A Multiobjective Optimization Approach for Coexisting Wireless Systems under Channel State Information Uncertainty

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***Abstract*—This paper considers an underlay access strategy for coexisting wireless networks where the secondary system utilizes the spectrum owned by the primary system to simultaneously support multiple secondary users. In the considered scenario, the throughput performance of each system is limited by the interference imposed by the other. Hence, improving the performance of one system conflicts with that of the other. We analyze the fundamental trade-off between the tolerance interference level at the users of the primary system and the total achievable throughput of the users of the secondary system. We introduce a beamforming design problem as a multi-objective optimization problem with contradictory objectives, i.e., minimizing the interference imposed on each primary user while maximizing the intended signal received at every secondary user, taking into account the uncertainty in the estimation of channel state information (CSI). Assuming the uncertainty of the CSI is confined in hyper-spherical sets, we then map the proposed optimization problem to a robust counterpart under the worst case of CSI estimation error. We finally transform the robust counterpart into a standard semidefinite programming form which is convex and can be solved by standard optimization packages. Simulation results confirm the effectiveness of the proposed scheme against various levels of CSI estimation error.**

I. Introduction

In an underlay spectrum access cognitive radio network, where the primary and secondary networks share the same radio resource [1], there exist fundamentally contradictory interests between the primary and secondary networks. The performance of each network, e.g. system throughput, is limited by the interference imposed by the other. Since the radio resource is owned by the primary network, the secondary system has to operate in a way that its interference inflicted on the primary users are less than the interference threshold/tollerance level defined by the primary system. The interference threshold is normally a fixed maximum tolerable level, see, e.g., [1] and references therein. It has been shown that beamforming is an efficient method for the secondary system to manage its interference [2]–[6].

Motivated by the fact that the primary system can tolerate a higher interference level in certain cases,[1] we recently introduced in [8] a multi-objective optimization problem (MOP), see e.g, [9], [10], that simultaneously optimizes two contradicting objectives. The first one is to minimize the interference due to the transmission of the cognitive base station (BS) on each primary user (PU). The second one is to maximize the intended signal received at each secondary user (SU). As the SUs share the same resource, maximizing received signal at any SU conflicts with that of the other. Therefore, we included a set of SINR constraints to ensure that each SU is provided with its required level at least. To protect the primary system, interference levels at PUs are kept below their thresholds.

Our previous work in [8] was based on the perfect channel state information (CSI). In many practical scenarios, the available CSI at BSs is imperfect due to several reasons, e.g., estimation error, delay and the quantization error that may arise as a result of limited feedback from a user terminal to a BS [11]. Since the performance of a normal beamforming approach deteriorates and the constraints in such beamforming optimization problem are usually violated under CSI estimation error, see e.g. [8] and [12], it is desirable to develop a scheme that is robust to the imperfect CSI.

This paper takes a further step by introducing a robust beamforming design to the original multiobjective optimization problem proposed in [8]. We model the uncertainty of CSI obtained by the transmitter confined in hyper-spherical sets. We then derive a robust counterpart for the proposed optimization problem for the worst case of CSI estimation error, and then transform the robust counterpart into a standard SDP form which is convex and can be solved using standard optimization packages. Simulation results indicate the trade-off between the interference tolerance at PUs and the total achievable throughput at SUs. The proposed approach provides robust against error in CSI estimation at the cost of a decease in the total attainable SUs' throughput. It however guarantees all the SUs' SINR and PUs' interference constraints.

Notations: $\mathrm{Tr}(\cdot)$: trace operator; $\mathbf{Y} \succeq 0$: a positive semi definite matrix; $\preccurlyeq$: element-wise inequality; $(y_i)_{i=1}^{U}$: $\begin{bmatrix} y_1 & y_2 & \cdots & y_U \end{bmatrix}^T$; $\mathbb{E}(x)$: expected value of x.

[1]For instance, when the primary system experiences a lower traffic load or the primary receiver has a sophisticated coding technique [7].

