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OPTIMAL GENERATION CAPACITY ALLOCATION AND NETWORK EXPANSION SIGNALING USING OPF

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ABSTRACT

This paper presents a novel method of allocating new generation capacity within existing grid using Optimal Power Flow (OPF). New generation capacity and connections to external networks are modelled as generators with quadratic cost functions. This allows preferences to be expressed regarding the location of new capacity and a focus on a specific part of the network whilst, at the same time, considering the technical and economic impact of exports and imports of power on the broader network. Sequential Quadratic Programming (SQP) is used to solve the OPF, as it produces signals that can be used in planning mechanisms for the efficient development of the network.

Keywords: Generation capacity, allocation, Optimal Power Flow, OPF, generation cost functions, Capacity Expansion Locations, Export Points, Import Points.

INTRODUCTION

Electricity networks are called to accommodate more and more generation capacity in order to supply the increasing demand. However, environmental and social concerns often prohibit the expansion of the current networks. Furthermore, in most cases, the way deregulation of the electricity sector was carried out failed to attract adequate investment for the reinforcement of the existing infrastructure. The long-term catastrophic consequences of underinvestment on networks are visible already [1]. Therefore, the efficient use of the limited economic resources is vital. Network Operators (NOs) are solely responsible for the operation and development planning of the network. The first-come-first-served policy currently enforced by most NOs in issuing generation connection permits protects the system from crossing its operating limits, but has no theoretical ground in terms of efficiency of allocation. Consequently, there is a need for a generation capacity allocation mechanism that exploits the potential capabilities of the existing network, but at the same time provides signals for the efficient development of the network.

Several coordination strategies have been proposed for a more effective allocation of new capacity [2]-[3]. We suggest a novel mechanism, which seeks the optimal allocation of new generation capacity to predetermined connection points, with respect to the constraints imposed by the existing network infrastructure [4]. It is based on a well-documented tool in power engineering, Optimal Power Flow (OPF). The way new generation capacity is modeled allows us to express preferences over locations for the allocation of new capacity. If these locations are also connected to specific generation technologies (e.g. wind energy resources) the preferences may reflect the societal support for them (e.g. renewables). The results can be easily adapted by the NO for new capacity planning mechanisms, which direct the financial incentives (e.g. subsidies) for new capacity to specific locations. Subsequently, the investment force implicitly allocates the new capacity in an efficient manner. The allocation can focus on a specific part of the network, where both the financial and technical impact of exports and imports of power on the rest of the network are considered.

In order to solve the OPF we use Sequential Quadratic Programming (SQP). The LaGrange multipliers, sensitivity byproducts of SQP attached to each constraint explicitly described in the OPF, can be used to facilitate network planning decisions. In most practical cases they indicate which line, transformer or other equipment would improve the overall potential connection capacity, if upgraded first.

DEFINITIONS

Capacity allocation is the problem of defining the capacity of new generation on predetermined connection points of the existing network.
Optimal capacity allocation is the capacity allocation that maximises an objective function, usually the total new capacity, with respect to network constraints.
Network constraints are constraints imposed by statutory regulations (e.g., voltage limits on buses) and equipment specifications (e.g., thermal limits on transmission lines and transformers).

OPTIMAL POWER FLOW

OPF is the process of dispatching the electric power system variables in order to minimize an operation criterion, while attending load and feasibility. The mathematical formulation of the problem is briefly described below.

It is assumed that the following are known: active and reactive power generation capabilities and costs, sizes of fixed loads, transmission and distribution line capacities, specification of fixed transformers and other power system devices. The control variables *c*, which are regulated during optimization, are usually the ones at the disposal of system operators:

a) generators' active and reactive power output.
b) tap ratios and/or phase shifts of transformers with tap changers and/or phase shifting capabilities.
c) settings of switched shunt devices, e.g., capacitor banks.
d) active power transferred from DC links.
e) shedding of interruptable loads.

However, in order to completely define the state of the system, more variables have to be introduced. The state variables *s* are:

a) voltage phase angles at every bus.
b) voltage magnitudes at load buses.

