An Optimal Power Allocation for D2D Communications Over Multi-User Cellular Uplink Channels

Adeola Omorinoye, Quoc-Tuan Vien, Tuan Anh Le, and Purav Shah

Middlesex University, The Burroughs, London NW4 4BT, UK
`AO991@live.mdx.ac.uk;{Q.Vien;T.Le;P.Shah}@mdx.ac.uk`
https://www.mdx.ac.uk

Abstract. Device-to-Device (D2D) communications has emerged as a promising technology for optimizing spectral efficiency, reducing latency, improving data rate and increasing system capacity in cellular networks. Power allocation in D2D communication to maintain Quality-of-Service (QoS) remains as a challenging task. In this paper, we investigate the power allocation in D2D underlaying cellular networks with multi-user cellular uplink channel reuse. Specifically, this paper aims at minimizing the total transmit power of D2D users and cellular users (CUs) subject to QoS requirement at each user in terms of the required signal-to-interference-plus-noise ratio (SINR) at D2D users and base station (BS) over uplink channel as well as their limited transmit power. We first derive expressions of SINR at the D2D users and BS based on which an optimization framework for power allocation is developed. We then propose an optimal power allocation algorithm for all D2D users and CUs by taking into account the property of non-negative inverse of a Z-matrix. The proposed algorithm is validated through simulation results which show the impacts of noise power, distance between D2D users, the number of D2D pairs and the number of CUs on the power allocation in the D2D underlaying cellular networks.

Keywords: Device-to-device communication · uplink · power allocation.

1 Introduction

One of the fundamental motivation behind using Device-to-Device (D2D) communication underlaying cellular networks is to enable direct connection between a pair of proximity devices without involvement of Base Station (BS). Although in the current cellular systems, D2D communications along with the development of small cells can cover a large area providing an enhanced Quality-of-Service (QoS), this may require a considerably increased operating expense [1–5].

In spite of the benefits of D2D communication in cellular networks, energy efficiency and interference management have become the fundamental requirements [6] to control the interference caused by the D2D users, while simultaneously extending the battery lifetime of the User Equipment (UE). Cellular links only suffer from cross-tier interference from D2D transmitter, whereas D2D links not only deal with inter-D2D interference, but also with cross-tier interference from cellular transmission. Channel allocation and power allocation have been bestowed in the literature as strategies to diminish interference in cellular networks. In [7], close-loop and open-loop power control schemes used in LTE were investigated with an optimization based approach aimed at reducing total power consumption and increasing spectrum efficiency for D2D communications.

Additionally, green communication has been proposed attracting a number of research works with various power control and resource allocation approaches to enhance energy efficiency (EE) of D2D-aided heterogeneous network. In [1], the aim of controlling and limiting the interference of a D2D communication to the cellular network was investigated. There are basically two extensive categories of power control in D2D underlaying cellular networks which include distributed [8, 9] and centralized approaches [10, 11]. In the distributed approach resource allocation and power control are performed independently by the UEs, whereas they are both carried out at the BS in the centralized approach.

Considering an interference limited environment, resource allocation for D2D communications has been investigated in various research works, e.g. [12]. In this paper, we investigate the resource allocation in D2D underlaying cellular networks where the D2D users exploit multi-user cellular uplink channels.[1]

We first develop an optimization problem to find the optimal power for D2D users and CUs so as to minimize the total power consumption of the system subject to per-user QoS constraints in terms of the required signal-to-interference-plus-noise ratio (SINR) and limited transmit power at each user. In order to solve the developed problem, the property of Z-matrix is exploited to find the optimal power allocation at all users. The impact of the number of CUs and D2D users, noise power and the distance between D2D users are investigated and validated through the simulation. The proposed algorithm is shown to be able to allocate power to all D2D users and CUs achieving the minimum total power subject to various QoS constraints, while not affecting the performance of the CUs. Given a low QoS requirement, it is shown that a considerable transmit power of the D2D users can be saved for an increased energy efficiency of the overall system. In particular, the number of CUs is shown to have a significant impact on the average transmit power of the D2D users due to the interference from the CUs.

2 System Model

2.1 System Description

Figure 1 illustrates the system model of a D2D underlaying cellular network where we focus on a multi-user cellular uplink channels for D2D communication consisting of a BS, K CUs $\{CU_1, CU_2, \ldots, CU_K\}$ and N pairs of D2D users. The D2D communication exploits the uplink resource of cellular networks, i.e.

---

[1] This paper is different from [12] which considered only a CU in the uplink channel.

