# Breaking the Speed Limit for Perfect Quantum State Transfer

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We describe a protocol for perfectly transferring a quantum state from one party to another under the dynamics of a fixed, engineered Hamiltonian. Our protocol combines the concepts of fractional revival, dual rail encoding, and a rare glimpse of the anti-Zeno effect. Remarkably, this non-deterministic transfer can happen on average faster than the speed limit for perfect quantum state transfer [1, 2].

## I. INTRODUCTION

Current generations of quantum computers are extremely promising, being at the cusp of proving quantum advantage [3, 4], while not yet being capable of implementing arbitrary algorithms of useful sizes. Critical to their performance is the ability to apply as many operations as possible before the inevitable decoherence-induced limits are reached. Each step of a quantum computer requires controlling a quantum system to create the desired evolution. Any such task has a spectrum of possible solutions, spanning a range from full time control of every conceivable parameter, to (essentially) no control during the evolution. These will trade different properties, including ease of implementation, likelihood of introducing error (as a rule of thumb, the more one can interact with the system, the worse decoherence gets) and speed. A specific experiment, with specific hardware and noise constraints, will choose the most appropriate solution from within that range. Central to that body of knowledge is the enumeration of the extreme cases.

Theoretically, it is crucial to understand the ultimate bounds – how long *must* we take to perform a specific operation, and can such a bound be saturated? For example, in the protocol of perfect state transfer introduced in [5, 6], there is a speed limit for the protocol: there cannot exist a solution that takes less than a certain time [1, 2]. Hence, the solution in [6] is optimal, operating at twice the speed of the consecutive swap gates that would be specified in the gate model.

With such limits in place, the challenge is to understand the conditions under which a no-go theorem is proved and whether, by relaxing any of them, the limit can be broken. State transfer, for example, imposes perfect transfer, and requires no time control of the system Hamiltonian. Relaxing either one permits faster protocols that may still be of practical relevance [7–9].

Minimising the theoretical assumptions is also important for experiments, each of which have their own challenges; having a range of options for what constraints to relax due to ease of implementation is vital. Experimentally, state transfer schemes have now been implemented in a range of systems. Optical experiments [10, 11] have been particularly successful at tuning the required coupling strengths, while in NMR [12] and, more recently, superconducting systems [13] and quantum dots [14], short transfers have been demonstrated with chains of three or four qubits. The superconducting results are particularly promising with regards to scalability.

In this paper, we investigate another potential loophole: an assumption of the unitarity of evolution. The anti-Zeno effect suggests the possibility that repeated measurement at appropriate times *could* speed up the evolution [15]. As a consequence of relaxing this assumption, the arrival of the transferring state will no longer be deterministic. We will demonstrate a protocol, taking inspiration from the heralded, non-deterministic arrival protocols of [16–18], improving them for the specific purpose of use with an engineered spin chain. This will allow us to demonstrate a violation of the speed limit thanks to the anti-Zeno effect. The protocol becomes particularly elegant if the engineered chains that we choose exhibit fractional revival [19, 20], introduced in Section II.

Our main protocol is detailed in Section III. In contrast to [16, 17], which required a dual-rail encoding across two (arbitrary, preferably identical) parallel spins chains, we carefully tune the properties of the spin chain that we use, such that our protocol utilises only a single chain. We refer to this as a "monorail". Moreover, should our state fail to arrive, it behaves like a complete reset of the protocol, which is computationally far easier than the repeated updates and recalculations required in [16, 17]. In Section IV, we shall prove that the analytic solutions of [21] saturate an equivalent speed limit for fractional revival, and are hence the optimal choice. We show that,

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for even length chains, they yield a state transfer protocol whose expected arrival time is less than the fastest possible perfect state transfer time. The advantage is subtle, however, as illustrated by the fact that there is no advantage in the case of odd chain length. While we emphasise the theoretical aims of this result — understanding that the speed limit can be breached — in Section V, we make a few practical comparisons of our monorail scheme to the dual rail scheme. While we require a carefully tuned Hamiltonian that will be susceptible to perturbations, its noise tolerance is much better thanks to a much faster transfer time.

