5-2023

Optimal R&D investment in the management of invasive species

William Haden Chomphosy

Dale T. Manning

Stephanie A. Shwiff

Stephan Weiler

Ecological Economics 211 (2023) 107875

Contents lists available at ScienceDirect

Ecological Economics

journal homepage: www.elsevier.com/locate/ecolecon

Analysis

Optimal R&D investment in the management of invasive species

William Haden Chomphosy [a,*], Dale T. Manning [b], Stephanie Shwiff [c], Stephan Weiler [b]

[a] Hendrix College, 1600 Washington Ave, Conway, AR, United States of America
[b] Colorado State University, Fort Collins, CO, United States of America
[c] USDA - National Wildlife Research Center, Fort Collins, CO, United States of America

ARTICLE INFO

Keywords:
Invasive species
Dynamic optimization
Bioeconomic modeling
Endogenous technological change
Boiga irregularis

ABSTRACT

Invasive alien species (IAS) threaten world biodiversity, ecosystem services, and economic welfare. While existing literature has characterized the optimal control of an established IAS, it has not considered how research and development (R&D) into new removal methods or technologies can affect management decisions and costs over time. R&D can lower the costs of control in a management plan and creates an intertemporal trade-off between quick but costly control and gradual but cheaper removal over time. In this paper, we develop and solve a continuous time dynamic optimization model to study how investment in R&D influences the optimal control of an established invasive species. After characterizing the dynamic model solution, we solve the model numerically to study the benefits from R&D in the management of the brown tree snake (*Boiga irregularis*), and explore how optimal solutions vary across economic and biological conditions. We find that the introduction of R&D significantly reduces overall costs of IAS and management and that the cost reductions substantially outweigh research expenditure. These results imply that policymakers seeking to control IAS should consider R&D as a vital component of cost effective control strategies.

1. Introduction

Harmful invasive alien species (IAS) pose immediate danger to the global environment, threatening international biodiversity, ecosystem services, and economic welfare (Wilcove et al., 1998; Chew, 2015). IAS are capable of imposing significant economic damages through predation of agricultural commodities, property damage, and disease transmission (Pimentel et al., 2005; Shwiff et al., 2017). In the economic literature studying invasive species and their management, researchers typically assume convex control costs, but do not account for the possibility that R&D investment in new and better control methods may lower future costs of population reduction. Ongoing patterns of international trade and climate change are likely to increase the probability of invasive species distribution (Perrings et al., 2002; Hellmann et al., 2008; Hulme, 2017), in spite of greater focus on prevention. In light of this, devoting attention to cost-effective population management, in addition to prevention, will become even more important. In this paper, we introduce R&D investment in control technology efficacy as an endogenous management decision and examine its impact on optimal management paths, total management costs, and the benefits of research spending relative to the cost.

When specifying a management strategy, the planner must consider the reduced cost of population reduction in the future from research conducted in the present. The addition of a dynamic R&D decision highlights the important role of research investment in IAS management that has yet to be studied rigorously in the economic literature on invasive species. After characterizing the model analytically, we apply it to the management of a specific invasive species, the brown tree snake (*Boiga irregularis*, BTS). Labeled a “catastrophic” invasive species (Burnett et al., 2012), the brown tree snake is a historically damaging invader on the island of Guam that has imposed both ecological and economic harm, and threatens a similar invasion on Hawai’i (Savidge, 1987; Burnett et al., 2008; Shwiff et al., 2010). The United States government has targeted this species for eradication from the island of Guam (US Congress, 2004), and has pursued this goal with both population control and active research investment (USGS Brown Tree Snake Lab, USDA National Wildlife Research Center, USDA Wildlife Services). We find that when R&D is endogenous to the management decision,targeted population reduction can be achieved for substantially lower costs than in its absence. Additionally, these results show that the cost-savings can outweigh the R&D investment, suggesting that research can be a valuable part of an efficient of an IAS management program.

The focus of this article is measuring benefits from research on invasive species removal efficacy. The current literature on the economics of IAS management has yet to provide a precise examination of this

* Corresponding author.
E-mail address: HadenChomphosy@Hendrix.edu (W. Haden Chomphosy).

https://doi.org/10.1016/j.ecolecon.2023.107875
Received 16 December 2020; Received in revised form 30 March 2023; Accepted 1 May 2023
Available online 26 May 2023
0921-8009/

aspect of real-world invasive species management. Additionally, we demonstrate how adding research spending to management plans can influence the timing and magnitude of IAS removal.