K CUs operating together with N D2D pairs. Specifically, N D2D transmitters, i.e. $\{DT_1, DT_2, \ldots, DT_N\}$ send their data to N desired D2D receivers, i.e. $\{DR_1, DR_2, \ldots, DR_N\}$. The D2D receivers suffer the interference from not only other D2D transmitters, but also the CUs. Similarly, over the uplink channels, the BS receives unwanted signals from the D2D transmitters in addition to those from other CUs in the network.

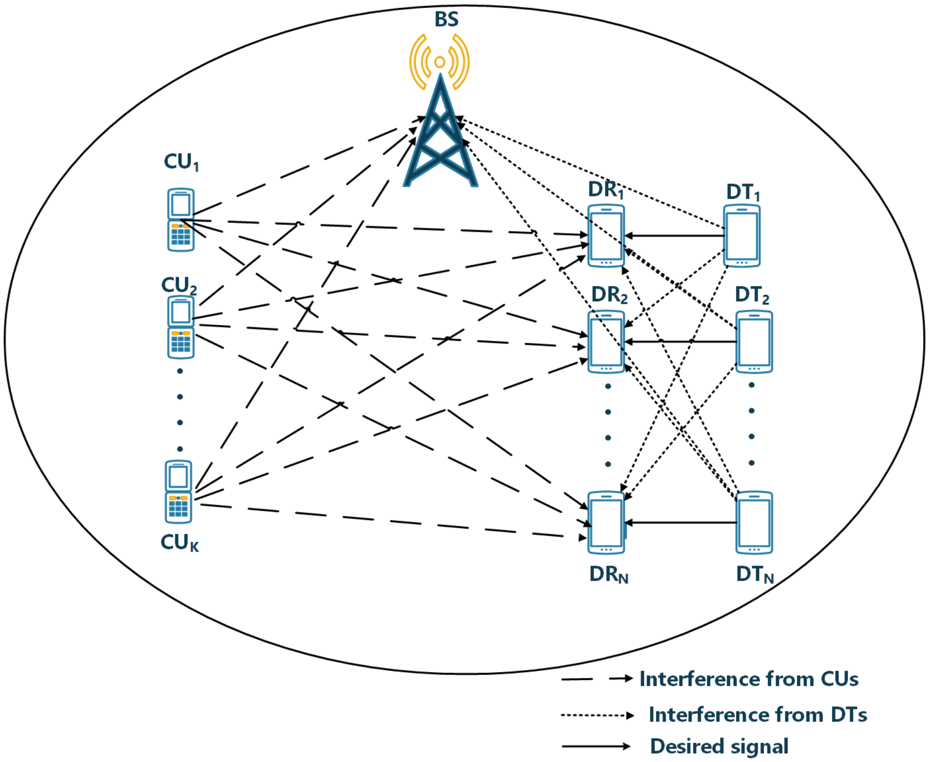

Fig. 1. System Model.

Let d_{b,c_k}, $d_{i,j}$, d_{j,c_k}, $d_{b,i}$, $\{i, j\} \in \{1, 2, \ldots, N\}$ and $k = 1, 2, \ldots, K$, denote the distances between CU_k and BS, between DR_i and DT_j, between CU_k and DR_j, and between DT_i and BS, respectively. The links $CU_k \rightarrow$ BS, $DT_j \rightarrow DR_i$, $CU_k \rightarrow DR_j$, and $DT_i \rightarrow$ BS, $\{i, j\} \in \{1, 2, \ldots, N\}$, $k = 1, 2, \ldots, K$, are assumed to experience Rayleigh flat fading channels having channel coefficients h_{b,c_k}, $h_{i,j}$, h_{j,c_k}, and $h_{b,i}$, respectively, with $E[|h_{b,c_k}|^2] = 1/d^{\alpha}_{b,c_k}$, $E[|h_{i,j}|^2] = 1/d^{\alpha}_{i,j}$, $E[|h_{j,c_k}|^2] = 1/d^{\alpha}_{j,c_k}$, and $E[|h_{b,i}|^2] = 1/d^{\alpha}_{b,i}$. Here, $E[\cdot]$ and α denote the expectation operator and pathloss exponent, respectively.

2.2 Channel Model

Over shared uplink channels, the channels dedicated for CUs can be reused by D2D transmitters. The received signal at BS is thus given by

$$y_b = \sum_{k=1}^{K} \sqrt{p_{c_k}} h_{b,c_k} x_{c_k} + \sum_{i=1}^{N} \sqrt{p_i} h_{b,i} x_i + n_b, \tag{1}$$

where $x_{c_k}, k = 1, 2, \ldots, K$ and x_i, $i = 1, 2, \ldots, N$, are signals transmitted from CU_k and DT_i with transmit power p_{ck} and p_i, respectively, and n_b is an independent circularly symmetric complex Gaussian (CSCG) noise having zero mean and variance of $E[|n_b|^2] = N_0$.