The equation linking variables *c* and *s* is the power system load flow equation:

$$EqCon(c,s) = 0 \quad (1)$$

It can be analysed for a number of equality constraints, one for each bus, expressing the power balance between the active/reactive power injected and withdrawn from the bus.

Inequality constraints describe the power system's operating limits:

$$IneqCon(c,s) < 0 \quad (1)$$

Such constraints include line thermal limits, active and reactive power generation capability bounds, range of tap changers or phase shifters and voltage limits at buses. Additional operational constraints may be easily added, using expressions of the *c* and *s* variables describing operating violations, if they are valued as critical for the system security.

The "objective function" defines the operating criterion of the power system. In most cases, it is the short-term cost of electricity production. The OPF optimization process aims to find the variables *c* and *s* that minimise the objective function, subject to the equality and inequality constraints. It is a Nonlinear Programming (NP) problem:

$$\min f(c,s) \quad (2)$$

subject to (1) and (2).

CAPACITY ALLOCATION USING OPF

In the following paragraphs, we present how OPF can be used to allocate, optimally, new generation capacity.

Modeling Sinks and Sources

New generation capacities are simulated as generators with quadratic cost functions with negative coefficients:

$$C_g(P_g) = a \cdot P_g^2 + b \cdot P_g + c \quad (3)$$

w.r.t. $a,b,c < 0$ and $P_g > 0$, where C_g is the operational cost of generator *g* at output level P_g.

These generators are connected to predetermined locations in the network, the "Capacity Expansion Locations" (CELs), with the output of generators simulating the allocated capacity at the CEL. Different sets of coefficients between cost functions declare preferences for the allocation of new capacity between CELs.

Energy transfers from/to external networks are also simulated as generators with quadratic cost functions. We will refer to them as Export/Import Points (E/IPs). The coefficients of the cost functions are negative for exports and positive for imports. The outputs of the generators are negative when they represent exports and positive when they represent imports.

$$C_T(P_T) = a \cdot P_T^2 + b \cdot |P_T| + c \quad (4)$$

w.r.t. $a,b,c < 0$ and $P_T < 0$ for exports or $a,b,c > 0$ and $P_T > 0$ for imports, where C_T is the operational cost of the generator at output level P_T simulating the E/IP.

Existing generation capacities are simulated as generators with constant active power output ($P_{g,installed}$) and given reactive power ($Q_{g,installed}$) injection capabilities:

$$\begin{aligned} &P_{g,installed} = const. \\ &Q_{g,installed}^{\min} < Q_{g,installed} < Q_{g,installed}^{\max} \end{aligned} \quad (5)$$

Loads are simulated as sinks of constant active (P_D) and reactive (Q_D) power:

$$L_D = P_D + jQ_D \quad (6)$$

System Constraints

The amount of active and reactive power injected into any system bus *k* must equal the amount withdrawn from it. The complex power balance on the buses is formulated:

$$\sum_t (P_{tk} + jQ_{tk}) + \sum_g (P_{gk} + jQ_{gk}) + \sum_d (P_{dk} + jQ_{dk}) = 0 \quad (7)$$

where *t*=all lines, *g*=all generators and *d*=all loads connected to bus *k*. P_{tk}, P_{gk}, P_{dk} and Q_{tk}, Q_{gk}, Q_{dk} are the active and reactive power injected into bus *k*. If the bus is also an E/IP, then the complex power transferred from/to bus *k* from the external grid must be added to (for import) or subtracted from (for exports) the above sum.

Proper operation of the power system equipment and quality of supply requires the maintenance of bus voltages close to their nominal values:

$$V_b^{\min} < V_b < V_b^{\max}, \text{ for all buses } b \quad (8)$$

where $V_b^{\min}$ and $V_b^{\max}$ are the lower and upper bounds of the bus voltage V_b around the rated value.

The installation of new generation capacity is limited by statutory regulations on quality of supply, environmental concerns, planning policies, technological limitations or system constraints imposed by stability, fault or other security analyses. Here, only restrictions resulting from statutory regulations are used:

$$LB_g < P_g < UB_g \tag{9}$$

where LB_g and UB_g are the lower and upper bounds respectively for the generation output P_g at CEL g.

