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Inclinations of Egyptian Pyramids and Finding of the Divine Essence

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Abstract

The aim of this research is discovery of astronomical reasons in orientation of slopes of Egyptian pyramids used as tombs for pharaohs of Ancient Egypt. The article contains results of statistical analysis of change in inclination of slopes of the pyramids (3^{rd} – 2^{nd} millennia BC) depending on time of their building. The first year of the corresponding pharaoh's reign has been accepted, as usually it is considered that building of pyramids ones started during either the first or second year of the reign. On the base of the obtained results a conclusion has been drawn that the average annual change of the angle of slopes of pyramids was close to value of the precession of the equinoxes. The sides were directed to the Sun at culmination, but a day for this procedure was chosen by the acronical rising of some stars after the autumnal equinox.

In the course of research days of heliacal and acronical risings of some mythologically important stars have been determined for the first year of pharaohs reign. Within framework of the suggested hypothesis, the received days have been compared with days when the Sun was at culmination at height equal to the angle of slopes of a corresponding pyramid. Such comparison has made possible to discover that the inclination of the slopes of the earliest pyramids was connected with acronical rising of star Betelgeuse that has been connected with Osiris cult. And, the inclination of slopes of pyramids built after the 3^{rd} dynasty of pharaohs was connected with acronical rising of the star Aldebaran that has been connected with Horus cult.

And, this choice of this or that star depended on aspiration of a pharaoh to emphasize significance of this or that elite group from Upper Egypt or his belonging to it.

On the base of the evidences obtained in the course of research a conclusion about gradual deviation from stellar orientations and transition to solar orientations of pyramids is drawn. The sense of all these actions was ritual one, and not only to guarantee the ascension of the pharaoh to the sky after his death, but above all for sacralization of his power, finding of the divine essence, and maintenance of the Cosmic Order at the beginning of his reign.

Keywords: pyramids, ancient Egypt, inclination of sides, solar and stellar orientations, cults of Osiris and Horus.

Introduction

During the last decades we see a lot of works explaining these or those features of the Egyptian pyramids from positions of numerology. And these games with figures lead even to conclusions about knowledge in ancient Egypt of the Golden Ratio, gravitational constant, distances to the

Moon and the Sun and many other things [1]. Usually ones do it by recalculation of Egyptian linear measures. However multiple numbers in these measures allow to judge about their unconditional use, and the heuristic value of numerology comes to its end here. But often researchers go forward. In particular, concerning the problem designated in the title of this article, there is a popular belief that as the angle of inclination of pyramids depended on their height, the latter was calculated according to a formula $h = 4RC/2\pi$, where RC is length of one side. The number π here was important because it was transcendental and irrational [2]. However actually only the Great Pyramid and some others show close correlation with this formula (but it is not absolutely accurate); the vast majority of pyramids has other ratio of their height and perimeter. But the most important is that the pyramids were neither laboratories of ancient mathematicians nor observatories of astronomers, they were cult objects. Therefore they could be not as accurate as it can be shown by means of modern measuring equipment, but they had to be included into some mythological context, and identical cult objects had similar contents and had to reflect the same principles. And if we see some changes, we must start discussing some changes in ideology. And just this is perfectly reflected in the angle of inclination of Egyptian pyramids.

Orientation of pyramids

Thirty years ago S. Haack has demonstrated that orientation of pyramids of the 4th dynasty was extremely close to the true pole, but it was gradually changed clockwise at a speed of about 20" a year. As the axis of Earth's rotation precesses counterclockwise, and stars seem to be displacing clockwise, he has explained it by orientation of pyramids to some rising star and the precession. There are two exceptions: the earliest pyramid of Djoser, with a deviation of about 180' and two later pyramids of Djedefre and Sahure whose deviation from this trend (not from the pole!) is about 50' counterclockwise. The last has been explained by a possible choice of two other stars for orientation [3].

Table 1. "Errors" in orientations of the pyramids. Deviations from the north.

Pharaoh	**First year of the reign (BC)**	**Error, (')**
Hotepsekhemuy	2740	90.0
Djoser	2667	180.0
Huni/Sneferu (Pyramid at Meidum)	2637	-20.0
Sneferu (Bent Pyramid)	2613	-17.3
Sneferu (Red)	2600	-8.7
Khufu	2589	-3.0
Djedefre	2566	-48.7
Khafre	2558	-6.0
Menkaure	2532	14.0
Sahure	2487	-23.0
Neferirkare	2475	30.0
Unas	2375	17.5
Senwosret I	1956	-90.0
Amenemhat III	1831	15.7

Actually, the later pyramid of Unas, the pharaoh of the 5th dynasty, also gets to the same line with the orientation of these pyramids (tab. 1, fig. 1: hereinafter information on orientation is taken from [4-6]). Therefore, either it reflects orientation, taking into account the precession, only to one star, or it may be explained by negligence, although the negligence is improbable because three objects are situated along a single line.

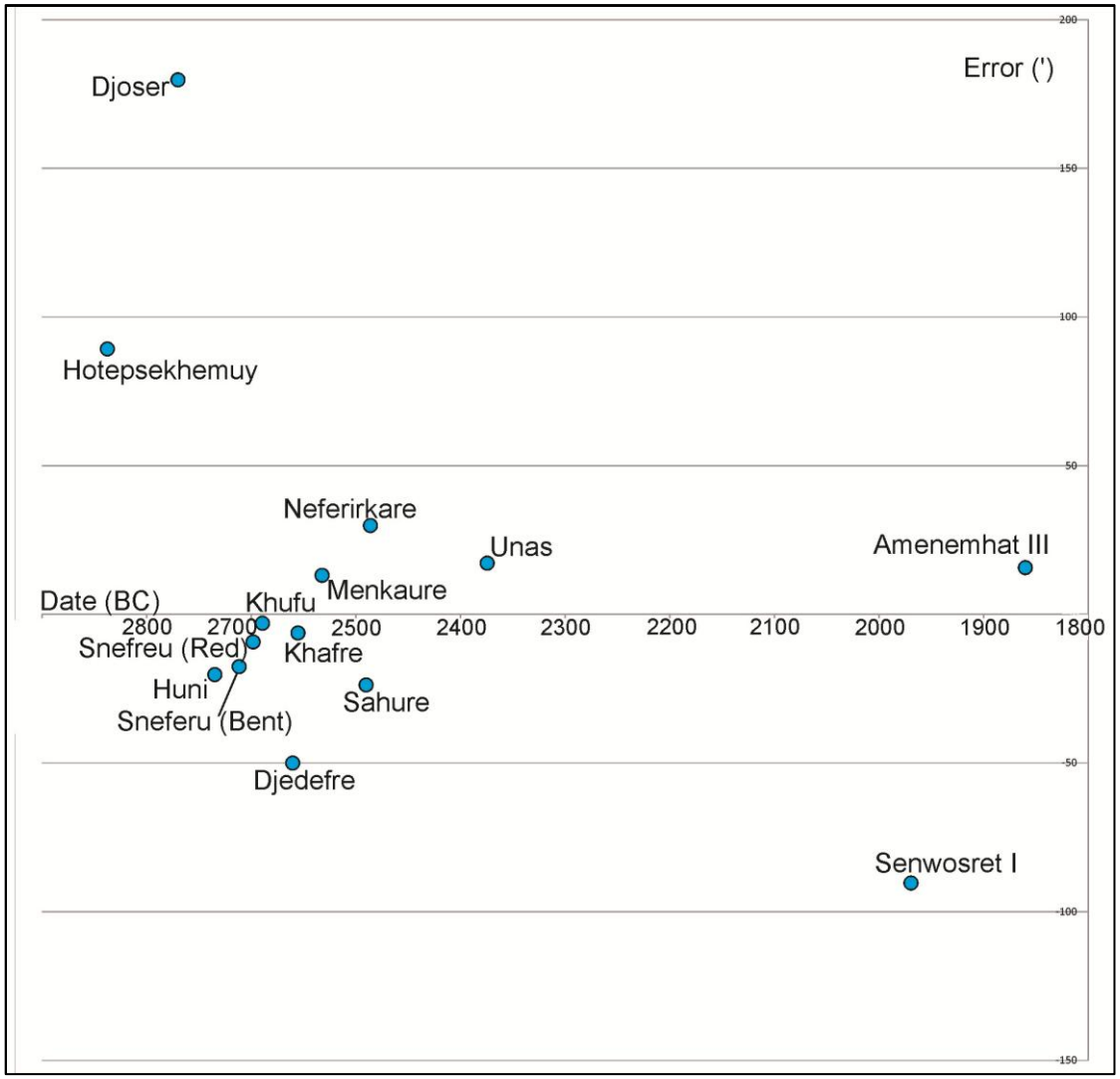

Figure 1. Deviations in orientations of the Egyptian pyramids from the north. Two precession lines are clearly visible: between deviations of the pyramids of Huni and Neferirkare, and the pyramids of Djedefre and Unas.

As there were no bright stars exactly at the pole at that time, it was impossible by this way to achieve this accuracy of orientation, and there are no clear descriptions of a method in ancient texts [7]. There are records of stars observation, and about a ceremony of "stretching the cord" which the pharaoh made together with the goddess Seshat [8, 9]. K. Spence having investigated this problem in details has supposed that during this period the Egyptians used two stars, Kochab from the Ursa Minor and Mizar from the Ursa Major, because they formed a vertical line in the north [10]. This work caused a discussion: other methods or other stars were suggested [11-16], but the principle of orientation to stars at the pole was not called in question as the precessional shift in orientation of pyramids of the 4th dynasty was rather obvious. Another version is the orientation to the vertical

line between the stars Phecda and Megrez in the constellation Plough (Ursa Major) which in the period of the 4th dynasty gave the vertical line in the north too [17]. But all these problems have been rather in details discussed in the abovementioned works, and we won't touch them here. We only state again that the orientation of pyramids had been made to the circumpolar stars with high accuracy. Some changes are not excluded: a season, a method of measurement or the used stars. In any case, the existence of the second precession line between the pyramids of Djedefre, Sahure and Unas (fig. 1) points to one of these possibilities, and not to errors.