The remainder of the paper is organized as follows: Section 2 reviews the two main bodies of literature that will be used to inform the present study. Section 3 outlines the bioeconomic model, its analytical foundations, and characteristics of optimal solutions. Section 4 provides numerical results for the BTS specification as well as a categorical analysis of the benefits of R&D based on species characteristics. Finally, Section 5 discusses the model solutions and their implications for policy and future work

2. Literature review

Bioeconomic models introduce biological processes into models of economic decision-making. While this type of model has been used in a variety of environmental economic studies, they have taken a prominent role in the literature regarding invasive species (Epanchin-Niell, 2017). The contribution of this paper is the introduction of additional management options in the form of R&D investment, which influences the cost of population reduction over the management horizon. The practice of modeling R&D and its economic impacts is common in macroeconomic growth models (Romer, 1990; Grossman and Helpman, 1994), microeconomic study of patents (Griliches, 1990; Hall et al., 2001), and analyses of environmental protection (Jaffe et al., 2003; Acemoglu et al., 2012). Despite the growing role of technological change in environmental economics, this practice has not extended to economic study of invasive species.

Of the limited examples on the topic of R&D in IAS management, Kim et al. (2010, 2012) demonstrate that technological change lowering IAS management costs generate welfare gains relative to conventional methods. While there are many opportunities for R&D to improve management outcomes, focusing attention on reducing costs allows us to build on the work of those existing studies. The present research differs in the use of a deterministic link between research investment and cost-saving technology, via a knowledge production function. This function allows our model to incorporate research investment as an explicit choice on the part of resource managers and characterize intertemporal tradeoffs between population reduction and research activity.

2.1. Economics of invasive species

Economic study of IAS has experienced a period of substantial growth in the past few decades. There are several comprehensive reviews of the literature (Lovell et al., 2006; Olson, 2006; Marbuah et al., 2014; Lodge et al., 2016; Epanchin-Niell, 2017), but the main branches of research have focused on damage estimation (OTA, 1993; Pimentel et al., 2005; Shwiff et al., 2010), the use of dynamic optimization to inform management strategies (Eiswerth and Johnson, 2002; Olson and Roy, 2002; Leung et al., 2002; Mehta et al., 2007; Burnett et al., 2008; Epanchin-Niell et al., 2012; Jardine and Sanchirico, 2018), and economic analysis of invasive species policy (Margolis et al., 2005; McAusland and Costello, 2004; Akter et al., 2015; Bartkowski et al., 2015).

The present study is most relevant to the second branch of the literature, relating to the use of dynamic optimization for invasive species management. Existing research has demonstrated the use of continuous time optimal control (Eiswerth and Johnson, 2002; Burnett et al., 2008; Haight and Polasky, 2010) and discrete dynamic programming (Olson and Roy, 2002; Mehta et al., 2007; Hyytiäinen et al., 2013). Epanchin-Niell (2017) suggests that the main contributions of this literature to policy design are enhancement of prevention efforts, cost-effective surveillance and monitoring, optimal management of established invasions, private control of spread, and accounting for uncertainty.

Within the IAS literature, studies that focus on changing control costs typically have not incorporated R&D within their models. Discussions of changing control costs through environmental and spatial factors (Burnett et al., 2007; Epanchin-Niell and Hastings, 2010), case studies of adopting pre-defined technologies (Adams and Lee, 2012), or sensitivity analyses are more common (Hyytiäinen et al., 2013). The dearth of research in this area may be due to a preference for analyzing IAS prevention, as it has been shown to be a more cost-effective method of IAS management, particularly given the challenge of eradicating established IAS (Leung et al., 2002). However, changes in the climate are expected to increase the likelihood of IAS dispersal, making study of control costs increasingly important (Hellmann et al., 2008; Hulme, 2017). This paper examines the introduction of R&D investment, knowledge accumulation, and endogenous control cost reductions within the dynamic model. This is an apt extension as economically valuable knowledge is often structured as a dynamic stock that changes over time with additional research.

