Metaheuristic Optimization of Reinforced Concrete Footings

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Abstract. The primary goal of an engineer is to find the best possible economical design and this goal can be achieved by considering multiple trials. A methodology with fast computing ability must be proposed for the optimum design. Optimum design of Reinforced Concrete (RC) structural members is the one of the complex engineering problems since two different materials which have extremely different prices and behaviors in tension are involved. Structural state limits are considered in the optimum design and differently from the superstructure members, RC footings contain geotechnical limit states. This study proposes a metaheuristic based methodology for the cost optimization of RC footings by employing several classical and newly developed algorithms which are powerful to deal with non-linear optimization problems. The methodology covers the optimization of dimensions of the footing, the orientation of the supported columns and applicable reinforcement design. The employed relatively new metaheuristic algorithms are Harmony Search (HS), Teaching-Learning Based Optimization algorithm (TLBO) and Flower Pollination Algorithm (FPA) are competitive for the optimum design of RC footings.

Keywords: Reinforced concrete footings, Optimization, Metaheuristic algorithms, Harmony search algorithm, Teaching-Learning based optimization, Flower pollination algorithm.

1. Introduction

Reinforced concrete (RC) spread footing is one of the major components of the structures as a type of foundation. Since it directs the structural loads to the ground and supports a compressive member (column), the failure of the member causes the total collapse of the structure. An economical, safe design under structural loads is not sufficient if the stability of footings is not ensured with respect to soil bearing capacity. In additional to non-linear behavior of RC members, the footing design is a complicated one since the design variables are depending on each other in the consideration of geotechnical and structural state limits.

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Several optimum design methods of RC spread footings have been previously developed, but new methodologies are in need for practical and detailed optimum designs. The cost optimization is the main idea of studies about RC spread footings. Wang and Kulhawy (2008) developed an optimum design methodology and considered the ultimate limit state, serviceability limit sate and cost for a spread footing supporting a column under axial loading. A reliability-based economic design optimization of RC spread foundations was proposed by Wang (2009). Zhang et al. (2011) developed an indirect method for reliability-based optimization of geotechnical systems including spread footings and retaining walls.

Metaheuristic algorithms are suitable for optimization of RC spread footings since these algorithms have been employed in several methodologies. A modified particle swarm optimization was employed by Khajehzadeh et al. (2011) in order to optimize RC spread footings and retaining walls and differently from the previous studies, the biaxial flexural moment of the supported RC column is considered. A gravitational search (GS) is employed in the optimization of shallow foundations by Khajehzadeh et al. (2012). Also, a new type of GS called global-local gravitational search algorithm was developed for the optimization of RC footings considering a multi-objective approaches including CO_2 emissions and cost (Khajehzadeh et al. 2014). A hybrid Big Bang-Big Crunch (BB-BC) algorithm was employed by Camp and Assadallahi (2013) for multi-objective optimization of RC footings supporting axial loaded columns. Then, uniaxial flexural moments were taken into consideration by using the hybrid BB-BC algorithm (Camp and Assadallahi 2015). Also, Khajehzadeh et al. (2013) developed a hybrid firefly algorithm for multi-objective optimization of RC footings and uniaxial flexural moment of the column was considered.

In the previously developed studies, RC footings with rectangular cross-sections are optimized. The present study proposes a trapezoidal shape in order to save from the volume of the concrete. Another novelty is the consideration of biaxial flexural moments of the column supported by the footing because biaxial flexural moments generally occur in the columns as optimized by Nigdeli et al. (2015). An important factor directly related to the reduction of internal forces and indirectly related to the optimum cost is the orientation of the supported column and the presented approach considers the orientation of the column as two design variables in two direction. The other optimized design variables are quantity of reinforcements and the base area of the spread footings. The design code developed by American Concrete Institute (ACI 318) is considered in the development of the design constraints related to geotechnical and structural limit states. In additional to these novelties, several methods are applied to cost optimization of RC footings. Three of the presented metaheuristic algorithms are relatively new methods including the Harmony Search (HS) algorithm developed by Geem et al. (2001), the Teaching-Learning Based Optimization (TLBO) developed by Rao (2011) and the Flower Pollination Algorithm (FPA) developed by Yang (2012). Since the previously documented RC footing problems w not optimized in detail, the employed algorithms are compared with well-known classical algorithms including Particle Swarm Optimization (PSO) developed by Kennedy and Eberhart (1995) and Differential Evolution (DE) developed by Storn and Price (1997) for the computational performance and applicability.

The paper is organized as follows. In Section 2, the general design methodology and design of RC footings are explained. The employed metaheuristic algorithms are summarized in Section 3. Then, numerical examples with several loading cases are presented in Section 4. Finally, discussions and conclusions are presented in Section 5.