II. System Model

Consider a primary system in cellular communication network with N PUs. Utilizing underlay spectrum access [1], a cognitive BS servers U SUs sharing the spectrum of the primary network subject to the interference temperature constraint at the PUs. The beamforming technique is adopted at the cognitive BS which is equipped with M antennas. We assume single antenna setting at the SUs and PUs. The received signal at the ith SU is

$$y_i = \mathbf{h}_{s,i}^H \mathbf{w}_i s_i + \sum_{j=1, j\neq i}^{U} \mathbf{h}_{s,i}^H \mathbf{w}_j s_j + n_i, \tag{1}$$

where $\mathbf{h}_{s,i}^H = \widetilde{\mathbf{h}}_{s,i}^H + \mathbf{e}_{s,i}^H$ is the true channel between the cognitive BS and ith SU, $\widetilde{\mathbf{h}}_{s,i}^H \in \mathbb{C}^{1\times M}$ and $\mathbf{e}_{s,i}^H \in \mathbb{C}^{1\times M}$ are, respectively, the estimated channel and its corresponding estimation error, $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ is the beamforming vector for the ith SU, s_i is the data symbol to be sent to the ith SU and n_i is a zero mean circularly symmetric complex Gaussian noise with variance σ_i^2, i.e., $n_i \sim \mathcal{CN}(0, \sigma_i^2)$. The primary system imposed interference at the SUs is considered as an additive background noise [13]. For brevity the average transmitted symbol energy to SU i at the cognitive BS is assumed to be unity. Let $\mathbf{R}_{s,i} = \mathbb{E}\left(\mathbf{h}_{s,i}\mathbf{h}_{s,i}^H\right)$, then $\mathbf{R}_{s,i} = \widetilde{\mathbf{R}}_{s,i} + \boldsymbol{\Delta}_{s,i}$ where $\widetilde{\mathbf{R}}_{s,i} = \mathbb{E}_{\widetilde{\mathbf{h}}_{s,i}}\left(\widetilde{\mathbf{h}}_{s,i}\widetilde{\mathbf{h}}_{s,i}^H\right)$, $\boldsymbol{\Delta}_{s,i} = \mathbb{E}_{\mathbf{e}_{s,i}}\left(\mathbf{e}_{s,i}\mathbf{e}_{s,i}^H\right)$. Furthermore, let $\mathcal{W} = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_U\}$ be the set of candidate beamforming vectors in the cognitive BS for all SUs. The SINR at SU i is

$$g_i(\mathcal{W}) = \frac{\mathbf{w}_i^H \left(\widetilde{\mathbf{R}}_{s,i} + \boldsymbol{\Delta}_{s,i}\right)\mathbf{w}_i}{\sum_{j=1, j\neq i}^{U} \mathbf{w}_j^H \left(\widetilde{\mathbf{R}}_{s,i} + \boldsymbol{\Delta}_{s,i}\right)\mathbf{w}_j + \sigma_i^2}. \tag{2}$$

Let $\mathbf{R}_{p,t} = \mathbb{E}\left(\mathbf{h}_{p,t}\mathbf{h}_{p,t}^H\right)$ where $\mathbf{h}_{p,t} = \widetilde{\mathbf{h}}_{p,t}^H + \mathbf{e}_{p,t}^H$ is the true channel between the cognitive BS and the tth PU, $\widetilde{\mathbf{h}}_{p,t}^H \in \mathbb{C}^{1\times M}$ and $\mathbf{e}_{p,t}^H \in \mathbb{C}^{1\times M}$ are, respectively, the estimate channel and its corresponding error. We can write $\mathbf{R}_{p,t} = \widetilde{\mathbf{R}}_{p,t} + \boldsymbol{\Delta}_{p,t}$ where $\widetilde{\mathbf{R}}_{p,t} = \mathbb{E}_{\widetilde{\mathbf{h}}_{p,t}}\left(\widetilde{\mathbf{h}}_{p,t}\widetilde{\mathbf{h}}_{p,t}^H\right)$, $\boldsymbol{\Delta}_{p,t} = \mathbb{E}_{\mathbf{e}_{p,t}}\left(\mathbf{e}_{p,t}\mathbf{e}_{p,t}^H\right)$. We aim to design beamforming vectors for the cognitive BS such that the total interference imposed on every PU t, i.e., $\sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{R}_{p,t}\mathbf{w}_i$, is kept below its threshold I_t. We assume that the primary system is able to update its interference thresholds, i.e., I_t, and provide that information to the secondary system. Methods for calculating, providing I_t and robust solution to inaccuracies of I_t are out of scope of this paper. Hereafter, if otherwise stated, $i \in \{1, \cdots, U\}$ and $t \in \{1, \cdots, N\}$.