With respect to CU_k, $k = 1, 2, \ldots, K$, the instantaneous received SINR at the BS can be obtained by

$$\gamma_{b_k} = \frac{p_{c_k} |h_{b,c_k}|^2}{\sum_{j=1,j\neq k}^{K} p_{c_j} |h_{b,c_j}|^2 + \sum_{i=1}^{N} p_i |h_{b,i}|^2 + N_0}. \tag{2}$$

Over D2D channels, the expected received signal at DR_i, $i = 1, 2, \ldots, N$, is given by

$$y_i = \sum_{j=1}^{N} \sqrt{p_j} h_{i,j} x_j + \sum_{k=1}^{K} \sqrt{p_{c_k}} h_{i,c_k} x_{c_k} + n_i, \tag{3}$$

where x_j, $j = 1, 2, \ldots, N$, and x_{c_k} are signals transmitted from DT_j and CU_k with transmit power p_j and p_{c_k}, respectively, and n_i is an independent CSCG noise having zero mean and variance of N_0.

In (3), DR_i, $i = 1, 2, \ldots, N$, is only interested in x_i from DT_i. The instantaneous SINR at DR_i can be obtained by

$$\gamma_i = \frac{p_i |h_{i,i}|^2}{\sum_{j=1,j\neq i}^{N} p_j |h_{i,j}|^2 + \sum_{k=1}^{K} p_{c_k} |h_{i,c_k}|^2 + N_0}. \tag{4}$$

Let g_{b,c_k}, $g_{i,j}$, g_{j,c_k} and $g_{b,i}$, $\{i, j\} \in \{1, 2, \ldots, N\}$, $k = 1, 2, \ldots, K$, denote the channel gains of the links $CU_k \to BS$, $DT_j \to DR_i$, $CU_k \to DR_j$, and $DT_i \to BS$, respectively, i.e. $g_{b,c_k} = |h_{b,c_k}|^2$, $g_{i,j} = |h_{i,j}|^2$, $g_{j,c_k} = |h_{j,c_k}|^2$, and $g_{b,i} = |h_{b,i}|^2$. The instantaneous SINR at BS and DR_i, $i = 1, 2, \ldots, N$, in (2) and (4) can be accordingly rewritten as

$$\gamma_{b_k} = \frac{p_c g_{b,c_k}}{\sum_{i=1}^{K} p_i g_{b,i} + \sum_{j=1,j\neq}^{k} p_{c_j} g_{b,c_j} + N_0}, \tag{5}$$

$$\gamma_i = \frac{p_i g_{i,i}}{\sum_{j=1,j\neq i}^{N} p_j g_{i,j} + \sum_{i=1}^{N} p_{c_k} g_{i,c_k} + N_0}. \tag{6}$$

3 Proposed Optimization Problem for Both D2D and Cellular Communications

In this section, we first formulate the optimization problem to minimize the total transmit power of all users in a D2D underlaying cellular network as illustrated in Fig. 1. A QoS-driven power allocation scheme is then developed for all users subject to constraints of the required SINR and limited transmit power at these users in the network.

i) D2D communications: Given constraints of SINR and transmit power, the optimization problem to minimize the total transmit power in D2D communications can be formulated as

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N} p_i, \\ \text{s. t.} \quad & \gamma_i \geqslant \bar{\gamma}_i, \forall i = 1, \ldots, N, \\ & p_i \leq p_i^{\max}, \forall i = 1, \ldots, N, \end{aligned} \tag{7}$$

where γ_i is given by (6), $p_i^{\max}$ is the maximum transmit power at DT_i, $\bar{\gamma}_i$ is the required SINR level at DR_i.

ii) Cellular communications: The optimal problem for uplink cellular communications can be expressed as

$$\begin{aligned} \min_{p_{c_k}} \quad & \sum_{k=1}^{K} p_{c_k}, \\ \text{s. t.} \quad & \gamma_{b_k} \geqslant \bar{\gamma}_{b_k}, \forall k = 1, 2, \ldots, K, \\ & p_{c_k} \leq p_{c_k}^{\max}, \forall k = 1, 2, \ldots, K, , \end{aligned} \tag{8}$$

where γ_{b_k} is given by (5), $p_{c_k}^{\max}$ is the maximum transmit power at the CU_k, and $\bar{\gamma_{b_k}}$ is the required SINR level at the BS for the uplink channel from CU_k.

