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## Cooperative Retransmission for Wireless

## Regenerative Multi-Relay Networks

Quoc-Tuan Vien, Brian G. Stewart, Huaglory Tianfield, and Huan X. Nguyen


#### Abstract

This paper investigates retransmission mechanisms in wireless regenerative multi-relay networks. Conventionally, the retransmission can be realised in a cooperative manner with the assistance of all available relays. However, this may result in a high overall power consumption due to the retransmission of the same packets across the nodes, especially when the number of relays is large. We propose a cooperative retransmission (CR) scheme based on relay cooperation and binary XOR operations to significantly reduce the number of packets retransmitted to produce a more power efficient system with non-overlapped retransmissions. Significantly, we also derive the error probability of retransmission decisions at the source and relays and show that the proposed CR scheme improves the reliability of the retransmissions. Furthermore, by deriving the average number of packets to be retransmitted at the source and relays, we not only show that the proposed CR scheme reduces the number of retransmissions and removes overlapped retransmitted packets, but also determine the optimised number of relays used for the retransmission phase. Finally, simulation results are presented to demonstrate the validity of the analytical expressions.


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## Index Terms

Relay network, cooperative retransmission, relay cooperation.

## I. Introduction

Relay technologies [1]-[3] are continuing to attract interest in wireless communications thanks to their potential to enhance throughput and improve service quality. Examples of relay-assisted communications exist in a variety of networks, e.g. cellular [4], ad hoc [5], sensor [6], ultrawideband body area [7] and storage [8] networks. In general, within relay networks, data transmission from a source node to a destination node is carried out with the aid of one or multiple relays. The issue of relay selection (RS) is often considered so that only the "best" relay is chosen for forwarding packets according to different selection criterion (e.g. minimizing bit error rate or maximizing throughput) [9]-[13].

The utilisation of relay-assisted communications provides opportunities for potential new solutions and new methods of improving data transmission in a number of areas. One of these areas is the well-documented positive acknowledgement (ACK) protocol with retransmission, which is widely used in wireless networks. A more advanced version of the protocol is the wellknown block ACK aggregation method. In this method, small-sized ACK packets are aggregated into a single block ACK packet to acknowledge a group of received data packets at the one time. This leads to overall throughput enhancement by reducing the arbitrary inter-frame spacing periods, the backoff counter time and the acknowledgement time [14], [15]. The employment of block ACK packets in wireless multi-relay networks can be rather complicated since transmission of information packets is required to be acknowledged for a potentially large number of links which exist between the source, destination and multiple relays. This also leads to the issue of simultaneous retransmissions of the same packets, that can considerably degrade the network throughput. To solve this problem, the retransmissions can be carried out in a cooperative manner [3], [16], referred to as cooperative retransmission (CR). In the application of CR, the relays can
help the source retransmit the corrupted packets whereas the source retransmits only the packets corrupted at all the relays and also the destination.

In multi-relay networks, two relaying and retransmission strategies can be considered. Firstly, only the "best" relay is chosen for forwarding the data packets and retransmitting the corrupted packets according to various relay selection criteria. This is referred to as the best-relaying CR (BCR) scheme in this paper. Secondly, multiple relays, rather than just the best relay, can participate in the retransmission phase. This group-relaying CR (GCR) scheme relies on a group of relays which are able to determine and retransmit the corrupted packets. However, the overall throughput and power consumption of the system using the GCR scheme suffer from the problem of sending the same packets at different relays due to the lack of mutual information shared between the relays.

In this paper, we propose a new GCR scheme for wireless regenerative multi-relay networks based on relay cooperation (RC) and binary XOR operations, namely an XOR and RC-based GCR (i.e. XRGCR) scheme.

In relation to this new XRGCR scheme, the contributions of this paper may be summarised as follows:

1) A novel cooperative retransmission mechanism is designed including two key elements: i) relay cooperation: the acknowledged information can be shared among the relays to avoid overlapping in retransmissions; and ii) XOR operations: the destination combines all acknowledged information to form one single block ACK packet. This novel design will lead to a significantly improved throughput, particularly when the number of relay nodes is large. Using these methods, the smallest number of packets to be retransmitted will be determined in a cooperative way across both the relays and the source itself.
2) Closed-form expressions for the retransmission decision error probability (RDEP) across the source and relays are derived for Rayleigh flat fading channels. Our analysis shows that the XOR combination helps improve the reliability of the determination of packets to
be retransmitted at the source and the relays, which leads to a reduced number of overall retransmissions. The average number of packets to be retransmitted (or average number of retransmissions (ANRs)) across the nodes is then derived, which helps to understand and quantify the level of packet retransmission overlapping in any relaying approach. Importantly, the derived ANRs motivate us to propose two RS schemes for high power efficient retransmission by determining the optimised number of relays in the XRGCR scheme. The first RS scheme is identified based on the constraint of frame length (i.e. the number of data packets in a data frame) and the second scheme is designed based on the constraint of total power consumption at the relays.

The rest of this paper is organized as follows: Section II describes the system model of a typical wireless regenerative multi-relay network and discusses details and examples of various CR schemes. Sections III and IV present the formulation of the mathematical expressions for RDEP and ANR at both sources and relays, respectively. Section V presents two RS schemes for the retransmission. Numerical and simulation results are presented in Section VI to validate the concepts and finally Section VII draws the main conclusions from the paper.

## II. Cooperative Retransmission

Fig. 1 illustrates a typical regenerative relay system model. The data transmission from a source node $\mathcal{S}$ to a destination node $\mathcal{D}$ is accomplished by a two-hop protocol with the assistance of a best relay in a group of $N$ relays $\mathcal{R}^{(N)}=\left\{\mathcal{R}_{1}, \mathcal{R}_{2}, \ldots, \mathcal{R}_{N}\right\}$. There are three phases during the data transmission: broadcasting (BC), forwarding (FW) and retransmission (RT) phases. Source $\mathcal{S}$ transmits data sequences continuously to $\mathcal{R}^{(N)}$ and $\mathcal{D}$ in the BC phase. Then, in the FW phase, all $\mathcal{R}^{(N)}$ decode the received data sequences but only the best relay is selected to forward the decoded data to $\mathcal{D}$ (see Fig. 1). In the RT phase, only the best relay or group of best relays will carry out retransmissions depending on whether BCR or GCR is used. Then, $\mathcal{S}$ retransmits the data packets which are not correctly decoded at both $\mathcal{R}^{(N)}$ and $\mathcal{D}$. If these packets are still


Fig. 1. System model of two-hop multi-relay network.
lost or corrupted at $\mathcal{D}$ but are received successfully at $\mathcal{R}^{(N)}, \mathcal{R}^{(N)}$ can help $\mathcal{S}$ retransmit these packets since $\mathcal{R}^{(N)}$ also receive the retransmitted packets from $\mathcal{S}$.

We make the following assumptions:
(A1) A half-duplex system is considered where all nodes can either transmit or receive data, but not simultaneously.
(A2) Without loss of generality, the order of the relays in the group $\mathcal{R}^{(N)}$ is based on the quality of the received signal at the relays, i.e. $\mathcal{R}_{1}$ denotes the best quality relay while $\mathcal{R}_{N}$ represents the relay with the lowest signal quality.
(A3) The relays are located within the transmission range of each other in a rather dense network, thus each relay is able to overhear the ACK information from all other relays.
(A4) Source $\mathcal{S}$ sends each data sequence in the form of aggregated frames, with every frame consisting of $W$ data packets.
(A5) An aggregated ACK packet, i.e. block ACK packet, of length $K$ (in bits) is used to report the status of each frame, where bits ' 0 ' and ' 1 ' represent the data packet being correctly received and the packet being lost or erroneously received, respectively.
(A6) The length of each block ACK packet, in bits, is equal to the number of packets in a data
frame, i.e. $K=W$. The bits used for overheads and other signalling information in block ACK packets are omitted for the sake of simplicity.
(A7) The channels for all forward, backward and cooperation links are Rayleigh flat fading channels.
(A8) The channels for the backward links and the links between relays are time-invariant over the whole transmission of block ACK sequences and known to all the nodes in the network.
(A9) The transmission of data and block ACK packets between the nodes is perfectly synchronised and coordinated.

For convenience, the main notation used in the paper is listed in Table I.

TABLE I
SUMMARY OF MAIN NOTATION

| Notation | Meaning |
| :---: | :---: |
| $\Theta_{A B}$ | $W$-bit block ACK packet that is generated at node $\mathcal{B}$ and sent to node $\mathcal{A}$ to acknowledge a frame of $W$ packets that are sent from $\mathcal{A}$ to $\mathcal{B}$ |
| $\mathfrak{X} \in\{\mathrm{B}, \mathrm{G}, \mathrm{X}\}$ | superscript letter in parentheses corresponds to the first letter in the name of CR scheme, e.g. B, G and X represent $\mathrm{BCR}, \mathrm{GCR}$ and XRGCR, respectively |
| $\Omega_{S}^{(\mathcal{X})}$ and $\Omega_{R_{j}}^{(\mathfrak{X})}$ | $W$-bit retransmission indication packets (RIPs) generated at $\mathcal{S}$ and $\mathcal{R}_{j}$, respectively, using various CR schemes in which bit ' 1 ' indicates that the corresponding data packet needs to be retransmitted while bit ' 0 ' indicates otherwise |
| $\Gamma_{A B}$ | power level for transmission link $\mathcal{A} \rightarrow \mathcal{B}$ |
| $h_{A B}$ | channel gain for transmission link $\mathcal{A} \rightarrow \mathcal{B}$ |
| $\mathbf{x}_{A B}$ | binary phase shift keying (BPSK) modulated signal of $\Theta_{A B}$ |
| $\mathbf{n}_{A B}$ | independent circularly symmetric complex Gaussian (CSCG) noise vector of transmission link $\mathcal{A} \rightarrow \mathcal{B}$ with each entry having zero mean and variance of $N_{0}$ |
| $\gamma_{A B}$ | average signal-to-noise ratio (SNR) of transmission link $\mathcal{A} \rightarrow \mathcal{B}$ |
| $\otimes$ | bitwise AND operator |
| $\oplus$ | bitwise XOR operator |
| $\bar{\Theta}_{A B}$ | bitwise complement of $\Theta_{A B}$ |
| $\hat{\Theta}_{A B, 0}$ and $\hat{\Theta}_{A B, j}$ | detected $\Theta_{A B}$ at $\mathcal{S}$ and $\mathcal{R}_{j}$, respectively. |



Fig. 2. Example of proposed XRGCR scheme in two-relay network.

