Theory of Price Formation in Experimental Markets

By

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Based on Joint Work with Vernon L. Smith

A Context Statement

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Abstract:

In this context statement I will overview joint theoretical work on competitive market price formation inspired by our reappraisal of experimental market findings and the classics of value theory.

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Finally, I dedicate this work to my Parents, to whom my warmest gratitude goes.

1 Introduction

Laboratory market experiments conducted mid-1950s established the stability and efficiency of competitive markets organized notably under the double-auction trading institution (V. L. Smith, 1962, 1964, 1965). These experimental findings (reviewed, e.g., in Davis & Holt, 1993; Holt, 1995; Plott, 1982; V. L. Smith, 1982; V. L. Smith & Williams, 1990), replicated many times around the world (Lin et al., 2020), challenge core tenets of neoclassical value theory (the requirements for a large number of traders, market clearance, complete information of supply and demand, and most notably passive price-taking trader behavior, which bypasses the fundamental problem of price discovery). But while the stability and efficiency of laboratory markets are robust to various supply and demand conditions, they do not hold for goods that can be re-traded for capital gains (Dickhaut, Lin, Porter, & Smith, 2012), for then the stabilizing virtue of competition is counteracted by speculation.

In retrospect, the laboratory results called for a new theory of competitive markets rooted in the old, classical, view of competition as a collective higgling and bargaining process, founded on reservation prices (an operational substitute for the utility function), as the authors' recent reappraisal of the experiments and the history of value theory suggests (Inoua, 2018a, 2018b; Inoua & Smith, 2020a, 2020b, 2020c, 2021b, 2022a, 2022b, 2022c; V. L. Smith & Inoua, 2019).[1] The fundamental principle of this theory says that a competitive market evolves to minimize the potential surplus available to traders (which reduces transaction after transaction, as the traders compete to extract the most of it). As it applies to a lab market context, this principle

[1] One of the most recent references, Inoua and Smith (2022c), is itself an overview of the theory as articulated around the distinction between perishable goods and retradable assets. This Context Statement, a more general summary of our work, follows closely the structure and content of sections in that chapter, forthcoming in the *Handbook of Experimental Finance* [Sascha Füllbrunn and Ernan Haruvy (eds), Edward Elgar Publishing]. I intend to make this Context Statement as self-contained as possible, hence provide more background materials and mathematical details all along.

can be interpreted in financial terms: the experimenter distributes to subjects, value and cost units that are *trade options*, identified with their payoffs and constituting a cost from the experimenter's viewpoint. The minimum potential surplus principle then says that the total cost of realizing a competitive equilibrium in the lab is minimized. Thus, a lab market involving trade options on an abstract (intrinsically valueless) commodity representation is isomorphic to a classical market, that is, one in which consumers (producers) are willing to buy (sell) units of an actual commodity to which they assign use-values (production costs), at prices below (above) their use-values (costs), these transaction prices emerging from the traders' collective higgling and bargaining, or competition (formally, buyer-buyer outbidding, seller-seller underselling, and buyer-seller haggling). Now, instability emerges when each trade option is supplemented with a *speculative retrade option* (the option to resell a unit for capital gains, implicit in the purchase of most financial assets), also identified with its payoff.[2]

In this Context Statement I will describe the price theory inspired by this reappraisal of the experiments and the classical literature on value theory. We start by briefly situating the theory in a broad perspective of history of ideas (Section 2), reexamining the history of value theory from Adam Smith to Jules Dupuit, and its replacement in the 1870s by the neoclassical school (Inoua & Smith, 2020a, 2020b, 2020c, 2022b; V. L. Smith & Inoua, 2019). Then we will present the mathematical theory of price formation (Sections 3 and 4) as it applies to a single-market (partial-equilibrium) competitive dynamics, only briefly mentioning in Section 2, elements for a multiple-market (general-equilibrium) extension, developed in greater details in a forthcoming book (Inoua & Smith, 2022a, ch. V).

[2] The concept of speculative retrade option is anticipated in Haruvy, Noussair, and Powell (2014, p. 686).

More specifically, we will focus on the following core themes of the theory and its methodological implication as regard the link between the lab and the field markets:

1. A mathematical formulation of competitive dynamics suggested by the standard experiments involving a perishable good (Subsection 3.3).
2. The problem of path dependency of competitive equilibrium (Subsection 3.4).
3. An illustration of the theory as it applies to the double auction (Subsection 3.5).
4. Sketch of a classical induced value theory (Subsection 3.6).
5. A special treatment to deal with speculation as suggest the retrade experiments (Section 4).
6. Conclusion: a summary along with a partial list of unexplored aspects of the theory that call for future theoretical and experimental research (Section 5).
7. An Appendix sketching a more general probabilistic formulation of the model (the subject of ongoing research) that partially answers some of the unexplored aspects of the theory mentioned in the Conclusion.

2 Background: Situating the Theory in the History of Economic Thought

The theory's foundations are classical in that we build on the classical economists' intuitive exposition of the mechanics of supply and demand (notably Adam Smith in *Wealth of Nations* 1776 [1904], Chapter VII of Book I), although the classical articulations of price formation tended to focus on long-run, natural, value (cost of production) and the old view on competition is often eclipsed in the old treatises by obscure controversies on value theory pertaining to the invariant measure of value and the ultimate cause of value (Inoua & Smith, 2020a).

The demand side of the old view of markets is more explicit in a relatively unknown French and Italian tradition inspired by Adam Smith (Inoua & Smith, 2020c), notably Germain Garnier (1796 [1846]), Jean-Baptiste Say (1828 [1836], e.g.), Jules Dupuit (1844, 1849), Pelligrino Rossi (1840 [1865]), but also Augustin Cournot (Cournot, 1838 [1897]), whose view on demand is classical, although on supply theory, he went on to inaugurate the foundation of neoclassical theory based on price-taking behaviors.

The old conception of competition culminated in the Austrian tradition on price theory (Böhm-Bawerk, 1888 [1891]; Menger, 1871 [1950]; von Wieser, 1889 [1893]), which insisted on the centrality of marginal utility in value theory, yet offered a general articulation of competitive market price formation based on traders' valuations (see, e.g., Böhm-Bawerk, 1888 [1891], Book IV) and classical competition, a trend that would be overshadowed, however, by the Jevons-Walras approach based on the utility function.

Alfred Marshall (1890 [1920]) formalized the methodology implicit in the old view of competition, emphasizing the centrality of the concept of reservation price in it; but in his attempt to reconcile the old view with the new-born marginalist doctrine, he lost a fundamental aspect of the old concept of reservation price as a critical, limit, price, the maximum (minimum) price a buyer (seller) would be willing to pay (accept) in exchange for a unit of a commodity:

this concept of reservation price (which Marshall replaced with that of marginal willingness to pay or accept) is crucial for the definition of supply and demand as distribution functions, as emphasized formally below in the mathematical theory (Section 3).

Throughout the neoclassical tradition, the concept of surplus is commonly relegated to the status of an approximate welfare measure, a proxy for the utility function in partial equilibrium analysis, whenever wealth effects are negligible (e.g., when consumer preferences are quasi-linear). In other words, the concept of reservation price, or surplus, plays at best a secondary role in economic theory, and was dismissed or downplayed early, during the marginal revolution, notably by Walras (1874, Lesson 61) in his critique of Dupuit's view on demand, and, inadvertently, by Marshall, through his famous clause of constant marginal utility of wealth (Marshall, 1890 [1920]). This neoclassical view on the concept of surplus is not correct upon scrutiny, however, as our reexamination of the old tradition on supply and demand suggests (Inoua & Smith, 2020b, 2020c, 2022b): the concept of reservation price offers in fact an alternative foundation of supply and demand that is as general as that of the utility function; and it offers, moreover, the additional advantage of allowing a natural formulation of competitive dynamics, as we will emphasize below. In a general-equilibrium context, for example, the consumer wealth, viewed as the maximum amount the consumer would be willing to pay for all units of all goods needed, plays the same role as that of the reservation price for a unit of good in partial equilibrium. A hint to a multiple-market extension of the reservation price is thus straightforward: for the demand side, for example, suffice it to notice that a budget constraint $w \geq q \cdot p$ and a realistic rationality on the quantity needed q (mostly given in practice) defines for each commodity, a reservation price that is depends not only on wealth but also the price of related goods. For a general-equilibrium extension along these lines, see, again, Inoua and Smith (2022a, ch. V).

Although the classical school was displaced in the 1870s by the marginal revolution, our intuitive understanding of the price mechanism in a competitive market (and the standard equilibrium analysis and comparative statics based on the supply and demand curves as given, as well explained in the modern introductory microeconomics textbooks) owes mostly to the old intuitions on competitive market price formation.[3] Thus any introduction to disequilibrium dynamics invokes traders' outbidding or underselling one another; yet these intuitions are mostly lost in the next step, consisting of deriving the supply and demand curves from passive price-taking trader behaviors, a negation of actual competitive behaviors introduced by Cournot (1838 [1897], ch. VIII).

Following Cournot's typology of markets in terms of the number of traders, the marginal revolution in the 1870s replaced the old view on competition by the axiom of price-taking behavior as a foundation for utility and profit maximization. It followed an inherent difficulty of formalizing competitive price formation in the neoclassical theory, which turns prices into exogenous parameters constraining trade rather than emerging from it. Here lies the core logical gap of neoclassical price theory. Jevons filled the gap by postulating a "theoretically perfect market" in which every trader has complete information on supply and demand and the consequent equilibrium price (Jevons, 1871 [1888], p. 87); Walras, realizing the difficult path-dependency problem emphasized by Bertrand (1883, p. 505), added the trade suspension clause to his initial formulation of tatonnement price adjustements (1874, e.g., Lesson 48), thus setting the stage for the modern interpretation of tatonnement as a virtual adjustment process executed as if by a fictional auctioneer.

[3] It is important to inisit on this point: that our basic intuition about supply and demand analsysis is not called into question by the conceptual difficulties of utility theory.

Despite a few able attempts at articulating more realistic price and trade formation processes, the fundamental problem of price discovery remains essentially unsolved in the neoclassical, Jevons-Wlaras tradition, despite its prevalence in modern economics. As states a review of the nontatonnement models: “we shall have to conclude that we still lack a satisfactory descriptive theory of the invisible hand.” (Hahn, 1982, p. 746) More recently: “we do not have an adequate theory of value, and there is an important lacuna in the center of microeconomic theory. Yet economists generally behave as though this problem did not exist.” (Fisher, 2013, p. 35)

Since the price-taking utility and profit maximization axiom is the root of the inherent difficulty of a neoclassical theory of competitive price discovery, one may try to build on the older tradition, a modern theory of competitive price founded on reservation prices as an alternative to the utility function. In retrospect, it is the early market experiments (Chamberlin, 1948; V. L. Smith, 1962) that laid the foundations for such revival of the old view of competition and for an alternative theory of competitive markets.

3 Theory

3.1 Motivation: From Institution to Competition

One of the most important contributions of the experimental research on market behavior vis-à-vis standard, abstract, neoclassical theorizing, is to emphasize the centrality of a market institution, the set of rules organizing trade in a market. Thus, unlike the original, experiments involving isolated trades (Chamberlin, 1948), the double-auction experiments (V. L. Smith, 1962) achieved competitive equilibrium only by adopting more realistic trading rules: the bid-ask continuous double auction protocol inspired by the rules of trades in the Chicago Commodity and the New York Stock Exchanges (Leffler, 1951). The pivotal role of institutions is reinforced by experiments comparing them, especially the double-auction versus the posted-offer ones (e.g., Ketcham, Smith, & Williams, 1984; Plott & Smith, 1978; V. L. Smith, 1964, 1976; V. L. Smith, Williams, Bratton, & Vannoni, 1982), or even by experiments comparing variations of the double auction (V. L. Smith, 1976) in terms of efficiency and speed of convergence to competitive equilibrium. The superior efficiency of the double auction (particularly the variant enforcing bid-ask reduction and rank-queue rules) stimulated a great deal of research on that specific institution, motivating also models of price equilibration in markets organized by these rules (Anufriev, Arifovic, Ledyard, & Panchenko, 2013; Asparouhova, Bossaerts, & Ledyard, 2020; Cason & Friedman, 1996; Friedman, 1991; Gjerstad & Dickhaut, 1998; Wilson, 1987). The central hypothesis in the experimental literature seems to be therefore: *the laboratory regularities (convergence and efficiency results) are shaped by the specific trading institution*, notably the double auction; or at least these regularities are not due to a superior rationality of the traders in that institution, as the striking market simulations involving zero-intelligence traders prove (Gode & Sunder, 1993).

