver$m/Wilkinson/Thomas/////Burnley

book$Hutton/Davies////vol 2 prop xvii; p 121

book$Anal. Geom./Young/vol 1//p190

journal$Diary////question, 1715

This shows two books and one journal being referred to in the same question.

> *Example 6*
> **q$o/51/051/cgeom/rgeom**
> **contrib$m/Wilkinson/Thomas/////Burnley**
> **journal$Phil. Mag//4//sec 3, No 239; Professor Davies's Geometry and Geometers**

The above shows reference to the *Philosophical Magazine* - a journal of note.

> *Example 7*
> **q$o/57/057/num**
> **contrib$m/Harley/R///Mr//Taunton///Dissenters' College/Mathematical Tutor**

This gives an example of both a contributor's institution and their occupation.
instn is Dissenters' College
occpn is Mathematical Tutor

> *Example 8*
> **q$o/44/044/alg**
> **contrib$m/Wilson////Dr**

This gives an example of Dr as **title**.

4.5 Error checking and data integrity in the *ET* database

The above sections concentrate upon the creation of a sound database structure. However, equally vital is the integrity of the data entered into the database. Thorough procedures have been put in place to ensure data integrity. The whole area of data consistency and error checking will be described now in detail.

Attempts have been made to discover established common practice regarding error checking and data consistency. It appears more detail is given regarding joint projects as opposed to individual ones. The creation of [*The Hartlib papers on CD-ROM* 1995] involved data input by a very large number of people. A specific method was used to check the integrity of the input data which involved work input by one person being double checked by another. Similarly, in the Master and Servant Project at

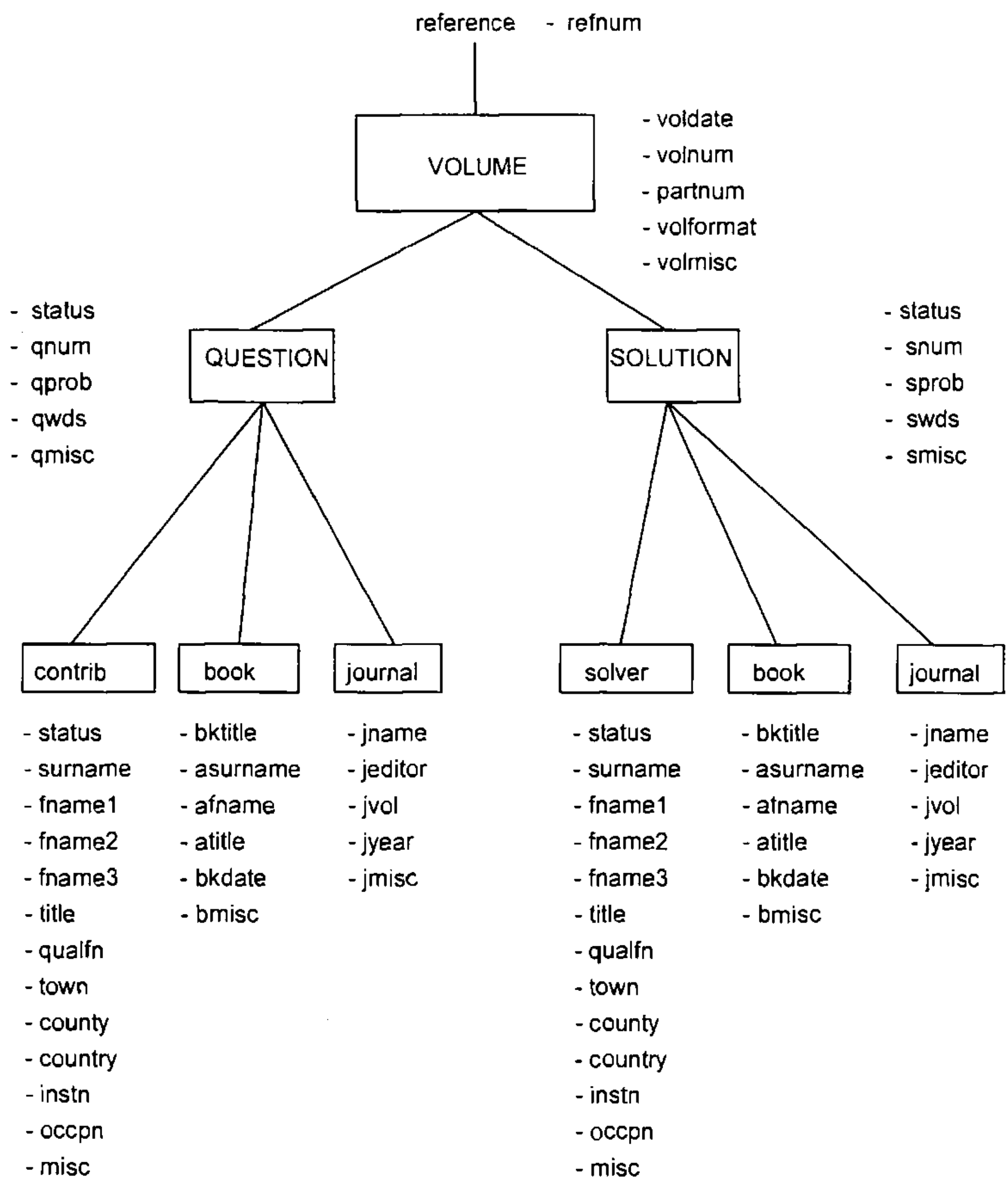
York University (Canada)[Craven and Hay 1995 71-4] a team of four data-inputters devised a method of double-checking each other's data entering and coding accuracy. These activities offer little practical assistance to the lone database creator.