III. Proposed Approach

We design the beamforming vector $\mathbf{w}_i$ for each SU in the cognitive BS considering their required SINR. In this paper, our objective is to maximize the intended signal power received at each SU i, i.e., $\mathbf{w}_i^H \mathbf{R}_{s,i}\mathbf{w}_i$, while minimizing the corresponding interference inflicted at each PU t, i.e., $\sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{R}_{p,t}\mathbf{w}_i$. Let

$$f_{s,i}(\mathcal{W}) = -\mathbf{w}_i^H \mathbf{R}_{s,i}\mathbf{w}_i = -\mathbf{w}_i^H \left(\widetilde{\mathbf{R}}_{s,i} + \boldsymbol{\Delta}_{s,i}\right)\mathbf{w}_i \tag{3}$$

and

$$f_{p,t}(\mathcal{W}) = \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{R}_{p,t}\mathbf{w}_i = \sum_{i=1}^{U} \mathbf{w}_i^H \left(\widetilde{\mathbf{R}}_{p,t} + \boldsymbol{\Delta}_{p,t}\right)\mathbf{w}_i, \tag{4}$$

the objective vector is then defined as

$$\begin{aligned} \mathbf{f}(\mathcal{W}) \;=\; & [f_{p,1}(\mathcal{W}), \cdots, f_{p,N}(\mathcal{W}), \\ & f_{s,1}(\mathcal{W}), \cdots, f_{s,U}(\mathcal{W})]. \end{aligned} \tag{5}$$

We now define the decision space

$$\begin{aligned} \mathcal{D} \triangleq & \Big\{\mathcal{W} \mid (\gamma_i)_{i=1}^U \preccurlyeq (g_i(\mathcal{W}))_{i=1}^U, \\ & (f_{p,t}(\mathcal{W}))_{t=1}^N \preccurlyeq (I_t)_{t=1}^N, \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i \le P_{\mathrm{m}} \Big\} \end{aligned} \tag{6}$$

where γ_i is the required SINR level at SU i and P_{m} is the cognitive BS maximum transmit power. We propose the following MOP:

$$\min_{\mathcal{W}\in\mathcal{D}} \quad \mathbf{f}(\mathcal{W}). \tag{7}$$

In (7), the set of SINR constraints guarantees each SU being served with its required level at least. The optimization problem then tries to tune the beam to further improve each SU's received signal strength and thus to raise the achievable throughput above the required level as far as possible.

Let $\lambda_{p,t} > 0\, \forall t$, $\lambda_{s,i} > 0\, \forall i$ and $\sum_{t=1}^{N} \lambda_{p,t} + \sum_{i=1}^{U} \lambda_{s,i} = 1$. According to [10], the Pareto optimal solution[2], i.e., $\widehat{\mathcal{W}}$, to the MOP defined in (7) can be obtained as the optimal solution to the following SOP

$$\begin{aligned} \min \quad & \sum_{t=1}^{N} \lambda_{p,t} f_{p,t}(\mathcal{W}) + \sum_{i=1}^{U} \lambda_{s,i} f_{s,i}(\mathcal{W}), \\ \text{s. t.} \quad & g_i(\mathcal{W}) \ge \gamma_i, \ \forall i \\ & f_{p,t}(\mathcal{W}) \le I_t, \forall t \\ & \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i \le P_{\mathrm{m}}. \end{aligned} \tag{8}$$

