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Prediction of Acoustical Behavior of Granular Material Stacks as Measured in a Standing Wave Tube by using a Biot Theory-based Model

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PREDICTION OF ACOUSTICAL BEHAVIOR OF GRANULAR MATERIAL STACKS AS MEASURED IN A STANDING WAVE TUBE BY USING A BIOT THEORY-BASED MODEL

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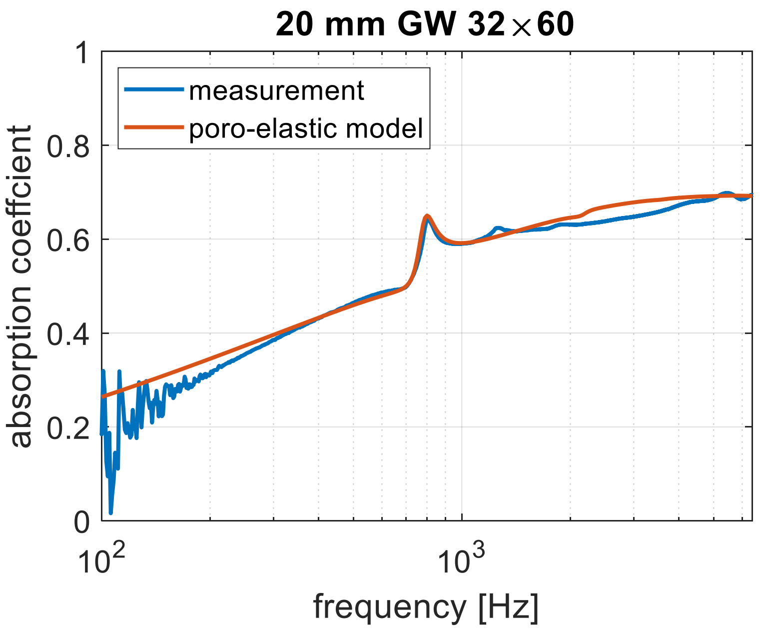
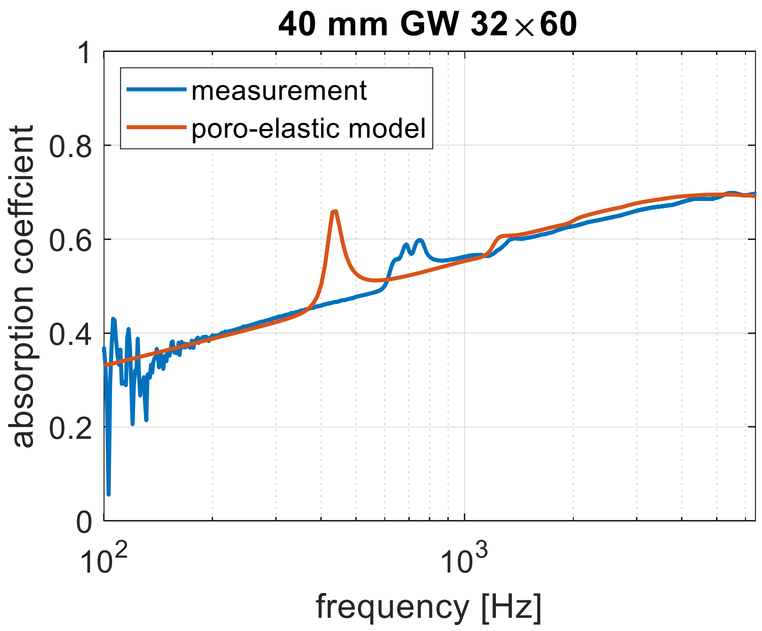
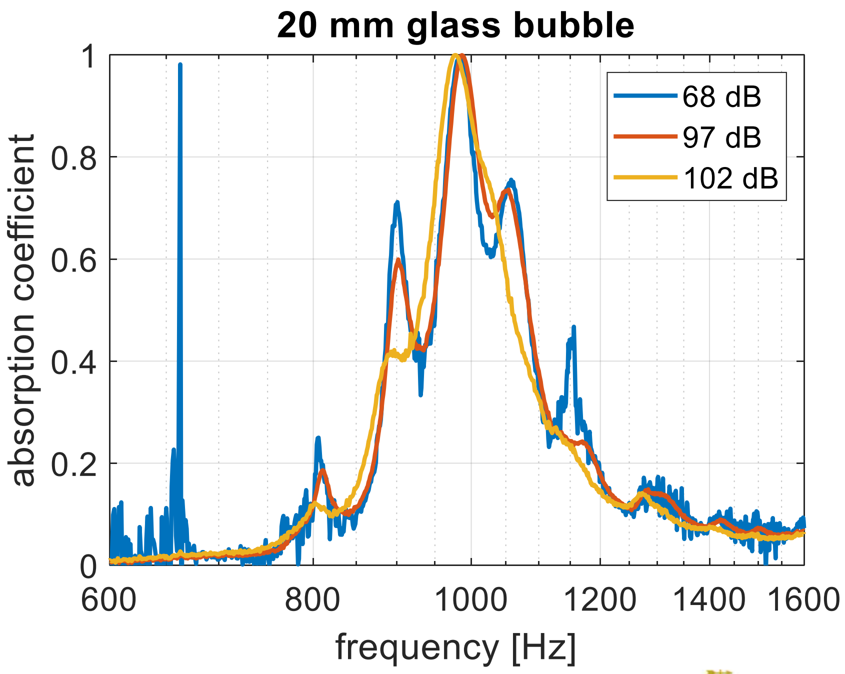
Introduction