2. Design and Optimization Methodology

The design of RC footings involves two different limit states, namely, geotechnical and structural limit states. These states are separately considered in order to save from the computational effort. If the design constraints about geotechnical state limits are not provided after the selection of dimensions of the footing, the following optimization (optimum reinforcement design ensuring structural state limits) is not conducted in order to minimize the computation time.

In the proposed study, several design variables are randomized in different states of the proposed method. Thus, the nonphysical design variables are eliminated. For example, the orientation of the bars must be checked. If the number of bars is assigned before, the assigned design variables may not be suitable and to continue the design procedure is pointless.

In metaheuristic algorithms, an iterative random search process is done in two ways and global and local searches are carried out. The global search process is stand to prevent the possibility of being trapped to a local optimum value, while the local search increases the effectiveness of the method in finding precise optimum solutions.

The design variables of the proposed study cover dimension parameters including the base (X_1 and X_2) and cross section of the footing (X_3), the orientation of the column (X_4 and X_5) and detailed reinforcement design (not only the required area; X_6, X_7, X_8 and X_9) as shown in Fig. 1. C_f and C_c are the centers of footing and column, respectively. The base dimensions (X_1, X_2) are also shown as L and B in x and y directions, respectively. The height of the footing at the face of the column (H) is the third design variable (X_3) and the minimum height of the cross section is H_{min}. The location of the column is also optimized since the supported column is under the effect of the biaxial moments in x (M_x) and y (M_y) directions in additional to the axial force (P). The eccentricity of the column is defined with two distances; e_x and e_y which are the fourth (X_4) and fifth (X_5) design variables, respectively. The size (diameter) of the reinforcement bar is defined as X_6 and X_7 in two dimensions and the distances between bars are X_8 and X_9.

In the methodology, the design constants and ranges of design variable are first defined and then an initial matrix is generated. This matrix is constructed by merging vectors containing a set of variables. The design variables in vectors are randomly assigned and the number of these vectors is defined with a parameter. The name of this parameter is different according to the inspiration of the algorithm, but it is generally population. The preselected range of design variables are used to reduce the optimization time and to express the preferences of the designer in practice. In the optimum design of RC member, discrete variables must be used because a sensitive production cannot be provided during construction and the reinforcements can be supplied with fixed sizes in local markets.

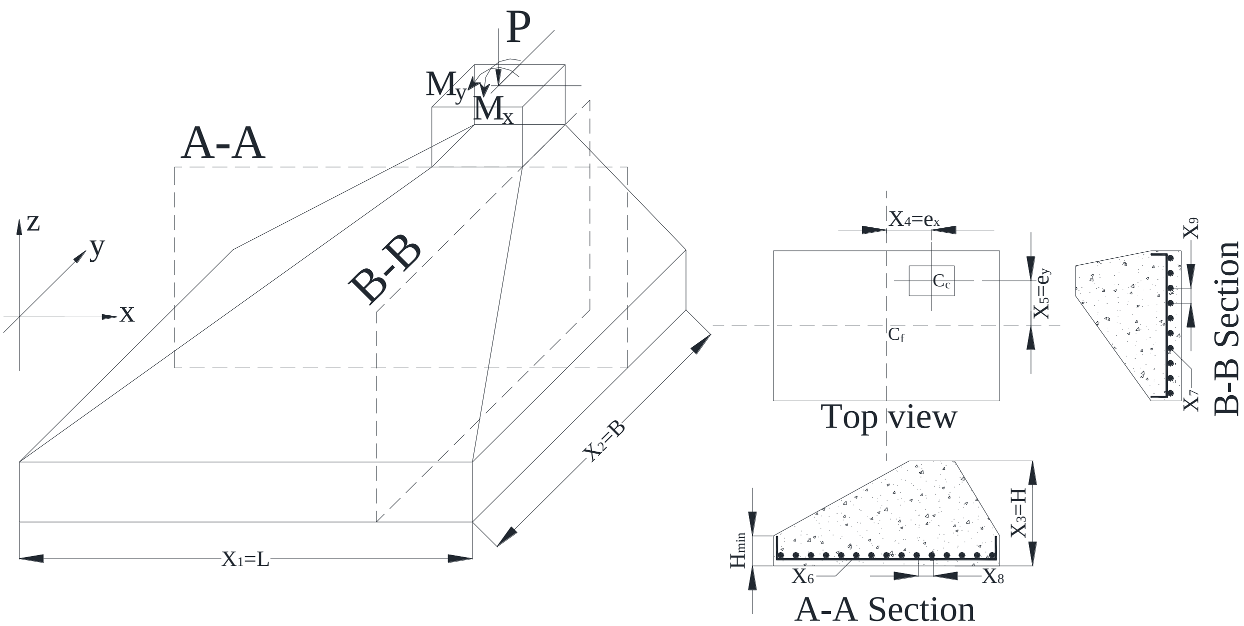

Fig. 1. The optimization problem with design variables.

The random generation process of design variables is done with a multi-step procedure. The required analyses for control of design constraints are carried out between the generations of different design variables.

