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A Transfer-Matrix-Based Approach to Predicting Acoustic Properties of a Layered System in a General, Efficient, and Stable Way

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A transfer-matrix-based approach to predicting acoustic properties of a layered system in a general, efficient, and stable way

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PURDUE MECHANICAL ENGINEERING

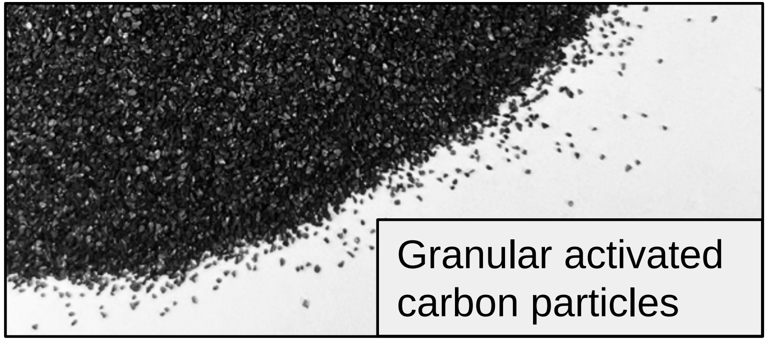
RAY W. HERRICK LABORATORIES

Agenda

- Motivation
- Methodology
- Applications: minimum-weight sound package design
- Conclusions

Motivation

Stacks of particles are known to be poro-elastic (Mo *et al.*, 2021)

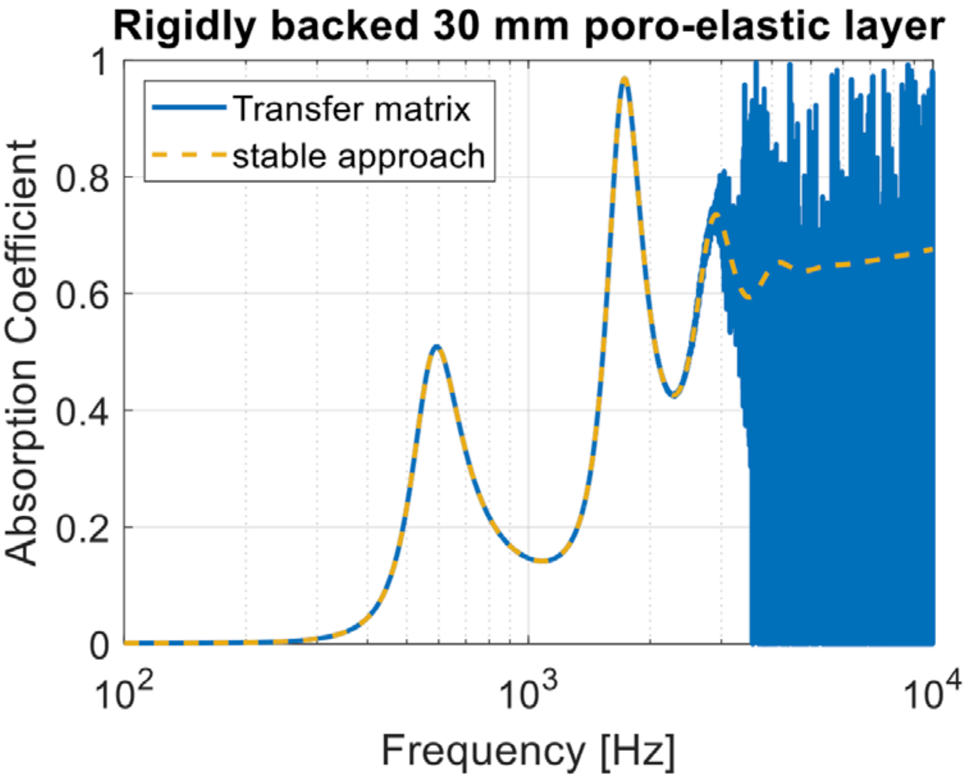


Granular activated carbon particles

3M™ Flexile Acoustic Material

Microspeakers with zeolite particles

σ [Rayls/m]	ϕ	α_∞	ρ_b [kg/m³]	E [Pa]	η	ν	θ
1.5×10^6	0.92	1.3	24	6000	0.004	0.27	0°



Previously-proposed methods

	Variables	General method?	Efficient to solve?	Effort to redesign the system	Stability
Arbitrary coefficient method (ACM) [2,3]	Amplitude of waves	√	x	Time-consuming	Unstable
Global transfer matrix method (GTM) [4,5]	State vector	√	x	Easy	Unstable
Xue *et al.*'s method [6]	State vector	x	x	Easy	Unstable
Dazel *et al.*'s method [7]	Information vector	√	√	Easy	Stable

Previously-proposed methods

	Variables	General method?	Efficient to solve?	Effort to redesign the system	Stability
Stabilized TMM method [8]	State vector	√	x	Easy	Stable
Proposed approach	State vector	√	√	Easy	Stable

In proposed approach, a large, complicated layered system is divided into a series of simple systems, which is therefore more efficient.

Proposed approach allows computationally expensive tasks:

- Inverse characterization of material properties
- Optimization of acoustical treatments

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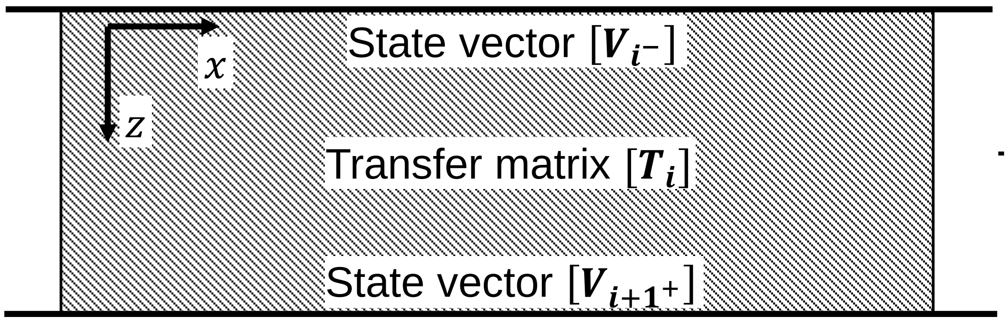
Goals to achieve in the proposed method:

