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Study of the Impact of Boundary Conditions on Acoustical Behavior of Granular Materials and their Implementation in the Finite Difference Method

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STUDY OF THE IMPACT OF BOUNDARY CONDITIONS ON ACOUSTICAL BEHAVIOR OF GRANULAR MATERIALS AND THEIR IMPLEMENTATION IN THE FINITE DIFFERENCE METHOD

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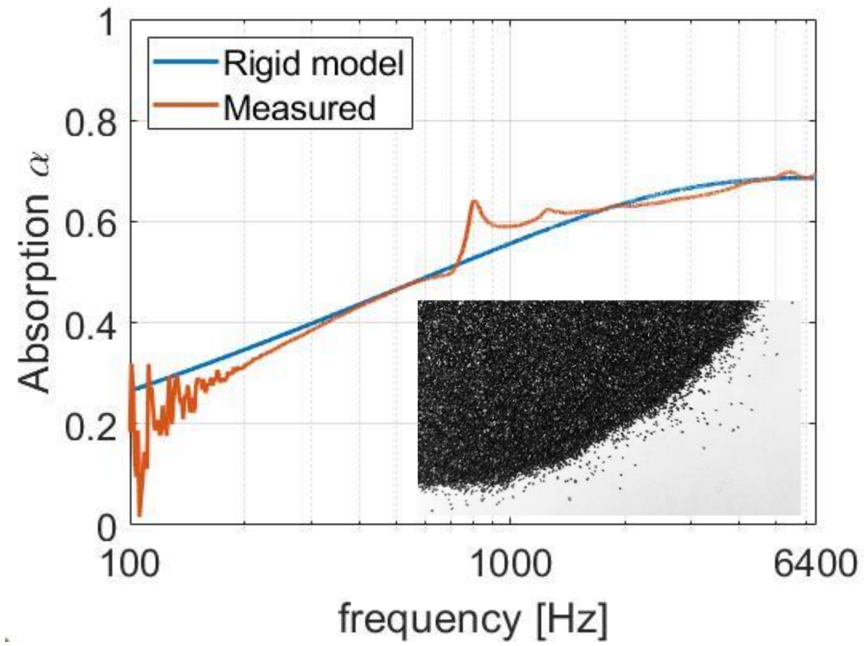
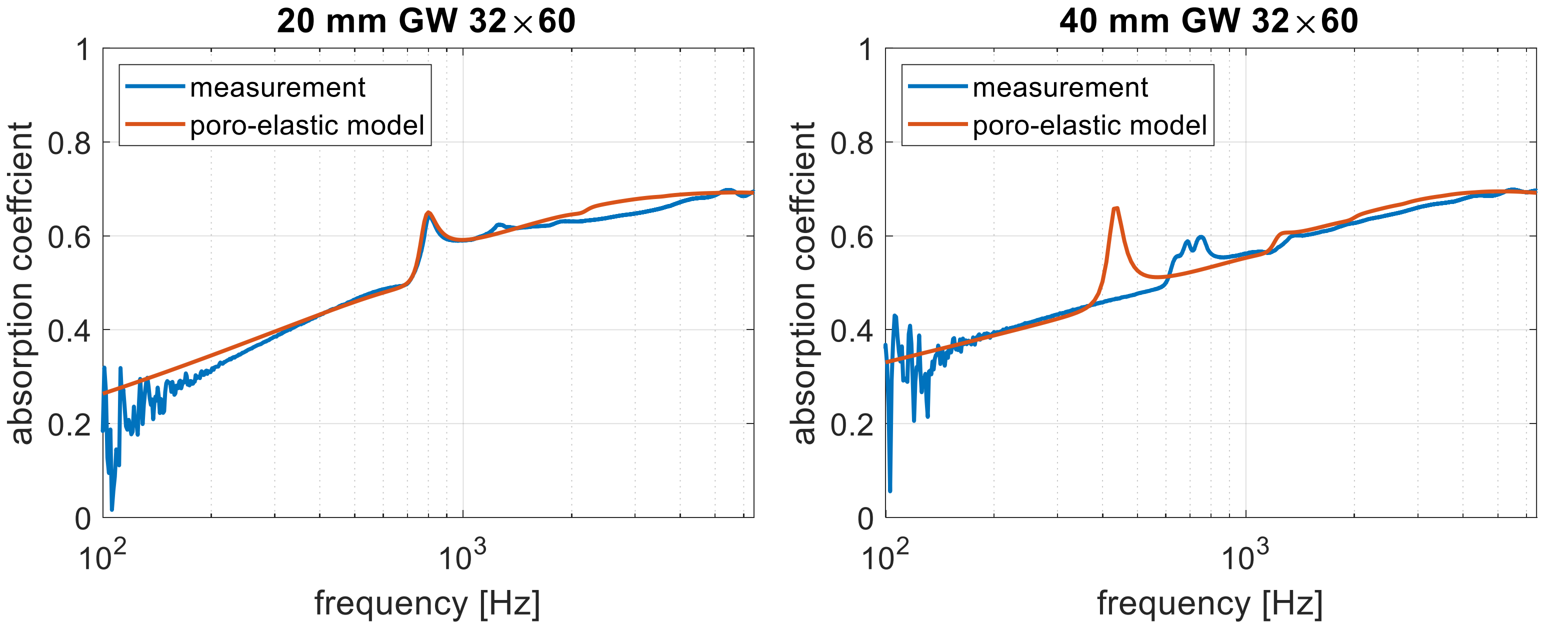
PURDUE UNIVERSITY | Ray W. Herrick Laboratories

Introduction

Porous granular materials have drawn attention due to their good performance at low frequencies, such as activated carbons:

