The siren call of probability: Dangers associated with using probability for consideration of the future

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Abstract

Many tools for thinking about the future employ probability. For example, Delphi studies often ask expert participants to assign probabilities to particular future outcomes. Similarly, while some scenario planners reject probability, others insist that assigning probabilities to scenarios is required to make them meaningful. Formal modelling and forecasting methods often also employ probability in one way or another. The paper questions this widespread use of probability as a device for considering the future, firstly showing that objective probability, based on empirically-observed frequencies, has some well-known drawbacks when used for this purpose. However, what is less-widely acknowledged is that this is also true of the subjective probability used in, for example, Delphi. Subjective probability is less distinct from objective probability than proponents of its use might imply, meaning it therefore suffers from similar problems. The paper draws on the foundations of probability theory as set out by Kolmogorov, as-well-as the work of Keynes, Shackle, Aumann, Tversky and Kahneman, and others, to reassert the essential distinction between risk and uncertainty, and to warn about the dangers of inappropriate use of probability for considering the future. The paper sets out some criteria for appropriate use.

Keywords: uncertainty; probability; Delphi; scenario planning; Keynes; Shackle

1 Introduction

Probabilistic methods for thinking about the future abound. In formal modelling and forecasting, an obvious example is the use of 'fan charts' to provide an indication of the range of possible future outturns, usually based on a normal probability distribution (Wallis, 2003; Gneiting, 2008). However, probability is also widely used in non-formal modelling techniques for consideration of the future. For example, in many Expert Knowledge Elicitation methods, such as Delphi, the focus is on quantifying experts' single-point estimates of uncertain quantities (Derbyshire and Wright, 2017; Morgan, 2014), which are sometimes given in the form of probabilities. Similarly, in scenario planning expert participants are often asked to assign probabilities to considered future outcomes[1] (Gary and von der Gracht, 2015). Probability, then, is employed as a device for consideration of the future in a wide-range of tools used for this purpose.

However, in light of the discussion set out in this paper, this widespread use of probability for thinking about the future is somewhat worrying. The paper outlines why the use of probability contains important dangers, not least among which is a downplaying of the full extent of the uncertainty of the future. A downplaying of this sort is something Stern (2016) has recently noted, when accusing climate-change modellers of understating the 'big risks' associated with climate change. By this Stern (2016) actually means that they understate the full extent of the *uncertainty* associated with climate change, since risk and uncertainty are two different things. This paper argues that this downplaying of uncertainty, both in relation to climate change and more broadly, results from the widespread failure to distinguish between the appropriate use of probability in circumstances characteristic of *risk*, and its inappropriate use in circumstances characteristic of *uncertainty* that is irreducible to risk. Under the latter circumstances, probability not only has little or nothing to contribute, its use is outright dangerous.

The full extent of the danger associated with the use of probability for thinking about the future is perhaps most obvious in the domain in which the reduction of uncertainty to risk is most evident and consistently enacted: mainstream economics. Skidelsky (2009, p.38-p.44) places the central blame for the Great Recession on inappropriate use of probability to consider aspects of reality subject to fundamental, irreducible uncertainty.

[1] Although, tellingly for the subsequent argument set out in this paper, some scenario planners suggest assigning probabilities to be inadvisable, because it draws attention away from potential high impact, but low-probability futures (Derbyshire, 2016).

Skidelsky (2009) argues that a central danger stemming from the use of probability is that it provides those using it with a comforting, but spurious accuracy that facilitates the self-delusion that the uncertainty of the future is fully measureable and has been tamed. In this paper, we refer to this self-delusion as the 'siren call of probability'. To understand the full extent of the danger to which it leads requires a detailed understanding of the problems associated with using probability to think about the future. As such, this paper discusses the following problems with probability:

1) *Determinism*. Probability is often portrayed as an antidote to determinism because its use acknowledges that there are multiple possible outcomes of varying likelihood, rather than just one possible, pre-determined future. This quality of probability, it is sometimes implied, renders it a device for thinking about the future that is able of take account of uncertainty (see, for example, Gneiting, 2008). Yet, the use of probability, while much preferable to the *absolute* determinism of single-track forecasting, can nevertheless contain a type of determinism of its own. According to the Stanford Encyclopaedia of Philosophy, determinism is 'the idea that every event is necessitated by antecedent events and conditions, in combination with the laws of nature'. Therefore, an individual scenario, in which a set of antecedent causes results in a single, specific outcome, is still *individually* deterministic, even if it is part of a wider group of scenarios, each of which is assigned a probability, such that multiple possible futures are considered (Derbyshire and Wright, 2017). Furthermore, the part of this definition of determinism that states '…in combination with the laws of nature' implies that, even if a consideration of the future includes a stochastic, random element, it could still be deterministic if the randomness is viewed as aleatory in nature, since the randomness is then part of 'the laws of nature'.
2) *Non-stationarity*. According to Skidelsky (2009, p.43), it is only possible to employ probability to think about the future by accepting the 'ergodic axiom', which holds that the future is merely a shadow of the past and present. To use probability for considering the future therefore requires the assumption of a certain level of stability over time. 'Non-stationarity' is commonly taken to refer to the absence of this stability, although its true meaning is somewhat more complicated than this (Koutsoyiannis and Montanari, 2015), highlighting the care that must be taken when dealing with this problem with the use of probability for consideration of the future. This contrasts with the lack of care commonly taken in actuality, and the inefficacy of solutions commonly implemented for overcoming this problem, such as the use of different models for time-periods exhibiting different variances following step-changes (Koutsoyiannis and Montanari, 2015). Sensible consideration of future outcomes cannot simply take probability distributions estimated using frequency of past occurrence, or based on the past variance of a given variable, as a true representation of what might occur in the future.
3) *Openness*. The use of probability assumes that the universe can be 'closed', such that all possible outcomes of a focal aspect of the future are fully-specifiable in advance, so that probabilities adding to unity (i.e. to 1) can be assigned to them. Under the Kolmogorov axioms (Kolmogorov, 1956) that underpin probability theory, if the 'event space' of all possible outcomes cannot be fully specified, then the probability of any single outcome, or even of any subset of outcomes, cannot be defined. Because of the requirement for 'closure', reducing uncertainty to risk by using probability leaves us exposed to the danger of failing to take account of the possibility of new outcomes - these being the central problem we really face when considering the future, because the future is inherently 'open'. Using probability to think about the future is therefore dangerous because it forces us to base our view of the future on what has happened in the past.
4) *Additivity*. Because of the aforementioned problem of 'openness', new possibilities occur to us as we feel our way into an opaque and emerging future, and to accommodate these requires us to constantly downgrade the probabilities we have assigned to the possibilities we have already considered, since they must add to unity (Shackle, 1955; 1961). This renders us susceptible to potential high-impact outcomes which are assigned low probabilities of occurrence, because of the need to accommodate all possible outcomes while still summing to unity.
5) *Crucial decisions*. The mid-20th century economist G. L. S. Shackle (Shackle, 1955; 1961), whose work focuses on the deficiencies of probability as a device for dealing with uncertainty, coined the phrase 'crucial decision'. These are one-off decisions that have major consequences, in which the very act of making the decision destroys forever the circumstances in which the decision is taken, such that no future decision can ever be taken in the same circumstances again. By contrast, the foundations of probability theory lie in repeated experiments that are sufficiently similar to represent an aggregable class from which relative probabilities can be derived. This logic is not only useless, but can be highly dangerous under circumstances in which a decision can only be taken once because it is a crucial one, the taking of which destroys the circumstances in which it is taken. Under these circumstances the focus must be on the potential impact of the various possible outcomes, not on their relative likelihoods