1. Generality: Use the general transfer matrix notation.
2. Stability: Decompose the transfer matrix
   Reformulate the equations
3. Efficiency: Introduce a merge layer operation

(Goals 1–2: Song *et al.*, 2022)

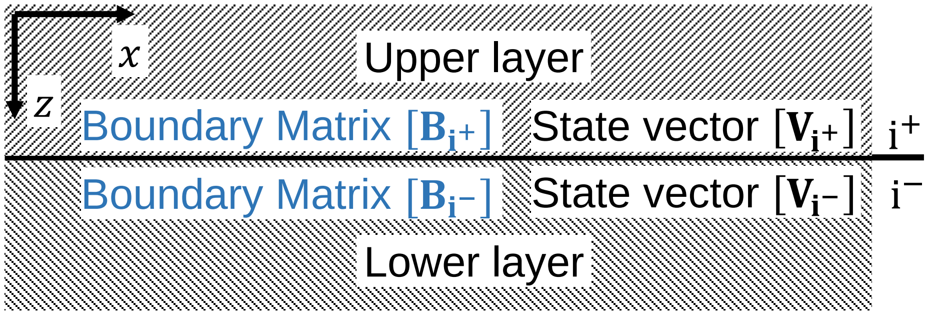
General expression:



Interface i

Interface i + 1

$$[\mathbf{V}_{i^-}] = [\mathbf{T}_i]\,[\mathbf{V}_{i+1^+}]$$



Interface i

$$[\mathbf{B}_{i^+}][\mathbf{V}_{i^+}] = [\mathbf{B}_{i^-}][\mathbf{V}_{i^-}]$$

Methodology – Stability

In the traditional TMM:

When there is a significant disparity between the magnitudes of the waves: i.e.,

- at higher frequencies
- for a thick layer
- for extreme parameter values

The most attenuated wave's contribution masked by numerical errors.

→ Instability occurs when inverting the global matrix

Decomposition – **extract wave attenuation terms**

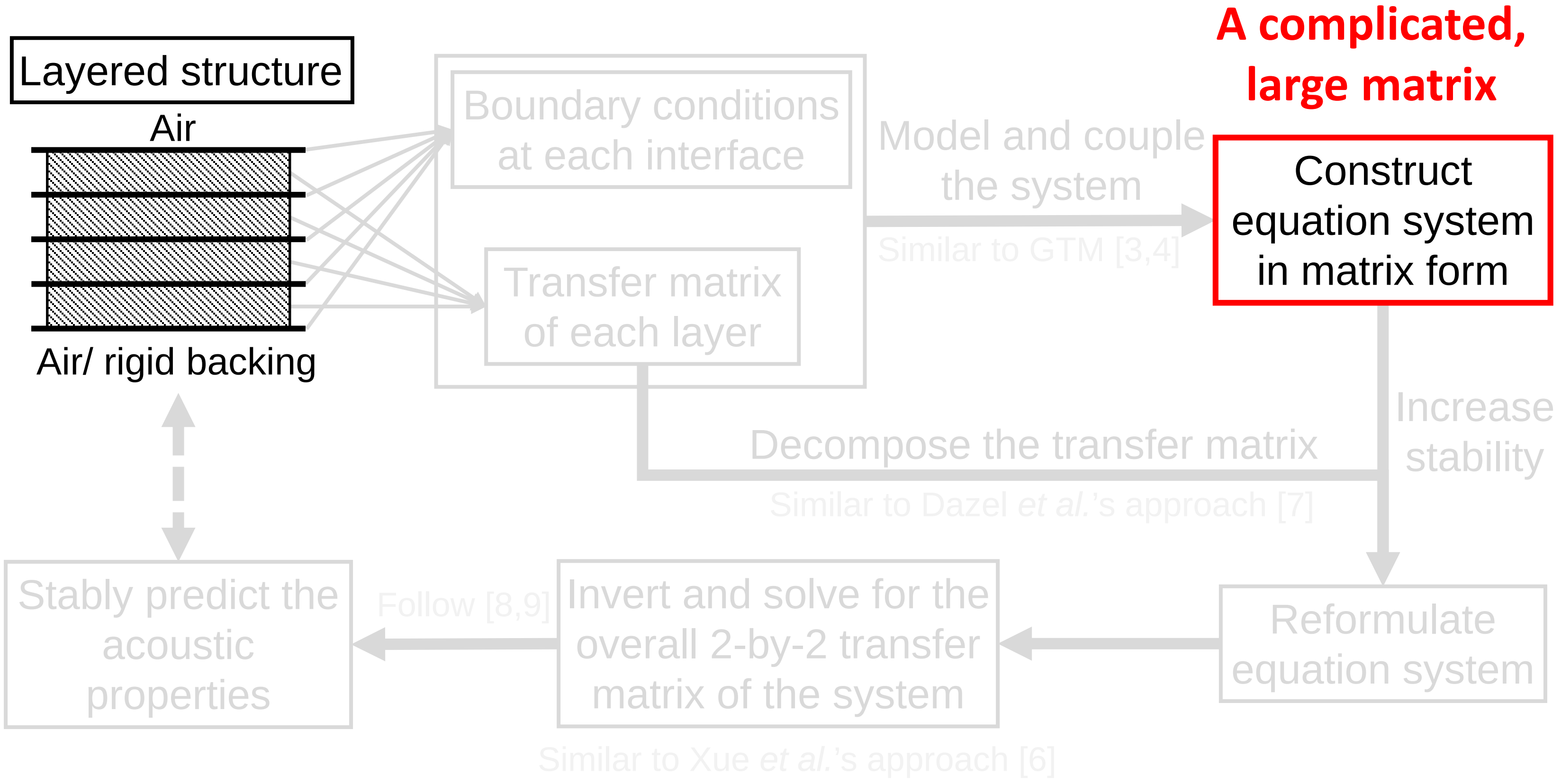
$$[\mathbf{T_i}] = [\mathbf{\Phi_i}][\mathbf{\Lambda_i}][\mathbf{\Phi_i}]^{-1}$$