### A. Perfect State Transfer

Underlying most physical realisations of a quantum computer is a Hamiltonian, describing how the individual components interact. One of the main tasks in creating localised gates for computation is to switch off that interaction, except between the specific pair of qubits where it's needed. Instead, the study of state transfer aims to embrace this multi-qubit interference, using it to gain speed-ups in the desired protocol. For example, let $H$ be the Hamiltonian of an $N$-qubit spin chain,

$$H = \frac{1}{2}\sum_{n=1}^{N-1} J_n(X_nX_{n+1} + Y_nY_{n+1}) - \frac{1}{2}\sum_{n=1}^{N} B_nZ_n, \quad (1)$$

where $Z_n$ denotes application of the Pauli-$Z$ matrix on qubit $n$. The standard concept of perfect state transfer, as introduced in [5, 6, 20], involves a unitary evolution of an unknown single-qubit state $|\psi\rangle$ from one end of the chain to the other,

$$e^{-iH\tau}|\psi\rangle \otimes |\underline{0}\rangle = |\underline{0}\rangle \otimes \left(e^{-iZ\phi/2}|\psi\rangle\right)$$

in the state transfer time $\tau$, up to a corrective phase rotation by angle $\phi$. Here, we denote $|\underline{0}\rangle = |0\rangle^{\otimes(N-1)}$ and $|\underline{n}\rangle = |0\rangle^{\otimes(n-1)}|1\rangle|0\rangle^{\otimes(N-n)}$. The representation of $H$ in the basis $\{|\underline{n}\rangle\}_{n=1}^{N}$ is denoted by $H_1$, and is an $N\times N$ tridiagonal matrix with ordered eigenvalues $\lambda_n > \lambda_{n+1}$.

It is well-known (see [1, 2]) that on any such $N$-qubit spin chain, there is a speed limit to the transfer:

$$J_{\max}\tau_{\min} \geq \frac{\pi}{4}\sqrt{N^2 - \frac{1}{2}(1-(-1)^N)} \quad (2)$$

where $J_{\max} = \max_n |J_n|$ is the maximum coupling strength [22]. This is saturated by the engineered chains introduced in [6].

## II. FRACTIONAL REVIVALS

Our aim is to specify a Hamiltonian $H_1$ and a corresponding non-deterministic protocol such that the expected transfer time is lower than the time imposed by the speed limit. Central to our construction is the concept of *fractional revival* (FR). Specifically, we say that $H$ has a $\theta$-revival between 1 and $N$ at time $\tau_0$ if the evolution satisfies

$$e^{-iH\tau_0}|\underline{1}\rangle = \cos\theta|\underline{1}\rangle + \sin\theta e^{i\phi}|\underline{N}\rangle, \quad (3)$$

up to a global phase. This is a very specific requirement, but constructions of an appropriate $H$ do exist [19–21]. The phase $\phi$ is irrelevant to our study (it will typically be $-\frac{\pi}{2}$, and will always be known, so that it can be compensated for). For such a revival, the state does not transfer, but gets entangled between extremal sites:

$$|\psi\rangle|\underline{0}\rangle \longrightarrow \cos\theta|\psi\rangle|\underline{0}\rangle + \sin\theta|\underline{0}\rangle\left(e^{-iZ\phi/2}|\psi\rangle\right).$$