2.2. R&D modeling in environmental economics

Technological change has become an important topic of study within environmental economics, much as it has been in agricultural economics for some time (Babcock et al., 1992). Jaffe et al. (2003) present a general overview of studies on technological change within the literature. Like economic study of IAS, this body of literature has made great use of dynamic optimization modeling (Parry et al., 2000; Goulder and Mathai, 2000; Popp, 2004; Vogt-Schilb et al., 2018). Most commonly, research has focused on carbon dioxide reduction, and technological change or investment is motivated by private firms' incentive to avoid fines or other penalties related to their emissions. In these studies the CO_2 imposes damages and exhibits natural decay, while economic activity adds to the stock. When studying invasive species there is also a harmful stock pollutant, but it exhibits the opposite dynamics: growing naturally and being reduced by management activity. A common outcome in these models is that optimal investment occurs early in the planning stage (Goulder and Mathai, 2000; Popp, 2004; Vogt-Schilb et al., 2018). This result is intuitive as these investments lower the cost of future abatement, and this early action has the greatest long-run impact.

Despite the methodological similarity between studying invasive species and other environmental stock pollutants, the IAS literature has done little to introduce dynamic R&D decisions into the bioeconomic models. Kim et al. (2010, 2012) appear to be the only examples of scholarly work focusing on technological development in the context of IAS management. This pair of papers studies the effect that technological development has on IAS management, but their model does not incorporate research investment as a choice variable within their of the model.

By introducing R&D to the management model we depart from the typical invasive species literature by relaxing the control cost function. As discussed in Jardine and Sanchirico (2018) the convention within the IAS literature is to specify a convex control cost that reflects the diminishing returns of control as effort is scaled up. This convexity reflects that at higher levels of control, it becomes more challenging to capture the reduced number of species. However, such an approach does not account for R&D that can lower control costs through investments in productivity. In light of this, we develop a model with convex control costs but with endogenous R&D that lower control costs over time. This represents a synthesis of the two research categories while also providing a more accurate portrayal of invasive species management.

3. Model

3.1. Problem statement and Pontryagin conditions

To examine optimal R&D strategies in the control of IAS, we develop a bioeconomic model in which a manager chooses control levels and R&D expenditure to minimize the present value of total social costs, including control, R&D, and damage costs. In each period $s \in [t, T]$ the manager chooses levels of population control x_s and research investment I_s, anticipating the impact these decisions have on future costs. The stock of IAS is represented by n_s while knowledge stock is given by K_s. In the present context we assume that the species has an established population in the ecosystem, but there are no additional introductions or migrations. IAS management can be initiated at different levels of species establishment but we focus our attention to the case where the population is at carrying capacity to demonstrate the incentives for research at their greatest potential benefit.

Total IAS costs are given in (1) as the sum of population control costs $C(x_s, K_s)$, the damages caused by invasive species, $D(n_s)$, and the cost of R&D investment, $R(I_s)$.

$$TC(x_s, n_s, I_s, K_s) = C(x_s, K_s) + D(n_s) + R(I_s) \tag{1}$$

Control costs are a function of the control effort[1] and knowledge stock; higher levels of population control increase costs at an increasing rate ($C_x > 0, C_{xx} > 0$), while the stock of economically valuable knowledge lowers total and marginal control costs ($C_K < 0, C_{xK} < 0$). The convex costs of population control reflect that, in a given time period, increasing control effort creates higher expenses such as overtime pay, additional resources, etc. Developing new knowledge can lead to lower-cost control methods. We assume increasing costs of IAS damage and research investment, but do not make further assumptions as these will be specific to the management context ($D_n > 0, R_I > 0$).

Population growth is composed of a biological growth function, $g(n_s)$ net of population control harvest $h(n_s, x_s)$:

$$\dot{n} = g(n_s) - h(n_s, x_s) \tag{2}$$

Biological growth is strictly a function of the IAS population in time s while the harvest is a function of the population as well as control effort. Marginal growth of the species may be increasing or decreasing, given the population $g_n \gtrless 0$. It is assumed that harvest is a generally increasing function of the effort and population $h_x > 0, h_n > 0$, and $h_{xn} = h_{nx} > 0$. Additional information about the exact shape of these functions will depend on the species being managed. An important characteristic of this formulation is that the harvest rate is a function of the IAS population, and as populations decrease, the marginal success of control falls. Effectively, this means that at low population levels there must be additional effort expended in order to continue harvesting IAS. Pairing this with the convex control costs featured in $C(x_s, K_s)$ creates a circumstance of stock-dependent costs in line with theory established by Olson and Roy (2008).