First, the variables about dimensions (X_1-X_5) are defined. Then, the geotechnical limit states; the bearing pressure on the soil and the settlement (δ) are checked. The bearing pressures at four sites of the foundation ($q_{1,2,3,4}$; from 1 to 4) are calculated according to Eq. (1), where W_f is the total weight of the foundation including the soil on the top of the foundation.

$$q_{1,2,3,4} = \frac{P+W_f}{BL} \pm \frac{6(M_y - Pe_x)}{BL^2} \pm \frac{6(M_x - Pe_y)}{B^2L} \quad (1)$$

In order to ensure the stability of the foundation, the minimum pressure must be over 0 because soil cannot carry tensile forces. Additionally, a factor of safety (FS) is defined for the maximum pressure. These two constraints are formulated as Eq. (2) and Eq. (3).

$$q_{1,2,3,4} \geq 0 \,, \quad (2)$$

$$FS < \frac{q_{ult}}{\max(q_{1,2,3,4})} \,. \quad (3)$$

The ultimate bearing capacity of soil (q_{ult}) for a cohesionless soil with no ground slope is calculated Eq. (4);

$$q_{ult} = \gamma D N_q F_{qs} F_{qd} + 0.5\gamma B N_\gamma F_{\gamma s} F_{\gamma qd} \,, \quad (4)$$

where γ is the unit weight of the soil. The bearing capacity factors (N_q and N_γ) and shape depth factors (F_{qs}, $F_{\gamma s}$, F_{qd}, $F_{\gamma d}$) are shown as Eqs. (5-11). The internal friction angle is shown with ϕ.

$$N_q = e^{\pi \tan\phi'} \tan^2(\frac{\pi}{4} + \frac{\phi'}{2}) \tag{5}$$

$$N_\gamma = 2(N_q + 1)\tan\phi' \tag{6}$$

$$F_{qs} = 1 + \frac{B}{L}\tan\phi' \tag{7}$$

$$F_{\gamma s} = 1 - 0.4\frac{B}{L} \tag{8}$$

$$F_{qd} = \begin{cases} 1 + 2\tan\phi'(1-\sin\phi')^2\left[\arctan(\frac{D}{B})\right] & if\ \ D > B \\ 1 + 2\tan\phi'(1-\sin\phi')^2(\frac{D}{B}) & if\ \ D < B \end{cases} \tag{9}$$

$$F_{\gamma d} = 1.0 \tag{10}$$

The cohesion has an additional increasing effect to the ultimate bearing capacity. For that reason, the numerical example was done for cohesionless soil in order to investigate the most critical case of the bearing capacity. The effect of cohesion can be also included in the optimization process.

The settlement (δ) is calculated according to the elastic solution of Poulus and Davis (1974) as shown in Eq. (11) and it must be equal or lower than the maximum allowed settlement (δ_{max}). The elasticity modulus and Poisson ratio of soil are defined as E and ν, respectively.

$$\delta = \frac{P + W_f(1-\nu^2)}{\beta_z E\sqrt{BL}} \le \delta_{max} \tag{11}$$

The shape factor (β_z) given by Whitman and Richart (1967) is calculated as Eq. (12).

$$\beta_z = -0.0017(\frac{L}{B})^2 + 0.0597(\frac{L}{B}) + 0.9843 \tag{12}$$

If one of the constraints given in Eqs (2), (3) and (11) are not satisfied, the total cost is penalized and the further processes are not taken into action. The total cost is assigned with a big value (10^6 \$ in numerical example). If the geotechnical limit states are satisfied, the process continues by assigning design variables about reinforcements and controlling structural state limits. The required flexural moment capacity in the critical sections of two directions, shear force capacity of the footing and two way shear capacity (punching) of the footing is checked in structural state limits of a spread footing. The total cost is penalized as done in the previous stage of geotechnical state limits.

In control of structural state limits, the axial force and flexural moments are factored with a ϕ value and it is taken as 1.2 and 1.6 for dead and live loads, respectively.

The critical sections for flexural are the sections along the face of the column where the pressure is highest in two directions. Tension controlled based design is considered for the maximum reinforcement area in flexure and the compressive block of the concrete is taken as an equivalent rectangular stress block.

For the critical one way shear force, the critical section is at the distance d_{ave} away from the face of the column and the average (d_{ave}) is the average of the effective depth of the foundation in two directions.The effective depth is different for two directions because the reinforcements cannot lie in the same plane. Since the column tends to punch through the spread footing because of the shear stresses, the two-way shear force of the footing must be also controlled. The critical punching perimeter is located $d_{ave}/2$ away from the column face and it is defined as

$$b_{perim} = 4(b_{column} + d_{ave}) \tag{13}$$

where b_{column} is the breadth of the column for corresponding direction.