Granular activated carbon

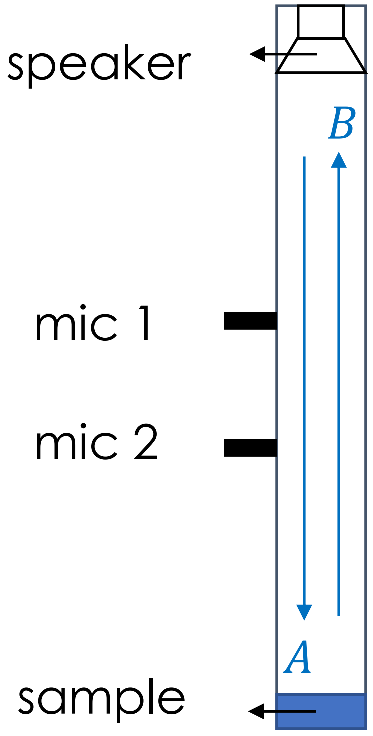
Glass bubbles
https://www.3m.com/3M/en_US/p/d/b40064606/

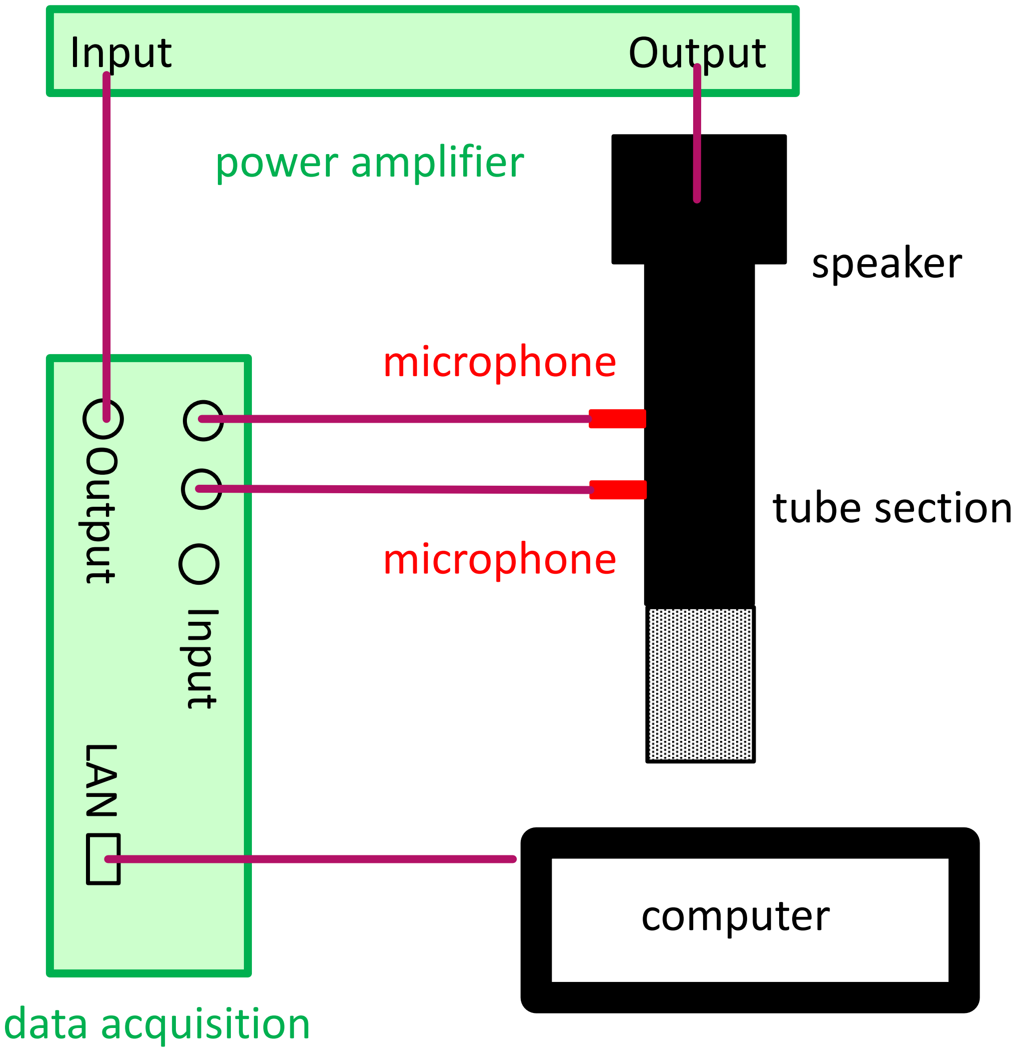






Testing Procedure



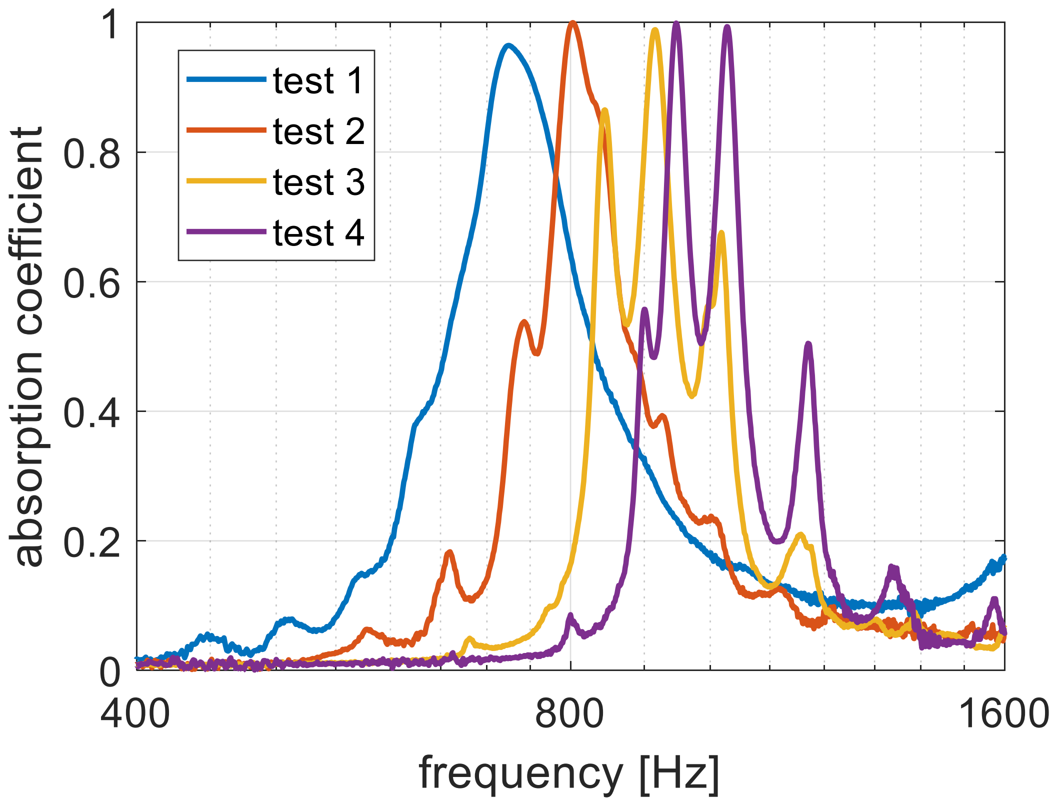


- Impedance tube: B&K Type 4206
- Power amplifier: B&K 2716C
- Data acquisition: B&K 3560-B
- Software: B&K PULSE™ LabShop