The state equation for knowledge is referred to as the knowledge production function (KPF):

$$\dot{K} = \eta(K_s, I_s; A) \tag{3}$$

[1] The choice to model IAS removal in terms of effort and subsequent successful harvest is made to more accurately depict the decision-making process of IAS managers. In other IAS studies, the choice variable is modeled as the number of IAS removed rather than the effort devoted to said removal. Our modeling choice is made to capture the imperfect relationship between removal effort and how productive that effort is. Examples of other studies that opt for this approach include Sanchirico et al. (2010) and Kling et al. (2017). In the numerical model, we use a rich data set for brown tree snake management to estimate the efficacy of population control behavior translating to successful culling.

The KPF describes the ability of researchers to generate new knowledge as a function of investment and the current knowledge stock, conditional on a research productivity parameter A. We assume that investment has a non-negative impact on knowledge creation $\eta_I \geq 0$. Depending on the relevant research characteristics, a wealth of prior knowledge may contribute to the growth of knowledge as researchers "stand on the shoulders of giants", alternatively if there appears to be some finite quantity of valuable knowledge, the research can experience diminishing returns with respect to the stock, or there could be no impact on future innovation at all $\eta_K \gtreqless 0$. Finally, we treat research productivity as an exogenous scaling parameter $A > 0$.

Combining the model components above, the IAS manager's dynamic optimization problem is:

$$\min_{x_s, I_s} \int_t^T [e^{-rs}[C(x_s, K_s) + D(n_s) + R(I_s)]]ds \tag{4}$$

$$s.t. \quad \dot{n} = g(n_s) - h(n_s, x_s) \tag{5}$$

$$\dot{K} = \eta(I_s, K_s; A) \tag{6}$$

$$n(t) = n_0 > 0, given \tag{7}$$

$$K(t) = K_0 > 0, given \tag{8}$$

At $s = t$ there is a positive IAS population stock since we are considering an established invasive species and ecosystem closed to additional introduction. Noting that minimization is equivalent to maximizing the negative of costs, the problem's current-value Hamiltonian is:

$$H = -[C(x_s, K_s) + D(n_s) + R(I_s)] - \lambda_s[g(n_s) - h(n_s, x_s)] + \mu_s \eta(I_s, K_s; A) \tag{9}$$

Invasive species impose harm upon society and the environment and are recognized as a social "bad", implying that their shadow value be negative. Consequently, $\lambda_s > 0$ is the marginal social benefit of reducing the IAS stock. $\mu_s \geq 0$ is the marginal social benefit of the stock of knowledge. The Pontryagin (necessary) conditions for optimality, assuming an interior solution, are given as:

$$\frac{\partial H}{\partial x_s} = -C_x + \lambda_s h_x = 0 \tag{10}$$

$$\frac{\partial H}{\partial I_s} = R_I - \mu_s \eta_I = 0 \tag{11}$$

$$\frac{\partial H}{\partial n_s} = -D_n - \lambda_s[g_n - h_n] = \dot{\lambda} - r\lambda_s \tag{12}$$

$$\frac{\partial H}{\partial K_s} = -C_K + \mu_s \eta_K = r\mu_s - \dot{\mu} \tag{13}$$

$$\dot{n} = g(n_s) - h(n_s, x_s)$$

$$\dot{K} = \eta(I_s, K_s; A)$$

Solving the model also requires transversality conditions that describe the state and co-state values at time T, but these depend on management goals and are left for later discussion.

The Pontryagin conditions provide the necessary requirements for an optimal solution, but do not ensure that the model will generate a cost-minimizing solution. The second-order conditions for an optimum solution require that (9) be concave for all relevant variable combinations. We detail this process in Appendix A, with the conclusions that optimality is dependent on control and research cost functions being strictly convex, while the damage function must be non-concave. In addition, we find that if there are increasing returns to knowledge growth from research investment that the second order conditions cannot be fulfilled. The combination of convex investment costs and non-convex research growth is intuitive; if these conditions were not fulfilled, then there would be incentive for endless investment, which is unrealistic.