Yet other institutions such as the call market, posted offer, or even a variation of Chamberlin's "pit market" enforcing publicity of all transaction prices (Holt, 2019, Section 2.1; List, 2004) also tend to converge to competitive equilibrium. All these institutions, and the double auction especially, are fundamentally different ways of organizing trade in a *competitive market*. This suggests a more unified theoretical approach (complementary to an institution-specific one) in which competition plays a more fundamental role than institution. It turns out that such theory is suggested by the above-described old view of a competitive market as a collective higgling and bargaining process, whose mathematical characterization was hinted, moreover, in the early experiments under the name of "excess-rent hypothesis", the "hypothesis found to be most successful in these experiments." (V. L. Smith, 1962, p. 112)[4] This hypothesis, as we generalize it, is the core principle of our theory of competitive markets, described in the next Subsections. It is a generalized law of supply and demand.

None of the implications of the theory holds, however, under the standard axioms of neoclassical theory, notably the paradoxical definition of a competitive market as one involving a "large number" of passive price-taking traders, and the consequent supply and demand concepts (optimal quantity choices at given prices). Rather we take competition, in its intuitive sense, taken for granted throughout the classical school.[5] Formally, competition means buyer-buyer outbidding, seller-seller underselling, or buyer-seller haggling; and a market is competitive if at least one of the three forms of competition operates in it. It is a good exercise

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[4] The hypothesis was inspired by an idea going back to Samuelson (1952), an equivalent minimum principle based on the integral of excess demand. But Samuelson saw in this minimum principle no special economic interpretation, let alone a fundamental principle of value theory, but merely invokes it as an artefact for computing equilibrium, and not as it would be used to explain equilibrium convergence pattern in the experiments (V. L. Smith, 1962).

[5] More precisely, the classical economists usually mention only two forms, competition on the side of buyers versus that on the side of sellers (see, e.g., A. Smith, 1776 [1904]: 1:58-59).

to classify market institutions in terms of the competition forms they allow or forbid: the double-auction institution is unique by allowing all of the three forms of competition; Chamberlin's original market simulations involve mostly isolated buyer-seller haggling, which in an extreme case of complete privacy of negotiated prices would amount in all rigor to a collection of isolated markets, unless publicity of the going transaction prices is enforced, in which case we can treat the individual trades as participating to a unique, organized, market; the English auction involves only buyer-buyer competition; the posted-offer institution, mostly seller-seller and buyer-seller competition (the buyers may refrain from buying if the posted price is judged too high, leading the seller to lower the price). The double auction is superior not merely competitively (by the intensity of the competition it allows), but also informationally (by the richness of its message space—the "bid", "ask", and "accept" messages—and the accumulation of this information throughout a market session).

It seems natural therefore to build a general price theory centered on competition and information, which varies across institutions in terms of intensity. Any market is governed by a mix of:

1. Competition: a combination of the three forms.
2. Institution: from custom (as in a casual, informally regulated, haggling) to the double auction (with more complex and explicitly codified trading rules).
3. Regulation: for example, price controls (treated as constraints on competitive price).[6]

[6] A regulatory constraint on transaction price can be written generically as $g(p) \leq 0,$ where g is usually an affine function for State intervention: thus a price control setting a floor $p \leq p_{\min}$ corresponds to $g(p) = p - p_{\min}$, whereas a price ceiling constraint $p \geq p_{\max}$ corresponds to $g(p) = p_{\max} - p.$ Therefore the theory readily generalizes to constrained competitive price dynamics, in view of the linear-programming formation of the theory: see Propositions 2 and 3 in Subsection 3.4 below. Such generalization is so far only explored

So, a general theory of competitive markets would be centered on competition as unifying principle, *a theory that is in essence a qualitative (nonparametric) description of competitive markets*, however organized. For such theory to be general, it must be based on concepts that apply to all markets: thus, as regards price (which can take various forms depending on the institution: posted price, bid, ask, and so on), such theory would be based on the *transaction price*. An institutional rule (e.g., the bid-ask reduction one), or a treatment, is theoretically treated by the extent to which it fosters or hinders the intensity of competition, as reflected by the speed of convergence to competitive equilibrium. The theory is to be specialized parametrically for each institution, to reflect notably the specific informational intensity of the trade and quote message accumulation that the institution allows. In what follows, we overview a candidate theory along these lines.

Before we come to the general formulation of the theory, it may be useful to start with a specific example, going into various details that may not be easy to emphasize from the general, abstract, formulation.

3.2 Heuristic Example of Price Formation[7]

passingly, however, in our work, and is hinted in terms of the notion of frictions against an otherwise "purely competitive dynamics" in Inoua and Smith (2021b, Definition 3).

[7] Based on Inoua and Smith (2022a, Ch. 3, Sec. 3.1).

Consider a market in which units of a certain commodity (a good or a service) are traded. Let m be the total number of units that can be supplied by all sellers in the market (the total production capacity) and let n be the total number of units that buyers would be willing to buy (total purchase capacity).

1. Each trader may trade multiple units; thus, multiple unit values or costs may be associated with the same trader (although most commonly in consumer markets each buyer requires a single unit, and units are vended one unit per transaction).[8] The convention throughout this book shall be to list values and costs individually, as in the example above, rather than re-grouping them according to the identity of their holders: this is the natural and convenient convention mathematically, given that we shall deal with all units collectively as one distribution. (There is no point in isolating and regrouping multiple units according to the identity of their holders anyway, at least because these units might be acquired at different transaction prices or even involve different trading counterparties.) Thus, we consider:

- The distribution of costs $\{c_1, \ldots, c_m\}$, ranked by increasing order.
- The distribution of values $\{v_1, \ldots, v_n\}$, ranked by decreasing order.

Costs	3	5	8	10	15
Values	12	8	7	5	2

Table 1. Example of values and costs.

2. Traders' actions are guided by a profit motive. A trade involving the matching of a buyer with a seller can be identified by the value and cost involved, abstractly (v_i, c_j). Let p_{ij} be the price at which a trade (v_i, c_j) is concluded if trade occurs between this pair. Trades are voluntary. A person would be willing to trade only if there is a gain from trade. This imposes a constraint on the possible trading price: for any trade (v_i, c_j), we must have $c_j \leq p_{ij} \leq v_i$. If

[8] A buyer's shopping basket contains a quart of milk, a pound of butter, a pound of bacon and so on.

we assume that all trades occur as random isolated transactions (isolated value-cost pairings), then we cannot in all rigor treat these trades as forming a unique market, but rather should treat each as an isolated "higgling and bargaining" market on its own right. Every price in the interval $[c_j, v_i]$ is a possible competitive equilibrium price of the isolated exchange involving (i, j), as a basic supply-demand diagram would reveal (Figure 1).[9]

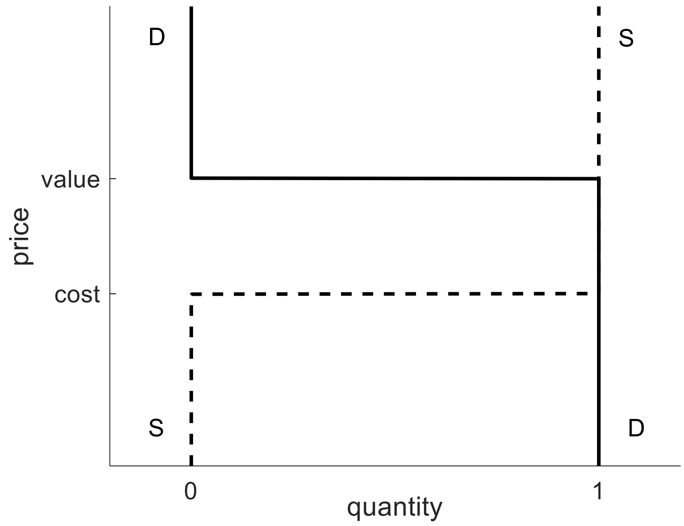

Figure 1. Supply and Demand functions for an Isolated Exchange.

3. Assume, in contrast with this isolated bilateral matching, a hypothetical market in which trade is organized according to trading rules applying to all transactions, and by means of which, all traders can potentially interact and compete. Then we can treat all values and costs as participating contiguously in a unique organized market. We know there cannot be more trades than there are values or costs; thus, we are led to truncate the distribution of reservation prices to the trade-relevant portion. The buyer-seller pairing depends on the specific trading institution.

[9] The interval being closed implies that a seller (buyer) is willing to vend at her cost (pay his value) rather than forego a trade. Consequently the $[c_j, v_i]$ include a profit sufficient to induce a trade at either limit.

4. Competition. Assume the trading institution allows for the following two forms of competition: (1) buyer-seller negotiation narrowing the spread between revealed buyer bid and seller ask, depending on who, among all the buyers and sellers, is the more eager or patient in the haggling; and (2) buyer-buyer outbidding and seller-seller underselling, according to which higher-value buyers outbid lower-value buyers, so that they can trade before (or to the exclusion of) these latter; likewise, lower-cost sellers undersell (that is, submit lower asks than) higher-cost sellers so that they can trade to the exclusion of these latter. Now suppose, hypothetically, that we imagine trades occurring in accord with a strict priority rule. Thus, highest-value buyer trades first, followed by the second-highest value buyer, and so on; similarly, the lowest-cost seller trades first, followed by the second-lowest seller: trades occur in sequence of increasing costs and decreasing values. (This is a simple assumption about the implication of competition: below and in Chapter 4 is presented a general formulation.) Let T be the maximum number of trades that can take place $[T \leq \min(m,n)]$, and consider the first T values and first T costs arranged as follows: costs on the left, by increasing order; values on the right, by decreasing order, assuming no tie to simplify the discussion:

$$c_1 < c_2 < ... < c_T \qquad v_T < ... < v_2 < v_1.$$

The transactions can now be denoted simply (c_t, v_t), and the transaction price p_t, *t*=1,…,*T*. Under the above strict competitive trade priority rule, we have the following sequence of trades: $(c_1, v_1) \to (c_2, v_2) \to \cdots (c_T, v_T)$, where the lowest-cost unit is sold to the highest-value buyer, then the second-lowest-cost unit sold to the second-highest-value buyer, and so on. Thus the standing transaction price p_t will end up somewhere in the set $[c_T, v_T]$. The important point is that, round after round, transactions involve units nearer and nearer to the middle, or center, of the distribution of values and costs, which is $[c_T, v_T]$.

5. The fundamental principle. Every step in this heuristic illustration of competitive market price formation has a natural distributional formulation: (a) ranking of costs from lowest to highest means we are considering the (cumulative) *distribution function of unit costs*; (b) ranking values from highest to lowest means we are considering the (complementary) *distribution function of unit values*; (c) the movement of the transaction price, round after round, toward the middle of the distribution of values and costs, means convergence to a *median* of the distribution of reservation values. Let the distribution of values and costs be written as a vector $\mathbf{r} = [r_1, ..., r_{m+n}]^{\mathrm{T}}$, where r_h denotes a reservation price generically considered, whether it is a buyer value or a seller cost.[10] That is:

$$\mathbf{r} = [v_1, ..., v_n; c_1, ..., c_m]^{\mathrm{T}},$$

We know from statistics that a median is a best summary of a distribution, here $\mathbf{r}$, in the sense of minimizing the distance function

$$p \mapsto \sum_h | r_h - p | .$$

But, at every step, the distribution of reservation prices is being truncated to exclude traders that cannot afford the standing price (the profit motive), because they are outbid or undersold by a rival. Thus, in modeling competitive price dynamics, we need to define a concept that associates to any standing transaction price p_t, the number of units involved in the competition (those that can afford the standing price), which we write as $D(p_t)$ and $S(p_t)$. These are by construction, the number of units that traders can afford to trade, the number of which is given by the (nonnormalized) cumulative distribution functions of costs and values, defined as

[10] Throughout this book, the superscript "T" denotes vector or matrix transposition, following standard notation.

$$D(p) = \#\{i : v_i \geq p\} = \sum_{i=1}^{n} I(v_i \geq p), \tag{1}$$

$$S(p) = \#\{i : v_i \geq p\} = \sum_{j=1}^{m} I(c_j \leq p), \tag{2}$$

which use the indicator functions:

$$I(v_i \geq p) = \begin{cases} 1, v_i \geq p, \\ 0, v_i < p, \end{cases} \tag{3}$$

$$I(c_j \leq p) = \begin{cases} 1, c_j \leq p, \\ 0, c_j > p. \end{cases} \tag{4}$$

(We will also at times denote the indicator fucntion as $\mathbf{1}_{\{\cdot\}}$.) Using traditional terminology, the distribution functions (1) and (2) can be interpreted as demand and supply functions; and the indicator functions (3.3) and (3.4), as demand and supply units. Consequently, the fundamental summary property of competitive market price dynamics is reflected in the function

$$V(p) = \sum_i | v_i - p | I(v_i \geq p) + \sum_j | c_j - p | I(c_j \leq p). \tag{5}$$

To make transparent the fact that we are fundamentally dealing with one distribution (of reservation prices) expressed in two partitions (the supply side and the demand side), one can rewrite (5) as

$$V(p) = \sum_h | r_h - p | a_h(p), \tag{6}$$

where we introduce the affordability or profitability indicator function

$$a_h(p) = \begin{cases} I(v_h \geq p), h \in \{1, \ldots, n\}, \\ I(c_h \leq p), h \in \{n+1, \ldots, m+n\}. \end{cases} \tag{7}$$

We shall refer to the function V as the *price-value distance*, where value means here both buyers' values and sellers' costs (costs being the sellers' valuations, more precisely).