The capacity of the RC footing for one-way shear ($V_{n,one\text{-}way}$) and two-way shear ($V_{n,two\text{-}way}$) are calculated as given in Eqs. (14) and (15), respectively.

$$V_{n,one-way} = 0.75(0.17 w d_{ave} \sqrt{f_c^{'}}) \tag{14}$$

$$V_{n,one-way} = 0.75 * \min \begin{cases} 0.17(1+\dfrac{2}{\beta})\sqrt{f_c^{'}} b_{perim} d_{ave} \\ 0.083(\dfrac{4d_{ave}}{b_{perim}}+2)\sqrt{f_c^{'}} b_{perim} d_{ave} \\ 0.33\sqrt{f_c^{'}} b_{perim} d_{ave} \end{cases} \tag{15}$$

The shear capacity is also investigated for two directions and the value of w is equal the length of the footing in the corresponding direction. This value is B in x direction and L for y directions. β is the ratio of the long side to short side of the column. The compressive strength of concrete is symbolized by $f_c^{'}$.

After the structural state limits are checked, the objective function defined as the total cost of the footing is calculated and it is formulated in Eq. (16).

$$f_{\cos t} = \mathrm{V_{concrete}} \mathrm{C_{concrete}} + \mathrm{W_{steel}} \mathrm{C_{steel}} \tag{16}$$

In Eq. (16), the total cost (f_{cost}) is calculated in terms of the volume of concrete ($V_{concrete}$), cost of concrete for unit volume ($C_{concrete}$), the weight of the reinforcements (W_{steel}) and the cost of reinforcement for unit weight (C_{steel}). The volume of concrete and the weight of the reinforcements are depended to the design variables. After an initial matrix is generated, the iterative optimization is carried out according to the rules of the employed metaheuristic algorithm. The iterative optimization process is done for several iterations and the optimization process ends at the maximum number of iterations. The iterative process of different algorithms is described in the Section 3 and the flowchart of the methodology is shown in Fig. 2.

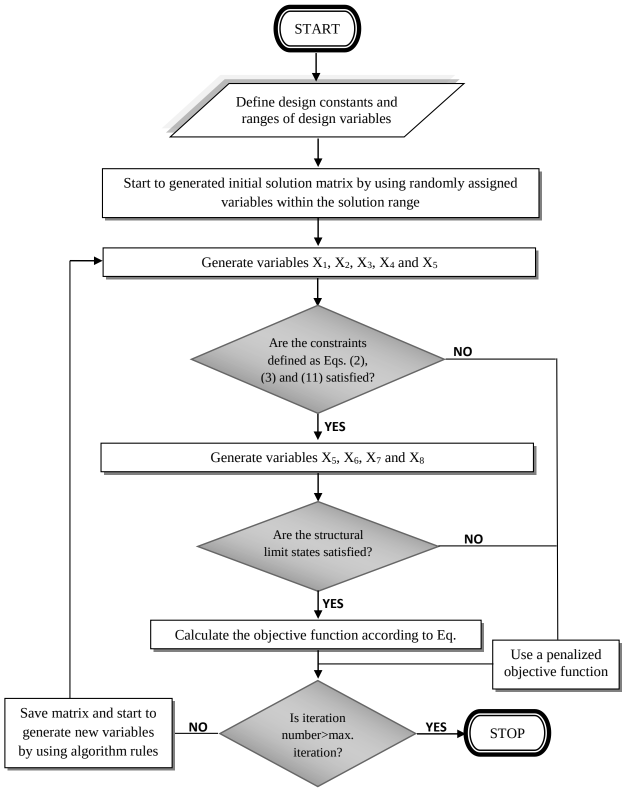

Fig. 2. The flowchart of the optimization methodology.

3. Brief Description of Metaheuristic Algorithms

In this section, three of the employed algorithms including HS, FPA and TLBO are briefly explained. Since the other employed algorithms including DE and PSO are well-known, these methods are not presented in the paper.

3.1. Harmony Search Algorithm

The music inspired metaheuristic algorithm, namely harmony search (HS), has been applied for several multidisciplinary applications by demonstrating the effectiveness of the algorithm. Structural engineering is one of the application areas of HS. In Bekdaş et al. (2016), the studies related with the optimum design of reinforced concrete members are summarized.

Firstly in HS, an initial harmony memory matrix is constructed and then, this matrix is updated according to the algorithm rules. The number of vectors constructing a HM is defined with a parameter called Harmony Memory Size (HMS). After the generation of an initial HM matrix, the iterative optimization is started by generating a new harmony vector in two ways.