## A. Examples of Cooperative Retransmission Schemes

Examples of retransmission schemes are considered for two-relay and three-relay networks as illustrated in Figs. 2 and 3, respectively. These will help clarify the generation of block ACK packets along with different CR schemes in determining the RIPs at the source and relays.

1) Example 1-Two-Relay Network: We consider the example as depicted in Fig. 2 where $\mathcal{S}$ wishes to deliver a data frame of $W=8$ packets $\{s[1], s[2], \ldots, s[8]\}$ to $\mathcal{D}$ with the assistance of $\mathcal{R}_{1}$ and $\mathcal{R}_{2}$. Suppose that the packets with a crossthrough are lost or have errors. In this example, we assume that the erroneous packets received at $\mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{D}$ in the BC phase are $\{s[3], s[5]\},\{s[2], s[5], s[7]\}$ and $\{s[2], s[3], s[5], s[6]\}$, respectively. Then, $\mathcal{R}_{1}$ is selected to forward its correctly decoded packets $\{s[1], s[2], s[4], s[6], s[7], s[8]\}$ to $\mathcal{D}$ in the FW phase. Assume that the erroneous packets of link $\mathcal{R}_{1} \rightarrow \mathcal{D}$ are $\{s[1], s[2], s[6]\}$. Since the data frame includes 8 packets, the block ACK packet for the acknowledgement is 8 bits in length. Based on the received data packets, $\mathcal{R}_{1}$ generates $\Theta_{S R_{1}}=‘ 00101000 ', \mathcal{R}_{2}$ generates $\Theta_{S R_{2}}={ }^{\prime} 01001010$ ',
and $\mathcal{D}$ generates $\Theta_{S D}=' 01101100$ ' and $\Theta_{R_{1} D}=' 11101100$.
$\underline{B C R}$ Scheme: Following the BCR scheme, only the best relay (i.e. $\mathcal{R}_{1}$ ), which has most correctly received packets, is used in the FW and RT phases. The RIPs can be obtained as follows: $\Omega_{S}^{(\mathrm{B})}=\Theta_{S R_{1}} \otimes \Theta_{S D}=‘ 00101000$ ' and $\Omega_{R_{1}}^{(\mathrm{B})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{1}}={ }^{\prime} 01000100$ '. In this case, $\mathcal{S}$ and $\mathcal{R}_{1}$ need to retransmit $\{s[3], s[5]\}$ and $\{s[2], s[6]\}$, respectively. It is obvious that $\mathcal{R}_{1}$ helps resend the packets (i.e. $\left.\{s[2], s[6]\}\right)$ that $\mathcal{D}$ fails to decode while $\mathcal{S}$ resends the packets that are lost at both $\mathcal{R}_{1}$ and $\mathcal{D}$ (i.e. $\{s[3], s[5]\}$ ).

GCR Scheme: In the GCR scheme, $\mathcal{R}_{2}$ helps $\mathcal{R}_{1}$ in the RT phase. The RIPs at $\mathcal{S}, \mathcal{R}_{1}$ and $\mathcal{R}_{2}$ can be obtained as follows: $\Omega_{S}^{(\mathrm{G})}=\Theta_{S R_{1}} \otimes \Theta_{S R_{2}} \otimes \Theta_{S D}={ }^{‘} 00001000^{\prime}, \Omega_{R_{1}}^{(\mathrm{G})}=\Theta_{R_{1} D} \otimes$ $\Theta_{S D} \otimes \bar{\Theta}_{S R_{1}}=‘ 01000100$ ' and $\Omega_{R_{2}}^{(\mathrm{G})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{2}}=‘ 00100100^{\prime}$. In this case, $\mathcal{S}, \mathcal{R}_{1}$ and $\mathcal{R}_{2}$ retransmit $\{s[5]\},\{s[2], s[6]\}$ and $\{s[3], s[6]\}$, respectively. It can be seen that $\mathcal{S}$ only retransmits one packet $s[5]$ with the help of $\mathcal{R}_{2}$ in the retransmission of $s[3]$. However, there is one overlapped packet in the RT phase (i.e. $s[6]$ ).

Proposed XRGCR Scheme: In the proposed XRGCR scheme, only one combined block ACK packet $\Theta_{D}$ is generated and sent from $\mathcal{D}$ instead of two separate packets $\Theta_{R_{1} D}$ and $\Theta_{S D}$. In particular, $\Theta_{D}=\Theta_{R_{1} D} \otimes \Theta_{S D}=‘ 01101100$ '. The RIPs at $\mathcal{S}, \mathcal{R}_{1}$ and $\mathcal{R}_{2}$ can be obtained as follows: $\Omega_{S}^{(\mathrm{X})}=\Theta_{S R_{1}} \otimes \Theta_{S R_{2}} \otimes \Theta_{D}={ }^{‘} 00001000 ', \Omega_{R_{1}}^{(\mathrm{X})}=\Theta_{D} \oplus\left(\Theta_{S R_{1}} \otimes \Theta_{D}\right)={ }^{‘} 01000100$ ' and $\Omega_{R_{2}}^{(\mathrm{X})}=\Lambda_{2,1} \oplus\left(\Lambda_{2,1} \otimes \Omega_{R_{1}}^{(\mathrm{X})}\right)=‘ 00100000$ ', where $\Lambda_{2,1}=\Theta_{D} \oplus\left(\Theta_{S R_{2}} \otimes \Theta_{D}\right)=‘ 00100100$ 。. Thus, the packets that $\mathcal{S}, \mathcal{R}_{1}$ and $\mathcal{R}_{2}$ require to retransmit are $\{s[5]\},\{s[2], s[6]\}$ and $\{s[3]\}$, respectively. It can be seen that there is no overlapped packet in the RT phase with our proposed XRGCR scheme.
2) Example 2-Three-Relay Network: The example depicted in Fig. 3 contains three relays. Let the erroneous packets received at $\mathcal{R}_{1}, \mathcal{R}_{2}, \mathcal{R}_{3}$ and $\mathcal{D}$ in the BC phase be $\{s[3], s[5]\},\{s[2]$, $s[5], s[7]\},\{s[1], s[4], s[8]\}$ and $\{s[2], s[3], s[5], s[6]\}$, respectively. Similar to the example of the two-relay network, $\mathcal{R}_{1}$ is selected to forward its correctly decoded packets $\{s[1], s[2], s[4], s[6]$, $s[7], s[8]\}$ to $\mathcal{D}$ in the FW phase and the erroneous packets of link $\mathcal{R}_{1} \rightarrow \mathcal{D}$ are $\{s[1], s[2], s[6]\}$.


Fig. 3. Example of proposed XRGCR scheme in three-relay network.

In order to acknowledge the received data packets, $\mathcal{R}_{1}$ generates $\Theta_{S R_{1}}={ }^{\prime} 00101000$ ', $\mathcal{R}_{2}$ generates $\Theta_{S R_{2}}=‘ 01001010 ', \mathcal{R}_{3}$ generates $\Theta_{S R_{3}}=' 10010001$ ' and $\mathcal{D}$ generates $\Theta_{S D}={ }^{\prime} 01101100$ ' and $\Theta_{R_{1} D}=' 11101100$ '.
$\underline{B C R}$ Scheme: Since the BCR scheme does not depend on the number of relays, the determinations of packets to be retransmitted at $\mathcal{S}$ and $\mathcal{R}_{1}$ are carried out in the same way as the BCR scheme for the two-relay network, and thus the RIPs at $\mathcal{S}$ and $\mathcal{R}_{1}$ are $\{s[3], s[5]\}$ and $\{s[2]$, $s[6]\}$, respectively.

GCR Scheme: In this scheme, $\mathcal{R}_{2}$ and $\mathcal{R}_{3}$ help $\mathcal{R}_{1}$ in the RT phase. The RIPs at $\mathcal{S}, \mathcal{R}_{1}$, $\mathcal{R}_{2}$ and $\mathcal{R}_{3}$ can be obtained as follows: $\Omega_{S}^{(\mathrm{G})}=\Theta_{S R_{1}} \otimes \Theta_{S R_{2}} \otimes \Theta_{S R_{3}} \otimes \Theta_{S D}={ }^{‘} 00000000$, $\Omega_{R_{1}}^{(\mathrm{G})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{1}}=‘ 01000100^{\prime}, \Omega_{R_{2}}^{(\mathrm{G})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{2}}=‘ 00100100$ ' and $\Omega_{R_{3}}^{(\mathrm{G})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{3}}=‘ 01101100$ '. In this case, $\mathcal{S}$ does not require to retransmit any packets while $\mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{R}_{3}$ need to retransmit $\{s[2], s[6]\},\{s[3], s[6]\}$ and $\{s[2], s[3], s[5]$, $s[6]\}$, respectively. It can be seen that $\mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{R}_{3}$ assist $\mathcal{S}$ in the retransmission of lost packets. However, there are four overlapped packets in the RT phase including two $s[6]$ packets,


Fig. 4. Timing graph of proposed XRGCR scheme.
one $s[2]$ packet and one $s[3]$ packet.
Proposed XRGCR Scheme: With only one combined block ACK packet $\Theta_{D}=$ ' 01101100 ' at $\mathcal{D}$, the RIPs at $\mathcal{S}, \mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{R}_{3}$ can be obtained as follows: $\Omega_{S}^{(\mathrm{X})}=\Theta_{S R_{1}} \otimes \Theta_{S R_{2}} \otimes$ $\Theta_{S R_{3}} \otimes \Theta_{D}=‘ 00000000 ', \Omega_{R_{1}}^{(\mathrm{X})}=\Theta_{D} \oplus\left(\Theta_{S R_{1}} \otimes \Theta_{D}\right)={ }^{‘} 01000100^{\prime}, \Omega_{R_{2}}^{(\mathrm{X})}=\Lambda_{2,1} \oplus\left(\Lambda_{2,1} \otimes\right.$ $\left.\Omega_{R_{1}}^{(\mathrm{X})}\right)=‘ 00100000$ ' and $\Omega_{R_{3}}^{(\mathrm{X})}=\Lambda_{3,2} \oplus\left(\Lambda_{3,2} \otimes \Omega_{R_{2}}^{(\mathrm{X})}\right)={ }^{\prime} 00001000^{\prime}$, where $\Lambda_{2,1}=\Theta_{D} \oplus\left(\Theta_{S R_{2}} \otimes\right.$ $\left.\Theta_{D}\right)={ }^{\prime} 00100100 ', \Lambda_{3,2}=\Lambda_{3,1} \oplus\left(\Lambda_{3,1} \otimes \Omega_{R_{1}}^{(\mathrm{X})}\right)=‘ 00101000 ’$ and $\Lambda_{3,1}=\Theta_{D} \oplus\left(\Theta_{S R_{3}} \otimes\right.$ $\left.\Theta_{D}\right)={ }^{\prime} 01101100$ '. Thus, $\mathcal{S}$ does not require to retransmit any packet and the packets that $\mathcal{R}_{1}$, $\mathcal{R}_{2}$ and $\mathcal{R}_{3}$ need to retransmit are $\{s[2], s[6]\},\{s[3]\}$ and $\{s[5]\}$, respectively. It can also be observed, as in the example for the two-relay network, that there are no overlapped packets in the RT phase with our proposed XRGCR scheme.