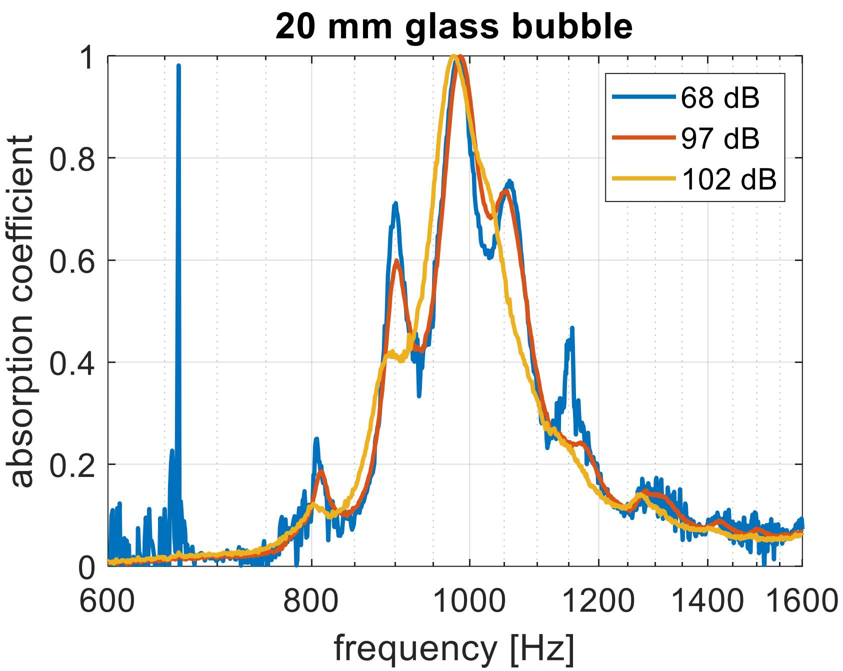


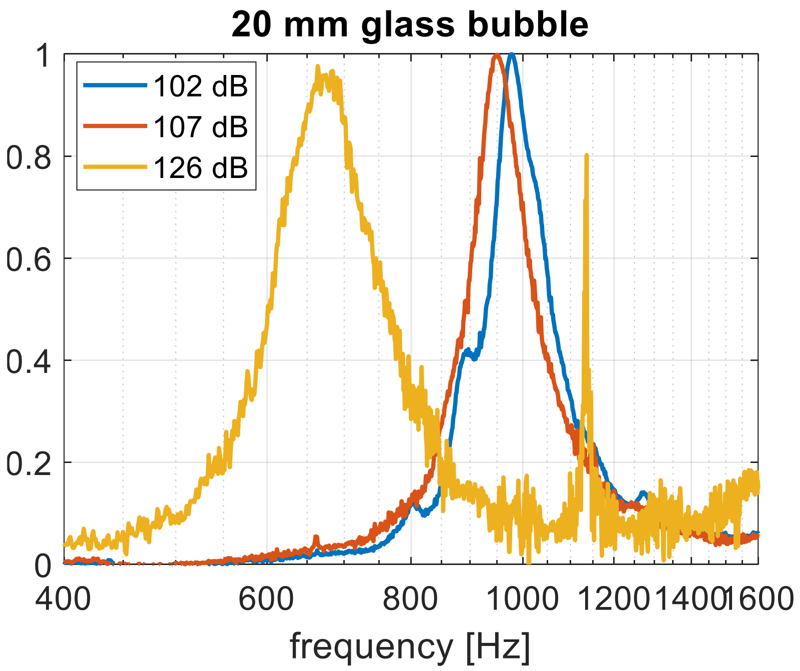


- Rigid model does not predict the resonance
- 1-D response does not follow the trend with varying thickness

Introduction

Glass bubbles
https://www.3m.com/3M/en_US/p/d/b40064606/





- Light weight glass bubbles show complex behavior under different input

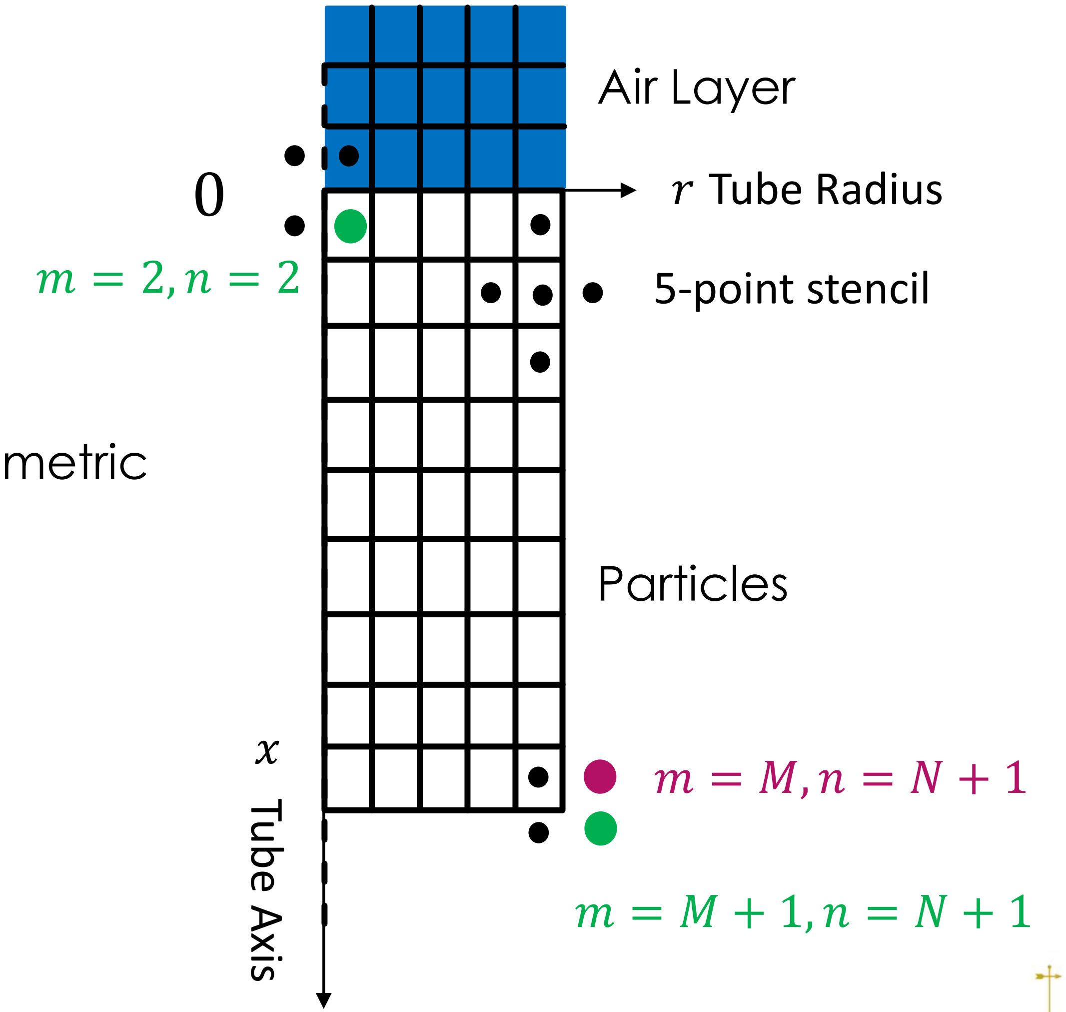
Finite Difference Approach

Introduce poro-elastic model (Biot, 1956):

