Index Tracking with Utility Enhanced Weighting

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Abstract

Passive index investing involves investing in a fund that replicates a market index. Enhanced indexation uses the returns of an index as a reference point and aims at outperforming this index. The motivation behind enhanced indexing is that the indices and portfolios available to academics and practitioners for asset pricing and benchmarking are generally inefficient and, thus, susceptible to enhancement. In this paper we propose a novel technique based on the concept of cumulative utility area ratios and the Analytic Hierarchy Process (AHP) to construct enhanced indices from the DJIA and S&P500. Four main conclusions are forthcoming. First, the technique, called the utility enhanced tracking technique (UETT), is computationally parsimonious and applicable for all return distributions. Second, if desired, cardinality constraints are simple and computationally parsimonious. Third, the technique requires only infrequent rebalancing, monthly at the most. Finally, the UETT portfolios generate consistently higher out-of-sample utility profiles and after-cost returns for the fully enhanced portfolios as well as for the enhanced portfolios adjusted for cardinality constraints. These results are robust to varying market conditions and a range of utility functions.

Keywords: Investments; Portfolio Allocation; Asset Management; Utility Functions; Index Tracking; AHP

JEL Classification: G1, G11

Word Count: 9554 including references

1 Introduction

1.1 Motivation

A combination of high fees and disappointing performance in actively managed investment funds has generated the burgeoning investor interest in index funds. This passive strategy involves investing in a fund that replicates or, more often, tracks a market index. These funds typically have no front end loads and very low management fees, often as low as 7-25 basis points (Morningstar, 2018). If these index funds are managed correctly, they should yield a return equal to the return on the market index less the management fees plus or minus a small "tracking error". It is argued that because of these low fees and the difficulty of consistently outperforming the market, index tracking provides investors with higher returns than those achieved in the active management strategies. In the US in 2014, index funds and index-based exchange-traded funds (ETFs) accounted for 20.2% of total equity mutual fund assets. From 2007 through 2014, domestic equity index mutual funds and ETFs had net inflows of $1 trillion, including reinvested dividends.[1] In contrast, over the same period, actively managed domestic equity mutual funds experienced a net outflow of $659 billion, including reinvested dividends.[2]

The rationale behind passive index investing is the efficient market hypothesis and the argument that no one can consistently "beat the market". The practical implication is that financial indices achieve the best returns over time.[3] Passive index investing starts from a reference portfolio,

[1] Interestingly, Levy and Lieberman (2016) find that investment flows and returns are affected by investment styles. A passive investment style compared with an active style weakens the relationship between flows and returns.
[2] See: 2015 Investment Company Factbook, Investment Company Institute, http://www.icifactbook.org/fb_ch2.html#popularity.
[3] Interestingly, even Warren Buffet, the exception that confirms the rule, seems to have capitulated to this philosophy. In his 2014 letter "To the Shareholders of Berkshire Hathaway Inc.," (page 20) Buffett revealed his instructions in his will to the trustee for his wife's benefit: "Put 10% ... in short-term government bonds and 90% in a very low-cost S&P 500 index fund. (I suggest Vanguard's.) I believe the trust's long-term results from this policy will be superior to those attained by most investors – whether pension funds, institutions or individuals – who employ high-fee managers."

usually a well-established financial index, and constructs a portfolio designed to replicate the returns on the reference portfolio. Traditionally, this is done by minimizing the tracking error defined as the standard deviation of the differences between the portfolio and index returns (Roll, 1992).[4] In practice some portfolio tracking strategies, such as Vanguard, aim to include all the stocks with the same weights as those of the reference portfolio. Others rely on tracking models that seek to minimize the tracking error as well as the number of stocks in the tracking portfolio (e.g. Acosta-Gonzalez et al., 2015). The latter strategies are plagued by their computational difficulty when implementing cardinality constraints.[5] They also have the obvious shortcoming that as the number of stocks in the tracking portfolio decreases, the stock picking risk increases.

In this paper we develop a methodology for enhanced index tracking and test its effectiveness over a seven year sample period, including five years of out-of-sample testing. We focus on enhancing investor utility with respect to a benchmark index but we also address the specific issues outlined above of computational difficulty, diversification and rebalancing. First, we develop a criterion based on investor utility that is relevant for any return distribution. The criterion is based on the concept of cumulative areas calculated over empirical distribution functions (EDFs) that can be used to measure investor preferences. For example, the cumulative area ratio developed by Leshno & Levy (2002) measures almost stochastic dominance.[6] We propose cumulative areas that reflect specific investor preferences and then use them to construct a ratio, called the CUAR (cumulative utility area ratio) criterion that measures the incremental utility of one asset compared to another. The insight behind this criterion is that, given two assets

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[4] For a review of other methods, see Canakgoz and Beasley (2008). More recently, Wu and Tsai (2014) propose a technique based on Fuzzy Goal Programming to minimise tracking error.

[5] Cardinality constraints refer to implementing regulatory or trading constraints that limit the number of stocks in the chosen portfolio.

[6] See also Denuit et al (2014) for an extension of "almost stochastic dominance" to "almost marginal conditional stochastic dominance".

F and G, other things being equal, all investors with monotonically increasing utility functions will prefer the asset with the higher expected incremental utility. Since the criterion is measured using EDFs, it avoids the problems associated with distribution identification, moment estimation and subjective moment weightings.

To find the enhanced index weights we use the CUAR criterion in the Analytic Hierarchy Process (AHP) developed by Saaty (1980a and 1980b). This is a well-known technique used in multi-criteria decision making that allows diverse elements to be compared to one another in a rational and consistent way using pairwise comparison values taken from a set of criteria, subjective and/or objective. AHP has been used in a broad range of applications. Cheng (1996) used it to evaluate the performance of naval tactical missiles. Forgionne et al. (2002) used it to analyze the quality of academic journals. Berrittella et al. (2007) to find optimum transport solutions in order to reduce climate change impacts. Ishizaka et. al. (2011) use AHP as a choice support system in a real decision problem. In the recent past there have been some applications of AHP in the finance area, such as, Levary and Wan (1999), who apply AHP to take into consideration the risks related to Foreign Direct Investment (FDI) decisions made by individual firms, Xu and Zhang (2009), who present an online credit evaluation model based on AHP and Tas et. al. (2010), who use AHP as a weighting tool in a credibility score model.