Competitive dynamics, as the heuristic derivation above seems to suggest intuitively, can be characterized by the general principle:

$$V(p_{t+1}) \leq V(p_t), \tag{8}$$

whereby the tansaction price evolves, transaction after transaction, to reflect the reservation prices better and better. It turns out this dynamic inequality is more than just an intuition: it is the fundamental principle of value theory as understood in this book. It generalizes and reinterprets a hypothesis suggested empirically by the analysis in the early market experiments (V. Smith, 1962, 1965). It is a natural formulation of competitive market dynamics (provided that market “competition” is taken in its ordinary and familiar sense of “rivalry” of buyers and sellers, as it used to be understood in the classical tradition, and not in the paradoxical sense of “price-taking behavior” that it became during the marginal revolution). This can be established through a more general formulation of competitive dynamics.

3.3 The Competition Principle[11]

With the emphasis on competition, one sacrifices some institutional specifics and parameters for a general principle. Then, as it turns out, one can characterize a single-market price mechanism as convergence to competitive equilibrium defined (more generally than market-clearing price) as the minimum of the total potential surplus available to all buyers and sellers in a market at any arbitrary standing transaction price p and defined by the function:

$$V(p) = \sum_{v \geq p} (v - p) + \sum_{c \leq p} (p - c), \tag{9}$$

[11] The motivation of the competition principle follows closely, while extending it, Inoua and Smith (2022c, Section 2).

where the sum symbols denote summation of all profitable value and cost units, namely all values $v \geq p$ and all costs $c \leq p$. It is easy to see that the potential surplus, viewed as a function of transaction time, decreases as trade unfolds (with transacted units withdrawing) and through competition (buyer-buyer outbidding tends to raise the standing transaction price, reducing therefore potential buyer surplus; seller-seller underselling tends to lower the standing transaction price, thus reducing potential seller surplus; and buyer-seller haggling means conceding potential surplus to the counterparty, to come to a mutually advantageous transaction price). Thus, competitive dynamics can be characterized by the following principle: *if a transaction price sequence* $\{p_t : t = 1,...,T\}$ *emerges from the competition of traders, then it corresponds to a non-increasing sequence* $\{V(p_t) : t = 1,...,T\}$ *of potential surplus:*

$$V(p_{t+1}) \leq V(p_t),\ t = 0,1,...,T-1. \tag{10}$$

Figure 2 illustrates the minimum potential surplus, or competition, principle.

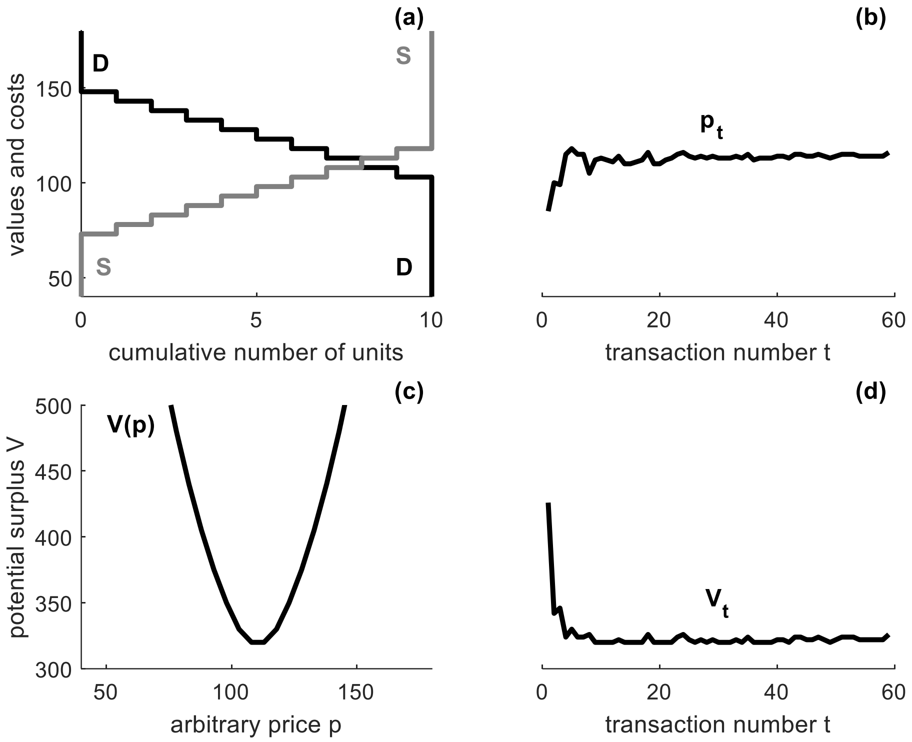

Figure 2. The Minimum Potential Surplus Principle illustrated using lab data: (a) supply and demand (value and cost distributions); (b) transaction price dynamics; (c) potential surplus function V; (d) dynamics of potential surplus V. Data source: Ikica, Jantschgi, Nax, Nunez Duran, and Pradelski (2021, "Same" treatment).

The potential surplus function is therefore a fundamental concept of value theory. In its form (9), it is the classical version of the famous "area under the supply and demand curve", formally an integral of excess supply more commonly known as consumers' and producers' surplus since Alfred Marshall (1890 [1920]), but which is more accurately a "virtual surplus" (V. L. Smith, 1962, Section V; 1965), or better yet, a potential surplus (as opposed to realized surplus, introduced in Subsection 3.4), since supply and demand are merely willingness to trade, not necessarily fulfilled into actual trade and surplus:

$$V(p) = \int_0^p S(x)dx + \int_p^\infty D(x)dx, \tag{11}$$

where S and D denote the supply and demand functions. The potential surplus function measures the maximum monetary payment that the subject-traders would earn, were all transactions to occur at a price p. From the viewpoint of the experimenter, it measures therefore the overall potential cost (or rent) of implementing a competitive equilibrium in the lab. (More on this cost interpretation in Subsection 3.6.) As announced earlier, the minimum potential surplus principle is a generalization and reinterpretation of a characterization of convergence to equilibrium in the early experiments, then referred to as the “excess-rent hypothesis” (V. L. Smith, 1962, Section V). Interpreted from the viewpoint of the experimenter, it uncovers an emergent rationality of the market in the form of a minimum principle: the experimental subjects unthinkingly minimize the monetary cost of them collectively realizing a competitive equilibrium in the lab.[12]

The simplest formulation of the minimum principle follows if we assume smooth supply and demand curves (as we would get approximately in a market that involves a sufficiently large number of dispersed values and costs). Then it is easily seen that the potential surplus function is indeed an integral of excess supply:

$$\frac{dV}{dp} = S(p) - D(p). \tag{12}$$

Therefore, it is minimized at a market-clearing price in this specific smooth case, provided the excess supply function is upward sloping. Moreover, by the chain rule:

$$\frac{dV}{dt} = \frac{dV}{dp}\frac{dp}{dt} = [S(p) - D(p)]\frac{dp}{dt}.$$

Hence a decreasing potential surplus

[12] “I think the formulation of competitive market equilibrium as a rent minimization problem makes the "Invisible Hand" distinctly more visible and more teleological.” (V. L. Smith, 1962, p. 133) “The discovery of the excess-rent hypothesis draws me nearer to the camp of "Invisible Hand" enthusiasts, but only because of the greater visibility of the Hand.” (Footnote 18).

$$\frac{dV}{dt} \leq 0 \tag{13}$$

is equivalent to the law of supply and demand (which holds that a competitive price change has the same sign as the corresponding excess demand). More generally, the potential surplus function has all the nice properties of a Lyapunov function associated with a competitive price dynamics:

Proposition 1 *(Properties of the Potential Surplus Function). The function V is an integral of excess supply and a convex Lyapunov function for a competitive market, provided the price evolves continuously. Besides continuity and convexity, more specifically, the function satisfies the following properties:*

$$V(p) = V(0) + \int_0^p [S(x) - D(x)]dx. \tag{14}$$

$$V(p) \to \infty,\ |p| \to \infty. \tag{15}$$

$$V(p_{t+1}) \leq V(p_t). \tag{16}$$

$$V \geq 0. \tag{17}$$

Proof. See Inoua and Smith (2021b, Lemma 1).

The function V being a continuous and convex function with the additional property that $V(p) \to \infty$ as $|p| \to \infty,$ it always has a nonempty minimizing set

$$C \equiv \arg\min_{p\in\mathbb{R}} V(p), \tag{18}$$

provided the set $[\min(c), \max(v)]$ is nonempty. We call this set the *center of value*. It is the relevant competitive equilibrium concept in a lab market, extending the market-clearance equilibrium concept. Experimental demand and supply being the cumulative distribution functions of values and costs, respectively, which are (discontinuous) step functions, the smooth case holds approximately only for “large market”. In practice, we have discrete value and cost distributions.

Formally, consider a market involving $m+n$ reservation prices, with m cost units and n value units (where multiple units may belong to the same trader):

$$\mathbf{r} = [c_1, \ldots, c_m;\ v_1, \ldots, v_n],$$

which we write generically as $\mathbf{r} = [r_u]$. To fix ideas, consider the following example in Table 1, which corresponds to

Costs	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Values	3.25	3.00	2.75	2.50	2.25	2.00	1.75	1.50	1.25	1.00	0.75

Table 2. Example of value and cost distributions (V. Smith, 1962, Chart 1).

The cumulative value and cost distribution functions define the supply and demand functions:

$$D(x) = \#\{i : v_i \geq x\} = \sum_{i=1}^{n} \mathbf{1}_{\{v_i \geq x\}}, \tag{19}$$

$$S(x) = \#\{j : c_j \leq x\} = \sum_{j=1}^{m} \mathbf{1}_{\{c_j \leq x\}}, \tag{20}$$

where $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function.

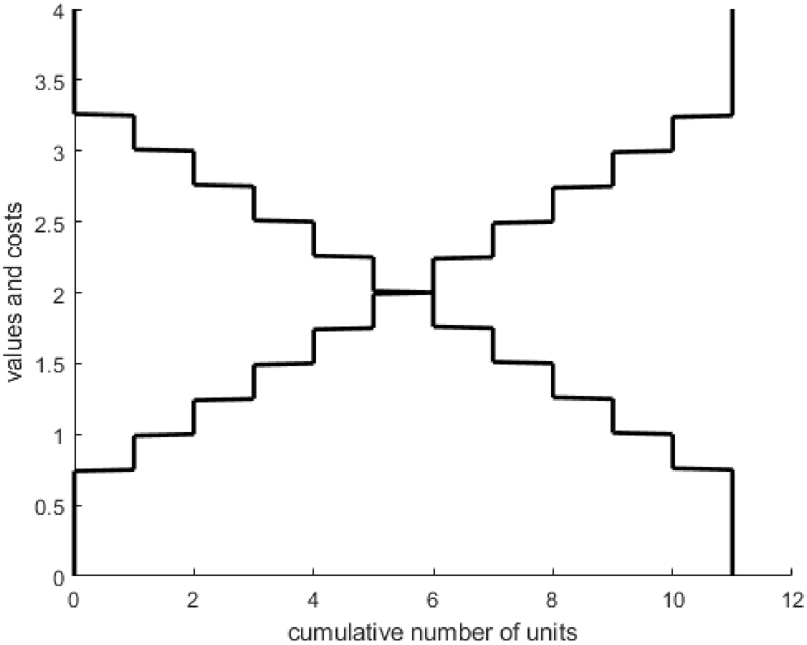

Figure 3. An Example of Experimental Supply and Demand Curves: Cumulative Value and Cost Distributions in Table 1.