In most distributed generation applications, synchronous generators perform Automatic Voltage Regulation in power factor control mode [5]. Thus, in order to simplify our analysis we assume that CELs have constant power factors:

$$\cos\phi_g = P_g \Big/ \sqrt{P_g^2 + Q_g^2} = const. \tag{10}$$

However, in more general cases, the production of reactive power is more flexibly controlled or additional reactive power sources are utilized (e.g., FACTS). Then, this restriction can be relaxed providing higher generation capacities.

The thermal capacity of a line sets a limit to the maximum apparent power (MVA) transfer:

$$S_t < S_t^{\max},\ for\ all\ lines\ t \tag{11}$$

where S_t is the apparent power and $S_t^{\max}$ is the thermal limit of line t.

Each E/IP represents a physical connection to an external network. The capacity of the connection sets a limit to the maximum amount of power that can be transferred to and from the external network. Furthermore, in cases where the quantity of the exported or imported power has a significant impact on the operation of the external grid, more conservative bounds than the connection capacity must be applied to limit the voltage rise or drop at buses within the external network. These limits are expressed as:

$$|P_T| < |P_T^{\max}|,\ for\ all\ E/IPs\ T \tag{12}$$

where $P_T > 0$ for imports and $P_T < 0$ for exports.

In addition, we must provide of the maximum reactive power $Q_T^{\max} > 0$ the external network can feed into the system and the minimum $Q_T^{\min} < 0$ it can absorb:

$$Q_T^{\min} < Q_T < Q_T^{\max},\ for\ all\ E/IPs\ T. \tag{13}$$

OPF Objective Function

The OPF Objective Function f is the total cost of all simulated generators. It includes the negative cost of generation at CELs and exports at EPs, as well as the cost of imports at IPs. The cost of losses could also be taken into account, but is ignored at this stage of research to confine the model to explore optimization of generation capacity allocation .

$$f(P_g, P_T) = \sum_g C_g(P_g) + \sum_T C_T(P_T) \tag{14}$$

OPF Target Function

The OPF Target Function g is the minimum of the objective function f, subject to (8)-(14):

$$g(P_g, P_T) = \min f(P_g, P_T) \tag{15}$$

Thus, the OPF problem reflects the optimal allocation of new generation capacity at CELs and the setting of energy transfers at E/IPs, with respect to the power system constraints.

SEQUENTIAL QUADRATIC PROGRAMMING

We use SQP to solve the OPF, because it outperforms every other tested NP method in terms of efficiency, accuracy, and percentage of successful solutions, over a large number of test problems [5]. The solution of NP problems generally requires an iterative procedure to establish a direction of search at each major iteration. Basically, SQP sequentially forms a Quadratic Programming (QP) problem at each iteration based on the quadratic approximation of the Lagrangian function. The Lagrangian function is practically the objective function $f(x)$ to be minimized, augmented with the constraint functions $g_i(x)$ penalized by parameters λ_i (called LaGrange multipliers):

$$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i \cdot g_i(x) \tag{16}$$

where i=1,...,m and m the number of constraints.

QP concerns the minimization of a linearly constrained quadratic objective function; the LaGrangian in this case. Thus, the nonlinear constraints described in (2) are linearized in the beginning of each iteration. The QP part of the iterative process consists of two stages: a) it finds a feasible starting point, if the initial point given in the SQP problem is not feasible and b) calculates an iterative sequence of feasible points. The sequence of feasible points finally converges to the solution of the subproblem, as it attempts to satisfy the Kuhn-Tucker equations.

The solution to the QP subproblem produces a vector d_k, which is used to form a new iterate $x_{k+1}=x_k+a.d_k$. The step length parameter a is determined in order to produce a sufficient decrease in a merit function, such as the one stated in [6]:

$$M(x) = f(x) + \sum_{i=1}^{m_e} [r_i g_i(x)] + \sum_{i=m_e+1}^{m} [r_i \cdot \max\{0, g_i(x)\}] \tag{17}$$

where r_i is a penalty parameter for constraint i. [6] recommends setting the penalty parameter of the next iteration equal to:

$$(r_{k+1})_i = \max_i \left\{ \lambda_i, \frac{1}{2}\left[(r_k)_i + \lambda_i \right] \right\} \tag{18}$$

The next QP subproblem is formed with x_{k+1} as the new feasible starting point and a new approximation of the LaGrange function is calculated around this point. The overall process converges to the optimum when d_k gets sufficiently small.