  For random signals, source level and frequency range can be selected.

Testing Procedure

Glass bubbles are sensitive to sample preparation



Test 1: not compacted
+ 1000 m Vrms 0-6400 Hz signal exposure

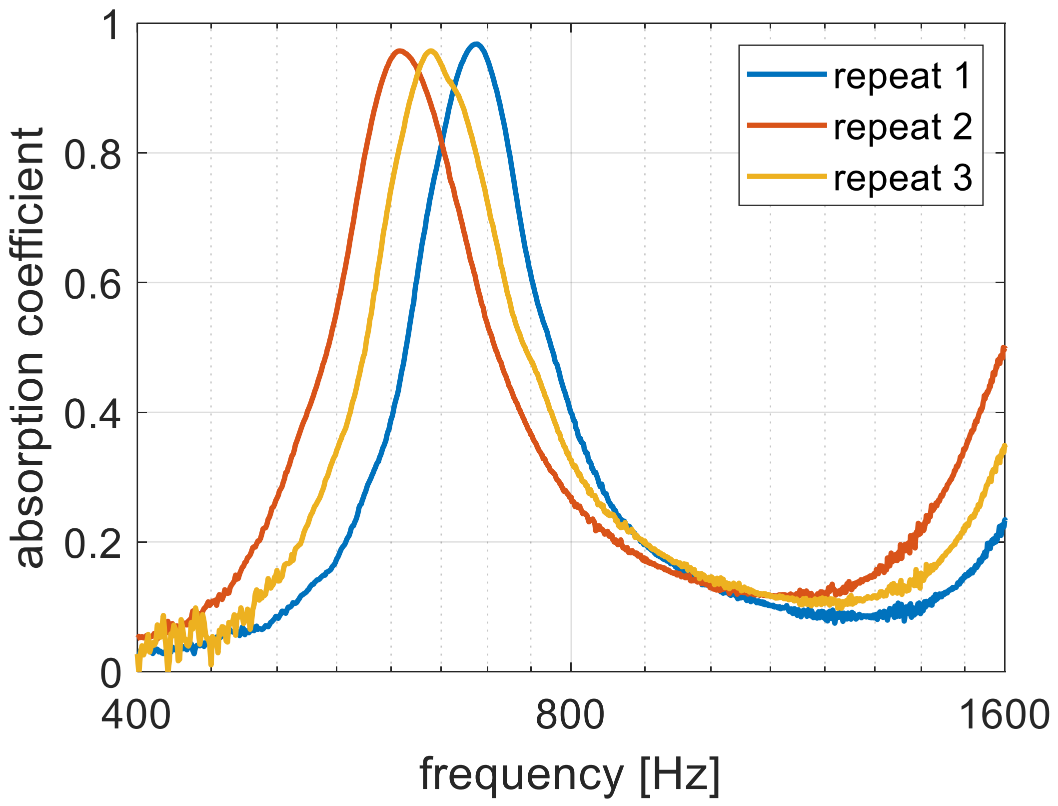
Test 2: compacted
+ 1000 m Vrms 0-6400 Hz signal exposure

Test 3, 4: compacted
+ 1000 m Vrms 100-1700 Hz signal (12 dB amplifier)