The value center *C* contains any existing market-clearing price and it is easily obtained graphically as the intersection of the supply and demand curves viewed as if they were continuous curves. See Figure 3 and Figure 4 for illustrations. The standard example of a non-clearing market was studied experimentally (Inoua & Smith, 2022b, fig. 4; V. L. Smith, 1965), the so-called "swastika" supply and demand configuration illustrated in Figure 4.

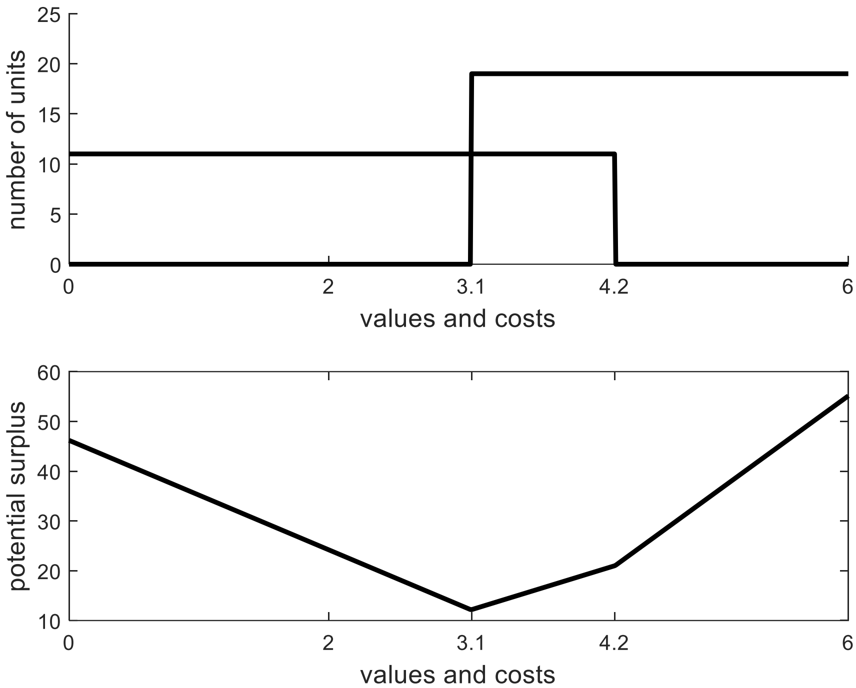

Figure 4. A non-clearing market: An example of a "swastika design" (V. L. Smith, 1965, p. 389, Session 5).

It follows by simple integration of the supply and demand step functions, (19) and (20), that the potential surplus function is a distance-like function:

$$V(p) = \sum_{v \geq p} |v - p| + \sum_{c \leq p} |c - p|, \tag{21}$$

which we took as a definition (without the absolute-value bars) in formula (9). Thus, the center of value C is a *generalized median of the values and costs* (a simple median being obtained by removing the profitability conditions for trade under the sum symbols). This links value theory to the robust statistics literature, notably quantile regression (Koenker, 2005; Koenker & Bassett Jr, 1978). The minimum principle is thus also an informational characterization of a competitive market (Inoua & Smith, 2021b, 2022b), at least because it says that any competitive price (any point in the center of value C) is by construction a generalized median of the traders' values and costs, which echoes Hayek's famous intuition about the competitive price system as synthesizing (or aggregating) a large amount of private and dispersed information in the economy (Hayek, 1937, 1945). Formulated probabilistically, moreover, the minimum principle becomes an informational characterization of a competitive market in a more fundamental way, with information as quantified à la Shannon (1948), namely in terms of log-probabilities:

A transaction concluded at a price p reveals existence of a cost unit $c \leq p$ and a value unit $v \geq p,$ which are private information: in probabilistic terms, the trade is formally the realization of the event $\{p \text{ accepted}\},$ implying realization of the two events $\{v \geq p\}$ and $\{c \leq p\}.$ Thus every trade that takes place in a market provides uncertainty-reducing knowledge of net economic worth. According to Shannon's theory (Shannon, 1948), the amount of information the realization of an event reveals, is measured (up to a multiplicative constant that fixes the unit of information) by the log-probability of the event. In standard information theory, the total information (or entropy) is obtained by integrating (averaging) over a (whole) probability distribution of possible realizations. For our purpose, however, the total information a trade reveals about the distribution of values and costs is obtained by integrating across the relevant range of the cost and value distributions revealed by the trade event.

One can indeed show that the competition principle can be written as a maximization of the following information integral function:

$$I(p) = -\int_{c_{\min}}^{p} \log \text{prob}\{c \leq x\} dx - \int_{p}^{v_{\max}} \log \text{prob}\{v \geq x\} dx, \tag{22}$$

where $c_{\min} = \min\{c_j\}$ and $v_{\max} = \max\{v_i\}.$ Supply and demand being the cumulative distribution functions involved in (22), we can rewrite the information function (22) as

$$I(p) = -\int_{c_{\min}}^{p} \log \frac{S(x)}{m+n} dx - \int_{p}^{v_{\max}} \log \frac{D(x)}{m+n} dx. \tag{23}$$

This information is maximized in a competitive market, as Figure 5 shows. [See Inoua and Smith (2021b), for greater details about the information function (23) and its properties.]

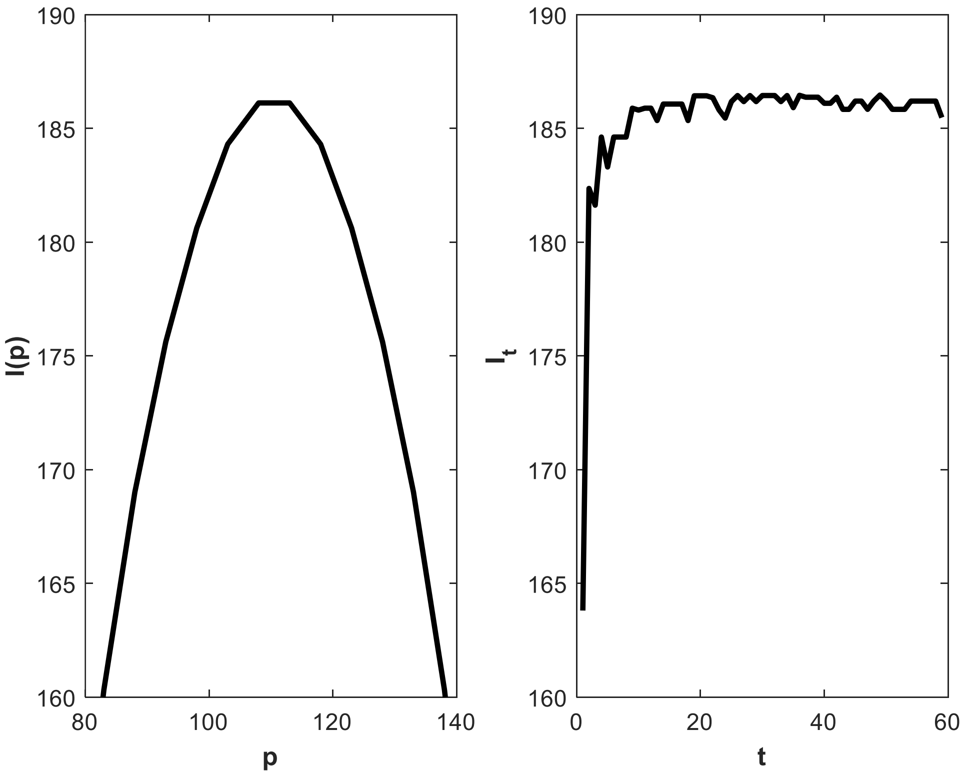

Figure 5. Competition maximizes the information integral function I(p). Illustration using the lab data in Figure 6.

To clarify: the information aggregated into price is the traders' values and costs: a competitive price (the Center of Value) is a best summary of the reservation prices in the following senses, contrasted with similar concepts in standard economic theory:

1. A competitive price is the *best summary* of the traders' values and costs in the L^1 sense (corresponding to a median) whereas standard economic models of information aggregation, e.g. the efficient model hypothesis (Fama, 1970), appeals instead to the mean or expectation (namely the L^2 norm). The difference is that a median is a best summary that

is robust to outliers: for example, median [1,2,3] = median [1,2,300] = 2; whereas mean[1,2,3] = 2 but mean [1,2,300] = 101.[13]

2. On the other hand, models of the new information economics (e.g., S. Grossman, 1976; S. J. Grossman & Stiglitz, 1976) hold that a competitive price aggregates information about supply and demand in the sense of being a sufficient statistics for these latter; similarly, in the Appendix, I sketch a probabilistic formulation of the model in which realized trades are a s*ufficient statistics for the traders' unknown values and costs*. Although this informational implication is qualitatively equivalent to the one in terms of the function I(p) introduced in equation (22), the one in terms of sufficient statistics can be shown to be more general by allowing an additional parameter that can be interpreted as subjects' cross-period learning parameter: this aspect of the theory is still the subject of ongoing work, however, that I merely sketch in the Appendix.

[13] Median-based statistics (based on minimum absolute-deviations criterion, or the "L_1 norm") preceded historically the now more popular mean-based regression (based on minimum least-square deviations, or the "L_2 norm"): it first appeared in the 18th century in the median regression work by Boscovich (1757), but was eclipsed by the simpler to implement and still more popular mean-based statictics (based on square deviations), through the infuencial contribution of Legendre (1805) and Gauss (1809) in the early 19th century. Despite a brief resurgence in late 19th century through (1888a, 1888b)"L_1 statistics" will truly enjoy an important revival only in the 1950s-60s in (Hampel, 1968; Huber, 1964; Tukey, 1960), aided with the development of computational techniques, notably the linear programing formulation of (Wagner, 1959), and it expanded enormously in the late 1970s due notably the quantile regression extension (Koenker, 2005; Koenker & Bassett Jr, 1978), and is now of course a mature industry, included in standard statistical packages. On the fascinating early development of statistics, see Stigler (1986). [This footnote is borrowed from Inoua and Smith (2022a, Ch. 3).]

3.4 The Problem of Path Dependency of Competitive Equilibrium[14]

So far, we bypassed for simplicity the path dependency problem, which was lurking from the outset: that competitive equilibrium price, defined as the set of points of minimum potential surplus, be invariant to the transaction sequence: otherwise, competitive equilibrium is indeterminate, that is, more precisely, it would depend on the arbitrary buyer-seller pairing. Provided the units are competitively allocated [namely an allocation such that no higher value (resp. cost) units are excluded from trade by lower value (resp. higher cost) ones], the competitive equilibrium price is determinate, and is indeed the value center *C*, by a fundamental duality between price and quantity:

As transactions occur, the distributions evolve, and the new distributions are obtained by simply removing the value and cost units that traded. Let D_t and S_t be the distributions of values and costs (or supply and demand functions) at time *t*, namely after the t^{th} transaction occurs (*t* can be viewed as denoting time, or better a transaction number). If the t^{th} transaction involves the value and cost c_t and v_t then the demand and supply curves shift as follows (that is, the new curves are obtained by simply removing the elementary step functions corresponding to the value and cost units that traded):[15]

$$S_{t+1}(x) = S_t(x) - \mathbf{1}_{\{c_t \leq x\}}, \tag{24}$$

$$D_{t+1}(x) = D_t(x) - \mathbf{1}_{\{v_t \geq x\}}, \tag{25}$$

where the initial distributions are $S_0 = S$ and $D_0 = D,$ namely (19) and (20).

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[14] For the general presentation with the proofs, see Inoua and Smith (2021b), which bypasses, however, the problem of path dependency of competitive equilibrium addressed here.

[15] If *t* denotes a transaction period, then one of course simply removes all value and cost units that traded during that period.