For clarity, the timing process of data transmission and block ACK reporting for a tworelay network using the proposed XRGCR scheme with time division multiple access (TDMA) protocol is illustrated in Fig. 4. The transmission protocol of an $N$-relay network, $N>2$, can be readily extended. In the BC phase, $\mathcal{S}$ transmits $W$ packets sequentially to $\mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{D}$. Then, $\mathcal{R}_{1}$ forwards the correctly received packet to $\mathcal{D}$ in the FW phase. After decoding and error-checking all of the $W$ packets received from $\mathcal{S}$, the nodes $\mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{D}$ generate
block ACK packets $\Theta_{S R_{1}}, \Theta_{S R_{2}}$ and $\Theta_{S D}$, respectively. Meanwhile, $\mathcal{D}$ also attempts to decode signals forwarded from $\mathcal{R}_{1}$ and then generates $\Theta_{R_{1} D}$ after checking all the $W$ data packets. In our proposed XRGCR scheme, the block ACK packet $\Theta_{S R_{1}}$ can be received by $\mathcal{R}_{2}$ over the cooperation link. Additionally, instead of sending $\Theta_{S D}$ and $\Theta_{R_{1} D}$ separately, $\mathcal{D}$ generates only one combined block ACK packet $\Theta_{D}$ and broadcasts it to $\mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{S}$. Based on the received block ACK packets, $\mathcal{R}_{1}, \mathcal{R}_{2}$ and $\mathcal{S}$ determine the retransmission indication packets and then sequentially retransmit these packets to $\mathcal{D}$ in the RT phase.

## B. Cooperative Retransmission Schemes

The BCR, GCR and XRGCR cooperative schemes may be described as follows:

1) BCR: Since only $\mathcal{R}_{1}$ is used in the FW and RT phases, the RIPs at $\mathcal{S}$ and $\mathcal{R}_{1}$ can be obtained as follows:

$$
\begin{gather*}
\Omega_{S}^{(\mathrm{B})}=\Theta_{S R_{1}} \otimes \Theta_{S D},  \tag{1}\\
\Omega_{R_{1}}^{(\mathrm{B})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{1}} . \tag{2}
\end{gather*}
$$

Note that (1) and (2) are based on the principle of CR, i.e. the source node retransmits the packets that are lost at the selected relay and destination nodes, whereas the selected relay node retransmits only those packets that it correctly decodes but the destination node fails to decode.
2) GCR: The RIPs at $\mathcal{S}$ and $\mathcal{R}_{j}, j \in\{1,2, \ldots, N\}$, can be obtained by

$$
\begin{gather*}
\Omega_{S}^{(\mathrm{G})}=\Theta_{S R_{1}} \otimes \Theta_{S R_{2}} \otimes \cdots \otimes \Theta_{S R_{N}} \otimes \Theta_{S D}  \tag{3}\\
\Omega_{R_{j}}^{(\mathrm{G})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{j}} \tag{4}
\end{gather*}
$$

The principle of CR in (3) and (4) is that the source node retransmits the packets that are lost at all the relay and destination nodes, whereas each relay node retransmits only those packets that it correctly decodes but the destination node fails to receive.
3) Proposed $X R G C R$ : Instead of sending 3 block ACK packets $\Theta_{S R_{1}}, \Theta_{S D}$ and $\Theta_{R_{1} D}$ as in the BCR and GCR schemes, our proposed XRGCR scheme only requires to send 2 block ACK packet $\Theta_{S R_{1}}$ and $\Theta_{D}$, at $\mathcal{R}_{1}$ and $\mathcal{D}$, respectively, where $\Theta_{D}$ is created as follows:

$$
\begin{equation*}
\Theta_{D}=\Theta_{R_{1} D} \otimes \Theta_{S D} \tag{5}
\end{equation*}
$$

The RIPs at $\mathcal{S}$ and $\mathcal{R}_{1}$ can be obtained as

$$
\begin{gather*}
\Omega_{S}^{(\mathrm{X})}=\Theta_{S R_{1}} \otimes \Theta_{S R_{2}} \otimes \cdots \otimes \Theta_{S R_{N}} \otimes \Theta_{D}  \tag{6}\\
\Omega_{R_{1}}^{(\mathrm{X})}=\Theta_{D} \oplus\left(\Theta_{S R_{1}} \otimes \Theta_{D}\right) \tag{7}
\end{gather*}
$$

In (6), the determination of packets to be retransmitted at $\mathcal{S}$ follows the principle that $\mathcal{S}$ retransmits the packets that are lost at all the relays $\left\{\mathcal{R}_{1}, \mathcal{R}_{2}, \ldots, \mathcal{R}_{N}\right\}$ as well as $\mathcal{D}$. The idea behind (7) is originated from the sense that $\mathcal{R}_{1}$ resends the packets that are correctly decoded at $\mathcal{R}_{1}$ but $\mathcal{D}$ fails to decode and are not resent by $\mathcal{S}$. Thus, the packets that $\mathcal{R}_{1}$ needs to retransmit are determined by the XOR operation of $\Theta_{D}$ and $\left(\Theta_{S R_{1}} \otimes \Theta_{D}\right)$.

Since $\mathcal{R}_{2}$ can overhear the block ACK $\Theta_{S R_{1}}$ from $\mathcal{R}_{1}$, the RIPs at $\mathcal{R}_{2}$ can be obtained by

$$
\begin{equation*}
\Omega_{R_{2}}^{(\mathrm{X})}=\Lambda_{2,1} \oplus\left(\Lambda_{2,1} \otimes \Omega_{R_{1}}^{(\mathrm{X})}\right), \tag{8}
\end{equation*}
$$

where $\Lambda_{2,1} \triangleq \Theta_{D} \oplus\left(\Theta_{S R_{2}} \otimes \Theta_{D}\right)$. The idea behind (8) is also based on the principle that $\mathcal{R}_{2}$ resends the packets that are correctly decoded at $\mathcal{R}_{2}$, but both $\mathcal{R}_{1}$ and $\mathcal{D}$ fail to decode in both the BC and FW phases, and are not resent by $\mathcal{S}$. Generally, the RIPs at $\mathcal{R}_{j}, j \geq 2$, can be obtained by the inductive method as follows:

$$
\begin{equation*}
\Omega_{R_{j}}^{(\mathrm{X})}=\Lambda_{j, j-1} \oplus\left(\Lambda_{j, j-1} \otimes \Omega_{R_{j-1}}^{(\mathrm{X})}\right), \tag{9}
\end{equation*}
$$

where

$$
\begin{gather*}
\Lambda_{j, j-1}=\Lambda_{j, j-2} \oplus\left(\Lambda_{j, j-2} \otimes \Omega_{R_{j-2}}^{(\mathrm{X})}\right)  \tag{10}\\
\Lambda_{j, 1}=\Theta_{D} \oplus\left(\Theta_{S R_{j}} \otimes \Theta_{D}\right) \tag{11}
\end{gather*}
$$

## C. Some Observations

(O1) Higher Reliability: The combination of block ACK packets at the destination in the proposed XRGCR scheme improves the reliability of the determination of the packets to be retransmitted. For convenience, let us refer to the XRGCR scheme without such combination as the non-combined XRGCR scheme. The RIPs at $\mathcal{S}$ and $\mathcal{R}_{1}$ using the non-combined XRGCR scheme can be determined as

$$
\begin{gather*}
\Omega_{S}^{\prime(\mathrm{X})}=\Theta_{S R_{1}} \otimes \Theta_{S R_{2}} \otimes \cdots \otimes \Theta_{S R_{N}} \otimes \Theta_{S D}  \tag{12}\\
\Omega_{R_{1}}^{\prime(\mathrm{X})}=\Theta_{R_{1} D} \otimes \Theta_{S D} \otimes \bar{\Theta}_{S R_{1}} \tag{13}
\end{gather*}
$$