The global optimization is carried out by using the same way used in the generation of an initial harmony matrix according to Eq. (17). X_{new} is the newly generated set of design variables while X_{min} and X_{max} are the minimum and maximum bounds of the solution ranges, respectively. The probability of using local optimization is defined with an algorithm parameter called harmony memory considering rate (HMCR). A random number between 0 and 1 is generated and the generated number is compared with HMCR. In the current study, HMCR is linearly changed from 0 to 1 according to iteration number.

$$X_{new} = X_{\min} + rand(0,1) \cdot (X_{\max} - X_{\min}) \tag{17}$$

In local optimization, the results of an existing vector (X_j) are used and the random generation is done in a smaller range than the previously defined one. This range is generated around the values of the existing design variables, but the randomly assigned new design variables must also be in the limits of the user defined range. If not, these values are assigned with the limit values. The ratio of a small range and the user defined range is determined by the parameter called Pitch Adjusting Rate (PAR). The local optimization of HS can be formulized as

$$X_{new} = X_j + rand(-1/2, 1/2) \cdot PAR \cdot (X_{\max} - X_{\min}). \tag{18}$$

After the generation of a new vector, the results are compared with the existing vectors in the HM matrix. If the total cost of the new random design is better than the worst one in HM, the new one is replaced with the worst one. The newly generated and the worst existing vector may have a penalized cost. In that case, the elimination is done according to the violation of geotechnical and structural state limits. The comparison is done according to the last violation since the following analyses are not conducted in the methodology. The pseudocode of HS algorithm is given in Fig. 3.

```
Objective minimize f(cost), X=(X1, X2, X3, X4, X5, X6, X7, X8,X9)^T
Define harmony memory size (HMS), pitch adjusting rate (PAR) and ranges
Define harmony memory considering rate (HMCR)
Generate initial harmony matrix
while ( t<Max number of iterations )
        if (rand<HMCR), choose an existing harmonic randomly
        adjust pitch for a new range around an existing one
        Generate new harmony vector (Eq. (18))
        else generate new harmony vector via randomization (Eq.(17))
        end if
        Accept the new harmony vector if better
end while
```

Fig. 3. Pseudocode of HS algorithm.

3.2. Teaching-Learning Based Optimization

TLBO is a newly generated metaheuristic algorithm and it imitates the teaching and learning process of a class. In structural engineering, TLBO is employed in optimization of truss structures (Camp and Farshchin 2014; Dede and Ayaz 2013; Degertekin and Hayalioglu 2013), RC retaining walls (Temur and Bekdaş 2016) and tuned mass dampers (Nigdeli and Bekdaş 2015). The global and local optimizations are consequently conducted in TLBO. In global optimization called teacher phase, the newly generated design variables ($X_{new,i}$) are generated according to the best existing solution marked as a teacher ($X_{teacher}$) and mean of the all solutions (X_{mean}). Thus, a good convergence is obtained. This phase is formulated as Eq. (19) and all old solutions ($X_{old,i}$) in the class with a population of n are updated.

$$X_{new,i} = X_{old,i} + rand(0,1) \cdot (X_{teacher} - T_F \cdot X_{mean}) \quad for\ i = 1\ to\ \mathrm{n}\ , \tag{19}$$

The main parameter is T_F in the algorithm. It is also a random number which can be 1 or 2 as an integer number. After the teacher phase, the student phase is started and all existing design variables are updated according to Eq. (20).

$$X_{new,i} = \begin{cases} X_{old,i} + rand(0,1)(X_j - X_k) & if\ j^{th} \text{ solution is better than } k^{th} \text{ solution} \\ X_{old,i} + rand(0,1)(X_k - X_j) & if\ k^{th} \text{ solution is better than } j^{th} \text{ solution} \end{cases} \quad (20)$$

The comparison between the newly generated result and existing results is done by the same methodology described for HS algorithm. Differently from HS, the whole set of design variables in the matrix (class) are updated. Since the two phases are also considered, the number of analyses is 2n times of iteration numbers. The pseudocode of TLBO is shown in Fig. 4.

*Objective minimize f(*cost*),* X=*(*$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9)^T$
Define class population and ranges
Randomly generate the initial students
while *(t<Max number of iterations)*
    (Teacher Phase)
    Calculate the mean of each design variable
    Identify the best student as teacher
    Generate new solutions (Eq.19)
    Accept the new solution if better
    (Learner Phase)
    Select any two solution randomly [j, k]
    Generate new solutions (Eq.20)
    Accept the new solution if better
end while

Fig. 4. Pseudocode of TLBO.

3.3. Flower Pollination Algorithm

Flower pollination algorithm is a newly developed algorithm, based on the pollination characteristics of flowering plants (Yang 2012), and the effectiveness of FPA on structural optimization has been shown for the weight optimization of truss structures (Bekdaş et al 2015) and optimization several structural elements (Nigdeli et al. 2016). After the initial values of the design variables are generated in FPA, global and local optimization stages are used. Global optimization is inspired from biotic (or cross) pollination and pollinators obey the rule of Lévy flights. As given in Eq. (21), a new design variable is found according to a Lévy distribution (LD) and the best existing solution (X_{best}). The Lévy distribution is defined by Eq. (22) where r(λ) is the standard gamma function. The Lévy distribution is valid for large steps s > 0. λ is taken as 1.5 in the simulations.

$$X_{new,i} = X_{old,i} + LD(X_{best} - X_{old,i}) \quad (21)$$

$$LD \sim \frac{\lambda\Gamma(\lambda)\sin(\pi\lambda/2)}{\pi}\frac{1}{s^{1+\lambda}} \quad (22)$$

Here, the notation ~ means that the step size *LD* should be drawn from a distribution governed by the right-hand side. A switch probability (p) is used to choose the optimization type in the iterative search. This switch probability is taken between 0 and 1.