(which are, anyway, meaningless for a non-repeatable, one-off decision of a crucial nature), since it might be desirable to avoid some outcomes at all costs.

6) *Accurate aggregation*. Use of frequency-based probabilities therefore depends on the property of being aggregable thereby exposing us to the danger of inappropriate aggregation, leading to inaccurate relative frequencies, which are then used to plan for the future. Tversky and Kahneman (1974) showed aggregation into categories of outcome to be problematic because of the tendency for individuals to use representativeness based on similarity as a heuristic when assigning events to classes.
7) *Innate subjectivity*. The logic of subjective probability, as used in Delphi and other expert-based approaches to consideration the future, assumes that differences in the subjective probabilities assigned to particular outcomes stem from individuals' inherently bounded and incomplete access to information. Under this logic, when individuals' engage in multiple exchanges of their inherently biased information, the bias is reduced such that all have access to the same information. The result is expected to be a convergence of opinions, both between the participant experts, and between the mean assigned probabilities and the 'true' state of the universe. However, as noted by Morris (1995) and Aumann (1976), this logic does *not* hold true where individuals' subjective opinions are not based on differences in available information, but on differing ontologies, which cause them to interpret the same information differently. Under these circumstances, no amount of exchange of information will resolve the differences and bring about convergence. Essentially, what this amounts to is a criticism of approaches based on Bayesian updating, which are of limited usefulness under circumstances in which individuals have heterogeneous prior beliefs despite access to the same information.

While the paper therefore outlines a number of important problems with probability as a device for consideration of the future in general - covering both probability that is 'objective' and probability that is 'subjective' - a central purpose of the paper is to emphasise that the use of subjective probability, as in Delphi and other expert-based approaches, can be as problematic as can the use of objective probability in formal modelling. These two types of probability are not nearly as distinct from each other as proponents of subjective probability might suggest. Ultimately, when constructing the foundations of subjective probability, Savage (1954) sought to abide by the same axioms which underpin probability theory more generally, including that which is frequency-based, as subsequently set out by Kolmogorov (1956). As a result, some of the problems associated with objective, frequency-based probability as a device for consideration of the future also apply to subjective probability.

To set out these problems, the paper draws on, among others, Kolmogorov (1956), Keynes (1921; 1936; 1937), Shackle (1955; 1961) and Tversky and Kahneman (1974). It concludes by setting out some criteria for identifying circumstances in which it is safe to use probability for consideration of the future. It is argued that circumstances in which probability can safely be used to consider the future are a minority, rather than representing the vast majority, as one might assume given the widespread use of probability. The paper concludes with a plea to avoid what Hacking (1990, p.5) refers to as 'the imperialism of probability', which in this paper is characterised as the comforting but dangerous siren call of probability, that lulls us into the self-delusion that the future is knowable based on present knowledge, and that uncertainty is tameable through reduction to risk. The next section describes the two main interpretations of probability which have already featured in the above discussion, both of which are central to the argument subsequently set out: the objective/frequentist view and the subjective view.

2 Two interpretations of probability

2.1 The objective/frequentist view

Morris (1995, p.231) draws on Savage (1954) to identify two main interpretations of 'probability': the frequentist and the subjective. The first corresponds with what most people might intuitively assume probability to be, if asked: frequency-based probability, based on past empirical occurrence, representing 'the long run frequencies of repeated events' (Morris, 1995, p.231), which is referred to in this paper as 'objective probability' to contrast it with 'subjective probability', which is not based on the frequency of repeated events. A widely-used example of objective probability is that which results from coin-tossing, because an important requirement of frequentist-based probability can be easily conveyed using this example: that each outcome in a series from which a probability is derived is independent of each other outcome. In coin-tossing this is self-evident, since the outcome from a single coin-toss is entirely independent of the outcome from prior and subsequent tosses.

2.2 The subjective view

The second understanding of probability identified by Morris (1995, p.231) and Savage (1954) is the 'personalistic' or 'subjective Bayesian' view, which we here refer to simply as 'subjective probability'. Subjective probability, in contrast to frequentist probability's basis in objective empirical occurrence, is a measure of the confidence an individual has in the truth of a particular proposition (Morris, 1995). However, while subjective, and therefore personal, this confidence is not based on mere whimsy, but influenced by available information, and is, in that respect, also empirically-based. Its distinctiveness from objective probability lies in subjective probability's recognition that individuals can hold differing views with regards to particular propositions because of differences in the information available to them. Subjective probability can therefore be measured by individuals' willingness to wager, representing the confidence they have in their information, which reflects, under this view of probability, a proposition's 'truth' (Guo and Tanaka, 2010; de Finetti, 1937).

3 The nature of probabilistic risk vis-à-vis non-probabilistic uncertainty

3.1 Kolmogorov's probability axioms

The Kolmogorov axioms (Kolmogorov, 1956) form the foundations of modern probability theory. Kolmogorov's second and third axioms are very salient to the argument developed in this paper. The second states that, when using probabilities, it must be certain that one event will occur out of all the possible events constituting the 'event space'. This space must therefore comprise all possible future outcomes. The third states that the sum of all the various probabilities in the event space constituting all possible outcomes must be 1. Crucially for the subsequent discussion, the second axiom means that if the event space cannot be closed and fully specified in advance, the probability of any single outcome, or even any subset of outcomes, cannot be defined. The third axiom means that, if there is a failure to achieve the requirements of axiom two, and a new event occurs not currently incorporated in the event space, to use the event space for further consideration of the future from that point on requires that the existing events of which it is comprised have their probabilities downgraded to accommodate the new possibility. We subsequently outline the dangers which these requirements for valid use of probability lead to.