For small pyramids and temples there are evidences of other methods of orientation – by the sun (to find the lines north – south at noon and west – east at the equinox), but the sun was used as an additional target also in orientation of large pyramids. And, meticulous studying of pyramids on the Giza plateau has demonstrated that at the benchmarking of the pyramids the Egyptians used both the direction to the sunrise or sunset at equinox, and the direction to the north, to circumpolar stars [18]. Therefore, this operation had to be done at equinox. It is not excluded that these directions were made more accurate by observation of the sun at culmination. In this sense it is interesting that the hieroglyph designating the goddess Seshat looks like the Roman *groma*, a tool for determination of the line north – south at noon. Some images of this goddess have a similar tool over her head too [19].

Table 2. Azimuths of orientations of pyramids of the 3rd – 12th dynasties.

Pyramid	**Dynasty**	**Date (BC)**	**a (°)**
Sneferu (Med)	4th	2637	94
Sneferu S (Bent)	4th	2613	89.75
Sneferu N (Red)	4th	2600	90
Khufu	4th	2589	90
Djedefre	4th	2566	90.75
Khafre	4th	2558	89.75
Menkaure	4th	2532	90.25
Shepseskaf	4th	2503	90.5
Khentkaus	5th	2500	90.25
Userkaf	5th	2494	90.25
Sahure	5th	2487	91.5
Neferirkare	5th	2475	90.25
Neferefre	5th	2460	92.25
Niuserre	5th	2445	90.75
Djedkare	5th	2414	89.75
Unas	5th	2375	90.25
Teti	6th	2345	80.75
Pepi I	6th	2321	90.25
Pepi II	6th	2278	90
Iput II	6th	2265	88.75
Qakare Ibi	8th	2170	77
Amenemhat I	12th	1985	91.75
Senuseret I	12th	1956	90.75
Amenemhat III	12th	1831	88

Use of sunrise/sunset at equinox for orientation of pyramids is well visible from table 2 and figure 2. From Sneferu pyramid to Neferefre pyramid the azimuths of southern and northern sides of pyramids (in other words their orientation along the line west – east) turned smoothly clockwise with a speed about 70" a year. It does not mean that orientation of these sides was made by some star, as it was the case with the northern direction. Perhaps, after the solar orientation of a side the builders made additional control using stars, though this value is slightly more than the annual precession.

Possibly, the role of the use of stars in the orientations decreased over time, and the use of the sun increased. For example, between pyramids of Djedkare and queen Iput II (i.e. between 2414 and 2265 BC) we do not see a precession change of orientation, although their orientations are very close to the east (tab. 2, fig. 2). Thus, along the line west – east it could be done using only the sun, without correcting by means of more accurate stellar orientation of the line north – south. But if deviations in orientation of pyramids of pharaohs are no more than 0.25° or 15', and these deviations are directed to both sides, the deviation of the queen's pyramid is 1.25° counterclockwise. It was possible in case of less careful preparation of the place for the construction and orientation of the construction using the sun by means of a vertical stake and shadows from the sun, and if the slope goes down from the west to the east [20]. Lesser accuracy in the orientation of the queen's pyramid than that of the pharaohs is quite explainable, and all told above confirms that during this period another principle of pyramids' orientation was used: it was not stellar as before, it was solar.

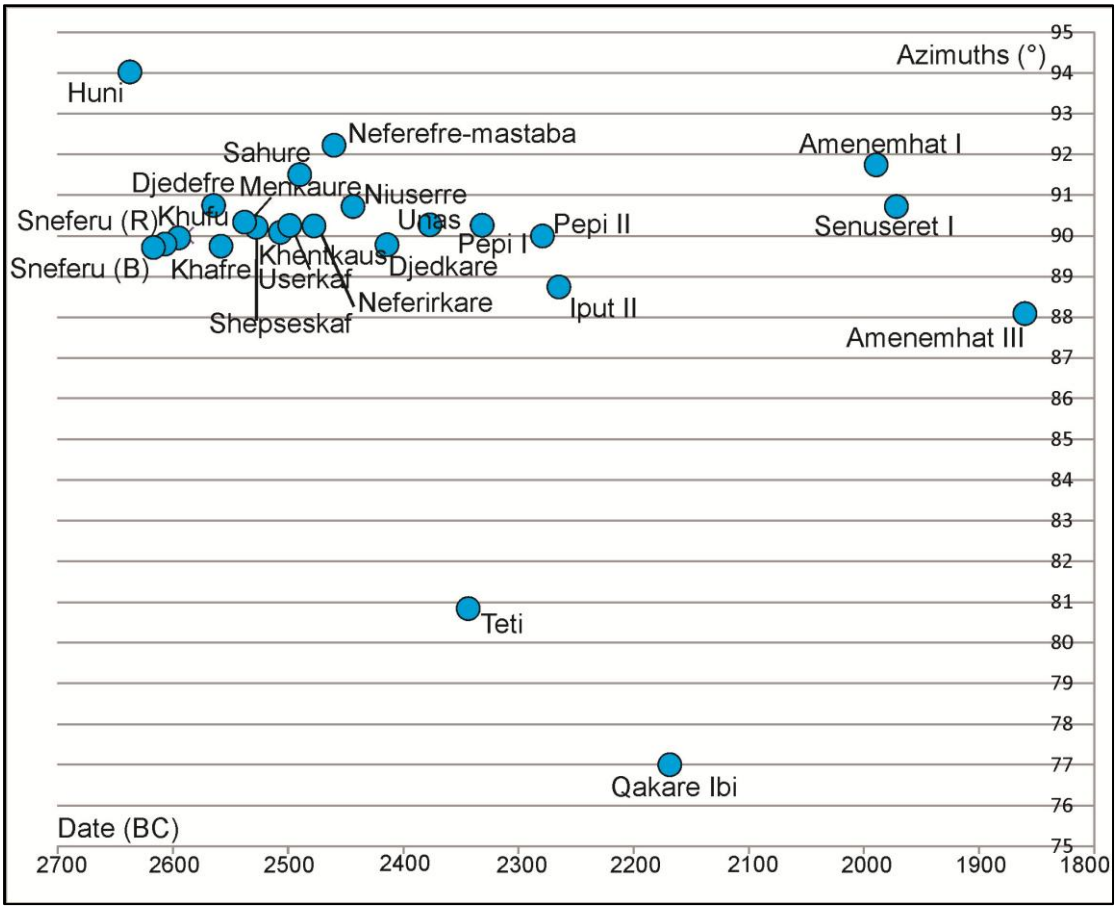

Figure 2. Orientations of pyramids of the 3rd – 12th dynasties to the east.

There were also two pyramids with essential deviations at this time: Teti (9.25°) and Qakare Ibi (13°). The latter is small, and obviously it had been made hurriedly. Therefore, possibly, just the carelessness was the reason of these deviations. And pyramids of pharaohs of the 12th dynasty have deviations no more than 2° to both sides. Therefore, the reason of deviations here was not in the carelessly prepared place, but in the carelessness in orientation. It is not excluded that it was connected already with the sun, and using the sun it was impossible to orient with such accuracy as using stars.

The difference in orientation of pyramids of the first pharaoh of the 2nd dynasty Hotepsekhemuy and his descendant Djoser, the founder of the 3rd dynasty, is 90' (tab. 1, fig. 1) that corresponds to the shift of 77" a year. It is some more than that of later pyramids and more than the value of precession; perhaps, some way of correlation of both stellar and solar orientations was used.

But the stellar orientations of pyramids took place already in the period of the 2nd dynasty. There is a description of the king Khasekhemwy hammering boundary poles into the ground for the ritual of pyramids' orientation (the ceremony of "stretching the cord") [21]. The ritual arose, probably, in the period of the 1st dynasty [22]. In any case, already the tomb of Narmer, the founder of the 1st dynasty, is directed to the azimuth of 314¾º. It means a typical for this dynasty orientation of opposite corners along the line north – south that gives for the sides the turn of 45º [23].

Thus, we see that originally pyramids were oriented to the circumpolar stars and the sun at equinox, with a possible use of additional target, the sun at culmination. About the middle of the 5th dynasty importance of the solar orientations started increasing sharply, and the stellar orientations became less important. It quite corresponded to the changes which happened during this period in the Egyptian society. Primary orientation to the stars and the sun reflects not simply an aspiration to get a more accurate result. The ideas of a stellar essence of the monarch had been combined with the dominant solar theology. The sun-god travels across the celestial ocean with the Imperishable Stars and the pharaoh [24]. In the period of the 5th dynasty the solarization of the cult intensified, the great importance gets a cult of the solar god-creator Ra from Heliopolis (Egypt. *Iwnw*) in Lower Egypt that is also reflected in names of some pharaohs (Sakhura, Ramesses). Ra became almost the main god of the kingdom [25-27]. Then the process of solarization of the cult continued. In the Middle Kingdom, after the Theban dynasty came to power, the local Theban god of the sky Amun gradually becomes very important in the Egyptian pantheon and becomes a solar God. According to the Theban theogamy the pharaoh is considered as Amun's son because Amun impregnates his mother. The name of this god is often included in names of pharaohs (Amenemhat, Amenhotep, Hatshepsut of Khenemet-Amun). In the New Kingdom all this brings to the predominant Amun-Ra's cult already under the pharaohs of the 18th dynasty. Under Amenhotep III his deification increases, obtaining its maximal complete expression under his son Amenhotep IV (Akhenaten) in the form of worship centered on the solar god Aten [28, 29]. And in orientations of sides we see the increasing deviations that may be explained probably by mainly solar orientations.

Inclinations of pyramids

In the course of work with the database I have paid attention that the angle of inclination of the pyramids was changing too (tab. 3, fig. 3). To be convinced of it diagrams of relations between the angle of inclination and the time of erection of the pyramids have been done. In the provided tables the most accepted dates of pharaohs' reigns are given from the Oxford History of Ancient Egypt [30], and the evidences on angles of inclination are taken from the Mark Lehner's publication [31].

Unfortunately, the last publication was inaccessible for me, and these evidences are cited from another work [32].

Since the pyramids of the 3rd dynasty it is possible to see that the angle of inclination gradually increases (with some small deviations towards reduction). Between the Sekhemkhet's pyramid (2648 BC, 50°36') and the Merenre's pyramid (2287 BC, 57°7'48") the annual change of the angle of inclination is 65" that is close to the annual precession. Then to the 12th dynasty serial data are absent, and pyramids of the 12th dynasty show the same regularity with two cases of deviation. In this period orientations of pyramids were not so accurate too.