We use the Dow Jones Industrial Index (DJIA) to present our methodology, which we then test on the DJIA and the Standard and Poor 500 (S&P 500) for an out-of-sample period of four years. We find that the enhanced indices consistently outperform the actual indices by an average annual return of 5.7% and 2.7% respectively. The outperformance holds when transactions costs are considered. Additional performance tests suggest that the whole distribution of returns is improved as the enhanced index offers not only higher returns, but also has a higher probability of achieving positive returns than the actual index. This improvement does not come with higher risk as the standard deviation of returns for the enhanced and the actual index are statistically equal. These results are robust to a range of utility functions that include decreasing, increasing and constant absolute risk aversion (DARA, IARA, CARA) as well as to varying market conditions.

There are several advantages to the methodology we propose. First, it is computationally parsimonious and easy to implement, even for a large set of stocks. Second, cardinality conditions are simple and straightforward, requiring only a few seconds of computational time. In the empirical work we test enhanced portfolios that retain most of the stocks in the reference index, thereby preserving their diversification dimension, as well as portfolios with strict cardinality conditions. Third, the enhanced portfolios do not require frequent rebalancing and thus, transaction costs are negligible.

1.2 Previous Research

Enhanced indexation lies between active and passive fund management. It relates to passive fund management in the sense that the fund manager uses an index as a reference point. It relates to active fund management in the sense that it seeks to exploit the evidence that the indices and portfolios available to academics and practitioners for asset pricing and benchmarking are generally inefficient with respect to conventional risk averse utility functions and, thus, susceptible to enhancement (e.g. Shanken (1987), Gibbons et al. (1989), Shalit and Yitzhaki (1994), Anderson (1996), Fama and French (1998), Post (2003), Kuosmanen (2004), Linton et. al. (2005), Post and Versijp (2007)).[7] The area of enhanced indexation is relatively new and research in this area is

[7] For example, using first order stochastic dominance rules on the CRSP all share index, a value weighted index of the stocks listed on the NYSE, AMEX and NASDAQ, Kopa and Post (2009) conclude that no investor with a strictly increasing utility function, including a quadratic utility investor, would hold this portfolio.

scarce. There is no generally accepted methodology to enhance an index. Some studies use only historical stock returns as inputs in their analysis (e.g. Dose and Cincotti, 2005; Roman et. al., 2013), while others consider stock fundamentals like sales, earnings or book value (e.g. Patari et. al., 2012; Arnott et. al., 2005) or diversity weighted indices that weight the securities according to some measure of diversity such as size (see: Fernholz et al., 1998).[8] O'Doherty et. al. (2016) propose a model combination approach to index replication that pools information from a diverse set of pre-specified factor models. Clark et al. (2011) have used marginal conditional stochastic dominance (MCSD) to make inefficient indices efficient and, Fabian et al. (2011) and Roman et al. (2006) have developed computationally parsimonious tracking models based on second order stochastic dominance (SSD).[9] Kwon and Wu (2017) optimize returns of a benchmark index based upon the Fama-French three factor model, subject to risk and tracking error limits.

The empirical evidence on the performance of enhanced index funds is thin and results are mixed. For example, of the 30 enhanced index funds studied by Ahmed and Nanda (2005) only one showed positive abnormal performance. Of the 70 enhanced index funds considered by Krause (2009) none showed any positive abnormal performance. On the other hand, Clark et al. (2011) show that their MCSD enhanced portfolio is more efficient than their reference indices, and Roman et al. (2013) show that the computationally parsimonious tracking models they developed based on SSD consistently outperform the reference indices as well as the traditional index trackers over a short, out-of-sample period of 147 days. Both of these latter studies have advantages as well as drawbacks. One of the advantages of the efficient indices developed by Clark et al. (2011) is that

[8] Canakgoz and Beasley (2008) provide a review of the relevant literature.
[9] Hodder, Jackwerth and Kolokolova (2015) compare SSD-related strategies to standard portfolio choice approaches and find that the SSD-related choices outperform these portfolios based on the Sharpe ratio, equal weights, and the information ratio.

they retain most of the stocks in the reference index, thereby preserving the diversification dimension of the portfolio. However, although they do improve the shareholder utility, they do not necessarily improve his wealth. The SSD portfolios of Roman et al. (2013) improve shareholder wealth with daily rebalancing over a 147 day out-of-sample test period. Although the 147 day out-of-sample period is very short, it is nevertheless impressive. Daily rebalancing, however, could be a practical problem. Also, although the low number of stocks reduces computational difficulty, this lack of diversification is a potential drawback.

We contribute to the enhancement literature by developing a parsimonious methodology for assigning weights to the set of assets in the benchmark index such that the resulting enhanced tracking index provides increased returns as well as an enhanced utility profile. The results of our study over a range of distinct utility functions and a relatively long out-of-sample testing period that includes varying market conditions provide evidence that the methodology does, in fact, achieve the stated purpose of enhanced indexation with respect to increased utility and wealth.

2. Theoretical Background

In this section we explain the key concepts behind our methodology. The goal is to enhance investor utility with respect to a benchmark index. First, we develop the concept of the cumulative utility area ratio (CUAR) criterion and show how it is constructed. We then present the AHP technique and show how it can be used to construct the enhanced tracking indices.

2.1.The CUAR Criterion

Following Levy (2006), define x as the investment outcome and let $f(x)$ and $g(x)$ denote the density functions of two random variables with

$$F(x) = \int_{-\infty}^{x} f(t)dt \tag{1}$$

and

$$G(x) = \int_{-\infty}^{x} g(t)dt \tag{2}$$

as the corresponding cumulative distribution functions with $F(-\infty) = G(-\infty) = 0$ and $F(+\infty) = G(+\infty) = 1$.

Consider the continuous utility function $U(x)$ such that $U'(x) \geq 0$ with a range where $U'(x) > 0$ to avoid the trivial case of the utility function coinciding with the horizontal axis. Using the definition of expected utility gives:

$$\Delta = E_F\ U(x) - E_G\ U(x) = \int_a^b [f(x) - g(x)]U(x)dx = \int_a^b [G(x) - F(x)]U'(x)dx \tag{3}$$

where $a \leq x \leq b$. [10]

Let Δ_F and Δ_G represent the change in expected utility when $F < G$ and $F > G$ respectively:

$$\Delta_F = \int_a^b \max(G(x) - F(x), 0)U'(x)dx \tag{4}$$

and

$$\Delta_G = \int_a^b \max(F(x) - G(x), 0)U'(x)dx \tag{5}$$

[10] Integrating $\int_a^b [f(x) - g(x)]U(x)dx$ by parts gives: $(F(x) - G(x))U(x)\Big|_a^b - \int_a^b [F(x) - G(x)]U'(x)dx$. The first term is equal to zero because the integration goes from 0 to 1. When $x = b$, $F(b) - G(b)$ = 1 - 1 = 0 and when $x = a$, $F(a) - G(a)$ = 0, which leaves the solution in (3).