3.2 The distinction between probabilistic risk and non-probabilistic uncertainty

There was a time in which the limitations of probability as a device for thinking about the future were widely acknowledged. Derbyshire (2016) shows that there were three early-to-mid twentieth century economists that emphasised a crucial distinction between risk and uncertainty, and who showed that probability has severe limitations in relation to the latter. Knight (1921) is perhaps the most widely-known. However, Keynes (1921; 1936) also made an important distinction between risk and uncertainty, stating that:

> 'By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable…The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence…About these matters there is no scientific basis on which to form any calculable probability whatever' (Keynes, 1937, p.113-114).

The third economist to emphasise a distinction between risk and uncertainty is more obscure: G. L. S. Shackle (1955; 1961). Yet, Shackle's obscurity is undeserved, because his is the most persuasive and detailed argument on the dangers of probability and the importance of uncertainty (Zappia, 2014). Shackle was writing on the theme of uncertainty at around the same time that the subjective approach to probability based on Savage (1954), Ramsey (1931) and de Finetti (1937) was coming to prominence - the approach to probability that has subsequently come to dominate mainstream decision theory and economics. Subjective probability renders risk and uncertainty indistinguishable because, even where objective probabilities cannot be created, an individual's estimate of the probability of any event can be elicited from the odds at which she would be prepared to bet for or against that event (Feduzi et al., 2014). For this reason, following the advent of the subjective probability approach, important insights into the limitations of probability as a means for dealing with uncertainty were shifted to the margins, where they have remained for many decades (Basili and Zappia, 2009; Basili and Zappia, 2010).

Yet, the distinction between risk and uncertainty insisted upon most forcefully by Shackle, but also made prominent by Knight and Keynes, while perhaps wounded by the advent and then dominance of subjective probability, was not fatally so. It has never really gone away and has the tendency to return to prominence in times of heightened uncertainty (Feduzi et al., 2014). We now live in times in which many would consider uncertainty more palpable than ever before, and in which the range and interconnectedness of challenges faced makes the continued denial of uncertainty untenable (Derbyshire, 2016). Under these circumstances of increased uncertainty, the dangers associated with inappropriate use of probability for considering the future are still more pronounced. The next section describes these dangers in detail.

4 Problems associated with using probability to think about the future

4.1 Probability and the problem of determinism

The use of probability to consider the future, it might be argued, is an acknowledgement that there is no single, absolutely certain future outcome, as inherent in probability's representation of a set of *multiple* possible outcomes, each with its own likelihood. Therefore, probability, this line of reasoning might continue, represents an *opposite* to determinism, and, concomitantly, an acknowledgement of uncertainty. Yet, Davidson (1991, p.132) suggests the opposite: probability implies *certainty* and the presence of absolute foreknowledge, as is characteristic of determinism, under circumstances in which probabilities, calculated on the basis of past empirical occurrence, remain relevant over time (the so-called 'ergodic axiom', which we come to subsequently). Davidson (1991, p.132) explicitly states that, under these circumstances '…*probability is knowledge, not uncertainty*!'.

The implication is that where future probabilistic outcomes converge with those observed in the past there is no longer any uncertainty. Even if certain knowledge of individual outcomes cannot be known, and all that can be known is the probabilities of differing possibilities in the aggregate, this still represents a form of certainty and absolute knowledge. The uncertainty of not knowing with certainty *the* future outcome in any one instance, and instead only knowing the long-run frequencies, is not really *uncertainty* at all - it is *risk*. Indeed, Shackle (1955; 1961) makes essentially the same point when identifying divisibility and seriability as the qualities characteristic of probabilistic risk, as distinct from non-probabilistic uncertainty (Derbyshire, 2016). The coin-tossing example is again useful here, because there *is* certainty, interpreted as Davidson (1991) does, in relation to a single outcome (an individual toss) - the certainty of knowing the probabilities of the two possible outcomes, 0.5 for heads and 0.5 for tails.

Rather than representing an antidote or opposite to determinism then, as it is sometimes claimed to, probability can itself be deterministic. Furthermore, this remains true even if outcomes are thought of as dependent on an element of randomness. Approaches which seek to account for aleatory randomness, which refers to the intrinsic uncertainty associated with natural variability (Maier et al., 2016, p.155), could also be considered deterministic if this natural variability is viewed as part of the fundamental laws of nature. Determinism, according to the Stanford Encyclopaedia of Philosophy, is 'the idea that every event is necessitated by antecedent events and conditions together *with the laws of nature*' (emphasis added). If natural variability is a fundamental part of reality (i.e. it is ontological), then an outcome deemed to be dependent on a set of antecedent conditions together with natural variability in the form of aleatory randomness - the latter, for example, perhaps being taken account of by including a stochastic element in a forecasting model - is, nevertheless, still deterministic.

In relation to approaches which allow consideration of multiple possible futures, and which assign a probability to each, as is sometimes done in scenario planning: while multiple futures are considered, if each future is nevertheless *individually* determined by a single set of antecedent causes, or a single set of parameter values in formal modelling, then each individual scenario remains deterministic, even if the exercise as a whole is not because it considers multiple such outcomes (Derbyshire and Wright, 2014). If each individual considered future results from a single set of causes or parameter settings, determinism is still present. This determinism at the level of the individual scenario or outcome can give the misleading impression that all that is necessary in order to avoid an undesirable future is to be alert to the occurrence of a particular set of causes, or the empirical values of a set of indicators, and, if such are identified as occurring, to take action to avoid the undesirable outcome (Derbyshire and Wright, 2017). This misleading view is prominent in both the horizon scanning and scenario planning literature, and is associated with identification of 'weak signals' and 'early warnings' (Derbyshire and Wright, 2014).

The point to be emphasised is that probabilistic methods can themselves contain strong deterministic assumptions, despite their use of probability. Probability cannot, then, be considered a straightforward antidote for, or opposite to, determinism. Approaches to consideration of the future which examine multiple possible futures are not *necessarily* engaging with the problems of indeterminism or uncertainty if each of those individual futures is anyway necessitated by a set of antecedent conditions (Derbyshire and Wright, 2017), even if the considered outcomes are partly random in nature.