Fractional revival chains have some key properties that will be of use to us. In particular, Eq. (3) implies that

$$e^{-iH\tau_0}|\underline{n}\rangle = \cos\theta|\underline{n}\rangle + \sin\theta e^{i\phi}|\underline{N+1-n}\rangle \quad (4)$$

for any $n \neq \frac{N+1}{2}$. To see this, observe, for example, that $|\underline{2}\rangle = (H|\underline{1}\rangle - B_1|\underline{1}\rangle)/J_1$ such that

$$\begin{aligned} e^{-iH\tau_0}|\underline{2}\rangle &= \frac{1}{J_1}(H - B_1\mathbb{1})e^{-iH\tau_0}|\underline{1}\rangle \\ &= \frac{1}{J_1}(H - B_1\mathbb{1})(\cos\theta|\underline{1}\rangle + \sin\theta e^{i\phi}|\underline{N}\rangle) \\ &= \cos\theta|\underline{2}\rangle + \sin\theta e^{i\phi}\left(\tfrac{J_{N-1}}{J_1}|\underline{N-1}\rangle + \tfrac{B_N-B_1}{J_1}|\underline{N}\rangle\right). \end{aligned}$$

Since this output must be orthogonal to the output of the evolution Eq. (3), and must also be normalised, we see that $B_1 = B_N$ and $J_1 = J_{N-1}$.

While these are the key properties for our protocol, some further detail is beneficial for the optimality proofs of Section IV. Following through the consequences of this argument shows that if the chain length, $N$, is even, then the chain must be symmetric. If $N$ is odd, it is either a symmetric chain, or the central two coupling strengths $J_{(N\pm1)/2}$ could be asymmetric.

In the case of FR on a symmetric chain, satisfying $SH_1S = H$ with

$$S = \sum_{n=1}^{N}|\underline{n}\rangle\langle\underline{N+1-n}|, \quad (5)$$

we can be more explicit about the phases. A symmetric chain divides into symmetric and anti-symmetric subspaces. These have eigenvalues $\{\lambda_{2n-1}\}$ and $\{\lambda_{2n}\}$ respectively. The initial state $|\underline{1}\rangle$ and its $\theta$-revival are both supported only on the state $(|\underline{1}\rangle + |\underline{N}\rangle)/\sqrt{2}$ in the symmetric subspace. Hence, within that subspace, we must have a perfect revival of that state (which, in fact, means a perfect revival of all symmetric states), up to a global phase. The same also holds for the antisymmetric space. Thus, the only relevant parameter is the relative phase

between these two perfect revivals, which we call $2\theta$. We have

$$\frac{1}{\sqrt{2}}(|\underline{1}\rangle + |\underline{N}\rangle) \mapsto \frac{1}{\sqrt{2}}(|\underline{1}\rangle + |\underline{N}\rangle)$$
$$\frac{1}{\sqrt{2}}(|\underline{1}\rangle - |\underline{N}\rangle) \mapsto \frac{e^{i2\theta}}{\sqrt{2}}(|\underline{1}\rangle - |\underline{N}\rangle).$$

We immediately learn that for a symmetric FR chain,

$$|\underline{1}\rangle \mapsto \cos\theta\,|\underline{1}\rangle - i\sin\theta\,|\underline{N}\rangle\,, \quad |\underline{N}\rangle \mapsto \cos\theta\,|\underline{N}\rangle - i\sin\theta\,|\underline{1}\rangle \tag{6}$$

up to a global phase.

## III. ALL ABOARD THE MONORAIL

We now come to the key part of our protocol. This has many aspects in common with [16–18], which used a dual rail encoding – an encoding of the qubit entirely within the single-excitation subspace, but split across two parallel spin chains. Here, we will use the same encoding procedure, but on a single rail — a chain that has a $\theta$-revival between 1 and $N$ at time $\tau_0$.