Considering an overall system, the problems in (7) and (8) can be combined as in the following form:

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N+K} p_i, \\ \text{s. t.} \quad & \frac{p_i g_{i,i}}{\sum_{j=1, j\neq i}^{N+K} p_j g_{i,j} + N_0} \geqslant \bar{\gamma}_i, \forall i = 1, \ldots, N+K, \\ & p_i \leq p_i^{\max}, \forall i = 1, \ldots, N+K, \end{aligned} \tag{9}$$

Here the notation is slightly abused by using the index $N+K$ to represent both D2D and cellular communications, i.e. $p_{N+k} = p_{c_k}$, $g_{i,N+k} = g_{i,c_k}$, $g_{N+k,j} = g_{b,j}$, $g_{N+k,N+k} = g_{b,c_k}$, and $\bar{\gamma}_{N+k} = \bar{\gamma}_{b_k}$.

The optimization problem in (9) can be rewritten by rearranging the SINR constraints in (9) in the following equivalent form:

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N+K} p_i, \\ \text{s. t.} \quad & p_i g_{i,i} - \bar{\gamma}_i \sum_{j=1, j\neq i}^{N+K} p_j g_{i,j} \geqslant N_0 \bar{\gamma}_i, \forall i = 1, 2, \ldots, N+K, \\ & p_i \leq p_i^{\max}, \forall i = 1, 2, \ldots, N+K, , \end{aligned} \tag{10}$$

The scalar form of (10) can be further simplified by introducing the following vectors:

$$\mathbf{G} \triangleq \begin{bmatrix} g_{1,1} & -\bar{\gamma}_1 g_{1,2} & \cdots & -\bar{\gamma}_1 g_{1,N} & -\bar{\gamma}_1 g_{1,c_1} & -\bar{\gamma}_1 g_{1,c_2} & \cdots & -\bar{\gamma}_1 g_{1,c_K} \\ -\bar{\gamma}_2 g_{2,1} & g_{2,2} & \cdots & -\bar{\gamma}_2 g_{2,N} & -\bar{\gamma}_2 g_{2,c_1} & -\bar{\gamma}_2 g_{2,c_2} & \cdots & -\bar{\gamma}_1 g_{2,c_K} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\bar{\gamma}_N g_{N,1} & -\bar{\gamma}_N g_{N,2} & \cdots & g_{N,N} & -\bar{\gamma}_N g_{N,c_1} & -\bar{\gamma}_N g_{N,c_2} & \cdots & -\bar{\gamma}_N g_{N,c_K} \\ -\bar{\gamma}_{b_1} g_{b,1} & -\bar{\gamma}_{b_1} g_{b,2} & \cdots & -\bar{\gamma}_{b_1} g_{b,N} & g_{b,c_1} & -\bar{\gamma}_{b_1} g_{1,c_2} & \cdots & -\bar{\gamma}_{b_1} g_{1,c_K} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\bar{\gamma}_{b_K} g_{b,1} & -\bar{\gamma}_{b_K} g_{b,2} & \cdots & -\bar{\gamma}_{b_K} g_{b,N} & -\bar{\gamma}_{b_K} g_{b,c_1} & g_{b,c_K} & \cdots & -\bar{\gamma}_{b_K} g_{b,c_K} \end{bmatrix}, \tag{11}$$

$$\mathbf{p} \triangleq \left[p_1 \; p_2 \cdots p_N \; p_{c_1} \; p_{c_2} \cdots p_{c_K}\right]^T, \tag{12}$$

$$\mathbf{n} \triangleq \left[N_0\bar{\gamma}_1 \; N_0\bar{\gamma}_2 \cdots N_0\bar{\gamma}_N \; N_0\bar{\gamma}_{b_1} \; N_0\bar{\gamma}_{b_2} \cdots N_0\bar{\gamma}_{b_K}\right]^T, \tag{13}$$

$$\mathbf{p}_{\max} \triangleq \left[p_1^{\max} \; p_2^{\max} \cdots p_N^{\max} \; p_{c_1}^{\max} \; p_{c_2}^{\max} \cdots p_{c_K}^{\max}\right]^T. \tag{14}$$

Hence, the problem (10) can be rewritten as

$$\begin{aligned} \min_{p_i} \quad & \sum_{i=1}^{N+K} p_i, \\ \text{s. t.} \quad & \mathbf{G}\mathbf{p} \succeq \mathbf{n}, \\ & \mathbf{p} \preceq \mathbf{p}_{\max}. \end{aligned} \tag{15}$$

where $\succeq$ and $\preceq$ denote the element-wise greater and less operators, respectively.