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_x\\ \tau_y\\ \tau_z\\ s\end{bmatrix} = \begin{bmatrix}P & & & & & & Q\\ & P & & & & & Q\\ & & P & & & & Q\\ & & & N & & & \\ & & & & N & & \\ & & & & & N & \\ Q & Q & Q & & & & R\end{bmatrix}\begin{bmatrix}e_x\\ e_y\\ e_z\\ \gamma_x\\ \gamma_y\\ \gamma_z\\ \epsilon\end{bmatrix}$$

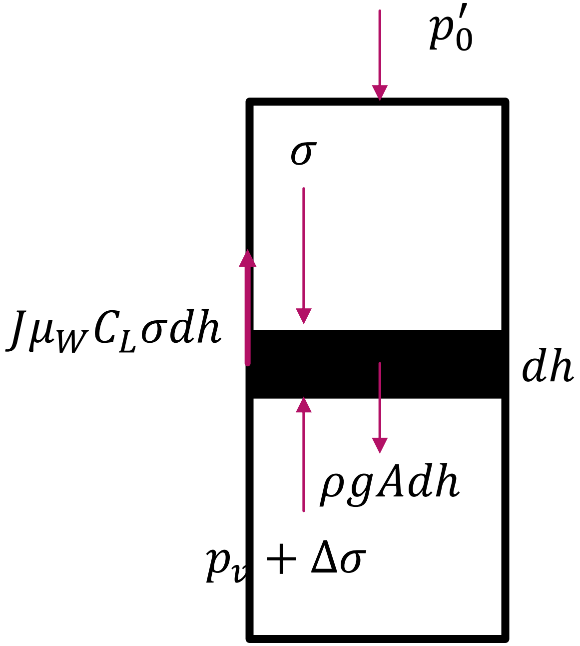
- The granules contacting each other are regarded as the “frame”
- The fluid phase can be described by the corresponding rigid model

Axisymmetric



Finite Difference Approach

Jassen's model – Force deflection in cylindrical container and friction on container wall (Duran, 2000, Springer)



$$A\Delta\sigma + J\mu_W C_L \sigma dh = \rho g A dh$$

$$\sigma = \frac{\rho g}{\beta}\left(1 - e^{-\beta x}\right) + p_0' e^{-\beta x}$$

β is the Jassen factor:

$$\beta = 4J\mu_W/d$$

Hertzian contact – effective stiffness increases with the contact surface area (Fischer-Cripps, 1999)

$$E = E_0 \sigma^{1/3}$$

With Jassen's model and Hertzian contact theory, the stiffness of particle stack can be expressed as a function of depth, which has been applied in previous studies, e.g., Matchett and Yanagida, 2003; Tsuruha et al., 2020

$$E = E_0 \left[\frac{\rho g}{\beta}\left(1 - e^{-\beta x}\right) + p_0' e^{-\beta x}\right]^{1/3}$$

$$\frac{\partial E}{\partial x} = \frac{1}{3} E_0 \left[\frac{\rho g}{\beta}\left(1 - e^{-\beta x}\right) + p_0' e^{-\beta x}\right]^{-2/3} \left(\rho g - \beta p_0'\right) e^{-\beta x}$$

Finite Difference Approach

For activated carbon, three levels of pores are assumed to exist in the material:

$$B = \left(\frac{1}{B_p} + \frac{1-\phi_p}{B_u} F_d\right)^{-1}$$

$$B_u = \left(\frac{1}{B_m} + \frac{1-\phi_m}{B_n} F_{nm}\right)^{-1}$$

Ref: Venegas and Umnova, 2016
Boutin and Geindreau, 2008
Boutin and Geindreau, 2010

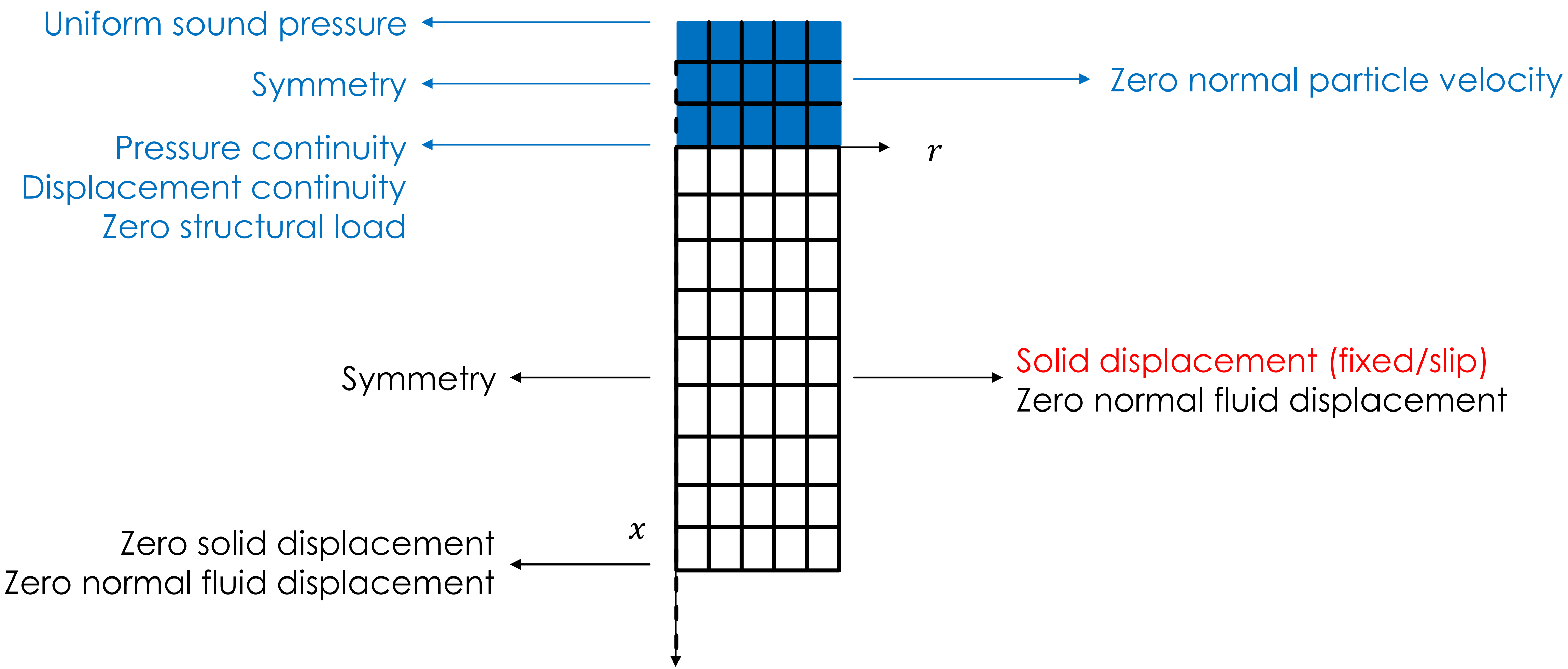
For glass bubbles, only the interstitial pores need to be considered:

$$k_p = -j\delta_v^2 (1 - 3C/x^2)^{-1}$$

$$k_p' = -j\delta_t^2 \left(1 - \zeta^3 + \frac{3\zeta}{x_t^2}\left(\zeta x_t \frac{1 + x_t + \tanh\left(x_t(\zeta-1)\right)}{x_t + \tanh\left(x_t(\zeta-1)\right)}\right) - 1\right)$$

where $\zeta = (1-\phi)^{1/3}$, and all other parameters follow the definitions in the references.

Finite Difference Approach



Model Predictions

Slip boundary condition:

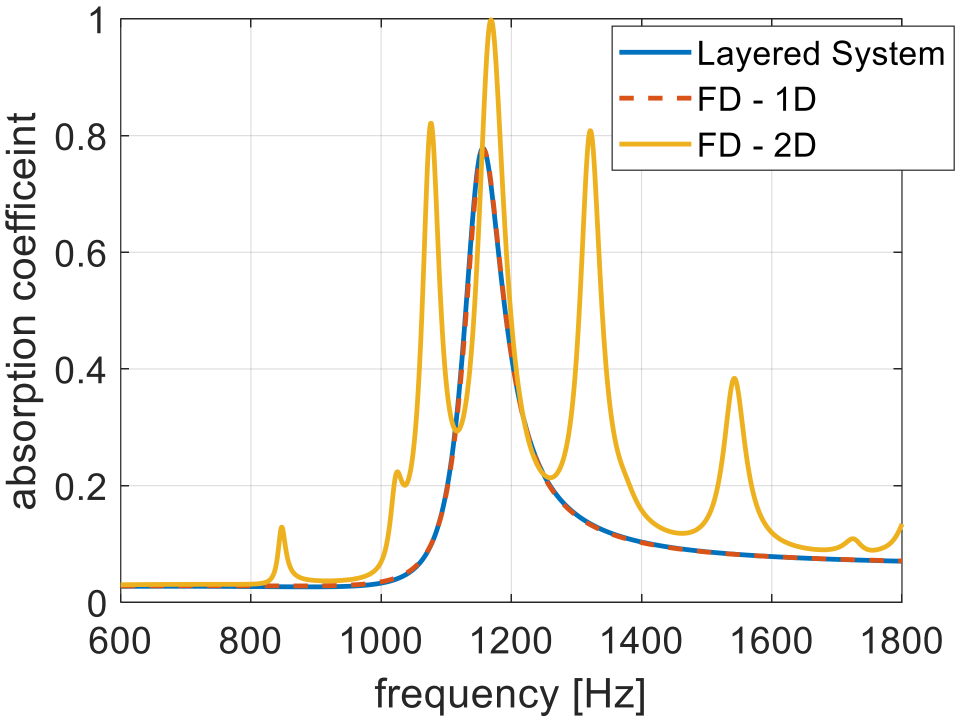
$$\left.\frac{\partial u^x}{\partial r}\right|_{r=R} = 0, \left.u^r\right|_{r=R} = 0$$

Fixed solid displacement:

$$\left.u^x\right|_{r=R} = 0, \left.u^r\right|_{r=R} = 0$$

If the slip boundary condition is applied all along the wall, the response will be purely 1D, which is equivalent to an infinite layer.
If the fixed boundary condition is applied, the response will be 2D.

20-mm-thick glass bubble simulation
Varying stiffness achieved with 20 layers in analytical model (Dazel et al., 2013)



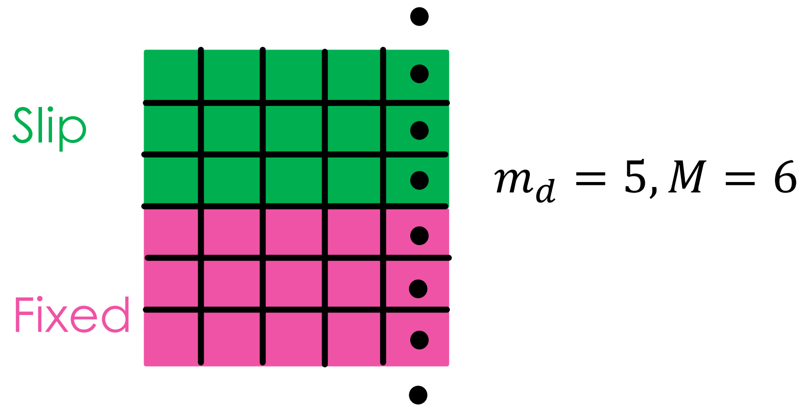
Model Predictions

Mixed boundary condition:

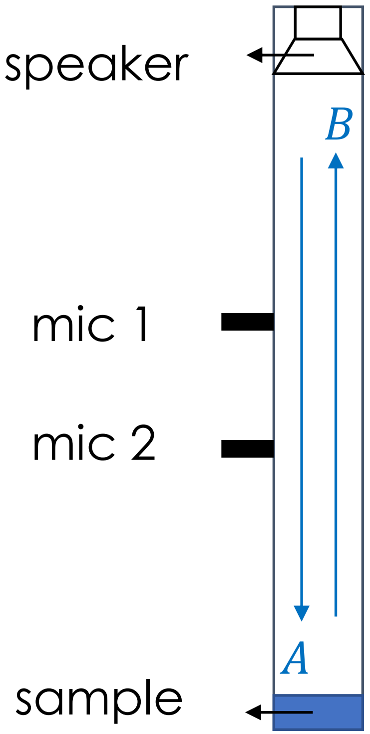
$$\left.\frac{\partial u^x}{\partial r}\right|_{r=R} = 0 \ (m < m_d)$$

$$\left.u^x\right|_{r=R} = 0 \ (m \geq m_d)$$

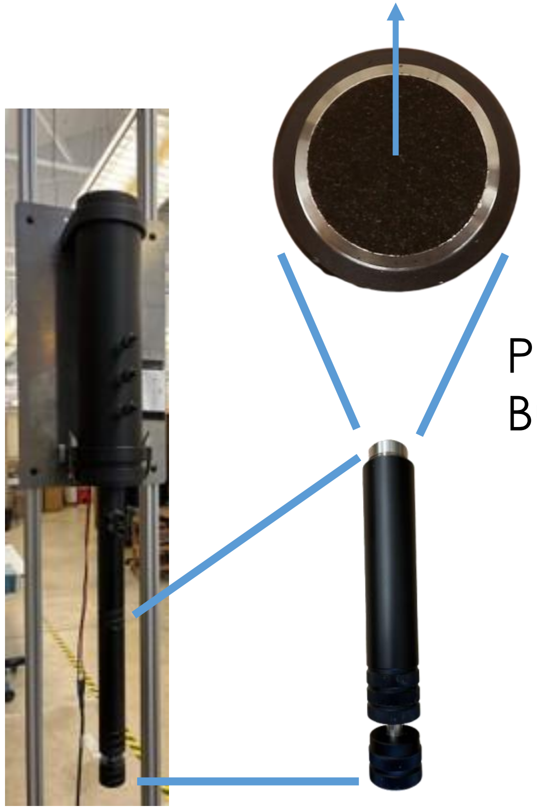
m_d is the row number before which slip boundary condition is applied, and after which fixed boundary condition is applied.



