2021

Exploring how teachers deal with students' errors in mathematics: a case of a secondary school in Siaya County, Kenya.

John Robert Owala Ouko

THE AGA KHAN UNIVERSITY

Institute for Educational Development, East Africa

EXPLORING HOW TEACHERS DEAL WITH STUDENTS' ERRORS IN MATHEMATICS: A CASE OF A SECONDARY SCHOOL IN SIAYA COUNTY, KENYA.

BY

OWALA JOHN ROBERT OUKO

A Research Project Submitted to the Institute for Educational Development, East Africa in Partial Fulfilment of the Requirements for the Degree of Master in Education

(Mathematics Education)

Dar es Salaam, Tanzania

November 2021

APPROVAL

THE AGA KHAN UNIVERSITY

Institute for Educational Development East Africa

OWALA JOHN ROBERT OUKO

I hereby give my permission for the research project of the above-named student, for whom I have been acting as supervisor, to proceed to examination.

Peter Kajoro

(Research Project Supervisor)

Date: 29th November, 2021

The members of the Research Project Evaluation Committee appointed to examine the research project of the above-named student find it satisfactory and recommended that it be accepted.

(Internal Examiner)

Date: 5th June 2022

DEDICATION

This project is dedicated to my wife Josephine Adhiambo Ouko, my children Richy Robert Omondi Ouko, Cassy Conslate Ouko, Adrian Trevor Ouko, and Joseph Anwyll Abonyo Ouko for enduring my absence while I was studying in a country far away from home. I also dedicate it to my late mother Millicent Linnet Ajwang' (Nyaki) and my late father David Owala for their support during my formative years and my late wife Irene Ouko, my siblings; Nancy Owala, Clackson Owala, Sharon Owala, Bilclinton Owala, Patrick Owala and my Sister-in-law Emma Bong'o for their support.

Abstract

Making errors in mathematics during learning by students is a common occurrence in our classrooms. Errors signal gaps in students' understanding, competencies or knowledge that require to be addressed. Teachers employ a wide range of approaches to identify these gaps to provide interventions. The purpose of this study was to explore how teachers deal with students' errors in mathematics during teaching and learning inside and outside the classroom in a secondary school in Siaya County, Kenya. Specifically, the study sought to understand how teachers identify, communicate and follow up on students' errors and the challenges that teachers faced while following up on the students' errors. This qualitative study used interviews, lesson observation, focussed group discussions and document analysis to collect data. Data analysis involved organising, coding, seeking patterns and interpreting data. The study's findings show that when students make verbal mathematical errors in class, teachers usually respond with verbal cues and questions directed either to the specific student or to the entire class. The findings revealed that teachers' communication of students' errors was provided in the form of verbal and written communication provided by teachers to learners to achieve various objectives regarding the errors made by learners. The findings also revealed that while teachers followed up on students' errors, they used a variety of strategies to help students correct and understand their mistakes. The study recommended that mathematics teachers need to re-evaluate their current understanding of students' mathematical errors, their causes, and their importance in mathematics teaching and learning, as this will guide their teaching strategies, which will greatly aid in improving learners' mathematical understanding since by examining and discovering errors together with the students, teachers develop in the students the capacity to identify, ascribe, and rectify their own mistakes.

ACKNOWLEDGEMENTS

I wish to thank the Almighty God for granting me the energy to complete this study. It was not an easy task, but God helped me go through the steps despite the challenges that came my way.

I would also like to thank Aga Khan University for granting me the partial scholarship without which I would have never completed this Master's Degree Programme and the Government of Kenya through the Teachers Service Commission for granting me study leave to be away to study.

My special gratitude goes to my Supervisor Prof. Peter Kajoro for the immense support he has given me throughout this project by reading and correcting my work, giving me necessary advice and listening to me several times in the course of this project and during our Mathematics Education lectures. I would also like to thank Ms Veronica Sarungi for her immense support during our Mathematics Education lectures and her constant support while orientating me into the course.

I would also like to thank my colleagues in our cohort who assisted me in one way or another during the course. Special thanks go to my roommates in Room 1C who ensured that we had a home away from home. My greatest thanks go to Daniel Odhiambo Omollo whose guidance was so invaluable during this course and Rachael Kulwenza who read through my work several times to ensure it that was edited well.

I would also like to thank Damary Luvayo Kisiangani (Aga Khan University alumni and Professional Development Tutor) who informed me about this course and took me through the process of the application successfully.

Lastly, I would like to thank the Kenya Union of Post Primary Education Teachers (KUPPET) Siaya Branch and the Executive Secretary Mr Samuel Okoth Opondo for his support and for allowing me to be away to study while I was supposed to be serving the members and the office.

DECLARATION OF ORIGINALITY

I, **OWALA JOHN ROBERT OUKO**, hereby declare that this research project is my own work. It represents my own effort and has not been taken in whole or in part, without reference to whom or from where the information was attained.

Signature

Date: 29th November, 2021

Table of Contents

LIST OF FIGURES

LIST OF ABBREVIATIONS

FGD	Focus Group Discussion
HoD	Head of the Department
INSET	In-Service Education and Training
MOE	Ministry of Education
MOICNG	Ministry of Interior and Coordination of National Government
NACOSTI	National Commission for Science, Technology, and Innovation
OECD	Organisation for Economic Co-operation and Development
PISA	Program for International Students Assessment
SMASE	Strengthening of Mathematics and Science Education
TSC	Teachers Service Commission

CHAPTER ONE

INTRODUCTION

1.0 Introduction

This is a report of a study that was done in Siaya County, Kenya. The study was aimed at finding out how teachers of mathematics in a public secondary school deal with learners' errors during the teaching, learning, and assessment of mathematics. This chapter presents the background and context of the study, statement of the problem, the rationale and significance of the study, the purpose of the study, the research questions, the scope of the study, assumptions of the study, limitations of the study, the definition of key terms, and organisation of the study report.

1.1 Background and Context

Mathematics is an important subject for national development. Mathematics as a subject provides a great way of developing mental discipline and of fostering logical reasoning, as well as mental rigour, and it is essential for understanding the contents of other subjects in schools such as science, social studies, and even music and art (Hansson et al., 2020). As a result, the Kenyan government has recognized this role that mathematics has in national development and emphasized its importance in schools by making it compulsory in secondary schools under the current educational system and even made the number of lessons for mathematics higher than any other subject (Adino, 2015; Mbugua et al., 2012).

The assessment of mathematics in the Kenyan curriculum is done at the national level, school level, and class level. The Kenya National Examinations Council (K.N.E.C.) assesses secondary school mathematics at the end of four years of schooling during the Kenya Certificate of Secondary Education (K.C.S.E.) examination, whereas at the school level the subject is assessed through end of term examinations, mid-term examinations, and scheduled continuous assessment tests, and at the class level, it is assessed through class assessments through random tests, assignments, verbal and written questions, black/white-board workings and others (Tennant & Sarungi, 2016).

Learners’ performance in mathematics is still low in most countries. Despite the importance placed on mathematics, a large number of learners still find it difficult

to understand the subject, which is reflected in students' poor performance in the subject during Tanzanian national examinations (Mazana et al., 2020; Mazana et al., 2019). In addition, despite the wide potential application and importance of mathematics, learners in Uganda, like in most countries, consistently underachieve in the subject, because its students lag behind their contemporaries in mathematics in national examinations (Kiwanuka et al., 2015). In Kenya, the situation is similar, and performance in mathematics by secondary school students, as reflected by K.C.S.E. results, has remained poor over a time spanning several years (M'Kiambi, 2013; Mbugua et al., 2012).

Teachers have a huge task in assisting learners in their quest to achieve better performance in mathematics. When teachers use appropriate teaching strategies in class it assists the learners to make meaningful connections between ideas, skills, and real-life situations thereby making learners understand concepts being taught in class and hence they end up performing exemplarily in the subject (Njiru et al., 2020). It is therefore important to note that teachers' involvement in-class activities, which help the learners, need to be enhanced for good performance in mathematics by the learners.

Errors made by learners in the course of learning mathematics contribute significantly to their poor performance in the subject. Students make errors when they interact with new knowledge and these errors end up limiting their performance in mathematics while at the same time these errors make the learners find a lot of difficulty in learning mathematics (Khan & Chishti, 2011).

Since teachers come across learners' errors not only in exams but also in their daily interactions in mathematics teaching and learning in classrooms, they have to respond to these errors while teaching as well as after teaching and this requires that teachers use students' errors to diagnose the learners' difficulties (Sapire et al., 2016). If these learners' errors are responded to well depending on their nature and frequency, then the learners would benefit significantly in their studies. However, some teachers end up not handling these errors well by either ignoring the sources of the errors or ineffectively handling them thus resulting in poor understanding of concepts in mathematics leading to students' poor performance in the mathematics subject (Hansen et al., 2020).

1.2 Statement of the Problem

Mathematics is not only a core subject in the Kenyan curriculum for secondary schools but also a very important pillar for other subjects in the same curriculum because it provides skills for problem-solving, communication, and reasoning which are important life tools that are used in school and even in life after school. Even though the subject is very important it has been noted that students usually make mathematical errors that make them not achieve the best grades in the subject (Chege, 2015). According to Hansen et al. (2020) while citing Ball and Bass (2003) and Barmby et al. (2009) these mathematical errors committed by learners are reasoned, reasonable and are part of learning mathematics for the students therefore teachers are supposed to identify them, focus and build on them to help the learners develop mathematical knowledge. Whence teachers who are effective in their teaching are usually supposed to create a culture in their classes where learners do not fear making mistakes because mistakes/errors are tools for learning.

There is inadequate information available on how teachers deal with students' errors in mathematics in secondary schools in Kenya and little research has been done in this area to provide a framework for good teacher practices regarding how to deal with these learners' errors in schools. Chege (2015) carried out a study in Gatanga Sub-County, Murang'a County, Kenya to examine and establish the errors made by secondary school students that affect success in solving word problems in mathematics. This study's findings revealed that students made a wide range of errors when tackling mathematics word problems. These mistakes included computational errors, incorrect equations, incorrect facts, premature approximations, and incorrect formulation. In addition, Simiyu (2012) also conducted a study that looked into mathematics teachers' feedback on students' errors in a form two-level class at a public secondary school in Bungoma County, Kenya, and the findings showed that providing feedback in a large class is difficult.

These studies did not examine how teachers deal with and respond to errors that learners make during the learning of mathematics hence the teachers may not know what prompted those errors and they may end up not identifying areas where the learners need more support in their course of learning mathematics (Brodie, 2014). This consequently brings to the fore the question: how do teachers in our Kenyan school system deal with the students' errors in mathematics? This research project,

therefore, considered this question and explored various ways on how teachers deal with students' errors in mathematics in a secondary school in Siaya County, Kenya intending to describe how the teachers deal with these errors in order to propose how the current practices may be improved.

1.3 The Rationale and Significance of the Study

The study was guided by the following rationale and significance;

1.3.1 The Rationale of the Study

Several scholars have discussed the significance of mathematics, citing several ways in which it affects other subjects in the curriculum and life in general. Having taught mathematics in secondary schools in Kenya for over sixteen years and having gone through rigorous training on teacher practices in the classroom during our mathematics education classes aroused my interest in learning how teachers deal with students' errors during teaching, learning, and assessment of mathematics. This is because it is demotivating for teachers to teach the students with an expectation that they will achieve the best in their studies, but as a result of errors made in mathematics, watch them as they attain low levels of achievement in mathematics. As a result, the researcher was inspired to conduct this study to find solutions to students' errors and how teachers can deal with them.

1.3.2 The Significance of the Study

The investigation in this study was expected to result in the contribution to knowledge about student errors and how teachers deal with them. The findings could also be a source of information for teacher educators to use in planning for professional development to improve teachers' ability to deal with students' errors. The study could also inform teacher educators about the importance of emphasising the skills for dealing with errors made by students when preparing teachers during teacher education. The study's findings could also be a source of knowledge for other researchers planning to replicate the study in other contexts to help develop more knowledge and awareness of students' errors and how teachers should deal with them.

1.4 The Purpose of the Study

The purpose of this study was to explore how teachers identify, communicate and follow up students' errors in mathematics during teaching and learning in class and outside the classroom.

1.5 Research Questions

To explore how teachers deal with students' errors in mathematics and to propose ways of how to improve how teachers deal with these errors in the study context, the following questions guided the study:

1.5.1 Main Research Question

How do teachers in a secondary school in Siaya County deal with students' errors in mathematics?

1.5.2 Subsidiary Questions

1. How do teachers identify students' errors in mathematics?
2. How do teachers communicate the errors to students in mathematics?
3. How do teachers follow up the errors with the students in mathematics?
4. What challenges do teachers encounter in following up on students' errors?

1.6 Scope of the Study

1. The study focused on how teachers deal with individual student errors.
2. The researcher concentrated on identifying, communicating, and following up on students' errors in mathematics content.
3. The researcher worked with two class levels, form two and three, at a rural public mixed day and boarding secondary school.

During the research, it was challenging to determine the magnitude to which FGD, document analysis, observations, and interviews as data collection methods impacted how teachers dealt with students' errors.

1.7 Assumptions of the Study

The study was based on the assumption that mathematics teachers in secondary schools in Kenya, particularly in the study area, identified, communicated, and followed up on learner errors while teaching mathematics, and that the presence of the researcher in the context of the study did not have a significant impact on their usual pedagogical practices.

1.8 Limitations of the Study

The study was carried out at a time when students and teachers have come back to school after the lockdown caused by the coronavirus pandemic that necessitated that the school calendar be restructured to have four school terms in a year as compared to the traditional three terms in a year and these terms were very

short therefore teachers had a lot of work to cover within a very short time hence making them not to available to the researcher at all times when needed.

1.9 Definition of Key Terms

This study adopted the definition of mathematical error as defined by Verwey (2011) which states that mathematical error is a learners' response, question, or contribution that is different from what is expected. The study also adopted the definition of assessment as a "technique of collecting facts and data-both qualitative and quantitative, to track a student's progress helping in planning future educational course of action" (Mohan, 2016).

1.10 Organisation of the Study Report

The study report is organised into five chapters. Chapter one presents the background and overview of the study as chapter two presents a review of literature on the concept of students' mathematical errors while chapter three presents the discussion on the methodology used. Chapter four presents the findings, gives an analysis based on the findings and discusses the findings and chapter five summarises the findings, presents conclusions and implications based on the findings and suggests possible areas for future research.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This study explored how teachers deal with learners' errors in mathematics and this chapter focused on the review of relevant literature that has a significant role in this study intending to justify why this study had to be undertaken.

2.1 The Concept of Learners' Errors in Mathematics

The understanding of how students learn mathematics is essential for understanding the mistakes, errors and misconceptions they make, as well as how one can incorporate these into their teaching to assist every child's mathematical mental development as well as examining how children learn mathematics effectively and what teachers can do to effectively and efficiently support learning of mathematics. In addition, there are several proverbs and sayings from different communities around the world that carry a lot of encouragement to people and learners to make mistakes and learn from them, for instance, Mahatma Gandhi once said that *"Freedom is not worth having if it does not include the freedom to make mistakes"* (Shallenberger, 2015, p. 248). On the other hand, even though making errors while learning is prevalent, it is also regularly viewed as something negative, shameful, and self-threatening, preventing people from seeing errors as learning opportunities (Steuer et al., 2013). From this, it is notable that errors can be very beneficial for learning as long as individuals make educated guesses because when their errors are corrected, they are very likely to recall that information later. However, this is heavily reliant on responses given by the teachers and students must not only receive the correct answer but also pay attention to it and process it. If an individual processes not only the correct answer but also additional explanation or analysis, the amount of learning from errors increases and learning from these mistakes is also more effective if the student makes the error rather than being given incorrect information and an explanation for why it is incorrect (Metcalfe, 2017). In this light, therefore, it was critical to examine what these errors are, how they are made, why they are made, and how teachers can deal with them while teaching in a mathematics class.