We take the unknown single-qubit state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ and encode it as $\alpha\,|\underline{2}\rangle + \beta\,|\underline{1}\rangle$, i.e. across the first two sites of a spin chain. After waiting the time $\tau_0$, test to see if the single excitation has arrived without looking at the state itself. As in [16, 17], the key to this working is that the arrival amplitude for $1 \to N$ and $2 \to N-1$ should be equal. By construction, Eq. (4), an FR chain at the time $\tau_0$ has this property. If the excitation has arrived, the state has successfully transferred.

$$e^{-iH\tau_0}(\alpha\,|\underline{2}\rangle + \beta\,|\underline{1}\rangle) = \cos\theta(\alpha\,|\underline{2}\rangle + \beta\,|\underline{1}\rangle) + \sin\theta e^{i\phi}(\alpha\,|\underline{N-1}\rangle + \beta\,|\underline{N}\rangle) \xrightarrow{\text{measure}} \begin{cases} \alpha\,|\underline{N-1}\rangle + \beta\,|\underline{N}\rangle & p = \sin^2\theta \\ \alpha\,|\underline{2}\rangle + \beta\,|\underline{1}\rangle & p = \cos^2\theta \end{cases}$$

If the excitation is not present, due to the properties of FR, we know it must be back at the start, and the protocol can just repeat. Repetition until success completes after an expected time

$$\mathbb{E}(J_{\max}T) = J_{\max}\tau_0 \sum_{n=1}^{\infty} n\cos^{2n-2}\theta\sin^2\theta = \frac{J_{\max}\tau_0}{\sin^2\theta}.$$

Since the success of each repetition is independent, one can apply Chernoff bounds to show that the need for many repetitions vanishes exponentially.

The full protocol is summarised as follows, with two illustrative cases being depicted in Fig. 1 and Fig. 2. Including the encoding steps, we have

1. Prepare an arbitrary single-qubit state $|\psi\rangle$ in the first qubit of the chain.
2. Apply a controlled-NOT gate on the first two qubits (with the first qubit being the control, and applying the target operation if the control qubit is in $|0\rangle$).
3. Evolve the FR chain up to its revival time $\tau_0$.
4. Apply the standard CNOT gate on the last two qubits (with the last qubit being the control).
5. Measure the second-last qubit in the $Z$ basis.
6. If the measurement result is 1, halt: $|\psi\rangle$ has arrived on the last qubit. Otherwise, repeat from step 3 (the state is the same as it was then).

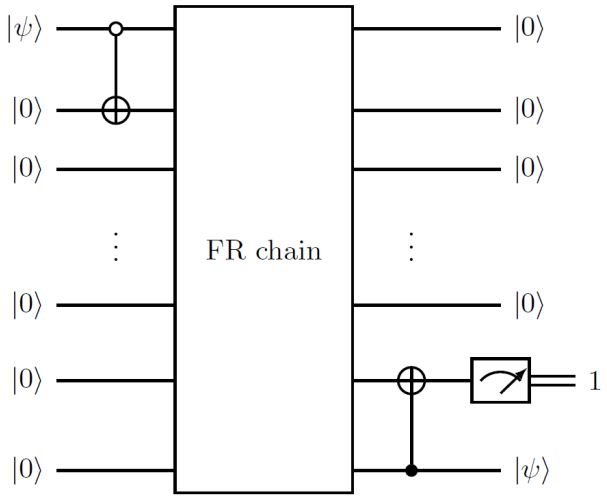


Figure 1. PST spin-chain with heralded anti-Zeno effect (successful arrival). If 1 is measured on the second last qubit at FR time, the state $|\psi\rangle$ has arrived at the last qubit.

## IV. SPIN CHAIN CONSTRUCTION

In the previous section, we gave a protocol for transferring a state based on a chain the the FR property. In this section, we evaluate the efficacy of the scheme for a specific FR construction. We will see that for an even length $N$, the construction yields an expected transfer time that is lower than any deterministic $N$-qubit spin