In local optimization, abiotic (or self) pollination is the inspiration. Abiotic pollination is the self-fertilization of a flower. In this type of optimization, two solutions are chosen and a new solution is found according to Eq. (23).

$$X_{new,i} = X_{old,i} + rand(0,1)(X_j - X_k) \tag{23}$$

The updating and comparison of the design variables are done according to the same way with other algorithms. The optimization process is summarized in the pseudocode of FPA (Fig. 5).

*Objective minimize f(*cost*),* X=*(*X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8,X_9*)*^{T}
Define ranges, flower population and a switch probability (*p*)
Initialize population of n flowers with random number
Find the best solution (g_*) *of the initial population*
while *(t<Max number of iterations)*
    if *(rand<p), global pollination*
    Generate new solutions (Eq.21)
    else *local pollination*
    Select any two solution randomly [j, k]
    Generate new solutions (Eq.23)
    end if
    Find the current best solution (X_{best})
end while

Fig. 5. Pseudocode of FPA.

4. Test problems and optimization results

As numerical examples, a footing with the design constants and solution ranges shown in Table 1 are optimized for four different cases of the loading conditions. The dead and live loads of the cases are given in Table 2. The dead and live loads are denoted by subscripts G and Q, respectively. The applicable production in the construction yard is provided by assigning discrete design variables with 5cm differences for dimensions. Cases 2 and 3 are similar cases which must have the same optimum results. These cases are investigated for validation of the developed methodology and checking the robustness of the optimum results. For one million analyses, the optimum design variables and costs are presented in Table 3 for Cases 1-4. All algorithms are effective to find the same results in these cases. For that reason, the methods will be compared for a fixed number of analyses and fixed accuracy. Thus, the only difference can be seen in standard deviation values and computation time, because the same results are not found for 50 independent runs. Also, the algorithms reach the optimum values before one million analyses. Also, the number of analyses for obtaining optimum results are also presented in order to compare the computational effort. All analyses are conducted by using Matlab (2010) and a computer with 32 GB Ram and 8 core 3.6 GHz processor. The time for 1000 iterations is between 16.8 s and 128.11 s for different cases. The number of vectors is taken as 50.

Table 1 Design constants and ranges of design variables.

Definition	Symbol	Unit	Value
Yield strength of steel	f_y	MPa	420
Compressive strength of concrete	f'_c	MPa	25
Concrete cover	c_c	mm	100
Max. aggregate diameter	D_{max}	mm	16
Elasticity modulus of steel	E_s	GPa	200
Specific gravity of steel	γ_s	t/m^3	7.86
Specific gravity of concrete	γ_c	kN/m^3	23.5
Cost of concrete per m^3	C_c	$\$/m^3$	40
Cost of steel per ton	C_s	$/t	400
Internal friction angle of soil	ϕ'	°	35
Unit weight of base soil	γ_B	kN/m^3	18.5
Poisson ratio of soil	v	-	0.3
Modulus of elasticity of soil	E	MPa	50
Maximum allowable settlement	δ_{max}	mm	25
Factor of safety	FS	-	3.0
Minimum footing thickness	h_{min}	m	0.25
Column breadth in two direction	b/h	mm/mm	500/500
Range of width of footing	B	m	2.0-5.0
Range of length of footing	L	m	2.0-5.0
Range of height of footing	H	m	h_{min}-1.0
Range of diameter of reinforcement bars of two direction	ϕ	mm	16-24
Range of distance between reinforcement bars	s	mm	5ϕ-250

Table 2 The loading cases for the optimization example.

	P (kN)		M_x (kNm)		M_y (kNm)	
	P_G	P_Q	M_{xG}	M_{xQ}	M_{yG}	M_{yQ}
Case 1	750	500	300	200	300	200
Case 2	750	500	300	200	400	300
Case 3	750	500	400	300	300	200
Case 4	750	500	400	300	400	300
Case 5	1000	750	600	400	600	400

Table 3 The optimum results and comparison of different algorithm (Case 1-4)

		Case Number			
		1	2	3	4
Optimum design variables	B (m)	2.75	2.60	3.20	3.05
	L (m)	2.65	3.20	2.60	3.05
	H (m)	1.00	1.00	1.00	1.00
	e_x (m)	0.75	0.75	0.75	0.90
	e_y (m)	0.55	0.75	0.75	0.80
	ϕ_x (mm)	16	16	20	16
	S_x (mm)	150	140	230	140
	ϕ_y (mm)	16	20	16	20
	S_y (mm)	160	230	140	210
	Best Cost ($)	217.3235	252.2302	252.2302	285.4123
DE	Av. Cost ($)	218.5794	253.3985	253.3545	287.1336