According to Nebel (2019), an error means that an incorrect association has been made and will need to be replaced with the correct association and this is

supported by Verwey (2011) who stated that a mathematical error is a learners' response, question, or contribution that is different from what is expected. In addition, most of the literature in mathematics define errors in relation to what causes them and Hansen et al. (2020, p. 1) while citing Ryan and Williams (2007) noted that "an error is principally formed within the surface levels of knowledge and as such a child's response to a task is procedural and can be corrected by the teacher while providing correct alternatives" (p.1). It is also important to highlight misconception alongside errors since the two terms are related in their application in mathematics and according to Barmby (2009) as cited by Hansen et al. (2020) misconception is used when a student's idea is said to be in disagreement with the recognized meanings and understandings in mathematics.

In mathematics, there are at least four types of errors made by learners: **procedural errors,** which occur when a student does not follow the correct steps or procedure, **factual errors**, which are computational, occur when a student is unable to recall a fact needed to solve a problem because they have not mastered basic facts, **careless errors,** which occur when students do not pay attention or work too fast in doing mathematics and **conceptual errors,** which occur when a student has misconceptions or does not understand a basic mathematics concept or applications (Lai, 2012; Oktaviani, 2017).

According to Wijaya et al. (2014), errors are made by learners due to numerous reasons that comprise; an error made as a consequence of carelessness, misinterpretation of symbols or texts, failure to understand what a task is testing, misunderstanding the initial instructions, error in converting a word problem to a mathematical problem, misunderstanding a keyword, an error in choosing the right information to use, taking into account the problem's context without regard to the mathematics, using incorrect operations, lack of pertinent experience or knowledge connected to the mathematical concept, error in providing the solution in the correct context, leaving a task unfinished, a lack of awareness or inability to check the solution given or the result of a misconception. These propositions are supported by Lai (2012) who argues that most of the learners' errors in mathematics are caused by lack of conceptual knowledge, poor attention, carelessness, inability to process information at the rate of instructional pace, a lack of adequate opportunities to practice the given mathematical tasks, a lack of specific feedback from teachers

regarding misunderstanding or non-understanding, anxiety about mathematics, and visual and/or auditory processing difficulties.

Teachers use a variety of strategies to deal with errors in the classroom, and how a teacher handles an error is determined by the attitudes of the learning community; thus, some teachers try to avoid errors made by learners at all costs, while others try to help students learn from them (Demirdag, 2015). According to Metcalfe (2017) teachers in the United States of America avoid errors in the classroom and set methods or procedures for dealing with specific types of difficulties that are taught explicitly hence making these proper methods or procedures practised and emphasized as errors are avoided or ignored and this ends up not allowing students to think critically, criticize, reconsider, evaluate, or explore their thought processes. In contrast, the norm in Japan is extensive discussion of errors, including the reasons for them and how they may appear plausible but still lead to the incorrect answer, as well as discussion of the route and reasons to the correct answer, and such in-depth discussion of the thought processes underlying both actual and potential errors encourages exploratory approaches by students (Metcalfe, 2017). This is evident according to the OECD PISA reports of 2018 and 2015 indicating that Japan leads the US in the average score on the mathematics literacy scale by the education system (Peña-López, 2016; Schleicher, 2019). A review of the literature on errors reveals that errors are important in mathematics learning and a teacher's perspective on student errors influences their error-handling practice (Bray, 2011). In contrast to strategies that emphasize error avoidance, making learning more challenging by allowing for false starts and errors followed by feedback, conversation, and correction may finally result in a better and more adaptable transfer of skills to later critical situations. This current study is aimed at describing current teachers' practices in dealing with learners' errors during mathematics teaching and learning.

With this background information on errors and why they are made, the discussion now shifts to how teachers can deal with these errors while teaching in a mathematics class through identification of the errors, communication about the errors to learners and follow up of the errors with the learners.

2.1.1 Identification of Students' Errors in Mathematics

Identification of specific errors in learners work during the teaching and learning of mathematics is incredibly significant for students with learning difficulties and underperforming learners since by identifying a student's errors, the teacher can provide teaching that is tailored to the student's specific needs and it is usually the first step in providing remedial or corrective instruction (Lai, 2012). Teachers should identify students' errors for several reasons, including identifying trends of errors or mistakes that learners make in their work, understanding why learners make the errors, and providing focused teaching to address the errors (Oktaviani, 2017). Errors can occur in students' verbal responses or written work and the errors may recur or persist in the same topic or concept.

Some strategies to help in the identification of these errors have been suggested and they include: collecting and collating student's work for a particular type of problem, question, task or concept that errors are arising from; allowing the student to think aloud or to verbalize or to write without the teacher providing any help, prompt or cues; and the teacher records all the student's responses given in verbal or written format usual in a table (Lai, 2012). These strategies are not fixed and may vary from one teacher to another and this leads the researcher to the next discussion about how errors are communicated in a mathematics class.

2.1.2 Communication of Students' Errors in Mathematics

The provision of communication on learner errors is viewed through the formative assessment perspective of the teaching and learning process and the provision of feedback.

2.1.2.1 Formative Assessment

Assessment is widely accepted as one of the most essential components of teaching and learning and according to Mohan (2016), it is the technique of collecting facts and data that can be both qualitative and quantitative, to track a student's progress, helping in planning future educational course of action. The method of gathering, reviewing, as well as using the information to enhance current performance is referred to as assessment, whereas the method of making judgements based on defined requirements and the proof is referred to as evaluation (Weurlander et al., 2012; Yambi, 2018). These two key objectives are inextricably related and, to varying degrees, apply to most assessment tasks. Formative assessment is characterised by

those activities that are undertaken by teachers and/or their learners and in which judgments about the quality of students' achievements are made and this information is used as feedback to facilitate and modify the learning and teaching activities that the students and teachers are engaged in (Trumbull & Lash, 2013).

2.1.2.2 Feedback on Learner's Errors

Feedback is defined as the process by which students interpret information regarding their performance and use it to improve the quality of their work or learning approaches or it is the information provided to learners about their performance concerning learning goals or outcomes (Henderson et al., 2019). By aligning effort and activity with an outcome, feedback redirects or refocuses either the teacher's or the learner's actions to achieve a goal and it can be about the activity's output, the activity's process, the student's management of their learning or self-regulation. This feedback can be given verbally, in writing, through tests, or through digital technology and it could come from a teacher, someone taking a class, or a peer.

Descriptive feedback is a communication that can either be oral or written which highlights gaps in understanding and especially inform learners on how they can enhance their learning rather than enumerating what they got wrong, enabling a mutual learning experience among students and teachers (Filsecker & Kerres, 2012). In addition, the formative assessment allows teachers to gather data that indicates students' needs and this allows teachers to create a comprehensive learning environment that will encourage every learner to develop once they recognize what learners need to be able to succeed (Mandinach, 2012). Based on this, it is important to note that formative assessment is key in the process of communicating learners' errors since the teachers would identify the gaps between the expected correct work from the student and the work given by the learners which at times contains errors that are identified, communicated to the learners and used by the teachers and learners to build on the correct ideas.

2.1.2.3 Verbal Feedback on Errors of Learners

Verbal feedback is beneficial since it is frequently provided during or immedia tely following the learning or task, and it allows for even more discussion between the teacher and student, ensuring that they comprehend the feedback and allowing each o ther to respond to it and act on it right away. There are various forms of verbal feedback serving various purposes as described in the next section.

2.1.2.3.1 Evaluative verbal feedback

Evaluative verbal feedback is a type of feedback on learners' errors that judges the learners' performance on a task. It is one of the most commonly used types of verbal feedback in classrooms by teachers because it involves the use of phrases such as ‘no,’ ‘is it true?’, ‘not really?’ and others which are common words in our classrooms hence assisting the learners to perform well in their next tasks (Brookhart, 2017; Molloy & Boud, 2014).

2.1.2.3.2 Corrective feedback

Corrective feedback is a form of oral feedback that is given as a response to a learner’s utterances that contain an error (Li, 2014). Its purpose is to indicate the error, provide the correct version of the anticipated response or provide detailed information about the nature of the error.

2.1.2.3.3 Repetition

Repetition is a type of verbal feedback in which a teacher repeats a learner's response that contains an error to evoke more details, and the teacher may use prompts to help the learner correct the incorrect response (Lyster et al., 2013).

2.1.2.3.4 Scaffolded feedback

Scaffolded feedback was a method designed to take advantage of the benefits of retrieval practice by providing incremental hints until the correct answer could be self-generated and has the following features: it should be graduated with no more help than is needed; it should be contingent on learners' needs; and it should be dialogic, with both the learner and the interlocutor collaborating to solve the problem and these three features distinguish scaffolded feedback from the other kinds of prompts (Rassaei, 2014).

2.1.2.4 Written Feedback on Errors of Learners

At the centre of effective learning is good feedback that is written. It is all about the teacher bridging the gap between what the students already know, comprehend, and can be able to do and what they will know, understand, and be able to do in the future. The teacher's role is not to close this gap for their students, but to assist them in closing the gap for themselves. Teachers can provide this assistance because of the effective marking and feedback that they give to learners during the

process of teaching and learning. There are various ways of giving written feedback to learners that serve various purposes;

2.1.2.4.1 Marks, Grades and Scores

Grades, marks and scores assist students in determining where they are on their journey to mastery of important learning goals (Guskey, 2019). Grades, marks and scores are usually given to students after assessment and evaluations have been done and they help students together with their teachers know how to rate the students in terms of their academic achievement. Marks, grades and scores provide a very open and direct way of letting learners know what they have got right and what they have got wrong hence providing a very good way of pinpointing errors of students.

2.1.2.4.2 Comments

Comments that identify what learners did well, what advancements they require to make, and how to make those advancements, when provided with sensitivity to essential contextual aspects, can guide learners on their pathways to learning success and ensure that all students learn excellently (Guskey, 2019). Comments are also a good way of pointing out parts of students' work in which they have made mistakes or errors and also a way that teachers use to provide help in supporting error correction.

2.1.2.4.3 Descriptive Written Feedback

This is detailed written feedback that provides students with specific information and explanations about why their responses were incorrect and how it will help them improve their learning (Cheruiyot, 2014). Furthermore, it is linked to the expected learning and addresses defective interpretations and a lack of understanding. It also provides learners with noticeable and controllable next stages predicated on an assessment of the task at hand and a picture of what great work looks like, enabling them to start to assume responsibility for self-assessing and self-correcting.

2.1.2.4.4 Error Flagging

As a feedback provision strategy, error flagging involves making marks such as encircling the error, underlining, placing question marks or a dot that locates the error, incomplete solutions, or a faulty strategy used by the learner (Simiyu, 2012). It assists the teacher to signal to the learners where an error has been committed without

necessarily providing the correct answer or method to the students and at the same time it helps the students to know the particular point where they made a mistake thus they can make corrections (Jackson, 2015).

2.1.3 Following up on Learners' Errors

For one to follow up on something or action or happening means that the person has to try to get more information about it or to do something in response to it or to take appropriate action about its happening (Merriam-Webster, n.d.). It is important to note that the kind of response that would be given to students once they have made errors in their learning depends on every individual teacher therefore these responses may not be the same.

According to Lai (2012), once the teacher has gone through the process of identifying errors of the students, the teacher analyses the responses as they look for patterns of common errors in the responses as the teacher also looks for exceptions in the patterns that could signal other pointers to the error. The teacher then describes the patterns observed in the simplest language possible while giving possible reasons for the errors noted and talking with the learner to allow them to explain how they solved the problem intending to confirm the error that is suspected (Lai, 2012). This is possible once the teacher has collected information from various students regarding the errors committed in a particular concept or question which are then recorded (Nuraini et al., 2018).

Another important strategy of following up on errors is by allowing an atmosphere in the classroom that allows the making of errors and allowing the students to discuss their errors. By allowing this kind of atmosphere and the discussion on errors the perspective of the student who made the error and the perspective of their peers about errors are changed and the situation ends up providing a learning point about the concept in which the students had made an error (Arani et al., 2017). However, most teachers are reluctant to discuss errors in their classes because they believe that the discussion is time-consuming and they also fear that errors may lead their students into making those same errors hence getting the same incorrect responses that they are trying to eradicate (Rushton, 2018).

Explaining the reasons why the responses given by the students are incorrect or having errors by the teacher is also an important aspect of following up on learners'

errors (Arani et al., 2017). In addition, the explanation on why errors are made assists the learners to understand the difficulty that made them make the errors and assists the teacher to make the learners not fear making mistakes as there is an opportunity for correction as well as allowing teachers to provide interventions for the errors made by the students (Nuraini et al., 2018).

During the process of following up on learners' errors, it is also crucial for the teachers to open up a discussion forum to help the learners justify their answers which are incorrect as this helps the learners to interact with one another allowing each of them to interrogate each other's responses to help them learn from their mistakes (Lai, 2012). Furthermore, discussion-based teaching and learning lead to significant learning outcomes such as assisting learners in reconciling opposing arguments, reducing fear of sharing personal views by building relationships among fellow students, and integrating fresh concepts from divergent viewpoints and fields of study hence helping in bridging the gap between errors made and correct responses (Ying, 2020).

2.2 Theoretical Framework

This section presents the theories that this study was hinged on and these theories included; Constructivism Theory, Mediation Theory, Recursive Reminding Theory, Prediction Error Theory, Negative Knowledge Theory and Reconsolidation Theory

2.2.1 Constructivism Theory and Learners' Errors in Mathematics

The proponents of the constructivism theory postulate that new knowledge is developed by linking it to existing knowledge and it eventually accentuates that learners generate meaning rather than acquiring it (Clark, 2018). In the scope of this research, constructivism was considered as a process in which learners play an important and active role in their learning by constructing their appreciation of new knowledge and making sense of whatever information is available to them based on previous knowledge and individual experiences (Kalpana, 2014).

Human beings are actively involved in the creation of their knowledge, according to social constructivism, a social learning theory developed by Russian psychologist Lev Vygotsky who held that learning occurs predominantly in social and cultural contexts rather than exclusively within the individual (Schreiber & Valle,

2013). According to Johnson and Bradbury (2015), the social constructivism theory focuses profoundly on dyads (something that consists of two elements or parts, in this case, the teacher-learner dyad) and groups. Therefore, this theory was significant because it underpins the fact that students learn mainly from their interactions with their teachers and through discussions created within the classroom environment by their teachers.

The Zone of Proximal Development is a central concept in Vygotsky's theory of social constructivism, emphasizing the role of the teacher in an individual's learning while it also distinguishes between activities that a student can perform without the assistance of a teacher and those that the student cannot perform without the assistance of a teacher, therefore, it implies that, with the assistance of a teacher, students can comprehend and master skills and knowledge that they would not be able to do on their own and once students have mastered a skill, they can complete it on their own while, the teacher, rather than being a passive figure, plays an active role in the students' acquisition of knowledge (Davis et al., 2017; Schreiber & Valle, 2013). This was important in this study since the teacher has a role in identifying, communicating and following up on learners' errors during the teaching and learning of mathematics.

Psychological constructivism is attributed to Piaget who held that new knowledge is actively constructed in light of students' current knowledge and understanding of the world, as well as their subjective interests, past experiences, and emotions (Van Bergen & Parsell, 2019). In addition, the central idea of psychological constructivism is that a person learns by mentally organizing and reorganizing new information or experiences and that this organization is facilitated in part by relating new experiences to prior knowledge that is already meaningful and well understood (Seifert & Sutton, 2019). It is on this background that this study hinged itself on this theory since the role of the teachers is to create an interactive class but still focus on the individual learners as they build on what the learner had already known previously and while the teachers are doing this, they have to ensure the learners take an active role in the learning process rather than being passive recipients of information from the teacher as this will help them acquire skills that are required in the 21st century such as problem-solving and collaboration (Keiler, 2018).