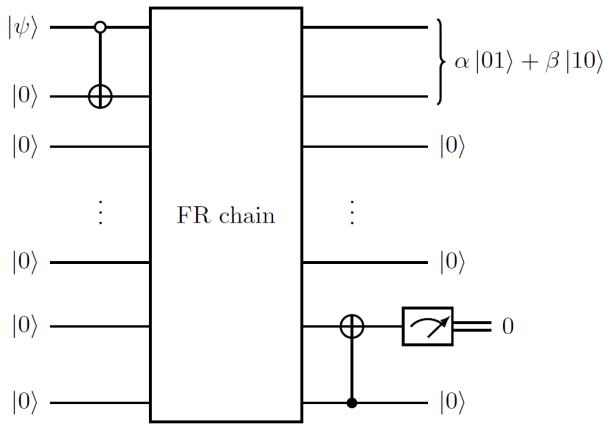


Figure 2. PST spin-chain with heralded anti-Zeno effect (waiting for arrival). If 0 is measured on the second last qubit, the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ has not arrived at the last qubit at FR time, but we can simply wait for the next FR cycle (as the protocol automatically resets itself).

chain with the same maximum coupling strength, and will prove that this is the optimal FR construction for these purposes.

Consider the FR chain constructed by Genest *et al.* ([21], Section 6). Written using our indexing convention, the coupling strengths that give a $\theta$-revival at time $\tau_0 = \pi/2$ are

$$J_n = \sqrt{\frac{n(N-n)((N-2n)^2 - \frac{4\theta^2}{\pi^2})}{(N-1-2n)(N+1-2n)}}, \qquad B_n = 0. \quad (7)$$

We will be particularly interested in

$$\theta_c = \frac{\pi}{2}\sqrt{1 - \frac{3}{N^2-1}},$$

which is extremely close to possessing perfect state transfer ($\theta = \frac{\pi}{2}$). This choice fixes that $J_{\max} = J_{N/2\pm1} = J_{N/2} = \theta_c N/\pi$.

If $T$ is the arrival time of the state (which is a geometric random variable), the expected transport time is

$$\mathbb{E}[J_{\max}T] = \frac{\pi}{2}\frac{\theta N}{\pi}\frac{1}{\sin^2\theta} = \frac{N\theta}{2\sin^2\theta},$$

which should be compared to the perfect transfer speed limit of Eq. (2). At our chosen value of $\theta_c$,

$$\frac{\mathbb{E}[J_{\max}T]}{J_{\max}\tau_{\min}} = \frac{2\theta_c}{\pi\sin^2\theta_c} = \frac{1-\epsilon}{\cos^2\frac{\pi\epsilon}{2}}$$

where $\theta_c = \frac{\pi}{2}(1-\epsilon)$. This is evidently smaller than 1 for $\epsilon < \frac{1}{2}$, which is certainly true for $\theta = \theta_c$ and $N > 2$.

### A. Fastest Possible FR

We will now justify that the chain we have used, defined by Eq. (7), yields the smallest possible value of $J_{\max}\tau_0$ for any given $\theta$. This proof strongly parallels the speed limit proof of [1]. Since $N$ is even, we saw in Section II that the chain is symmetric and the evolution is given by Eq. (6). Since the symmetric subspace has perfect revivals in time $\tau_0$, its eigenvalues must satisfy $\lambda_1 - \lambda_{2n-1} = \frac{2\pi}{\tau_0}k_n$ for some integers $k_n$. Similarly, the antisymmetric subspace must satisfy $\lambda_1 - \lambda_{2n} = \frac{2\pi}{\tau_0}k'_n + \frac{2\theta}{\tau_0}$, in order to get the correct relative phase. Most critically for us,

$$\lambda_{2n-1} - \lambda_{2n} \geq \frac{2\theta}{\tau_0}. \quad (8)$$

Recall from Eq. (5) that the symmetry operator has the property $S|\lambda_n\rangle = (-1)^{n+1}|\lambda_n\rangle$. Now,

$$2J_{\max} \geq 2J_{N/2} = \mathrm{Tr}(H_1 S) = \sum_n \lambda_{2n-1} - \lambda_{2n} \geq \frac{N}{2}\frac{2\theta}{\tau_0},$$

where we took the trace using the eigenvector basis $\{|\lambda_n\rangle\}$ and applied Eq. (8). We conclude that

$$J_{\max}\tau_0 \geq \frac{N\theta}{2}.$$

This is exactly the value that we achieved above. Hence, the solution from [21] is optimal for any given $\theta_c \leq \theta \leq \pi - \theta_c$, i.e. such that $J_{\max} = J_{N/2}$ in the chosen scheme. This recovers the previously known results for perfect state transfer when $\theta = \pi/2$ [1, 6, 20].