The optimal solution to problem (15) can be found by using the following lemma

Lemma 1. *If matrix* $\mathbf{G}$ *defined in* (11) *satisfies*

$$g_{i,i} > \bar{\gamma}_i \sum_{j=1, j\neq i}^{N+K} g_{i,j}, \forall i = 1, 2, \ldots, N+K, \tag{16}$$

then there exists a unique lower bound for the power allocation for problem (15) *as*

$$\mathbf{p}_{min} = \mathbf{G}^{-1}\mathbf{n}. \tag{17}$$

Proof. The proof follows the same approach as in [12] where the basic idea is to treat (11) as a Z-*matrix* [15, 16]. By observing (11), one can conclude that all the off-diagonal elements of matrix $\mathbf{G}$ are non-positive. Hence, as shown in [13, 14], matrix $\mathbf{G}$ is called a Z-*matrix*. If $\mathbf{G}$ satisfies the condition in (16), then $\mathbf{G}$ is strictly diagonally dominant matrix. According to [13, chapter 6, Theorem 2.3], all principal minors of $\mathbf{G}$ are positive. Since $\mathbf{G}$ is a Z-*matrix*, according to [14, theorem 3.11.10], $\mathbf{G}^{-1}$ exist and all of its elements are non-negative. In addition, all the elements of vector $\mathbf{n}$ in (13) are non-negative, and thus $\mathbf{p}$ is lower bounded by $\mathbf{p}_{\min} = \mathbf{G}^{-1}\mathbf{n} \succeq 0$. The proof is complete.

Remark 1. Notice that if $\mathbf{p}_{\min}$ defined in (17) satisfies $p_{\min} \preceq \mathbf{p}_{\max}$ then $\mathbf{p}_{\min}$ is the optimal solution to the optimization problem (15).

4 Simulation Results

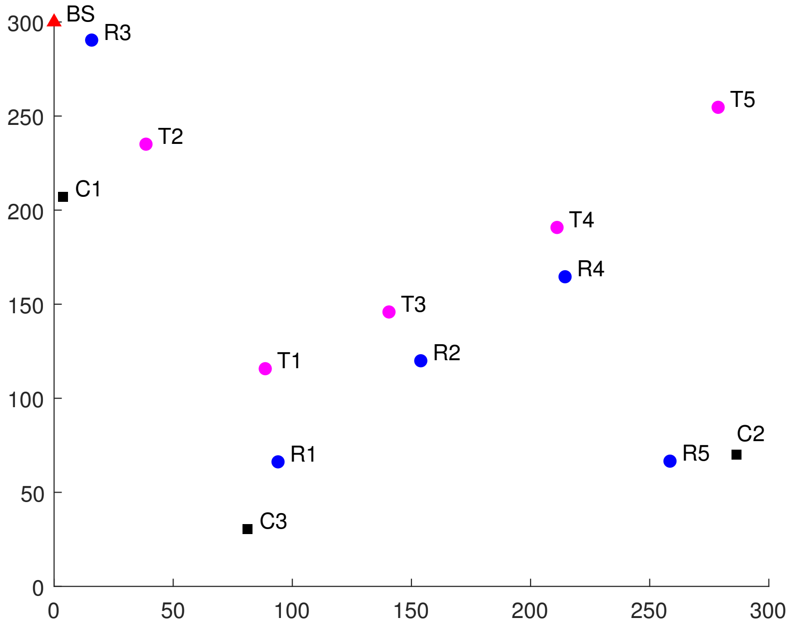

Fig. 2. A typical example of simulation model for a D2D underlaying cellular network consisting of a BS, 3 CUs, and 5 pairs of D2D users within an area of 300 m × 300 m.

In this section, we provide numerical results for the D2D underlaying cellular network. We implement the Monte Carlo method in MATLAB to evaluate the performance of the proposed power allocation algorithm. In the simulation, the nodes are located within an area of 300 m × 300 m, the pathloss exponent is set as $\alpha = 2$, the required SINR of all D2D users and CUs are equally set and varies as $\bar{\gamma}_i = \bar{\gamma}_{b_k} \in [-20, 5]$ dB, $\forall i = 1, 2, \ldots, N$, $\forall k = 1, 2, \ldots, K$, and the maximum transmit power is $p_i^{\max} = p_c^{\max} = 30$ dBm. It is assumed that BS is at the top left corner, i.e. $\{x_{BS}, y_{BS}\} = \{0, 300\}$ m, while the locations of other nodes, i.e. CUs and D2D users, are uniformly distributed in the range $[0, 300]$ m. Due to the requirement that mobile devices should be in short range for D2D communications, the distance between the D2D transmitter and D2D receiver are limited in $[d_{\min}, d_{\max}]$, where $10 \text{ m} \leqslant d_{\min} < d_{\max} \leqslant 50$ m. An illustration of the simulation settings is shown in Fig. 2 where 5 pairs of D2D users and 3 CUs are plotted with $d_{\min} = 10$ m and $d_{\max} = 25$ m.