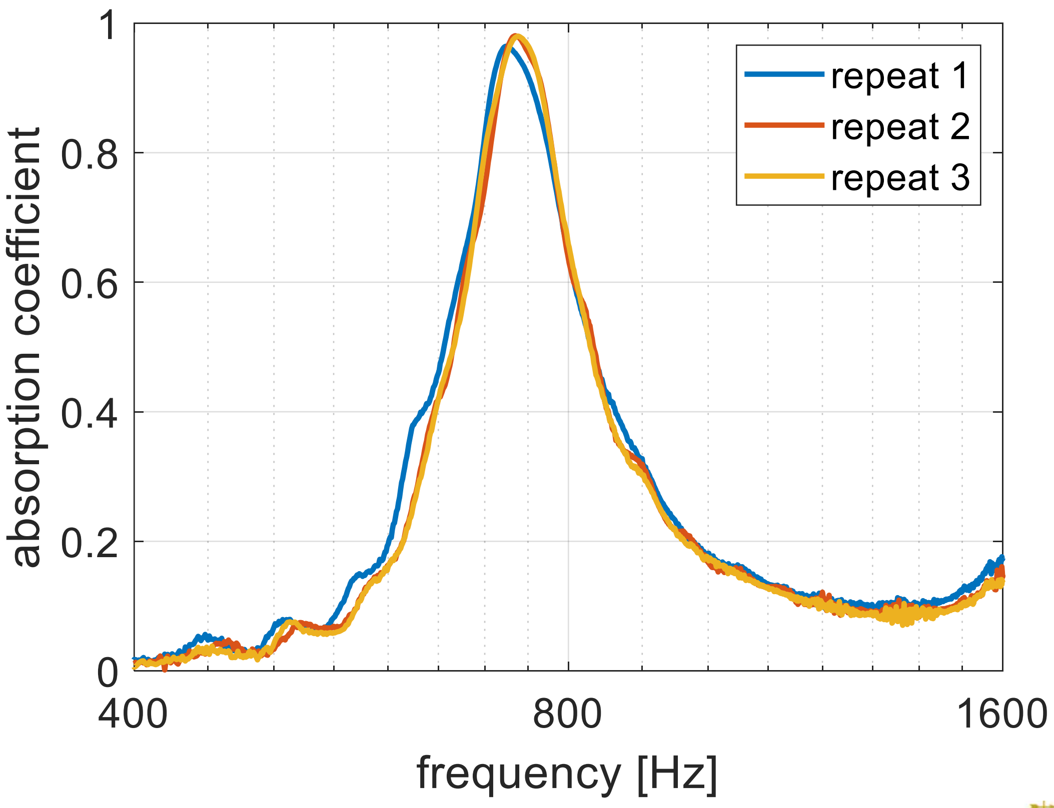
Testing Procedure

Exposure to strong signal improves consistency

No treatment

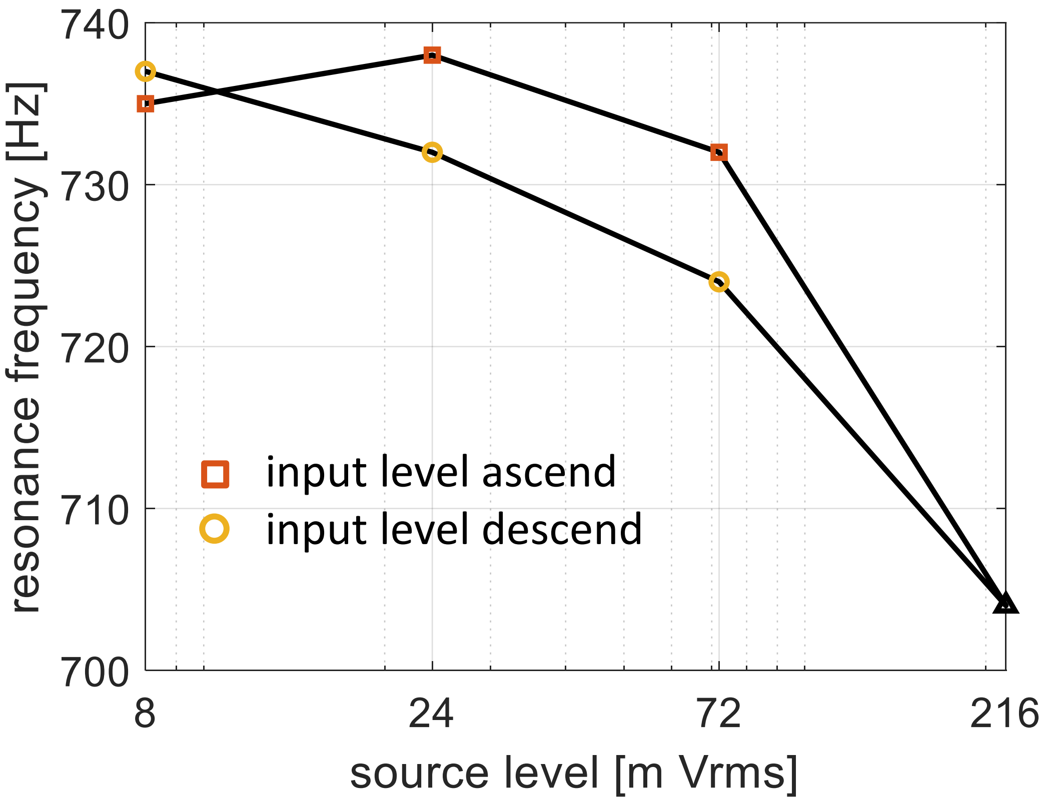


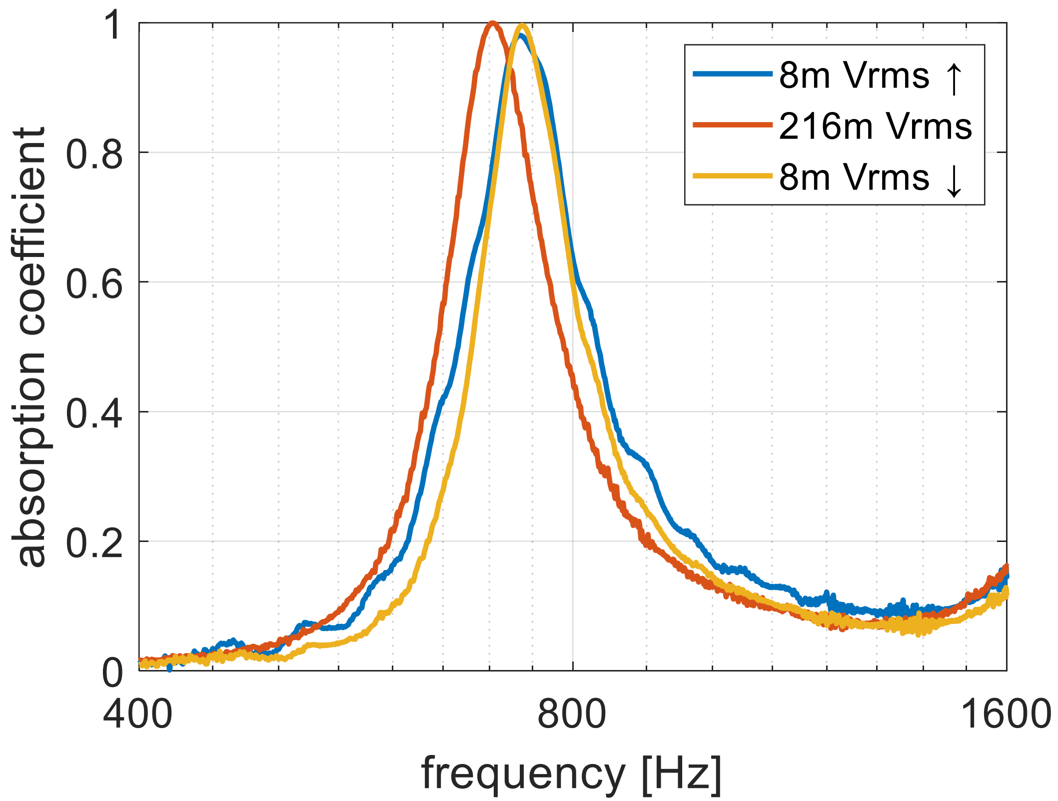
Exposed to 5min of 1000 m Vrms 0-6400 Hz signal