With wave attenuation terms

E.g., for a solid layer:

$$[\mathbf{\Lambda^s}] = \begin{bmatrix} e^{jk_{13}d} & 0 & 0 & 0 \\ 0 & e^{-jk_{13}d} & 0 & 0 \\ 0 & 0 & e^{jk_{33}d} & 0 \\ 0 & 0 & 0 & e^{-jk_{33}d} \end{bmatrix}$$

Methodology – Efficiency – Overview of Stabilized TMM [8]

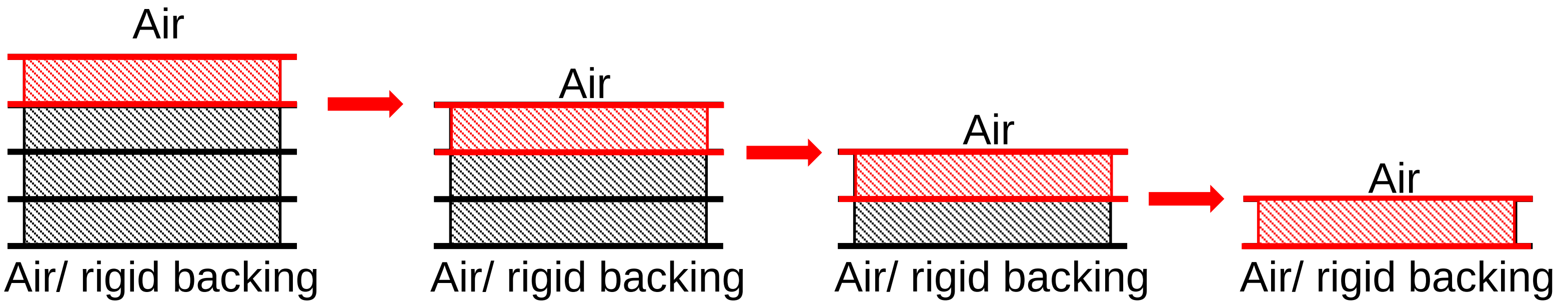


Methodology – Efficiency 1/2

To make the method efficient:

Splitting a large, complicated system into a series small systems.

Merge layer operation

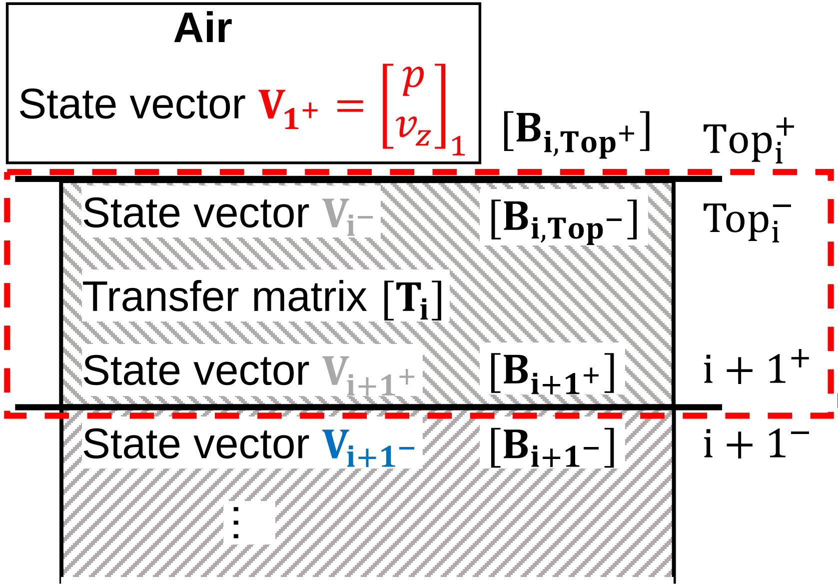


Execute the merge layer process repetitively

A multi-layered system → A single layered system

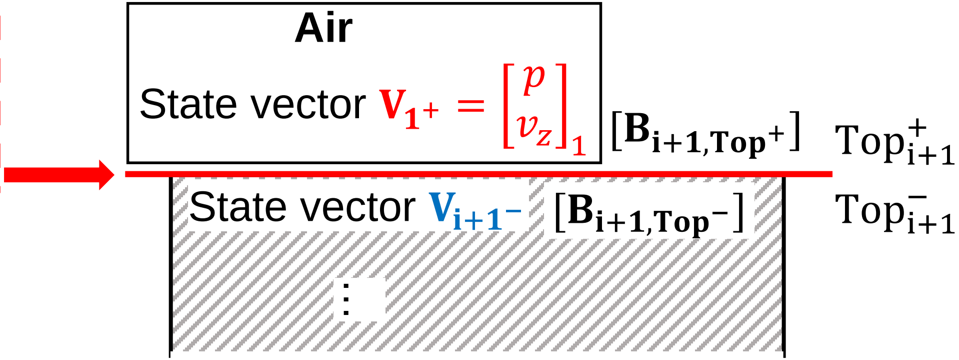
Solve for a single-layered system

Methodology – Efficiency 2/2



For a multi-layered system with 10 poro-elastic layers:

- Proposed method: <u>9.9 sec per 50 iterations</u>
- Stabilized TMM [8]: <u>30.7 sec per 50 iterations</u>



Two interfaces:

$$[\mathbf{B}_{i,Top^+}][\mathbf{V}_{1^+}] = [\mathbf{B}_{i,Top^-}][\mathbf{V}_{i^-}]$$

$$[\mathbf{B}_{i+1^+}][\mathbf{V}_{i+1^+}] = [\mathbf{B}_{i+1^-}][\mathbf{V}_{i+1^-}]$$