Based on the above submissions, the teacher, while dealing with learners' errors in mathematics, has a role of ensuring that an interactive environment is created for learners in the constructivist classroom while at the same time ensuring that learners create their knowledge from their sources. In this process of dealing with learners' errors, it will be significant for the teachers to recognise the relevance of the individual learner to prepare, plan and execute a programme that will add value to the learner's perspectives as an individual during the teaching and learning sessions. Additionally, as a constructivist pedagogue, the teacher has to adapt teaching and learning strategies that address learners needs during interactions in class as the teacher deals with the learners' errors to ensure that learners learn from the social happenings in class. Thus, as the teachers deal with learners' errors, they should keep in mind the constructivists' core concept that learners construct their knowledge from their internal sources and the experiences of the situations around them is pivotal to their learning improvement and hence their errors should be identified well, communicated appropriately and meticulously followed up throughout the course of teaching and learning of mathematics to use this negative knowledge learnt through experience about what is wrong and what should be avoided while performing a given task and to acquire positive knowledge about correct facts and procedures (Rach et al., 2013). Lastly, the teacher should be able to be a co-learner in the classroom by being able to deal with the errors of the learners, understanding the errors and offering a solution to the errors of the learners for a better learning experience for the students.

According to Osborne (2021), the constructivist viewpoint has very little to suggest about the nature of effective pedagogy and this is supported by Bagonza (2015) who suggests that it is important to highlight the fact that the constructivism theory places more importance on learning than teaching in the teaching and learning process. It is therefore critical to recognize that this reveals an inadequacy because these two processes of teaching and learning are interdependent, and one may not succeed without the other. In this regard, both teachers' and students' contributions to the effective teaching and learning process should be recognized for learning amplification. While disagreeing with the points of view above, Fernando and Marikar (2017) predicate that teaching entails the transmission of knowledge but it is also much more and teaching theories must be sensitive to the processes by which learners gain knowledge, or how students learn therefore constructivist learning and

teaching theory has a lot to offer when it comes to the processes by which learners obtain knowledge. Based on the fact that teachers contribute to the processes by which learners gain knowledge, this theory would be significant to the processes by which teachers follow up on learners errors including re-teaching a concept, consultation of teachers by learners and provision for error awareness.

2.2.2 Mediation Theory and Learners' Errors in Mathematics

When a mistake is made, one of the strategies for effective knowledge or skill acquisition is the learning error mediation process, which essentially consists of presenting a help message to the learner (Ogawa et al., 2018). Making the error the subject of educational activity, that is, problematizing it and incorporating it into a development process, is what mediating the error entails (Ogawa et al., 2018). An error indicates a contradiction in the thinking process, an inconsistency that the learner is unaware of, and it is then up to the teacher to point out this contradiction, making it the gateway to a higher level of awareness (Ogawa et al., 2018).

Furthermore, in an attempt to explain why testing aids learning, it was proposed by Carpenter (2011) that when people are tested, they are more likely to produce effective mediational retrieval cues. An intriguing prospect is that erroneous answers serve as guideposts or stepping stones rather than as competitors to the right answer and this hypothesis is consistent with findings demonstrating that only errors linked to the target help assist later recall, and it is also indicative of the fact that the errors have to be self-generated to help (Metcalfe, 2017). However, in most educational settings, unfavourable material conditions can prevent or impede error mediation. The knowledge about mediation is important in this study since it will assist the teachers to provide the learners with messages that would assist in scaffolding the students' learning whenever they make mistakes.

2.2.3 Recursive Reminding Theory and Learners' Errors in Mathematics

Creating long-lasting and transferable knowledge from individual episodes is critical to success in an ever-changing world; therefore, to thrive in a complex world, we must apply knowledge gained in previous situations to new and different situations, and at the same time, being reminded has been shown to generate generalizable knowledge from individual episodes, as well as to support generalization in problem-solving and category-learning (Tullis & Goldstone, 2016).

There is a possibility that errors, as well as elements of the context and situation in which an error is made, have a facilitative effect insofar as they are related to trying to retrieve the original episodic incident in which both the correct answer and the mistake were embedded and individuals may be able to remember the context in which they made an error and thus consider not only the error but also the situation in which it was stated clearly that the error had occurred and that the correction was the preferred solution (Jacoby & Wahlheim, 2013). The knowledge about recursive reminding will therefore assist the teacher to ensure that the learner is repeatedly reminded about the context and situation in which an error was made for the learner to be able to remember the mistake and to correct it with the help of the teacher.

2.2.4 Prediction Error Theory and Learners' Errors in Mathematics

A prediction error is primarily defined as a mismatch between a prior expectation and reality and because the world is constantly changing and is an environment that is having patterns that are not easily predicted, these predictions must also be constantly changed and fine-tuned in response to new information that may contradict the organism's prior expectations (den Ouden et al., 2012). Designs of prediction error define how we gain knowledge where the bad things prowl and where the good things live, as well as what actions are needed to avoid or seek them out (den Ouden et al., 2012).

2.2.5 Negative Knowledge Theory and Learners' Errors in Mathematics

People build two complementary types of knowledge, according to the theory of negative expertise: positive knowledge on correct facts and processes and negative knowledge on incorrect facts and processes therefore learning by making mistakes is regarded as acquiring negative knowledge (Heinze & Reiss, 2007). In addition, Oser et al. (2012) posit that knowing what is wrong helps one to understand what is right and define negative knowledge as "memories that are related to events, things, procedures or strategies that are experienced as false, inadequate or even ineffective" (p.54).

Negative knowledge aids in the creation of a helpful alert against repeating the same error (protection), the differentiation of contrasts (bad and good), the orientation of opposite characteristics, and the production of certainty in addressing a problem using a specific procedure (Oser et al., 2012). Knowledge about negative knowledge

is important in this study since it would help teachers to understand how their learners think and how to use this understanding to respond to learners' errors to help the learners to prevent further errors. The knowledge of negative knowledge is also important since it will assist the learners to have the motivation to learn by overcoming their errors.

2.2.6 Reconsolidation Theory and Learners' Errors in Mathematics

A framework for reconsolidation was developed earlier before the error correction paradigm to address the fear conditioning paradigm by looking at methods that help in overcoming conditioned fears (Dudai, 2012). Even though there are differences in the two paradigms they also have similarities that point to assisting in making corrections to errors in the learning environment.

Some errors are made because of conditioned fear and for them to be altered or eradicated then the wrong response has to be evoked first and when the wrong response has been evoked there is a chance that there is a limited time frame in which the undesirable response can be eliminated, altered, and reconsolidated but if the erroneous response is not evoked then it is said to be buried or unchanged (Metcalfe, 2017). The reconsolidation structure is coherent with the general thrust of the findings of numerous studies, which show that the likelihood of generating the correct answer is higher when the error is first retrieved in conjunction with the new, overwriting stimulation, as opposed to when the pre-existing error is not evoked and only the correct answer is provided (Metcalfe, 2017).

This knowledge on reconsolidation is significant in this study because it would provide the teachers with an understanding of the learners' errors and why they are made. Once a teacher has identified that the learners' errors are made due to conditioned fear then the teacher would evoke the erroneous responses to eliminate, alter, and reconsolidate the error.

2.3 Gaps in Existing Knowledge

In Kenya, there is very little literature on students' errors and how teachers deal with the errors in a mathematics class. The study done by O'Connor et al. (2016) on Sources of Student Errors and Misconceptions in Algebra and Effectiveness of Classroom Practice Remediation in Machakos County, Kenya, found out that the major challenge appears to be the teachers' ability to use the knowledge they have on

student error, rather than their awareness of the errors. This resulted in instructional strategies that did not address students' difficulties, as well as the discovery of flaws in algebra instruction. According to the findings of the study, teachers require assistance not only in identifying errors but also in understanding how errors are constructed throughout the learning process. In addition, Chege (2015) carried out another study in Gatanga Sub-County, Murang'a County, Kenya to examine and establish the errors made by secondary school students that affect success in solving word problems in mathematics but this study only focussed on errors in word problems and the study's findings revealed that students made a wide range of errors when tackling mathematics word problems. Another study carried out by Simiyu (2012) only focus on providing feedback on the errors made by the learners in mathematics. However, there is a need to build on this literature and provide clear information on how teachers deal with the errors that learners make during the course of the teaching and learning process through identification of these errors, communicating them to the learners and finally following them up to help the learners use their errors to gain more conceptual understanding in mathematics.

2.4 Chapter Summary

In this chapter, the researcher has provided an overview of the key concepts from the literature on mathematical errors and how teachers deal with students' mathematical errors. The researcher also reviewed the literature on how teachers identify, communicate, and follow up on students' mathematical errors, revealing a gap in the literature on error handling in the classroom in the Kenyan context. As a result, this literature review establishes the need to understand current practices in mathematics on how teachers deal with learners' errors as a first step toward closing the gap and improving mathematics teaching and learning in this context. In the following chapter, the researcher described the methodology used to gather data for this study.

CHAPTER THREE

RESEARCH METHODOLOGY

3.0 Introduction

In this chapter, the researcher describes the methodology that was used in the study on exploring how teachers deal with students' errors in mathematics. A discussion about the appropriateness of the qualitative research approach and case study research design is undertaken, followed by a brief description of the school where data was collected from, the sampling procedure and the participants. The data collection methods employed in the study which included interviews, FGD, lesson observation and document analysis are discussed as well as the data collection tools. Thereafter, the researcher embarks on explaining the data analysis procedure and how ethics and trustworthiness were upheld**.** A summary of the chapter is finally provided.

3.1 Research Approach

This study was underpinned by the constructivism philosophical worldview that assumes that human beings are interested in understanding the world they live and work in while they develop their experiential subjective meanings (Creswell, 2014). This study, therefore, used a qualitative research approach since the study was an exploration of how teachers deal with errors that studied a group of people while collecting variables that cannot easily be measured and the need to have a complex and detailed understanding of the issue to be investigated (Creswell, 2013). Furthermore, the researcher intended to delve deeper into people's experiences and practices for dealing with learners' errors in a mathematics class, and Boddy (2016) demonstrates that the primary goal of qualitative research is to gain a deeper and better understanding of a phenomenon.

3.2 Research Design

The researcher employed the case study design as it was the most appropriate for facilitating the researcher's engagement in the context of the school while obtaining an in-depth appreciation of the issue being investigated (Crowe et al., 2011). In addition, this design was used because it helped the researcher to find reasons for patterns observed, particularly invisible or surprising patterns during the study (Busetto et al., 2020). The researcher interacted with teachers and students in a realistic natural classroom setting (as well as in a school setting) to better understand

how teachers dealt with errors made by students in the course of the teaching and learning of mathematics. This was consistent with the reality that a case study is a deliberate choice made by a researcher in order to capture contextual details that may be critical for a successful study.

3.3 Context of the Study and Demographic Information of Participants

The site for this study was a multi-streamed mixed day and boarding secondary school in a rural setting in Siaya County, Kenya, conveniently sampled because the school was easily accessible from where the researcher lives hence lowering the cost of the study (Etikan et al., 2016).

3.3.1 Context of the Study

Mwenge Secondary School (pseudonym) is a County, Mixed, Day and Boarding Secondary School in Siaya County, Kenya with four streams in all classes from form one to form four with a total population of about 1000 students and is endowed with average learning facilities including classes and science laboratories. The school has a catchment area of students who learn in primary schools around it and also from primary schools all over the country who get admission as day scholars or boarders.

Out of the 4 classes (form one to four) in the school, two classes were chosen for the study: form two and form three and in each class, there were four streams of which a stream was chosen in each case; Form 2 East and Form 3 North. Form 2 East has a total of 57 students out of whom 32 were boys and 25 were girls while Form 3 North had a total of 66 students out of whom 40 were boys and 26 were girls. A total of 12 students were chosen for FGD, 6 each from Form 2 East and Form 3 North.

3.3.1 Demographic Information of Participants

Out of the 11 teachers who teach mathematics in the school, 9 are males and 2 are females. From this team of mathematics teachers, 2 were selected to participate in the study because they were teaching the selected classes.

Bahati (pseudonym) is the mathematics subject teacher in Form 2 East. He is a graduate who is employed by TSC and is currently the HoD of Mathematics in the school and he has attended several SMASE INSETs over the years.

Zawadi (pseudonym) is the mathematics subject teacher in Form 3 North class. He is a graduate employed by the TSC who has attended INSET in SMASE, and he is a trained and practising National Examiner of Mathematics Paper Two.

3.4 Sample and Sampling Procedure

The study concentrated on Form Two and Form Three classes in the chosen school because teachers in Form Two and Form Three class levels have had significant time with the students in the previous years to predict areas where students make errors. Furthermore, teaching, learning, and assessment practices in the examination class of Form Four were frequently influenced by the pressure of high-stake external examinations and the Form One class had just reported to the school, and it was assumed that the Form Two and Form Three classes were free of any pressure and were the best for the study. Therefore, the participating teachers and the classes were purposively sampled since this would help in identifying and selecting information-rich cases to make the best use of limited resources (Palinkas et al., 2015). Stratified random sampling was used to select the twelve participating students since the goal of stratified random sampling was to select a sample in which the target sub-groups were represented in the sample in the same proportion that they exist in the population (Omona, 2013).

3.5 Data Collection Strategies

A variety of qualitative data collection methods, including observations, document analysis, FGD and interviews were used, given that data from multiple data methods and sources were very useful to select the themes arising from the study (Busetto et al., 2020).

3.5.1 Interviews

The study involved conducting one-on-one semi-structured interviews with the participants to establish the participants' bio-information naturally, as well as to address the four subsidiary questions of this research on how learners' errors are identified, communicated, and followed up, the challenges teachers face in following up learners' errors, and how they overcome these challenges. This method typically consists of a dialogue between the researcher and the participant, guided by a flexible interview protocol and supplemented by follow-up questions, probes, and comments, and it allowed the researcher to collect open-ended data, explore participant thoughts, feelings, and beliefs about a specific topic, and delve deeply into personal and

sometimes sensitive issues while using it to verify and refute impressions gained from observations and document analysis (DeJonckheere & Vaughn, 2019).

3.5.2 Observation during Lessons

Classroom lesson observations were employed in this study to address three subsidiary questions of this research on how learners' errors are identified, communicated, and followed up. The observation was chosen for this study because it aided in the collection of data in a more natural and social setting (De Chesnay, 2014). Furthermore, observation was selected because it allowed the investigator to come into contact with the participants while gathering rich data and perspectives on the respondents' beliefs and values (Schneider et al., 2016).

3.5.3 Document Analysis

The learners' written and marked notebooks and marked examination scripts were an important source of data because they contain written errors as well as teachers' written communication about the errors. Although they were not written and marked specifically for the research study, they contributed data that directly answered two subsidiary questions for this study on the identification and communication of learners' errors. Analysis of documents such as school records, learners' written and marked exercise books, or scripts, can provide valuable contextual information to supplement data collected through interviews and observation (Yin, 2011).

3.5.4 Focus Group Discussion

FGDs are a more specific in-depth group interview with a discussion that is done in a structured and organized way with the help of a facilitator or moderator where the selected topics are explored and it assists in data collection because it promotes interaction among participants with spontaneity (Gundumogula, 2020).

3.6 Data Collection Instruments/Tools

In this study lesson observation guide (Appendix I), interviews guide (Appendix F), focussed group discussion guide (Appendix G), and document analysis framework (Appendix H) were used as data collection tools. An audio recorder and a camera were also used during the collection of data in this study.

3.7 Data Analysis

Data analysis started as soon as the data collection began and it occurred after field access had been obtained, sampling decisions had been made, and data had been collected, recorded, and changed to text type (transcribed) into protocols and transcripts (Flick, 2013). The protocols and transcripts were then coded, using labels or tags having one or more short descriptions of the content in a sentence or paragraph and this coding was being done manually (Busetto et al., 2020).

3.8 Trustworthiness, Rigour, Credibility and Transferability

Throughout the study, the researcher maintained high standards and rigour in data collection, analysis, and reporting. Multiple data collection methods served to reinforce the evidence from the research results and ensured validity from convergence from the data (Crowe et al., 2011). In addition, to address trustworthiness, the researcher involved triangulation of the data, self-reflection and provision of thick description of the research site, participants, and research process was provided to make the study more realistic and to allow readers who want to replicate the study in a similar context to do so (Anney, 2014). Furthermore, before reporting, the researcher conducted member checks with the participants for all transcribed data to ensure the veracity of the transcribed data (Birt et al., 2016). Finally, the researcher ensured the data's security and backup of data records.