Given two assets and the density functions of their outcomes, Δ_F measures the incremental expected utility of asset F with respect to asset G over the intervals where $F < G$ and Δ_G measures the incremental expected utility of asset G over asset F over the intervals where $F > G$. The ratio of Δ_F to Δ_G gives the CUAR criterion:

$$CUAR\ criterion = \frac{\Delta_F}{\Delta_G} \tag{6}$$

The CUAR criterion measures the incremental utility of one asset compared to another. In the extreme case where $(F(x) \leq G(x))$ or $(F(x) \geq G(x))$, $\forall x$, such that the CUAR criterion equals infinity or zero, the asset with zero incremental utility is eliminated from the set. As noted above, the insight behind the CUAR criterion is that given two assets, F and G, all investors with increasing utility functions will prefer the asset with the higher expected incremental utility. Thus, for each pair of assets the CUAR criterion uses the ratio of incremental utility to measure how much asset F is better than asset G. These pairwise comparison values are the type of input required for AHP analysis.

2.2. The Analytic Hierarchy Process and Construction of the Enhanced Tracking Index

AHP analysis makes it possible to convert pairwise comparison values into a weight for each element of the input matrix, thereby allowing diverse elements to be compared to one another in a rational and consistent way.

To construct the enhanced tracking index we start with a market index comprised of N assets. The CUAR criteria for each pair of assets in the index are the pairwise comparison values that are arranged in an N x N matrix and serve as the input for the AHP. For an index containing

N stocks, this involves the calculation of $N(N-1)$ CUAR criteria. The general format of an AHP pairwise comparison matrix of N assets for a given utility function is given as follows:

$$[a_{ij}] = \begin{cases} CUAR\ criterion(i,j), & i > j \\ 1, & i = j \\ CUAR\ criterion\ (i,j)^{-1}, & i < j \end{cases} \quad ; \text{i, j} = 1,2,\ldots,N \tag{7}$$

The principal eigenvector of this matrix gives the relative outcomes of the individual assets expressed as a percentage of total incremental utility accruing to all of the assets taken together.

To avoid problems associated with rank reversal, the comparison values must be consistent. To ensure the robustness of the AHP technique, we follow Saaty (1980a) and test for consistency of the pairwise matrix by checking the Consistency Ratio (CR). The CR is defined as the ratio of the Consistency Index (CI) to the Random consistency Index (RI), i.e. CR=CI/RI, where:

$$\mathrm{CI} = \frac{\lambda_{\max} - \mathrm{N}}{\mathrm{N} - 1} \tag{8}$$

and λ_{max} is the largest eigenvalue of the pairwise comparison matrix while RI is determined from a standard table. A pairwise matrix is consistent if CR<0.1 (see Saaty, 1980a).

The construction of the utility enhanced index involves normalizing costs and incorporating them in the analysis. Normalized costs represent the percent of total investable funds allocated to each asset. To capture the total amount of incremental utility reflected in each asset, the percentage of funds allocated to each asset should just be equal to the percentage of incremental utility associated with each asset in the index. On a benefit/cost basis this is equivalent to equalizing the relative outcomes of the individual assets (percent of incremental utility/ percent of investable funds = 1) so that the hierarchy across the assets is eliminated. On a benefit/cost basis no asset is preferred to another and all can be purchased with the available investable funds. Thus, the utility enhanced index includes all the assets in the original market index, but with weights that differ according to the utility function used to calculate the CUAR criteria.

2.3 Calculation of the CUAR Criterion for a DARA Utility Function

In this section we illustrate the calculation of the CUAR criteria for a discrete random variable via a simple instructive example for two assets. Let A and B be two stocks with historic returns for the past four periods as presented in Table 1. Since all periods are equally weighted, the outcome probabilities are 0.25. After sorting all returns from the smallest to the largest, the cumulative probability distributions of these two stocks are given in Table 2.

[Please insert Tables 1 and 2 here]

For any investor, outcomes must reflect their incremental utility as set out in equations (4) and (5). In the instructive example that follows, we calculate the CUAR criterion for a risk averse investor with Decreasing Absolute Risk Aversion (DARA). We use the function $y = \ln(x)$ as an example of a DARA type of utility function. To calculate the CUAR criteria, we calculate the differences of both the outcomes and the cumulative probabilities. Reading left to right, Table 3 shows the steps involved in these calculations.

[Please insert Table 3 here]

The first column in Table 3 gives the outcomes for assets A and B. The second column, denoted by ln(R), gives the natural logarithm of each outcome. Column 3 gives the difference in the logarithmic outcomes. For example, the first entry in column 3 is 0.05716 which is the difference between ln(.90) and ln(.85) = -0.10536 – (-0.16252). The columns denoted F(A) and G(B) show the cumulative distribution functions of assets A and B. The DARA_Δ column is the product of the ln(R)_D and F(A)-G(B) columns. It represents the incremental utility of A compared to B over the interval. Positive values mean that F(A) is greater than G(B) and represent the incremental utility accruing to B. Negative values mean F(A) is smaller than G(B) and represent the incremental utility accruing to A over the interval. The incremental utility of A over B over the whole distribution is represented by the sum of the absolute values of the negative entries in the DARA_Δ column, while the incremental utility of B over A is represented by the sum of the positive values. From equation (6) the CUAR criterion for A with respect to B can be calculated from the DARA_Δ column in Table 3 as the sum of all the absolute values of the negative areas divided by the sum of all positive areas. This gives us the following CUAR criterion: $\frac{0.02502}{0.01429+00.01163+0.01021} = 0.6925.$ The latter implies that A is 0.6925 times as good as B or that B is 1.444 times better than A. The same procedure can be applied to all strictly increasing utility functions.