As shown in (6), besides the requirement of block ACK packets from $\mathcal{R}^{(N)}$, the determination of RIPs at $\mathcal{S}$ requires a combined block ACK packet $\Theta_{D}$ from $\mathcal{D}$ instead of a single block ACK packet $\Theta_{S D}$ as shown in (12). It can be observed in (5) that $\Theta_{D}$ is generated by combining the block ACK packets of links $\mathcal{R}_{1} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{D}$. This means that the creation of $\Theta_{D}$ depends on the decisions of these two different links, and thus the decision reliability of the packets to be retransmitted at $\mathcal{S}$ is improved with the proposed XRGCR scheme. Additionally, only one block ACK packet, $\Theta_{D}$, needs to be known in the proposed XRGCR scheme as shown in (7) to determine the RIPs at $\mathcal{R}_{1}$. In the non-combined XRGCR scheme as shown in (13), the determination of RIPs at $\mathcal{R}_{1}$ requires two block ACK packets $\Theta_{R_{1} D}$ and $\Theta_{S D}$ from $\mathcal{D}$. Therefore, the proposed XRGCR scheme has a lower probability of error in the determination of RIPs at $\mathcal{R}_{1}$.
(O2) Reduced Number of Retransmissions: With the proposed XRGCR scheme, the number of packets to be retransmitted at the source and relay nodes is reduced compared with the non-combined XRGCR scheme. It can be seen that the detection of packets to be retransmitted depends on the quality of the backward links and block ACK schemes. As noted in observation (O1), the reliability in the determination of RIPs in the proposed XRGCR scheme is higher than
that in the non-combined XRGCR scheme, and thus, over the same backward environment, the proposed XRGCR scheme requires a lower number of data retransmissions.
(O3) Reduced Number of Retransmissions at $\mathcal{S}$ and Non-Overlapping Retransmissions at $\mathcal{R}_{j}$ : The number of packets to be retransmitted at the source is significantly reduced in the GCR and the proposed XRGCR schemes compared to the BCR scheme thanks to the help of multiple relays in the RT phase. In the GCR scheme, it can be observed that the relays retransmit many overlapped packets due to the lack of cooperation between the relays. Instead, there are no overlapped retransmission packets at the relays in the proposed XRGCR scheme with the RC between the relays. In fact, with binary XOR and AND operations as shown in (9), the relays can determine the packets to be retransmitted with no overlap.
(O4) Complexity Analysis: Let us investigate the computational complexity, which is measured by the number of binary operations (e.g. XOR, AND and complement). It can be observed in (1) and (2) that the BCR scheme requires a total of 4 binary operations, including 1 operation at $\mathcal{S}$ and 3 operations at $\mathcal{R}_{1}$. With the GCR scheme, as expressed through (3) and (4), $\mathcal{S}$ and $\mathcal{R}_{j}, j=1,2, \ldots, N$, perform $N$ and 3 binary operations, respectively. Thus, the GCR scheme requires a total of $4 N$ binary operations. With our proposed XRGCR scheme, as shown in (5) and (6), 1 binary operation and $N$ binary operations are implemented at $\mathcal{D}$ and $\mathcal{S}$, respectively. For the operations at the relays, let us denote $p_{j}$ as the number of binary operations carried out at $\mathcal{R}_{j}$. From (7)-(11), we have $p_{1}=2, p_{2}=4$ and $p_{j}=2+\sum_{k=1}^{j-1}\left(2+p_{k}\right)$ for $j>2$. Therefore, in total, $\left(N+1+\sum_{j=1}^{N} p_{j}\right)$ binary operations are required in our proposed XRGCR scheme.

## III. Error Probability Analysis of Block ACK Transmission

In this section, we first present signal models for the transmission of block ACK packets through the backward links. Then, we will derive the retransmission decision error probability (RDEP), i.e. the probability of error in the determination of packets to be retransmitted, at the relay and source nodes in our proposed XRGCR scheme.

After receiving a frame of $W$ packets from $\mathcal{S}$ in the BC phase, each $\mathcal{R}_{j}$ creates a block ACK packet $\Theta_{S R_{j}}, j \in\{1,2, \ldots, N\}$, and sends it back to $\mathcal{S}$. Over the wireless medium, the other relays, i.e. $\mathcal{R}_{j^{\prime}}, j^{\prime} \in\{2,3, \ldots, N\}, j^{\prime}>j$, can also receive the block ACK packet from $\mathcal{R}_{j}$ through the cooperation links $h_{R_{j} R_{j^{\prime}}}$. The signals received at $\mathcal{S}$ and $\mathcal{R}_{j^{\prime}}$ from $\mathcal{R}_{j}$ can be written as

$$
\begin{gather*}
\mathbf{y}_{R_{j} S}=\sqrt{\Gamma_{R_{j} S}} h_{R_{j} S} \mathbf{x}_{S R_{j}}+\mathbf{n}_{R_{j} S},  \tag{14}\\
\mathbf{y}_{R_{j} R_{j^{\prime}}}=\sqrt{\Gamma_{R_{j} R_{j^{\prime}}}} h_{R_{j} R_{j^{\prime}}} \mathbf{x}_{S R_{j}}+\mathbf{n}_{R_{j} R_{j^{\prime}}} \tag{15}
\end{gather*}
$$

respectively. From $\mathbf{y}_{R_{j} S}$ and $\mathbf{y}_{R_{j} R_{j^{\prime}}}, \mathcal{S}$ and $\mathcal{R}_{j^{\prime}}$ can detect $\Theta_{S R_{j}}$ as $\hat{\Theta}_{S R_{j}, 0}$ and $\hat{\Theta}_{S R_{j}, j^{\prime}}$, respectively.

Meanwhile, $\mathcal{D}$ generates $\Theta_{S D}$ corresponding to the error of the packets received from $\mathcal{S}$. The data packets forwarded from $\mathcal{R}_{1}$ in the FW phase are acknowledged by packet $\Theta_{R_{1} D}$. Then, $\mathcal{D}$ generates a new block ACK packet $\Theta_{D}$ as described in (5). This block ACK packet is sent to $\mathcal{S}$ and all $\left\{\mathcal{R}_{j}\right\}$. The received signals at $\mathcal{S}$ and $\mathcal{R}_{j}, j=1, \ldots, N$, can be written as

$$
\begin{gather*}
\mathbf{y}_{D S}=\sqrt{\Gamma_{D S}} h_{D S} \mathbf{x}_{D}+\mathbf{n}_{D S},  \tag{16}\\
\mathbf{y}_{D R_{j}}=\sqrt{\Gamma_{D R_{j}}} h_{D R_{j}} \mathbf{x}_{D}+\mathbf{n}_{D R_{j}}, \tag{17}
\end{gather*}
$$

respectively. From (16) and (17), $\mathcal{S}$ and $\mathcal{R}_{j}$ can detect $\Theta_{D}$ as $\hat{\Theta}_{D, 0}$ and $\hat{\Theta}_{D, j}$, respectively.
The RIPs at $\mathcal{S}$ and $\mathcal{R}_{j}$ are given by

$$
\begin{gather*}
\hat{\Omega}_{S}=\hat{\Theta}_{S R_{1}, 0} \otimes \hat{\Theta}_{S R_{2}, 0} \otimes \cdots \otimes \hat{\Theta}_{S R_{N}, 0} \otimes \hat{\Theta}_{D, 0}  \tag{18}\\
\hat{\Omega}_{R_{1}}=\hat{\Theta}_{D, 1} \oplus\left(\Theta_{S R_{1}} \otimes \hat{\Theta}_{D, 1}\right),  \tag{19}\\
\hat{\Omega}_{R_{j}}=\hat{\Lambda}_{j, j-1} \oplus\left(\hat{\Lambda}_{j, j-1} \otimes \hat{\Omega}_{R_{j-1}, j}\right), j=2,3, \ldots, N \tag{20}
\end{gather*}
$$

where

$$
\begin{gather*}
\hat{\Lambda}_{j, j-1}=\hat{\Lambda}_{j, j-2} \oplus\left(\hat{\Lambda}_{j, j-2} \otimes \hat{\Omega}_{R_{j-2}, j}\right),  \tag{21}\\
\hat{\Lambda}_{j, 1}=\hat{\Theta}_{D, j} \oplus\left(\Theta_{S R_{j}} \otimes \hat{\Theta}_{D, j}\right) \tag{22}
\end{gather*}
$$

$$
\begin{equation*}
\hat{\Omega}_{R_{i, j}}=\hat{\Theta}_{D, j} \oplus\left(\hat{\Theta}_{S R_{i, j}} \otimes \hat{\Theta}_{D, j}\right), i=1,2, \ldots, N, i<j \tag{23}
\end{equation*}
$$

Next, we derive a closed-form expression for the $\operatorname{RDEP}$ at $\mathcal{S}$ and $\mathcal{R}_{j}$ in our proposed XRGCR scheme. The RDEP at $\mathcal{S}$ and $\mathcal{R}_{j}$ can be defined as the bit error probability (BEP) of $\Omega_{S}$ given by (18) and the BEP of $\Omega_{R_{j}}$ given by (20), respectively.

Over a Rayleigh flat fading channel, the BEP for signal transmission through link $A \rightarrow B$ is given by [17]

$$
\begin{equation*}
P_{b}\left(E_{A B}\right)=\phi\left(\gamma_{A B}\right), \tag{24}
\end{equation*}
$$

where $\phi(x) \triangleq \frac{1}{2}\left(1-\sqrt{\frac{x}{1+x}}\right)$.
Theorem 1. The RDEPs at $\mathcal{S}$ and $\mathcal{R}_{j}, j=1,2, \ldots, N$, in our proposed XRGCR scheme are given by

$$
\begin{gather*}
P_{b}\left(E_{\Omega_{S}}\right)=\left[1-\prod_{i=0}^{N}\left(1-\beta_{i}\right)\right] \prod_{i=0}^{N} \alpha_{i}+\sum_{\mathbb{P}} \prod_{i=0}^{N} \delta_{i} \epsilon_{i},  \tag{25}\\
P_{b}\left(E_{\Omega_{R_{j}}}\right)=\left(1-\alpha_{j}\right)\left[1-\left(1-\zeta_{j}\right) \prod_{i=1}^{j-1}\left(1-\eta_{i j}\right)\right] \prod_{i=0}^{j-1} \alpha_{i} \\
+\left(1-\alpha_{j}\right)\left[\zeta_{j}\left(1-\alpha_{0}\right)+\left(1-\zeta_{j}\right) \alpha_{0}\right] \sum_{\mathbb{P}^{\prime}} \prod_{i=1}^{j-1} \delta_{i}^{\prime} \epsilon_{i}^{\prime}, \tag{26}
\end{gather*}
$$

where $\alpha_{i}=\phi\left(\gamma_{S R_{i}}\right), \beta_{i}=\phi\left(\gamma_{R_{i} S}\right), \zeta_{i}=\phi\left(\gamma_{D R_{i}}\right), \eta_{i j}=\phi\left(\gamma_{R_{i} R_{j}}\right),\{i, j\} \in\{1,2, \ldots, N\}, i<j$, $\alpha_{0}=\alpha_{00} \alpha_{01}, \alpha_{00}=\phi\left(\gamma_{S D}\right), \alpha_{01}=\phi\left(\gamma_{R_{1} D}\right), \beta_{0}=\phi\left(\gamma_{D S}\right), \mathbb{P}=\left\{(\delta, \epsilon) \mid \delta_{i}=\beta_{i}\right.$ or $1-\beta_{i}, \epsilon_{i}=$ $1-\alpha_{i}$ if $\delta_{i}=\beta_{i}$ and $\epsilon_{i}=\alpha_{i}$ if $\left.\delta_{i}=1-\beta_{i}\right\}$ and $\mathbb{P}^{\prime}=\left\{\left(\delta^{\prime}, \epsilon^{\prime}\right) \mid \delta_{i}^{\prime}=\eta_{i j}\right.$ or $1-\eta_{i j}, \epsilon_{i}^{\prime}=$ $1-\alpha_{i}$ if $\delta_{i}^{\prime}=\eta_{i j}$ and $\epsilon_{i}^{\prime}=\alpha_{i}$ if $\left.\delta_{i}^{\prime}=1-\eta_{i j}\right\}$.