4.2 Probability and the problem of non-stationarity

Objective probability, based on frequency of past occurrence, is closely associated with ‘rational expectations’ about the future, as incorporated in mainstream economics (Davidson, 1991, p.132). The central assumption lying behind the use of this type of probability is that ‘the same probabilities which determined past outcomes will continue to govern future events’ (Davidson, 1991, p.132). Put more formally: the ‘time averages calculated from past data will converge with the statistical averages computed from any future time series’ (Davidson, 1991, p.132). This is the so-called ‘ergodic axiom’ that is a central component of econometric-based forecasting methods, but which can also feature, more surreptitiously, in the assumptions used when applying other approaches to consideration of the future, such as horizon scanning or Delphi.

If ergodicity is present, knowledge of the future merely involves projecting forwards average outcomes based on past and current occurrences, since the future is simply a reflection of the past - and time is, essentially then, of no importance. In formal modelling, in relation to time series of observations, this stability is known as ‘stationarity’, which at its most basic can simply be taken to mean that probability distributions based on frequency of past occurrence remain stable over time. On the face of it then, stationarity is a fundamental requirement for the valid use of probability for consideration of the future.

However, Koutsoyiannis and Montanari (2015) show the issue of stationarity to be much thornier than this: even a stationary process describes a system changing in time. As a result, the meaning of non-stationarity, as commonly employed, has become confused. To be strictly valid, ‘non-stationarity’ must mean that a probability density function changes *as a deterministic function of time* (Koutsoyiannis and Montanari, 2015). It does *not* simply mean that probability cannot be used for consideration of the future, or that non-stationarity and uncertainty are synonymous. Indeed, as Koutsoyiannis and Montanari (2015) state, the improper or unjustified claim of non-stationarity can itself result in underestimation of variability and risk, highlighting the full extent of its thorniness as an issue when considering the future. What these confusions around the concept’s meaning highlight is the great care that must be taken in relation to this issue when considering the future – a level of care which, as Koutsoyiannis and Montanari, (2015) discuss, it seldom receives.

Koutsoyiannis and Montanari (2015) highlight a common problem with the way the issue of non-stationarity is dealt with, by reference to a time-series that contains obvious step changes over time. In common statistical practice, these step changes would be described as non-stationarities and a non-stationary model with different properties for each of the stable time-periods would be created. However, such a model would be of little use for consideration of the future, and overlooks the fact that overall, the stochastic process which generated the series *is*, in fact, a stationary one (Koutsoyiannis, 2011). This chimes with an essential point made by Wynne (1992 p.115) when discussing uncertainty in relation to environmental learning: scientific knowledge can only be produced by reducing ‘the framework of the known to that which is amenable’ to the parochial methods and models at hand.

Nevertheless, while perhaps not strictly having the meaning it is commonly attributed, the broader implications of non-stationarity are relatively intuitive. It is undeniable that circumstances change over time, such that what was once true no longer is. The present author is writing from the UK in which this tendency has been rendered stark by the result of the recent referendum that will lead eventually to Britain leaving the European Union. This will undoubtedly have many ramifications, such that some past possibilities no longer apply, while other, new possibilities are opened up. As a result, the probability distributions on which future outturns for economic growth, or on which an individual firm planned its investment in the UK, may no longer be valid. The result is an absence of stability. For reasons such as this, under circumstances of non-stationarity, probability, such as that based on frequency of past occurrence, or in the form of a distribution based on past variance, is a misleading guide to the future.

To return to the more formal language of statistics for a moment, the broad implication is that for aspects of the future ‘past probability distribution functions can differ persistently from the time averages that will be generated as the future unfolds and becomes historical fact’ (Davidson, 1991, p.133). What needs to be emphasised is that under these circumstances, as Davidson (1991, p.133) goes on to imply, sensible consideration of future outcomes cannot - and, therefore, *should* not – simply take probability distributions estimated using frequency of past occurrence, or based on the past variance of a given variable, as a given and true representation of what might occur in the future.

Stern (2016) has recently discussed how climate change models drastically understate the possible extremity of future temperature changes (i.e. they underestimate the full extent of climate-change uncertainty). One reason for this is that in climate-change modelling future temperature outcomes are partly based on what has occurred in the past (i.e. past variance). 'Climate *change*', however, implies that the future is *changed* compared to the past; so there is no reason to think the past is an accurate guide to the full range of possible future climate outcomes. Even using the past as a partial guide to the future - one which is then manually adjusted to reflect possible, more extreme outcomes - could still significantly underplay the full uncertainty associated with climate change. This is especially so when we consider that climate change results from a complex interaction of driving forces that can result in compound, non-linear (and, therefore, accelerating) change over time (Maislin, 2013). Herein we see the problem of non-stationarity writ large.

4.3 Probability and the problems of accurate aggregation and openness

4.3.1 The problem of aggregation

Shackle (1955; 1961) was earlier highlighted as one of three early-to-mid twentieth century economists to have made an explicit distinction between probabilistic risk and non-probabilistic uncertainty. Shackle showed frequentist-based objective probability to hinge on an essential quality: the quality of outcomes being aggregable, which in turn depends on them being replicable and serialable (i.e. they can be made into a series).

Shackle (1955; 1961) showed that probabilities can only be calculated based on frequency of occurrence if the 'events' being considered form a homogenous class such that each is an accurate - and, importantly, an *independent* - replication of each other, making them a series. Coin-tossing is the classic example of these criteria, because it provides the simplest possible example in which they are met. Each toss of a coin in a series is independent from each other toss, yet is also an accurate replication of each other toss, such that all tosses taken together can be aggregated to form a series from which a frequency-based probability distribution can be calculated (Derbyshire, 2016). Obviously, the long-run average of this aggregation is a probability distribution representative of 50% occurrence of heads and 50% tails. This property of being aggregable, which is in turn related to the properties of independence and seriability, is crucial to the creation of valid objective probabilities based on frequency.

However, accurate classification of events into categories of outcome has been shown by Tversky and Kahneman (1974) to present significant judgemental problems, rendering individuals susceptible to representativeness biases in which irrelevant information is employed to assess likeness so as to form the categories. Tversky and Kahneman (1974, p.1124) show that when answering questions of the type 'What is the probability that event A originates from process B?' the heuristic device which people intuitively fall back on is to consider the degree to which A resembles B. While this may seem like a reasonable approach to classifying types of outcome so as to derive probability frequencies, an important problem is its tendency for irrelevant information to corrupt the categorisation process rendering it inaccurate. Tversky and Kahneman (1974) give the example of an exercise to assess the probability that someone called Steve is engaged in a particular occupation. The participants are given information that 'Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail'. The result is that Steve is very readily given a high probability of being a librarian. This 'representativeness' approach to categorisation can lead to severe errors because the representativeness is *not* influenced by factors that should affect judgements of probabilities, such as base rate estimates, which are much less intuitively relied upon.