Meaning of LaGrange multipliers in OPF solution

As we already mentioned, the solution of the QP subproblem at each major iteration of the SQP complies with the Kuhn-Tucker equations. The Kuhn-Tucker equations are necessary conditions for optimality of a constrained optimization problem:

$$\nabla f(x)+\sum_{i=1}^{m}\lambda_i\cdot\nabla g_i(x)=0 \qquad (19)$$

$$\lambda_i\cdot g_i(x)=0 \qquad (20)$$

$$\lambda_i\geq 0 \qquad (21)$$

Equation (19) describes a cancelling of the gradients between the objective function and the active constraints at the solution point. For the gradients to be cancelled, the LaGrange multipliers are necessary to balance the deviations in magnitude of the objective function and constraint gradients. Generally, the magnitudes of gradients represent the effect marginal changes of function variables have on the function value. Therefore, each multiplier reflects the impact of a marginal relaxation of the respective constraint on the objective function value.

A constraint is considered 'active' when the solution lies on the constraint boundaries. Only active constraints are included in this cancelling operation. Constraints that are not active must not be included in this operation and so are given LaGrange multipliers equal to zero. This is stated implicitly in equations (20) and (21).

During the final iteration of SQP, the QP subproblem produces the LaGrange multipliers connecting the overall optimum of the objective function with the constraints. When we use SQP to solve the OPF, not only we optimally allocate new generation capacity at CELs and set the energy transfers at E/IPs, but also calculate the LaGrange multipliers for all the active system constraints. By examining the multipliers we can decide which constraint, if marginally relaxed, would result in greatest increase of the objective function. When OPF is utilised for generation capacity allocation a higher value of the objective function means higher total new generation capacity. Subsequently, when OPF allocates new generation capacity the LaGrange multipliers indicate which system constraint would result in even higher total generation capacity, if it was marginally relaxed.

We have to stress the fact that LaGrange multipliers reflect only the impact of marginal changes of the constraints on total new generation. They do not directly specify how much these constraints should be relaxed and, in which order, to optimally increase the potential connection capacity. Such a sophisticated planning mechanism requires an iterative process, which would marginally relax the constraint with the higher multiplier and reallocate capacities at each iteration. An additional cost should be attached to each marginal constraint relaxation, reflecting the cost of reinforcing the existing infrastructure in reality. This is the target of future work. However, as we will see later in the test cases, usually one multiplier is much higher than the others clearly indicating that the respective constraint should be relaxed first.

Nevertheless, not all constraints can be relaxed in real world power systems. For instance, some transmission lines can be upgraded and their thermal limit increased, but none of the bus voltage limits can be relaxed as they are imposed by statutory regulations. This limits the number of multipliers considered during network reinforcement planning.

EXAMPLE

A 12-bus 14-line network has 3 available CELs at buses 1, 10 and 11 (Figure 1). It also has an E/IP to an external network at bus 12. A 50 MW generator is installed on bus 5. It can consume or provide up to 34 MVAr of reactive power. The network has a common rated bus voltage level at 33 kV, except for the CEL buses which have a rated voltage of 11 kV and the E/IP bus at 132 kV.