Proof: See Appendix A.

Lemma 1. The RDEPs at $\mathcal{S}$ and $\mathcal{R}_{j}, j=1,2, \ldots, N$, in the non-combined XRGCR scheme can be similarly derived as

$$
\begin{equation*}
P_{b}\left(E_{\Omega_{S}^{\prime}}\right)=\left[1-\prod_{i=0}^{N}\left(1-\beta_{i}\right)\right] \alpha_{00} \prod_{i=1}^{N} \alpha_{i}+\left[\beta_{0}\left(1-\alpha_{00}\right)+\left(1-\beta_{0}\right) \alpha_{00}\right] \sum_{\mathbb{P}} \prod_{i=1}^{N} \delta_{i} \epsilon_{i} \tag{27}
\end{equation*}
$$

$$
\begin{align*}
P_{b}\left(E_{\Omega_{R_{j}}^{\prime}}\right) & =\left(1-\alpha_{j}\right)\left[1-\left(1-\zeta_{j}\right)^{2} \prod_{i=1}^{j-1}\left(1-\eta_{i j}\right)\right] \prod_{i=0}^{j-1} \alpha_{i} \\
& +\left(1-\alpha_{j}\right)\left[\zeta_{j}\left(1-\alpha_{01}\right)+\left(1-\zeta_{j}\right) \alpha_{01}\right] \sum_{\mathbb{P}^{\prime}} \prod_{i=1}^{j-1} \delta_{i}^{\prime} \epsilon_{i}^{\prime} . \tag{28}
\end{align*}
$$

We may make the following observation in relation to (25), (26), (27) and (28):
(O5) Lower RDEPs: Our proposed XRGCR scheme has lower $P_{b}\left(E_{\Omega_{S}}\right)$ and $P_{b}\left(E_{\Omega_{R_{j}}}\right), j=$ $1,2, \ldots, N$, than the non-combined XRGCR scheme. This confirms the statement in observation (O1). It is noted that $0<\phi(x) \leqslant 1 / 2 \forall x$. Thus, we get $0<\alpha_{00} \leqslant 1 / 2,0<\alpha_{01} \leqslant 1 / 2$, $0<\beta_{0} \leqslant 1 / 2,0<\zeta_{j} \leqslant 1 / 2, \alpha_{0}<\alpha_{00}, \alpha_{0}<\alpha_{01}$ and $\left(1-\zeta_{j}\right)^{2}<\left(1-\zeta_{j}\right)$. Also, we can deduce that $\beta_{0}\left(1-\alpha_{00}\right)+\left(1-\beta_{0}\right) \alpha_{00}>\beta_{0}\left(1-\alpha_{0}\right)+\left(1-\beta_{0}\right) \alpha_{0}$ and $\zeta_{j}\left(1-\alpha_{01}\right)+\left(1-\zeta_{j}\right) \alpha_{01}>$ $\zeta_{j}\left(1-\alpha_{0}\right)+\left(1-\zeta_{j}\right) \alpha_{0}$. Thus, $P_{b}\left(E_{\Omega_{S}^{\prime}}\right)$ and $P_{b}\left(E_{\Omega_{R_{j}}^{\prime}}\right)$ in (27) and (28) are greater than $P_{b}\left(E_{\Omega_{S}}\right)$ and $P_{b}\left(E_{\Omega_{R_{j}}}\right)$ in (25) and (26), respectively.

## IV. Average Number of Packets in Retransmission Phase

In this section, we derive the average number of retransmissions (ANR) at $\mathcal{S}$ and $\mathcal{R}_{j}, j=$ $1,2, \ldots, N$, in our proposed XRGCR scheme. Here, the ANR at $\mathcal{S}$ and $\mathcal{R}_{j}$ can be defined as either the average number of data retransmissions required to transmit one packet or the probability of packet retransmissions from $\mathcal{S}$ to $\mathcal{D}$ and from $\mathcal{R}_{j}$ to $\mathcal{D}$, respectively.

At first, the expression of ANRs is derived over error-free backward links. In this error-free environment, the RDEPs are omitted, i.e. $P_{b}\left(E_{\Omega_{S}}\right)=0$ and $P_{b}\left(E_{\Omega_{R_{j}}}\right)=0, j=1,2, \ldots, N$.

Theorem 2. Over error-free backward links, the ANRs at $\mathcal{S}$ and $\mathcal{R}_{j}, j=1,2, \ldots, N$, in the XRGCR scheme are given by

$$
\begin{gather*}
\lambda_{S}^{(\text {free })}=\alpha_{00} \prod_{j=1}^{N} \alpha_{j}  \tag{29}\\
\lambda_{R_{j}}^{(\text {free })}=\left(1-\alpha_{j}\right) \alpha_{01} \alpha_{00} \prod_{i=1}^{j-1} \alpha_{i} \prod_{i=1}^{j-1}\left(1-\eta_{i j}\right), \tag{30}
\end{gather*}
$$

where $\lambda_{A}^{(\text {free })}, A \in\left\{S, R_{j}\right\}$, denotes the ANR at node $A$. Here, $\alpha_{00}, \alpha_{01}, \alpha_{i}$ and $\eta_{i j},\{i, j\} \in$ $\{1,2, \ldots, N\}$, are defined as in Theorem 1.

Proof: See Appendix B.
Some important points may be observed in relation to (29) and (30):
(O6) Reduced ANR at $\mathcal{S}$ : The ANR at $\mathcal{S}$ in the GCR and the proposed XRGCR schemes is significantly reduced compared to the BCR scheme when the number of relays is larger than one. This confirms the statement in observation (O3). In fact, following the BCR scheme, the ANR at $\mathcal{S}$ depends only on the links $\mathcal{S} \rightarrow \mathcal{R}_{1}$ and $\mathcal{S} \rightarrow \mathcal{D}$, and thus can be derived easily as

$$
\begin{equation*}
\lambda_{S}^{(\mathrm{B}, f r e e)}=\alpha_{00} \alpha_{1} . \tag{31}
\end{equation*}
$$

Similar to the proposed XRGCR scheme, the ANR at $\mathcal{S}$ in the GCR scheme is given by

$$
\begin{equation*}
\lambda_{S}^{(\mathrm{G}, \text { free })}=\alpha_{00} \prod_{j=1}^{N} \alpha_{j} . \tag{32}
\end{equation*}
$$

From (29), (31) and (32), it can be seen that $\lambda_{S}^{(\text {free })}=\lambda_{S}^{(\mathrm{G}, \text { free })}<\lambda_{S}^{(\mathrm{B}, \text { free })}$ when $N>1$.
(O7) Reduced ANR at $\mathcal{R}_{j}$ : The ANR at $\mathcal{R}_{j}, j>1$, in the XRGCR scheme is lower than that in the GCR scheme. Following the GCR scheme, the ANR at $\mathcal{R}_{j}, j=1,2, \ldots, N$, depends only on the links $\mathcal{S} \rightarrow \mathcal{R}_{j}, \mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{R}_{1} \rightarrow \mathcal{D}$. Thus, its ANR is simply given by

$$
\begin{equation*}
\lambda_{R_{j}}^{(\mathrm{G}, \text { free })}=\left(1-\alpha_{j}\right) \alpha_{01} \alpha_{00} \tag{33}
\end{equation*}
$$

Comparing (30) and (33), it can be observed that $\lambda_{R_{j}}^{(\text {free })}<\lambda_{R_{j}}^{(\mathrm{G}, \text { free })}$. In fact, in the GCR scheme, there is lack of cooperation between the relays and thus there are various overlapped packets in the RT phase compared with the proposed XRGCR scheme which has non-overlapped packets. The overlapped packets at $\mathcal{R}_{j}, j>1$, in the GCR scheme can be quantified as

$$
\begin{equation*}
\Delta_{j}=\lambda_{R_{j}}^{(\mathrm{G}, \text { free })}-\lambda_{R_{j}}^{(\text {free })}=\left(1-\alpha_{j}\right) \alpha_{01} \alpha_{00}\left[1-\prod_{i=1}^{j-1} \alpha_{i} \prod_{i=1}^{j-1}\left(1-\eta_{i j}\right)\right] . \tag{34}
\end{equation*}
$$

This confirms the statement in observation (O3) concerning the overlapped packets at the relays in the RT phase.

Lemma 2. Over erroneous backward links, the ANRs at $\mathcal{S}$ and $\mathcal{R}_{j}, j=1,2, \ldots, N$, in the XRGCR scheme are given by

$$
\begin{gather*}
\lambda_{S}=\lambda_{S}^{(\text {free })}+P_{b}\left(E_{\Omega_{S}}\right)  \tag{35}\\
\lambda_{R_{j}}=\lambda_{R_{j}}^{(\text {free })}+P_{b}\left(E_{\Omega_{R_{j}}}\right), \tag{36}
\end{gather*}
$$

where $P_{b}\left(E_{\Omega_{S}}\right)$ and $P_{b}\left(E_{\Omega_{R_{j}}}\right)$ are given by (25) and (26), respectively.
(O8) Lower ANRs: Over unreliable backward links, the ANRs at $\mathcal{S}$ and $\mathcal{R}_{j}, j=1,2, \ldots, N$, in the proposed XRGCR scheme are reduced compared to that in the non-combined XRGCR scheme due to the improved RDEPs (see observations (O1) and (O5)). This confirms the statement in observation (O2) regarding the reduced number of retransmissions. In fact, the ANRs at $\mathcal{S}$ and $\mathcal{R}_{j}$ in the non-combined XRGCR scheme can be similarly derived as

$$
\begin{gather*}
\lambda_{S}^{\prime}=\lambda_{S}^{(\text {free })}+P_{b}\left(E_{\Omega_{S}^{\prime}}\right),  \tag{37}\\
\lambda_{R_{j}}^{\prime}=\lambda_{R_{j}}^{(\text {free })}+P_{b}\left(E_{\Omega_{R_{j}}^{\prime}}\right), \tag{38}
\end{gather*}
$$

where $P_{b}\left(E_{\Omega_{S}^{\prime}}\right)$ and $P_{b}\left(E_{\Omega_{R_{j}}^{\prime}}\right)$ are the RDEPs at $\mathcal{S}$ and $\mathcal{R}_{j}$ in the non-combined XRGCR scheme given by (27) and (28), respectively. Thus, from (35), (36), (37), (38) and observation (O5), we can deduce that $\lambda_{S}<\lambda_{S}^{\prime}$ and $\lambda_{R_{j}}<\lambda_{R_{j}}^{\prime}$.