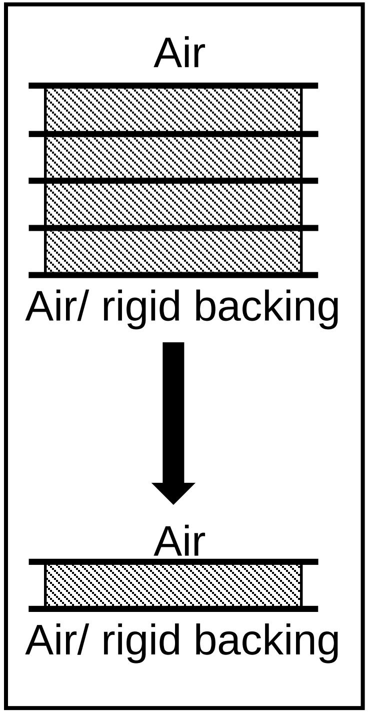
With $[\mathbf{V}_{i^-}] = [\mathbf{T}_i][\mathbf{V}_{i+1^+}]$

One new interfaces:

$$[\mathbf{B}_{i+1,Top^+}][\mathbf{V}_{1^+}] = [\mathbf{B}_{i+1,Top^-}][\mathbf{V}_{i+1^-}]$$

Eliminates all the intermediate variables and forms an equivalent interface.

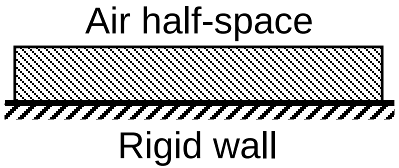
Methodology – A single layered system [8]



Use stabilized TMM [8] to solve for the 2x2 transfer matrix: $V_{1^+} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} V_{n+1^-}$

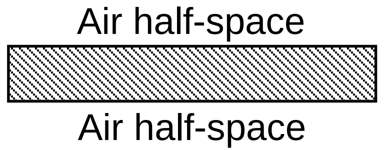
Fixed on a rigid wall:

- $R = \frac{T_{11}\cos\theta/(T_{21}\,\rho_0 c)-1}{T_{11}\cos\theta/(T_{21}\,\rho_0 c)+1}.$



Fluid on both sides:

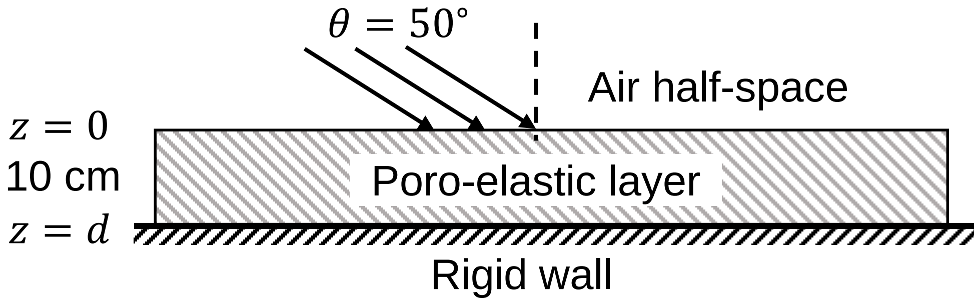
- $T = \frac{2e^{jk_z d}}{T_{11}+T_{12}\cos\theta/\rho_0 c+T_{21}\,\rho_0 c/\cos\theta+T_{22}},$
- $R = \frac{T_{11}+T_{12}\cos\theta/\rho_0 c-T_{21}\,\rho_0 c/\cos\theta-T_{22}}{T_{11}+T_{12}\cos\theta/\rho_0 c+T_{21}\,\rho_0 c/\cos\theta+T_{22}},$
- $\alpha = 1-|R|^2,$
- $TL = 20\log_{10}\frac{1}{|T|}.$

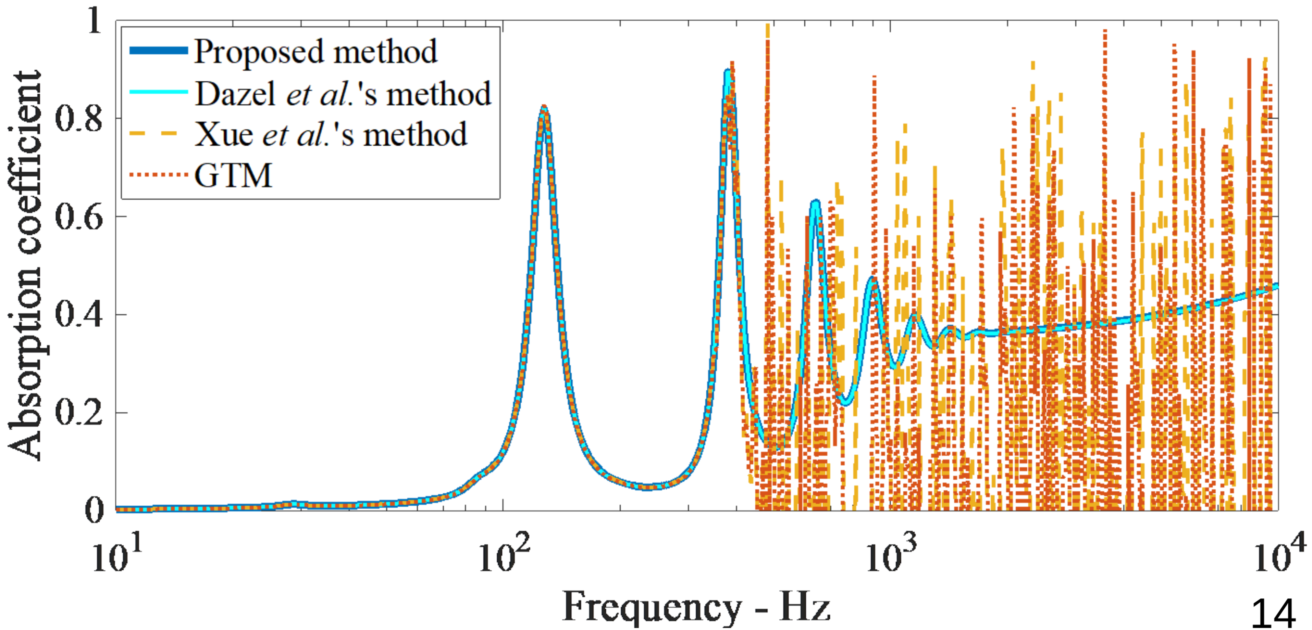


Verification

Poro-elastic layer

σ – Rayls/m	4×10^6
ϕ	0.4
α_∞	1.75
Λ – m	9.3×10^{-6}
Λ' – m	2.0×10^{-5}
ρ_1 – kg/m^3	120
E - Pa	4×10^4
η	0.2
ν	0.3