3.9 Ethical Considerations

Ethical issues are important in research because research deals with and intrude into people's lives (Carter et al., 2013). The researcher was guided by three ethical considerations throughout the study: informed consent, anonymity, and reciprocity. Before beginning research, the researcher sought approval from the Aga Khan University, Institute for Educational Development (East Africa) for ethical clearance (Appendix M), as well as a research permit from NACOSTI (Appendix N), clearance from Kenya's MOE (Appendix O) and MOICNG (Appendix P).

To gain access to the school, the researcher explained the nature of the research to the Principal and handed him the Information Sheet (Appendix A), to read and sign. The Principal then assisted the investigator in getting in contact with the teacher(s) in the school who participated in the study, who were given a Participant Information Sheet (Appendix B) to read and who then signed an Informed Consent (Appendix C) indicating their voluntary acceptance to participate.

The study included minors as participants whose participation were approved by their parents by signing the Parental Consent (Appendix D). The investigator also sought the learners' assent through Assent Forms (Appendix E), while assuring them that their identities, as well as the identities of the school, would be protected through the use of pseudonyms during report writing. Hard copies of data were being stored in a lockable cabinet, while soft copies were protected by computer passwords.

To address reciprocity and as a way of giving back, the researcher intended to share a summary of findings and recommendations with the school, disseminate the results of the study to the school, provide a copy of the complete dissertation to the school and researcher also participated in any of the activities of the school that did not affect the research process.

3.10 Chapter Summary

The researcher presented the methodology and design of the study in this chapter, as well as the data collection methods and data analysis. Furthermore, the researcher has also presented the steps taken to ensure the study's trustworthiness as well as the ethical considerations. The following chapter presented findings and discussed findings derived from the analysed data.

CHAPTER FOUR

DATA ANALYSIS, FINDINGS AND DISCUSSIONS

4.0 Introduction

In this chapter, the data analysis, findings and discussions of the project will be presented. The chapter will present the key findings and discussions on how teachers deal with students' errors in mathematics which was the main question of the study. The research data collections strategies of interviews, FGDs, lesson observations and document analysis were appropriate in answering the subsidiary questions of the study culminating into the findings presented herein. The chapter will present the perspective on students' errors, teachers' identification of students' errors, teachers' communication of students' errors, teachers' follow up on students' errors, challenges of following up on students' errors and a summary of the chapter.

4.1 The Perspectives on Students' Errors

The perspective of teachers on students' errors in mathematics was based on the data that was collected from interviews and backed up by data from lesson observations that showed that students made different kinds of errors which were both verbal and written. The verbal learners' errors were made in class during teaching and they were responded to by the teachers in various ways while written errors were made by learners on the board, on their notebooks and on their examination scripts that were either marked in class or marked later on. The teachers understand errors as mistakes made by students in the course of doing mathematics and this was confirmed by Bahati who said that "Students make a lot of errors. Sometimes we call them mistakes", and this was supported by Zawadi who posed that "From my view, I would say that these are the mistakes that learners often make when doing maths".

The report on the perspective or views on students' errors was linked to the sources of errors and their importance in teaching and learning of mathematics as brought up in teachers' interviews, lesson observations and FGDs with learners

4.1.1 Sources Students' Errors in Mathematics

The data that was collected from interviews of the teachers and classroom observation of actual teaching showed that teachers pointed the cause of students' errors to; insufficient conceptual knowledge, not following the correct procedures while calculating, being careless or being in a hurry to complete a task, not being able

to remember facts required for a particular task and lack of preparedness for an examination or assignments.

On inadequate conceptual understanding, Zawadi during his interview pointed out that lack of understanding of a concept is responsible for the causes of errors by students in mathematics and said that "One of the causes I could say is maybe lack of understanding of a concept…". In addition to the interview, it was noted during classroom observation that while working on Sine Rule the students were finding the sine of the length of sides of triangles instead of the sine of angles of the triangles and the teacher had to take the learners through the concept and the process of how to handle the task. From this observation, it was noted that lack of understanding of the concept made the learners find the sine of lengths instead of sine of angles hence making errors in their work.

From the observation made during another lesson the teacher gave the following task;

Find θ if $2\ \mathrm{Cos}\ \theta = -1.81$ for $0° \leq \theta \leq 720°$.

A student worked on it as follows

$$2\ \mathrm{Cos}\ \theta = -1.81$$

$$\mathrm{Cos}\ \theta = -0.91$$

$$\theta = 155.5°$$

To find the angle in the third quadrant the student subtracted $155.5°$ from $360°$ giving an angle of $204.5°$. Even though this was one of the correct angles it was identified by the teacher as a wrong concept. The teacher then explained that the angle has to be found in its equivalent acute angle in the first quadrant by taking $\mathrm{Cos}\ \theta = 0.91$ giving $\theta = 24.5°$ then to get the angles the student has to identify that the cosine is negative hence the angles fall in the second and third quadrants and are obtained as follows; 2nd quadrant: 180° - 24.5° = 155.5° and 3rd quadrant: 180° + 24.5° = 204.5°. While the other angles are 515.5 °and 564.5° obtained by adding 360° to the angles found and also fall in the 2nd and 3rd quadrants respectively.

Finally, the teacher explained to the student that even though he obtained a correct answer his method was not correct, and he cannot be able to get correct responses for other tasks using the same method because it was a wrong concept. Data from this observation supported claims from the teachers during interviews about errors that arise due to a lack of conceptual understanding by the students.

From the interviews, it was also noted that errors by students are caused by them not following the correct procedures of handling mathematical tasks. This came during the interview with Zawadi who noted that while students are working on quadratic equations and a negative sign is placed before a variable say x^2 to give $-x^2$, then the value of x should be squared first then multiplied by the negative as a procedure but when faced with a negative value of x the students usually do not follow this procedure ending up making an error as he said;

> If the question is $-x^2$ and then the value of x is negative. The learners only square this negative, but then this other x they forget that it should also be multiplied by the negative that is there. So…you get that the learner gives a wrong answer… if x is -3, you get the learners answering it as 9, but in the real sense, it should be -9.

Further to these, it was also noted from the interviews that some of the learners' errors were caused by the carelessness of the students or being in a hurry to complete a task. During the interview with Bahati he noted that those students who are not composed, panicking or in a hurry usually make errors while working on mathematical tasks and he had this to say;

> I would attribute that to students who are not composed, maybe panicking or in a hurry.., they just scan through the digits like that 23 and they see it as 32..., so that when you present the script to them later on and they see it and they wonder "Wow! How did I see this?" What did I do?

Another source of errors of students in mathematics is not being able to remember facts required for a particular task which usually arise from lack of preparedness for an examination or assignment. The teachers while being interviewed noted that lack of preparation for examinations and assignments makes learners make mistakes that make them not get good grades. They also noted that lack of exposure to different kinds of questions before these examinations and assignments also contributed to these errors that they make as supported by Bahati who said that "…the other apart from being in a hurry is for students who have not prepared well…if a candidate or a student has not been exposed to many different questions, then they tend to make such errors." This view was also held by Zawadi who added that "I could say that lack of exposure to many questions of different natures…I think if

learners can always after being taught and be exposed to these questions. So many questions of variety, then these errors could be eliminated".

From the data, it was noted that teachers blamed students' errors on a lack of conceptual knowledge, not following the correct procedures while calculating, being careless or in a hurry to complete a task, being unable to remember facts required for a specific task and a lack of preparedness for an examination or assignments. These views are consistent with those of Lai (2012) and Oktaviani (2017) who contend that there are at least four types of errors made by students in mathematics: Factual errors, which occur when a student is unable to recall a fact needed to solve a problem because they have not mastered basic facts, careless errors, which occur when students do not pay attention or work too quickly in doing mathematics, and conceptual errors, which occur when a student has misconceptions or does not understand a basic mathematics concept.

4.1.2 Importance of Students' Errors in Mathematics

Data from interviews, FGDs and lesson observations indicated that mathematics teachers and learners had high regard for students' errors as a basis for their teaching and learning of mathematics. The errors that the students make in mathematics assist the teachers to reflect on their teaching to evaluate if they are teaching well or not as pointed out by Bahati

> As a teacher when you give an assignment or some task to the students…you'd find some errors that are cutting across that many of them are making. If you're a good teacher, you should reflect back and probably you as a teacher could be the source of the problem - how they got the lesson when you are teaching. So if you find an error that is cutting across then you have to review how the lesson was delivered because you could have been the cause of that error.

From this, it was noted that the teacher would look at the errors that have been made by the learners and if it is an error that cuts across all learners then the teacher would reflect on this and understand that the concept that was taught was not understood by the learners hence the teacher has to make plans of re-teaching the concept or changing his/her teaching approach. This was further supported by the fact that during a lesson observation of Zawadi in Form 3 North he had to re-teach when

students in the class were finding sine of lengths instead of sine of angles of a triangle. From this observation, it was noted that teachers usually decide to re-teach a concept when they have realised that a great number of learners have made a similar error in their work.

In addition, the teachers also use the errors committed by the learners as a platform for correcting and teaching other students in the same class or other classes as learning points as backed up in this excerpt;

> A few errors that are just isolated. You would identify such and then use those mistakes to teach the others or even the classes that may be coming after that class has gone, yes, so we can identify those errors and use them as learning points, especially if they are not errors made in the final exam.

According to the learners, the errors they have made in their work motivated them to put more effort into their academic work because when they finally get the task right it is very satisfying. This was confirmed by Furaha (pseudonym) during the FGD with the form two students who said;

> They have written for you in your book something like *poor*, yes, you will end up saying why my book has been written *poor*. I'm not poor like that, so you will end up going and consulting for that question until you know that question…you'll get satisfied because you will be saying that you will never get another *poor*. So you can do the maths and get even a *good* and that will please you.

Furthermore, these errors help the teachers to understand areas where the students have difficulty and also allow the teachers to identify the students who need academic assistance in the subject, and this was confirmed by Bahati when he said that;

> What I do mostly when I identify an error written…I usually like addressing that error one on one with the student if it is possible. Sometimes when I mark and I realise that the student is making an error, it pains me when that error is going to cost a student some marks…I would call such a student. Let them look at their work. Some of the students will identify that mistake just by setting their eyes on the paper. When they have already seen their mistake and you see from their faces they are regretting.

This was also evident during various classroom observations where the teachers would move from one student to the other in the classroom assisting them in their areas of difficulty. While having discussions with students they also confirmed that their teachers usually walk around in the class as they teach to check their work and assist them in areas in which they have made errors as stated by Neema as follows;

> The teacher can let you know about an error by the time let's say the way the teacher was walking in class then he found that you have done something very wrong, so he will direct you through the procedure of doing it and then you do it now very well.

Although the teachers considered the students' errors as very important in the teaching and learning of mathematics, it was noted that they did not give reasons to the learners at all times when the learners made errors and sometimes chose to explain it extensively for the candidate class as supported by Bahati who stated that "For the candidates … sometimes we even organise a day that we do quite a lot, we organise a day where we just talk about the mistakes that they make that cost them marks in the final exams." This was supported by Zawadi who said that "… I feel if there is an error, maybe the student is the one who should tell me why that error has been committed."

These data revealed that the errors that students make in mathematics help teachers reflect on their teaching to determine whether or not they are teaching well, as it also serves as a platform for correcting and teaching other students in the same class or other classes as learning points, while it also helps teachers understand areas where the students have difficulty and allows teachers to identify the students' difficulty areas and at the same time these errors assist the learners to improve in their work. These views were consistent with those of Simiyu (2012), who asserted that mathematics teachers viewed students' errors as essential in assisting them to know; students' areas of weakness, identify students who require assistance and focus their teaching process on them. As a result, teachers' practices and teaching choices, such as remedial work, were informed by students' errors.

4.2 Teachers' Identification of Students' Errors

The data collected from the various methods of interviews, FGDs, lesson observation and document analysis indicated that teachers identified learners' errors in both verbal and written ways.

4.2.1 Verbal Identification of Students' Errors

Data collected through lesson observations and interviews indicated that teachers identified verbal errors made by learners during the actual teaching and learning in class. It was noted that most of the errors committed verbally by learners were made when learners were asked a question or given a task to explain verbally or when learners were working on a task on the board and explaining verbally at the same time.

While observing Bahati teaching his class on Similarity and Enlargement he gave the following task;

Task

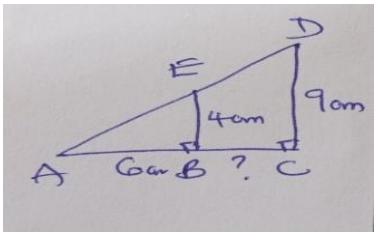


Figure 2: Sample Question

Find the length of BC in the figure above.

A student responded by saying that we subtract the AC from AB and the teacher responded by saying that "not really" and asked another student to respond to the same question. The second student responded by saying that we subtract AB from AC and the teacher replied, "that's right" and reinforced what the student said by repeating what the second student said while explaining that AC is greater than AB. From this, it is noted that the teacher identified the error made by the learner and used the words 'not really' to show that the response given by the learner was erroneous.

While observing Zawadi in class he asked the class what a right-angled triangle is. One student replied that a right angle adds up to 90^{o}. The teacher then asked the student again what adds up to 90^{o} but the teacher did not give the student chance to answer this and asked another student what a right-angled triangle is. The second student replied by saying that they add up to 180^{o}. The teacher once again

asked the student what adds up to 180^{o} but again he did not give the student chance to respond but proceeded to ask another student the same question and the student replied that a right angle triangle has the base and the height intersecting at $90^{o}$and the other two angles are 45^{o}each. The teacher realised that all the students who gave responses did not give the right answer and finally the teacher gave the correct relationships in a right-angled triangle to the whole class.

From the data, it was noted that teachers identify the verbal errors made by learners by using comments that would let the learners know that they made mistakes and also use the questioning technique to either let the student know about the error that they have made or to solicit a response from another student indicating that the initial response was not correct.

4.2.1.1 Questioning

Data from interviews and lesson observations indicated that questioning was the main technique used by teachers to identify verbal errors made by students during teaching and learning of mathematics. While interviewing Zawadi he noted that when a student makes a mistake, he usually sought the opinions of other students about the answer given to ascertain whether students in the class have understood the concept being taught. He contended that some students would disagree with the wrong responses given by their colleagues and some students may also not know the concept. He also noted that when questioning is employed when a student makes a mistake then it assists the students to have peer learning in class and he said;

> So when answering questions verbally and I realise a student has made a mistake. I get the opinion from the others first, like I can ask the other students 'do you think what the boy has just said is true?' Then I get the reactions from the students. Now some students may say yes. That is when I will realise that maybe a majority did not know this concept. Some may say no. Now you identify the student who has said no that whatever has been said is wrong. Then he can try to explain…but then you know that is peer learning…so the student gets to know very fast.

During all the lesson observations it was observed that when students made a mistake the teacher either asked them further questions to assist them to understand and to help them give the correct response or ask other students the same question or

other related questions that help in getting the correct response to the initial mistake made.

From the data, it was revealed that when students make verbal mathematical errors in class teachers usually respond by giving verbal cues that are either directed to the particular student or the whole class and the teachers also respond through the use of questions that are also directed to the particular student or all students in the class. These verbal cues or questions assist the learners to realise that they made mistakes/errors in their responses and assist them to correct.

4.2.3 Written Identification of Students' Errors

From the data collected from the interviews, FGDs, lesson observations and document analysis it was noted that errors committed by learners in written form were identified by the teachers during the lesson or while marking assignments after lessons.

4.3 Teachers' Communication of Students' Errors

Teachers' communication of students' errors was discussed in terms of the verbal communication and the written communication given by teachers to learners and the objectives of such communication as presented hereafter.

4.3.1 Verbal Communication of Students' Errors

Data collected from lesson observations indicate that while communicating verbally about learners' errors to the students teachers have various reasons why they communicate in a particular way. They communicate to assess, correct, assist learners to give more information and provide hints for learners when learners make mistakes.

Verification communication entailed determining whether or not learners' responses were correct and evidence from observations revealed that teachers' verbal communication was primarily focused on verifying learners' responses. Teachers used phrases like 'not really' when verifying students' incorrect responses. During the lesson observation for Bahati, a student used a wrong scale and the teacher commented verbally by saying "this is wrong, your scale is not good, where is zero?" and the teacher then assisted the student to draw the correct scale.