## V. Relay Selection for Retransmission

In multi-relay networks, various RS schemes are considered in the FW phase to help the source forward data to the destination [9]-[13]. In our work, we have investigated various CR schemes where multiple relays are used to help the source retransmit the corrupted packets to the destination. This naturally requires an efficient RS mechanism in the RT phase.

In this Section, based on the derived ANR at the relays in Section IV, we propose two RS schemes for the RT phase. The first is based on the constraint of the total number of packets in a frame and the second is based on the constraint of the total power consumption at the relays.

The RS process can be carried out by a scheduler of a coordinator node in a centralized manner [18], [19], i.e. each relay informs the coordinator its ANR through a specific feedback channel and then the coordinator selects the relays for the retransmission based on this information.

Let $N_{1}^{*}$ and $N_{2}^{*}$ denote the number of relays required for the RT phase using the first and second RS schemes, respectively. Regarding the frame length (i.e. $W$ ), the first RS scheme is defined through

$$
\begin{equation*}
N_{1}^{*}=\arg \max _{j=1,2, \ldots, N}\left\{\lambda_{R_{j}} \geqslant \lambda_{\mathrm{threshold}} \triangleq \frac{1}{W}\right\} . \tag{39}
\end{equation*}
$$

With limited total power consumption at the relays for the RT phase, the second RS scheme is determined by

$$
\begin{equation*}
N_{2}^{*}=\arg \max _{j=1,2, \ldots, N}\left\{\sum_{i=1}^{j} W \lambda_{R_{j}} P_{R} \leqslant P_{R, \text { tot }}\right\}, \tag{40}
\end{equation*}
$$

where $P_{R}$ and $P_{R, \text { tot }}$ are the power required at each relay node to retransmit a packet and the total power constraint at the relays for the retransmission, respectively. The algorithms corresponding to the two RS schemes are summarized in Tables II and III.

## TABLE II

RS BASED ON FRAME LENGTH


Step 2. Compare $\lambda_{R_{j}}$ with $\lambda_{\text {threshold }}$ :
If $\lambda_{R_{j}}$ is larger than or equal to $\lambda_{\text {threshold }}$, then assign $N_{1}^{*}$ as $j$.
Back to Step 1 with the next relay $\mathcal{R}_{j+1}$.
Otherwise, stop the RS process.
(O9) High Power Efficiency: The first RS scheme is helpful for the proposed XRGCR scheme to reduce the power consumption in the RT phase since the ANR of $\mathcal{R}_{j}$ decreases as $j$ increases. Specifically, when $W$ is small, the proposed XRGCR scheme requires a lower number of relays in the RT phase compared to the GCR scheme. With the second RS scheme, it can be seen that the proposed XRGCR scheme is preferred for a limited $P_{R, \text { tot }}$ while the GCR scheme is

TABLE III
RS BASED ON POWER CONSTRAINT

```
Step 1. Calculate ANR at relay }\mp@subsup{\mathcal{R}}{j}{},j=1,2,\ldots,N (i.e. 㞰的)
```



```
    with the total power constraint (i.e. }\mp@subsup{P}{R,\mathrm{ tot }}{}\mathrm{ ):
    . If }\mp@subsup{\sum}{i=1}{j}W\mp@subsup{\lambda}{\mp@subsup{R}{j}{}}{}\mp@subsup{P}{R}{}\mathrm{ is smaller than or equal to }\mp@subsup{P}{R,\mathrm{ tot , then assign }\mp@subsup{N}{2}{*}\mathrm{ as }j\mathrm{ .}}{\mathrm{ .}
    Back to Step 1 with the next relay }\mp@subsup{\mathcal{R}}{j+1}{}\mathrm{ .
    Otherwise, stop the RS process.
```

beneficial to achieve a higher diversity gain in the RT phase if $P_{R, \text { tot }}$ is large enough. In fact, the proposed XRGCR scheme can exploit all the relays to help the source in the RT phase even with a low $P_{R, \text { tot }}$ since the relays can help each other to retransmit the corrupted packets without any packet overlapping. In other words, our proposed XRGCR scheme is more power efficient than the GCR scheme.

## VI. Numerical and Simulation Results

In this section, we present both analytical evaluation and simulation results of the RDEP and the ANR at the source and relay nodes using different CR schemes. The simulations are carried out for a network consisting of a source node $\mathcal{S}$, five relay nodes $\left\{\mathcal{R}_{1}, \mathcal{R}_{2}, \mathcal{R}_{3}, \mathcal{R}_{4}, \mathcal{R}_{5}\right\}$ and a destination node $\mathcal{D}$. For clarity in presentation, different line types and markers are used to distinguish between cases, which are defined as follows:

- BCR scheme: black square marker (simulation result) and black solid curve (analytical result),
- GCR scheme: red round marker (simulation result) and red solid curve (analytical result),
- Non-combined XRGCR scheme: blue upper-triangular marker (simulation result) and blue dash curve (analytical result),
- Proposed XRGCR scheme: magenta lower-triangular marker (simulation result) and magenta


Fig. 5. RDEP at $\mathcal{S}$ versus $\operatorname{SNR}_{R_{1} S}$.
solid curve (analytical result).
We observe that the analytical results of BCR and GCR schemes are consistent with the simulation results, and thus, for simplicity, we represent both simulation and analytical results of BCR and GCR schemes by a curve with marker in the figures shown below. Without any loss of generality, the SNRs of the forward links $\mathcal{S} \rightarrow \mathcal{R}_{i}, i=1, \ldots, 5$, are assumed to be $5 \mathrm{~dB}, 2 \mathrm{~dB}$, $-1 \mathrm{~dB},-4 \mathrm{~dB}$ and -7 dB , respectively. Thus, $\mathcal{R}_{1}$ is selected as the best relay to forward the data in the FW phase. In the RT phase, $\mathcal{R}_{1}, \mathcal{R}_{2}, \mathcal{R}_{3}, \mathcal{R}_{4}$ and $\mathcal{R}_{5}$ sequentially help $\mathcal{S}$ retransmit the lost packets to $\mathcal{D}$. The SNRs of the remaining forward inks $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{R}_{i} \rightarrow \mathcal{D}, i=1, \ldots, 5$, are assumed to be -20 dB and 0 dB , respectively. At the source and relay nodes, errors occur if the packets required to be retransmitted are different from the actual retransmitted packets.

Let us first investigate the RDEP with various CR schemes for both analytical expression and simulation results. As shown in Fig. 5, the RDEP at $\mathcal{S}$ is plotted as a function of the SNR of


Fig. 6. Sum-RDEP versus $\mathrm{SNR}_{R_{1} S}$.
the backward link $\mathcal{R}_{1} \rightarrow \mathcal{S}^{1}$. The SNRs of the remaining backward links $\mathcal{R}_{j} \rightarrow \mathcal{S}, j=2, \ldots, 5$, $\mathcal{D} \rightarrow \mathcal{S}$ and $\mathcal{D} \rightarrow \mathcal{R}_{i}, i=1, \ldots, 5$, are assumed as follows: $\gamma_{R_{j} S}=\gamma_{R_{1} S}, \gamma_{D S}=0 \mathrm{~dB}$ and $\gamma_{D R_{i}}=10 \mathrm{~dB}$. It can be seen that the proposed XRGCR scheme achieves better performance than the non-combined XRGCR scheme in terms of RDEP. This confirms the statement in observations (O1) and (O5) regarding the higher reliability in the determination of packets to be retransmitted with the combination of block ACK packets at the destination. With the GCR and the proposed XRGCR schemes, the RDEPs at $\mathcal{S}$ are shown to be significantly improved thanks to the combination of various block ACK packets from various relays in the RT phase. Also, the derived analytical RDEPs at $\mathcal{S}$ for the proposed XRGCR and the non-combined XRGCR

[^0]

Fig. 7. ANR at $\mathcal{S}$ versus $\operatorname{SNR}_{R_{1} S}$.
schemes given by (25) and (27) are consistent with the simulation results.
Considering the reliability of the retransmissions in the whole system, Fig. 6 shows the sum$\operatorname{RDEP}^{2}$ against various values of the SNR of the backward link $\mathcal{R}_{1} \rightarrow \mathcal{S}$. We can observe that the summations of the derived RDEPs at $\mathcal{S}$ and $\mathcal{R}^{(N)}$ for the proposed XRGCR and the noncombined XRGCR schemes given by the analytical expressions (25), (26), (27) and (28) are consistent with the simulation results. Also, it can be seen that our proposed XRGCR scheme achieves the best performance in terms of sum-RDEP. In fact, with the cooperation between the relays, the RDEPs at the relays are considerably improved and this results in the improvement of the sum-RDEP for the whole system. This can be easily seen when comparing the sum-RDEPs of the XRGCR scheme with the GCR scheme.

[^1]

Fig. 8. ANRs at the 5 relays.


Fig. 9. Sum-ANR versus $\mathrm{SNR}_{R_{1} S}$.