One source of practical help was the 'ESRC Data Archive History Unit Guidelines For Documenting Historical Data, 1993' In this document it is suggested that methodological information should comprise, methods used to create the dataset, consistency checks, error corrections and other relevant information. These guidelines inspired the following summaries of consistency checking throughout the entire database production cycle. The life cycle of this project will now be visited.

4.5.1 Revision of the database structure

The pilot database (mentioned in section 4.3 and fig. 4.6) is substantially the same structurally as the final *ET* database. The groups **CONTRIB**, **SOLVER**, **BOOK** and **JOURNAL** are identical in both the pilot and final versions. The major differences lie higher up the hierarchy with **QUESTION**, **SOLUTION** and **VOLUME**. In the pilot study key words were entered in to the database via the element **qwds** for the questions and **swds** for the solutions. These were a sample of up to three mathematical key words selected from the text of the problems, for example, circle, triangle, and logarithm. The rationale behind this key wording was to preserve the integrity of the data, even though the key words were only a sample of the words in each problem. In the pilot study, a codebook was created within the κλειω DBMS to allocate codes to these key words. This was unsuccessful as it was impossible to attach meaningful codes to the key words out of the context of the originating question/solution. Also, the production of the codebook was a slow, time-consuming task, and was deemed inappropriate given the large amount of data entry involved in the *ET* database. The retention of partial key words would have allowed subsequent users of the database to impose their own coding systems in the light of new research or changes in methodology, and this would have been desirable. The information gleaned from the codebook in the pilot study was so poor that the decision to code the mathematical content of the questions/solutions at source was taken. This practice is second best in terms of stated source-oriented and κλειω methodology [Woollard and Denley 1993 xiv], but every effort has been made to ensure the coding itself is as comprehensive and accurate as possible, and also that it is fully and unambiguously described (4.4.1.). The key words **qwd** and **swd** for the questions and solutions respectively were replaced by **catl**, **cat2**, **cat3** and **meth** as described above.

Fig 4.7 Structure diagram for version 1 of κλειω *ET* database



The group **VOLUME** was subdivided following the pilot study. The elements **voldate, volnum, partnum** and **volformat** remained in the group **VOLUME** whilst the element **volmisc** was transformed into a separate group **VOLINF**. This was due to the fact that the contents of **volmisc** were becoming too voluminous and diverse to analyse meaningfully. The pilot study revealed a wealth of pertinent information each month so this was scrutinised and sub-divided into the elements of **VOLINF**, namely **edcomment, bookinfo, articles, mathns, instns, exams** and **volmisc**.

Following the creation and analysis of the database, minor alterations were made to the names of the elements as this greatly facilitated the querying of the database. Both **qnum** and **snum** were changed to the generic term **num.** This was prompted by a query, for example, to locate a particular person in the database, irrespective of whether they posed or solved a problem. Having located the person, a generic term, **num** is needed to locate either the question they posed or the solution they solved. Similarly, the generic term **misc** was used instead of **qmisc** and **smisc** to quickly locate terms of interest in the miscellaneous sections of either the questions or solutions. Finally, the groups **QUESTION** and **SOLUTION** were reduced to **Q** and S respectively for economy.