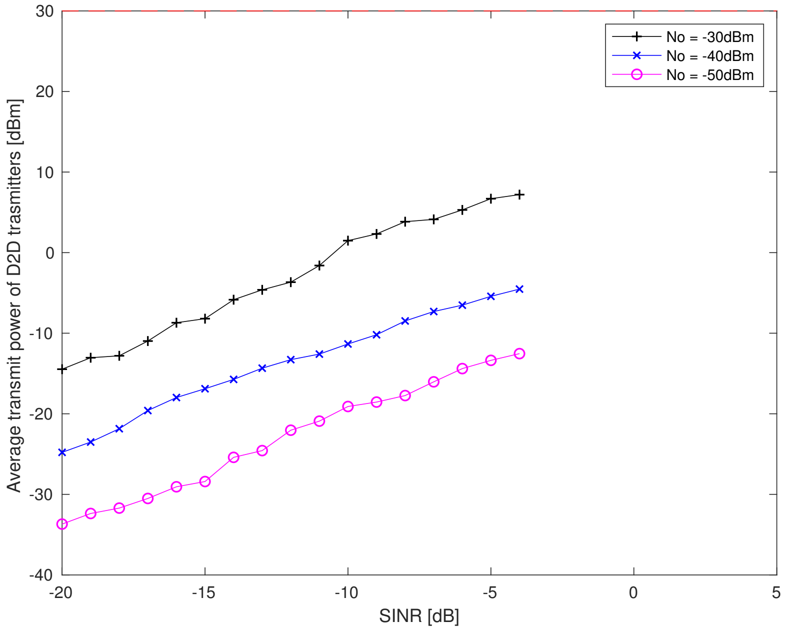

Fig. 3. The average transmit power of D2D transmitters versus required SINR with respect to different noise power.

Investigating the impacts of noise power, Fig. 3 shows the average transmit power of the D2D transmitters, i.e. $E[\mathbf{p}_{\min}]$ against the required SINR, i.e. $\bar{\gamma}$, with three scenarios of noise power $N_0 \in \{-30, -40, -50\}$ dBm. We considered five pairs of D2D users, i.e. N=5, three CUs and the distance between each D2D pairs and CUs are uniformly distributed in the range [10,25] and [30,90]m respectively. One can observe that the average transmit power increase with an increase in the SINR requirement, which follows the intuition that increased in noise power contribute to a higher transmit power. For instance, when the required SINR is -5 dBm, the average transmit power required is -12 dBm with noise power of -50 dBm compared to when the noise power -30 dBm with an average transmit power of 8 dBm. The fluctuating in the graph is due to the fact that the instantaneous SINR is considered over different fading generations, among which some cause the matrix **G** defined in (11) is not convertible, i.e. does not satisfy the condition in Lemma 1.

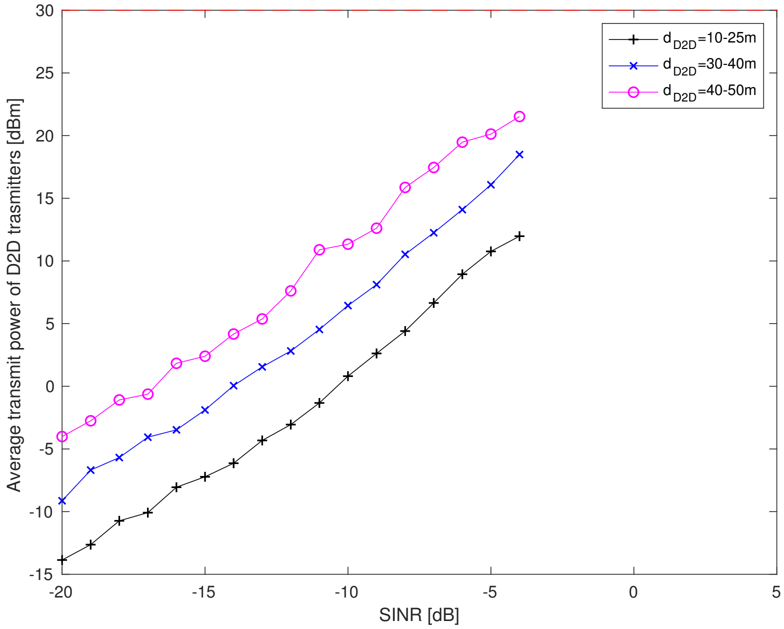

Fig. 4. The average transmit power of D2D transmitters versus required SINR with respect to distance between D2D users.