Model Predictions



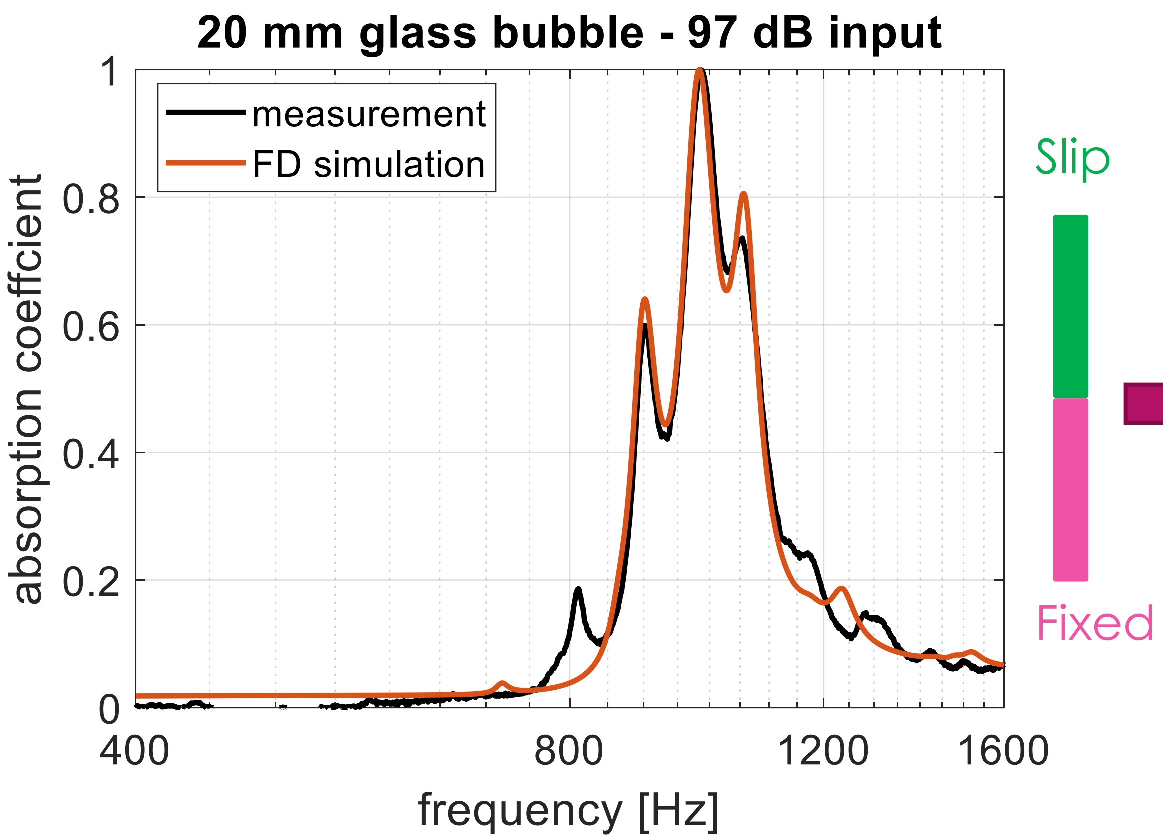
Activated carbon particles



Particle size: ~0.6 mm
Bulk density: ~500 kg/m^3

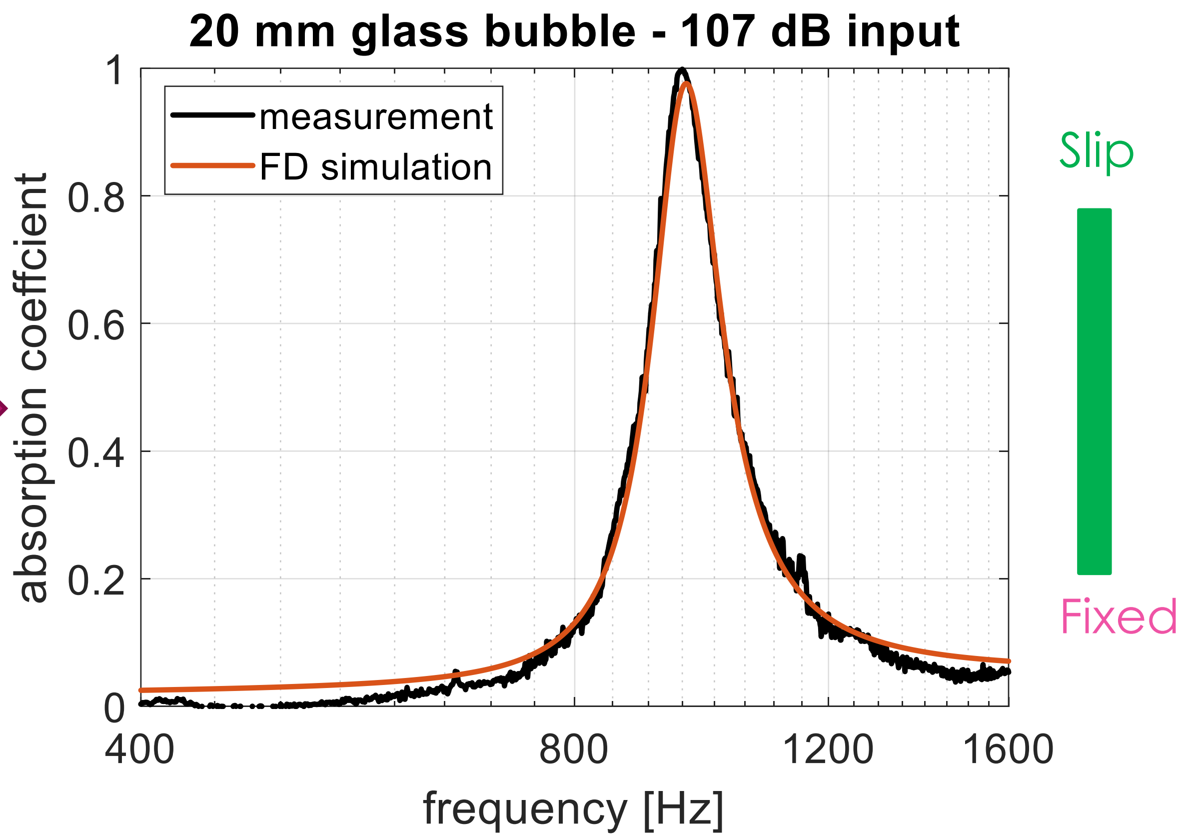
Particle size: 60 um
Bulk density: ~120 kg/m^3