Two small changes were also made in the mod file at this stage. The function 'alias' did not appear to be working, and after much investigation, a semi-colon was found to be missing in the mod file. In the course of discussing this problem with colleagues with κλειω expertise, it was noted that 'alias' could be used with elements as well as groups. This was utilised, and a generic term **cat** was used in the mod file via a logical object to cover **cat1, cat2,** & **cat3**. This was extremely useful during subsequent querying.

4.5.2. Revising the mathematical categories

The categories went through many versions before reaching the final form. Initially, the data used in the pilot study was re-cast in the new data structure containing the elements **cat1, cat2, cat3,** and **meth**. These were then checked for appropriateness in consultation with the Director of Studies for the thesis. Revisions were made, and this process repeated until categories were seen to be being applied to problems with consistency and accuracy. There were 47 categories finally, representing the refinement needed during the revision iterations. An intermediate stage of 37 categories was reached and several years data entered using these categories. One of these categories was **alg** for algebra. This turned out to be a large category over several years and had to be sub-divided. This involved a significant amount of work, locating all the problems containing **alg** categories and reclassifying these problems into the new categories **appeqns**, **algmisc**, **algexp**, **seqns**, **seqnsq**, **reseqns** and **exp** as described in 4.4.1. There were approximately 500 such problems. Any problems containing the category **geom** were also revisited to ensure a subdivision into **rgeom** or **cgeom** could not be made. Where **geom** has been left as a category, it is in circumstances where the problem is so general as to preclude subclassification. **angeom** was too broad a category and all problems containing this category were located and recategorised as one of the **pangeom** categories, or **sangeom**. Similarly, an earlier classification of **trig** was refined into **ptrig** or **strig**. The classification **pangeom** was checked to ensure it accompanied **curv** or **conic** and that if the subject matter were pertinent to **pangeomc**, **pangeomr** or **pangeomcr** then these categories were used instead. Originally **calc** was not amongst the methods used in the element **meth**. This was rectified and all the problems containing the categories **diffc**, **intc** or **de's** were isolated and **calc** was replaced in the database as the method for that problem. The above process was lengthy and demanding but was vital to ensure the quality of the categorising process itself, and the subsequent analysis built upon these categories.

4.5.3 Data entry methods

Data entry on a large scale is repetitive and monotonous and it is difficult to ensure that erroneous data does not infiltrate the database. Surprisingly, no specific reference is made in classic database texts [Date 1995] concerning data entry methods. For the *ET* database, a data entry routine was devised that contained an element of double-checking. Firstly, details were entered into the group **VOLUME**, then **VOLINF**. Second all the solution numbers were entered, then the question numbers, each time entering the same value for the element **prob**. The solutions were then checked for diagrams, and 'i' was inserted where necessary. As this was carried out, the entries for **num** and **prob** were double-checked. Simultaneous to the search for diagrams, each solution was inspected to see whether it had been solved by the proposer. If so, an entry of 'd' was made for the **status** element. Very few questions possessed a diagram.

The solvers details were then inserted into the database. The database was then searched to see if the solver had appeared previously, and if so, their details would be copied and pasted to the current month, taking careful account of any variations (and there were many). The same was then undertaken for the proposers. Lastly, the categories and respective methods were entered, firstly for the solutions, then the questions. Any book or journal details were entered at this stage. This routine was carefully observed, and although not infallible, it did provide some degree of double-checking.

4.5.4 Spot checks

Regular, random spot checks were made on the data entered into the database by the Director of Studies for the thesis to inspect its accuracy.

4.5.5 Compilation errors

The κλειω DBMS reports on errors located during the compilation process. These were mainly of the following types,

1. There was a extra / character in a row of data, causing the κλειω DBMS to perceive a surplus data entry field.
2. The entry for say, the element **bkdate** was in the wrong format as prescribed by the logical object *dates*.
3. A / character was missed out before the element **surname**, causing an incorrect value for the element **status**, which can only take the values prescribed in the logical object *abbreviations*.
4. A character was used which is reserved exclusively for the κλειω DBMS, for example, an = sign.

The database was compiled several times, normally after the addition of a couple of years of data. Each time the data was compiled, there were only approximately 11 compilation errors.

4.5.6. 'Data position errors'

It is possible to go through the process of rectifying any errors produced during compilation and to believe the data may be error free. This is unfortunately untrue as not all errors are made manifest during the compilation process. For example, the error of placing a solver's surname in the position reserved for a solver's town would not show up during compilation. This type of error is far more insidious and can only be eradicated by the systematic investigation of the contents of each element. This was achieved by writing and recording a series of κλειω queries to give a count of the contents of each element in turn. Rogue data would then stand out, as would mis-spellings. These would then be corrected in the database, and at the end of the process it would be re-compiled. Many errors were located by this method, which again took some time to complete as κλειω stated at the end of the compiled version of the data that there were 26,510 elements of data.