Testing Procedure

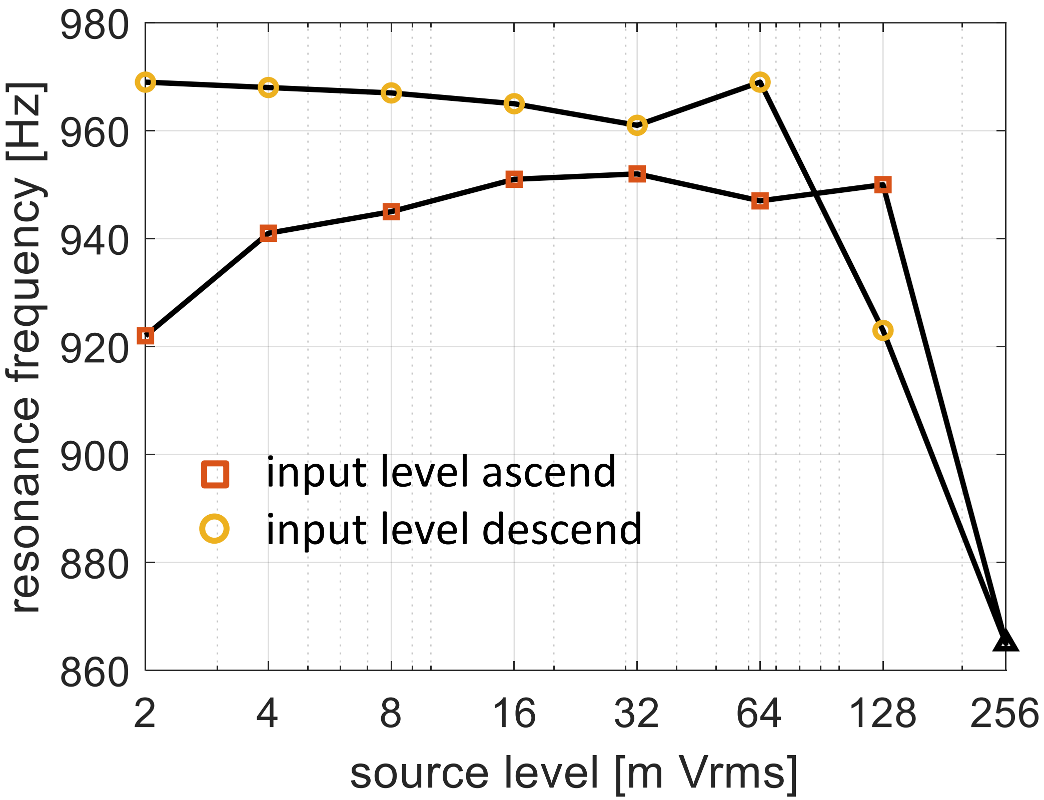
Softening with increasing input level

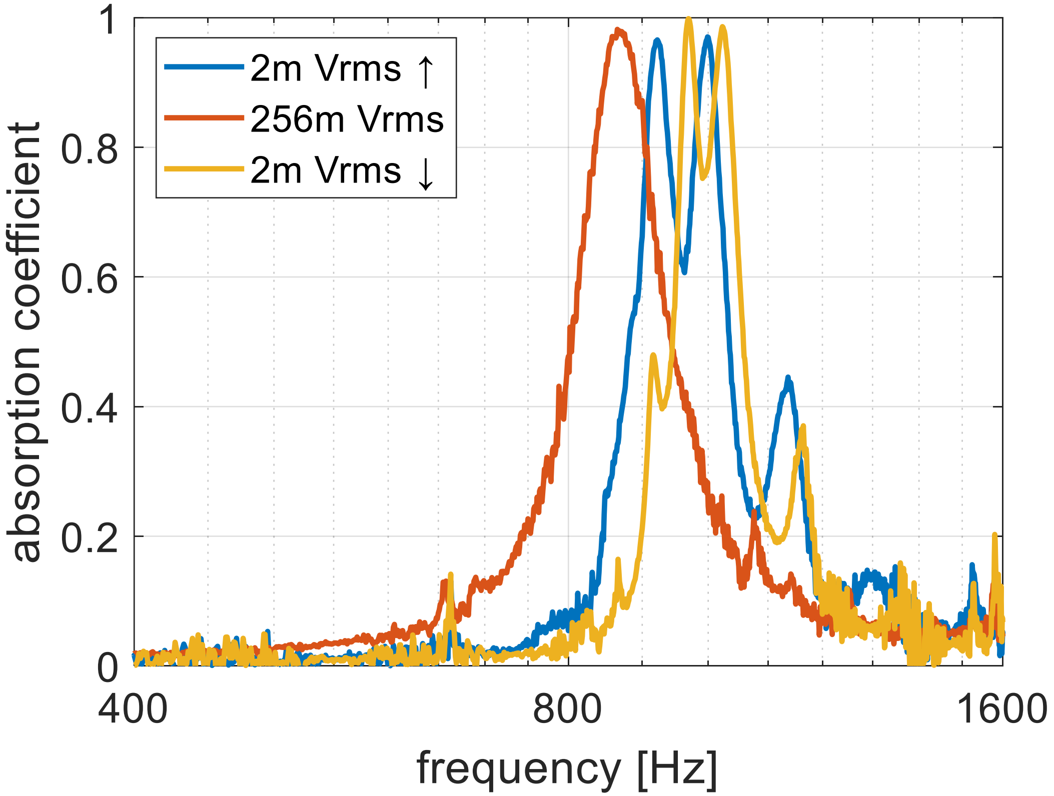




Testing Procedure

Softening with increasing input level



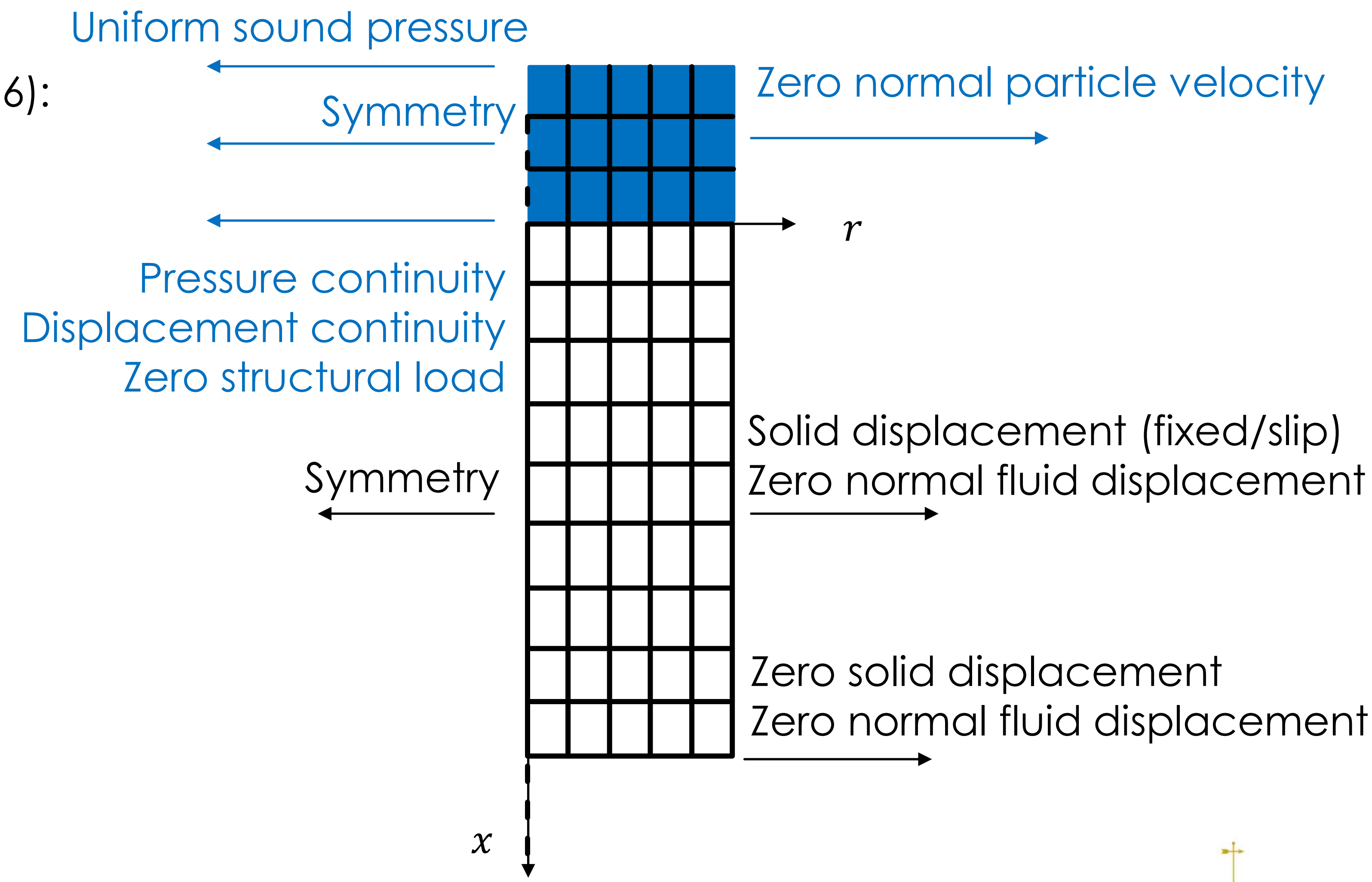


Finite Difference Approach

Introduce poro-elastic model (Biot, 1956):

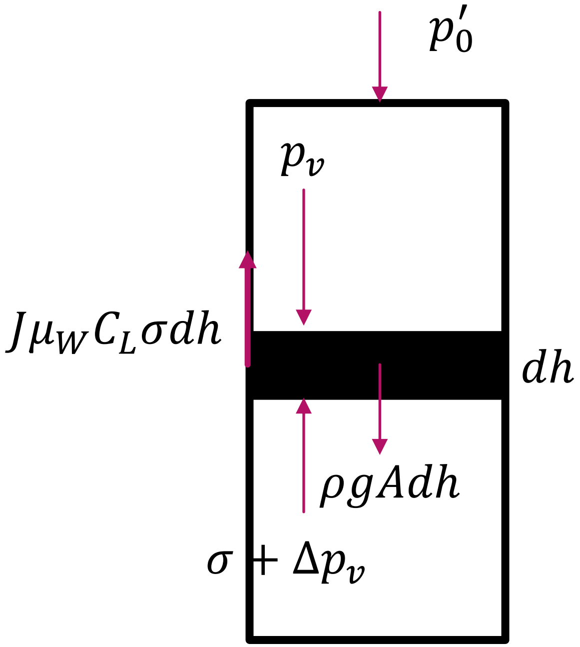
$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_x\\ \tau_y\\ \tau_z\\ s\end{bmatrix} = \begin{bmatrix}P & & & & & & Q\\ & P & & & & & Q\\ & & P & & & & Q\\ & & & N & & & \\ & & & & N & & \\ & & & & & N & \\ Q & Q & Q & & & & R\end{bmatrix}\begin{bmatrix}e_x\\ e_y\\ e_z\\ \gamma_x\\ \gamma_y\\ \gamma_z\\ \epsilon\end{bmatrix}$$