3. Application of the Utility Enhanced Tracking Technique

In this section we apply our methodology to the Dow Jones Industrial Average, one of the main stock indices of the U.S. market, but, with only 30 stocks small enough for expository clarity. We collect daily data from DataStream over the period 30/09/2010 to 30/09/2015 to apply the UETT to the DJIA. The DJIA consists of 30 stocks from various industries and is a price weighted index. Throughout the analysis, we assume that investors have a DARA utility function. An investor with a DARA utility function "attaches a smaller risk premium to any given risk the greater his wealth" (Arditti, 1967, p. 21). Pratt (1964) and Arrow (1965) suggest it is quite reasonable to assume that investors have a DARA utility function and Arditti (1967) and Cremers et. al. (2003) provide empirical evidence in favor of this.

We proceed in two steps. First, we calculate daily returns including dividends for a 1-year period (calculation period), which are applied in the UETT to calculate weights for each asset in the index. In step 2, we use the weights derived from step 1 to calculate the enhanced index returns for the 1-month period right after the calculation period.

3.1. Modified Consistency Ratio

An issue of concern with this type of analysis is the robustness of the output matrix. The Consistency Ratio (CR) defined by Saaty (1980a) to test the robustness of the AHP application gives values only up to N=15. Since the number of stocks in our sample is higher than 15, we follow Alonso and Lamata (2006), who developed a modified CR definition (MCR) that is more flexible than the classical CR definition:

$$\mathrm{MCR} = \frac{\lambda_{\max} - \mathrm{N}}{\mu_{\max} - \mathrm{N}} \text{ where } \mu_{\max} = (2.7699 * \mathrm{N}) - 4.3513 \tag{9}$$

Substituting the value $\mu_{\max}$ into the definition of MCR we have

$$\mathrm{MCR} = \frac{\lambda_{\max} - \mathrm{N}}{(1.7699 * \mathrm{N}) - 4.3513} \tag{10}$$

The consistency criterion is the same in this modified definition, i.e. MCR must be less than 0.1 for an acceptable consistency. Furthermore, the closer the MCR is to zero the more consistent is the pairwise matrix. So, each time we calculate weights, we also calculate the MCR statistic. If we obtain a value higher than 10%, this indicates that for one or more pairs of stocks there is first degree dominance or almost first degree dominance which is why some elements in the matrix are extremely high (and low). In this case we eliminate from the sample the dominated stock(s) and re-calculate.[11]

3.2. Construction of the Utility Enhanced Index

In this section we demonstrate in detail how we apply the UETT to the DJIA and we report the weights of the enhanced index for the first period of our sample. Our sample period begins on 30/9/2010, so we calculate daily returns for each stock for the period 30/9/2010 to 30/9/2011. Then, using these returns, we calculate the CUAR criterion values and organize them in matrix form.[12] An examination of this matrix shows that there are no cases where $(F(x) \le G(x))$ or $(F(x) \ge G(x))$, $\forall x$, and in more than 98% of cases the pairwise comparison values are compatible with the Saaty Scale that ranges between 1/9 and 9 for both indices. In one case only the CUAR has a very large value. This is the case of the comparison between the returns of American Express and Hewlett Packard. The CUAR is very large because Hewlett Packard almost dominates American Express in the first degree so, we eliminate American Express from the sample and we re-calculate the CUARs.[13] We then use this matrix as input in the AHP process to obtain the enhanced weights. The results are reported in table 4.

[Please insert Table 4 here]

[11]There are procedures to make the Saaty matrix consistent without eliminating stocks (e.g. Ergu et. al. 2014). We choose not to follow such a procedure for two reasons. Firstly, these procedures alter the elements of the matrix in order to make it consistent, thus distorting the initial rankings. Secondly, from a theoretical point of view, it makes sense to eliminate dominated or almost dominated stocks in the first degree because these stocks would be rejected by all risk-averse investors. Throughout our sample, there are only 21 cases of dominance or almost dominance.

[12] These matrices are available on request.

[13] Even if American Express was left in the sample, its estimated weight would be very close to zero. However, it would distort the estimation of the weights of the other stocks.

Table 4 shows that the enhanced index is well diversified and that there are no obvious extreme or non-intuitive weights for any of the assets. We apply the above procedure to the DJIA for the period 30/9/2010 to 30/9/2015. We use one year of daily returns including dividends to calculate the stock weights, which we hold for the next month.[14] We then roll the data forward one month and calculate the weights for the next month and so on until the end of the sample. Thus, we use the first year of the sample period (i.e. 30/9/2010 to 30/9/2011) for the estimation of the stock weights, which we apply to the next month (i.e. 1/10/2011 to 30/10/2011). Then, we use the period 31/10/2010 to 31/10/2011 to estimate stock weights for the period 1/11/2011 to 30/11/2011, and so. The only exception is when there is a change in the composition of the index, in which case we re-estimate the stock weights using the new constituents. It should be noted that in order to make our portfolios practical investment vehicles, we apply the estimated weights on the 1st day of each holding period (month) and we calculate buy-and-hold returns until the end of the month when the portfolios are re-balanced based on the new estimated weights. This means that the returns reported below assume re-balancing only every month or when the index constituents change.

3.3. Performance Evaluation

The results for the actual and the enhanced index are reported in table 5. The CUAR, which measures the relative increase in utility with respect to the benchmark index, is very high at 2.72. The average daily return of the actual index is 0.053% while for the enhanced index 0.073%. On an annual compounded basis, the enhanced index gives about 5.7% more return than the actual index. The tracking error at 0.22% is very low and the correlation coefficient between the two

[14] The rebalancing period of 1 month (4 weeks) was determined empirically. Rebalancing periods from 1 week to 5 weeks were tested. Results from 1 to 4 weeks are quantitatively and qualitatively the same but begin to degenerate after 4 weeks.

indices is about 97%, which means that although the enhanced index is outperforming the benchmark, it is also tracking it very closely. In Table 5 we also report the annual returns for each index, which show clearly that the enhanced index outperforms the DJIA over the whole period as well as over all sub-periods. These results are summarized in Figure 1 where we plot the value of a $1 investment in the actual and the enhanced index. As an additional measure of performance we also report the Omega (Ω) ratio (see Keating and Shadwick, 2002) which involves partitioning returns into loss and gain above and below a given threshold. The Ω ratio is the ratio of the probability of having a gain by the probability of having a loss. The ratio is calculated as:

$$\Omega(r) = \frac{\int_r^{\infty}(1 - F(x))dx}{\int_{-\infty}^{r} F(x)dx} \tag{11}$$

where F is the cumulative distribution function and r is the threshold defining the gain versus the loss. The higher the ratio, the better it is. The threshold rates we use in the analysis are -25 basis points, 0% and +25 basis points[15]. Like the CUAR, this ratio has the advantage that it takes into account all moments of the return distribution. The Ω ratio suggests there is higher probability of positive returns with the enhanced index than the actual index. Overall, all performance measures suggest that the enhanced index clearly outperforms the DJIA.