There was one error located which was not corrected - the elements **exams** and **instns** in the group **VOLINF** had changed position during one of the many iterations. Unfortunately, data entry did not change to reflect this alteration and some **exams** data was entered into **instns** and vice versa. It would have been possible to go through each entry for these two elements and correct it, but as there were not many of these elements and there was considerable overlap in the subject matter of the data for these two elements, this was not undertaken.

4.5.7 'Data logic errors'

These are the most difficult of all to trace, and comprise, for example, a blank entry in each of the elements **catl, cat2 & cat3** for a particular problem. Similarly, the element **meth** must contain an entry for all solutions. These were checked via an appropriate query to ensure this was the case.

4.5.8 Miscellaneous errors

The entire database was inspected from beginning to end to check the problem numbers (the elements **num** and **prob** were found to be correct with the exception of a couple which were found with extra digits). All the volume dates were printed out and checked against the originals in the Institute of London Library, as they were irregular. Missing pages of data were identified, obtained from this library, and entered into the database.

4.6 Analysing the *ET* database

4.6.1. κλειω querying

In order to obtain information from the *ET* database, it was necessary to write κλειω queries. These are not dissimilar in nature to Structured Query Language (SQL) queries written to interrogate a relational database. Once written, the query is compiled and the result placed in a named DOS file. The query files can be documented to aid subsequent analysis. Many of the queries used were similar in structure.

An example of a typical simple query is now given with an explanation of its constituent parts.

```
note This query prints out the contents of the element 'town'
query name=et11;part=:town=not null
index part=:query[];type=count
stop
```

This query investigates the contents of the element **town** belonging to the group **CONTRIB** or **SOLVER**. The κλειω command **note** allows comments to be written after it in a κλειω query.

query name=et11 means that κλειω will query the database entitled et11.

part=:town=not null Within the et11 database, κλειω will select all the instances where the contents of the element **town** are not null.

index part=:query[];type=count means that κλειω will index the selection (denoted by the query function:-**:query[]**), and will count the instances of each separate term.

stop indicates the end of the query.
Further examples of κλειω queries are given in Appendix A, A5.1-A5.5.

4.6.2. Excel$_{TM}$ spreadsheets and charts

κλειω does not contain its own large-scale statistical functions, so it was necessary to export κλειω query results to the spreadsheet package Microsoft Excel$_{TM}$ version 5.0. Microsoft Excel$_{TM}$ is a constituent program of the Microsoft Office$_{TM}$ suite of programs and allows the creation of worksheets. These are spreadsheets, containing cells arrayed in rows or columns. The cells may contain textual data, numerical data or formulae which can manipulate data from other cells in the spreadsheet. This information can then be analysed or modified in a variety of ways, including sorting, filtering, and making charts from some or all of the cells. An Excel$_{TM}$ file is called a workbook and may contain many worksheets.

Before data could be transferred to Excel$_{TM}$, the κλειω results file needed editing, sometimes substantially. The final data would be left comma-delimited so it could be imported straight into Excel$_{TM}$. Once there, it was possible to perform statistics like average or count, or sort the results. Charts could then be created from sections of the results. Mostly the transfer process went smoothly between the two software packages. Once the results were in Excel$_{TM}$ format, further possibilities for analysis became clear and were undertaken.

4.7 Summary

This chapter has reviewed the differences between the relational model for creating databases, and the source-oriented model as exemplified by the κλειω DBMS. Both models are suitable for the construction of the *ET* database, but the latter was chosen as it was possible to enter the data in an unrestricted way, and also keep all the data pertaining to a particular month in the same place in the data file. This allowed for easy manual reference after the database contents were printed out, as well as easy reference within the computer data file itself. Also, when this project began, the κλειω software appeared more user-friendly than dBaseIV which was available to implement the relational model. Data modelling with κλειω appeared more straight-forward than using the relational model, however, querying with a RDBMS is much faster than with κλειω, which needs to be exported to a spreadsheet for analysis. The structure and working of the *ET* database was set out, plus a description of error-checking undergone. The method of creating κλειω queries was visited, together with the methods used to import the results of such a query into an Excel_{TM} workbook. Results from the database analysis will now be looked at.