- The granules contacting each other are regarded as the "frame"
- The fluid phase can be described by the corresponding rigid model



Finite Difference Approach

Jassen's model – Force deflection in cylindrical container and friction on container wall (Duran, 2000, Springer)



$$A\Delta\sigma + J\mu_W C_L p_v dh = \rho g A dh$$

$$\sigma = \frac{\rho g}{\beta}\left(1 - e^{-\beta x}\right) + p_0' e^{-\beta x}$$

β is the Jassen factor:

$$\beta = 4J\mu_W/d$$

Hertzian contact – effective stiffness increases with the contact surface area (Fischer-Cripps, 1999)

$$E = E_0 \sigma^{1/3}$$

With Jassen's model and Hertzian contact theory, the stiffness of particle stack can be expressed as a function of depth, which has been applied in previous studies, e.g., Matchett and Yanagida, 2003; Tsuruha et al., 2020

$$E = E_0 \left[\frac{\rho g}{\beta}\left(1 - e^{-\beta x}\right) + p_0' e^{-\beta x}\right]^{1/3}$$

$$\frac{\partial E}{\partial x} = \frac{1}{3} E_0 \left[\frac{\rho g}{\beta}\left(1 - e^{-\beta x}\right) + p_0' e^{-\beta x}\right]^{-2/3} \left(\rho g - \beta p_0'\right) e^{-\beta x}$$

Finite Difference Approach

See (Venegas and Umnova, 2016) for details

For glass bubbles, only the interstitial pores need to be considered:

$$k_p = -j\delta_v^2(1 - 3C/x^2)^{-1}$$

$$k_p' = -j\delta_t^2\left(1 - \zeta^3 + \frac{3\zeta}{x_t^2}\left(\zeta x_t \frac{1 + x_t + \tanh\big(x_t(\zeta - 1)\big)}{x_t + \tanh\big(x_t(\zeta - 1)\big)}\right) - 1\right)$$

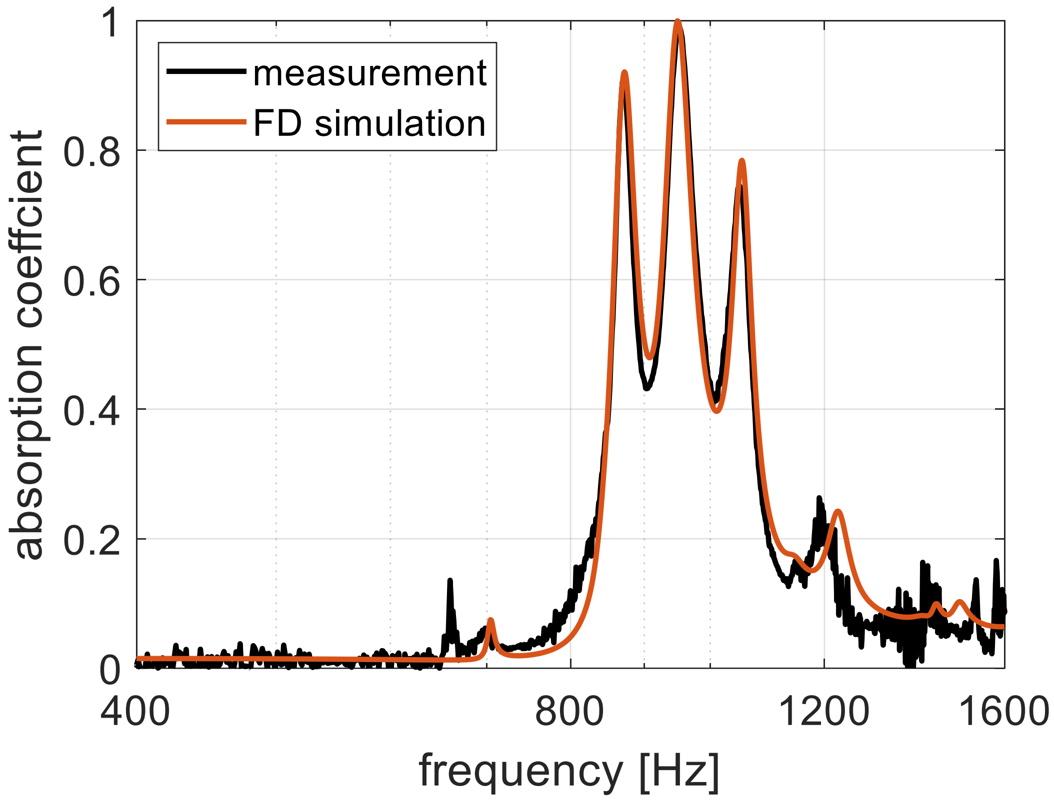
where $\zeta = (1 - \phi)^{1/3}$, and all other parameters follow the definitions in the references.