[Please insert Table 5 here]

3.4. Imposing cardinality constraints

As discussed in the introduction, it is known that most index as well as enhanced index tracking models naturally select a very large number of stocks in the composition of their portfolios.

[15] Since we are working with daily returns, which are typically very close to 0%, we choose threshold rates for the Omega ratios as -0.25%, 0% and +0.25%

Strategies that seek to reduce the number of assets in the tracking portfolio are plagued by their computational difficulty when implementing cardinality constraints (see, for example, Beasley et. al., 2003, Canakgoz and Beasley, 2008). Cardinality constraints in the context of UETT are simple and straightforward to apply, requiring only a few seconds of computation time.

The application of cardinality constraints involves deciding on the desired level of correlation of the tracking portfolio with the reference index. Once this has been decided, the UETT methodology is applied to obtain a preliminary enhanced portfolio that includes all the stocks. We need this portfolio to deduce the weights of the cardinality adjusted portfolio (CAP). To preserve the tracking feature of the CAP, the weight of the stocks in the CAP should reflect their relative weights in the preliminary UETT portfolio. The CUARs of each stock with respect to the reference index are then calculated and ranked from highest to lowest. The CAP is constructed in a stepwise methodology. The first stock that enters the CAP is the stock with the highest CUAR. All the weight of the stocks excluded from the preliminary UETT portfolio is allocated to this stock in the CAP. The correlation of this single stock portfolio with the reference index is then calculated. If it is below the desired correlation level, the stock with the second highest CUAR is added to the CAP and the weights of the excluded stocks are apportioned according to their relative weights reflected in the preliminary UETT portfolio. For example if stock 1 represented 3% of the preliminary UETT portfolio and stock 2 represented 2%, stock 1 would have a weight of 60% in the CAP and stock 2 would have a weight of 40%. The correlation of the returns on this portfolio is then calculated and so on, until a CAP with the desired level of correlation is attained. The final CAP is composed of the lowest number of stocks with the highest CUARs that render the desired level of correlation with the reference index[16].

We used the foregoing methodology to construct and test an enhanced DJIA with a constant number of stocks and a minimum correlation target of 0.9. This worked out to a portfolio that included 10 stocks.[17] We used a one year burn in period. To construct the CAP for October 2011, we calculated the UETT portfolio and the CUARs of each stock against the DJIA using returns from 30/9/2010 to 30/9/2011. The enhanced portfolio for October 2011 includes the 10 stocks with the highest CUARs against the index over the reference period, weighted with respect to their relative weights in the UETT portfolio. On the 31/10/2011 we repeat the procedure to determine the 10 stocks and their weights that enter the portfolio in November 2011 and so on. The results for the CAP over the four year out-of-sample period are reported in Table 6.

[Please insert Table 6 here]

The 10-stock CAP enhanced portfolio also outperforms the DJIA. The CUAR is far above 1. The Omega ratios are also higher than that of the DJIA. Interestingly, the return on the CAP enhanced portfolio is considerably higher than the DJIA as well and slightly higher than the return on the original enhanced index. The tracking error is small at 0.38% and, although the standard deviation is slightly higher, there is no statistical difference between the two. Interestingly, the out-of-sample correlation with the reference index is as high as the minimum in-sample correlation. All the performance measures suggest that the 10-stock CAP enhanced portfolio convincingly outperforms the actual DJIA reference portfolio. We also tested the performance of a 20-stock portfolio with a minimum correlation target of 0.95 and the results are similar. This 20-stock CAP

[16] See the appendix for a stepwise procedure for the construction of the CAP.
[17] The double constraint facilitates performance comparisons.

enhanced portfolio also outperforms the DJIA. The CUAR is far above 1, the Omega ratios are higher than that of the DJIA with higher returns and a very small tracking error. All these results suggest that reducing the number of stocks in the enhanced portfolio can preserve the enhancement and tracking properties of the UETT with respect to CUAR, the Omega ratio, and returns. Thus, another advantage of the UETT procedure is that it allows a fund manager to build an enhanced portfolio with as many or as few stocks as he wants, depending on his preferences with respect to risk and return, and how closely he wants to follow the proxy index.

3.5. The S&P 500

We now turn our attention to the S&P 500 to examine how our methodology performs on an index with a large number of stocks. We consider the same sample period as for the DJIA and proceed as follows. We estimate stock weights at the end of each month except when there is a change in the composition of the index. In this case, we re-estimate the weights using the sample that includes the new entries and drops the exiting stocks. Changes to the composition of the index during our sample period required re-estimating the weights a total of 90 times. We follow exactly the same procedure as for the DJIA. First we eliminate all stocks which are dominated or almost dominated by other stocks. On average there are about 122 dominated or almost dominated stocks so on average we work with a sample of 378 stocks.[18] Next, we estimate the AHP matrix and check for consistency. The minimum and maximum MCR for the 90 AHP matrices are 2.6% and 4.77% respectively, such that all matrices are consistent at the 5% level. The results for the enhanced S&P 500 index as well as the statistics for the actual S&P 500 index are reported in columns 2 and 3 of Table 7. The results are qualitatively similar to the results for the DJIA. The CUAR is far above 1

[18] The number of stocks in the S&P 500 during our sample period ranges from 500 to 505. After elimination of the dominant stocks, the average, median, maximum and minimum number of stocks each month is respectively 377.8, 387.5, 446 and 261.

(1.351) and the Omega ratios for the enhanced portfolio are higher than that of the benchmark. The enhanced index also outperforms the actual S&P 500 by about 2.7% annually with a tracking error of 0.19% and a correlation coefficient with the actual index of 98.1%.