Historical outcomes can be still more difficult to categorise than the simple case of categorising Steve, because the past is open to interpretation, is opaque, and tends to get re-interpreted to suit particular perspectives (Bradfield et al., 2016). As a result, accurately categorising past outcomes, such that they form homogenous classes from which frequency-based objective probabilities can be calculated, is even more susceptible to bias.

4.3.2 The problem of openness

The property of being aggregable is related to a further, important requirement for the creation of accurate objective probabilities: that a full set of possible outcomes can be fully-specified in advance. In probability theory this is known as the quality of being 'closed'.

The quality of being aggregable and serialable requires that the 'experiment' is a closed one in which all possible outcomes are known in advance. If new outcomes can occur over time that have not previously occurred, there is no valid way to construct objective, frequency-based probabilities, because there is no way to aggregate unlike 'experiments' in which the possible outcomes are not the same in each instance. The implication is that an 'event space' representing all known outcomes must be identified in advance in order for valid probabilities about the future to be calculated. Indeed, it was earlier noted that a fundamental assumption of the Kolmogorov (1956) axioms which underpin probability theory is that all possible outcomes must be fully-specifiable. If the event space cannot be fully specified in advance, then the probability of any one outcome, or even of any subset of outcomes, cannot be defined (Derbyshire, 2016).

Yet, one of the problems associated with considering the future is that we can never know what we do not know - the so-called problem of 'unknown unknowns'. What this means in relation to the present discussion is that we do not - and, indeed, cannot - ever know for certain that there are no new possibilities that have not previously occurred, apart from in very simple circumstances such as in coin-tossing. When it comes to the future, things change over time, and new possibilities occur, such that surprise is always possible, and to ignore this possibility is highly dangerous. The use of probabilistic methods for considering the future subtly coaxes us into epistemological hubris: the assumption that we know more than we really know, and can close the universe as it has occurred up to now, such that we can make assumptions about the future based on present knowledge. This tendency to make us rely on what we know presently, and to ignore consideration of what we do not know, or how things may change in the future rendering our present knowledge inaccurate, is an inherent by-product of the use of probability and is highly dangerous.

4.4 Probability and the problem of additivity

Related to the problem of 'openness' is that of 'additivity', referring to the requirement that probabilities add to unity (i.e. 1). It is in turn connected to Kolmogorov's requirement for an a priori fully-specifiable set of outcomes (i.e. a complete 'event space', or the achievement of 'closure'), because the central reason for this requirement in Kolmogorov's axioms is that, only by knowing all possible outcomes can each be attributed a probability, such that there is a complete set that sums to unity.

Again, there is an obvious problem with this assumption that does not require any special training to grasp. Namely, because of openness, new possibilities occur as we feel our way into an opaque and emergent future; and these new possibilities, if we are to accommodate them within the event space of all possibilities, inevitably requires the downgrading of previously considered outcomes currently part of that space. The danger is that, as new possibilities are added, attention to existing possibilities is diminished, because their probabilities are increasingly small, as we reduce them to accommodate new possibilities. Indeed, it was this problem that led Royal Dutch Shell to explicitly disavow the assignment of probabilities to created scenarios about the future in the 1970s, which was the period in which they used scenario planning to consider the possibility of, and plan for, the forthcoming oil crises. Shell discovered that attributing small probabilities to potentially highly impactful events diminished the attention management was willing to give them. The occurrence of these events would be so damaging that they deserved attention despite their low likelihood, but the assignment of probability would act against their receiving this due attention (Jefferson, 2012; 2014; Derbyshire, 2016).

The use of a 'residual' is sometimes countered as a means to overcome the additivity requirement by proponents of the use of probability, meaning that an amount of probability would initially be left unallocated to accommodate new possibilities as they arose (Derbyshire, 2016). Indeed, in the 1950s, after he set out his critique of probability as a device for dealing with uncertainty in which the problem of additivity featured prominently, Shackle's contemporary opponents made exactly that response: the problem of additivity can be overcome simply by using subjective probabilities along with a residual, the latter essentially amounting to a subjective assessment of one's own ignorance. Shackle responded to this suggestion, as detailed by Basili and Zappia (2009), by insisting that the use of a residual does *not* help in overcoming the problem of additivity (Derbyshire, 2016, p.16: footnote). As Runde (2000) describes, Shackle's logic was that to use a residual must inevitably mean that a perfectly possible outcome is represented by a 'lower numerical probability' (Runde, 2000, p.226).

This problem can be thought of in the following way: an individual decision-maker, on the basis of presently-existing circumstances (which, incidentally, Shackle suggested are the *initial* basis for consideration of future subjectively-imagined possibilities) imagines four possible future outcomes ('scenarios') in relation to a focal issue of concern. The decision-maker assigns likelihoods of 0.30, 0.20, 0.25 and 0.15 to each respectively, leaving a residual of 0.10, which is the decision-maker's assessment of her own present ignorance, representing an acknowledgement that alternative possible outcomes may have been overlooked. Indeed, as time moves on, a new possibility does indeed occur to the decision-maker, subjectively imagined, but perhaps stimulated by changes in then presently-existing, objective circumstances. This new, previously-unconsidered possibility appears stark in the mind of the decision-maker, who then assigns a likelihood of 0.25 to it. To accommodate this new possibility, the likelihoods of the originally-identified possible outcomes are now revised downwards to 0.20, 0.15, 0.20 and 0.10 respectively. Again, the decision-maker cannot be certain to have closed the problem, and therefore once again leaves a residual of 0.10 to reflect her ignorance, and the possibility for still further possibilities to be revealed over time. Indeed, some time later a further, new possibility does indeed occur to her, which now must also be accommodated. Therefore, the already-considered outcomes must again be revised down in order to accommodate the new one.

We will not try the reader with a further iteration of this tiresome logic as the point is now obvious. To accommodate further outcomes, existing, already-considered outcomes must constantly have their probabilities revised downwards, even where a residual is employed. This is true *unless* the residual determined right at the beginning of the process (in the first iteration) is so large as to be adequate to accommodate all possibilities that are revealed over time, or the residual is itself revised over time to represent the decision-maker's changed assessment of her own ignorance. Inevitably, however, because of the finite resource of assignable likelihood that results from the requirement for additivity, the adding of new possibilities over time must require a revising downwards of the likelihood assigned to perfectly possible outcomes that have already been considered. This is especially so if the problem is highly uncertain, and therefore requires the decision-maker to assign a large initial residual, and to maintain the large residual through the many iterations over time, because this large residual then leaves little remaining likelihood to assign to considered outcomes. They must therefore, inevitably, be assigned only small probabilities.