In addition to the solution for even chain lengths, [21] gives an equivalent symmetric solution for odd chain lengths. This works within our protocol (as does any chain with FR), but does not provide any speed advantage over state transfer. Moreover, following [2], one can prove that this is also the fastest symmetric chain with a given FR angle between the chain ends. One simply replaces the bounds

$$\lambda_{2n} - \lambda_{2n+1} \geq \frac{\pi}{\tau} \longrightarrow \lambda_{2n} - \lambda_{2n+1} \geq \frac{2(\pi-\theta)}{\tau_0}$$
$$\lambda_{2n-1} - \lambda_{2n} \geq \frac{\pi}{\tau} \longrightarrow \lambda_{2n-1} - \lambda_{2n} \geq \frac{2\theta}{\tau_0}.$$

We conclude that

$$J_{\max}\tau_0 \geq \frac{\sqrt{(N^2-1)\theta(\pi-\theta)}}{2},$$

which is saturated by the solution in [21] for any $\theta$ such that the largest coupling is the central one.

When $N$ is odd, solutions are not required to be symmetric. In the asymmetric case, a chain with a $\theta$-revival is symmetric apart from the middle two coupling strengths $J_{(N-1)/2} = \sqrt{2}J\cos\eta$ and $J_{(N+1)/2} = \sqrt{2}J\sin\eta$. $H_1$ is

similar to $H' = UH_1U^\dagger$ with

$$U = \left|\underline{\frac{N+1}{2}}\right\rangle\left\langle\underline{\frac{N+1}{2}}\right| + \sum_{n=1}^{(N-1)/2} \sin\left(\frac{\pi}{4}-\eta\right)(|\underline{n}\rangle\langle\underline{n}| - |\underline{N+1-n}\rangle\langle\underline{N+1-n}|) + \cos\left(\frac{\pi}{4}-\eta\right)(|\underline{N+1-n}\rangle\langle\underline{n}| + |\underline{n}\rangle\langle\underline{N+1-n}|).$$

$H'$ is a symmetric chain with central coupling $J$, and all other parameters equal to those of $H_1$. The evolution of the two are related by

$$e^{-iH_1\tau_0}|\underline{1}\rangle = U^\dagger e^{-iH'\tau_0}U|\underline{1}\rangle.$$

These can only be equal if $H'$ possesses FR by some angle $\theta'$. Thus, the right-hand side is entirely known thanks to Eq. (6). Hence, the two FR angles are related by

$$\sin\theta = \sin(2\eta)\sin\theta'.$$

The expected state transfer time can then be evaluated relative to that of the symmetric chain,

$$\mathbb{E}[J_{\max}T] = \mathbb{E}[J'_{\max}T'] \times \begin{cases} \frac{\sqrt{2}\cos\eta}{\sin^2 2\eta} & J_{\max} = \sqrt{2}J\cos\eta \\ \frac{\sqrt{2}\sin\eta}{\sin^2 2\eta} & J_{\max} = \sqrt{2}J\sin\eta \\ \frac{1}{\sin^2 2\eta} & \text{otherwise} \end{cases}$$

In all these cases, $\mathbb{E}[J_{\max}T] > \mathbb{E}[J'_{\max}T']$ – the symmetric solution has the best expected transfer time, which we have already shown cannot break the speed limit.