Table 3. Inclinations of pyramids.

Pharaoh	Dynasty	Date (BC)	Inclination
Joser	3rd	2667	43°30'/43.5°
Sekhemkhet	3rd	2648	50°36'/50.55°
Khaba	3rd	2640	68°/68°
Huni-Snefru	3rd	2637	51°50'35"/51.84°
Sneferu	4th	2613	54° 50' 35"/54.84°
Sneferu-up	4th	2607	43°22'/43.3°
Sneferu-Red	4th	2600	43°22'/43.37°
Khufu	4th	2589	51°50'40"/51.84°
Djedefre	4th	2566	52°/52°
Khafra	4th	2558	53°10'/53.18°
Menkaure	4th	2532	51°20′25″/51.34°
Userkaf	5th	2494	53°7'48"/53.13°
Sahure	5th	2487	50°11'40"/50.19°
Neferirkare Kakai	5th	2475	54°30'/54.5°
Neferefre-mastaba	5th	2460	78°/78°
Niuserre	5th	2445	51°50'35"/51.84°
Djedkare Isesi	5th	2414	52°/52°
Unas	5th	2375	56°/56°
Teti	6th	2345	53°7'48"/53.13°
Pepi I	6th	2321	53°7'48"/53.13°
Merenre	6th	2287	57°7'48"/57.13°
Pepi II	6th	2278	53°7'48"/53.13°
Qakare Ibi	8th	2170	53°7′/53.12°
Amenemhat I	12th	1985	54°27'44"/54.46°
Senusret I	12th	1956	49°24'/49.4°
Senusret II	12th	1877	56°18'35"/56.31°
Senusret III	12th	1870	56°18'35"/56.31°
Amenemhat III	12th	1831	56°18'35"/56.31°
Amenemhat III-Hawara	12th	1831	48°45'/48.75°
Ameny Qemau	13th	1790	55°/55°
Khendjer	13th	1760	55°/55°
Ahmose I	18th	1550	60°/60°

Therefore the increase in deviations is not surprising. In general, we see the same ascending trend, and also with small deviations towards reduction of inclination. If to count the axial line between the Amenemhat I's pyramid (1985 BC, 54° 27' 44") and the Ahmose I's pyramid (1550 BC, 60°), we will receive the annual change of 44". Thus, we see a gradual change of the angles of inclination; it is close, but not identical to the annual precession.

It also differs from the figure which was obtained earlier by Haack, but 20" is the annual shift of the Northern Celestial Pole, and the shift of the point of equinox relative to stars is about 50" a year. Thus, it is more probable that it was connected with the shift of the point of equinox, but not directly, that means, not as an orientation to some star near the horizon.

This obvious connection of change of angles of the pyramids' inclination with the precession shows that determination of the inclination was realized by means of some stars, as well as orientation of the pyramids. This means that the choice of the inclination was not a technological, it was a ritual choice.

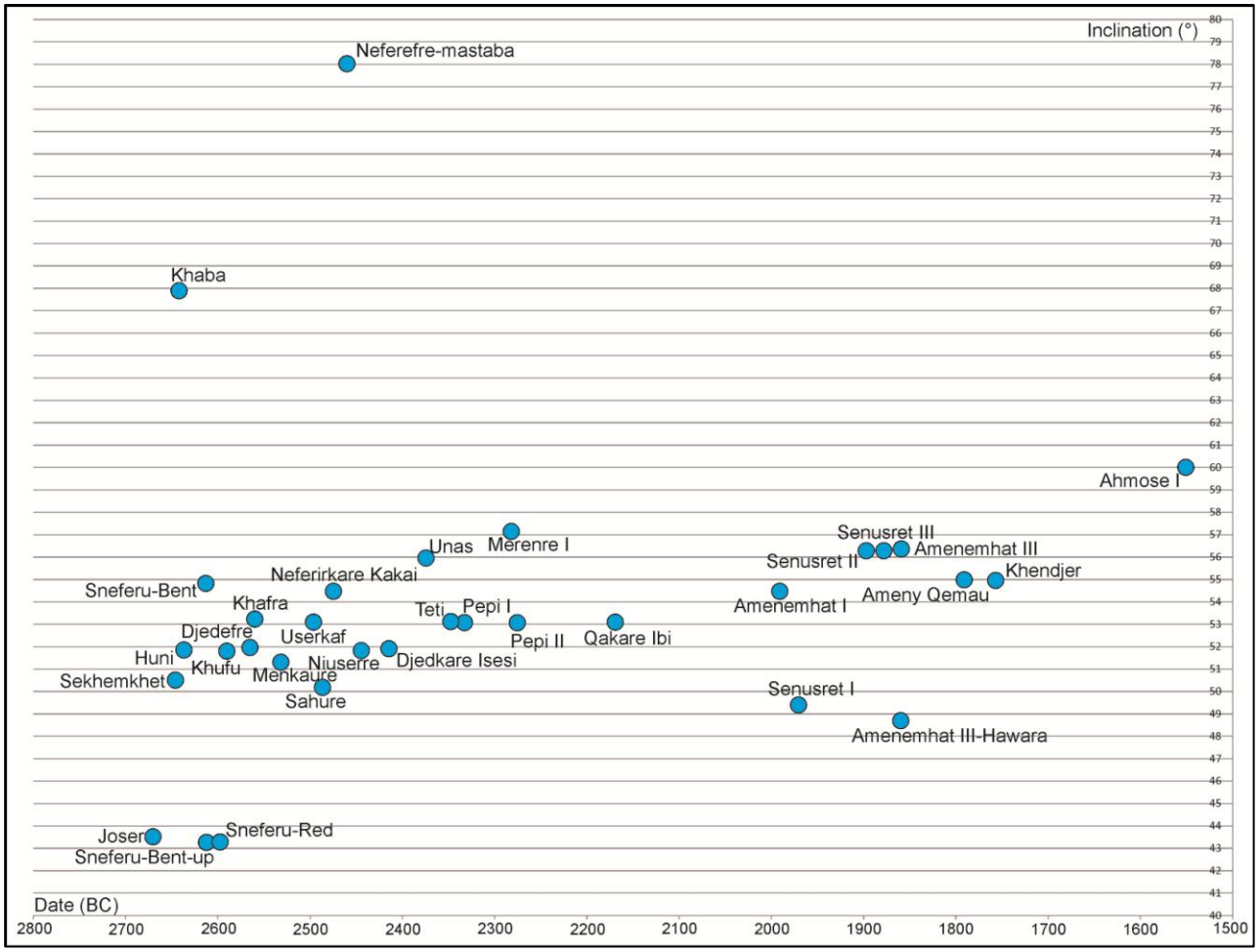

Figure 3. Lines of inclination of pyramids from Sekhemkhet to Merenre and from Senusret (Amenemhat?) to Ahmose. The last line can be continued to the pyramid of Djoser and the Red pyramid of Sneferu.

There are deviations from this trend, for example, sharply steep inclination of the pyramid of Khaba, but years of his reign are unreliable as well as belonging to him of this pyramid at Zawyet el-Aryan [33]. One more deviation is the earliest pyramid of Djoser, the pharaoh of the 3rd dynasty. The upper part of the Sneferu Bent pyramid at Dahshur and his Red pyramid have the same

inclinations as the Djoser's pyramid (fig. 4). It is remarkable that Sneferu completed the pyramid of his predecessor Huni who, probably, was not his father, and originally he built the lower part of his own pyramid with the same inclination as that of the Huni's pyramid. However, then the angle was changed and the top part was constructed with the same angle as that of the Djoser's pyramid. An idea is widespread that the reduction of the angle of inclination of the Sneferu's pyramid was caused by aspiration to precipitate the building of the pyramid as the pharaoh was afraid not to manage it to finish. But after that he constructed one more pyramid, with the same new angle of inclination.

Figure 4. Bent pyramid of Sneferu[1].

As it follows from the above, the angle of inclination was connected with orientation to some particular star, and from it we may suppose that originally Sneferu oriented the pyramid to the same star as his predecessor, but then he preferred a new star, and this choice was probably a ritual one.

Egyptian districts (*nomes*) had different gods-protectors who could be, at the same time, the all-Egyptian gods and gods-protectors in other districts [34]. But often the gods were associated with particular stars or constellations, and it is not excluded that the reasons of this change in inclination should be looked for in appearance of a dynasty with roots in another nome. It explains well the situation with the Bent pyramid of Sneferu: at first there was the aspiration to emphasize continuity with his predecessor, and then, after strengthening of the power, the orientation to "his" star was chosen. It is a normal behavior of a usurper with attempt to legitimize his power. Sometimes with the same purpose they married a daughter of a previous pharaoh. But in this case his son, Khufu, returned to the earlier inclination.

Of course, it is possible to explain the choice of a new star for the determination of inclination of the pyramids in the Middle Kingdom by purely technological reasons. In case of use of the former target from the very beginning of the Middle Kingdom the angle of the inclination would be about

[1] http://commons.wikimedia.org/wiki/Category:Bent_Pyramid#/media/File:01_bent_red_satellite.jpg

60°. But it did not confuse builders of the pyramid of Ahmose I in the very beginning of the New Kingdom. Of course, it would create problems to the following builders, but Egyptians had no idea of precession (in any case, we have no evidence about this), and they would hardly worry about this problem. Another circumstance is interesting here: taking into account the precession it is possible to believe that inclinations of pyramids of the Middle Kingdom and the beginning of the New Kingdom (Ahmose I's pyramid) get to the same precession line, as inclinations of the Djoser's pyramid and the last pyramid of Sneferu. It is a question, but the situation is not obvious. We see a huge temporal lacuna between the pyramids of Djoser and Sneferu and the pyramids of the Middle and New Kingdoms, during which we cannot show continuation of this line. Besides, taking into account increasing carelessness and deviations in orientation of later pyramids, the situation with angles of inclinations can be also presented as some dispersion. But obviously we may not speak about a single clearly visible precession line from the 3rd dynasty to Ahmose I.