To account for the imperfection of channel estimation, here we assume that the uncertainty in the estimation of channel covariance matrices $\delta_{s,i}$ and $\delta_{p,t}$ are confined within hyperspherical sets $\mathcal{E}_{s,i}$ and $\mathcal{E}_{s,i}$, respectively, with radius $\delta_{s,i}$ and $\delta_{p,t}$ defined as

$$\mathcal{E}_{s,i} = \left\{\boldsymbol{\Delta}_{s,i} \in \mathbb{C}^{M\times M} : \|\boldsymbol{\Delta}_{s,i}\| \le \delta_{s,i}\right\}, \forall i \tag{9}$$

$$\mathcal{E}_{p,t} = \left\{\boldsymbol{\Delta}_{p,t} \in \mathbb{C}^{M\times M} : \|\boldsymbol{\Delta}_{p,t}\| \le \delta_{p,t}\right\}, \forall t. \tag{10}$$

[2] *Properly Pareto optimal* solutions are defined as Pareto optimal solutions with bounded trade-offs amongst the objectives [10].

Furthermore, for any $M \times M$ Hermitian positive semidefinite matrix, $\mathbf{Y}$, $\|\mathbf{Y}\| \leq \delta$, and a $M \times 1$ arbitrary vector $\mathbf{x}$, we have

$$\mathbf{x}^H \mathbf{Y} \mathbf{x} \leq \mathbf{x}^H \delta \mathbf{I} \mathbf{x}. \quad (11)$$

Utilizing (11), we then evaluate the worst case effect of the channel estimation error on $f_{s,i}(\mathcal{W})$ and $f_{p,t}(\mathcal{W})$ as follows:

$$\max_{\|\boldsymbol{\Delta}_{s,i}\| \leq \delta_{s,i}} f_{s,i}(\mathcal{W}) = -\mathbf{w}_i^H \left(\widetilde{\mathbf{R}}_{s,i} - \delta_{s,i} \mathbf{I} \right) \mathbf{w}_i \quad (12)$$

and

$$\max_{\|\boldsymbol{\Delta}_{p,t}\| \leq \delta_{p,t}} f_{p,t}(\mathcal{W}) = \sum_{i=1}^{U} \mathbf{w}_i^H \left(\widetilde{\mathbf{R}}_{p,t} + \delta_{p,t} \mathbf{I} \right) \mathbf{w}_i, \quad (13)$$

Similarly, utilizing (11) we then write the worst case of error on $g_i(\mathcal{W})$ as (14)[3] given at the top of next page. Hence, in the worst case (8) can be cast as (15) given at the top of next page.

We proceed by defining beamforming matrix $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H$, where $\mathbf{W}_i \succeq 0$ and $\mathbf{W}_i$ is a rank-one matrix.[4] Then, by rearranging the constraints, using $\mathbf{x}^H \mathbf{Y} \mathbf{x} = \operatorname{Tr}\left(\mathbf{Y} \mathbf{x} \mathbf{x}^H\right)$, problem (15) is converted to the SDP form in (16) shown on next page, where $\{\mathbf{W}_i\} = \{\mathbf{W}_1, \cdots, \mathbf{W}_U\}$ is the set of beamforming matrices. When transforming (15) into (16), we have dropped the rank-one condition on $\mathbf{W}_i$. Following the same approach in the proof of Theorem 1 in [8], one can prove that the optimal solutions to problem (16) are rank one. Therefore, the transformed problem (16) maintains the optimality of the original problem (15). The optimization problem in (16) can be solved by the SeDuMi solver, provided by CVX optimization package [15], to obtain the set of optimal beamforming matrices $\mathbf{W}_i^\star$.

IV. Simulation Results

In this section, the performance of the proposed scheme is investigated and compared against a baseline introduced in [8]. We consider a cognitive cellular network with 2 PUs and 2 SUs. The PUs are located at -50° and 50° while the SUs are located at -10° and 10° relative to the array broadside. The distances from the SUs and PUs to the cognitive BS are 0.5km and 1km, respectively. The (p,q)th entry of the $M \times M$ channel matrice $\mathbf{R}_{s,i}$ or $\mathbf{R}_{p,t}$ is obtained using [16]:

$$\xi e^{\frac{j 2 \pi \Delta}{\ell}[(q-p)\sin\phi]} e^{-2\left[\frac{\pi \Delta \sigma_a}{\ell}\{(q-p)\cos\phi\}\right]^2}, \quad (17)$$