## V. ENCODING IN THE SINGLE EXCITATION SUBSPACE

A standard implementation of quantum state transfer encodes an initial state $\alpha|0\rangle + \beta|1\rangle$ across the 0- and 1-excitation subspaces,

$$(\alpha|0\rangle + \beta|1\rangle)|\underline{0}\rangle.$$

The dual rail approach provides the alternative encoding

$$\alpha(|1\rangle|\underline{0}\rangle)_A(|0\rangle|\underline{0}\rangle)_B + \beta(|0\rangle|\underline{0}\rangle)_A(|1\rangle|\underline{0}\rangle)_B$$

across two chains, $A$ and $B$. Our monorail instead encodes on a single chain as

$$(\alpha|01\rangle + \beta|10\rangle)|\underline{0}\rangle.$$

Both the monorail and dual rail encodings encode within the single-excitation subspace, which means they have many features and practical benefits in common. Indeed, if you allow the two chains in a dual rail scheme to be the same and have engineered couplings, instead of the default case of uniform couplings, one option is to use exactly the FR chains chosen here. We get exactly the same transfer fidelity and expected transfer time, and can therefore also beat the state transfer speed limit using the dual rail encoding. What are these benefits, and why might one select a monorail versus dual rail solution?

**Simplicity**. It is clearly easier to have just a single chain rather than two identical copies. While the dual rail scheme can function if the two chains are different, there is a severe cost to the expected transfer time, and the classical calculation to find optimal measurement times, which are no longer the simple regular measurements that the FR structure permits. In previous experimental realisations of state transfer [10, 11], the photonic nature of the implementation lends itself very naturally to being in the single excitation subspace. There would be no need for an encoding/decoding step.

**Stability against perturbations**. The dual rail method can function (eventually) whatever the two chains are, while the monorail is heavily dependent on the precise nature of the couplings. When compared to the uniformly coupled dual rail scheme, the unperturbed fidelity of the engineered scheme is so much higher, that there is a modest regime where the perturbed monorail outperforms it.

**Initial state of chain**. While we have dealt with the specific case that the rest of the chain is initialised as $|\underline{0}\rangle$, our encoding is exactly that used to tolerate any initial state [20, 23], and hence this carries over immediately so long as the sites $N$ and $N-1$ are prepared in $|0\rangle$. The Hamiltonian must be of the XX type, as specified in Eq. (1) and not include any $Z_nZ_{n+1}$ terms. The dual rail encoding does not benefit from this.

**Heralded arrival**. One can detect the presence or absence of a single excitation and determine whether the state has arrived without disturbing the contents of the state. This is exactly the point of the decoding procedure in both the dual rail and monorail systems. This can minimise the effect of a range of fault types, such as imperfect preparation of the Hamiltonian. Numerical tests indicate that this is insufficient to combat the fact that *both* components of the qubit have to transfer under the faulty Hamiltonian, whereas the usual state transfer protocol encodes one of those components in $|\underline{0}\rangle$, and is therefore unaffected by many such faults (e.g. if the perturbed Hamiltonian remains excitation preserving).

**Decoherence-free subspace**. As observed in [16], and formalised in [24], the evolution of a single excitation under the effects of amplitude damping noise can simply be modelled as its evolution under the Hamiltonian for time $t$, with probability $e^{-\gamma t}$, and otherwise is returned to the $|\underline{0}\rangle$ state. ($\gamma$ is the strength of the noise.) Hence *any* encoding of a qubit into the single excitation subspace either has the qubit arriving correctly, modulated by a probability $e^{-\gamma t}$, or fails in a manner that is detected by the heralded arrival (because there is no longer an excitation present). We are using a decoherence-free subspace. The simple fact that the monorail has a much faster runtime than the dual rail scheme (without engineering) means that it achieves vastly higher success

probabilities. The failure probability of the monorail is approximately (working in the limit where the transfer probability is close to 1)

$$\prod_{l=1}^{\infty}\left(1-e^{-\gamma l\pi N/(4J_{\max})}\right),$$

while the equivalent for the dual rail was given in [16]:

$$\prod_{l=1}^{\infty}\left(1-1.35N^{-2/3}e^{-\gamma lN/(2J_{\max})}\right).$$

For instance, if we take $\gamma N/J_{\max}=1$, then the failure probability of the monorail is about 0.36. That of the dual rail still depends on the size, but even for an extremely short chain, such as $N=11$, the failure probability is 0.93, only getting worse for larger $N$.