For this reason, Shackle (1955; 1961) noted that the use of a residual is an acknowledgement by the decision-maker that she does *not* have an exhaustive set of all possible future outcomes, which, as we have noted, is a central requirement for the creation of valid probabilities according to the Kolmogorov axioms. And secondly, given the radical uncertainty we face in relation to many aspects of the future, while it might be revised as we go along, at each point in time (i.e. at each point at which a revision might occur) there is no way to know in advance how large the residual would have to be to accommodate possibilities that are revealed in the future. If it were possible to know how many new outcomes may occur to us as the future unfolds, and approximately what probabilities they may require, then we *would* be able to specify a full set of possible outcomes (a complete 'event space') in advance, meaning we would not require a residual. The very reason a residual is necessary is that *we do not know*. As Runde (2000, p.227) goes on to comment, primarily for these reasons, 'The standard [subjective probability] model cannot accommodate a residual hypothesis in Shackle's sense, and subjective probability does not appear to be able to capture his notion of possibility'.

In short, the requirement for additivity is highly problematic when it comes to the use of probability for thinking about the future. Moreover, as has been shown, this is not a merely esoteric, theoretical or technical problem - it contains within it the very real danger of overlooking outcomes which could be highly impactful. The use of probability actively encourages a dangerous diminishing of attention to potentially highly impactful outcomes.

4.5 Probability and the problem of crucial decisions

Shackle is perhaps still better known for the concept which he called a 'crucial decision' (Shackle, 1955; 1961), which also has major implications for the usefulness of probability as a device for considering the future. A crucial decision is one which destroys forever the circumstances in which the decision is made in the first place, such that no decision can ever be made again *in the same circumstances*. Decisions of the same type can be taken once more, but the circumstances in which they are made are changed, not least often in relation to the person taking the decision, whose mental state and attitude is altered.

Take, for example, an individual's decision to marry. If he or she is lucky, only one such decision will be taken during a lifetime, and this decision will obviously change forever the circumstances in which it is taken for that reason. However, that such a decision changes the circumstances in which it is taken remains true even if the individual is initially unlucky in marriage, but lucky enough to have an opportunity to take the same decision once more, sometime subsequent to extrication from the first marriage. While the decision is the same, the circumstances are not; the individual is now perhaps more circumspect, or, alternatively, may be filled with appreciation for another chance at a happy life, having now experienced the opposite. The point is that crucial decisions change the circumstances in which they are made; the decision may be made multiple times, but the significance on each occasion is different (Shackle, 1955, p.6). The same holds in a less sublime setting, such as that pertaining to the development of new weapons (Derbyshire 2016). In this setting, a nation develops a new weapons system and potential enemies respond with their own new weapons systems designed to counter that developed by their enemies in turn. The nation taking the initial decision, thereby triggering this sequence, may of course make the same decision again, and develop a further new system in response, but this decision is made under the changed strategic circumstances in which the responding, counter weapon system now exists.

Shackle's distinction between crucial decisions, for which probability is *not* useful, and those of a more mundane type which are subject to risk, and for which probability *is* therefore useful, was based on the aforementioned property of being aggregable. The important distinction between crucial decisions and those which are more mundane is that crucial decisions *cannot* be aggregated into a reference class from which probabilities can be drawn, even by pooling their outcomes across the many different individuals or organisations which make them (Derbyshire, 2016).

Or, rather, they can be aggregated, and the resulting probabilities may be of interest from a purely statistical point of view, but are of no use to an individual making a decision in his or her particular case. Indeed, the above example of marriage illustrates this point. The divorce rate across all marriages can be, and is, calculated as a statistic, so an individual could indeed establish a probabilistic 'risk' for the success of their own forthcoming marriage if they wished to. But each individual is different, and marries in the unique circumstances of their own life, to another individual who is also unique. Probabilities which apply at the level of the population say nothing about their particular case, and it is doubtful how useful it would be for an individual to take them into consideration, especially because considering such information to be pertinent would probably reduce the likelihood of finding a willing partner to marry in the first place.

The example Shackle (1955, p.4) provides when initially setting out the concept of a crucial decision is that of a sentry who is caught up in a rebellion and must decide whether to follow the commands of his Captain who has treacherously joined the side of the rebels, or instead to remain loyal. The sentry considers that if he obeys his Captain two things could happen. Either the rebellion succeeds and he remains a soldier in the guard, or the rebellion fails, and he is beheaded. Whereas, if he remains loyal, two alternative scenarios could occur: the rebellion succeeds and he is beheaded, or it fails and he is rewarded handsomely for his loyalty. The soldier, considering these possible scenarios, finds only the last to be attractive and opts for that one, and is duly rewarded as the rebellion fails.

By contrast, as Shackle (1955, p.3) notes, a probabilistic analysis might have led to a very different decision. The sentry may have examined some historical records and found a thousand similar cases and that in six hundred the rebellion succeeded, under which circumstances those who had remained loyal were beheaded. On this basis the sentry may have made the opposite decision to join the rebellion. However, having one's head cut off is a rather final act. If the sentry had chosen to join the rebellion based on his probabilistic analysis, 'just before the axe fell, [he may have had time] to reflect that he would never, in fact, be able to repeat his experiment a thousand times, and thus the guidance given him by actuarial considerations had proven illusory' (Shackle, 1955, p.3). Probability, then, provides little useful guidance for one-off, crucial decisions with major consequences, which change the very circumstances in which the decision is made in the first place. A danger with using probability for such decisions is to place the focus on the relative likelihoods, rather than on the potential outcomes, their impacts, and how desirable these would be if they should transpire.

4.6 Probability and the problem of innate subjectivity

In response to the discussion thus far, it could be countered by an advocate of probability that the highlighted problems, while real, apply mainly to objective probability of the frequentist variety. Whereas, a widely used method for thinking about the future such as Delphi employs *subjective* probability, to which some of these problems do not apply, or, at least, apply less.

Delphi requires that expert participants form an initial 'belief' about something, such as a particular aspect of the future, based on their own presently available information, and then update this 'belief' as new information arrives in the form of others' opinions (e.g. such as their assigned probabilities) and the reasoning behind them. The logic here is that individuals' personal views, based on their own inevitably parochial and bounded information, are inherently biased. However, this bias is then countered by the introduction of other experts' opinions, and an exchange of the information on which these opinions were constructed, such that, when combined, the average of all expert participants' views provides a more accurate assessment of the focal issue than would any single participants'. Essentially, the logic is that the inherently bounded information available to any one participant, is compensated for by exchange of information with other experts, whose available information is also incomplete and biased but in different directions, such that the information available to all is then a much better reflection of reality. This can lead, the logic of Delphi implies, to more accurate probabilistic assessments of future outcomes. The logic of a tool such as Delphi is, then, grounded in information theory (Aumann, 1976).