In addition, data from lesson observations revealed that teachers also provided corrective communication on students' errors, guiding the students to the accurate response by assisting the students in detecting inconsistencies in their responses. This corrective communication defined what was untrue and presented the right solution;

in some cases, the corrective communication involved extensive teacher and student interaction in which the teacher expanded on the students' response or prompted additional details on the students' response. This was supported further by the fact that during a lesson observation of Zawadi in Form 3 North, he had to give the correct response when he realised that students in the class found sine of lengths instead of sine of angles of a triangle.

Furthermore, it was also noted from the various lesson observations that teachers repeated learners' responses that have errors, and this was done with the major purpose of evoking more details from the learners to help them correct the responses that they had given that were not correct. It was also noted that most of these repeated communications about learners' errors were accompanied by scaffolded communication about the same errors committed by students and these scaffolded communications served to provide incremental hints to the students who used them until a correct response is generated. This was evident from the lesson observation of Bahati where he assisted learners to correct their mistakes while determining the scale factor of enlargement.

However, according to the data obtained from FGD, the students noted that sometimes the teachers gave them negative verbal comments that made them get discouraged in doing mathematics and some of the verbal comments made other students laugh at them when they made mistakes which at times made them develop a negative attitude towards mathematics as a subject as posed by Furaha "Sometimes when the teacher gives you questions to do. Sometimes you did not arrange them very well, so he will tell you that you like doing ugly work and you may feel very bad." This view was supported by Vumilia who stated that *"Sasa unaezapata umefanya hesabu fulani na haujafanya vile Mwalimu alikuwa anataka atasema maneno yenye atazinakudiscourage so inafaa ukifanya kitu mbaya unafaa kupewa comments ambayo zinakuencourage hata kama haujafanya swali vizuri"* [sometimes you may have done a question in mathematics and you have not done it the way the teacher wanted it done then the teacher will give you verbal comments that are very discouraging, the comments given by the teachers should be encouraging even if you have not got something right]. In addition, Nuru supported the same views and stated that;

> Because like if a teacher insults you in class because you have made a mistake in your work. You will feel bad because others will laugh at you and they will

start making jokes about you, so you will have a negative attitude towards the teacher and the moment you have that attitude towards the teacher you'll never get the concept in mathematics.

This data showed that when students commit errors, teachers communicate verbally with them to evaluate, rectify, help, give more information, and provide hints. This is consistent with the points of view of Brookhart (2017), Molloy and Boud (2014) and Li (2014) who argue that a teacher's verbal response to a student who has made a mistake should be to judge the student's response to help them perform well and correct their incorrect responses.

4.3.2 Written Communication of Students' Errors

Data on teachers' written communication of students' mathematical errors were gathered through FGDs, interviews, and an examination of learners' notebooks, as well as marked examination scripts that the teachers had marked after the lessons and examinations. Written communication of students' errors was analysed and classified as follows: conventional marking (crosses, marking codes, grades, scores, and error flagging), comments, and complete solutions.

4.3.2.1 Conventional Marking

According to the data gathered, teachers used traditional marking as a written communication tool to signal learners' errors. Data gathered from document analysis of marked students' notebooks and examination scripts revealed that both teachers, Bahati and Zawadi, communicated student errors using traditional marking methods such as crosses, marking codes, grades, scores, and error flagging and these are supported by documents in Appendix L. In addition, these were supported by sentiments of learners during the FGDs and Amani said that "Maybe sometimes you have collected your books together so you went wrong at some point, so they will just circle or put a question mark to show that you have a problem." This view was also held by Imara during the FGD of the form three students who said that "When you make a mistake teacher *anaeza surround hapo penye ulikuwa umekosea sasa when utapitia utaona hapo ndio utajua hapo ndio ulifanaya mistake"* [when you make a mistake the teacher will circle that point so that when you will be going through your work you will notice that is where you made the mistake]. In addition, this view was also held by Bahati who said that;

> What I would do is to put a big question mark in red showing that you were disgusted as a teacher. If it is that kind of an error like 10 + 15 and then they write 12 or something. I will do…exclamation marks, I would circle, I would underline. I mean to make so many lines on the problem and put even put a star, asterisks on the mistake."

From these views it is noted that marks, grades, and scores provide a very open and direct way of letting learners know what they have gotten wrong, making them an excellent tool for pinpointing student errors (Guskey, 2019). Error flagging, on the other hand, entails making marks such as encircling the error, underlining, placing question marks, using an inverted V or a dot that locates the error, incomplete solutions, or a faulty strategy used by the learner, and it assists the teacher in signalling to the learners where an error has been committed without necessarily providing the correct answer or method to the students, and it also helps the students to know the specific point that they made a mistake thus they can make corrections (Jackson, 2015; Simiyu, 2012).

4.3.2.2 Comments

Data from FGDs and document analysis revealed that teachers made general and short comments on students' notebooks and examination scripts about the errors they make while performing mathematical tasks. While making remarks during the FGD, Lulu noted;

> "When you did the mathematic, but you did it wrong then the teacher will mark it wrong. Then sometimes you find the teacher writing *see me* down there. So that when you get your book you are supposed to go to the teacher who taught that subject so that he or she can explain it to you further."

While supporting the remarks made by Lulu, Imara also said that "the teachers also write *terrible* in the notebooks when a student has made a mistake." During their discussion, Furaha supported the above saying that "…when you have collected your book and the teacher is going through your work and seeing that you can do the work, but you have just made a small mistake. They can write for you like *avoid silly mistakes*." These sentiments were also supported by the comments that were noted in the notebooks of the learners and their marked examination scripts which showed that teachers accompanied their conventional marking with short comments on the

learners' work which had errors. These comments included; *see me, terrible, poor, do correction, simplify* among other comments as supported by documents in Appendix L.

As teachers gave these comments on learners written work it is important to note that these comments had positive effects on the learners as confirmed by the students' sentiments of Baraka, who stated that "So for example *umeandikiwa* 'terrible' *na* friends *zako nao wamepata* so *itabidi tu wewe pia* you work hard in your work *hili pia upate"* [you are written for terrible in your work and your friends have got it right it will force you now just to work hard so that you also get it right]. In addition, this was also supported by Furaha who said that;

> These comments assist us because by the time the teacher will write for you *poor* you will not be pleased with it. So after you have seen that it is *poor* and it is really *poor*, you will have to look for a way for consulting other people or teachers so that you make sure there is no other *poor* that will be written in your book again."

From these sentiments, it was noted that the learners take the comments from their teachers as a motivating factor in their studies and they may end up improving in their academic work because they would not like to see negative comments in their books again.

4.3.2.3 Complete Solution

Data from marked learner's notebooks and marked examination scripts revealed that teachers provided hints and complete solutions when students made mistakes and failed to complete tasks correctly. This was supported by comments as seen in documents in Appendix L.

The data collected suggested that teachers use a variety of written communication on the errors of their students. The written communication of student errors was analysed and classified as follows: traditional marking (crosses, marking codes, grades, scores, and error flagging), comments, and complete solutions. The use of traditional marking, on the other hand, was the most preferred form of written communication on learners' errors. These communications were important because learners understand the aim and significance of communication from their teachers when they make mistakes, and they prefer timely, structured, and constructive communication on their performance (Nerali et al., 2021). However, it was also noted

from the data that some negative comments from the teachers may end up making the students develop a negative attitude towards the subject hence making them have dismal performance.

4.4 Teachers' Follow Up on Students' Errors

The data collected using interviews, FGDs, document analysis and lesson observations indicated that teachers used various approaches to follow up on learners' errors including re-teaching a whole concept, consultation by the students and setting a special day to educate the students about the various errors that exist in mathematics and that are commonly committed by the students.

4.4.1 Re-teaching a Concept

Data from interviews and lesson observations showed that re-teaching a concept that learners made errors on is one of the most preferred ways that teachers use to follow up on learners' errors in mathematics. This was evident from the interview with Bahati who said that;

> "So if I realise that in a class of 60 you find 50 or 40 students or even 30 students making a similar mistake, then you would want to believe that that mistake is contingent on the teacher, so in such cases, I would go, if it is myself, and reteach the same thing so that we correct it for the specific errors."

This was further supported by the fact that during a lesson observation of Zawadi in Form 3 North he had to re-teach when students in the class were finding sine of lengths instead of sine of angles of a triangle.

4.4.2 Consultations of Teachers by the Students

Consultations are considered a wonderful source of teacher-student interaction that can be done for a group of students or individual students. Data from FGDs indicate that students benefit a lot from these meetings because the teachers find an opportunity to help the students at an individual level to work on academic areas that they make errors in. This was supported by Lulu in the remarks given during the FGD;

> Then maybe you got some questions wrong. So by taking your free time you can go to the teacher to consult. Then through consultation, you get compelled to know what the question was talking about and how to handle the question

so that next time when the same question is repeated you will be able to handle it correctly and get the correct answer.

The same sentiments were held by Furaha during their FGD who said that "The teachers let us know about the errors or mistakes that we make in mathematics by going and consulting them, and they show us how to work the math that we have a problem in." In addition, Bahati during his interview concurred with what the learners said saying that consultations help in correcting learners' errors by noting that "The ones that are written on the script we call the students and then tell them "this is this" then we correct them." From these propositions, it was noted that consultations between the students and the teachers offer a good opportunity for the teachers to follow up on the learners' errors and to correct them.

4.4.3 Setting up a Day for Errors Awareness

As a practice, the teachers usually organise a day to make students, especially the candidate class, be aware of the errors that they may commit. According to Bahati, the HoD of Mathematics, this is usually done as part of the preparation of the candidates for the impending national examinations and said;

> For the candidates we do, and sometimes we even organise a day that we do quite a lot we organise a day where we just talk about the mistakes that they make and cost them marks in the final exams.

From these data, it was revealed that when a day for error awareness is organised and the students are made aware of what they should avoid then they end doing well in their examinations because they have information of what could lead them to make mistakes hence they avoid them.

4.4.4 Follow up Quizzes and Tasks

The other way of following up on learners errors by the teachers is by giving follow up examples in terms of questions and tasks that come immediately after an error was corrected or periodically. Zawadi during the interview emphasised that this was a very good way of helping the students to correct the errors that they make and said;

> I always give a follow-up example so that I see if they got what I said or not? If not, I again readdress it, and again I give another example. So when I am now satisfied that they've gotten it correctly Then at the end of the week. I give follow up questions.

4.5 Challenges of Following up on Students' Errors

While following up on students' errors teachers face various challenges including large class sizes, lack of documentation of errors for future reference, poor or negative attitude from the students and little or lack of knowledge on errors that are given to teachers during their training.

4.5.1 Large Class Sizes

Data on enrolment of the school indicated a large number of students per class more than the normal pegged at 45 students by the MOE. Form 3 north, for example, had 66 students which is 21 students above the normal 45 students and this trend is replicated in almost all classes. These large numbers hinder the teachers from giving individualised attention to the students who have made errors in their work hence creating a big challenge for teachers in following up on students' errors.

4.5.2 Lack of Documentation of Students' Errors

Lack of documentation of the errors committed by learners on the part of the teachers posed a great challenge on the follow on learners' errors since the teachers do not have a reference point on errors created previously and how they were solved hence making the errors committed by the learner seem new every time they are committed as held by Bahati who said that "…we don't record these errors. A lot of times, to be honest, I don't record the errors…maybe I may need to start recording so that in future somebody…. would have something to refer to."

From this, it was noted that these records would offer a very valuable starting point for following up on learners errors by the teachers and their absence impacts negatively on this quest.

4.5.3 Poor or Negative Attitude from Students

From the data collected from FGD, it was noted that some students at times seem to develop negative attitudes when their teachers follow up on their errors or while making verbal and written comments on their errors to correct their mistakes. Nuru when asked about how they feel about teachers comments had this to say;

> Because like if a teacher insults you in class because you have made a mistake in your work. You will feel bad because others will laugh at you. And they will start making jokes about you, so you will have a negative attitude towards

the teacher and the moment you have that attitude towards the teacher it will, you'll never get the concept in mathematics.

From this, it was noted that in the process of identifying and communicating students' errors teachers may say or write something that may discourage the learners from focussing on the corrections of their errors and the students end up not getting the mathematical concepts that are intended for them to get.

4.5.4 Little or Lack of Knowledge on Errors during Teacher Education

From the interviews, it was revealed that most teacher education and training do not equip teachers with knowledge on errors of students and how they are supposed to be dealt with as a teacher. Teachers, therefore, meets errors of students in their course of teaching and fumble on how to deal with them. During the interview with Bahati when asked whether they were taught something about students' errors in college, he was very emphatic and said that "No, nothing that I can remember. I don't think I was ever taught." These sentiments were supported by Zawadi who said that most of the teaching they were given during teacher training was about mathematics and taught by lecturers who were not education-based oriented but only mathematics oriented and said that "…during our undergraduate training we were not being taught by subject specialists…. but what the training was all about is how you teach mathematics..."

From the data, it was noted that lack of or little knowledge of students' errors during teacher education makes teachers have a challenge in dealing with students' errors. However, it was also worth noting that even though this was lacking during training but teachers who have attended workshops, in-service training and joint markings have been exposed a little to students' errors and how teachers can deal with them.

4.6 Summary

Data from interviews, FGDs, lesson observations, and document analysis were presented and discussed in this chapter in connection to what literary works and other scholarly articles have documented about them. The summary of the findings, implications, and recommendations, as well as recommendations for further research, will be presented in the following chapter, and conclusions will be drawn.

CHAPTER FIVE

SUMMARY OF FINDINGS, IMPLICATIONS, RECOMMENDATIONS AND CONCLUSIONS

5.0 Introduction

The goal of this study was to investigate how teachers in a secondary school in Siaya County deal with students' mathematical errors. As a result, this chapter provides a summary of key findings, implications, and recommendations, as well as suggestions for additional research and conclusions.

5.1 Summary of Key Findings

5.1.1 Sources of Students' Errors

According to the study's findings, teachers ascribe students' errors to insufficient conceptual knowledge, not following the correct procedures while calculating, being careless or in a hurry to complete a task, being unable to remember facts required for a specific task and a lack of preparedness for an examination or assignments.

These findings imply that teachers should implement strategies that specifically address these sources of error. Mathematics teachers need to work efficiently and effectively to ensure that they use teaching methods that help students improve their conceptual understanding. Furthermore, mathematics teachers need to provide opportunities for students to justify their solutions as part of developing their confidence in the use of mathematical language and their ability to communicate mathematically to avoid incorrect procedures and forgetfulness while doing mathematics. Finally, teachers need to provide more examples to students for them to adequately prepare for examinations and assignments to prevent them from panicking that results in making errors.

5.1.2 Importance of Students' Errors

The study discovered that teachers and students both regard students' mathematical errors as important in the teaching and learning of mathematics, and their reasons for doing so were as follows:

1. The errors that the students make in mathematics assist the teachers to reflect on their teaching to evaluate whether they are teaching well or not.

2. Teachers usually decide to re-teach a concept after discovering that a large number of students have made a similar error in their work.
3. The teachers also use the errors committed by the learners as a platform and learning points for correcting and teaching other students in the same class or other classes.
4. According to the learners, the errors they have made in their work motivated them to put more effort into their academic work because when they finally get the task right it is very satisfying.
5. These errors help the teachers to understand areas where the students have difficulty and also allow the teachers to identify the students who need academic assistance in the subject.

These perspectives of teachers and students guide and shape teachers' responses to student errors. The implication of this is that when teachers' perspectives are divergent from students' learning needs, learning opportunities are likely to be lost, and students must also be able to understand their mistakes and accept the corrections that their teachers give them without developing a poor or negative attitude. Teachers should therefore be exposed to prevailing mathematics teaching methods, which will influence their perspectives on mathematics teaching and learning.