To examine the effect of cardinality constraints on the enhanced index we also create a CAP enhanced index using each month the 200 stocks with the highest CUAR against the S&P 500. As we did for the DJIA, the selection is carried out among the stocks which enter the initial enhanced index and we change the weights to reflect the initial weights. Column 4 of Table 7 reports the results. Again, we see that the CAP enhanced portfolios perform just as well as the initial enhanced index. Most importantly, the CUAR is much higher. The Omega ratio and the average returns are also higher than the actual index, the correlation coefficient is very high and the tracking error is very low.

[Please insert Table 7 here]

3.6. Transaction costs

Actively managed funds may have high transaction costs which erode the profitability of the investment strategy. The transaction costs of the indices we consider are very low because there is little change in the weight of each stock from month to month. For the S&P 500 enhanced index, the average required change on a monthly basis is 16.6% of the funds invested. The average bid-ask spread during our sample period is only 5.2 basis points because the stocks in the index are some of the most liquid stocks in the U.S market. On average, the bid-ask spread reduces the annual returns by 0.192%.[19] After accounting for transaction costs, the CUAR remains relatively

[19] During our sample period, the maximum annual cost is 0.223% and the minimum 0.149%.

unchanged and the average annual compounded return on the enhanced index is still 2.53% higher than the S&P 500. For the DJIA the average required change in the funds invested is 27.7% on a monthly basis and the average bid-ask spread reduces our annual returns by 0.202%. After accounting for these transaction costs, the CUAR declines only marginally and the average annual compounded return on the enhanced index is still 5.5% higher than the DJIA. Therefore, even accounting for transaction costs, the enhanced indices outperform the actual indices.

3.7. Robustness tests

3.7.1. Other utility functions

Having used the log function for investors with DARA in the preceding tests, we now turn to the performance of the UETT methodology using two alternative utility functions that represent investors with increasing absolute risk aversion (IARA) and those with constant absolute risk aversion (CARA). First, we use the familiar quadratic utility function that has the property of increasing absolute risk aversion. [20] For the utility function with the property of CARA we use the negative exponential.[21] The results for the DJIA are reported in Table 8.

[INSERT TABLE 8 ABOUT HERE]

[20] The conventional quadratic utility function is $(1+r)-b(1+r)^{(2)}$. In each sample we used a different risk aversion coefficient to reflect the preferences of an average risk averse investor. For example, suppose that the minimum variance portfolio for a sample has a standard deviation of returns of 3% and the maximum variance portfolio has a standard deviation of returns of 11%. Then for this sample we chose the risk aversion coefficient that gave an efficient portfolio with a standard deviation of (3% + 11%) / 2 = 7%. We consider this to be a good representative portfolio for an investor with a quadratic utility function.

[21] For the CARA, the utility function is $-exp^{-r}$.

The 2nd column of Table 8 reports statistics for the DJIA while the 3rd and 4th columns report statistics for the enhanced portfolios based on the quadratic and CARA utility functions respectively. It is obvious that the UETT methodology performs as well for these utility functions as it does for the DARA. Both the quadratic and CARA enhanced portfolios have high CUARs. Their returns are far higher than those of the DJIA and the tracking error is very small (0.17% and 0.19% respectively). The correlation with the DJIA is also very high at 0.978 and 0.976 respectively and the Omega ratios are higher than that of the DJIA. This is clear evidence that the UETT methodology can serve to provide enhanced portfolios over a wide range of risk averse utility functions.

3.7.2. Turning market conditions

The results presented so far refer to a period where the market is generally bullish. As a robustness test we now examine how the methodology performs under changing market conditions. Specifically, we employ a calculation period where the market was aggressively bear to construct the enhanced index for the following period when the market turned bullish. To this end, we incorporate the 2008 financial crisis. During this period, both the Dow Jones and the S&P500 reached their lowest points in the first week of March 2009. We start with the period 03/31/2008 to 02/28/2009 to derive weights for the enhanced index for March 2009. Then, we use the period 04/30/2008 to 03/31/2009 to derive weights for the enhanced index for April 2009, and so on. We repeat this process 13 times. The final calculation period is 03/31/2009 to 02/28/2010 where we derive weights for March 2010. Overall, we obtain 274 daily returns (excluding holidays) where the calculation period returns come from a bear market and the holding period is a bull market. The results are reported in Table 9.

The CUARs are far above 1, 1.329 for the enhanced DJIA and 1.474 for the enhanced S&P. The returns are also far higher, 6.37% higher for the enhanced DJIA and 17% for the S&P. The tracking errors for both indices are very small and the correlation with the benchmark indices is almost 99%. The Omega ratios are similar for both pairs of portfolios and there is not much difference in their volatilities. Overall, these results are evidence that the UETT methodology is robust to changing market conditions.

[INSERT TABLE 9 ABOUT HERE]

4. Conclusion

In this paper we develop a novel technique for reweighting a benchmark index to generate enhanced returns that reflect the preferences of a wide range of investor types, those with DARA, CARA and IARA. The UETT technique itself is very general and can accommodate any return distribution. The only assumption we make is that investors are risk averse. It is also simple and easy to implement. The only inputs are the returns on the assets included in the benchmark index and the utility function of the investor. We proceed in two steps. First, we develop the concept of the Cumulative Utility Area Ratio criterion, which measures the incremental utility of one asset compared to another. We then use these pairwise comparisons in the classical Analytic Hierarchy Process (AHP) developed by Saaty (1980a) to determine the enhanced index weights.

Besides its simplicity, the UETT procedure has the advantage that the enhanced index does not require frequent re-balancing, which means transaction costs are negligible. Our out-of-sample tests over a four year period show that with monthly rebalancing, the enhanced index outperforms the actual DJIA by about 5.7% on an annual basis. It outperforms the S&P 500 by 2.7%.

Accounting for transaction costs reduces these figures to 5.5% and 2.5% respectively. We achieve similar results for the bear market and transition to the bull market. The imposition of cardinality constraints is also simple and straightforward, requiring only a few seconds of computational time. The out-of-sample results show that with monthly rebalancing the cardinality adjusted portfolios perform just as well as the general UETT portfolio. Overall, the UETT portfolios generate consistently higher out-of-sample utility profiles and after-cost returns for the fully enhanced portfolios as well as for the enhanced portfolios adjusted for cardinality constraints. These results are robust to varying market conditions and a range of risk averse utility functions. Although risk aversion is the default assumption in conventional portfolio analysis, it does exclude some of the new developments in behavioural finance and Prospect theory. To address this shortcoming our ongoing research aims at extending the analysis to include a range of risk seeking utility functions and combinations of risk averting/risk seeking utility functions.