The impacts of the distance between D2D users are investigated in Fig. 4 where the average transmit power of D2D transmitters is plotted over the required SINR with respect to three cases of distance between D2D users, i.e. $\{[d_{\min}, d_{\max}]\} \in \{[10, 25], [30, 40], [40, 50]\}$ m. The noise power is fixed as $N_0 = -30$ dBm. There are five pairs of D2D users and their locations are similarly set as in Fig. 3. It can be observed that a higher transmit power is required with D2D users having distance within 40 m to 50 m compared to those with D2D users with shorter distance such as 10 m -25 m and 30 m to 40 m. This means that, the distance between the D2D users has a considerable impact on the average power at the D2D users.

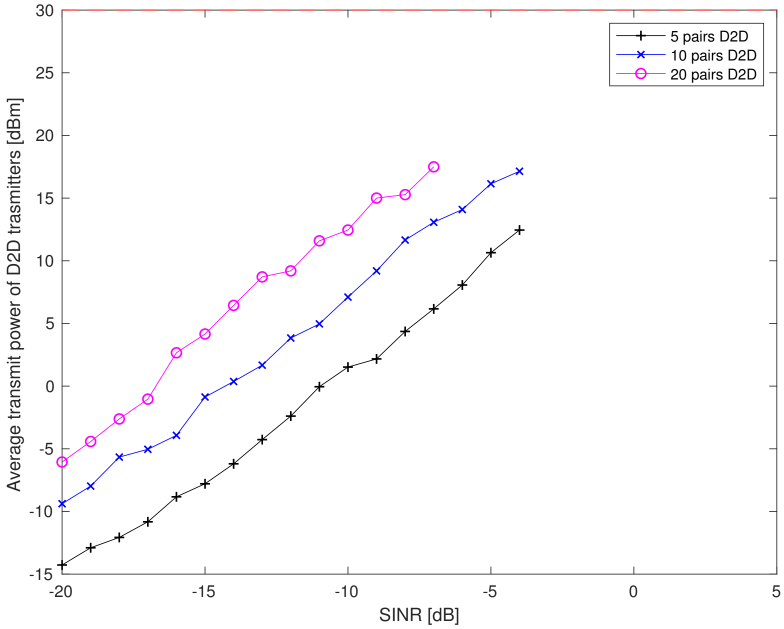

Fig. 5. The average transmit power of D2D transmitters versus required SINR with respect to the number of D2D users.

Taking into account the number of D2D users, Fig. 5 plots the average transmit power of D2D transmitters versus required SINR for three scenarios of the number of D2D pairs, i.e.$N \in \{5, 10, 20\}$. In this figure, the distance between D2D users and CUs are varied as in Fig. 3 in the range [10,20] and [30,90] respectively. The noise power parameter is set similarly as in Fig. 4, and the number of D2D pairs ranging from [5,10,20]. It can be observed that the proposed power allocation demonstrates that the number of D2D pairs has a significant impact on the transmit power due to the interference from both other D2D transmitters and other CUs.

Taking into account the impact of the number of CUs, Fig.6 plots the average transmit power of the D2D transmitter against the required SINR with respect to four scenarios of CUs (ranging from 1 to 4) with distance range $[0, 70]$ m. The parameter is set as in Fig. 3 i.e. the distance between D2D users $[10, 25]$m, and the number of D2D pairs N=5, the noise power is set similar to Fig. 4 $N_0 = -30$ dBm. It can be observed that there is much difference in the average transmit power required with four CUs compared to one CU due to the interference from other CUs and D2D transmitters.

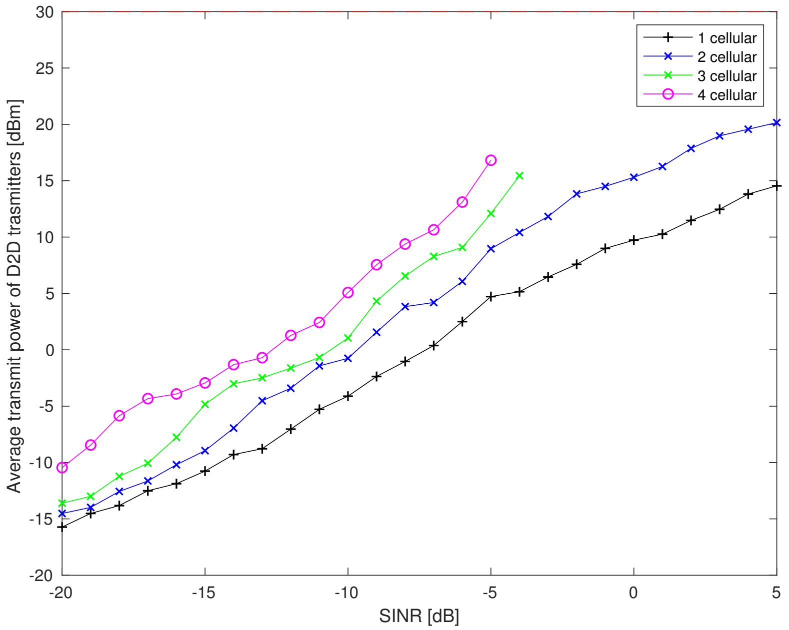

Fig. 6. The average transmit power of D2D transmitters versus required SINR with respect to the number of CUs.