where ξ represents the channel gain coefficient, ϕ is the angle of departure, Δ is the antenna spacing at the BS, σ_a is the angular spread and ℓ is the carrier wavelength. In (17), we set $\Delta = \ell/2$, $\sigma_a = 2^\circ$, $\xi = 34.5 + 35\log_{10}(d)$ captures the distance-dependent path-loss, where d is the distance in meters with $d \geq 35$m, a log-normal shadow fading with 8dB standard deviation, and a Rayleigh component for the multi-path fading channel. The noise power spectral density, the noise figure at each SU, and antenna gain are assumed to be -174dBm/Hz, 5dB, and 15dBi, respectively.

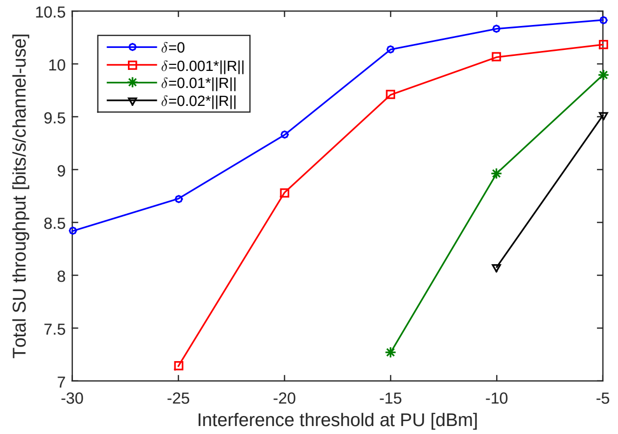

Fig. 1. Total throughput of the SUs *vs.* angle separation between PUs and SUs. SUs' SINR requirements $\gamma_i = 10$dB $\forall i$. $\delta = 0.01 * ||\mathbf{R}||$

Fig. 1 illustrates the approximation of Pareto frontiers of the proposed and the baseline scheme.[5] The proposed scheme is shown with different error levels δ.[6] Fig. 1 indicates the fact that there is a trade-off between the interference tolerance level at PUs and the total achievable throughput of SUs. The higher the interference tolerance level at the PU is, the higher total throughput can be attained by the SUs. Fig. 1 also shows that the performance of the proposed robust scheme decreases as the error level in the estimation of CSI increases. This can be explained as follows. As the uncertainty of users' CSI increases, the proposed approach has to reduce it transmit power to protect the PUs, consequently, the total throughput at SUs is reduced. The proposed scheme also maintains the SINR constraint of 10dB at each SU as it can be seen from the figure that the total throughput at SUs is always greater than $2\log_2(1+10) = 6.92$bits/s/channel-use. Furthermore, Fig. 1 reveals that the baseline scheme provides higher total SU throughput than the proposed approach. This higher performance, however, comes at a cost of harming the PUs as shown in Figs. 2 and 3.

In order to investigate the effect of transmission of PUs, let us define a normalized interference constraint value as $\frac{f_{p,t}(\mathcal{W})}{I_t}$ where $f_{p,t}(\mathcal{W})$ is given in (4) and I_t is the interference threshold/tollerance at tth PU. If the normalized interference constraint value is less than 1 then the interference constraint at each PU is maintained. Otherwise, interference constraint at each PU is violated.

Figs. 2 and 3 depict the histograms of the normalized interference constraint values, respectively, for PU 1 and PU

[3]This type of worst-case evaluation for SINR was first introduced in [14].

[4]A matrix is rank one if its largest number of linearly independent columns/rows is one.

[5]For a given I_m, Fig. 1 depicts the maximum achievable SU throughput.