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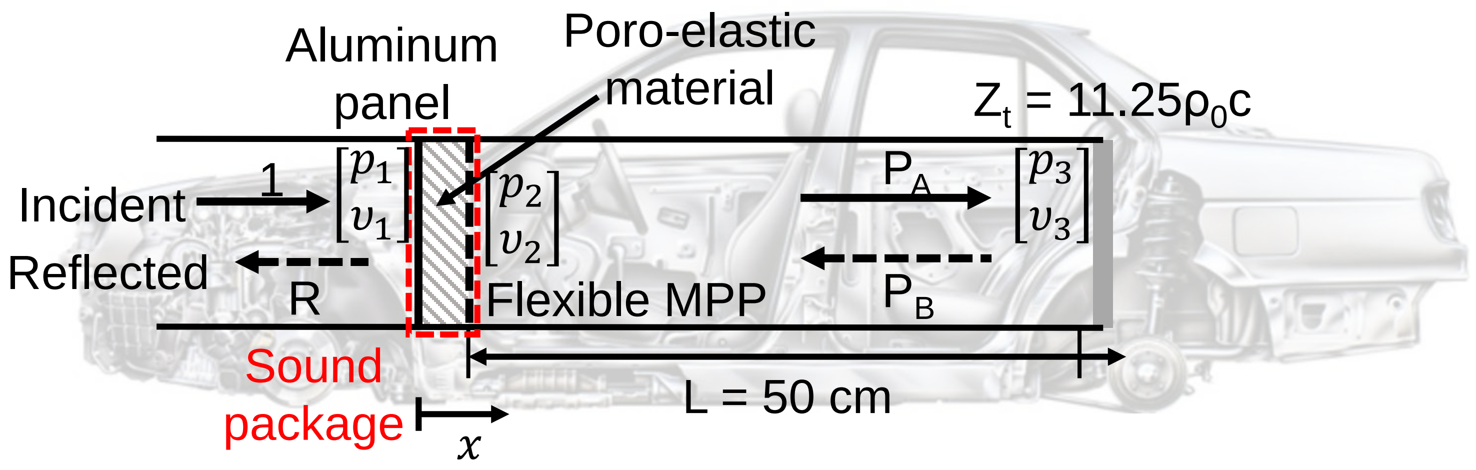
Applications – Problem statement [minimum weight sound package design]

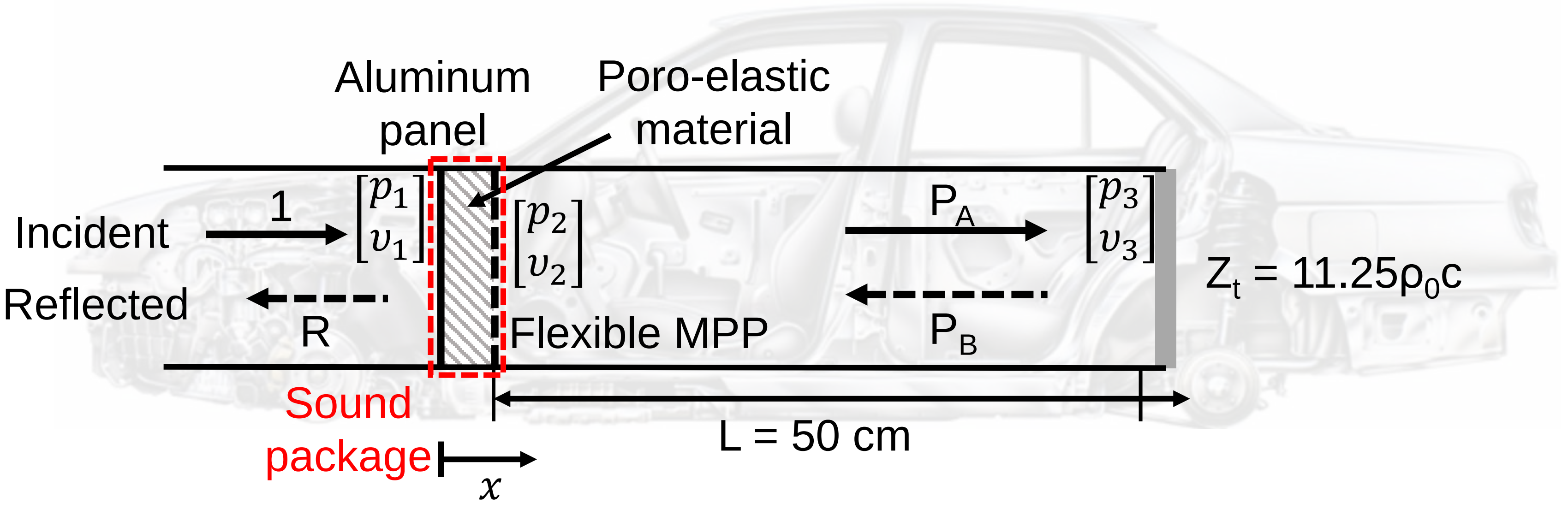
The configuration is similar to Shin and Bolton's work [9]



Important factors

- Safe
- Cost
- Weight
- Volume
- …

Applications – Problem statement [minimum weight sound package design]