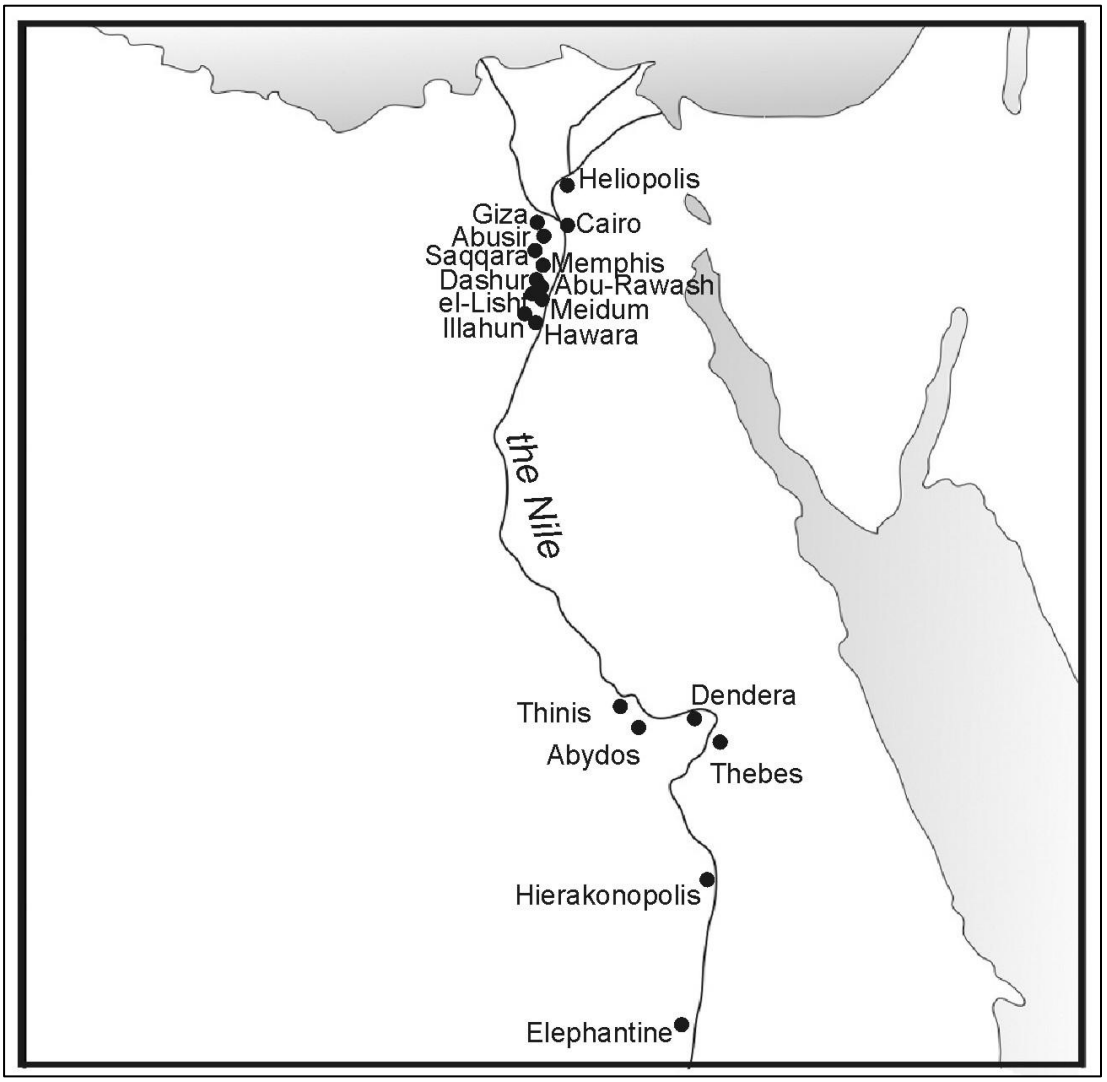

Figure 5. Map of Ancient Egypt. Two precession lines of inclinations of pyramids correspond to places of origins of ruling dynasties: originally Thinis near Abydos, then Memphis in the north and again Thebes in the south, near Abydos.

Djoser was the son of the last pharaoh of the 2nd dynasty; he finished integration of Upper and Lower Egypt. Thinis nearby Abydos in Upper Egypt (fig. 5) was the capital in the period of the 1st

dynasty. Even after removal of the capital to Memphis pharaohs of the 1st dynasty were buried in Abydos [35]. And this city was probably understood also as homeland by pharaohs of the 2nd dynasty. The founder of the 3rd dynasty, Djoser, probably still felt the connection with Upper Egypt. The situation with pyramids of Sneferu is more complicated. Initially for political or ideological reasons he evidently tried to emphasize the connection with previous king, and then he emphasized a connection with another elite group, but his successor Khufu preferred to follow the tradition of former dynasty. Reasons of these changes should be looked for in the Predynastic Period when in Upper Egypt three political centers existed: Naqada, Abydos and Hierakonopolis (*egyp*. Nekhen). Initial emergence of statehood was connected probably with Naqada, but by the Early Dynastic Period this city lost its significance and was replaced by Abydos, whose elite controlled the country in alliance with less powerful elite from Hierakonopolis [36]. It is remarkable that a full name of Khufu, Khnum-khufu, means "God Khnum protects me", and it was a local god from Elephantine, a city near the First cataract [37]. It was far to the south from Abydos and Thinis, but near Hierakonopolis (fig. 5). Therefore the change of orientation and even the pharaoh's name show an aspiration to emphasize the importance of another elite group from Upper Egypt.

Principles of orientation

It is more difficult to tell – whether the inclination of pyramids directed to a certain star was somehow connected with stellar orientation of sides of the pyramids. As we have seen, against the background of constancy of orientation to the circumpolar stars, a rather abrupt change of inclination of slopes took place.

Respectively, orientation of a pyramid and determination of its inclination were made using different stars. And if the angle of inclination was chosen by a star rising over the horizon and this star was identified with a deity connection with which a pharaoh wanted to emphasize, the orientation of the entire pyramid was made by the "Imperishable Stars" located at the North Pole which rotate round the pole and never disappear. Thus, absolutely different ideas were put in these orientations.

We can only surmise the choice of the stars, basing on their connections with gods Ptah (god of Memphis) and Amun (god of Thebes). In principle, the sacred bull Apis was associated with Ptah (and then with Osiris), and a ram was a symbol of Amun [38]. It is not excluded, therefore, the orientation to the brightest star of the Taurus constellation, Aldebaran, and probably to the constellation Aries. It has no bright stars; therefore it is difficult to assume a possible target, perhaps Hamal.

Besides, Abydos, mentioned above, was the important center of the worship of Osiris, and Hierakonopolis was a place of the worship of Horus symbolizing the living pharaoh [39].

But here a series of vital problems start. The Egyptian set of constellations not quite corresponded to the modern one going back to the Greek tradition. For example, Sheep or Goat (*srt*), corresponds to the constellation Capricorn with possible inclusion of a part of the Grus constellation, and not to the Aries constellation at all. The star cluster Hyades was united with Aldebaran, the brightest star of the Taurus, in the constellation *ʿrt* ("jaw"). Perhaps the Big Dipper ("Bovine Leg") was the only constellation connected with bull [40].

But the last constellation has also another interpretation: after the divine justice Seth was taken in the heavenly solar boat. He was responsible for thunder. He was "lord of the northern sky" and the constellation Ursa Major is the 'leg of Seth' [41].

Another author notes that the constellation *'rt* in all contexts was connected with the Taurus where Aldebaran is the brightest star. And this star is designated also as *uadjat*, "Eye of Horus" [42]. Actually, it is a well-known Egyptian myth how Horus lost his eye fighting against Seth. One fact is interesting here: after the eye was restored, Horus used it for Osiris's revival. Thus, this eye, and correspondently, Aldebaran, can be considered also as means of revival. But it was also the symbol for a state of soundness or perfection. Horus's eye supported the monarchy and protected it from Seth. However, Horus sometimes appears as the sky god whose right eye is the sun and left (lost) eye is the moon [43]. Taking into account the fact that this important for pharaohs cult was originally widespread in Hierakonopolis, and Khufu emphasized the connection with this area, the most part of orientations could be connected just with this cult and, respectively, with the Aldebaran.

At last, the Amun's connection with Thebes is already a later phenomenon. Osiris and Onuris, the god of hunting and war, were local gods near Abydos. And the Orion constellation, *sah*, was closely connected with the first of them [44]. Therefore in the case of Djoser it is more right to consider this connection.

But the builders could not orient the pyramid's sides using a star. It is extremely difficult to do in the night with high precision. Besides, precession changes of stars at culmination are not so great. We see changes about 45-65" a year, but in case of the culminated stars we would see changes about 10-20" a year.

Orientation of the sides' inclination to another celestial object, the sun, is very probable. Several years ago in the Ezidian temple of Lalish in Iraqi Kurdistan I paid attention that pyramids in whose basis fakirs light sacred fires, are crowned with a gold knob, and their sides symbolize sunbeams. Therefore, here a similar identification is not excluded too. Solar orientation imposes me also because of solarization of pharaohs' power, clearly visible already in the period of the 4th dynasty.

At first sight, it completely conflicts with all discussed above as conflicts with the discussed precession lines. Besides, now at latitude of Cairo height of the sun over the horizon on the day of summer solstice (June 22) is 70°, on the day of winter solstice (December 21) it is 32°, at the equinox it is 55°. The angles of inclinations of pyramids change from 48° to 56° (if not to consider rare inclinations of 43° and 60°). And, if to be based on the sun, it is a period between March 1 and the equinox (and a corresponding period after the autumnal equinox). Therefore there is the only variant to solve the problem: orientation of inclination of pyramids' sides to the sun at culmination, but a day of this procedure was determined by some stars or constellations.

As the rise of any star can be observed almost daily (except for periods when it crosses the sky in the solar day), the only possibility to connect this observation with a fixed date is heliacal or acronical rise of stars. After an invisibility period the star rises close to sunrise, and its heliacal rising happens in beams of the rising sun. For example, Sirius, the brightest star of the Northern hemisphere, after the period of invisibility of 70 days appears in beams of the rising sun again [45]. Then each day the star starts rising earlier, moving away from the sun, until it rises last time in beams of the setting sun, i.e., it is an acronical rise at sunset. As it happens always on a fixed day (insignificant shifts about 1-2 days are possible because of atmospheric conditions or individual visual acuity), these phenomena were an important temporal marker and they had some mythological meanings. The heliacal rising of Sirius from which the Egyptian New Year began and which preceded the Nile inundation is most known in this sense. Therefore the heliacal or acronical rises of stars during the period before and after equinoxes are surmised dates of observation.