	Sta. Dev. ($)	0.3229	0.3268	0.4763	0.9942
	Analyses for optimum	223700	118500	110300	93200
PSO	Av. Cost ($)	220.8968	254.2458	254.4545	288.1056
	Sta. Dev. ($)	1.9426	1.5769	1.5691	1.5778
	Analyses for optimum	358600	345750	152600	658400
HS	Av. Cost ($)	218.3516	253.2736	253.4752	287.3481
	Sta. Dev. ($)	0.4723	0.4146	0.4812	1.0326
	Analyses for optimum	901100	301005	545795	602117
FPA	Av. Cost ($)	218.2720	253.1640	253.1547	286.5305
	Sta. Dev. ($)	0.4852	0.6284	0.5691	1.1384
	Analyses for optimum	221800	116850	145150	98150
TLBO	Av. Cost ($)	219.1628	253.7319	253.4578	286.6425
	Sta. Dev. ($)	1.4007	0.8896	0.9886	1.6982
	Analyses for optimum	242200	276400	381700	126100

In order to adjust the minimum and maximum pressure, a non-symmetric design for Cases 1 and 4 are found as optima. In Cases 2 and 3, the optimum values are the same in opposite directions as expected. Because of the difference of the flexural moment in two directions, a significant difference in dimension and reinforcements in a direction is clearly seen.

The number of analyses for finding the optimum values are very different for cases and algorithms. DE and FPA are the fastest algorithms.

For different runs of the optimization process, the optimum results of Case 1-4 are always the same for one million analyses. It means that all these algorithms are able to obtain the optimal results for small external forces. Thus, the standard deviation values are given and the standard deviation values are generally low for the HS algorithm, because the worst result is always eliminated in HS. The others use a population based strategy in updating all design variables and the corresponding results are only updated. Thus, the worst results are only updated according to global or local optimization in FPA. In TLBO, teacher and student phases are consequently applied and the worst one can be updated for two times. As seen in Table 3, FPA has better standard deviation results than TLBO. DE is the algorithm with the best standard derivative results while PSO is the worst one. Also, t-test analyses were done by taking PSO approach results as the reference. In Table 4, the t_{stat} values define as Eq. (24) are shown for α=5% significance level and 49 degree of freedom. In that case, t critical value (t_{cr}) is -1.6765.

$$t_{stat} = \frac{\overline{x} - \mu}{s/\sqrt{n}} \tag{24}$$

$\bar{x}$ is the average value of the reference method (PSO), s is the standard deviation and n is the number of observation. According to the results given in Table 4, the average values (μ) of the methods are generally better than PSO approach with 95% confidence level.

Table 4 t_{stat} values for the algorithms and cases

	Case No			
	1	2	3	4
DE	-50.7478	-18.3332	-16.3304	-6.9131
HS	-38.1056	-16.5810	-14.3904	-5.1872
FPA	-38.2525	-12.1729	-16.1500	-9.7835
TLBO	-8.75364	-4.08478	-7.12900	-6.0921

The optimum results of Case 5 are different for the algorithms and the results are presented in Table 5. FPA, TLBO and PSO are effective to find the best results, but PSO has a big standard deviation value. Since the loading case 5 is symmetrical, the inverse optimum solutions of x and y directions are both optimums as seen in the results of FPA and TLBO. Also, the optimum solution of PSO in reinforcements is quite different with equal amount of reinforcement.

Table 5 The optimum results and comparison of different algorithm (Case 5)

Optimum design variables	DE	PSO	HS	FPA	TLBO
B (m)	4.15	4.15	4.20	4.15	4.15
B (m)	4.15	4.15	4.15	4.15	4.15
H (m)	1.00	1.00	1.00	1.00	1.00
e_x (m)	1.00	1.00	1.10	0.95	1.00
e_y (m)	0.95	0.95	0.85	1.00	0.95
ϕ_x (mm)	16	22	18	18	22
S_x (mm)	110	210	140	150	210
ϕ_y (mm)	20	18	18	22	18
S_y (mm)	180	150	150	210	150
Best Cost ($)	549.7689	549.2428	554.2151	549.2428	549.2428
Av. Cost ($)	554.0413	602.9062	556.9158	553.6568	554.7540
Sta. Dev. ($)	0.8603	69.08	2.33	2.5674	13.29
Analyses for optimum	474800	935900	846801	712050	394700

5. Discussions and Conclusions

The same results are found for the cases 1-4 of the optimization problem. For that reason, the comparison of the metaheuristic algorithms are done according to results of 50 runs.