5.1.3 Teachers' Identification of Students' Errors

These findings revealed that when students make verbal mathematical errors in class, teachers usually respond by giving verbal cues and questions that are either directed to the specific student or the entire class. These verbal cues or questions help learners recognize that they made mistakes/errors in their responses and help them correct them. Furthermore, the findings revealed that teachers identified errors committed by learners in written form during the lesson or while marking assignments after lessons.

5.1.4 Teachers' Communication of Students' Errors

The findings revealed that teachers' communication of students' errors was given in terms of verbal and written communication given by teachers to learners to achieve various objectives regarding the errors made by learners.

The findings indicated that when communicating verbally about learners' errors to students, teachers communicate for a variety of reasons, including communication to assess, correct, assist learners, provide more information, and provide hints for learners when learners make mistakes. The findings showed that teachers' verbal communication was primarily focused on verifying learners' responses, and teachers used phrases like 'not really' when verifying students' incorrect responses. Furthermore, findings revealed that teachers provided corrective communication on students' errors, guiding students to the correct response by assisting students in detecting inconsistencies in their responses, and this corrective communication defined what was untrue and presented the correct solution; in some cases, the corrective communication involved extensive teacher and student interaction in which the teacher expanded on the students' response or promulgated the correct solution. Finally, the findings revealed that teachers repeated learners' responses that contained errors, with the primary goal of eliciting more details from the learners to assist them in correcting the incorrect responses that they had provided. However, the students observed that sometimes teachers make negative verbal comments that discourage them from doing mathematics and that some of the verbal comments cause other students to laugh at them when they make mistakes, causing them to develop a negative attitude toward mathematics as a subject.

On the other hand, according to the findings, written communication of student errors was classified as conventional marking (crosses, marking codes, grades, scores, and error flagging), comments, and complete solutions. Teachers used traditional marking as a written communication tool to signal students' errors, whereas marks, grades, and scores provide a very open and direct way of letting students know what they have gotten right and wrong, making them an excellent tool for pinpointing student errors. Furthermore, it was discovered that teachers made general and brief comments like *“see me”, “poor”, “terrible”* on students' notebooks and examination scripts about errors they made while performing mathematical tasks and according to the students, these comments motivated them to put in extra effort in their academic work to perform better. Finally, the findings revealed that when students made mistakes or failed to complete tasks correctly, teachers provided hints and complete solutions to help the students to correct their work.

5.1.5 Teachers' Follow Up on Students' Errors

From the findings, it was revealed that teachers use various ways of following up students' errors including;

1. Re-teaching is a concept that most learners have made errors in. This is significant since the teacher can assist all the learners in the class to rectify areas that they were getting errors in.
2. Consultation of teachers by students is an interaction between teachers and students on an individual or group basis. Consultations are important because students benefit a lot from these meetings since the teachers find an opportunity to help the students at an individual level and help them to work on academic areas that they make errors in.
3. Setting up a day for errors awareness. This was significant since the teachers used this opportunity to assist learners to know areas that errors usually arise hence assisting the learners to prepare for examinations.
4. Follow up quizzes and tasks that are formulated in terms of examples and questions that expose learners to various concepts hence assisting them to know which areas do have errors.

5.1.6 Challenges of Following up on Students' Errors

The challenges that teachers faced during their quest to follow up on students' errors were revealed from the findings as follows;

1. Large class sizes
2. Lack of documentation of students' errors.
3. The poor or negative attitude from students when their errors are being dealt with by the teachers
4. Little or lack of knowledge on errors during teacher training

5.2 Implications and Recommendations

5.2.1 Teachers

Teachers of mathematics need to re-evaluate their current understanding of students' mathematical errors, their causes, and their importance in mathematics teaching and learning, as this will guide their teaching strategies, which will greatly aid in improving learners' mathematical understanding.

There is a need for mathematics teachers to gain more competencies required for identifying learner errors and to diversify strategies for identifying learner errors that are both verbal and written in nature. Teachers need also respond to students' verbal mathematical errors in a class by giving verbal cues that are either directed to the specific student or the entire class, as well as responding by using questions that are also directed to the specific student or the entire class, and these responses have to be directed at correcting the mistakes that the learners have made.

While communicating learners' errors that are verbal in nature teachers need to use comments that communicate for a variety of reasons, including communication to assess, correct, assist learners, provide more information, and provide hints for learners when learners make mistakes. These comments, however, need those that do not make learners feel looked down up thus making them have a negative attitude towards their mistakes in mathematics. On the other hand, while communicating with written learners' errors teachers need to use conventional marking, comments and complete solutions that are easily interpreted by the learners to make learners understand the mistakes they made in their work and to use these communications to correct their errors and these comments need to be motivating to the learners rather than being discouraging to them.

While following up on learners' errors teachers need to employ more strategies in addition to re-teaching a concept, consultation by the students, setting up a day for errors awareness and follow up quizzes and tasks as this would enable the teachers to diversify their methods of demystifying these errors and assisting the learners to correct their errors as they also gain a mathematical understanding of concepts. Teachers also need to expose themselves to current knowledge and practices that would enable them to deal with the challenges of following up on learners' errors.

5.2.2 Teacher Educators

Regardless of the numerous benefits of teacher identification, communication, and follow-up on students' errors brought to light by the study findings, mathematics teachers in the context of the study could not remember instances in their teacher training that highlighted how to deal with students' errors in mathematics. Pre-service teacher educators who prepare mathematics teachers must highlight the importance of embedding knowledge of how to deal with student errors in the teacher training curriculum. Furthermore, professional development training that concentrates on

current best methods for dealing with student errors would greatly benefit teachers who are already in the profession, and the exposure to the latest trend in practices for dealing with student errors would improve mathematics teachers' understanding of dealing with the difficulties of students who make mistakes.

5.2.3 Curriculum Developers

According to the findings, teachers of mathematics are not trained about learners' errors while undergoing teacher training; therefore, mathematics curriculum designers have to include strategies for how teachers deal with students' errors as a component of pre-service teachers' curriculum. These should provide mathematics teachers with current knowledge and skills for identifying, communicating, and following up on students' errors, allowing students to take action on their errors while also providing teachers with strategies for dealing with the challenges of following up on students' errors in large classes and a backdrop of negative attitudes from students.

5.3 Recommendations for Further Research

1. Since the study only included two teachers from one secondary school, a larger sample size is recommended, and a comparative study in different types of schools may also be conducted to gain a general understanding of how teachers deal with students' errors in Mathematics.
2. A study focusing on the effects of how teachers deal with students' mathematical errors on learners' mathematical performance and conceptual understanding is recommended.
3. It is also suggested that a study be conducted on how to improve how mathematics teachers deal with students' mathematical errors.

5.4 Conclusion

The study was designed to investigate how teachers deal with students' mathematical errors. The study's findings show that when students make verbal mathematical errors in class, teachers usually respond with verbal cues and questions directed either to the specific student or to the entire class. At the same time, the findings revealed that teachers' communication of students' errors was provided in the form of verbal and written communication provided by teachers to learners to achieve various objectives regarding the errors made by learners. While teachers followed up on students' errors, they used a variety of strategies to help students correct and understand their mistakes. The study recommended that mathematics teachers need

to re-evaluate their current understanding of students' mathematical errors, their causes, and their importance in mathematics teaching and learning, as this will guide their teaching strategies, which will greatly aid in improving learners' mathematical understanding. The reasoning behind this is that by examining and discovering errors with students, teachers develop in them the capacity to identify, ascribe, and rectify their own mistakes.

REFERENCES

Adino, A. (2015). *Factors influencing students performance in mathematics in Kenya certificate of secondary education in public secondary schools in Butere sub county, Kenya* University of Nairobi].

Anney, V. N. (2014). Ensuring the quality of the findings of qualitative research: Looking at trustworthiness criteria.

Arani, M. R. S., Shibata, Y., Sakamoto, M., Iksan, Z., Amirullah, A. H., & Lander, B. (2017). How teachers respond to students' mistakes in lessons: A cross-cultural analysis of a mathematics lesson. *International Journal for Lesson and learning studies*.

Bagonza, J. (2015). *Teachers' Provision of Written Formative Feedback to Enhance Learning in a Secondary School: The Case of An English Classroom In Uganda.* Aga Khan University]. Dar es Salaam.

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. *A research companion to principles and standards for school mathematics*, 27-44.

Barmby, P. (2009). *Primary Mathematics: Teaching for understanding*. McGraw-Hill Education (UK).

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2009). The array representation and primary children's understanding and reasoning in multiplication. *Educational studies in mathematics, 70*(3), 217-241.

Birt, L., Scott, S., Cavers, D., Campbell, C., & Walter, F. (2016). Member checking: a tool to enhance trustworthiness or merely a nod to validation? *Qualitative health research, 26*(13), 1802-1811.

Boddy, C. R. (2016). Sample size for qualitative research. *Qualitative Market Research: An International Journal*.

Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38.

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational studies in mathematics, 85*(2), 221-239.

Brookhart, S. M. (2017). *How to give effective feedback to your students*. ASCD.

Busetto, L., Wick, W., & Gumbinger, C. (2020). How to use and assess qualitative research methods. *Neurological Research and Practice, 2*(1), 14. https://doi.org/10.1186/s42466-020-00059-z

Carpenter, S. K. (2011). Semantic information activated during retrieval contributes to later retention: Support for the mediator effectiveness hypothesis of the testing effect. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 37*(6), 1547.

Carter, K., Banks, S., Armstrong, A., Kindon, S., & Burkett, I. (2013). Issues of Disclosure and Intrusion: Ethical Challenges for a Community Researcher. *Ethics and Social Welfare, 7*(1), 92-100. https://doi.org/10.1080/17496535.2013.769344

Chege, N. S. (2015). Assessment of errors made by secondary school students that influence achievement in solving word problems in mathematics in Gatanga Sub-County.

Cheruiyot, B. K. (2014). *Effects of descriptive written feedback on achievement in mathematics in secondary schools in konoin district of bomet county* University of Nairobi].

Clark, K. R. (2018, Nov). Learning Theories: Constructivism. *Radiol Technol, 90*(2), 180-182.

Creswell, J. W. (2013). *Qualitative Inquiry & Research Design: Choosing among Five Approaches* (3 ed.). Sage.

Creswell, J. W. (2014). *Research Design: Qualitative, Quantitative and Mixed Mehods Approaches* (4th ed.). SAGE.

Crowe, S., Cresswell, K., Robertson, A., Huby, G., Avery, A., & Sheikh, A. (2011). The case study approach. *BMC Medical Research Methodology*, *11*(1), 100. https://doi.org/10.1186/1471-2288-11-100

Davis, M. L., Witcraft, S. M., Baird, S. O., & Smits, J. A. (2017). Learning Principles in CBT. In *The Science of Cognitive Behavioral Therapy* (pp. 51-76). Elsevier.

De Chesnay, M. (2014). *Nursing research using phenomenology: Qualitative designs and methods in nursing*. Springer Publishing Company.

DeJonckheere, M., & Vaughn, L. M. (2019). Semistructured interviewing in primary care research: a balance of relationship and rigour. *Family Medicine and Community Health*, *7*(2).

Demirdag, S. (2015). Management of errors in classrooms: student mistakes and teachers. *International Journal of Humanities and Social Science*, *5*(7), 77-83.

den Ouden, H. E. M., Kok, P., & de Lange, F. P. (2012). How prediction errors shape perception, attention, and motivation. *Frontiers in psychology*, *3*, 548-548. https://doi.org/10.3389/fpsyg.2012.00548

Dudai, Y. (2012). The restless engram: consolidations never end. *Annual review of neuroscience, 35*, 227-247.

Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American journal of theoretical and applied statistics, 5*(1), 1-4.

Fernando, S. Y., & Marikar, F. M. (2017). Constructivist Teaching/Learning Theory and Participatory Teaching Methods. *Journal of Curriculum and Teaching, 6*(1), 110-122.

Filsecker, M., & Kerres, M. (2012). Repositioning formative assessment from an educational assessment perspective: A response to Dunn & Mulvenon (2009). *Practical Assessment, Research, and Evaluation, 17*(1), 16.

Flick, U. (2013). *The SAGE handbook of qualitative data analysis*. Sage.

Gundumogula, M. (2020). Importance of Focus Groups in Qualitative Research. *International Journal of Humanities and Social Science (IJHSS), 8*(11), 299-302.

Guskey, T. R. (2019). Grades versus comments: Research on student feedback. *Phi Delta Kappan, 101*(3), 42-47.

Hansen, A., Drews, D., Dudgeon, J., Lawton, F., & Surtees, L. (2020). *Children' s Errors in Mathematics* (A. Hansen, Ed. 5th ed.). Sage.

Hansson, L., Hansson, Ö., Juter, K., & Redfors, A. (2020). Curriculum Emphases, Mathematics and Teaching Practices: Swedish Upper-Secondary Physics Teachers' Views. *International Journal of Science and Mathematics Education*, 1-17.

Heinze, A., & Reiss, K. (2007). Mistake-handling activities in the mathematics classroom: Effects of an in-service teacher training on students' performance in geometry. Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education,

Henderson, M., Phillips, M., Ryan, T., Boud, D., Dawson, P., Molloy, E., & Mahoney, P. (2019). Conditions that enable effective feedback. *Higher Education Research & Development*, *38*(7), 1401-1416.

Jackson, E. (2015). *The Role of Written Feedback in Enhancing Students' Mathematics Learning in Tanzanian Lower Secondary Schools: An Intervention Study at a Lower Secondary School In Bukoba Municipality, Tanzania* Aga Khan University]. Dar es Salaam.

Jacoby, L. L., & Wahlheim, C. N. (2013). On the importance of looking back: The role of recursive remindings in recency judgments and cued recall. *Memory & Cognition, 41*(5), 625-637.

Johnson, M. D., & Bradbury, T. N. (2015). Contributions of social learning theory to the promotion of healthy relationships: Asset or liability? *Journal of Family Theory & Review, 7*(1), 13-27.

Kalpana, T. (2014). A constructivist perspective on teaching and learning: A conceptual framework. *International Research Journal of Social Sciences, 3*(1), 27-29.

Keiler, L. S. (2018). Teachers' roles and identities in student-centered classrooms. *International Journal of STEM education, 5*(1), 1-20.

Khan, S. B., & Chishti, S.-u.-H. (2011). Learners' Errors: Supporting Learners For Participating In Mathematics' classroom. *International Journal of Academic Research, 3*(1).

Kiwanuka, H. N., Van Damme, J., Van Den Noortgate, W., Anumendem, D., & Namusisi, S. (2015). Factors affecting Mathematics achievement of first-year secondary school students in Central Uganda. *South African Journal of Education, 35*(3), 1-16.

Lai, C.-F. (2012). Error Analysis in Mathematics. Technical Report# 1012. *Behavioral Research and Teaching*.

Li, S. (2014). Oral corrective feedback. *ELT Journal*, *68*(2), 196-198.

Lyster, R., Saito, K., & Sato, M. (2013). Oral corrective feedback in second language classrooms. *Language teaching, 46*(1), 1.

M'Kiambi, J. (2013). *Contribution of SMASSE in-service project on students' performance in mathematics in KCSE examination in Nkuene Division, Meru County, Kenya* Masters' thesis). Kenyatta University, Nairobi, Kenya].

Mandinach, E. B. (2012). A perfect time for data use: Using data-driven decision making to inform practice. *Educational Psychologist*, *47*(2), 71-85.

Mazana, M. Y., Montero, C. S., & Casmir, R. O. (2020). Assessing students' performance in mathematics in Tanzania: the teacher's perspective. *International Electronic Journal of Mathematics Education, 15*(3), em0589.

Mazana, Y. M., Suero Montero, C., & Olifage, C. R. (2019). Investigating students' attitude towards learning mathematics. *International Electronic Journal of Mathematics Education, 14*(1), 207-231.

Mbugua, Z. K., Kibet, K., Muthaa, G. M., & Nkonke, G. R. (2012). Factors contributing to students' poor performance in mathematics at Kenya certificate of secondary education in Kenya: A case of Baringo county, Kenya. *American International Journal of Contemporary Research, 2*(6), 87-91.