3.2. Optimal flow conditions

Using (10)–(13), we characterize the optimal solution paths analytically. These flow conditions describe the economic decision-making process of the IAS manager when choosing levels of population control and research investment.

3.2.1. IAS stock management

Condition (10) describes the manager's decision for population control, and can be rearranged to find the following:

$$\lambda_s^* = \frac{C_x(x_s^*, K_s^*)}{h_x(n_s^*, x_s^*)} \tag{14}$$

$\lambda_s^* > 0$ is the marginal social benefit of reducing the IAS stock in any period $s \in [t, T]$. (14) shows that the optimal level of population control corresponds to the per-unit marginal cost of IAS removal being equal to the marginal benefit of removal. It can be seen that marginal costs of population reduction are doubly affected by the population control choice. First, $C_x > 0$, so any additional control effort will increase costs, but increased effort further impacts costs via the stock effect of control efficacy. We have stated $h_x > 0$, but if the harvest function exhibits constant or diminishing returns, the marginal cost per-unit of IAS removal is increasing because the convexity of the control cost function.

Examining the marginal benefit of reduction more closely allows us to draw more insight from (14). We solve for λ_s from the Pontryagin conditions to show how different model components impact the marginal benefit of harvest directly. Co-state Eq. (12) can be arranged as the following first-order ordinary linear differential equation:

$$\dot{\lambda} + \lambda_s[g_n - h_n - r] = -D_n \tag{15}$$

First order ordinary differential equations can be solved analytically using an integrating factor (Simon and Blume, 1994), which is shown in detail in Appendix B.

$$\lambda_s = \int_s^T [e^{\int_s^u [g_n - h_n] d\tau} e^{-r(u-s)} D_n] du + \lambda_T e^{\int_s^T [g_n - h_n] d\tau} e^{-r(T-s)} \tag{16}$$

(16) shows that in any period, s, the marginal benefit of reducing IAS is equivalent to the discounted sum of marginal damages over time, accounting for changes in the growth rate of the population, and the marginal social value of IAS in the final management period. Considering both (14) and (16), we see that control efforts should be pursued to the point where the marginal cost per individual captured is equivalent to the present value of future IAS damages and the terminal marginal value of the IAS stock.

The first term in (16) summarizes the discounted benefit of population control via its impacts on marginal damage from the IAS stock. There are two distinct effects being shown, one reflecting the impact of current population on future marginal growth and the other showing the role of discounting over time. In (16), we see that the marginal social value of control is a function of the pace of species growth relative to harvest rate at that period in time, s. This is represented by the factor $e^{\int_s^u [g_n - h_n] d\tau}$ pre-multiplying the marginal damages from the population. When the population is growing quickly, represented by marginal growth outpacing marginal removal ($g_n - h_n > 0$), then the marginal social value of control is greater as it mitigates accelerating marginal damages. The opposite is true in periods when the rate of removal outpaces the species growth.

The first term in (16) summarizes the discounted benefit of population control via its impacts on marginal damage from the IAS stock. There are two distinct effects being shown, one reflecting the impact of current population on future marginal growth and the other showing the role of discounting over time. $e^{\int_s^u [g_n - h_n] d\tau}$ shows that because population control at time s impacts IAS stock, it will also have an impact on the marginal rate of IAS growth in future periods. We see that if population control in s leads to IAS populations growing quickly in some future period u ($g_n > h_n$), this leads to greater marginal damages in the future. On the other hand, if population control causes the rate of growth to diminish in u, ($g_n < h_n$), then marginal damages will be smaller in the future. The marginal growth rate for any given stock level, n_s, will vary based on the functional forms of $g(n_s)$ and $h(n_s, x_s)$.

The presence of non-constant marginal IAS growth illustrates the additional complexity in the manager's population control choice. By lowering IAS stock they will reduce the damages from the species, but depending on how this affects the growth of the species such action may lead to even higher damage in the future. The effect of time on the marginal value of control is determined by the discounting factor $e^{-r(u-s)}$, which is the discount rate r multiplied by the passage of time $(u - s)$.