The non-classical methods are also tested for different number of population. Table 6 shows the values of best costs, average costs and standard deviations for different harmony memory size, pollen or population. The results are given for the second case and the optimum results are found as the same for all solutions. By the increase of HMS, HS is generally effective to find the optimum results immediately. For TLBO, the increase of population has a small side effect. FPA is not generally affected by the change of pollen number.

The effectiveness of using a truncated pyramid shaped footing and optimizing the position of the footing were evaluated by investigating different states of the problem. The optimum costs for all cases are presented in Fig. 6.

Table 6 Optimization results for different vector numbers.

		HMS (for HS), Pollen (for FPA), Population (for TLBO) No				
		10	20	30	40	50
HS	Best Cost ($)	285.4123	285.4123	285.4123	285.4123	285.4123
	Av. Cost ($)	288.2548	288.1456	287.8542	287.6451	287.3481
	Sta. Dev. ($)	1.2658	1.2178	1.2458	1.1447	1.0326
	Analyses for optimum	858231	754525	657845	622254	602117
FPA	Best Cost ($)	285.4123	285.4123	285.4123	285.4123	285.4123
	Av. Cost ($)	286.8156	286.6954	286.7545	286.6858	286.5305
	Sta. Dev. ($)	1.1522	1.1489	1.1512	1.1456	1.1384
	Analyses for optimum	119800	109500	103550	110200	98150
TLBO	Best Cost ($)	285.4123	285.4123	285.4123	285.4123	285.4123
	Av. Cost ($)	287.1025	286.8925	287.2654	286.9254	286.6425
	Sta. Dev. ($)	1.7248	1.7056	1.7452	1.7254	1.6982
	Analyses for optimum	172800	158400	168600	145200	126100

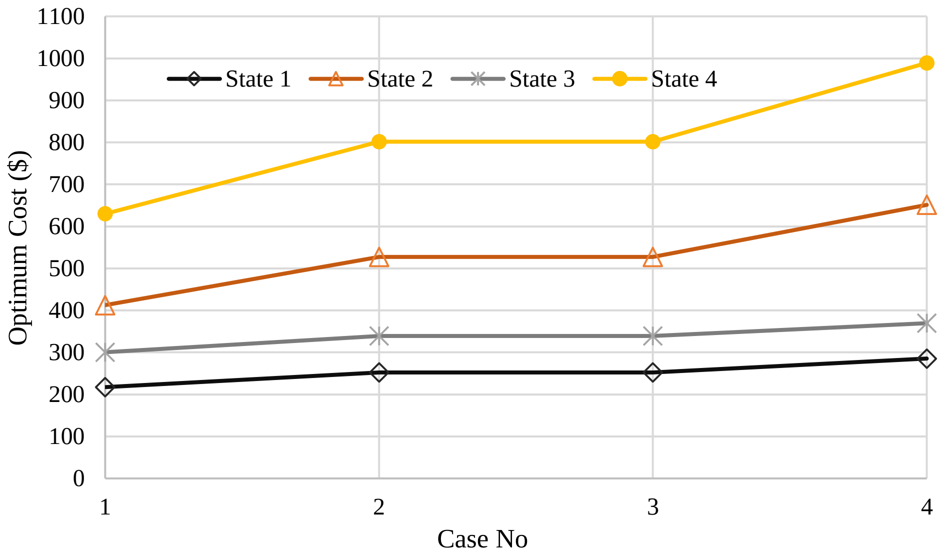

Fig. 6. The optimum costs for different states.

The first state is for the presented design in the numerical example section. For the second state, the column is mounted on the middle of the footing. By the increase of the flexural moments, the difference of the optimum cost increases in state 2. In state 3, a rectangular prism shaped footing was investigated and the height of the footing is taken as 1 m. In this state, a significant increase of the optimum cost is seen and the increase is constant for the internal force cases. In the last state, both position optimization and proposed shape were not considered. It is clearly seen that the optimum costs are too much comparing to the state 1. As a conclusion, the detailed optimization using a truncated pyramid shape and position optimization are effective on reducing of the optimum cost.

For cases 1-4, FPA is the best algorithm in finding the best average cost. DE has a little higher average cost value than FPA and DE is the robust algorithm on finding similar results in all runs because of the low standard deviation value. For the first two cases, FPA is a little faster than DE, while DE is better in Cases 3-4. The number of analyses values of FPA and DE are close, so both algorithms are the fastest ones. In case 5, TLBO is also effective in finding the best solution rapidly, but FPA has more positive standard deviation value. DE is the best on the standard deviation but it is weak on finding the best solution.

Though the results are promising, it can be expected that further studies are need to investigate if these methods can be applied to the large-scale problems of structural systems with more design variables and members. Parametric studies will also be useful to see if the computational times can be further reduced so that good solutions can be obtained with the minimum computational efforts. In addition, it can be fruitful to extend these methods to solve multi-objective footing optimization problems in a more realistic context with multiple design objectives.

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