Method

On the first stage information about localization of the pyramids and their geographical coordinates has been collected (tab. 4, 6) as these data should be inserted in all used programs although it has a noticeable impact only on a situation with the Ahmose I's pyramid. It is located far in the south where the sun is higher at culmination. In addition to this, at an advance southward by 1° a star rises heliacally by one day earlier [46].

Table 4. The beginning of reign of pharaohs, and localization of pyramids, days when the height of the sun corresponded to the angle of inclination of the pyramid (2nd column), heliacal (HR) and acronical (AR) risings of stars.

Pharaoh	Days of orientation	Hamal		Aldebaran		Betelgeuse		Rigel		Dabih	
Djoser, 2667 BC Saqqara		HR	AR	HR	AR	HR	AR	HR	AR	HR	AR
	25.2/27.11	12.3	26.8	29.4	24.10	1.6	24.11	9.6	3.12	17.12	5.6
Sekhemkhet, 2648 BC Saqqara	18.3/6.11	12.3	26.8	29.4	24. 10	31.5	24.11	9.6	3.12	16.12	5.6
Khaba, 2640 BC Zawyet el-Aryan	3.5/23.9	12.3	26.8	29.4	24. 10	31.5	24.11	9.6	3.12	16.12	5.6
Huni, 2637 BC Meidum	20.3/2. 11	12.3	26.8	29.4	24. 10	31.5	24.11	8.6	3.12	16.12	5.6
Sneferu-Bent, 2613 BC Dahshur	27.3/26.10	12.3	26.8	29.4	24. 10	31.5	24.11	8.6	3.12	16.12	5.6
Sneferu-Bent (up), 2607 BC Dahshur	26.2/27.11	13.3	26.8	29.4	24. 10	1.6	24.11	9.6	3.12	16.12	5.6
Sneferu-Red, 2600 BC Dahshur	26.2/27.11	13.3	26.8	29.4	24. 10	1.6	24.11	9.6	3.12	16.12	5.6
Khufu, 2589 BC Giza	20.3/3.11	12.3	26.8	28.4	25. 10	31.5	25.11	10.6	3.12	16.12	5.6
Djedefre, 2566 BC Abu Rawash	22.3/2.11	13.3	26.8	29.4	25. 10	1.6	25.11	9.6	3.12	17.12	5.6
Khafra, 2558 BC Giza	24.3/30.10	13.3	26.8	29.4	25. 10	1.6	25.11	9.6	3.12	17.12	5.6
Menkaure, 2532 BC Giza	20.3/4.11	13.3	26.8	29.4	25. 10	1.6	25.11	10.6	4.12	17.12	6.6
Userkaf, 2494 BC Saqqara	24.3/29.10	14.3	27.8	29.4	26. 10	2.6	26.11	9.6	4.12	17.12	6.6
Sahure, 2487 BC Abusir	16.3/6.11	14.3	27.8	29.4	26. 10	2.6	26.11	10.6	4.12	17.12	6.6
Neferirkare Kakai, 2475 BC Abusir	24.3/29.10	14.3	27.8	29.4	26. 10	2.6	26.11	10.6	4.12	17.12	6.6
Neferefre-mastaba, 2460 BC Abusir	1.6/26.8	14.3	27.8	29.4	26. 10	2.6	26.11	9.6	4.12	17.12	6.6
Niuserre, 2445 BC Abusir	19.3/1.11	14.3	27.8	30.4	26. 10	1.6	26.11	9.6	4.12	17.12	6.6
Djedkare Isesi, 2414 BC Saqqara	20.3/1.11	15.3	27.8	1.5	26. 10	2.6	26.11	10.6	4.12	18.12	7.6
Unas, 2375 BC Saqqara	31.3/21.10	16.3	28.8	29.4	26. 10	1.6	26.11	9.6	4.12	18.12	7.6
Teti, 2345 BC Saqqara	22.3/28.10	15.3	28.8	30.4	26. 10	1.6	26.11	9.6	4.12	18.12	7.6

Pepi I, 2321 BC Saqqara	22.3/28.10	15.3	28.8	30.4	26. 10	1.6	26.11	9.6	4.12	18.12	7.6
Merenre, 2287 BC Saqqara	2.4/18.10	16.3	28.8	1.5	26. 10	1.6	27.11	9.6	4.12	18.12	7.6
Pepi II, 2278 BC Saqqara	22.3/28.10	16.3	28.8	1.5	26. 10	1.6	27.11	9.6	4.12	19.12	7.6
Qakare Ibi, 2170 BC Saqqara	22.3/28.10	16.3	28.8	2.5	28. 10	2.6	28.11	10.6	5.12	20.12	8.6
Amenemhat I, 1985 BC el-Lisht	23.3/23.10	17.3	29.8	2.5	28.10	3.6	28.11	11.6	5.12	20.12	9.6
Senusret I, 1956 BC el-Lisht	9.3/5.11	17.3	29.8	2.5	28.10	3.6	28.11	11.6	5.12	20.12	9.6
Senusret II, 1877 BC Illahun	16.3/28.10	17.3	30.8	2.5	29.10	2.6	28.11	10.6	5.12	20.12	9.6
Senusret III, 1870 BC Dahshur	27.3/18.10	18.3	30.8	3.5	29.10	3.6	28.11	11.6	5.12	21.12	9.6
Amenemhat III, 1831 BC Dahshur	30.3/15.10	18.3	30.8	3.5	29.10	3.6	28.11	10.6	5.12	21.12	9.6
Amenemhat III, 1831 BC Hawara	7.3/5.11	18.3	30.8	3.5	29.10	3.6	28.11	10.6	5.12	21.12	9.6
Ameny Qemau, 1790 BC Saqqara	24.3/21.10	19.3	31.8	4.5	30.10	4.6	29.11	10.6	5.12	22.12	10.6
Khendjer, 1760 BC Saqqara	24.3/20.10	18.3	31.8	4.5	30.10	3.6	29.11	10.6	5.12	22.12	10.6
Ahmose I, 1550 BC Abydos	25.3/15.10	20.3	1.9	4.5	1.11	3.6	30.11	7.6	6.12	24.12	12.6

For modeling of the stellar sky in the ancient time the StarCalc 5.73 program was used. At the first stage this program calculated days when the sun was at culmination at the height corresponding to the angle of pyramids' inclinations. The first year of the corresponding pharaoh's reign was accepted. Respectively, for each year two possible dates, in the spring and in the autumn, have been found (tab. 4).

Then the star sky of these days at the time of sunrise and sunset were examined and the list of constellations for more careful study in these dates has been completed. These are the above-mentioned Aries, Taurus and Orion, and also Capricornus. For the work someone, as a rule, brightest or observable star was used: Hamal of Aries, Aldebaran of Taurus, Dabih of Capricornus and Betelgeuse of Orion. Then Rigel of Orion was added. For each chosen year the heliacal and acronical risings of these stars were defined, these data were inserted in the table (tab. 4). Initially altitude of the horizon was ignored and taken as 0°, but subsequently the corresponding examinations have been done (see below).

Problem of accuracy of measurements

It is necessary to understand that the evidences given below are certainly not absolute. We may concede a possibility of some inaccuracies in the ancient time caused by carelessness, different eyesight of observers, atmospheric conditions (for example, wind and dustiness of the horizon). Humidity fluctuations have not been considered too as their influence on refraction is small, although at that time, taking into account the inundations of the Nile, these fluctuations could be higher. But this factor could influence by another way – difficulty of visibility over the horizon in a certain day – the factor which is not predicted and it has not been taken into account.

There are also well-known problems of the Egyptian chronology. Here these data are given in one system as their relation is essentially important. Besides, we do not know exactly – when the benchmarking of place of any pyramid took place. Therefore here the first year of reign of a pharaoh is taken, as usually it is considered that building of pyramids ones started during either the first or second year of the reign. And when a pharaoh, like Sneferu, had built more than one pyramid, the date is conventional at all. Therefore this work is directed only to identification of general principles and regularities.

Some inaccuracies are caused also by that the altitude of the horizon along the azimuths of sunrise, sunset and rising of stars was not taken into account. Because of all these reasons it seems to me unreasonable to seek for exact specifications of concrete days and dates. Perhaps, in the long term, if the suggested hypothesis will be confirmed, it will be possible to use it for checking the Egyptian chronology. But now it obviously should not be done as the shift of a point of equinox each year is too small, and for the period of 70-100 years it is only about 1˚. In principle, people in antiquity could do a mistake of 0.5˚, and then another small mistake at the building. It could be small mistakes which reduce one another, but their superposition could increase a general mistake.

There is also a problem of accuracy of measurement of angles as it can be made by different ways yielding slightly different results. For the majority of cases this difference is noncritical. For example, the inclination of sides of the Great Pyramid (51 ° 50' 40") used in the majority of publications is an average result of F. Petrie's measurements of the northern side of the pyramid. In his publication he provided many values for different sides and different ways of measurements. The minimal value is 51 ° 44' 11", the maximal one is 51 ° 57' 30" [47]. Thus, the deviation to one side is 6' 50", and the deviation to another is 6' 29", and these deviations are not essential at all. But, as a matter of fact, I do not think that such high precision was really needed. For example, the difference of inclination between Huni pyramid completed by Sneferu and the lower part of the Bent pyramid of Sneferu is 3 ° that gives as a result a difference of only six days for the period after autumnal equinox. Correspondently, a deviation of 1 ° means a difference of two days, or 30' a day. The discussed differences between the risings of Betelgeuse and Aldebaran are about one month. Errors of one day and more could be caused by features of the horizon, besides, we do not know where this day was determined – on this place or in some special temple in another place. Possibly, in the long term all these figures will be defined more exactly, in case of appearance of any specific problem connected, for example, with the Egyptian chronology. But for the discussed problem even the deviations of 1 ° are not important.