Abbreviations

AHP	Analytic Hierarchy Process
CI	Consistency Index
CR	Consistency Ratio
DARA	Decreasing Absolute Risk Aversion
MCR	Modified Consistency Ratio
MV	Mean Variance
RI	Random consistency Index
UETT	Utility enhanced tracking technique
CUAR	Cumulative Utility Area Ratio

Table 1: Historic Returns of Stocks A and B

Period	Stock A	Stock B
1	-15%	-5%
2	5%	-10%
3	20%	25%
4	5%	10%

The table reports returns for two hypothetical stocks.

Table 2: Cumulative Probability Distributions of A and B

Outcomes (Sorted)	f(A)	g(B)	F(A)	G(B)
0.85	0.25	0	0.25	0
0.90	0	0.25	0.25	0.25
0.95	0	0.25	0.25	0.5
1.05	0.25	0	0.5	0.5
1.05	0.25	0	0.75	0.5
1.10	0	0.25	0.75	0.75
1.20	0.25	0	1	0.75
1.25	0	0.25	1	1

The table reports the cumulative probability distributions for the two hypothetical stocks of Table 1.

Table 3: Calculation of the Intersection areas for DARA utility function

R	ln (R)	ln(R)_ D	f(A)	g(B)	F(A)	G(B)	F(A)-G(B)	DARA_Δ
0.85	-0.16252	0.05716	0.25	0.00	0.25	0.00	0.25	0.01429
0.90	-0.10536	0.05407	0.00	0.25	0.25	0.25	0.00	0.00000
0.95	-0.05129	0.10008	0.00	0.25	0.25	0.50	-0.25	-0.02502
1.05	0.04879	0.00000	0.25	0.00	0.50	0.50	0.00	0.00000
1.05	0.04879	0.04652	0.25	0.00	0.75	0.50	0.25	0.01163
1.10	0.09531	0.08701	0.00	0.25	0.75	0.75	0.00	0.00000
1.20	0.18232	0.04082	0.25	0.00	1.00	0.75	0.25	0.01021
1.25	0.22314	0.00000	0.00	0.25	1.00	1.00	0.00	0.00000

The table shows how to construct the CUAR for the two hypothetical stocks of Tables 1 & 2. The notation of row 1 is as follows:

R: Sorted Outcomes
ln (R): Natural logarithm of the outcomes
ln(R)_D: Differences of natural logarithm of the outcomes
f(A): Probability distribution of asset A
g(B): Probability distribution of asset B
F(A): Cumulative probability distribution of asset A
G(B): Cumulative probability distribution of asset B
DARA_Δ: Intersection areas of F(A) and G(B) with respect to natural logarithm of outcomes

Table 4: Utility Enhanced Weights for the DJIA

Stock	Weights for the enhanced index
3M	1.680%
ALCOA	2.481%
AT&T	3.094%
BANK OF AMERICA	1.852%
BOEING	2.976%
CATERPILLAR	3.206%
CHEVRON	5.746%
CISCO SYSTEMS	1.496%
COCA COLA	4.661%
E I DU PONT DE NEMOURS	2.660%
EXXON MOBIL	6.953%
GENERAL ELECTRIC	3.381%
HOME DEPOT	3.714%
HP	0.917%
INTEL	5.105%
INTERNATIONAL BUSINESS MACHINES	7.886%
JOHNSON & JOHNSON	3.155%
JP MORGAN CHASE & CO.	2.296%
KRAFT FOODS	3.808%
MCDONALDS	4.969%
MERCK & COMPANY	2.008%
MICROSOFT	3.365%
PFIZER	3.700%
PROCTER & GAMBLE	3.364%
TRAVELERS COS.	2.399%
UNITED TECHNOLOGIES	2.967%
VERIZON COMMUNICATIONS	5.295%
WAL MART STORES	2.503%
WALT DISNEY	2.363%

The table reports the weight of each stock of the enhanced DJIA based on stock returns from 30/9/2010 to 30/9/2011.

Table 5. Results for the DJIA and the Enhanced index

	DJIA	Enhanced
CUAR	2.72	
Omega -25 bps	1.21	1.27
Omega 0%	1.19	1.25
Omega +25bps	1.21	1.28
Average daily return	0.053%	0.073%
Average annual compounded return	14.37%	20.08%
Annualized Standard Deviation	13.14%	13.87%
Annual return 30/9/11 - 30/9/12	26.52%	29.33%
Annual return 30/9/12 - 30/9/13	15.59%	19.11%
Annual return 30/9/13 - 30/9/14	15.28%	16.84%
Annual return 30/9/14 - 30/9/15	-2.11%	10.94%
Tracking error	0.22%	
Correlation coefficient	0.97	

Notes. The table reports statistics for the DJIA and the enhanced portfolio. The sample period is 30/9/2010 to 30/9/2015 and the enhanced portfolios are created out-of-sample. Row 1 reports the CUAR which measures the incremental utility of the enhanced portfolio compared to benchmark DJIA for an investor with a DARA utility function. Row 2 reports the omega ratio for each portfolio where the threshold returns are 0% and ±25 basis point. The annual return refers to the total return index, i.e. to the index returns including dividends.

Table 6. Results for the enhanced and the reduced enhanced index

	DJIA	Enhanced	Enhanced with 10 stocks
CUAR		2.72	1.85
Omega -25 bps	1.209	1.274	1.261
Omega 0%	1.195	1.252	1.248
Omega +25bps	1.210	1.275	1.264
Average daily return	0.0533%	0.0726%	0.0731%
Average annual compounded return	14.37%	20.08%	20.23%
Annualized Standard Deviation	13.14%	13.87%	14.24%
Tracking error		0.22%	0.38%
Correlation coefficient with DJIA		0.97	0.91

Notes. The table reports statistics for the DJIA, the enhanced portfolio and the enhanced portfolio including 10 stocks each period. The sample period is 30/9/2010 to 30/9/2015 and the enhanced portfolios are created out-of-sample. Row 1 reports the CUAR which measures the incremental utility of the enhanced portfolio compared to benchmark DJIA for an investor with a DARA utility function. Row 2 reports the omega ratio for each portfolio where the threshold returns are 0% and ±25 basis point. Returns include dividends.