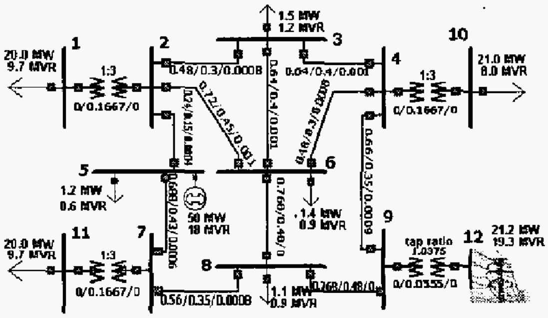

Figure 1 The 12-bus 14-line test case

The CEL buses connect to the network through 30 MVA transformers with fixed taps. The E/IP bus connects through a 90 MVA transformer with automatic tap changer, which regulates the voltage within a ±2% range of the rated voltage at the low voltage side with a ± 10 % tap range around the nominal tap ratio. The electric characteristics of transformers and lines are printed along those elements on Figure 1, in the form R/X/B (p.u.). Loads consume constant complex power on buses 1, 3, 5, 6, 8, 10 and 11.

Lines 2-5 and 4-9 are constrained by thermal limits of 37 MVA and 30 MVA respectively (specify (12)), while all other lines are considered to have unlimited capacity. We assume that the E/IP can exchange up to 100 MW with the external network without affecting its secure operation (specifies (13)). The external network is also capable of providing up to 60 MVAr of reactive power to the local network and consuming up to 50 MVAr (specifies (14)). To keep our example simple, we also assume that all new generators connected at CELs produce power at constant 0.9 lagging power factors (specifies (11)). A hypothetical government policy also restricts the maximum allocated capacity to 200 MW at each CEL (initializes (10)). Finally, statutory regulations limit bus voltage fluctuations to ±10% around the nominal values (specifies (9)).

Before the new capacity connects on the grid, the external network provides 21.2 MW and 19.3 MVAr, while the generator at bus 5 operates at full capacity (50 MW, 17.6 MVAr).

Results

Two test cases were examined. In Case A, cost functions were applied to CELs that expressed no preference for the allocation of new capacity. In Case B, the cost functions express a preference for the allocation of new capacity at bus 1. Table 1 contains the cost function coefficients for each CEL and the E/IP for both cases, which specify (4) and (5).

Table 1

	Case A			Case B		
CEL bus	**a**	**b**	**C**	**a**	**b**	**c**
1	0	-20	0	0	-30	0
10	0	-20	0	0	-20	0
11	0	-20	0	0	-20	0
E/IP bus	**a**	**b**	**C**	**a**	**b**	**c**
12	0	-20	0	0	-20	0

The OPF converged in less than 1 second (for an AMD Athlon XP 2100+ CPU) for both cases. The capacity allocation is presented in Table 2. A comparison of the allocations in Cases A and B proves that the preference for CEL 1 'shifts' the capacity from CEL 11 to that bus, in the expense of total new capacity and exports.

Table 2

CEL bus	Case A	Case B
1	11.5 MVA	37.6 MVA
10	44.5 MVA	27.5 MVA
11	19.9 MVA	9.5 MVA
Total	**75.9 MVA**	**74.6 MVA**
E/IP	**-31.4 MW**	**-24.5 MW**

In Case B, line 4-9 reached its thermal limit of 30 MVA. If the line is upgraded to over 35 MVA, the thermal limit becomes an inactive constraint for the OPF, which now allocates 82.6 MVA total new capacity. It is up to the NO to decide whether it is technically feasible and profitable to upgrade the line in order to increase the connection capacity of the network by 8 MVA.

In Case A, both lines 2-5 and 4-9 reach their thermal limits. The LaGrange multipliers connected with the constraints in OPF imposed by those limits are 2900 and 4000 respectively. The much higher multiplier connected with the thermal limit of line 4-9 indicates that a future upgrade of this line first will result to a higher connection capacity than an upgrade of 2-5. Indeed, if we consider line 4-9 unconstrained, the total generation capacity increases to 81.3MVA, whereas an upgrade of line 2-5 instead results in 79.4 MVA.

CONCLUSION

We developed a model for the simulation of new generation capacity in an existing network and the energy transfers from/to external networks, which can be easily integrated in the conventional OPF problem. The solution of this augmented OPF coincides with the optimal allocation of new generation capacity, subject to network constraints. The method allows preferences to be expressed for the connection of new capacity to specific locations. Finally, SQP is used for the solution of the OPF, because the LaGrange multipliers connected with the system constraints can be used as signals in planning mechanisms for the efficient reinforcement of the existing network.

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