However, this logic only holds true - and approaches to thinking about the future reliant on subjective probability such as Delphi can, therefore, only produce valid probabilities - if differences between individual experts' subjective opinions rest *only* on differences in the information available to them. If this is true, then exchanging this information, as in the Delphi process, resolves these differences, leading to common posterior probabilities (the probabilities arrived at after exchange), or, at least, posterior probabilities that more accurately reflect empirical reality than any one individual's estimated probability. In other words, posterior probabilities should, as several rounds of information exchange occur, converge on the 'true' probability, representing the 'true' state of the world.

This logic does not hold however, if differences in opinion are innate, by which is meant that individuals have access to exactly the same information, but interpret it differently because they hold inherently incommensurable ontologies (Aumann, 1976) - a criticism that is in turn based on the Harsanyi doctrine (Harsanyi 1967, 1968a, 1968b), which forms the basis of decision theory and of game theory under incomplete information. If the subjectivity in subjective probability is innate, rather than informational in nature, a technique such as Delphi cannot work. Fundamental ontological differences, leading to innate subjectivity, arise because different information can mean very different things to different actors, who can therefore 'frame' the problem, its context, and the information that is relevant to it, in highly distinctive ways (Dewulf et al., 2005). Moreover, the logic of convergence and eventual agreement inherent in Delphi may also not hold in the event that individuals' suspect the other experts of errors in the estimations they construct based on their available information, the commonality of biases when estimating probability having been highlighted by Tversky and Kahnemann (1974), a particularly important one being insensitivity to prior probabilities.

Aumann (1976) refers to both problems, implying that the exchange of information leading to revised subjective probabilities may not necessarily result in convergence towards more accurate probabilities that are a truer reflection of reality. Indeed, Aumann (1976) makes explicit reference to Delphi in setting out this argument. Morris (1995) makes similar reference to these problems, and shows a valid criticism of subjective probability to be that any outcome is possible (i.e. not only that in which there is a convergence on a true representation of reality) where individuals have heterogeneous prior beliefs despite access to the same information, implying innate subjectivity that is not informational in nature. On a similar theme, Franssen (2005) discusses Arrow's (1950; 1963; 1986) 'impossibility theorem' related to social choice. Arrow highlighted the deep problems associated with aggregating the preferences of several individuals over a set of options, such that the aggregated preferences are a 'faithful translation' of those of the group as a whole (Franssen, 2005, p.42). Arrow showed that as soon as the number of options is equal to or greater than three, it is impossible to create a collective decision function that is a 'faithful translation' of the preferences of the group as a whole. The implication is a lack of agreement and acceptance in relation to the aggregated outcome by some or all individuals involved. Indeed, an important aspect of so-called 'deep uncertainty', which is uncertainty of exactly the type that cannot be dealt with using probability, is situations in which the various stakeholders in a decision do not know *or cannot agree* on the relevant probabilities (Walker et al., 2012), as is reflective of Arrow's impossibility theorem.

These problems bring into question, then, the extent to which subjective probability genuinely assists in overcoming the problems associated with objective, frequentist-based probability. Subjective probability still depends on an accurate interpretation of objective, empirical information, even if the available information is initially partial. A process like Delphi that employs subjective probability is still, ultimately, dependent on individuals' accurate estimation of objective probabilities. It also requires them to interpret and frame reality in a common way. Ultimately then, subjective probability may not be much different from that which is objective, implying that the previously discussed problems apply to it also. Ultimately, when constructing the theoretical foundations of subjective probability, Savage (1954) sought to abide by the same axioms, as set out subsequently by Kolmogorov (1956), that underpin probability theory in general, including that which is frequency-based and here refered to as 'objective probability'. It is no surprise, then, if subjective and objective probability suffer similar drawbacks as devices for consideration of the future.

5 The appropriate use of probability for thinking about the future

5.1 Circumstances in which probability can be safely used

An implication of the highlighted problems with the use of probability is that the circumstances in which it is appropriate to use probability for consideration of the future are limited. Shackle (1955; 1961) argued such circumstances to be the minority, not the majority, contrary to the impression one might get from the commonality with which probability is employed for exactly this purpose (Earl and Littleboy, 2014). The circumstances in which it is appropriate to use probability to think about the future are those that are akin to risk, rather than uncertainty, as discussed earlier by reference to Knight (1921), Keynes (1937) and Shackle (1955; 1961). The requirement, then, is for those wishing to use probability for consideration of the future to identify the realm in which they are operating and whether it is one characteristic of probabilistic risk, or non-probabilistic uncertainty. In this regard, Makridakis et al. (2009) provide some guidance, but this guidance must be modified based on the discussion in this paper.

Makridakis et al. (2009) draw on the now famous quote of Donald Rumsfeld distinguishing various combinations of knowns and unknowns – known knowns, known unknowns and unknown unknowns. These in turn provide some indication of circumstances favourable to the use of probability. According to Makridakis et al. (2009, p.806), the return on an insured fixed term deposit in a bank is a *known known* in which we know exactly what we will receive in the future, and there is therefore an absolute absence of uncertainty. Whereas, accordingly to Makridakis et al. (2009), the example that has featured consistently throughout this paper - that of coin tossing - is characteristic of a *known unknown*, because it is *unknown* what any individual outcome will be, but we nevertheless *know* for sure that the result will be heads or tails. Herein we see the crossover with Davidson's (1991) implication that there can be certainty contained within the use of probability. Davidson (1991), it will be recalled, suggested that under the circumstances Makridakis et al. (2009) are here categorising as *known unknowns*, the unknown aspect only applies to an individual instance, not the long-run series, for which we know for sure the relative outcomes, meaning there is no genuine uncertainty at all. For this reason, based on Davidson (1991) and the argument set out in this paper, and contrary to Makridakis et al. (2009), coin-tossing more accurately belongs in the category *known knowns* because the only unknown aspect is the outcome in an individual instance. In other words, the longer-term distribution of outcomes is stable, will remain so, and no new possibilities can occur at any point. The outcomes are fully specifiable in advance and the problem is therefore a 'closed' one.