For the comparison of ANRs with various CR schemes, Figs. 7, 8 and 9 show the ANRs at the source, relays and for the whole system (in terms of sum-ANR ${ }^{3}$ ), respectively. The ANRs are also plotted as a function of the backward link $\mathcal{R}_{1} \rightarrow \mathcal{S}$ with respect to various CR schemes. As shown in Fig. 7, we observe that the ANR at $\mathcal{S}$ in the proposed XRGCR scheme is lower than the non-combined XRGCR scheme. In addition, the GCR and the proposed XRGCR schemes significantly reduce the ANR at $\mathcal{S}$ thanks to the help of all the relays in the RT phase. This confirms the statements in observations (O2), (O3), (O6) and (O8) regarding the lower ANRs at $\mathcal{S}$. In Fig. 8, it can be seen that the proposed XRGCR scheme significantly reduces the ANRs at $\mathcal{R}_{2}, \mathcal{R}_{3}, \mathcal{R}_{4}$ and $\mathcal{R}_{5}$ compared to the GCR scheme. The reduced ANRs at the relays confirm the statements in observations (O3) and (O7) in relation to the non-overlapped packets in the RT phase with our proposed XRGCR scheme. Therefore, summarising the ANRs at all the source and relay nodes for the evaluation of the whole system, Fig. 9 shows that the proposed XRGCR scheme achieves the best performance in terms of sum-ANR while a larger sum-ANR is required in the GCR scheme as a consequence of the overlapping packets in the RT phase. Also, in Figs. 7, 8 and 9, the derived expressions of ANRs at $\mathcal{S}$ and $\mathcal{R}^{(N)}$ for the proposed XRGCR and the non-combined XRGCR schemes given by (35), (36), (37) and (38) are consistent with the simulation results.

Taking the RS for the RT phase into consideration, Figs. 10 and 11 show the number of relays selected for the RT phase versus the frame length (i.e. $W$ [packets]) and total power constraint of the relays (i.e. $P_{R, \text { tot }}$ [Watts]), respectively, for both the GCR and the proposed XRGCR schemes. As shown in Fig. 10, if $W$ is smaller than 10000 packets, the proposed XRGCR scheme requires a lower number of relays for the RT phase compared to the GCR scheme. This arises since the relays in the XRGCR scheme can share the packets with each other in the RT phase without any overlapping packets. In Fig. 11, $W$ is fixed at 1000 packets and the power of each relay to

[^2]

Fig. 10. Number of relays selected for the RT phase versus frame length $\left(\log _{10} W\right)$.


Fig. 11. Number of relays selected for the RT phase versus total power constraint $\left(P_{R, \text { tot }}\right)$.
retransmit a packet (i.e. $P_{R}$ ) is assumed to be 1 Watt. It can be seen that the proposed XRGCR scheme can utilise all the relays for the RT with a lower $P_{R, \text { tot }}$ (e.g. 150 Watts). However, the GCR scheme requires a much larger $P_{R, \text { tot }}$ (e.g. 450 Watts) if all the relays are used for the RT. Thus, for a limited $P_{R, \text { tot }}$ (e.g. from 150 to 400 Watts), the proposed XRGCR scheme is better than the GCR scheme in the sense that all the relays can be used to help the source in the RT phase. This confirms the statement in observation (O9) regarding the high power efficiency of our proposed XRGCR scheme.

## VII. Conclusions

In this paper, we have proposed a cooperative retransmission scheme for wireless regenerative multi-relay networks based on XOR operations and RC. The XOR combination of block ACK packets at the destination results in a more reliable determination of retransmission and a decreased number of packets to be retransmitted at the source and relays compared to the non-combined-based scheme. The analyses of error probability of the determination of packets to be retransmitted and the average number of packets to be retransmitted have been carried out with respect to the SNRs of forward, backward and cooperation links. The derived expressions reflect well the impact of RC on the performance of the proposed scheme. Furthermore, two RS schemes have been proposed for the multi-relay-based CR based on frame length and total power constraint at the relays. The proposed XRGCR scheme is shown to be power efficient with a lower number of relays required for a small frame length, and a larger number of relays may join in the RT phase for the situation when the total power constraint is limited. For future work, we will investigate the throughput achieved with our proposed scheme taking into account the effects of both the number of the retransmission packets and the block ACK overhead. Also, we will consider a general network where the relays occasionally overhear the ACK information from the other nodes.

## Appendix A

## Proof of Theorem 1

Without loss of generality, let us consider only the first bit in each block ACK and RIP packet. For mathematical convenience, let $a_{S}, \hat{a}_{S}, a_{R_{j}}$ and $\hat{a}_{R_{j}}, j=1, \ldots, N$, denote the first bits of $\Omega_{S}, \hat{\Omega}_{S}, \Omega_{R_{j}}$ and $\hat{\Omega}_{R_{j}}$, respectively. Similarly, $b_{D}, \hat{b}_{D, 0}, \hat{b}_{D, j}, b_{S R_{j}}, \hat{b}_{S R_{j, 0}}, \hat{b}_{S R_{i, j}}, b_{S D}$ and $b_{R_{1} D}$ represent the first bits of $\Theta_{D}, \hat{\Theta}_{D, 0}, \hat{\Theta}_{D, j}, \Theta_{S R_{j}}, \hat{\Theta}_{S R_{j, 0}}, \hat{\Theta}_{S R_{i, j}},\{i, j\} \in\{1, \ldots, N\}, i<j, \Theta_{S D}$ and $\Theta_{R_{1} D}$, respectively. Then, the BEPs of $\Omega_{S}$ and $\Omega_{R_{j}}$ can be obtained as

$$
\begin{gather*}
P_{b}\left(E_{\Omega_{S}}\right)=\operatorname{Pr}\left(\hat{a}_{S}=0 \mid a_{S}=1\right) \operatorname{Pr}\left(a_{S}=1\right)+\operatorname{Pr}\left(\hat{a}_{S}=1 \mid a_{S}=0\right) \operatorname{Pr}\left(a_{S}=0\right)  \tag{41}\\
P_{b}\left(E_{\Omega_{R_{j}}}\right)=\operatorname{Pr}\left(\hat{a}_{R_{j}}=0 \mid a_{R_{j}}=1\right) \operatorname{Pr}\left(a_{R_{j}}=1\right)+\operatorname{Pr}\left(\hat{a}_{R_{j}}=1 \mid a_{R_{j}}=0\right) \operatorname{Pr}\left(a_{R_{j}}=0\right) . \tag{42}
\end{gather*}
$$

For convenience, let $\alpha_{0}^{\prime}=\operatorname{Pr}\left(b_{D}=1\right), \alpha_{00}^{\prime}=\operatorname{Pr}\left(b_{S D}=1\right), \alpha_{01}^{\prime}=\operatorname{Pr}\left(b_{R_{1} D}=1\right)$ and $\alpha_{j}^{\prime}=$ $\operatorname{Pr}\left(b_{S R_{j}}=1\right), j=1,2, \ldots, N$.

Let us first proceed with the calculation of $P_{b}\left(E_{\Omega_{S}}\right)$. We observe that $b_{S R_{j}}=1$ if there are errors in the data transmission over forward link $\mathcal{S} \rightarrow \mathcal{R}_{j}$ and $b_{D}=1$ if $b_{S D}=1$ and $b_{R_{1} D}=1$ (see (5)), i.e. if the data transmission over both links $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{R}_{1} \rightarrow \mathcal{D}$ has errors. Thus, $\alpha_{j}^{\prime}$ and $\alpha_{0}^{\prime}$ can be given by

$$
\begin{gather*}
\alpha_{j}^{\prime}=P_{b}\left(E_{S R_{j}}\right)  \tag{43}\\
\alpha_{0}^{\prime}=\alpha_{00}^{\prime} \alpha_{01}^{\prime}=P_{b}\left(E_{S D}\right) P_{b}\left(E_{R_{1} D}\right) \tag{44}
\end{gather*}
$$

Applying (24), we obtain

$$
\begin{gather*}
\alpha_{j}^{\prime}=\phi\left(\gamma_{S R_{j}}\right)=\alpha_{j}  \tag{45}\\
\alpha_{0}^{\prime}=\phi\left(\gamma_{S D}\right) \phi\left(\gamma_{R_{1} D}\right)=\alpha_{00} \alpha_{01}=\alpha_{0} . \tag{46}
\end{gather*}
$$

From (18), we can rewrite (41) as

$$
\begin{align*}
& P_{b}\left(E_{\Omega_{S}}\right)= \operatorname{Pr} \\
&\left(\hat{b}_{S R_{1,0}} \hat{b}_{S R_{2,0}} \ldots \hat{b}_{S R_{N, 0}} \hat{b}_{D, 0}=0 \mid b_{S R_{1}} b_{S R_{2}} \ldots b_{S R_{N}} b_{D}=1\right) \\
& \times \operatorname{Pr}\left(b_{S R_{1}} b_{S R_{2}} \ldots b_{S R_{N}} b_{D}=1\right)  \tag{47}\\
&+ \operatorname{Pr}\left(\hat{b}_{S R_{1,0}} \hat{b}_{S R_{2,0}} \ldots \hat{b}_{S R_{N, 0}} \hat{b}_{D, 0}=1 \mid b_{S R_{1}} b_{S R_{2}} \ldots b_{S R_{N}} b_{D}=0\right) \\
& \times \operatorname{Pr}\left(b_{S R_{1}} b_{S R_{2}} \ldots b_{S R_{N}} b_{D}=0\right) .
\end{align*}
$$

Note that

$$
\begin{gather*}
\operatorname{Pr}\left(\hat{b}_{S R_{j, 0}}=0 \mid b_{S R_{j}}=1\right)=\operatorname{Pr}\left(\hat{b}_{S R_{j, 0}}=1 \mid b_{S R_{j}}=0\right)=P_{b}\left(E_{\Theta_{S R_{j, 0}}}\right)=\phi\left(\gamma_{R_{j} S}\right)=\beta_{j},  \tag{48}\\
\operatorname{Pr}\left(\hat{b}_{D, 0}=0 \mid b_{D}=1\right)=\operatorname{Pr}\left(\hat{b}_{D, 0}=1 \mid b_{D}=0\right)=P_{b}\left(E_{\Theta_{D S}}\right)=\phi\left(\gamma_{D S}\right)=\beta_{0} \tag{49}
\end{gather*}
$$