Model Predictions



$E_0 = 1.45 \times 10^5$ Pa, $\nu = 0.29, \eta = 0.018$

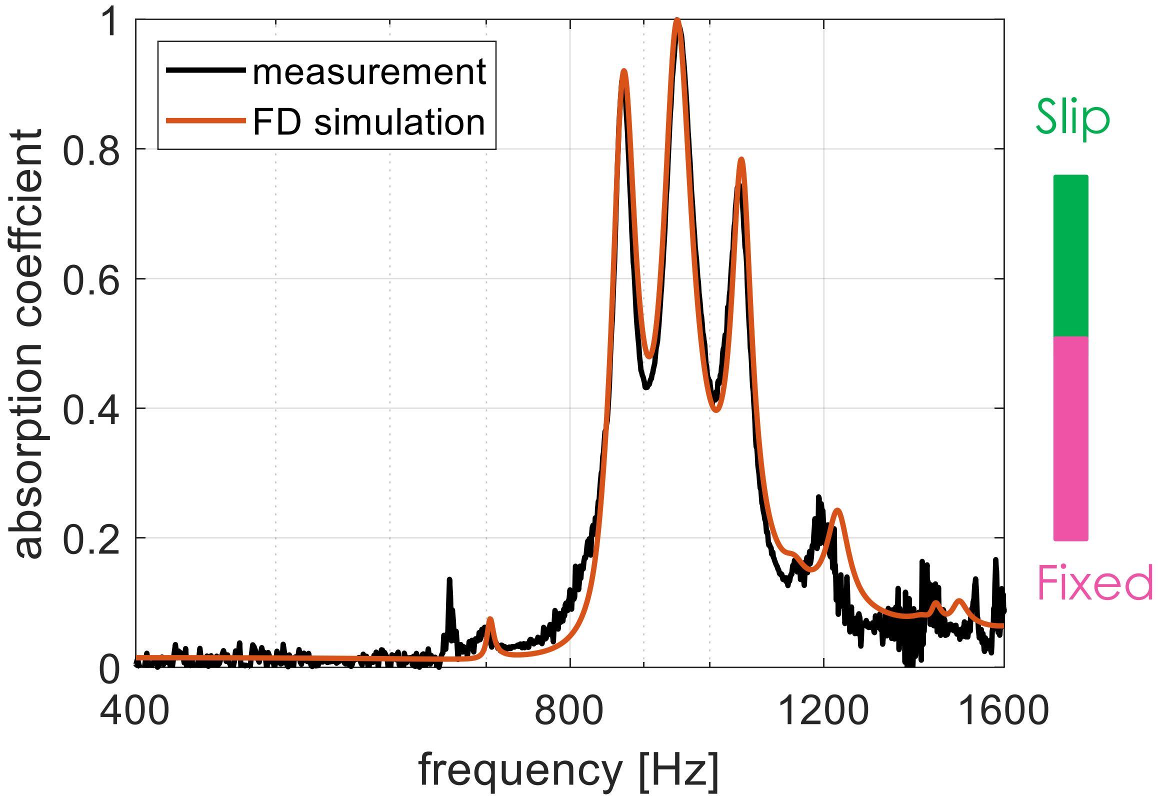
$\boldsymbol{m_d = 21}$



$E_0 = 1.45 \times 10^5$ Pa, $\nu = 0.29, \eta = 0.1$

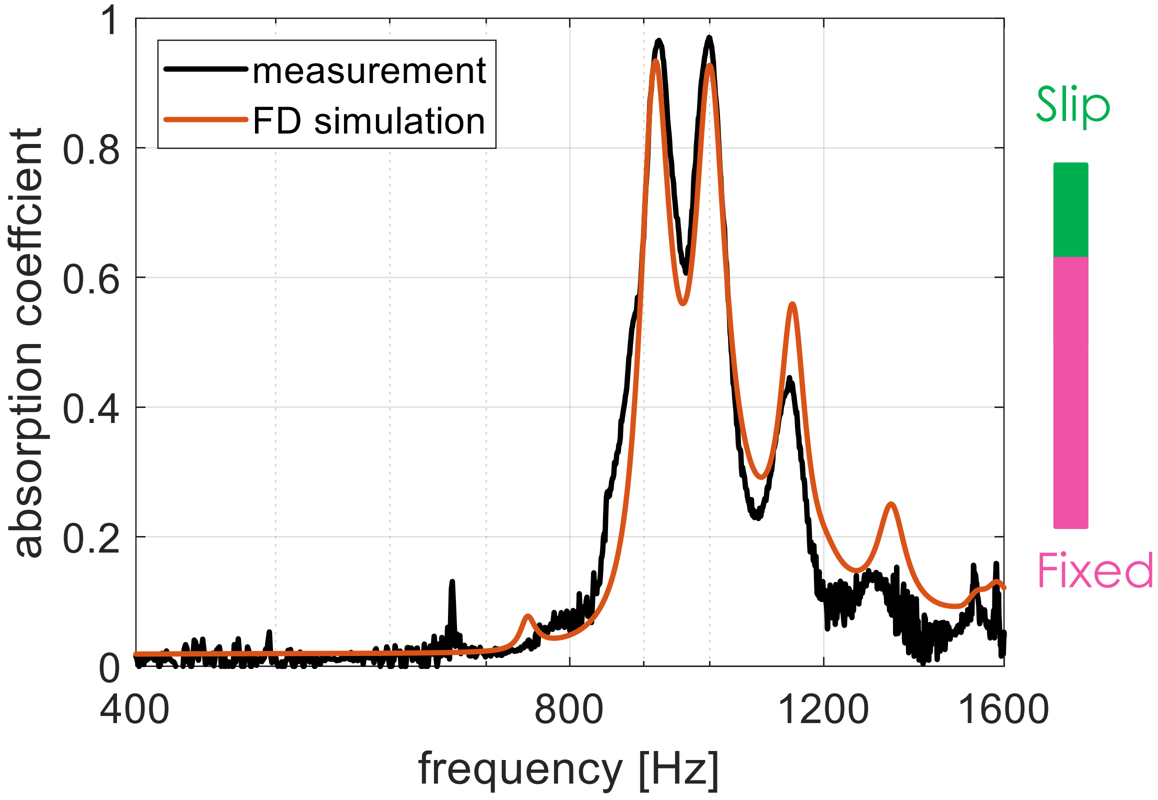
$\boldsymbol{m_d = 42}$

Model Predictions



$E_0 = 1.35 \times 10^5$ Pa, $\nu = 0.25, \eta = 0.004$

$\boldsymbol{m_d = 18}$



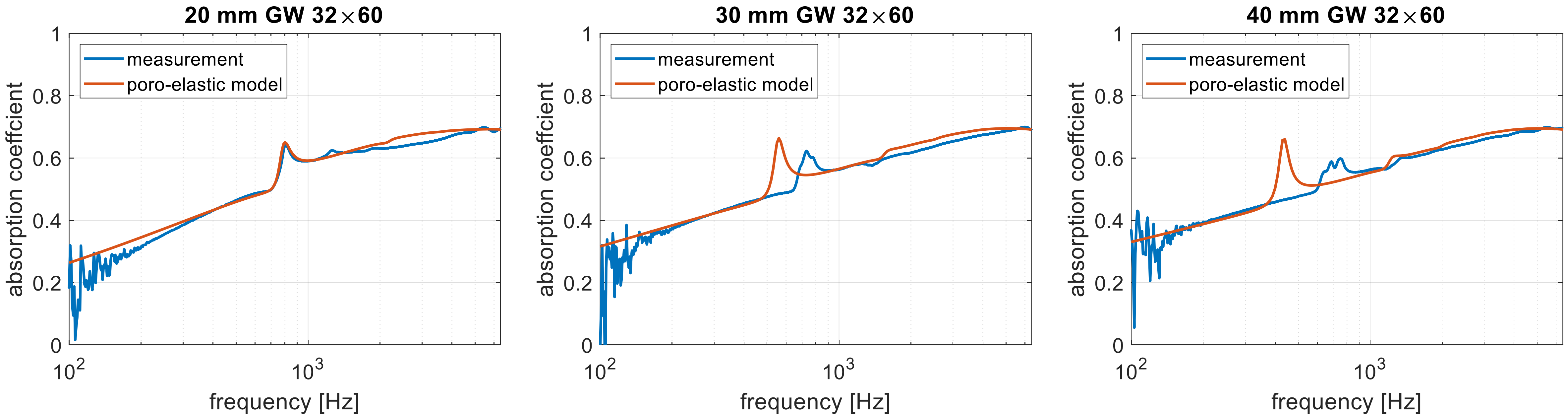
$E_0 = 1.52 \times 10^5$ Pa, $\nu = 0.26, \eta = 0.02$

$\boldsymbol{m_d = 13}$

Model Predictions

With fully slip boundary condition (1D response assumption), the shift of resonance cannot be captured with one set of parameters:

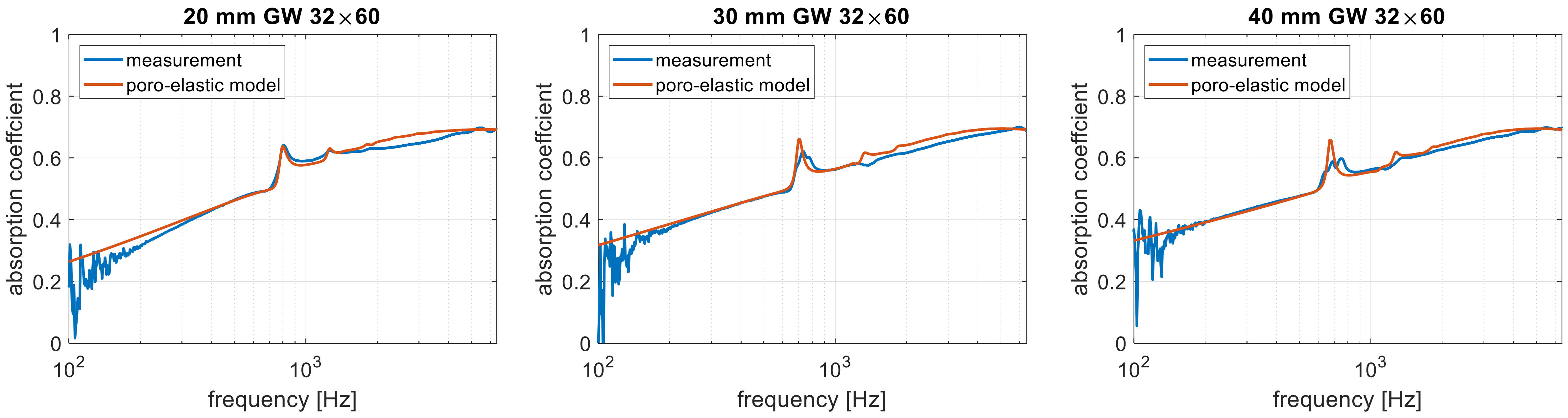
E_0 $[\mathrm{Pa}^{2/3}]$	3.0×10^5
ν	0.35
η	0.09