Based on the argument in this paper then, coin-tossing, the useful exemplar that has featured throughout this paper, rightfully belongs in the category *known knowns*, rather than the category *known unknowns* in which Makridakis et al. (2009) place it. That the latter is not the right category for coin-tossing is hinted at by the fact that Makridakis et al. (2009) also place bubbles in financial markets under this same category of *known unknowns*, since we *know* such bubbles and their bursting will occur in the future, but when or where is *unknown*. Financial bubbles, and the factors leading to their bursting, can take many different forms, such that they do not have the quality of being aggregable, as coin-tossing does. As shown in this paper, the quality of being aggregable is key to distinguishing circumstances of probabilistic risk from non-probabilistic uncertainty.

The point to be emphasised is that probability can only be used entirely safely in circumstances characteristic of *known knowns*, by which is meant - here at least, if not in Makridakis et al. (2009) - circumstances in which the past is an accurate guide to the future, in which all possible outcomes are known in advance, and in which the distribution of these outcomes is known, is stable, and will remain stable over time. The scarcity of such circumstances is evident in the stringency of these requirements. 'Known knowns' is the rightful realm of probability because in this realm uncertainty is not really uncertainty at all, it is risk.

It follows that a key to safe consideration of the future is to know in which realm (risk or genuine uncertainty) one is operating. A problem in this regard is that, often, we think we are operating in one realm, only to discover we are operating in another, by which time it is too late to do anything about it. For example, the financial industry had the tendency, leading up to the credit crunch, to think itself to be operating in circumstances of 'known knowns' and to have reduced the uncertainty to which the markets are subject to manageable risk, by using, for example, the Value at Risk and other models (Taleb, 2001; 2008; 2012). This turned out to be an incorrect assumption. Essentially, the finance industry was taken in by the siren call of probability.

5.2 The siren call of probability

Some might argue, however, that it is too stringent to confine the use of probability to the very limited circumstances in which the form of uncertainty is not really uncertainty at all, but risk. It might be argued, for example, that there are methods to take account of some of the problems set out in this paper, such that probability can still be employed safely. It could be argued that situations characteristic of non-stationarity, in which a future probability distribution is not guaranteed to remain stable as it is a deterministic function of time, can still be tackled using probability by applying approaches such as Extreme Value Theory (Embrechts et al., 1999). Actuarial, econometric and probability theory and methods, this argument might continue, have moved on since the early-to-mid-20th century in which Keynes, Shackle and others were writing, and in which a clear distinction between risk and uncertainty was set out. Indeed, this is an argument sometimes made by proponents of subjective probability as a device for consideration of the future, since subjective probability recognises no distinction between risk and uncertainty.

In response, it is worth making the obvious empirical observation that, while many such newer techniques exist - techniques which may be argued to extend the bounds of risk so that it encroaches on the territory we have here allocated to uncertainty - the efficacy of such techniques must be somewhat limited since the credit crunch still happened despite their availability, and uncertainty is perhaps a more pervasive problem than ever before. Furthermore, it can be contended that such approaches actually exacerbate the problem of uncertainty, rather than alleviating it. The very fact they are able to take better account of some of the problems with probability highlighted in this paper means their use is more likely to lull us into the false sense that we have tamed uncertainty by reducing it to risk - exactly the problem which led to the credit crunch according to Skidelsky (2009). A central danger with the use of probability is its tendency to lull us into such a false sense of 'knowing', leading us to place too much faith in our present knowledge, or present guesses, as to future outcomes and their extremity. Probability facilitates the self-delusion that the future is knowable based on present knowledge.

This is, essentially, the point made by Stern (2016) in highlighting the 'big risks' associated with climate change, by which we now see he actually meant the full extent of the uncertainty associated with it. Because of the dangerous siren call of probability it might perhaps be better, then, to avoid the use of probability altogether when considering the future. Indeed, recently, the Society for Decision Making Under Deep Uncertainty[2] was established to develop and disseminate new approaches for dealing with uncertainty not based on traditional, probabilistic decision theory. This suggests that after many years at the margins the distinction between risk and uncertainty is on its way back to the centre ground in economics, decision theory, and more broadly. In futures studies uncertainty never really went away as a topic of discussion, meaning those operating within that field have an important role to play in the present turn back towards uncertainty and how it can be mitigated.

6 Summarising remarks

The circumstances in which it is safe to use probability as a device for thinking about the future are far more limited than its widespread use for this purpose would imply. The full extent of the limited usefulness - and, indeed, outright danger - of probability as a device for considering the future only becomes evident when one gives full consideration to the criteria which must be met for its valid use for this purpose. These include, that all future outcomes are fully specifiable in advance, that their relative frequencies remain stable over time, and that like outcomes have been accurately classified into categories. Given the strictness of these requirements, circumstances allowing for valid use of probability are indeed rare. While new methods have been constructed that aim to take account of some of the issues discussed in this article, thereby allowing for continued use of probability for consideration of the future (Extreme Value Theory being one example), these developments may worsen the danger associated with probability's use, since they make more amenable the comforting self-delusion that we can tame uncertainty by reducing it to probabilistic risk, noted by Skidelsky as a central cause of the credit crunch.

Furthermore, a central contention of this paper has been that the use of subjective probability, as opposed to objective probability based on frequency, helps little to overcome the problems associated with probability as a device for consideration of the future. This will be contentious to many, especially those considering these issues from the perspective of mainstream economics or orthodox decision theory. Subjective probability has lain at the heart of these disciplines since the mid-20th century. Subjective probability does not recognise any distinction between risk and uncertainty because, it contends, even where there is no frequency-based means by which to construct valid probabilities, subjective probabilities can anyway be derived from individuals' willingness to wager. This paper has, however, suggested that subject probability anyway depends on accurate interpretation of objective, empirical information. This is particularly evident in Delphi, in which differences between individuals' personal opinions are simply assumed to result from differences in the information available to them, meaning that exchange of this information will result in a convergence of opinion on the

[2] http://www.deepuncertainty.org/

‘true’ state of the universe. Since subjective probability anyway depends on accurate interpretation of objective, empirical information then, many of the problems associated with the latter apply anyway to the former.

Perhaps the most central danger highlighted in this paper, however, is not any of those based on the theoretical technicalities of probability theory or econometrics. It is instead the danger of probability’s siren call. The gravest danger associated with the use of probability is its tendency to lull us towards epistemological hubris, by leading us to think that we have tamed the uncertainty of the future, based on what we know presently to be the relative outcomes of certain processes, or based on presently available information, or the dispersion of possible outcomes based on present and past variance. We are all too willing to accept the comforts of this siren call, as Skidelsky noted in relation to the credit crunch, and Stern has recently highlighted in relation to climate change. Let us resist by placing at the forefront of our thinking about the future the many problems associated with probability, and let us face the future with an epistemological humility that acknowledges that no matter how well we think we know the present or the past, this does not mean we know the future.

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