Results

The diagram shows some regularity (fig. 6). It is possible to claim with confidence that orientation of inclinations of pyramids was not carried out in the spring as the vernal equinox and heliacal risings of the most part of stars took place after the sun ascended to the height corresponding to the angles of pyramids' inclinations, and usually in a week after the heliacal rise of Hamal. But above we have discussed that orientation of the sides was made at equinox (and this operation had to precede the measurements of inclination of slopes), and the heliacal Hamal's rising occurred before the vernal equinox.

Inclination of the Khaba's pyramid could be determined on a day close to the heliacal rising of Aldebaran. But it is a single case. We have very few evidences about this pharaoh; his connection with this pyramid is also questionable. Therefore it would be better not to rest upon this fact.

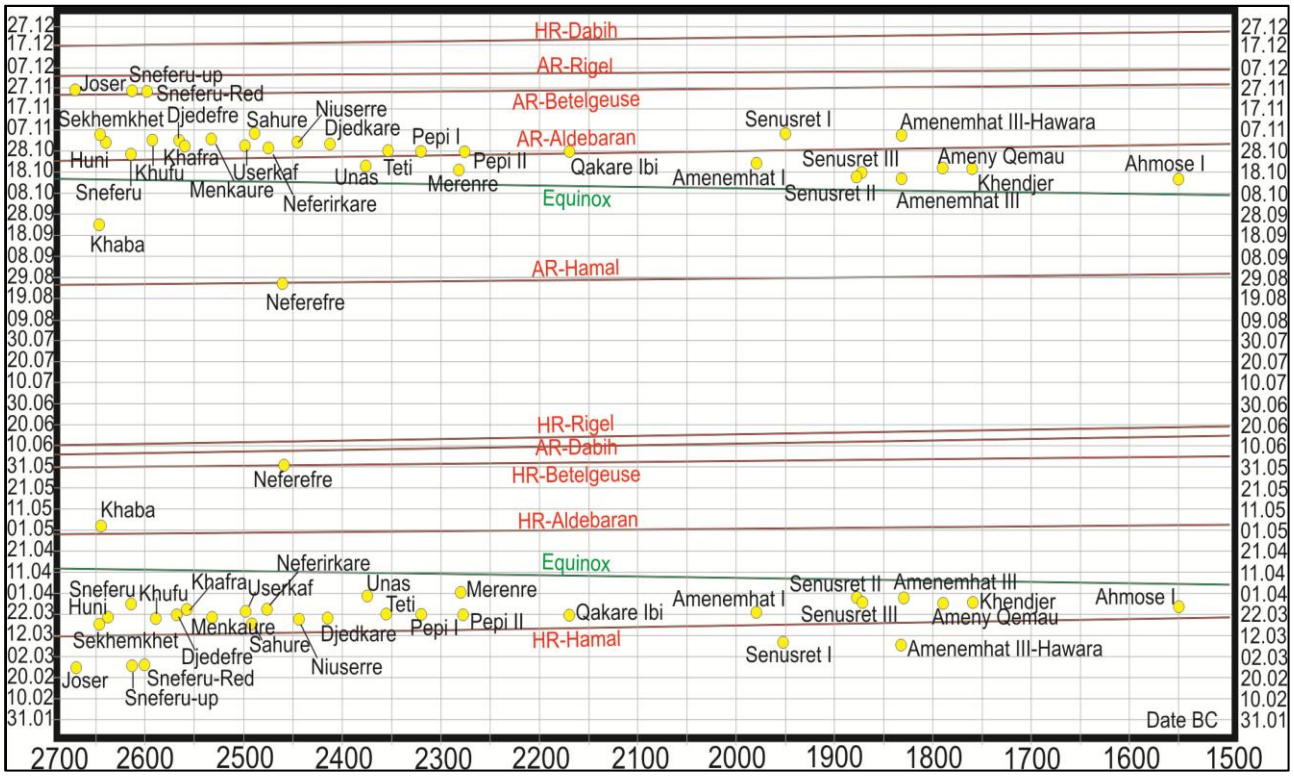

Figure 6. Autumnal and spring dates of orientations of pyramids' inclinations (round points), lines of equinoxes and dates of the heliacal and acronical risings of stars Hamal, Aldebaran, Betelgeuse, Rigel and Dabih on the chronological scale.

The second exception is the mastaba of Neferefre. Orientation of inclination of this unfinished pyramid coincided with the heliacal rising of Betelgeuse and the acronical rising of Hamal. But it is unsystematic data.

Orientations of all other pyramids were subordinated to strict regularities. In all other cases the inclinations of sides were determined after the autumnal equinox and, usually soon after the equinox, within 6-20 days. It is quite natural as above we have discussed that the orientation of sides of pyramids was done not only by the northern stars, but also by the rising or setting sun in the equinox. Respectively, the determination of inclination was technically possible and mythologically sensible after this procedure.

Exceptions are the already discussed pyramid of Khaba, mastaba of Neferefre, whose orientation had been done long before the autumnal equinox, and also the pyramids of Djoser and Sneferu whose orientation had been done 42-44 days after the equinox. It is obvious that the previous impression was false that the last pyramids get to the same precession trend with pyramids of the Middle Kingdom. They were absolutely definitely not connected with the risings of Hamal and Aldebaran and, respectively, with the constellations of Aries and Taurus which rose much earlier than the possible day of orientations of inclinations of these pyramids. But there is also no connection with the constellation Capricorn, as Dabih rose much later than the possible days of orientation of the inclinations. The only possible candidates are Betelgeuse and Rigel from the constellation Orion, whose acronical risings were observed after the autumnal equinox and, in general, they corresponded to the days of orientation of the inclinations. But the Rigel's rising happened by 9-10 days after the rising of Betelgeuse and did not correspond to the estimated days

of orientations of the pyramids' inclinations. Contrary to it, risings of Betelgeuse precisely correspond to it.

Inclinations of all other pyramids were determined before the Betelgeuse's rise. If to consider the general trend and to ignore individual situations, we see an obvious gradual approach of days of orientation of pyramids with the line of equinox. Lines of risings of stars demonstrate the inverse tendency. If to ignore small fluctuations and sharp deviations (which obviously are subject to another rule) we may present it as a gradual approach of a day of orientation of pyramid's sides to the equinox from 3 to 1.5 weeks. But we can also imagine the situation in another way: days of orientations can be presented not as a linear trend, but as three steps going down to the equinox.

Before 2414 BC (Djedkare Isesi) orientation of slopes of the pyramids could be made within 10 days after the acronical rising of Aldebaran. Then to the pyramid of the pharaoh Qakare Ibi of the 8th dynasty orientation of slopes could be made on a day close to the rising of Aldebaran, with small deviations. But actually, these deviations could be less if to consider the horizon altitude (see below), i.e., in general, the principle remained. Since the Middle Kingdom orientation of slopes was as a rule made before the acronical rising of Aldebaran. Exceptions are Senusret I and Amenemhat III's pyramids in Hawara which were oriented after this event as it had taken place in the previous dynasties of the Old Kingdom. We see also some closeness to the line of equinox. Partly, it can be considered as a compromise between two orientations, although another explanation is to be found. But then, in the New Kingdom, the pyramid of Ahmose I is oriented 17 days before the acronical rising of Aldebaran and 8 days after equinox, it is the solar orientation.

In principle, this entire picture has quite good logical explanations. Originally the pyramids were oriented to northern stars and the sun at the equinox, and the orientation of their inclinations was possible after that, on the day of acronical rise of Betelgeuse. It had technological reasons (several days needed to prepare the place after the benchmarking), but mainly the mythological reasons (see below). If it was explained only by the need to prepare the place before the following technological operation, we wouldn't have such exact coincidence of days after the long period from the equinox.

Then, still in the period of the 3rd dynasty, the date of the orientation of slopes was made on a day significantly closer to the equinox, on a day after the acronical rising of Aldebaran. And since this time, in the presence of some deviations, a day of determination of inclination of pyramids more and more corresponds to the rising of Aldebaran.

Then, we see two parallel processes. On the one hand, because of the precession the acronical rising of Aldebaran was further and further from the equinox. On the other hand, the further "solarization" of pharaoh's cult began, therefore from the ritual point of view it was undesirable to separate the day of orientation of pyramid determined in the equinox from the day of orientation of its sides' inclination, although in some cases the builders neglected this problem.

As we have seen above, the transition from the stellar orientation of pyramids to the solar one happened probably at the same time, or even slightly earlier, after the pharaoh Neferirkare, about 2475 BC (fig. 2). It is not excluded that it also was partly connected with the "solarization" of the cult, and partly with that the customary for the orientation stars displaced because of the precession; therefore it became impossible to combine precisely the orientation of the northern and southern sides to the sun at the equinox, and the western and eastern sides to the northern stars. Of course, it was possible to choose a new target, but it is necessary to remember that these targets were mythologically significant. And in this case a replacement of one couple of stars by another couple from the same constellation was quite admissible (for example, the discussed variants of orientations to Kochab from Ursa Minor and Mizar from Ursa Major or to Phecda and Megrez from

Ursa Major), but then there was a gap in this tradition, and only after the appearance of corresponding ancient texts we will know: what was initially here – the precession shift of stars or solarization of ideology.

In principle, a small transformation of some ideas and meanings caused by the precession was already discussed by the example of Sirius, whose heliacal rise coincided probably with the Nile inundations in the period when the calendar had started to be developed, then the day changed, but this connection was saved [48].

In our case there was also a period (about 3300 BC) when the acronical rising of Aldebaran coincided with the autumnal equinox. So, originally some exact astronomical coincidence had obtained a mythological understanding, but then people followed the new tradition and new ritual until divergences did not become too essential. This problem is already reflected in literature. As Egyptian gods were associated with stars, they were a metaphor for order, and in case of a disturbance of the order it was necessary to create a new myth to quiet fear [49].

Unfortunately, for the subsequent period of the New Kingdom all these conclusions cannot be examined as at this time the Egyptians stopped building of pyramids; and tombs of Valley of the Kings came to take their place. For the tombs of the 18th dynasty it is still possible to trace some regularity in the form of aspiration to orient to the rising or setting sun. Only three tombs of pharaohs with the name "Thutmose" are turned to the north. On example of the only in our list pyramid of Ahmose I we see that the determination of its inclination was made at the equinox. Thus, in the New Kingdom the solar orientations, probably, dominated.