Substituting $\alpha_{0}, \alpha_{j}, \beta_{0}$ and $\beta_{j}$ into (47), we obtain the closed-form expression of $P_{b}\left(E_{\Omega_{S}}\right)$ as

$$
\begin{equation*}
P_{b}\left(E_{\Omega_{S}}\right)=\left[1-\prod_{i=0}^{N}\left(1-\beta_{i}\right)\right] \prod_{i=0}^{N} \alpha_{i}+\sum_{\mathbb{P}} \prod_{i=0}^{N} \delta_{i} \epsilon_{i} \tag{50}
\end{equation*}
$$

where $\mathbb{P}$ denotes a set of $\left\{\beta_{i}, \alpha_{i}\right\}$ satisfying the condition that if one term is $\beta_{i}$ then there is another term $\left(1-\alpha_{i}\right)$, and if one term is $\left(1-\beta_{i}\right)$ then there is another term $\alpha_{i}$. In other words, we can represent $\mathbb{P}$ as

$$
\begin{equation*}
\mathbb{P}=\left\{(\delta, \epsilon) \mid \delta_{i}=\beta_{i} \text { or } 1-\beta_{i}, \epsilon_{i}=1-\alpha_{i} \text { if } \delta_{i}=\beta_{i} \text { and } \epsilon_{i}=\alpha_{i} \text { if } \delta_{i}=1-\beta_{i}\right\} \tag{51}
\end{equation*}
$$

Next, let us calculate the RDEP at $\mathcal{R}_{j}, j=1,2, \ldots, N$ (i.e. $P_{b}\left(E_{\Omega_{R_{j}}}\right)$ ) given by (42). We observe that $\mathcal{R}_{j}$ only retransmits the correctly received packets, thus $b_{S R_{j}}$ should be equal to zero. Otherwise, $\Omega_{R_{j}}=\hat{\Omega}_{R_{j}}=0$. From (19), (20), (22), (21) and (23), we can rewrite (42) as

$$
\begin{align*}
& P_{b}\left(E_{\Omega_{R_{j}}}\right)= \operatorname{Pr} \\
&\left(\hat{b}_{S R_{j-1, j}} \hat{b}_{S R_{j-2, j}} \ldots \hat{b}_{S R_{1, j}} \hat{b}_{D, j}=0 \mid b_{S R_{j-1}} b_{S R_{j-2}} \ldots b_{S R_{1}} b_{D}=1\right) \\
& \times \operatorname{Pr}\left(b_{S R_{j}}=0 \text { and } b_{S R_{j-1}} b_{S R_{j-2}} \ldots b_{S R_{1}} b_{D}=1\right)  \tag{52}\\
&+\operatorname{Pr}\left(\hat{b}_{S R_{j-1, j}} \hat{b}_{S R_{j-2, j}} \ldots \hat{b}_{S R_{1, j}} \hat{b}_{D, j}=1 \mid b_{S R_{j-1}} b_{S R_{j-2}} \ldots b_{S R_{1}} b_{D}=0\right) \\
& \times \operatorname{Pr}\left(b_{S R_{j}}=0 \text { and } b_{S R_{j-1}} b_{S R_{j-2}} \ldots b_{S R_{1}} b_{D}=0\right) .
\end{align*}
$$

Note that

$$
\begin{gather*}
\operatorname{Pr}\left(\hat{b}_{S R_{i, j}}=0 \mid b_{S R_{i}}=1\right)=\operatorname{Pr}\left(\hat{b}_{S R_{i, j}}=1 \mid b_{S R_{i}}=0\right)=P_{b}\left(E_{\Theta_{S R_{i, j}}}\right)=\phi\left(\gamma_{R_{i} R_{j}}\right)=\eta_{i j},  \tag{53}\\
\operatorname{Pr}\left(\hat{b}_{D, j}=0 \mid b_{D}=1\right)=\operatorname{Pr}\left(\hat{b}_{D, j}=1 \mid b_{D}=0\right)=P_{b}\left(E_{\Theta_{D R_{j}}}\right)=\phi\left(\gamma_{D R_{j}}\right)=\zeta_{j}, \tag{54}
\end{gather*}
$$

where $\{i, j\} \in\{1,2, \ldots, N\}$ and $i<j$. Substituting $\alpha_{0}, \alpha_{j}, \zeta_{j}$ and $\eta_{i j}$ into (52), we obtain the closed-form expression of $P_{b}\left(E_{\Omega_{R_{j}}}\right)$ as

$$
\begin{align*}
P_{b}\left(E_{\Omega_{R_{j}}}\right) & =\left(1-\alpha_{j}\right)\left[1-\left(1-\zeta_{j}\right) \prod_{i=1}^{j-1}\left(1-\eta_{i j}\right)\right] \prod_{i=0}^{j-1} \alpha_{i} \\
& +\left(1-\alpha_{j}\right)\left[\zeta_{j}\left(1-\alpha_{0}\right)+\left(1-\zeta_{j}\right) \alpha_{0}\right] \sum_{\mathbb{P}^{\prime}} \prod_{i=1}^{j-1} \delta_{i}^{\prime} \epsilon_{i}^{\prime}, \tag{55}
\end{align*}
$$

where $\mathbb{P}^{\prime}$ denotes a set of $\left\{\eta_{i j}, \alpha_{i}\right\}$ satisfying the condition that if one term is $\eta_{i j}$ then there is another term $\left(1-\alpha_{i}\right)$, and if one term is $\left(1-\eta_{i j}\right)$ then there is another term $\alpha_{i}$. Similarly, we can represent $\mathbb{P}^{\prime}$ as

$$
\begin{equation*}
\mathbb{P}^{\prime}=\left\{\left(\delta^{\prime}, \epsilon^{\prime}\right) \mid \delta_{i}^{\prime}=\eta_{i j} \text { or } 1-\eta_{i j}, \epsilon_{i}^{\prime}=1-\alpha_{i} \text { if } \delta_{i}^{\prime}=\eta_{i j} \text { and } \epsilon_{i}^{\prime}=\alpha_{i} \text { if } \delta_{i}^{\prime}=1-\eta_{i j}\right\} \tag{56}
\end{equation*}
$$

## Appendix B

## Proof of Theorem 2

It is noted that the ANR is corresponding to the error probability of the data transmission. We observe that $\mathcal{S}$, in the proposed XRGCR scheme, only retransmits the packet which is not correctly received by all $\left\{\mathcal{R}_{j}\right\}$ and $\mathcal{D}$, i.e. $b_{S D}=1$ and $b_{S R_{j}}=1 \forall j \in\{1,2, \ldots, N\}$. Thus, the ANR at $\mathcal{S}$ can be determined by

$$
\begin{equation*}
\lambda_{S}^{(\text {free })}=\operatorname{Pr}\left(b_{S D}=1\right) \prod_{j=1}^{N} \operatorname{Pr}\left(b_{S R_{j}}=1\right) . \tag{57}
\end{equation*}
$$

Substituting $\operatorname{Pr}\left(b_{S D}=1\right)=P_{b}\left(E_{S D}\right)=\alpha_{00}$ and $\operatorname{Pr}\left(b_{S R_{j}}=1\right)=P_{b}\left(E_{S R_{j}}\right)=\alpha_{j}, j=$ $1,2, \ldots, N$, (see Theorem 1) into (57), we have

$$
\begin{equation*}
\lambda_{S}^{(f r e e)}=\alpha_{00} \prod_{j=1}^{N} \alpha_{j}, \tag{58}
\end{equation*}
$$

In our proposed XRGCR scheme, $\mathcal{R}_{j}, j=1,2, \ldots, N$, retransmits a packet when the following conditions are satisfied:

- The packet is correctly received at $\mathcal{R}_{j}$,
- The packet fails to be received at $\mathcal{R}_{1}$ and $\mathcal{D}$ in both BC and FW phases,
- The packet fails to be received at $\mathcal{R}_{i}, i=1,2, \ldots, N, i<j$,
- The block ACK packets from $\mathcal{R}_{i}$ to $\mathcal{R}_{j}$ are correct.

Taking all these conditions into account, the ANR at $\mathcal{R}_{j}$ can be obtained by

$$
\begin{equation*}
\lambda_{R_{j}}^{(\text {free })}=\operatorname{Pr}\left(b_{S R_{j}}=0\right) \operatorname{Pr}\left(b_{S D}=1\right) \operatorname{Pr}\left(b_{R_{1} D}=1\right) \prod_{i=1}^{j-1} \operatorname{Pr}\left(b_{S R_{i}}=1\right) \prod_{i=1}^{j-1}\left[1-P_{b}\left(E_{\Theta_{S R_{i, j}}}\right)\right] . \tag{59}
\end{equation*}
$$

Substituting $P_{b}\left(E_{\Theta_{S R_{i, j}}}\right)=\phi\left(\gamma_{R_{i} R_{j}}\right)=\eta_{i j}, \operatorname{Pr}\left(b_{R_{1} D}=1\right)=P_{b}\left(E_{R_{1} D}\right)=\alpha_{01}, \operatorname{Pr}\left(b_{S D}=1\right)=$ $\alpha_{00}$ and $\operatorname{Pr}\left(b_{S R_{j}}=1\right)=\alpha_{j}, j=1,2, \ldots, N$, into (59), we obtain

$$
\begin{equation*}
\lambda_{R_{j}}^{(\text {free })}=\left(1-\alpha_{j}\right) \alpha_{01} \alpha_{00} \prod_{i=1}^{j-1} \alpha_{i} \prod_{i=1}^{j-1}\left(1-\eta_{i j}\right) . \tag{60}
\end{equation*}
$$

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[^0]:    ${ }^{1}$ It is noted that the wireless medium between $\mathcal{S}$ and $\mathcal{D}$ can be reasonably assumed to be unchanged during a period of transmission time due to the fixed locations of $\mathcal{S}$ and $\mathcal{D}$, while the wireless medium of the backward link $\mathcal{R}_{1} \rightarrow \mathcal{S}$ may vary due to different relay locations. Therefore, in this work, we fix the SNR of the backward link $\mathcal{D} \rightarrow \mathcal{S}$ and plot the performance as a function of the SNR of the backward link $\mathcal{R}_{1} \rightarrow \mathcal{S}$.

[^1]:    ${ }^{2}$ The sum-RDEP is defined as the summation of the RDEPs at $\mathcal{S}$ and $\mathcal{R}^{(N)}$ in the RT phase.

[^2]:    ${ }^{3}$ The sum-ANR is defined as the summation of the ANRs at $\mathcal{S}$ and $\mathcal{R}^{(N)}$ required for the RT phase.