Then the potential surplus function in the market at any arbitrary price p is more precisely

$$V_t(p) = \int_0^p S_t(x)dx + \int_p^\infty D_t(x)dx. \tag{26}$$

The potential surplus concept introduced previously is $V = V_0$ and the center of value was defined as $C = \arg\min_{p\in\mathbb{R}} V_0(p)$. After the t^{th} transaction takes place, the new potential surplus function is obtained by subtracting from the previous one, the realized surplus that the t^{th} trade extracts:

$$V_{t+1}(p) = V_t(p) - (v_t - c_t). \tag{27}$$

That is, the potential surplus function at any time is obtained by subtracting from the initial potential surplus function, the surplus realized through trade all along:

$$V_t(p) = V_0(p) - \sum\nolimits_{\tau\leq t}(v_\tau - c_\tau). \tag{28}$$

Denote the accumulated realized surplus up to time t as

$$\Lambda_t = \sum\nolimits_{\tau\leq t}(v_\tau - c_\tau). \tag{29}$$

The accumulated realized surplus can be written in terms the quantity allocation vector, given by trade indicators referenced collectively as

$$\mathbf{q} = [q_1, ..., q_n; q_{n+1}, ..., q_{m+n}],$$

where $q_i = 1$ if unit i is bought and $q_i = 0$ otherwise; $q_j = -1$ if unit j is sold and $q_j = 0$ otherwise. Then by construction we can define the realized surplus function

$$\Lambda(\mathbf{q}) = \mathbf{r}\cdot\mathbf{q}, \tag{30}$$

which to any arbitrary allocation vector associates the corresponding realized surplus. Thus, the potential surplus function at time t reads:

$$V_t(p) = V_0(p) - \Lambda(\mathbf{q}_t). \tag{31}$$

To emphasize the dependence on transactions, we can write the potential surplus more revealingly as a function of the standing transaction price and the state of the transaction indicator vector:

$$V(p_t, \mathbf{q}_t) = V(p_t) - \Lambda(\mathbf{q}_t). \tag{32}$$

Competitive dynamics reads more precisely:

$$V(p_{t+1}, \mathbf{q}_{t+1}) \leq V(p_t, \mathbf{q}_t). \tag{33}$$

We say that a market attains competitive equilibrium if attains minimum potential surplus:[16]

Definition 1 (Competitive equilibrium)*: It is a price and allocation pair* $(p^*, \mathbf{q}^*)$ *such that* $V(p^*, \mathbf{q}^*) = \min\{V(p, \mathbf{q}) : (p, \mathbf{q}) \in \mathbb{R} \times \mathbb{R}^{m+n}\}$.

The old problem of path dependency can be stated as follows:

Path dependency problem: *Is the transaction-dependent competitive equilibrium concept just defined path-independent? In particular: Is any competitive equilibrium price in the value center C, as determined from the initial supply and demand curves?*

The response is affirmative, by the following fundamental result:

Proposition 2 (Fundamental Duality Theorem)*. The potential and realized surplus functions* V *and* Λ *are dual in the sense of linear programming. That is,* $\min\{V(p)\}$ *and* $\max\{\Lambda(\mathbf{q})\}$ *are equivalent to the primal and dual of a linear program.*

Proof. Given $\mathbf{r} = [v_1, \ldots, v_n;\ c_1, \ldots, c_m] = [r_1, \ldots, r_n;\ r_{n+1}, \ldots, r_m]$, we can write

[16] Notice that the allocation vector **q** involves only the integer numbers -1, 0, and 1. One can show from linear programming theory that the competitive equilibrium allocation is indeed integral: technically, trades are a matching in a bipartite network (buyers versus sellers) and the corresponding linear program yields indeed an integral solution (Birkhoff, 1946; Dantzig, 1951), as is more simply indicated by the complementary slackness conditions stated below, equations (38)-(39), which yield integral solutions (putting aside the indeterminacy of zero-profit, or “marginal”, trades).

$$V(p) = \sum_{u=1}^{n} |r_u - p| I(r_u \geq p) + \sum_{u=n+1}^{m} |r_u - p| I(r_u \leq p).$$

Following standard procedure in linear programming, we decompose the relevant variables into positive and negative parts, corresponding to price $p = p_1 - p_2$ (we must not impose $p_2 = 0$ beforehand) and the potential unit surpluses $s_{1u} = \max\{r_u - p, 0\}$, $s_{2u} = \max\{p - r_u, 0\}$, so that by construction $r_u - p = s_{1u} - s_{2u}$ and $V(p) = \sum_{u=1}^{n} s_{1u} + \sum_{u=n+1}^{m} s_{2u}$. Let $\mathbf{1}_k = [1, ..., 1]$ and $\mathbf{0}_k = [0, ..., 0]$ be the $1 \times k$ vectors of 1 and 0, and let $\mathbf{I}_k$ be the $k \times k$ identity matrix. Let the vectors $\mathbf{s}_1 = [s_{11}, ..., s_{1m+n}]$, $\mathbf{s}_2 = [s_{21}, ..., s_{2m+n}]$, $\mathbf{s} = [p_1\ p_2,\ \mathbf{s}_1\ \mathbf{s}_2]^{\mathrm{T}}$, $\mathbf{e} = [0\ \ 0\ \ \mathbf{1}_n\ \ \mathbf{0}_m\ \mathbf{1}_m\ \ \mathbf{0}_n]$, and the $(m+n) \times [2(m+n)+2]$ matrix $\mathbf{J} = [\mathbf{1}_{m+n}^{\mathrm{T}}\ -\mathbf{1}_{m+n}^{\mathrm{T}}\ \ \mathbf{I}_{m+n}\ -\mathbf{I}_{m+n}]$, so that $V(p) = \mathbf{es}$ and the constraint $\mathbf{r} - p\mathbf{1}_{m+n}^{\mathrm{T}} = \mathbf{s}_1 - \mathbf{s}_2$ is equivalent to $\mathbf{Js} = \mathbf{r}^{\mathrm{T}}$. Thus the optimization $\min\{V(p) : p \in \mathbb{R}\}$ is equivalent to the linear program:

$$\min\{\mathbf{es} : \mathbf{Js} = \mathbf{r}^{\mathrm{T}}, \mathbf{s} \geq \mathbf{0}\}. \tag{34}$$

The dual program is:

$$\max\{\mathbf{rq} : \mathbf{J}^{\mathrm{T}}\mathbf{q} \leq \mathbf{e}^{\mathrm{T}}, \mathbf{q} \in \mathbb{R}^{m+n}\}. \tag{35}$$

The constraint in the dual program is equivalent (if written explicitly) to the requirements: $\mathbf{1}_{m+n}\mathbf{q} \leq 0, -\mathbf{1}_{m+n}\mathbf{q} \leq 0,\ \mathbf{q} \leq [\mathbf{1}_n\ \ \mathbf{0}_m]^{\mathrm{T}},\ \mathbf{q} \geq [-\mathbf{1}_m\ \ \mathbf{0}_n]^{\mathrm{T}}$, which means:

$$\mathbf{1}_{m+n}\mathbf{q} = \sum_{u=1}^{m+n} q_u = 0; \tag{36}$$

$$q_u \in \begin{cases} [0,1], u = 1, ..., n, \\ [-1,0], u = n+1, ..., m+n. \end{cases} \tag{37}$$

Assume the pair $(\mathbf{s}^*, \mathbf{q}^*)$ solves the two programs. Then the so-called complementary slackness condition implies (after some identification):

$$q_i^* = \begin{cases} +1, v_i > p^*, \\ \ \ 0, v_i < p^*, \end{cases} \tag{38}$$

$$q_j^* = \begin{cases} -1, c_j < p^*, \\ 0, c_j > p^*, \end{cases} \tag{39}$$

Hence, the dual program is surplus maximization. We know the first program has an optimal solution, and even characterized the optimal set (the value center *C*). It follows from the strong duality theorem of linear programming theory that $V(p^*) = \mathbf{es}^* = \mathbf{rq}^* = \Lambda(\mathbf{q}^*)$. ■

The duality theorem implies therefore:

Proposition 3 (Characterization of Competitive Equilibrium):

1. By the strong duality theorem of linear programming theory, the minimum potential surplus (a duality gap) is zero (hence all mutually beneficial gains from trades are realized) and is attained when both programs achieve their respective optimums. Thus, a competitive equilibrium $(p^*, \mathbf{q}^*)$ is a pair of solutions to the dual programs. The set of competitive-equilibrium prices coincides therefore with the value center C.

2. By the complementary slackness conditions if $(p^*, \mathbf{q}^*)$ is a competitive equilibrium, then

$$q_i^* = \begin{cases} +1, v_i > p^*, \\ 0, v_i < p^*, \\ \{0,1\}, v_i = p^*, \end{cases} \tag{40}$$

$$q_j^* = \begin{cases} -1, c_j < p^*, \\ 0, c_j > p^*, \\ \{0,1\}, c_j = p^*. \end{cases} \tag{41}$$

The implications (40)-(41) imply that, up to the indeterminacy, the equilibrium quantity traded is $Q = \sum_{i=1}^{n} q_i^* = \sum_{j=1}^{m} q_j^* = \min\{D(p^*), S(p^*)\}$.

Thus, so long as "inefficient units", namely values $v < p^*$ (respectively costs $c > p^*$) are out-bidding (undersold) by more efficient ones, the competitive equilibrium is indeed in the value center *C*, the "intersection" of the initial supply and demand curves, S_0 and D_0.

3.5 The Double Auction: A Brief Illustration of The Theory

Since transaction prices in the double auction are either a standing ask a or bid b accepted by a counterparty, the potential surplus function for this institution is more revealingly expressed in terms of the couple (a,b), thus:

$$V(a,b) = \sum\nolimits_{c \le b} (b - c) + \sum\nolimits_{v \ge a} (v - a). \tag{42}$$

This can be written as:

$$V(a,b) = \int_0^b S(x)dx + \int_a^\infty D(x)dx. \tag{43}$$

Assuming a large continuous double-auction market with smooth bid and ask competitive submissions and revisions for simplicity, we get by the chain rule:

$$\frac{d}{dt}V(a,b) = \frac{\partial V}{\partial b}\frac{db}{dt} + \frac{\partial V}{\partial a}\frac{da}{dt} = S(b)\frac{db}{dt} - D(a)\frac{da}{dt}.$$

Writing $a = b + (a - b)$ for the first term of the second equality, we get after basic manipulation:

$$\frac{d}{dt}V(a,b) = [S(b) - D(a)]\frac{db}{dt} + D(a)\frac{d}{dt}[a - b]. \tag{44}$$

Writing $b = a - (a - b)$ for the last term, we get similarly:

$$\frac{d}{dt}V(a,b) = [S(b) - D(a)]\frac{da}{dt} + S(b)\frac{d}{dt}[a - b]. \tag{45}$$

Or, adding the two equations, we get:

$$\frac{d}{dt}V(a,b) = [S(b) - D(a)]\frac{d}{dt}\left[\frac{a+b}{2}\right] + \frac{S(a) + D(b)}{2}\frac{d}{dt}[a - b]. \tag{46}$$

Competition in the double auction, as suggest the law of supply and demand and the bid-ask reduction process, reads simply, in terms of either one of the three formulas (44)-(46):

$$\frac{d}{dt}V(a,b) \leq 0. \tag{47}$$

Equivalently, a double-auction dynamics can be characterized informationally. A standing bid message b accepted reveals existence of a cost unit $c \leq b,$ and a standing ask message a accepted, a value unit $v \geq a.$ Thus, the information content that the standing bid and ask messages reveal about the private value and cost units is measured by the integral function:

$$I(a,b) = -\int_{c_{\min}}^{b} \log\frac{S(x)}{m+n}dx - \int_{a}^{v_{\max}} \log\frac{D(x)}{m+n}dx. \tag{48}$$