[6]Without any loss of generation, we have set the same error level for all users, i.e., $\delta = \delta_{s,i} = \delta_{p,t},\ \forall i, t$.

$$\min_{\|\boldsymbol{\Delta}_{s,i}\|\leq\delta_{s,i}} g_i\left(\mathcal{W}\right) = \frac{\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i} - \delta_{s,i}\mathbf{I}\right)\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\left(\widetilde{\mathbf{R}}_{s,i} + \delta_{s,i}\mathbf{I}\right)\mathbf{w}_j + \sigma_i^2} \tag{14}$$

$$\begin{aligned}
\min_{\{\mathbf{w}_i\}} \quad & \sum_{t=1}^{N}\lambda_{p,t}\sum_{i=1}^{U}\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{p,t} + \delta_{p,t}\mathbf{I}\right)\mathbf{w}_i - \sum_{i=1}^{U}\lambda_{s,i}\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i} - \delta_{s,i}\mathbf{I}\right)\mathbf{w}_i \\
\text{s. t.} \quad & \frac{\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i} - \delta_{s,i}\mathbf{I}\right)\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\left(\widetilde{\mathbf{R}}_{s,i} + \delta_{s,i}\mathbf{I}\right)\mathbf{w}_j + \sigma_i^2} \geq \gamma_i,\ \forall i \\
& \sum_{i=1}^{U}\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{p,t} + \delta_{p,t}\mathbf{I}\right)\mathbf{w}_i \leq I_t, \forall t, \\
& \sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{w}_i \leq P_{\mathrm{m}}
\end{aligned} \tag{15}$$

$$\begin{aligned}
\min_{\{\mathbf{W}_i\}} \quad & \mathrm{Tr}\left(\sum_{i=1}^{U}\left\{\left[\sum_{t=1}^{N}\lambda_{p,t}\widetilde{\mathbf{R}}_{p,t} + \sum_{t=1}^{N}\lambda_{p,t}\delta_{p,t} - \lambda_{s,i}\widetilde{\mathbf{R}}_{s,i} + \lambda_{s,i}\delta_{s,i}\right]\mathbf{W}_i\right\}\right), \\
\text{s. t.} \quad & \left(1 + \frac{1}{\gamma_i}\right)\mathrm{Tr}\left(\widetilde{\mathbf{R}}_{s,i}\mathbf{W}_i\right) - \sum_{j=1}^{U}\mathrm{Tr}\left(\widetilde{\mathbf{R}}_{s,i}\mathbf{W}_j\right) - \sum_{j=1,j\neq i}^{U}\delta_{s,i}\mathrm{Tr}\left(\mathbf{W}_j\right) - \frac{\delta_{s,i}}{\gamma_i}\mathrm{Tr}\left(\mathbf{W}_i\right) - \sigma_i^2 \geq 0,\ \forall i, \\
& I_t - \mathrm{Tr}\left(\widetilde{\mathbf{R}}_{p,t}\sum_{i=1}^{U}\mathbf{W}_i\right) - \mathrm{Tr}\left(\delta_{p,t}\sum_{i=1}^{U}\mathbf{W}_i\right) \geq 0,\ \forall t, \\
& P_{\mathrm{m}} - \sum_{i=1}^{U}\mathrm{Tr}\left(\mathbf{W}_i\right) \geq 0, \\
& \mathbf{W}_i \succeq 0,\ \forall i,
\end{aligned} \tag{16}$$

2 at $I_t = -15$dBm $\forall t$, SINR level at SU $\gamma_{s,i} = 10$dB $\forall i$ and $\delta_{p,t} = \delta_{s,i} = \delta = 0.01 \times ||\mathbf{R}||$, $\forall i, t$. It is clear from Figs. 2 and 3 that the proposed scheme effectively guarantees the imposed interference on each PU is less than the requirement while the baseline approach fails to protect the interference constraints for more than 70% of the occurrences. This confirm the effectiveness of the proposed scheme against the error in the channel estimation.

As shown in Fig. 4, the Pareto frontier obtained by the proposed approach can be designed by varying the number of antennas. With 4 antennas, the proposed approach can maintain the required throughput for 2 SUs while taking care the interference threshold for 2 PUs in the range from -10dBm to -5dBm. When the number of antennas increases, the proposed approach can provide much higher achievable throughput than the original SU requirement. For example, with 12 antennas, it offers 8 and 12.5bits/s/channel-use higher than the original SU requirement at the PU interference thresholds of -15dBm and -5dBm, respectively.