Table 7. Results for the enhanced and the reduced enhanced S&P 500 index

	S&P500	Enhanced	Enhanced with 200 stocks
CUAR		1.351	1.632
Omega -25 bps	1.244	1.255	1.273
Omega 0%	1.229	1.241	1.258
Omega +25bps	1.245	1.255	1.274
Average daily return	0.065%	0.074%	0.077%
Average annual compounded return	17.80%	20.52%	21.41%
Annualized Standard Deviation	13.95%	14.93%	14.49%
Tracking error		0.19%	0.26%
Correlation coefficient with SP500		0.981	0.959

Notes. The table reports statistics for the S&P 500 index, the enhanced portfolio and the enhanced portfolio including 200 stocks each period. The sample period is 30/9/2010 to 30/9/2015 and the enhanced portfolios are created out-of-sample. Row 1 reports the CUAR which measures the incremental utility of the enhanced portfolio compared to the benchmark S&P 500 for an investor with a DARA utility function. Row 2 reports the omega ratio for each portfolio where the threshold returns are 0% and ±25 basis point. Returns include dividends.

Table 8. Results for various utility functions

	DJIA	Quadratic	CARA
CUAR		1.863	2.183
Omega -25 bps	1.209	1.239	1.254
Omega 0%	1.195	1.222	1.237
Omega +25bps	1.210	1.238	1.253
Average daily return	0.0533%	0.0632%	0.0669%
Average annual compounded return	14.37%	16.99%	18.37%
Annualized Standard Deviation	13.14%	13.55%	13.79%
Tracking error		0.17%	0.19%
Correlation coefficient with DJIA		0.978	0.976

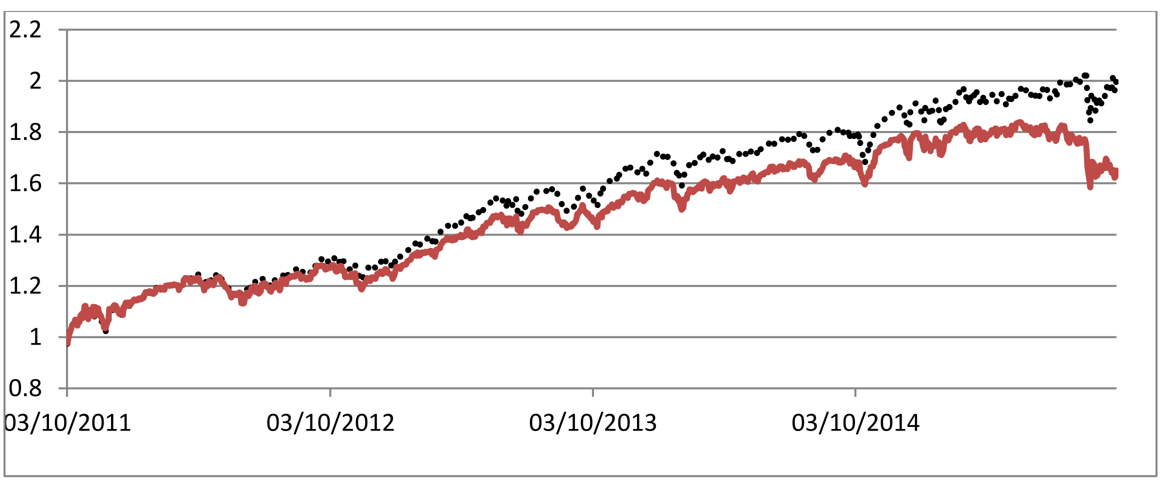
Notes. The table reports statistics for the DJIA and enhanced portfolios based on the following utility functions: quadratic and Constant Absolute Risk Aversion (CARA). The sample period is 30/9/2010 to 30/9/2015 and the enhanced portfolios are created out-of-sample. Row 1 reports the CUAR which measures the incremental utility of the enhanced portfolio compared to benchmark DJIA for an investor with each respective utility function. Row 2 reports the omega ratio for each portfolio where the threshold returns are 0% and ±25 basis point. Returns include dividends.

Table 9. Results for the DJIA and the S&P 500 and their respective Enhanced portfolios

	DJIA	Enhanced	S&P500	Enhanced
CUAR	1.329		1.474	
Omega -25 bps	1.50	1.49	1.46	1.47
Omega 0%	1.48	1.48	1.46	1.45
Omega +25bps	1.50	1.50	1.46	1.47
Average daily return	0.177%	0.193%	0.188%	0.229%
Average annual compounded return	56.05%	62.42%	60.73%	77.73%
Annualized Standard Deviation	20.53%	22.95%	26.36%	22.84%
Tracking error	0.28%		0.32%	
Correlation coefficient	0.986		0.989	

Notes. The table reports statistics for the DJIA and the S&P500 and their respective enhanced portfolios. The sample period is 03/31/2008 to 03/31/2010 and the enhanced portfolios are created out-of-sample. Row 1 reports the CUAR which measures the incremental utility of the enhanced portfolio compared to the benchmark for an investor with a DARA utility function. Row 2 reports the omega ratio for each portfolio where the threshold returns are 0% and ±25 basis point. The annual return refers to the total return index, i.e. to the index returns including dividends.

Figure 1. The performance of the DJIA and the enhanced index over a 4-year period



The graph shows the value of a $1 investment on the Dow Jones Industrial Average and the Enhanced Index from 30/9/2011 to 30/9/2015. The dotted line is the enhanced index. Daily returns are calculated including dividends.

APPENDIX: Procedure for the construction of the cardinality constrained portfolio (CAP)

- Step 1. Apply UETT methodology to construct unconstrained enhanced portfolio and derive weights (w_i) for each asset (w_1, w_2, w_3, …).
- Step 2. Decide the desired correlation between the benchmark and the CAP.
- Step 3. Calculate the CUAR for each asset included in the enhanced unconstrained portfolio and rank assets based on their CUAR from highest to lowest.
- Step 4. Allocate weight of 100% on the asset with the highest CUAR.
- Step 5. Calculate the correlation between this portfolio and the benchmark.
- Step 6. If the correlation is higher than the desired one, stop. If it is lower, include in the CAP the asset with the next highest CUAR and allocate weights as:

$$w_i' = \frac{w_i}{\sum_1^n w_i}$$

where w_i' is the weight of asset i in the CAP, w_i is the weight of asset i in the unconstrained enhanced portfolio, and n is the number of assets in the CAP. Repeat steps 5 and then 6 until the desired correlation is achieved.

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