But the subsequent tombs of pharaohs of the 19th and 20th dynasties have no definite orientation and were connected with free space, and only sometimes it is possible to assume an orientation to celestial bodies [50].

Paradoxically, the above described tradition somehow revived many years later. Kushite pyramids in Sudan having steeper inclination of sides (from 65 ° to 77 °, but usually about 68-69 °) are a good illustration of this, because these steep slopes can be explained by lower latitude. At this latitude the sun culminates on such angles one or two weeks after autumnal equinox. As it was a civilization secondary in relation to Egypt and borrowed not only technologies, but also the Egyptian religion, this angle demonstrates that the principle of orientation of sides of the pyramids described above had its continuation in Sudan, and it is too similar for a casual coincidence. But the Kushite pyramids in Meroe are dated to the 8th century BC. The chronological gap with the last in our list Ahmose pyramid is 800 years. But, if in the New Kingdom we see a gradual approaching of the day of orientation of slopes with equinox, here the interval increased. This principle had to be reflected in some other objects, but we do not know them. The only way of the transfer was a written tradition that seems improbable, but finds confirmation in other data. Kushite pharaohs felt themselves as successors of the Old Kingdom that found expression not only in the pyramids, but also in scenes on walls of temples that had been copied in temples of Sakkara and Abusir, titulature of pharaohs of that far epoch was reproduced, even texts of the Old Kingdom were copied and reproduced in temples [51]. Therefore this variant cannot be excluded, although it is impossible to find strict proofs.

Altitude of the horizon and possible deviations

As the altitude of the horizon in a concrete place could have essential impact on the date of rising, for some locations of the pyramids by means of the same program StarCalc 5.73 azimuths of risings of these stars were calculated (in Saqqara for 2667 and 1760 BC) (tab. 5). For other places

with pyramids near Cairo it has not been done as they are nearby and deviations of the points of rise for a theoretically plane area would be absolutely insignificant. But for located in the south Abydos this procedure was carried out for 1550 BC. And from the table it is visible that the chronological differences are more significant than the latitudes of the area. Therefore the chronological range of azimuths for risings of some stars has been determined. Taking into account points of sunrise and sunset (90° and 270°) we were interested in the following ranges for each area: 85-96°, 110-118° and 270°.

Table 5. Azimuths of rising of stars at Saqqara and Abydos; and general sectors of the azimuths.

Location	Hamal	Aldebaran	Betelgeuse	Rigel	Dabih
Saqqara, 2667 BC	91.38	93.58	96.30	117.88	107.65
Saqqara, 1760 BC	85.99	88.08	91.55	112.07	109.87
Abydos, 1539 BC	84.69	86.71	90.40	110.34	110.12
Azimuths	85-91	87-94	90-96	110-118	108-110

By means of another program (see http://www.heywhatsthat.com/) along these azimuths profiles of the area have been constructed. Generally the horizon is at the altitude of about 5°, except Hawara and Abydos, where the altitude of the horizon is about 2.5° and Abusir and El-Lishta where the altitude of the horizon reaches 7° (tab. 6). As the orientation was connected with the acronical rising of stars, the rising star and the setting sun were situated opposite to each other, in the azimuths close to 90° and 270°. We see from the table that the altitude of the horizon in these azimuths slightly differs, but very insignificantly. Higher horizon in both directions is explained by the location in the river valley.

In principle, for the heliacal risings the difference is not too essential. But under the conditions of acronical rising in case of the higher horizon in both directions the sun sets slightly earlier, and the star rises a bit later. Correspondently, the acronical rise has to happen when the sun is situated slightly closer to this star, i.e., in our case, slightly earlier and closer to the equinox. Therefore it is possible that the day of orientation of slopes of pyramids of the Middle Kingdom was closer to the acronical rising of Aldeberan.

Table 6. Altitude of the horizon in places of localization of pyramids. The blank section – the horizon along the azimuth is blocked.

Location	Coordinates	Azimuths		
		87-96°	110-118°	270°
Saqqara	29°52′16″N 31°12′59″E	5.2°	-	5.12°
Zawyet el-Aryan	29°54′N 31°12′E	-	-	-
Meidum	29°23′17″N 31°09′25″E	-	-	5.12°
Dahshur	29°48′23″N 31°12′29″E	5.16-5.07°	5.11-5.02°	4.93°
Giza	30°01′N 31°13′E	4.5°	4.5°	-
Abu Rawash	30°01′55″N 31°04′30″E	5.37-5.23°	5.32-5.28°	-
Abusir	29°54′N 31°12′E	7.07-7.5°	7.28°	-
el-Lisht	29°34′13″N 31°13′52″E	7.5-7.7°	7.5-7.51°	-
Illahun	29°14′N 30°58′E	-	-	4.83°
Hawara	29°16′N 30°54′E	2.28-2.32°	2.28-2.41°	-
Abydos	26°11′06″N 31°55′08″E	2.51°	2.42-2.51°	2.78°

But it is rather problematic to calculate it as rise of a star could be defined not on a place of predesigned pyramid, but in any remote temple located in an area with another line of the horizon. And it was usually so. In any case, the heliacal rising of Sirius was the all-Egyptian calendar event. And we really do not see this dependence. For example, the earlier (for 8 days) orientation of the pyramid of Merenre at Saqqara cannot be explained by the horizon in the area or the precession shift caused by a later date of the pyramid. Inclinations of sides of earlier and later constructions of this area (pyramids of Pepi I and Pepi II) had been oriented on close days. In areas with higher horizon, for example, Abusire, the pyramids of Neferirkare, Neferefre, Niuserre and Sahure had not been oriented after the general trend for this period at all, and the pyramid of Neferirkare had been oriented on even slightly earlier day of the year (we do not considered here the mastaba of Neferefre). The situation at El-Lisht is the same: the Amenemhat's pyramid quite corresponds to other pyramids of this period, and slopes of the Senusret's pyramid were oriented on a later day of the year. May be the deviation of the Senusret's pyramid was caused just by higher horizon at El-Lisht? Obviously not: the days of its orientation correspond to the days of orientation of the pyramid of Amenemhat III at Hawara, where, on the contrary, the horizon is lower. Therefore, reasons of the deviations are not in localization of the pyramids at all and not in local peculiarities of the horizon.

But, if not to consider individual deviations which could have many reasons, and to look at the general trend, it can be really explained by that since the 6th dynasty the builders made determination of a day for orientation of inclinations of pyramid's sides not in any single temple in a place convenient for observations any more, but directly near the pyramids. Everywhere here the horizon is slightly higher that was convenient, because the day of the rising star was closer to the equinox.

Possibly, more exact measurements of pyramids will allow us to understand some problems better. Small deviations in the antiquity are also not excluded. But errors in ancient measurements are unlikely. Measurements in this case are extremely simple: builders could put a vertical pole, and at noon connect its top with the end of shadow from the pole on the surface. The obtained line would give the angle of the pyramid's inclination with high accuracy. But at the subsequent building, subsidence of the structure and its facing small deviations were possible.

Mythological background

The calendar dates connected with heliacal rising of stars are well-known in Egyptology. For pyramids we have reconstructed the connection with acronical risings that seems to be quite justified in this case. The acronical rising could be chosen because it was funeral cult. Rising of a star, thus, coincided with that moment when the sun fell in the west to the Underworld. But here the connection of the cult of Osiris with the cult of Ra was important: it was interpreted as a concept of "Ra in Osiris and Osiris in Ra" [52]. But another explanation is also possible. Orientation of sides of a pyramid to the Sun was made on a day of the last (acronical) rising of a star. And if some star was connected with some local cults and, eventually, with a family of the pharaoh, the last acronical rise (i.e. death of the star) could mean also death of his human essence. The orientation to the sun was more universal. It reflected not the belonging to a family/clan, but the power over the world and lodgment of the person with divine essence. Some complicated synthesis of these two positions is also possible.

As a matter of fact, the connection of inclination of pyramids of Joser and Sneferu with the rise of the Orion constellation is quite explainable from the mythological point of view as this

constellation (*'bwt*) was identified with Osiris, the god of the Underworld and the reviving nature. On the late Egyptian Dendara Zodiac the Orion constellation is shown as a man with a stick that was characteristic for Osiris's images [53]. But it is remarkable that this constellation was identified not only with Osiris, but also with pharaoh. In the Afterlife the pharaoh reaches the firmament as Orion/Osiris who bestows on him the authority of a 'great force'. After his death the king becomes Osiris, ruler of the Underworld [54]. As a matter of fact, the entire funeral cult of pharaohs was connected with him. There are, however, different opinions about a star of this constellation which was connected with Osiris, and, sometimes, the preference is given to Rigel, but the connection of Orion with Osiris and the funeral cult is undoubted. Two stars had been mentioned in the Pyramid Texts: *Sopdet* (Sirius associated with Isis who was an important part of the myth about Osiris) and *Sah* (a part of the Orion constellation) [55]. But, if to take into account that the orientation of inclinations was made on a day after the acronical rising of Betelgeuse and before the acronical rising of Rigel, the Osiris's connection with Betelgeuse seems to be more acceptable although it is not excluded that it was this constellation in general, and Betelgeuse is simply its first bright star rising over the horizon.

The dead pharaoh was identified with Osiris, and this identification was necessary for his subsequent revival. There is a description that after his death the pharaoh of the 5th dynasty Unas traveled in the sky to become the star *Sah* (or Orion) [56].

The Osiris's cult is well known since the middle of the 5th dynasty [57], but, judging from the inclinations of the Joser's pyramid this cult existed earlier, already since the beginning of the 3rd dynasty. Above we have already mentioned that the cult originated from the area of Abydos where Joser came from. It is remarkable that the Osiris's crown *atef* is similar to the crown *hedjet* from Upper Egypt. The Sneferu's use of the orientation like that of the Djoser's pyramid was probably an attempt to emphasize his ties with Upper Egypt and Osiris.