5 Conclusion

In this paper, we have introduced a power allocation approach for D2D users over uplink in D2D underlaying cellular networks with multiple CUs. We have developed an optimization problem by incorporating the SINR constraints while maintaining the QoS of the system. An optimal power allocation has been proposed by taking into account the property of non-negative inverse of a Z-matrix. Moreover, the impact of the number of the CUs, noise power, the distance between D2D users and the number of D2D pairs have been evaluated for the considered system. Our simulation results have shown that deploying more number of either the CUs or D2D users has a significant impact on the transmit power of the D2D users due to the interference caused by both D2D and cellular communications. Also, the transmit power of the D2D users has shown to be dependent of not only their location but also the location of the CUs. For future work, we will consider a power control approach for an ultra-dense network with statistical modelling for multiple cells.

References

1. K. Doppler, M. Rinne, C. Wijting, C. B. Ribeiro, and K. Hugl.: Deviceto-device communication as an underlay to LTE-advanced networks. IEEE Commun **47**(12), 4249 (2009)
2. K. Ali, H. X. Nguyen, Q.-T. Vien, P. Shah, and Z. Chu, "Disaster management using D2D communication with power transfer and clustering techniques," IEEE Access, *6*, 14643-14654, (2018)
3. G. Fodor et al.: Design aspects of network assisted device-to-device communications. IEEE Commun. Mag **50**(3), 170177 (2012).
4. A. A. Omorinoye, and Q-T. Vien.: On the Optimisation of Practical Wireless Indoor and Outdoor Microcells Subject to QoS Constraints. Applied Science MDPI. *7*(9), 1-15, (2017).
5. X. Lin, J. Andrews, A. Ghosh, and R. Ratasuk.: An overview of 3GPP device-to-device proximity services. IEEE Commun. Mag., **52**(4), 4048 (2014).
6. P. Mach, Z. Becvar, and T. Vanek.: In-band device-to-device communication in OFDMA cellular networks: A survey and challenges. IEEE Commun. Surveys Tuts. **17**(4), 18851922 (2015)
7. G. Fodor, D. Della Penda, M. Belleschi, M. Johansson,and A. Abrardo.: A comparative study of power control approaches for device-to-device communication. IEEE International Conference on Communications (ICC). 6008-6013 (2013)
8. C.Yang, J.Li, P. Semasinghe, E. Hossain, S. M. Perlaza, and Z. Han.: Distributed Interference and Energy-Aware Power Control for Ultra-Dense D2D Networks: A Mean Field Game. IEEE International Conference on Communications (ICC). **16**(2), 1205-1217 (2017).
9. G. Zhang, J. Hu, W. Heng, X. Li, and G. Wang.: Distributed Power Control for D2D Communications Underlaying Cellular Network Using Stackelberg Game. IEEE Wireless Communications and Networking Conference (WCNC). 1-6 (2017).
10. Huang J., Xiong Z., Li J., Chen Q., Duan Q.,and Zhao Y.: A Priority-Based Access Control Model for Device-to-Device Communications Underlaying Cellular Network Using Network Calculus. Cham, Switzerland Springer int. 613-623 (2014).
11. H. ElSawy, E. Hossain, and M.S. Alouini.: Analytical Modeling of Mode Selection and Power Control for Underlay D2D Communication in Cellular Networks. IEEE Transactions on Communications. **62**(11), 4147-4161 (2014).
12. A Omorinoye, Q. Vien, and T.A. Le, and P. Shah.: On the Resource Allocation for D2D Underlaying Uplink Cellular Networks. IEEE ICT Proceedings (2019).
13. A. Berman and R. Plemmons.: Nonnegetive Matrices in the Mathematical Science. Society for Industrial and Applied Mathematics.(1994)
14. R. Cottle, J. Pang, and R. Stone.: The Linear Complementarity Problem. Society for Industry and Applied Mathematics. (2009)
15. A. Attar, M. R. Nakhai, and A. H. Aghvami.: Cognitive radio game for secondary spectrum access problem, IEEE Transactions on Wireless Communications, **8**(4), 21212131 (2009)

16. T. A. Le and M. R. Nakhai.: Downlink optimization with interference pricing and statistical csi, IEEE Transactions on Communications, **61**(6), 23392349 (2013)