One feature that the dual rail encoding offers [16] that is not present in the monorail is some protection against dephasing noise — if the dephasing noise is symmetric between the two chains of the dual rail, and if the Hamiltonians of the two chains are identical, then the quantum information is in a decoherence-free subspace. If the state arrives, it arrives without error. However, it must be noted that the arrival probability is still adversely affected by dephasing noise, with a dominant term of the form $e^{-\gamma t}$. Hence, the much longer run-time of the (uniform) dual rail still yields vastly inferior performance to the monorail.

## VI. CONCLUSIONS

In this work, we have designed a specific spin chain that works in concert with a probabilistic protocol for perfectly transferring an unknown quantum state with provable speedup over *any* deterministic perfect state transfer chain with the same maximum coupling strength.

The speedup that we achieve is minuscule, only reducing the state transfer time from $\tau$ by an amount $O(\tau/N^2)$, at the cost of non-deterministic arrival. Were we to use current experimental systems [11, 13], the difference between the heralded and perfect transfer systems is well inside the limits of experimental accuracy. Nevertheless, the essential insight is that the speed limit *can* be broken thanks to the anti-Zeno effect. FR provides a particularly elegant method allowing us to compress the dual-rail encoding onto a single chain, and also yielding information about what state the chain is reset to in the case of failure. Its disadvantage is that, after a failure, the state is reset such that it has to travel the full length of the chain again before the next possible arrival, which is why the use of FR for values of $\theta$ far from perfect state transfer is impractical.

While we have proven that we have found the most efficient FR-based monorail scheme, there may be other systems which are more efficient from this perspective, and yield greater advantages – now that we know this advantage exists, it would be interesting in the future to establish how large an advantage can be obtained. Equally, it is possible to achieve some speedup in the dual rail system and engineered chains (e.g. use two parallel FR chains). This has the potential to facilitate a fairer comparison between the two methods. Other particularly fast state transfer chains might be beneficial in this scenario, such as [7] — having an arrival probability of over 99% in only 76% of the comparable perfect transfer time appears very promising in terms of the expected arrival time. However, gathering the tails of the distribution in a reasonable time appears to be a challenging task. A simple numerical test was unable to find a sequence of measurement times that permits an expected arrival time below the perfect transfer time.

If we have more information to transport, such as a qutrit, one simply uses a superposition over the states $\{|\underline{1}\rangle, |\underline{2}\rangle, |\underline{3}\rangle\}$, and detects arrival by looking for the presence of a $|1\rangle$ on the final three sites. This requires a minimal update to our encoding/decoding procedures.

The speedup that we achieve is so small that if we account fully for the time required for the decoding operation before measurement, this outweighs the speedup. As an alternative option, the encoding and decoding operations can be built into the Hamiltonian without costing any time. Instead, we are required to use three-body terms, by making the following replacements:

$$\begin{aligned} X_1X_2 &\mapsto X_1, & X_{N-1}X_N &\mapsto X_N \\ Y_1Y_2 &\mapsto -X_1Z_2, & Y_{N-1}Y_N &\mapsto Z_{N-1}X_N \\ Y_2Y_3 &\mapsto -Z_1Y_2Y_3, & Y_{N-2}Y_{N-1} &\mapsto Y_{N-2}Y_{N-1}Z_N. \end{aligned}$$

These are just derived by defining a unitary $U$ that includes both the encoding and decoding operations and hence the overall evolution is $U^\dagger e^{-iHt}U = e^{-iU^\dagger HUt}$. Each term $h$ in the Hamiltonian is replaced by $U^\dagger hU$. As $U$ only acts on 4 qubits, the majority of the terms are unchanged. In doing so, we lose some of the protection against amplitude damping noise.

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