V. Conclusion

In this paper, we propose a multiobjective optimization problem for coexisting wireless networks adopting an underlay access strategy. Specifically, we formulate a beamforming design problem as the linear combination of two contradictory objectives, i.e., minimizing the interference imposed on each primary user while maximizing the intended signal received at every secondary user. The proposed beamforming approach takes into account the uncertainty in the estimation of channel state information. We then reformulate the proposed optimization problem to a robust optimization problem under the worst case of CSI estimation error. We finally transform the robust optimization problem into a standard semidefinite programming form which is convex and can be solved by standard optimization packages. Simulation results confirm the effectiveness of the proposed scheme against various levels of CSI estimation error. Although all the constraints imposed on the secondary system, i.e., SUs' SINR levels and PUs' interference thresholds, are maintained, the robustness comes at the cost of lower total SUs' throughput. Simulation results

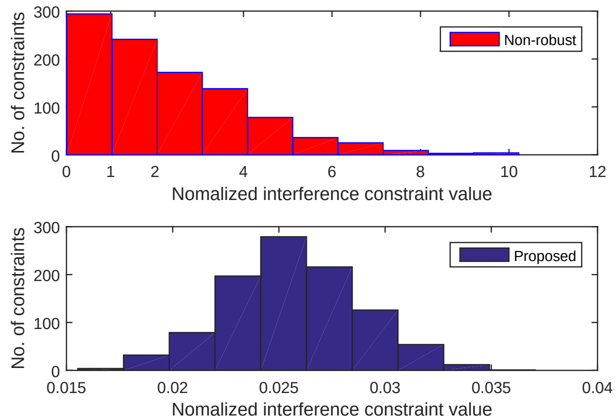

Fig. 2. $I_{\mathrm{m}} = -15$dBm. SUs' SINR requirements $\gamma_i = 10$dB $\forall i$. The number of antenna $Mt = 6$. $P_{\mathrm{m}} = 40$dBm. (a): Proposed against benchmarking scheme with 6 antennas. (b): Proposed scheme with different antenna settings.

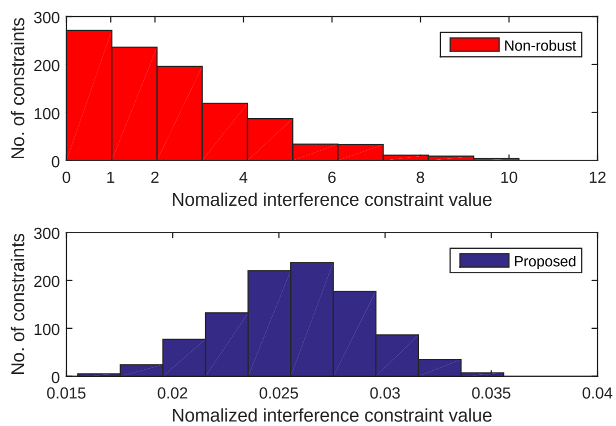

Fig. 3. Total throughput of the SUs *vs.* angle separation between PUs and SUs. SUs' SINR requirements $\gamma_i = 10$dB $\forall i$.

also indicate the fact that the baseline can provides higher total SUs' throughput than the proposed approach. However, the former fails to maintain the interference tolerance level of the PUs more than 70% of the occurrences.

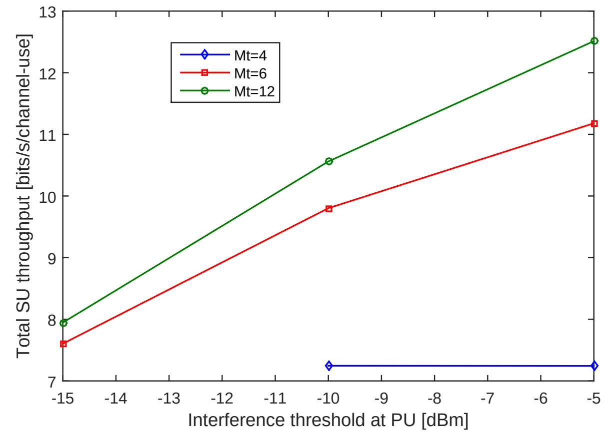

Fig. 4. Total throughput of the SUs *vs.* angle separation between PUs and SUs. SUs' SINR requirements $\gamma_i = 10$dB $\forall i$.

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