As bids and asks are sent and revised competitively (hence the quote revisions obey the law of supply and demand and bid-ask reduction), it follows from a similar manipulation that yielded (44)-(46), that the information the bid and ask messages reveal about the underlying value and cost distributions increases:

$$\frac{d}{dt}I(a,b) \geq 0. \tag{49}$$

3.6 Towards a Classical Induced Value Theory

The theory above summarized suggests a reappraisal of the theory of induced valuation (V. L. Smith, 1976), in retrospect a neoclassically inspired methodological foundation for market experiments that interprets a lab market as an arena in which the experimenter can use an appropriate reward medium to induce artificial neoclassical preferences and trade over subjects: but for the neoclassical induced preferences to truly hold in the lab market, all trades must be at exogenously given prices, for example, those that the experimenter would enforce (thus assuming the role of the Walrasian auctioneer), a condition that does not hold typically (at least in the double auction, all traders make the prices through their bid and asks, revising them competitively through a process that is precisely an outbidding, underselling, and haggling multilateral process). In contrast, classical induced value theory derives directly from an interpretation of the potential surplus function. Each value unit v (respectively each cost c) assigned to a lab subject-trader is formally an *option to buy (sell)* a unit of the abstract commodity at a competitively negotiated transaction price $p,$ with a strike price identified with v (respectively c), the assigned value (cost). These lab trade options, if identified with their payoffs, can be written respectively:

$$B_t^v = \max\{0, v - p_t\} \text{ and } B_t^c = \max\{0, p_t - c\}, \qquad (50)$$

where the trade option associated with value and cost units that already traded is identically zero. Thus, the potential surplus at a standing transaction price, or $V_t = V(p_t),$ can also be interpreted as the sum of trade options:

$$V_t = \sum_v B_t^v + \sum_c S_t^c. \qquad (51)$$

The experimenter organizes trade through a specific institution, notably the double auction, thus implementing in the process all three competition forms. The experimenter discovers that the total cost of implementing a competitive equilibrium in the lab is minimized, rediscovering by the same token a fundamental principle of value theory. It is in this form (more precisely a specific formulation of it) that the minimum potential principle reveals itself in the early experiments on the double auction: the “excess-rent hypothesis”. Thus (to repeat the summary announced in the introduction): *a competitively organized lab market involving trade options on an abstract (intrinsically valueless) commodity representation is isomorphic to a classical market, that is, one in which consumers (producers) are willing to buy (sell) units of an actual commodity to which they assign use-values (production costs), at prices below* *(above) their use-values (costs), with the prices determined through their collective competition.*

4 The Case of Retradable Assets and Speculation[17]

The minimum principle breaks down, however, if the experimenter allows for units to be retraded for capital gains (Dickhaut et al., 2012), for then stability due to competition can be counteracted by speculation. For example, Figure 2 contrasts No-Retrade (or trader specialization “SP”) with Retrade (“RT”), as experimental treatment conditions under the same supply and demand configuration (Table 3), demonstrating the effect of retrade on the price stability observed in No-Retrade (Dickhaut et al., 2012). The prospect of capital gains introduces price noise that diverts the market away from achieving convergence to the efficient equilibrium, which is exacerbated by high cash endowments.

[17] This Sections follow closely, while extending it, Section 3 in Inoua and Smith (2022c).

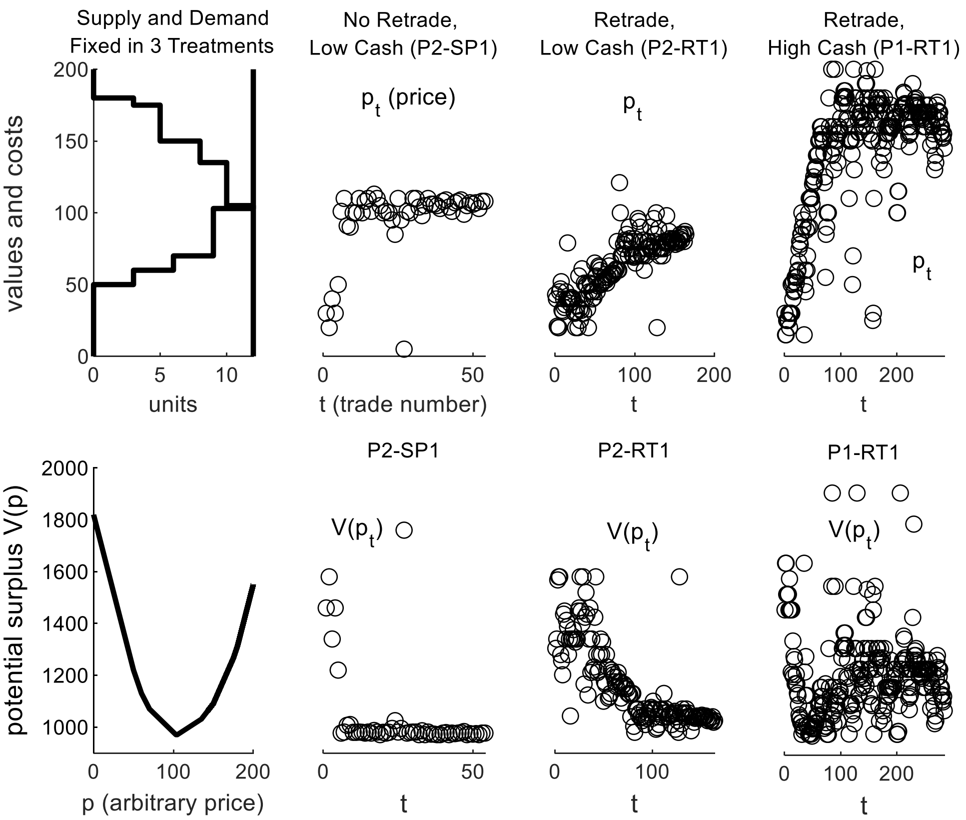

Figure 7. The minimum surplus principle and its breakdown in the presence of retrade and excess liquidity. Standard convergence to competitive equilibrium holds when No-retrade is allowed (Treatment "P2-SP1", Column 2); but stability is lost (counteracted by speculation) when units can be retraded (Treatment "P2-RT1"), and the instability is greater when subject had greater cash endowment for retrade (Treatment "P1-RT1"). Data source: Dickhaut et al. (2012).

Buyers' values	180	180	180	175	175	150	150	150	135	135	105	105
Seller's values	103	103	103	70	70	70	60	60	60	50	50	50

Table 3. Traders' dividend-values common to all treatments in Dickhaut et al. (2012). The competitive equilibrium price set, or Center of Value, is C = [103,105].

The values attached to units in these experiments are effectively dividend-values, that is, each unit a subject holds till the end of the trading period earns a dividend that equals the unit's assigned value. This supply and demand configuration, combined with cash endowments distributed to subjects, allows for enough flexibility to implement treatments that are intermediary between the standard supply-and-demand experiments (corresponding to a perishable consumption goods) and asset experiments that mimic more closely a field asset market operation (Friedman, Harrison, & Salmon, 1984; Noussair & Tucker, 2013; Palan, 2013; Plott & Sunder, 1982; V. L. Smith, Suchanek, & Williams, 1988).

The No-Retrade (or "SP") treatment echoes more precisely a field durable asset market in which all the participants are (long-run) dividend-value investors, trading the asset solely based on their valuations $\{v\}$ of the asset's fundamental present value, as reflecting the real economic prospect of the asset's issuer (and viewed as *exogenous* to the price dynamics), adopting a buy-and-hold trading strategy. The potential surplus function in this market is

$$V(p) = \sum_{v} | v - p |, \tag{52}$$

which is minimized for any price p that is a median of the values: the value center here is $C = \mathrm{median}\{v\}$. It is then reasonable to expect that a core tenet of the efficient market hypothesis (Fama, 1970) will hold in the market thus modeled, because the competitive price will approximate the asset's real value (as long as the subjective traders' valuations are not biased *in the aggregate*): any competitive equilibrium price is an optimal summary of traders' valuations, that, moreover, is robust to outliers.

It is otherwise, however, when the traders can resell asset units for capital gains, as the Retrade treatments in Figure 7 shows. Moreover, the price stability of non-retraded goods contrasts starkly with the prices observed in asset market experiments demonstrating the occurrence of asset price bubbles (Kujal & Powell, 2017; Palan, 2013; Porter & Smith, 2003; Powell & Shestakova, 2016; V. L. Smith et al., 1988). This experimental contrast between the stability of perishable final goods and the volatility of retraded assets echoes that in field markets, notably in finance, a contrast arising because consumer goods have value only in use, which governs their market price, whereas any good durable enough to retrade exhibits both a use value and a resale value: if such a good's resale price disconnects from its use-value price, the good may trade in a price bubble that deviates substantially, if unsustainably, from its fundamental use value.

Theoretically, the right to resell at a future period *t'*, a unit purchased in period *t*, can be interpreted as a *retrade option*, identified with its payoff, which in the simple case *t'*=*t*+1 (to which the general case comes down to) is simply[18]

$$R_t = p_{t+1} - p_t + d_t, \tag{53}$$

where p_t is the asset's price at the opening of period t, and d_t the dividend paid (per unit of asset held) during that period. (The retrade payoff is simply the net gain per asset unit that would realize a trader who buys units of the asset in period t to resell them the next period, t+1.) A (purely) speculative retrade option corresponds to the payoff $p_{t+1} - p_t$, the capital gain. Most financial assets come associated with them an implicit (speculative) retrade option.

The retrade option is of course a fundamental concept in financial theory, being conceptually equivalent to that of percent return

$$r_t = \frac{R_t}{p_t}.$$

The asset pricing theorem, for example, has a natural formulation in terms of the new concept (expressed in units of a numeraire): *an asset market model specified by a filtered probability space* $(\Omega, \mathbb{F}, \{\mathbb{F}_t\}, \mathbb{P})$ *is arbitrage-free if and only if there is a measure* $\mathbb{Q}$ *equivalent to* $\mathbb{P}$

[18] More generally, the retrade option's payoff is $R_{(t',t)} = p_{t'} - p_t + \sum_{t \le \tau \le t'} d_\tau$, $t' \ge t$.

under which the speculative retrade option on every asset is valueless: $\mathbb{E}_{\mathbb{Q}}(R_t \mid \mathbb{F}_t) = 0,\ t = 1,\ldots,T,$ where $\mathbb{E}_{\mathbb{Q}}$ means expectation under $\mathbb{Q}$, to be contrasted with $\mathbb{E}_{\mathbb{P}}$, expectation under $\mathbb{P}$. One can in fact characterize an arbitrage-free market as one that provides no incentives for risk-averse traders to speculate, controlling for risk through any risk-neutral measure:

Consider a speculator who buys merely for the prospect of gaining from retrade the next period: the speculator, by definition, has no fundamental motive for retrade such as hedging, hence assume the speculator's wealth W_t, prior to retrade, is constant (non-risky) given available information $\mathbb{F}_t$. The speculator's wealth, after (re)trading z_t unit of the asset in period *t*, becomes in the next period $W_{t+1} = W_t + z_t R_t$. If the speculator's utility function U is concave, then by Jensen's inequality, $\mathbb{E}_{\mathbb{Q}}[U(W_t + R_t) \mid \mathbb{F}_t] \leq U[\mathbb{E}_{\mathbb{Q}}(W_t + R_t \mid \mathbb{F}_t)] = U[\mathbb{E}_{\mathbb{Q}}(W_t \mid \mathbb{F}_t)]$. That is, $\mathbb{E}_{\mathbb{Q}}[U(W_{t+1}) \mid \mathbb{F}_t] \leq U[W_t]$, hence (by iterated expectations):

$$\mathbb{E}_{\mathbb{Q}}[U(W_{t+1})] \leq U(W_t),\ t = 0,\ldots,T. \tag{54}$$

Therefore, recursively, zero retrade is optimal in the sense of risk-discounted utility: a no-speculation result for arbitrage-free markets that adds to the other no-trade theorems (Gizatulina & Hellman, 2019; Milgrom & Stokey, 1982; Rubinstein, 1975; Tirole, 1982), which make it inherently hard to analyze the destabilizing power of the retrade option in the neoclassical approach to finance. Thus, we explore a classical approach to finance (Inoua & Smith, 2021a), which also consists of adopting a realistic approach to trader behavior, with asset supply and demand specified by the distribution of *minimum acceptable rate of return* across traders, assuming common practices of financial trading (notably trend-following speculation) under adaptive expectations, as is the case in practice, as mounting evidence, notably experimental data, suggest (Anufriev & Hommes, 2012; Chow, 2011; Colasante, Palestrini, Russo,

& Gallegati, 2017; Greenwood & Shleifer, 2014; Haruvy, Lahav, & Noussair, 2007; Hommes, 2021; Lahav, 2011; V. L. Smith et al., 1988), and deriving emergent regularities of these assumptions to explain financial stylized facts. A sketch of this classical model of financial volatility may be useful here:

Convergence to a fixed equilibrium point is of course at odds with the dynamics of speculative prices, which are prone to extreme (non-Gaussian) fluctuations. More precisely, speculative asset returns are power-law distributed, as is known since Mandelbrot's seminal work (Gabaix, Gopikrishnan, Plerou, & Stanley, 2003, 2006; Gopikrishnan, Plerou, Amaral, Meyer, & Stanley, 1999; Guillaume et al., 1997; Mandelbrot, 1963a, 1963b, 1967; Plerou, Gopikrishnan, Amaral, Meyer, & Stanley, 1999).[19] That is, the empirical cumulative distribution of a speculative asset's absolute return is

$$\mathrm{prob}\{|r| \geq x\} \sim \frac{K}{x^{\alpha}}, \text{for large } x, \tag{55}$$

where the exponent α is typically close to 3, and *K* is just a norming constant. A second universal regularity (which we do not discuss in greater detail here) is volatility clustering**:** large price changes tend to be clustered in time (i.e., small-magnitude price changes tend to be followed by small-magnitude price changes, and large-magnitude price changes by large-magnitude price changes): formally, while the return process is serially uncorrelated, its magnitude (or absolute value) is long-range correlated. These empirical regularities have also been observed in the lab and have been closely investigated experimentally (Kirchler & Huber, 2007, 2009).[20]

[19] The notation $f(x) \sim g(x)$ means $f(x)/g(x) \to 1$ as $x \to \infty$.