Model Predictions

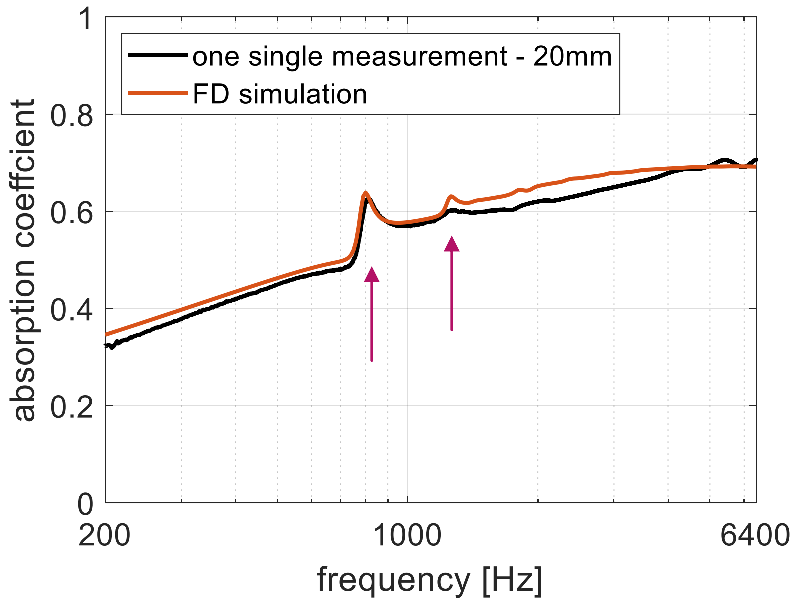
With fully fixed boundary condition, the activated carbon testing results can be reproduced with one set of consistent parameters:

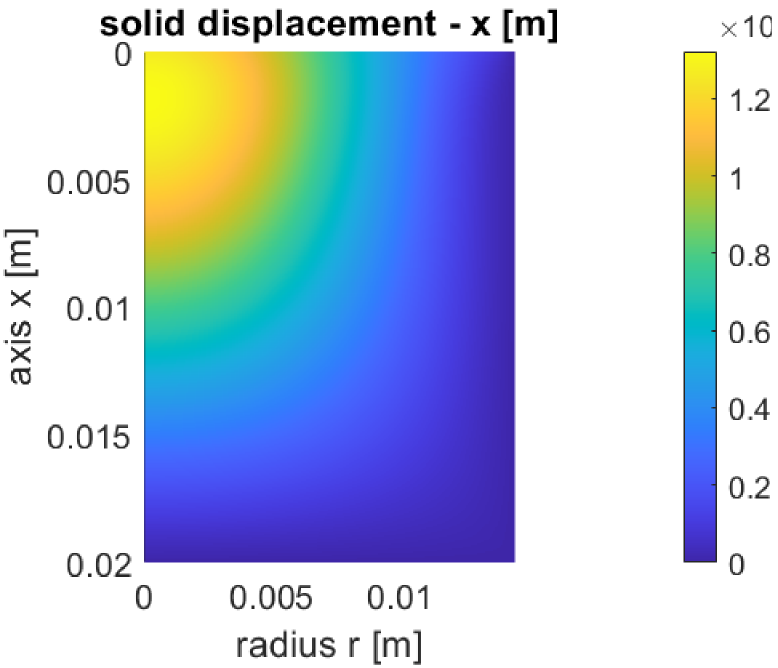
$E_0\ [\mathrm{Pa}^{2/3}]$	2.1×10^5
ν	0.35
η	0.06

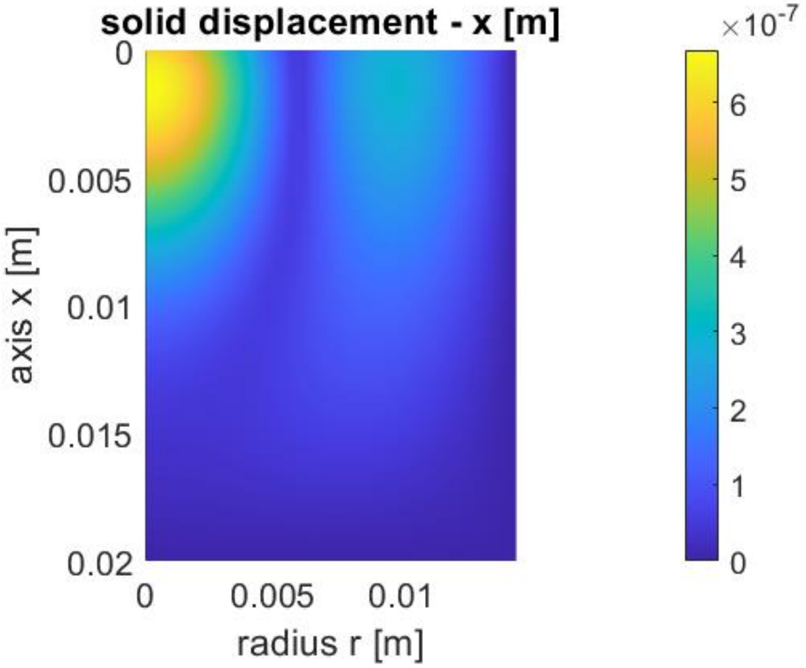


Model Predictions

With fully fixed boundary condition, the activated carbon testing results can be reproduced with one set of consistent parameters:







Conclusions

- The effect of different boundary conditions is studied with the proposed FD approach
  - A finite difference implementation of Biot theory is introduced, with consideration of cylindrical geometry of the test apparatus and different boundary conditions
  - The response of granular materials is well matched by the simulation results of finite difference scheme
  - By adjusting boundary conditions of solid phase, the response of granular materials can be better explained
- Future works
  - With consideration of non-linear behavior, a more complete model is needed, and correspondingly so are the boundary conditions

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