Merriam-Webster. (n.d.). *Follow up on (something)*. Retrieved August 5, 2021, from https://www.merriam-webster.com/dictionary/follow%20up%20on%20%28something%29

Metcalfe, J. (2017). Learning from errors. *Annual review of psychology*, *68*, 465-489.

Mohan, R. (2016). *Measurement, evaluation and assessment in education*. PHI Learning Pvt. Ltd.

Molloy, E. K., & Boud, D. (2014). Feedback models for learning, teaching and performance. In *Handbook of research on educational communications and technology* (pp. 413-424). Springer.

Nebel, C. (2019). Mistakes or Opportunities? Learning from Errors. *THE LEARNING SCIENTISTS*. https://www.learningscientists.org/blog/2019/10/17-1

Nerali, J., Chakravarthy Pishipati, V., Telang, L., & Telang, A. (2021). Dental students' perception towards feedback during clinical training [Original

Article]. *Archives of Medicine and Health Sciences, 9*(1), 62-67. https://doi.org/10.4103/amhs.amhs_212_20

Njiru, S. M., Karuku, S., & Nyaga, M. (2020). Performance rankings in education: Implications for policy and practice. *Educational Research and Reviews, 15*(12), 700-710.

Nuraini, N. L. S., Cholifah, P. S., & Laksono, W. C. (2018). Mathematics Errors in Elementary School: A Meta-Synthesis Study. 1st International Conference on Early Childhood and Primary Education (ECPE 2018),

O'Connor, M., Ndethiu, S., & Mulungye, M. M. (2016). Sources of Student Errors and Misconceptions in Algebra and Effectiveness of Classroom Practice Remediation in Machakos County--Kenya. *Journal of Education and Practice, 7*(10), 31-33.

Ogawa, R., Peres, L. M., Kutzke, A. R., & Silva, F. (2018). Promoting Mediation in Learning Error on Teaching Algorithms. CSEDU (1),

Oktaviani, M. (2017). Analysis of students' error in doing mathematics problem on proportion. Proceedings ofthe 2nd Asian Education Symposium (AES 2017),

Omona, J. (2013). Sampling in qualitative research: Improving the quality of research outcomes in higher education. *Makerere Journal of Higher Education, 4*(2), 169–185-169–185.

Osborne, J. (2021). Constructivism: Critiques. In R. Gunstone (Ed.), *Encyclopedia of Science Education* (pp. 1-6). Springer Netherlands. https://doi.org/10.1007/978-94-007-6165-0_344-2

Oser, F. K., Näpflin, C., Hofer, C., & Aerni, P. (2012). Towards a theory of negative knowledge (NK): Almost-mistakes as drivers of episodic memory amplification. In *Human Fallibility* (pp. 53-70). Springer.

Palinkas, L. A., Horwitz, S. M., Green, C. A., Wisdom, J. P., Duan, N., & Hoagwood, K. (2015). Purposeful sampling for qualitative data collection and analysis in mixed method implementation research. *Administration and policy in mental health and mental health services research, 42*(5), 533-544.

Peña-López, I. (2016). PISA 2015 results (Volume I). Excellence and equity in education.

Rach, S., Ufer, S., & Heinze, A. (2013). Learning from Errors: Effects of Teachers Training on Students' Attitudes towards and Their Individual Use of Errors. *PNA, 8*(1), 21-30.

Rambe, P., & Moeti, M. (2017). Disrupting and democratising higher education provision or entrenching academic elitism: towards a model of MOOCs adoption at African universities. *Educational Technology Research and Development, 65*(3), 631-651.

Rassaei, E. (2014). Scaffolded feedback, recasts, and L2 development: A sociocultural perspective. *The Modern Language Journal, 98*(1), 417-431.

Rushton, S. J. (2018). Teaching and learning mathematics through error analysis. *Fields Mathematics Education Journal, 3*(1), 4. https://doi.org/10.1186/s40928-018-0009-y

Ryan, J., & Williams, J. (2007). *Children'S mathematics 4-15: learning from errors and misconceptions: learning from errors and misconceptions*. McGraw-Hill Education (UK).

Sapire, I., Shalem, Y., Wilson-Thompson, B., & Paulsen, R. (2016). Engaging with learners' errors when teaching mathematics. *Pythagoras, 37*(1), 1-11.

Schleicher, A. (2019). PISA 2018: Insights and Interpretations. *OECD Publishing*.

Schneider, Z., Whitehead, D., LoBiondo-Wood, G., Faan, P. R., Haber, J., & Faan, P. R. (2016). *Nursing and midwifery research: Methods and appraisal for evidence-based practice*. Elsevier.

Schreiber, L. M., & Valle, B. E. (2013). Social constructivist teaching strategies in the small group classroom. *Small Group Research, 44*(4), 395-411.

Seifert, K., & Sutton, R. (2019). The learning process. *Educational Psychology*.

Shallenberger, D. (2015). Learning from our mistakes: International educators reflect. *Frontiers: The Interdisciplinary Journal of Study Abroad, 26*(1), 248-263.

Simiyu, K. N. (2012). *Exploring Mathematics Teachers Feedback on Learners' Errors: Insights From a Secondary School Classroom in Kenya.* The Aga Khan University]. Dar es Salaam.

Steuer, G., Rosentritt-Brunn, G., & Dresel, M. (2013). Dealing with errors in mathematics classrooms: Structure and relevance of perceived error climate. *Contemporary Educational Psychology, 38*(3), 196-210.

Tennant, G., & Sarungi, V. (2016). Achievement in mathematics: Comparative analysis from East Africa. In *Mathematics Education in East Africa* (pp. 27-39). Springer, Cham.

Trumbull, E., & Lash, A. (2013). Understanding formative assessment. *Insights from learning theory and measurement theory. San Francisco: WestEd*, 1-20.

Tullis, J. G., & Goldstone, R. L. (2016). Comparison versus reminding. *Cognitive Research: Principles and Implications, 1*(1), 20. https://doi.org/10.1186/s41235-016-0028-1

Van Bergen, P., & Parsell, M. (2019). Comparing radical, social and psychological constructivism in Australian higher education: a psycho-philosophical perspective. *The Australian Educational Researcher, 46*(1), 41-58.

Verwey, J. C. H. (2011). *Investigating the interaction of mathematics teachers with learners' mathematical errors* University of Pretoria].

Weurlander, M., Söderberg, M., Scheja, M., Hult, H., & Wernerson, A. (2012). Exploring formative assessment as a tool for learning: students' experiences of different methods of formative assessment. *Assessment & Evaluation in Higher Education, 37*(6), 747-760.

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555-584.

Yambi, T. d. A. C. (2018). *Assessment and evaluation in education*. Google Scholar. https://www.academia.edu/35685843/ASSESSMENT_AND_EVALUATION_IN_EDUCATION

Yin, R. K. (2011). *Qualitative research from start to finish*. Guilford publications.

Ying, J. (2020). The Importance of the Discussion Method in the Undergraduate Business Classroom. *Humanistic Management Journal*, *5*(2), 251-278. https://doi.org/10.1007/s41463-020-00099-2

APPENDIX A: INFORMATION SHEET FOR THE PRINCIPAL

Information Sheet for the Principal

The Aga Khan University

Institute for Educational Development, East Africa

Salama House, Urambo Street,

P.O. Box 125,

DAR ES SALAAM.

Date: ……...

The Principal

Dear Sir,

RE: REQUEST TO CONDUCT RESEARCH IN YOUR SCHOOL

I am a Master of Education Student at the Aga Khan University. I request to conduct a study in your school. The title of my study is: **EXPLORING HOW TEACHERS DEAL WITH STUDENTS' ERRORS IN MATHEMATICS: A CASE OF A SECONDARY SCHOOL IN SIAYA COUNTY, KENYA.** The purpose of this study will be to explore how teachers identify, communicate and follow up students' errors in mathematics during teaching and learning in class and outside the classroom.

To carry out this study, I will do the following:

1. Interview (one on one) the Form Two and Form Three teachers of Mathematics to get their views on how teachers deal with learners' errors in mathematics.
2. Examine and analyse the mathematics exercise books of Form Two and Form Three learners. I will be looking for teachers' written remarks, signs or symbols indicating how the teachers have identified, communicated and followed up the learners' errors.
3. Examine and analyse the marked mathematics scripts the Form Two and Form Three learners' tests done during the term for the teachers' written

comments, signs and symbols which are indicating how the teachers have identified, how they have communicated and followed up the learners' errors.

4. Photocopy/photograph some parts from the learners' work in their exercise books or scripts which are evidence of how the teachers identified, communicated and followed up the learners' errors. This is all subject to the participants' consent.
5. Conduct two Focus Group Discussions (FGD) Interviews; a total of twelve learners, that is, six Form Two learners and six Form Three learners.
6. For accurate information, I will need to audio record the one on one interviews with the teachers and the Focus Group Discussions (FGD) with the learners if the participants have consented.
7. Teachers' remarks in the learners' exercise books and scripts will be documented with the teachers' consent and your consent on behalf of and after learners have assented. This information is exclusively for data and I will not in any way make the content of the recordings public. The information will not be used for any other purpose other than the study.
8. Observe the Form Two and Form Three mathematics subject teachers while their classes are in progress to gain an understanding and experience on how teachers identify, communicate and follow up on learners' errors in mathematics during teaching and learning in class. With the teacher's consent, I will audio record these observations for correct information.
9. During the study which will last four (4) weeks, I will try as much as possible to work within your school program and timetable.

Participation in this study is voluntary and participants have a right to withdraw their participation at any point in the study for whatever reason. I will conceal the identity of the school, teachers and students during the study and in the final report, I will use pseudonyms. I will share a summary of the findings of the study with your school. Please kindly consider my request.

Owala John Robert Ouko

Phone No.: +254 721 889 637 (Kenya) / +255 747 192 587 (Tanzania).

Email: jroukoowala@gmail.com

APPENDIX B: PARTICIPANT(S) INFORMATION SHEET

Participant(s) Information Sheet

Owala John Robert Ouko

The Aga Khan University

Institute for Educational Development, East Africa

Salama House, Urambo Street,

P.O. Box 125,

DAR ES SALAAM.

Date: ……

I am a Master of Education Student at the Aga Khan University. I request to conduct a study in your school. The title of my study is: **EXPLORING HOW TEACHERS DEAL WITH STUDENTS' ERRORS IN MATHEMATICS: A CASE OF A SECONDARY SCHOOL IN SIAYA COUNTY, KENYA.** The purpose of this study will be to explore how teachers identify, communicate and follow up students' errors in mathematics during teaching and learning in class and outside the classroom.

To conduct the study, I will need to observe two mathematics teachers; one for Form Two and one for Form Three as they teach and have a one on one interview with the two. I will photocopy or photograph the sample of teachers' identification, communication and follow up on six randomly selected learners' notebooks and marked examination scripts for each teacher's class. The lessons and the interviews will be audio recorded. The audio recordings and photocopies and/or photographs will be for the accuracy of the information and for referring to them for details only.

The study is expected to run for four weeks and no one will be allowed access to the information collected apart from my Supervisor during discussions. Throughout the data collection and study report, I will conceal the identity of the participants and

the school by use of pseudonyms. I will share a summary of the findings of the study with the school.

For further information you can contact me on:

Phone No.: +254 721 889 637 (Kenya) / +255 747 192 587 (Tanzania).

Email: jroukoowala@gmail.com

Or contact

The Chairperson of the Ethical Review Committee

The Aga Khan University

Institute for Educational Development, East Africa

Salama House, Urambo Street,

P.O. Box 125,

DAR ES SALAAM.

Phone: +255-22-2152293/2150051
Email: iedea@akuied.ac.tz

APPENDIX C: INFORMED CONSENT FOR TEACHERS

Informed Consent for Teachers

I have read the participant information sheet. The nature and purpose of the study have been explained to me by OWALA JOHN ROBERT OUKO, a Master of Education Student at the Aga Khan University, Institute for Educational Development, East Africa. I fully understand what will be required of me as a participant and I hereby consent to take part in the study with the following understanding:

1. The purpose of this study will be to explore how teachers identify, communicate and follow up students' errors in mathematics during teaching and learning in class and outside the classroom.
2. The purpose of this study is NOT to judge me on the responses I give during the study.
3. The identity of the Research Participant will remain confidential and that my name or that of my institution will not be used in the study or the reporting of its findings at any point.
4. I hold the right to withdraw from the study at any point.
5. I hold the right to decline to answer any question which I do not feel comfortable with.
6. My voice will be audio recorded during interviews and lesson observations.
7. My written remarks in learners' notebooks and marked scripts will be photocopied and/or photographed.
8. I will receive a summary of the final report of the study.
9. Findings of this report may be used in conference presentations and academic publications.

I express my willingness to participate in the study by signing this form.

Name: …….. Designation: ……

Signature: …… Date: ……..

Name of Institution: ……..

Address: ……

Researchers' Name: Owala John Robert Ouko

Date: ……

Phone No.: +254 721 889 637 (Kenya) / +255 747 192 587 (Tanzania).

Email: jroukoowala@gmail.com

APPENDIX D: PARENTAL CONSENT LETTER

Parental Consent Letter

Owala John Robert Ouko

The Aga Khan University

Institute for Educational Development, East Africa

Salama House, Urambo Street,

P.O. Box 125,

DAR ES SALAAM.

Date: ……

Through

The Principal,

Dear Parent,

RE: REQUEST FOR YOUR SON/DAUGHTER TO TAKE PART IN A STUDY

I am a Master of Education student at Aga Khan University. I wish to conduct a study in your son's/daughter's school and particularly their class in the area of Mathematics Education. The purpose of this study will be to explore how teachers identify, communicate and follow up students' errors in mathematics during teaching and learning in class and outside the classroom.

As a researcher, I will not interact with the learners or the teacher during the learning period at all. My presence at the back of the class will be unobtrusive as possible so as not to interfere with teaching or hinder learning. I may attend your son's/daughter's class up to a maximum of three times each week during the period of the study. I will observe, take notes and audio record teacher-learner interactions for transcription purposes. I will have access to learners' marked written assignments and marked examination scripts which will be photocopied or photographed if they contain information that is crucial to the study. I will also

have a Focus Group Discussion (FGD) with the learners which will be audio recorded. The audio recordings and photocopies and/or photographs will be for the accuracy of the information and for referring to them for details only.

The study is expected to run for four weeks and no one will be allowed access to the information collected from your son/daughter apart from my Supervisor during discussions. Throughout the data collection and study report, I will conceal the identity of the participants and the school by use of pseudonyms. I will share a summary of the findings of the study with the school.

I have read and understood the intent and purpose of the study and my son/daughter can take part in the study. Tick one Agree ()

Disagree ()

Parent's Name…….

Signature…….

Date……...

Researcher's signature……... Date…….

Contact: Phone No.: +254 721 889 637 (Kenya) / +255 747 192 587 (Tanzania).

Email: jroukoowala@gmail.com

APPENDIX E: ASSENT FORM FOR LEARNERS

Assent Form for Learners

I have read the participant's information sheet, and the nature and purpose of the study have been explained to me by Owala John Robert Ouko, a Masters of Education student at Aga Khan University. I voluntarily accept that part of my written work may be photocopied or photographed, and I also voluntarily accept to be part of a Focus Group Discussion that will be audio recorded. I understand that the information collected during the period of research will be used for research purposes only and will not harm me in any way.

I also understand that while the information gained during the study may be published, I will not be identified anywhere in the study by my names but through the use of pseudonyms.

I am aware that I can decline to have my work included in the research study and I can also decline to participate in the Focus Group Discussion without penalty.

Name…….

Signature…….

Date……...

Researcher's signature…….. Date…….

Contact: Phone No.: +254 721 889 637 (Kenya) / +255 747 192 587 (Tanzania).