But the Osiris's cult was not only a funeral cult; it was also a cult of the productive power of nature which was closely connected with the pharaoh's cult too. And this connection can be also shown by the discussed inclinations of pyramids. R. Parker wrote that the first year of reign of pharaohs of the 12th dynasty began in November or December [58]. Above we have discussed the close days at the end of October and the beginning of November for orientation of inclinations of pyramids. We know also that it was practiced at the very beginning of reign of a pharaoh. Therefore it is not excluded that from the Egyptian point of view in full measure the pharaoh's reign began since the moment when all rituals connected with orientation of his pyramid had been completed. Thus, an aspiration to have time to build the pyramid was not a reason of the immediate beginning of this process. Probably the pyramid was not simply a tool for the following revival of the pharaoh. It was also a way of establishment of connections with the Sun and the Cosmos, and a ritual directed to maintain a cyclic natural order that quite fits the Osiris's cult. And this connection could be established in the course of ritual actions of orientation of pyramids. Therefore this was the sacralization of the pharaoh's power, finding of the divine essence, and exactly from this moment his reign began.

But what was the reason to change the star, whose acronical rising was the sign for a day of solar orientation of inclinations? As we have seen, then the rising of Aldebaran served as such a sign. It has been suggested that Seth was associated with the Hyades (and Egyptians united them in one constellation with Aldebaran) [59]. Therefore, in principle, we can try to interpret this situation as follows: after the acronical rising of "Seth", against the "dying" sun, ceremonies of solar orientation of inclinations of pyramids were carried out as an act of fighting against Chaos (Seth symbolized

powers of Chaos [see 60]). In principle, it can be explained as an act of union of the king with the solar deity, an act of lodgment with solar essence and the divine power. And it would be an esthetically beautiful theory. But the pyramids of Djoser and Sneferu were connected with another star, Betelgeuse, and, eventually, with the Osiris's cult. And discussing orientations we have to adhere to one principle for different instances. Therefore a star had to be connected with some positive image. At last, strict proofs of connection of Aldebaran with Seth are absent, and above we have discussed possible connection of Seth with the Big Dipper.

On the other hand (see above) there are some reasons to suppose that Aldebaran was associated with Horus. This falcon-god was a symbol of divine monarchy. He was son of Osiris and winner of Seth, and pharaoh was considered as living Horus on the throne of Egypt, and when crowning pharaohs obtained a name of Horus. Aldebaran was "Eye of Horus" used for Osiris's revival. It is remarkable that this concept was especially adopted after annexation of Lower Egypt [61], although sources of this cult were also in Upper Egypt and its worshipers belonged to another elite group came from the south. Therefore it quite corresponds to the picture that we see – after Djoser the choice of a new star connected with Horus. But then there was this strange attempt of Sneferu to return to the former star, Betelgeuse connected with Osiris. Osiris's cult originated in the south too. Therefore it is not excluded that at early stages of the united monarchy it was an attempt to emphasize importance of this or that elite group from Upper Egypt, perhaps, there was a gradual adaptation of both cults with one another. But for the pharaoh's cult the Horus's cult appeared more significant. Even when in the Middle Kingdom the Osiris's cult becomes extremely important we don't see its reflection in orientation on a day of acronical rising of Betelgeuse. It can be explained by that this cult becomes connected not so much with a family of the pharaoh, but with the population of Egypt in general. Researchers even write about "democratization" of this cult when complicated funeral ceremonies, including mummification, spread widely among the people, and now not only kings, but all people have spiritual force, *ba* [62]. Therefore the cult of Horus remains more significant for the power of pharaoh. But it was not a choice between two god-patrons. Horus was associated with the living, ruling pharaoh, and Osiris did with the dead. Therefore this change in orientations could be also provoked originally by inclination of pharaohs to this or that elite of Upper Egypt, but after this the religious component was the main reason.

And solarization of the pharaoh's cult was more important process. The solar cult existed in the Kingdom always, and its importance grew all the time, it had different expressions (Ra, Amun, Aten, Atum). This cult was closely connected with monarchy. The pharaoh was considered as son of the sun-god. And it is interesting that Atum "participated" in the ceremony of coronation. The sun-god takes his son, the dead pharaoh, to the sky. And architectural symbolism of the pyramid is that it was a stairway of sunshine to the sun-god [63]. It quite corresponds to the aspiration to direct slopes of pyramids to the sun. A long time it was combined with stellar concepts. But in process of increasing solarization of the cult and the divergence of times of equinox and acronical rising of the necessary star, the preference was given to the sun. As a matter of fact, we have also discussed the same processes by the example of orientations of sides of pyramids. Thus, it was a unified process. But it was not a development of a pure idea; it reflected processes happened in society. Already since the first dynasty legitimation of a new political order was realized through a concept of the king-god. Pharaoh was a mediator between people and gods, and the relations with the latters were his exclusive prerogative. After development of the cult of Ra in the Middle Kingdom he was a guarantor of stability of the World, repeatability of natural cycles, and all this was important for all Egyptians. After his death the connection of people with him saved. He continued to be their

defender. This resulted in a gradual rising of the solar cult of Ra. Local cults remained, and pharaohs remembered their roots and their gods that is reflected in stellar orientations. In the period of the Middle Kingdom the solarization of the pharaoh's cult intensified. At this time relative independence of nomes decreased. Imperial officials appeared in the nomes in addition to local rulers, and under Senusret III the nomarchs were replaced by officials at all. The role of local temples and local gods decreased for the sake of the solar cult [64]. And we see that the day of orientation of slopes of pyramids was closer to equinox in comparison with the period of the Old Kingdom. At the very beginning of the New Kingdom we see continuation of this process when orientation of slopes of the pyramid of Ahmose was closest to the equinox. This founder of the 18th dynasty was military official before accession, and his name shows a connection with the lunar god Ah. Nevertheless, in the process of building of his pyramid only the solar cult was taken into account. Former patrimonial connections of the pharaohs left behind, giving way to the cult dominating in the Kingdom which reaches its apogee under Amenhotep III and Akhenaten [65].

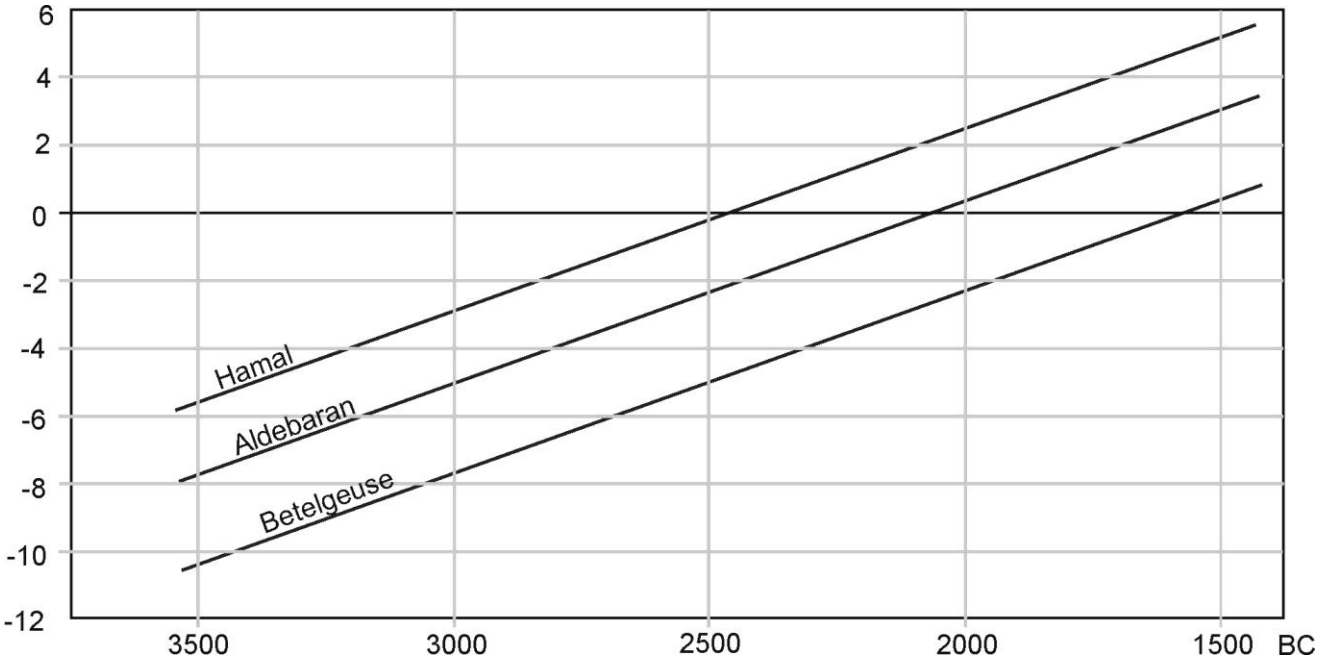

Figure 7. Lines of points of rising of stars at latitude of Cairo. Deviations from the east in different periods.

It is not excluded that in the course of solarization of this cult it was important that all stars discussed here rise in the east, close to sunrise at equinox (fig. 7). Because of the precession the point of their rising displaced all the time, but constellations occupy a large space in the firmament. Therefore even Orion during the considered period was rather close to the point of equinox and was certainly perceived as the east star connected with the sun.

Conclusions

Orientation of inclinations of the Egyptian pyramids, as well as orientation of their sides, was subordinated to combination of two principles – stellar and solar. Since the beginning of the Old Kingdom the orientation of the sides was made at the autumnal equinox to the circumpolar stars and the rising or setting sun. After that the orientation of inclination was made to the sun at culmination. Originally it was made on the day of acronical rising of Betelgeuse that was connected with the cult

of Osiris, but then we see the transition to orientation on a day after the rising of Aldebaran that was probably connected with growing importance of the cult of Horus and its connection with the pharaoh's cult. After the middle of the 5th dynasty the solarization of the pharaoh's cult increased. On the other hand, because of the precession former circumpolar stellar targets and solar targets drifted apart more and more, and the preference was finally given the latters.

In the period of the Middle Kingdom in view of the same developments of the solar cult of the pharaoh and the Amun's cult, and also in view of the precession, the orientation of sides of the pyramids, as well as their inclinations, was made only by the sun. But they tried usually not to separate a day for the last operation from the acronical rising of Aldebaran and the equinox. As a result, for the New Kingdom we can speak about the solar orientations only. Only more detailed studies of orientations and Egyptian texts can finally clear this question.

The sense of all these actions was ritual one, and not only to guarantee the ascension of the pharaoh to the sky after his death, but above all for sacralization of his power, finding of the divine essence, and maintenance of the Cosmic Order at the beginning of his reign.

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