[20] Plott and Sunder (1982) also found excess kurtosis in lab data.

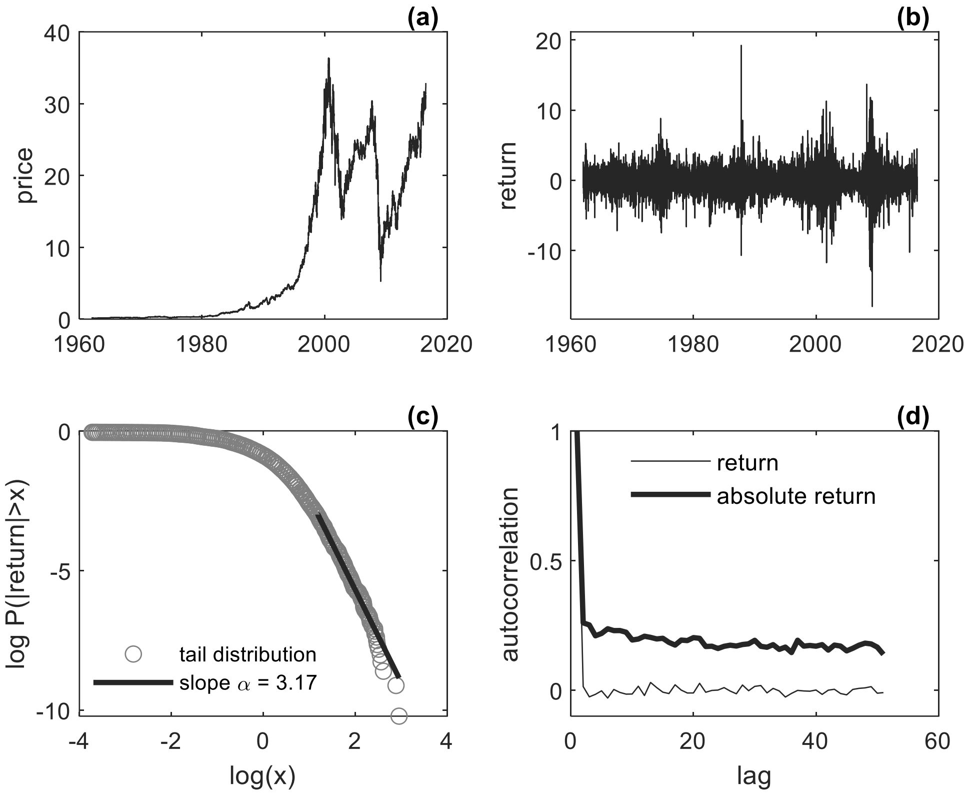

Figure 8. Power-law and clustered volatility illustrated: General Electric stock. (a) Price. (b) Return (in percent). (c) Cumulative distribution of volatility in log-log scale, and a linear fit of the tail, with a slope close to 3; (d) Autocorrelation function of return, which is almost zero at all lags, while that of volatility is nonzero over a long range of lags (a phenomenon known as volatility clustering).[21]

[21] The estimation of the power law is based on the maximum likelihood algorithm developed in Clauset, Shalizi, and Newman (2009).

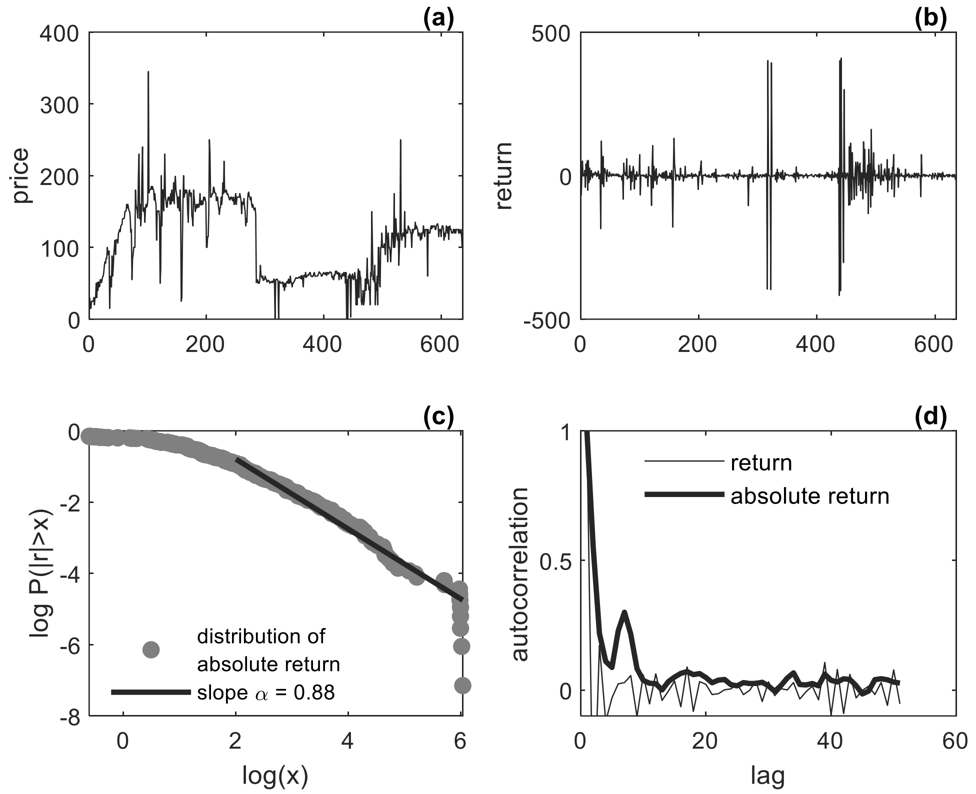

Figure 9. Price Volatility in the High-Cash-Retrade Treatment in Figure 7 (third column, but here all periods combined for statistical significance of the estimation of the power law tail exponent).

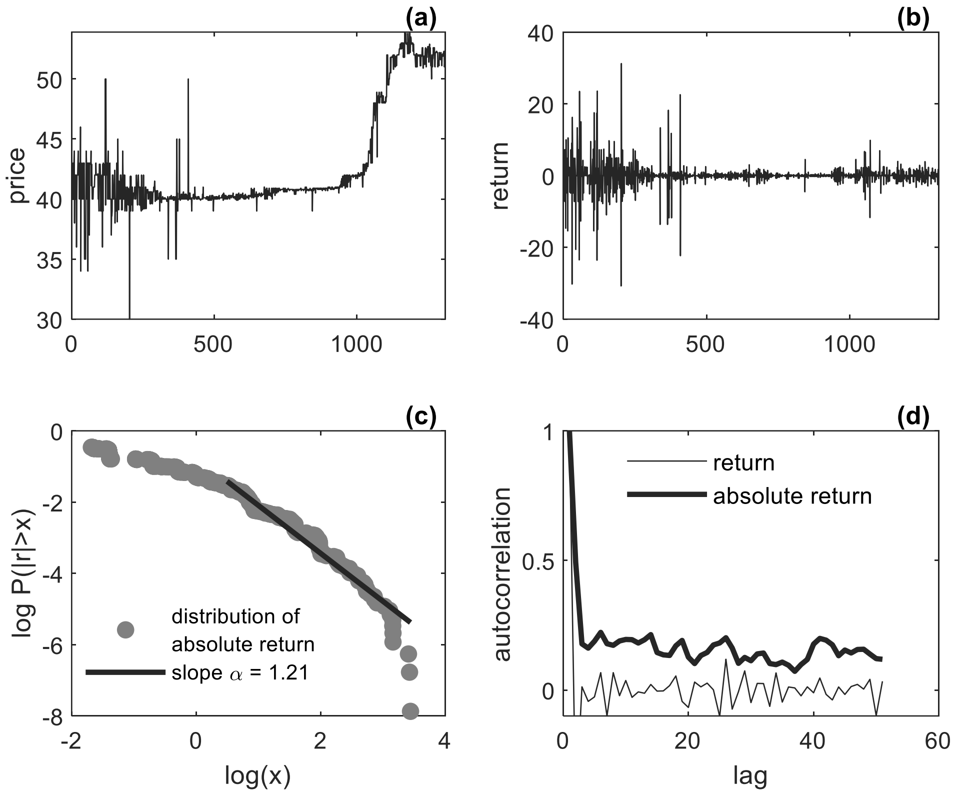

Figure 10. A Lab asset price volatility. Data source: Kirchler and Huber (2009, Market 5).

While the lab asset price volatility are clearly clustered and fat-tailed (non-Gaussian) upon graphical inspection, the price series (even when pooled across periods) are typically not long enough to allow for a fully comfortable statistical analysis of the tails of the price series (fat tails require a much longer data series for statistical significance than common distributions). In fact, there is at best only a weak graphical indication of a precise power-law behavior in the lab data we analyze. Notwithstanding this caveat, we observe that the lab asset price volatility is typically more extreme than its field counterpart (Figure 8 versus Figure 9 and Figure 10). (The lower the tail exponent α of a power law, the higher the volatility.)

There is as yet no consensus among experts as to a canonical explanation of the emergence of the power law of asset returns (and of volatility clustering), although theoretical models abound in this field, notably the interesting, if often mathematically intractable, agent-based models (reviewed, e.g., by Lux & Alfarano, 2016). Here we suggest a simple explanation found in the continuity of the foregoing classical characterization of the price mechanism.

Consider for simplicity of exposition a purely speculative market, where an asset has no other intrinsic value to a trader beyond the speculative retrade option attached to it. Assume, to further simplify, zero minimum acceptable rate of return. Then the speculator who expects a price increase of the asset, buys; the one who expects a price drop sells: thus, by construction, a speculator's effective reservation price for a good is his anticipated future resale price, say p^e. Thus, the potential surplus function becomes, for this purely speculative market model:

$$V_t = \sum_{p_{t+1}^e} | p_{t+1}^e - p_t |, \tag{56}$$

There is no reason why the speculative version (56) of the potential surplus function would be minimized (and that the market would converge to a fixed competitive equilibrium), for the median anticipated resale price needs not be given if the traders' expectations are self-reinforcing, which is the case if speculators follow short-run price trends, as they often do in practice. One can easily show that the price change of an asset traded in a competitive market populated by trend-following speculators, follows, at a first-order linear approximation, a random-coefficient autoregressive model (Inoua, 2020; Inoua & Smith, 2021a):

$$r_t = \sum_{h=1}^{H} \alpha_{ht} r_{t-h} + \varepsilon_t, \tag{57}$$

where $\{\alpha_{ht}\}$ and $\{\varepsilon_t\}$ are random variables. Random-coefficient autoregressive processes (also known as Kesten processes) are rigorously studied by mathematicians (Buraczewski, Damek, & Mikosch, 2016; Kesten, 1973; Klüppelberg & Pergamenchtchikov, 2004) and are perhaps the most natural class of power-law generating processes, where the tail exponent depends on the distribution of the feedback coefficients $\{\alpha_h\}$. But they cannot generate clustered volatility, which can be explained simply in terms of traders' reaction to exogenous news about the economy (Inoua, 2020; Inoua & Smith, 2021a).