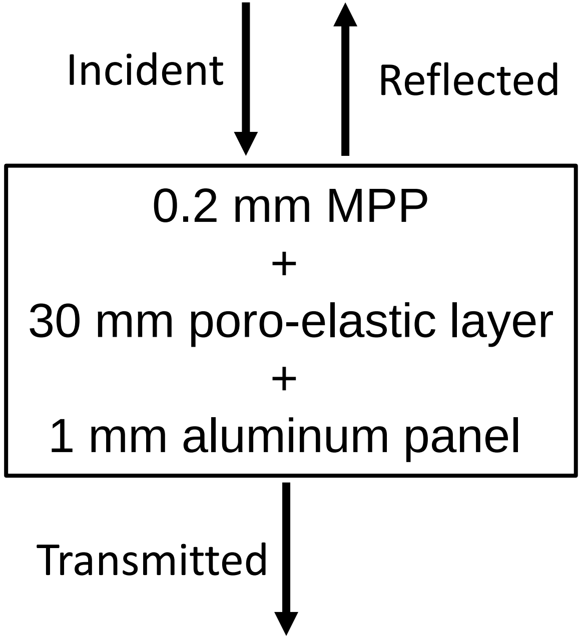


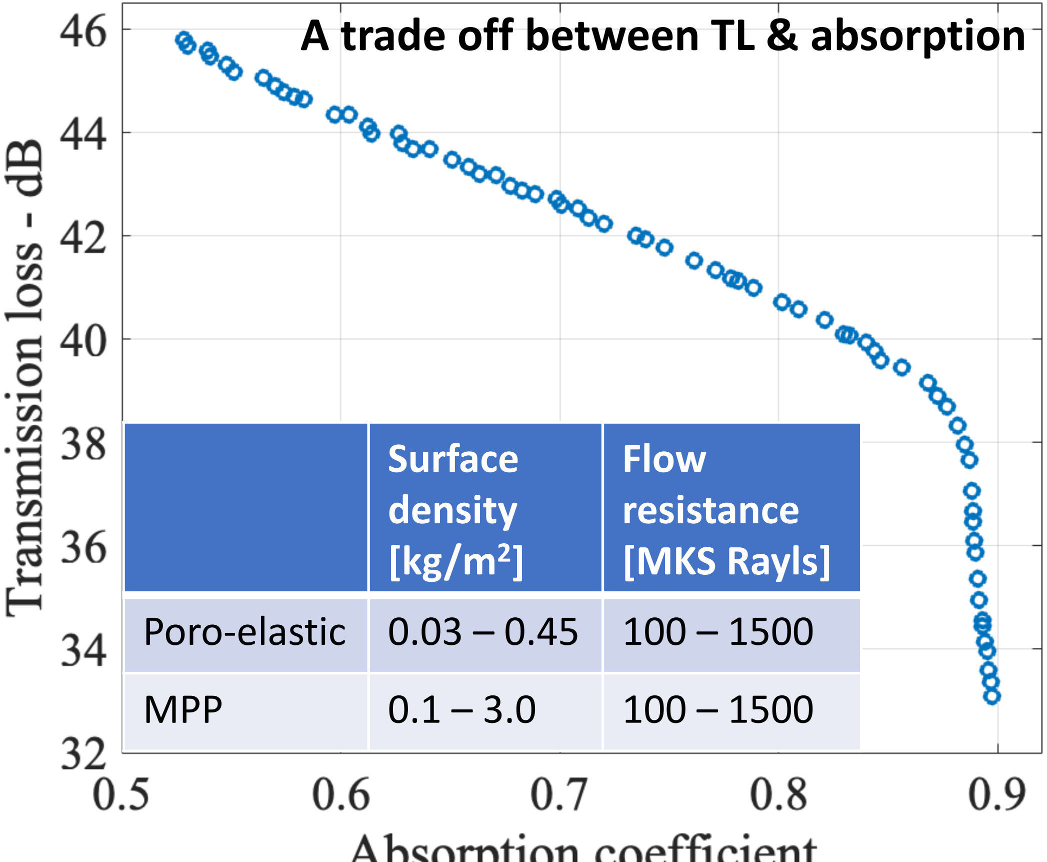
Preliminary study on the sound package:

- Absorption performance vs barrier performance [Multi-objective particle swarm algorithm]