Email: jroukoowala@gmail.com

APPENDIX F: INTERVIEW GUIDE FOR TEACHERS

Interview Guide for Teachers

1. Tell me about yourself (professional background, academic qualifications, professional development, INSET training, teaching experience).
2. What do you understand by students' errors in mathematics? (Probe for teacher's views on causes of learners' errors and the role of errors in learning mathematics).
3. How do you identify students' errors during classroom instruction? (Probe for teachers' strategies on formative assessment and how they identify learners' errors).
4. What happens if a learner in your class has difficulties in a given task what would you say, write or do? (Probe for teachers' provision of the reason behind learners' errors).
5. How do you communicate to the students the errors that you have identified in their work (Probe for teachers' strategies on how they communicate learners' errors)?
6. How do learners in your class know what they need to get better at? (Probe teacher's provision of feedback).
7. How do learners respond to your verbal and written feedback? (Are they embarrassed or are comfortable and use errors as an opportunity for learning) How do you know? What do they do? Action? Do they think feedback is useful?
8. What are your preferred practices when dealing with students' errors in mathematics? Why? (Probe for practices to individual, group or whole class/ immediately or delayed/evaluative or descriptive/task-oriented or ego-oriented/ written or verbal).
9. What do you see as the value of learners' errors for learning? Are there instances when you have identified, communicated and followed up learners' errors that you thought it was not useful? Explain.
10. How do you follow up on students' errors in mathematics?
11. Tell me about what you were taught in your teacher training about learners errors?

12. What challenges do you encounter while following up on students' errors?
13. How do you cope with these challenges? Please elaborate.
14. Is there any other comment you would like to make about learners errors?

APPENDIX G: FOCUS GROUP DISCUSSION GUIDE FOR STUDENTS

Focus Group Discussion Guide for Students

1. Please tell me about the different kinds/types/examples of errors (mistakes) that you encounter while learning mathematics.
2. How do your teachers let you know about the errors/mistakes that you have made in mathematics? (Probe for the practices used).
3. What kind(s) of comment(s)/remarks do you usually receive when you make these errors? Mention some (Probe for verbal and written communication about errors).
4. In your opinion, do these comments/remarks assist you in the learning of mathematics? If yes, please give reasons and if No, still give reasons for your answer.
5. Do you face any challenges with the verbal comments that teachers indicate about the errors/mistakes that you have made in mathematics? Please share with me about some of them.
6. Do you face any challenges with the written comments or symbols that teachers indicate in your written work? Please share with me about some of them.
7. How do you manage these challenges? Anyone is free to share their experiences with us.
8. Do you have any other observations that you would like to share with me?

APPENDIX H: DOCUMENT ANALYSIS FRAMEWORK

Document Analysis Framework

Document	Indicators	Comments
S1 **Learner's Notebook**		
S2 **Learner's Notebook**		
S3 **Learner's Notebook**		
S4 **Learner's Notebook**		
S5 **Learner's Notebook**		
S6 **Learner's Notebook**		
S7 **Learner's Notebook**		
S8 **Learner's Notebook**		
S9		

Learner's Notebook		
S10 **Learner's Notebook**		
S11 **Learner's Notebook**		
S12 **Learner's Notebook**		

Document	Indicators	Comments
S1 **Learner's Marked Script**		
S2 **Learner's Marked Script**		
S3 **Learner's Marked Script**		
S4 **Learner's Marked Script**		
S5 **Learner's Marked Script**		
S6		

Learner's Marked Script		
S7 **Learner's Marked Script**		
S8 **Learner's Marked Script**		
S9 **Learner's Marked Script**		
S10 **Learner's Marked Script**		
S11 **Learner's Marked Script**		
S12 **Learner's Marked Script**		

APPENDIX I: LESSON OBSERVATION GUIDE

Lesson Observation Guide

ITEM	OBSERVATION REMARKS
Lesson Introduction Setting an environment in which errors can be discussed.	
Lesson Development The teacher identifies the errors of students. The teacher communicates about errors to the students. Teacher follows up on errors of students.	

Lesson Conclusion Summarizes the lesson following up on errors of learners.	

APPENDIX J: PROPOSED RESEARCH BUDGET

Proposed Research Budget

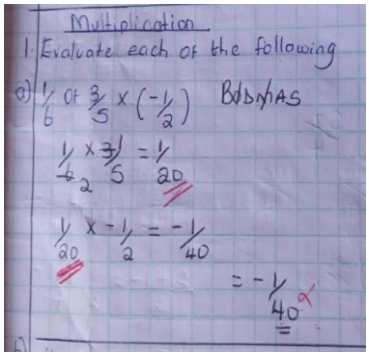
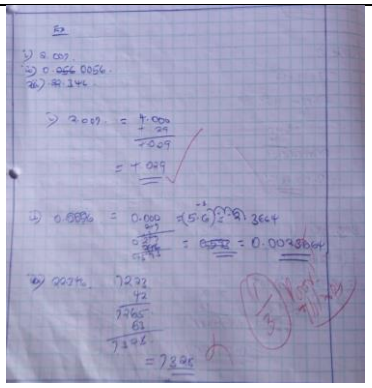
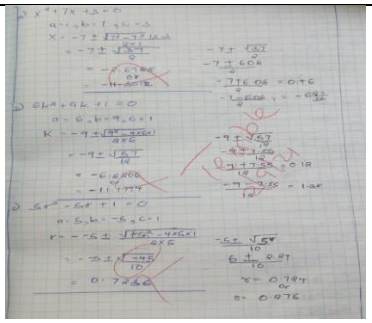
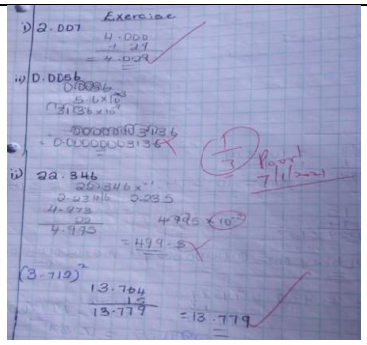
ITEM	QUANTITY	COST PER ITEM	TOTAL COST (Ksh)
Stationery -pens	**5**	**20**	**100**
Printing/ photocopying	**100 Pages**	**20**	**2,000**
Internet bundles	**40GB**	**100**	**4,000**
Travelling	**40 trips**	**100**	**4,000**
Spring Files	**2**	**75**	**150**
Total			**10,250**

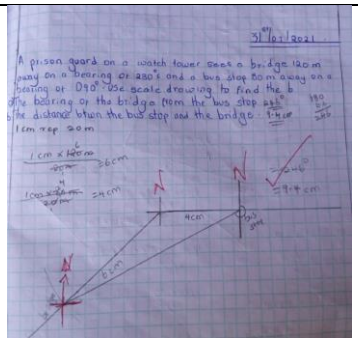
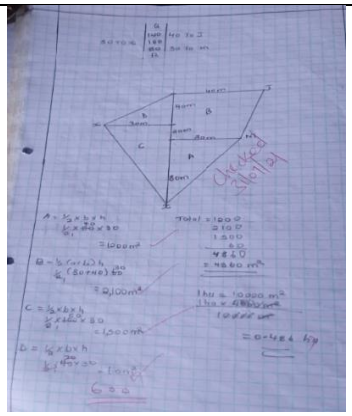
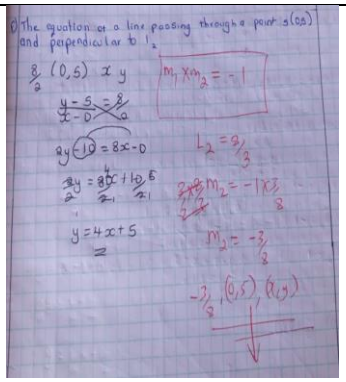
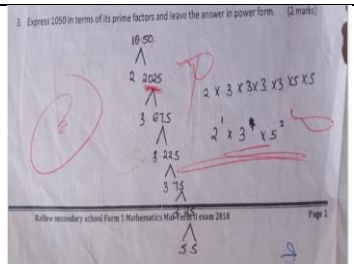
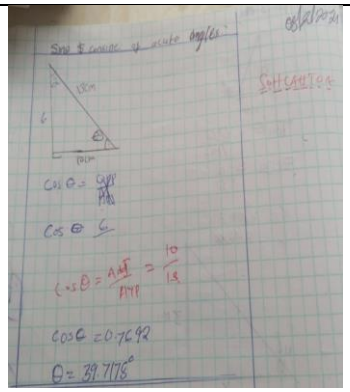
APPENDIX K: RESEARCH SCHEDULE

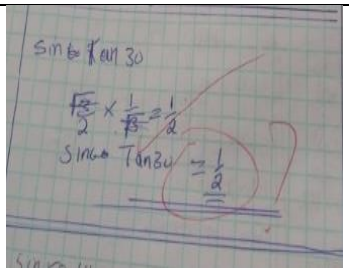
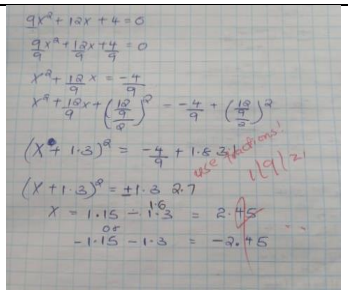
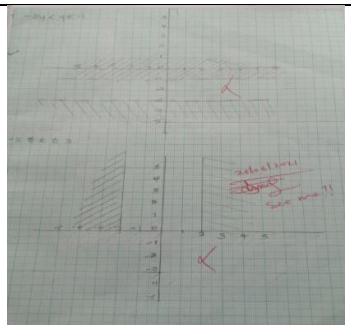
Research Schedule

Activity	Duration	Dates
Topic Selection	1 week	22nd May - 29th May, 2021
Seeking Literature	1 week	4th - 11th June, 2021
Annotation and Evaluation Of Sources	1 week	12th - 18th June, 2021
Constructing Research Questions	1 week	19th – 25th June, 2021
Completing Introduction	1 week	26th June – 3rd July, 2021
Writing Literature Review	1 week	4th - 10th July, 2021
Research Methodology	1 week	11th - 17th July, 2021
Completing Research Project Proposal	3 days	18th - 20th July, 2021
Proposal Defence	1 day	21st - 23rd July, 2021
Proposal Revision	1 week	24th July - 31st August, 2021
Submitting Proposal To ERC	5 days	1st - 5th August, 2021
Seeking ERC Certificate	2 days	15th - 16th August, 2021
Seeking NACOSTI, MOEST & Research Site Permit	10 days	21st - 30th August, 2021
Field/ Fieldwork Preparation	1 week	2nd - 6th September, 2021
Data Collection	4 weeks	9th September - 6th October, 2021
Data Analysis	4 weeks	9th September - 6th October, 2021
Project Write Up	8 weeks	7th October - 29th November, 2021
Submitting Research Project for Approval	1 day	29th November, 2021

APPENDIX L: SAMPLES OF WRITTEN COMMUNICATION

S. No	Document Type	Document	Evidence of Identification and communication of Errors of Students by the Teacher
1	Student notebook		Use of an X. Underlining.
2	Student notebook		Use of an X. Use of the word *poor* as a written comment. Use of marks
3	Student notebook		Use of an X. Use of the word *terrible* as a written comment.
4	Student notebook		Use of an X. Use of a circle. Use of marks. Use of the word *Poor* as a written comment.

5	Student notebook		The teacher added missing information to show errors.
6	Student notebook		Use of an X. Writing the correct answer to communicate error.
7	Student notebook		Partly working on the correct answer for the student.
8	Student marked script		Use of marking codes. Underlining. Use of marks.
9	Student notebook		Writing the correct step for the student.

10	Student notebook		Use of a circle. Use of question marks.
11	Student notebook		Use of instructions.
12	Student notebook		Use of the words *See me* as a written comment. Use of X.

APPENDIX M: ETHICAL CLEARANCE CERTIFICATE

THE AGA KHAN UNIVERSITY

Ref.: *AKU-IED, EA/2021/168/fB* **Date:** *August 16th, 2021*

Owala John Robert Ouko,
Aga Khan University,
Institute for Educational Development East Africa (IED EA),
P.O Box 125,
Dar es Salaam,
Tanzania.

ETHICAL CLEARANCE CERTIFICATE

Dear Owala John Robert Ouko,

This is to certify that your research project entitled, ***"Exploring How Teachers Deal with Students' Errors in Mathematics: A Case of a Secondary School in Siaya County, Kenya."*** undertaken as part of the dissertation project in the master of education program at IED EA has been approved for Ethical Clearance.

Yours Sincerely,

Dr. Fortidas Bakuza
Chair ERC – Tanzania

Cc: Dissertation Supervisor: Peter Kajoro

Salama House, 344 Urambo Street, P.O. Box 125, Dar es Salaam, Tanzania Tel: +255 22 215 2293, 22 215 0051, Fax: +255 22 215 0675; Email: iedea@aku.edu www.aku.edu

APPENDIX N: RESEARCH LICENSE

REPUBLIC OF KENYA

NATIONAL COMMISSION FOR SCIENCE,TECHNOLOGY & INNOVATION

Ref No: **530554**

Date of Issue: **23/August/2021**

RESEARCH LICENSE

This is to Certify that Mr.. OWALA ROBERT OUKO JOHN of Aga Khan University, has been licensed to conduct research in Siaya on the topic: EXPLORING HOW TEACHERS DEAL WITH STUDENTS' ERRORS IN MATHEMATICS: A CASE OF A SECONDARY SCHOOL IN SIAYA COUNTY, KENYA. for the period ending : 23/August/2022.

License No: **NACOSTI/P/21/12543**

530554

Applicant Identification Number

Director General
NATIONAL COMMISSION FOR SCIENCE,TECHNOLOGY & INNOVATION

Verification QR Code

NOTE: This is a computer generated License. To verify the authenticity of this document, Scan the QR Code using QR scanner application.

APPENDIX O: RESEARCH AUTHORISATION (MOE)

REPUBLIC OF KENYA
MINISTRY OF EDUCATION
State Department of Early Learning and Basic Education

COUNTY DIRECTOR OF EDUCATION
SIAYA COUNTY
P.O. BOX 564
SIAYA

E-mail:cdesiaya2016@gmail.com

MOE/SYA/CDE/URA/1/10/VOL.II/28 Thursday, August 26, 2021

TO WHOM IT MAY CONCERN

RESEARCH AUTHORIZATION - MR. OWALA ROBERT OUKO

The above-named person has been mandated to carry out research in Siaya County vide research License No. **NACOSTI/P/21/1243** dated 23rd July, 2021. This research study ends on 23rd August, 2022.

The research title is "***exploring how teachers deal with students errors in Mathematics: A case study of a secondary school in Siaya County, Kenya"***

Please accord him the necessary assistance in this County as he may require.

COUNTY DIRECTOR OF EDUCATION SIAYA COUNTY 26 AUG 2021 P.O. BOX 564 - 40601, SIAYA

SIFUNA N. M
COUNTY DIRECTOR OF EDUCATION
SIAYA COUNTY

SGS

APPENDIX P: RESEARCH AUTHORISATION (MOICNG)

REPUBLIC OF KENYA

THE PRESIDENT
MINISTRY OF INTERIOR AND COORDINATION OF NATIONAL GOVERNMENT

Fax No.
Tel.
Email: cc.siaya@yahoo.com
cc.siaya@interior.go.ke
When replying please quote Ref. & date

THE COUNTY COMMISSIONER
SIAYA COUNTY
P.O. BOX 83– 40600
SIAYA

Ref NO. CC/SC/A.31 VOL.IV/11

26th August, 2021

All Deputy County Commissioners
SIAYA COUNTY

RE: RESEARCH AUTHORIZATION – MR. OWALA ROBERT OUKO JOHN

The person referred to above from Aga Khan University has been licensed by the Director General, National Commission for Science, Technology and Innovation vide License No. NACOSTI/P/21/12543 dated 23rd August, 2021 to conduct research on the topic: **EXPLORING HOW TEACHERS DEAL WITH STUDENTS' ERRORS IN MATHEMATICS: A CASE OF A SECONDARY SCHOOL IN SIAYA COUNTY, KENYA** for the period ending: 23rd August, 2022.

The purpose of the letter therefore is to ask that you accord him the necessary support as he carries our research in your Sub County.

NOTE: ***Due to the prevailing COVID – 19 situation, he must observe containment protocols as directed by the Ministry of Health.***

RODAH CHERUS
For: COUNTY COMMISSIONER
SIAYA COUNTY

Copy to County Director of Education
SIAYA COUNTY

Mr. Owala Robert Ouko John
E-mail Address: jroukoowala@gmail.com