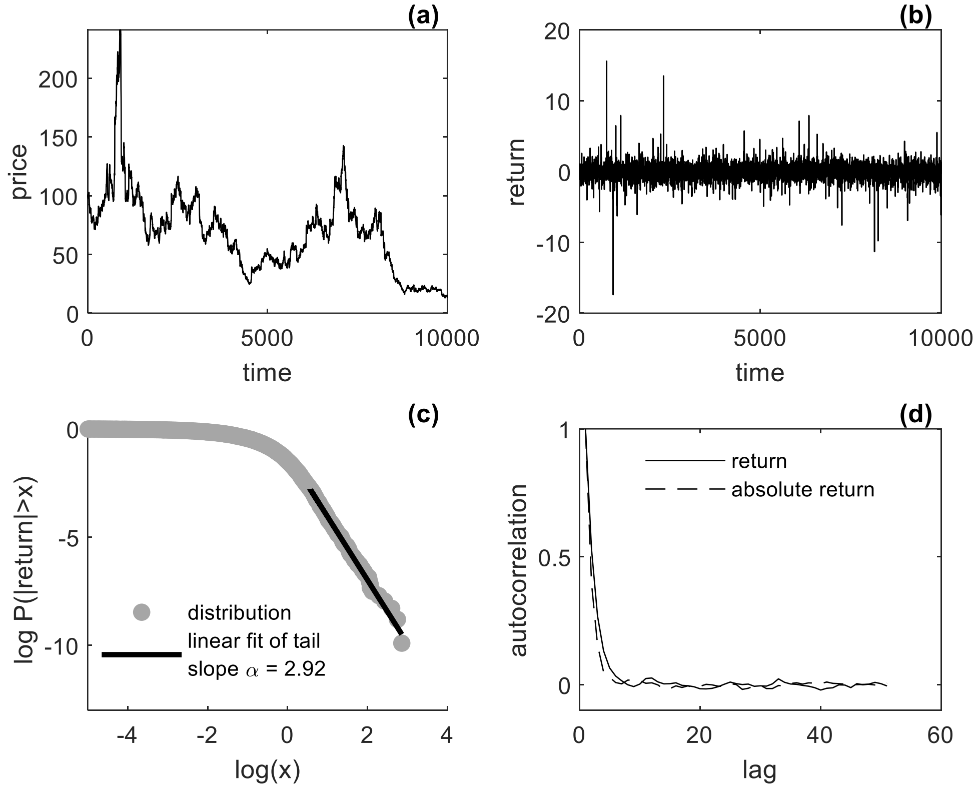

Figure 11. A simulation of the speculative asset market model: (a) price; (b) asset return follows a first-order random-coefficient auto-regressive process.; (c) power-law distribution of asset return; (d) autocorrelation functions of return and absolute return.

Thus, both experimentally and theoretically, the retrade option seems to be the most important impediment to price convergence and stability; two other classic cases in which a regular convergence to competitive equilibrium may fail are: [22]

[22] I thank Daniel Houser for pointing out the need for the nuance.

1. "Monopoly" in the classical sense, which includes collusion and price discrimination. Classically, "monopoly", like "competition", does not refer to the number of sellers per se, which hardly play any decisive role in price formation, as the "boundary experiments" show (V. L. Smith & Williams, 1990); monopoly classically means, not a single seller (as it come to be understood literally during the marginal revolution following Cournot's innovation) but rather the absence of competition on the supply side (a concept that can be extended to the demand side as well). An example of classical monopoly is a cartel, whereby sellers (whatever their number might be) collude to set a uniform price: standard supply and demand analysis still applies here as well, but with the competitive equilibrium concept corresponding to an effective market supply curve consisting, not of the sellers' private, individual, reservation prices, but of the cartel's uniform reservation price. (The word "monopoly" here is justified by the fact that the sellers collectively act as if they were one seller endowed with the cartel's reservation price.) Again the crucial point about the old view of monopoly (as we formalize it) is that supply and demand analysis still applies (see, e.g., Inoua & Smith, 2020a, Section 7), only with the supply and demand curves defined by effective reservation prices that differ from the sellers' private valuations (similarly to the retrade case in which the effective reservation price is the speculator's anticipated resale price).[23] The literal meaning of monopoly ("lone seller") appears also in the classical literature, but not when it comes to a lone supplier per se (since a lone-seller market such as the English auction is classically a regular competitive market), but rather

[23] On the subtle classical definition of "monopoly", see, e.g., Inoua and Smith (2020a, Section 6).

when, for example, it comes to a lone seller that artificially creates scarcity of a commodity or that price-discriminate.

2. Another important class of exceptions to the competitive price convergence model is of course the complex realm of nonmarket interactions. Whenever noneconomic considerations such social status or fellow feeling preclude, or interfere with, regular market competitive interactions, needless to say that the foregoing model is silent, not just because by definition a social exchange does not corresponds to a regular competitive institution, but because the subjects' reservation prices here also do not coincide with the subjects' private valuations as represented by a distribution of numbers, because the identity of each subject also matters.

More generally, the theoretical model suggests that there will be convergence problems whenever the price is determined by a noncompetitive institution or whenever the traders' effective reservation prices differ from their private valuations.

5 Conclusion

As we showed in the preceding sections, the early market experiments hinted to an idea (the excess-rent hypothesis) that, when generalized, turns out to be a natural formulation of competitive market dynamics, as our reappraisal of the experiments suggest. This reinterpretation and generalization of the hypothesis requires a major methodological departure from neoclassical theory, however: it requires that we go back to an old view of market competition from Adam Smith to Jules Dupuit and Alfred Marshall that is more or less explicitly based on the concept of reservation price (the traders' values and costs), treated as a primitive concept in partial equilibrium, and which is an operational substitute for the utility function with which Stanley Jevons and Leon Walras replaced the old tradition during the marginal revolution of the 1870s, adopting to that end the axiom of large number of price-taking traders, following a major conceptual innovation by Augustin Cournot that hinders any realistic articulation of competitive price discovery.

Hence, the experiments also motivated a reexamination of the history of value theory.

By its general, qualitative, nonparametric nature, the theory as presented briefly here is silent concerning the following properties of experimental markets that are directions for future research:

1. A parametrization of the theory to account for cross-period subject learning observed in the lab markets (for a sketch of work in progress in this direction, see the Appendix below).
2. A parametrization of the theory and a more detailed confrontation of the theory and experiments to account for different speeds of convergence to competitive equilibrium across institutions and treatments.

6 Appendix: A Probabilistic Formulation of the Model

This Appendix sketches a probabilistic formulation of the model (the subject of ongoing work) that complements the informational characterization of competitive price dynamics as overviewed in the main text. I emphasize here the recoverability of values and costs from observed price and trade data in the light of the model, focusing on the dual side, or trade data (to complement the informational aggregation in price, the focus of the discussion in the main text). An indication of the value and cost information aggregated into trade is in the very fact that the realized surplus $\Lambda = \Lambda(\mathbf{q})$ corresponds to the covariance between the reservation prices and the trade indicators (Inoua & Smith, 2021b, Equation 22); thus, the competitive trade allocation $\mathbf{q}^*$ reveals maximum information about the traders' valuations in the sense of being the most correlated with them. It turns out that realized trades also aggregate value and cost information in the following deeper sense:

First notice that a market session can be characterized as a series of competitively emerging trades, and the equilibrium of a market session, as a state of market at which the probability of any further mutually beneficial trade is the smallest. It turns out that the most natural formulation of this intuition consistent with the model (or more precisely the dual side) consists of postulating that the Shannon entropy of a trade sequence $\mathbf{q}$ (that is, the log-inverse-probability of observing that trade sequence) is proportional to the realized surplus $\Lambda(\mathbf{q})$. (Thus, the probability of trade is minimum when the realized surplus is maximum.) *Statistically, this comes down to saying that the vector of trade indicators* $\mathbf{q} = [q_u]$ *is a natural sufficient statistic for the underlying, private, values and costs* $\mathbf{r} = [r_u]$. By the Pitman-Darmois-Koopman theorem (see, e.g., Andersen, 1970), the probability of observing trades $\mathbf{q}$ belongs to the exponential family, which in our case reads as follows in canonical form, with values and costs $\mathbf{r} = [\mathbf{r}_u]$ as the distribution's natural parameters:

$$\mathrm{prob}(\mathbf{q} \mid \mathbf{r}) = \pi(\mathbf{r}) = \frac{\exp(-\mathbf{r} \cdot \mathbf{q})}{\sum_{\mathbf{x}} \exp(-\mathbf{r} \cdot \mathbf{x})} = \frac{\exp(-\Lambda(\mathbf{q}))}{\sum_{\mathbf{x}} \exp(-\Lambda(\mathbf{x}))}, \tag{58}$$

where $\Lambda(\mathbf{q})$, recall, is the realized surplus corresponding to the trades $\mathbf{q}$. A classic problem of statistical inference consists of inferring the distribution's unknown parameters, here the private values and costs $\mathbf{r}$, from a sample, here an observed trade sequence $\mathbf{q} = \mathbf{x}$. This problem turns out to have a nice solution, a maximum likelihood estimator. Under the model (58), the maximum likelihood estimator $\mathbf{r}^*$ of $\mathbf{r}$ given an observed trades $\mathbf{q} = \mathbf{x}$ is easily shown to be given by the system of equations:

$$\mathbf{E}_{\mathbf{r}^*}(\mathbf{q}) = \mathbf{x}, \tag{59}$$

where the expectation is taken under distribution (58) assuming $\mathbf{r} = \mathbf{r}^*$(note that by definition, $\mathbf{E}_{\mathbf{r}^*}(q_u)$ is the probability that a value or cost unit u will trade under the fitted model, since q_u is just the indicator function, indicating whether the unit u has traded or not). The maximum likelihood estimator $\mathbf{r}^*$, which is uniquely defined (when it exists) by the system of nonlinear equations (59), usually does not have a simple solution, but is readily solved numerically. More specifically, the type of nonlinear equations involved can be made more explicit as follows:

Since trade occurs in pairs (of buyers and sellers), we are implicitly modeling trade as a bipartite network linking buyers and sellers represented by the adjacency matrix $\mathbf{Q} = [Q_{uh}]$, where $Q_{uh} = 1$ if the value/cost pair (u,h) trades, and $Q_{uh} = 0$, otherwise; thus $q_u = \sum_h Q_{hu}$ for a value unit u and $q_u = -\sum_h Q_{hu}$ for a cost unit u (by the convention of counting sales negatively). Note that, in terms of the matrix $\mathbf{Q}$, the realized surplus can be written as

$$\Lambda(\mathbf{Q}) = \sum_{u,h=1}^{m+n} (r_u - r_h) Q_{uh}. \tag{60}$$

In terms of $\mathbf{Q}$, the probabilistic model reads more specifically:

$$\mathrm{prob}(\mathbf{Q} \mid \mathbf{r}) = \frac{\exp(-\Lambda(\mathbf{Q}))}{\sum_{\mathbf{X}} \exp(-\Lambda(\mathbf{X}))}. \tag{61}$$

According to this model, the probability of trade between the value and cost units v_i and c_j can be shown to equal (see, e.g., Park & Newman, 2004, Equation 24):[24]

$$\mathrm{prob}(Q_{ij} = 1) = \frac{\exp(v_i - c_j)}{1 + \exp(v_i - c_j)}. \tag{62}$$

To fix ideas about the estimation of values and costs from observed trades, assume the probability of trade between each value-cost pair is given by (62) independently of any other trade; then one can show that the maximum likelihood estimator of values and costs takes on the following more explicit form (where a hat denotes an estimator):

$$x_u = \sum_{h \neq u} \frac{\exp(\hat{\theta}_u + \hat{\theta}_h)}{1 + \exp(\hat{\theta}_u + \hat{\theta}_h)}, \tag{63}$$

where $\theta_u = v_u$ if u is a value unit, and $\theta_u = -c_u$ if u is a cost unit (the different sign is due to the fact that x_u is counted negatively for a sale). For the technical details and a discussion of the properties of the estimators (63), see, e.g., Chatterjee, Diaconis, and Sly (2011, Equation 3); for an overview of the theory of sufficient statistics and exponential families (and a proof of the Pitman-Darmois-Koopman theorem for a discrete probability space), see, e.g., Andersen (1970).

[24] A negative sign is associated to costs because a sale is counted negatively.

More generally, the exponential family with reservation prices as natural parameters and trades as observations has the more general canonical formulation, with one additional parameter:

$$\mathrm{prob}(\mathbf{q} \mid \mathbf{r}) = \frac{\exp(-\lambda\Lambda(\mathbf{q}))}{\sum_{\mathbf{x}} \exp(-\lambda\Lambda(\mathbf{x}))}. \tag{64}$$

Following standard interpretation in discrete-choice models, the additional parameter λ, which measures the speed of adjustment to equilibrium, can also be interpreted as a collective *trader learning parameter*. The main lead for future research mentioned in the Conclusion then reads more precisely, in terms of experimental hypotheses, as follows:

1. Cross-institution variation of the speed of adjustment to equilibrium: λ is on average smaller for the double auction compared to the posted price, for example.
2. Cross-period subject learning in a lab market: λ tends to decrease across periods (as subjects make more and more precise decisions, reducing decision errors across repetitions).

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