Optimized design – sound package 1/3

Sound package: 0.2 mm MPP + 30 mm poro-elastic layer + 1 mm aluminum panel



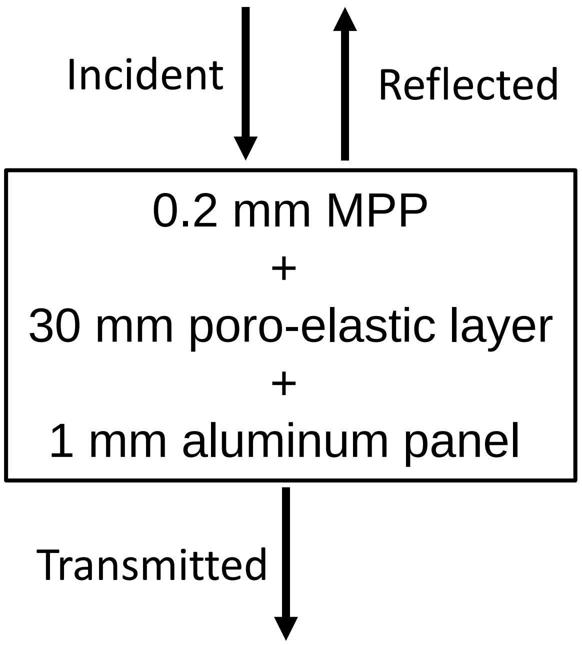


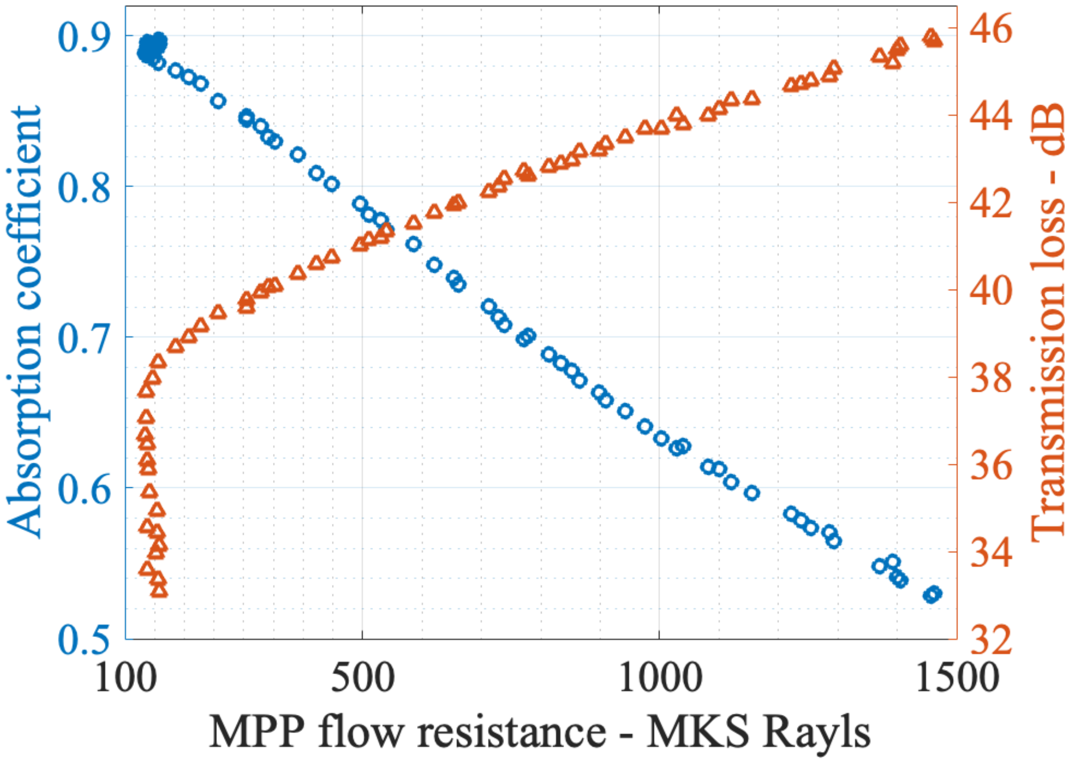
	Surface density [kg/m²]	Flow resistance [MKS Rayls]
Poro-elastic	0.03 – 0.45	100 – 1500
MPP	0.1 – 3.0	100 – 1500

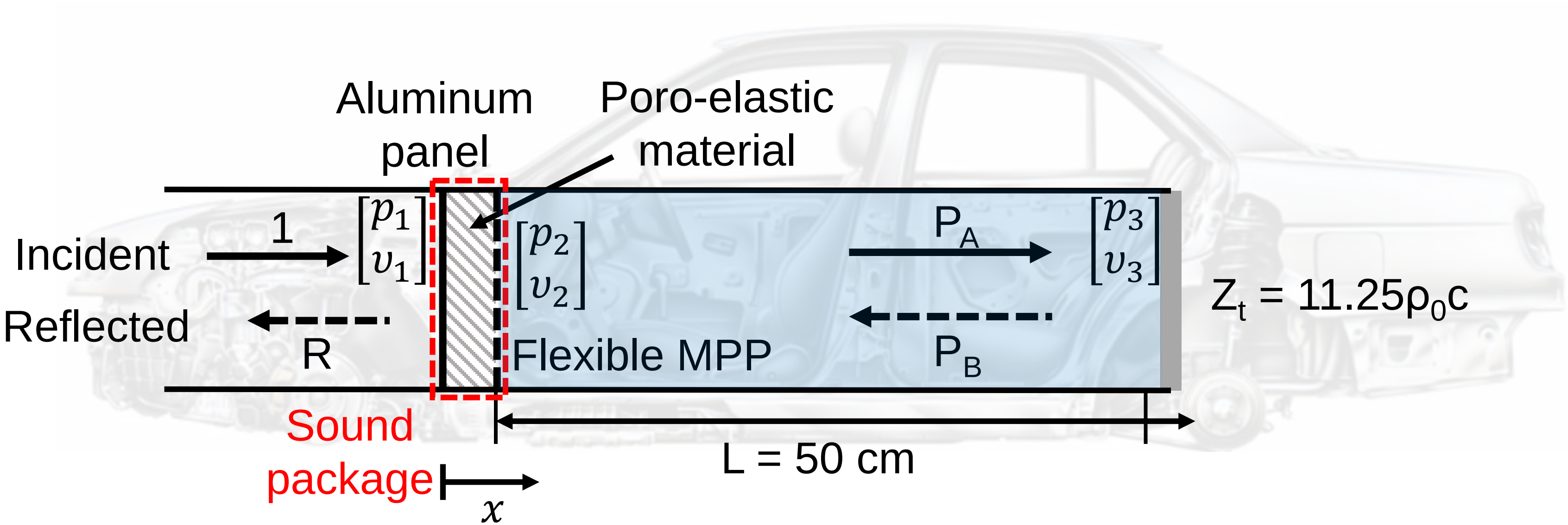
Optimized design – sound package 3/3

An open-pore MPP → MPP + porous + backing → absorption performance
A closed-pore MPP → double panel system → barrier performance

Sound package:



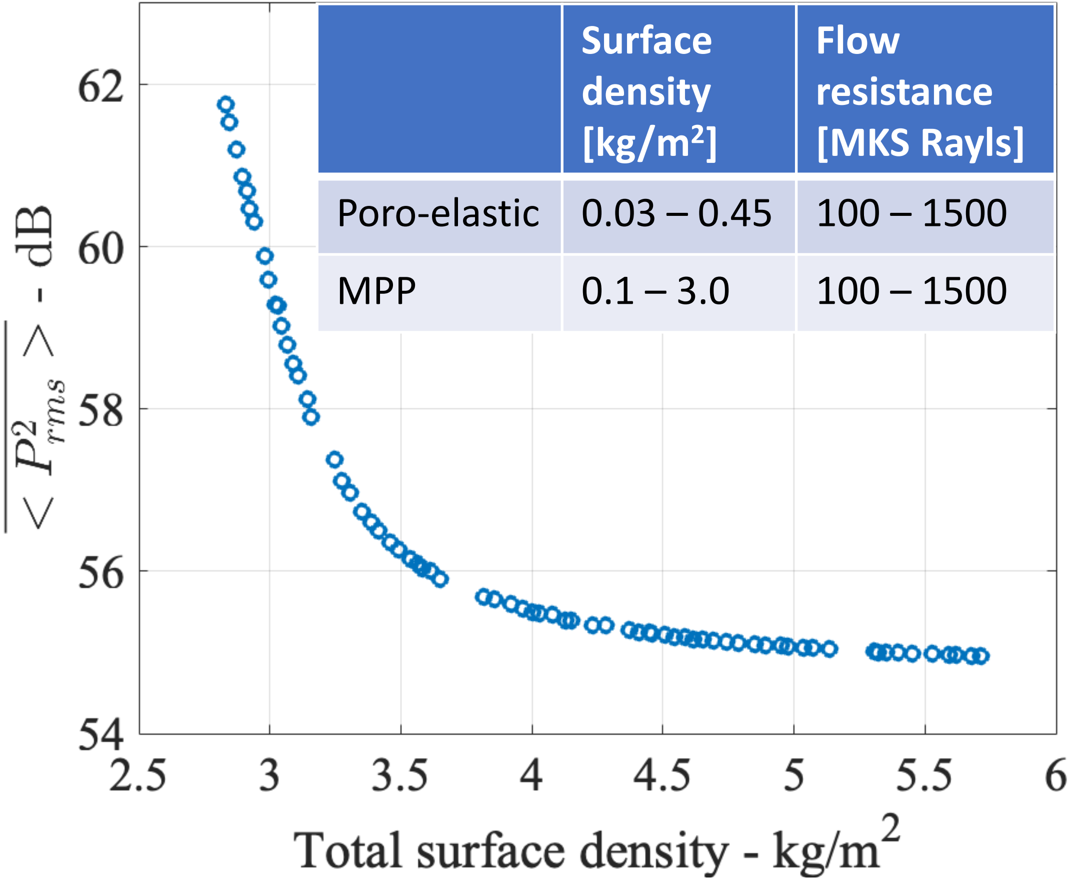




Minimum weight sound package design target:

Optimize the design so that the mean-square pressure in the internal air cavity, $\overline{< P_{rms}^2 >}$, is minimized with multi-objective particle swarm algorithm in MATLAB.

Optimized design – minimum-weight design 1/2



	Surface density [kg/m²]	Flow resistance [MKS Rayls]
Poro-elastic	0.03 – 0.45	100 – 1500
MPP	0.1 – 3.0	100 – 1500

Higher requirements on internal SPL → Heavier sound package

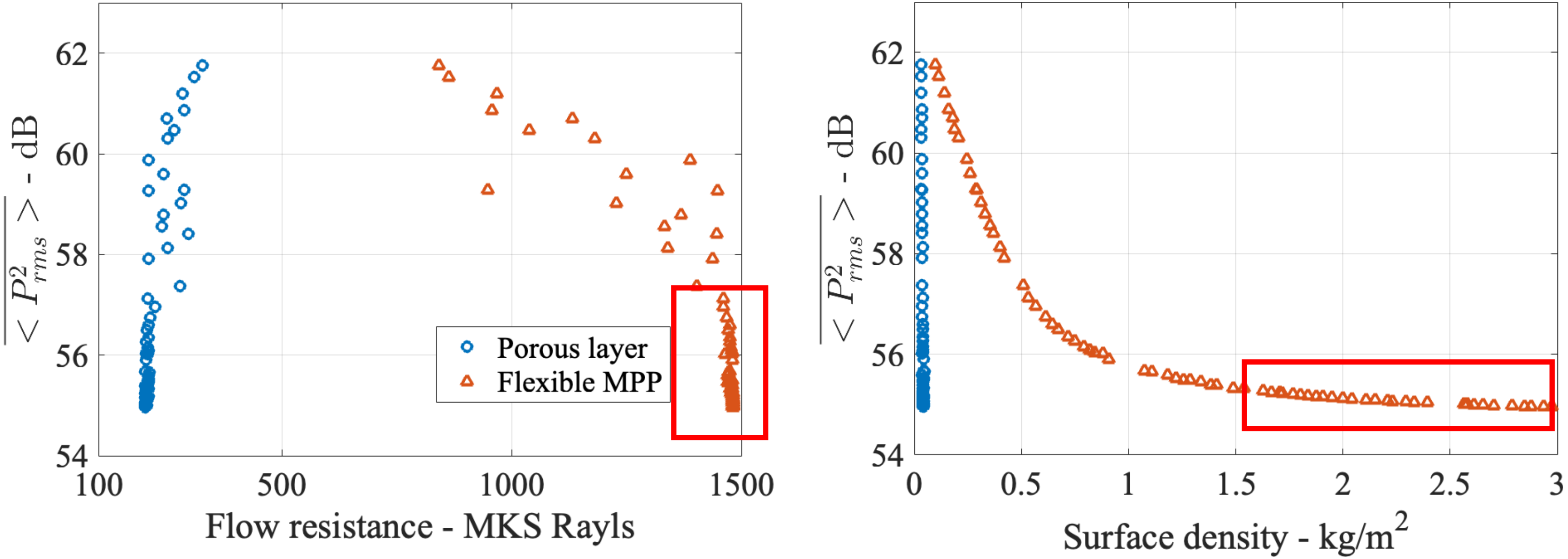
Optimized design – minimum-weight design 2/2

In order to reduce the mean-square pressure in the internal air cavity, $\overline{< P_{rms}^2 >}$

→ increase MPP flow resistance and surface density

→ yield a double panel system

→ good barrier performance



Conclusions

- An approach is proposed to modeling and coupling layered acoustical systems in a general, efficient, and stable way.
  Helpful for tasks like inverse characterization of material properties and optimization of acoustical treatments.

- When optimizing the design of a sound package. It was found that there is a trade-off between absorption and barrier performance.

- A heavier and more resistive MPP favors barrier performance, while a lighter and less resistive MPP favors absorption performance.
  The minimum-weight design emphasizes the sound package's barrier performance.

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