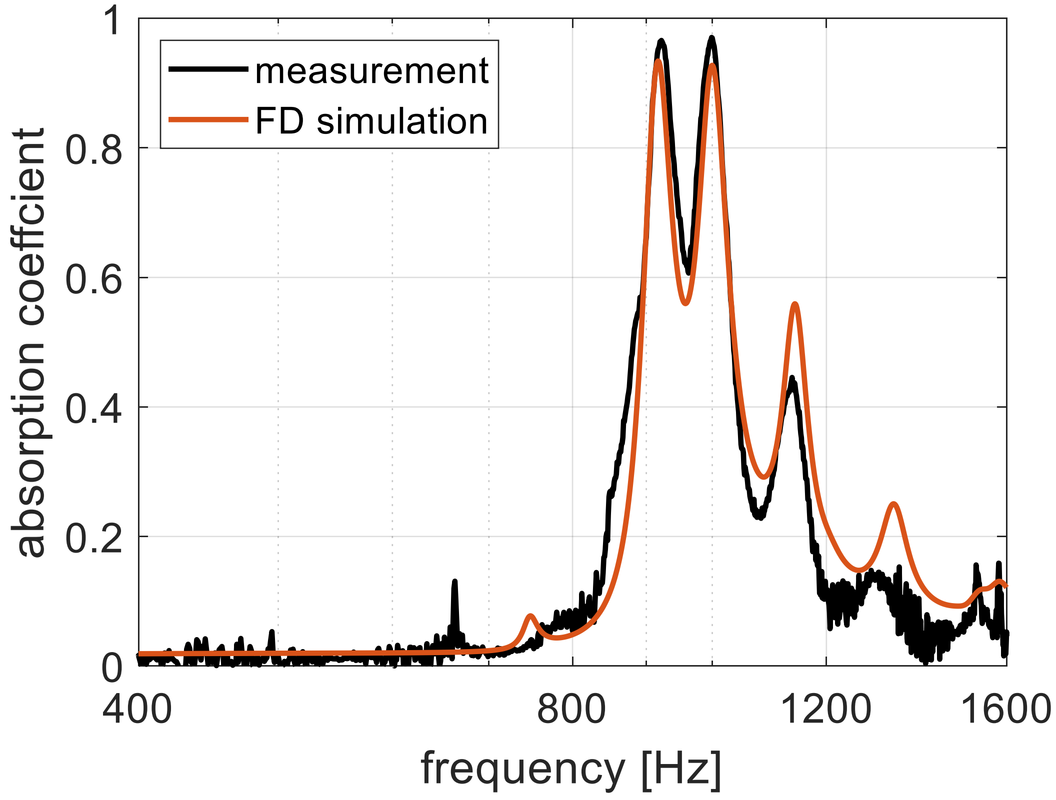
For granular activated carbon, triple porosity model is applied:

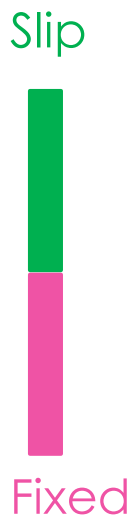
$$B = \left(\frac{1}{B_p} + \frac{1 - \phi_p}{B_u} F_d\right)^{-1}$$

$$B_u = \left(\frac{1}{B_m} + \frac{1 - \phi_m}{B_n} F_{nm}\right)^{-1}$$

Model Predictions Compared with Measurements

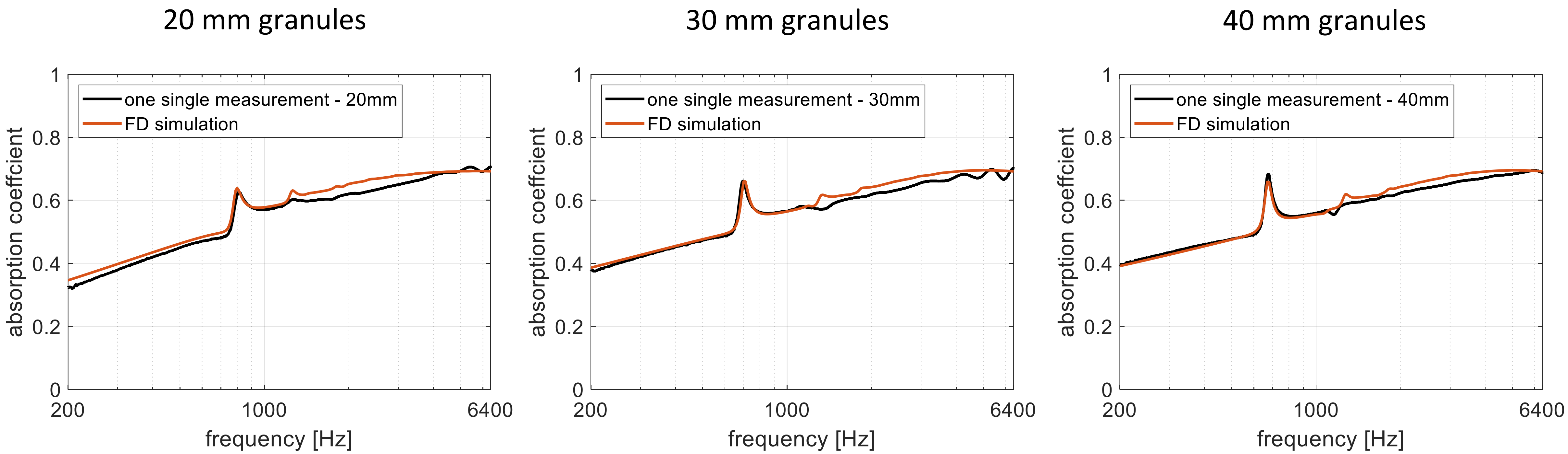






Model Predictions Compared with Measurements

The finite difference model provides predictions that match different thicknesses:



Conclusions

- The measurement and model description of granular materials are introduced
  - Testing procedure is developed for acoustic measurement of granular materials
  - A numerical implementation of Biot theory is introduced
  - The model prediction match very well with the measurement
- Future work
  - Improve the testing procedure, pursuing better consistency
  - Establish more complete model for forward prediction of material properties

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