The presence of λ_T in (16) provides insight into how different management goals impact the optimal behavior of the manager and is dependent on the transversality conditions that correspond to management goals associated with $n(T)$. If $n(T)$ is chosen optimally, then management will be suspended when $\lambda_T = 0$. If, instead, the manager has a specific population target such as, but not limited to, eradication then λ_T is solved endogenously within the model when the current-value Hamiltonian (9) is equal to zero in the terminal period:

$$\lambda_T = \frac{\mu_T \eta(I_T, K_T) - [C(x_T, K_T) + D(n_T) + R(I_T)]}{g(n_T) - h(n_T, x_T)} \tag{17}$$

3.2.2. Research investment

The manager's optimal investment decision is characterized in a similar way as control effort. (11) describes the optimal investment behavior, and shows that optimal R&D spending equates the marginal social benefit of knowledge creation with its marginal cost:

$$\mu^* = \frac{R_I(I^*)}{\eta_I(I^*, K^*; A)} \tag{18}$$

Rearranging (18) as an ordinary first-order differential equation then integrating allows us to describe the marginal benefit of knowledge accumulation (details found in the mathematical Appendix):

$$\mu_s = -\int_s^T [e^{\int_s^u [\eta_K] d\tau} e^{-r(u-s)} C_{K_u}] du + \mu_T e^{\int_s^T [\eta_K] d\tau} e^{-r(T-s)} \tag{19}$$

Keeping in mind our assumption that knowledge corresponds to lower control costs ($C_K < 0$), (19) shows that the marginal social value of knowledge for IAS management is equal to the discounted sum of its future cost-savings, adjusted for the impact of investment on marginal knowledge accumulation, plus a discounted terminal marginal value. Together, (18) and (19) show that optimizing research investment rests on equating the present value of future cost-savings, adjusted for the management goal, with the marginal costs of knowledge production.

The first term in (19) shows that marginal benefits of research investment have a direct impact in terms of reducing control costs, but also have an indirect impact since investment may affect future knowledge production. Depending on the KPF, it is possible that a wealth of knowledge contributes to more rapid technological development $\eta_K > 0$, which would make the marginal benefits of investment even greater. On the other hand, if this growth effect diminished, or if there is some limit to the ability for research to lower costs, it might restrain the potential benefits of R&D and discourage investment.

The relationship between stock-dependent knowledge growth η_K and the discount rate r, plays an important role in determining the magnitude of the marginal social benefit of R&D spending. In the special case that discounting is exactly offset by the growth of knowledge, the marginal benefit is simplified to the sum of all future cost-savings (plus the terminal value determined by the transversality condition). When this equality does not hold, it drives a wedge between the marginal benefit of R&D spending and future costs savings. The comparison of η_K and r amounts to whether the potential returns of building up a stock of knowledge outweigh the effect of discounting those future benefits. When $\eta_K < r$ the potential savings overstate the marginal benefit since they are not experienced immediately. Alternatively, if $\eta_K > r$ R&D is even more valuable since it builds a stock of knowledge that yields high returns in the future that are increasing even in the presence of discounting. We return to this relationship when examining the steady state condition for knowledge below.

3.3. Optimal steady state

The previous section described the behavioral rules that determined the manager's population control and research choices at any point in the planning horizon. We turn our attention to identifying the optimal steady state solutions of the model.

The solution of a dynamic bioeconomic model is characterized by a series of differential equations that dictate how the state and co-state values change over time. The equations of motion for the stock variables (IAS population and knowledge) are given in the problem set up, (2) and (3), while the co-state equations can be found from (12) and (13). As a final step, we apply the optimality conditions (14) and (18) to the co-state equations to find the dynamic system in state and co-state space:

$$\dot{n} = g(n_s) - h(n_s, x_s) \tag{20}$$

$$\dot{\lambda} = (\frac{C_x}{h_x})[r - (g_n - h_n)] - D_n \tag{21}$$

$$\dot{K} = \eta(I_s, K_s) \tag{22}$$

$$\dot{\mu} = (\frac{R_I}{\eta_I})[r - \eta_K] + C_K \tag{23}$$

When this system of equations equals zero, there is no incentive to increase or decrease the stock of IAS or knowledge and the system is at rest. It is clear that IAS populations are constant ($\dot{n} = 0$) whenever natural growth is exactly equal to the rate of IAS removal, while the steady state condition for knowledge production ($\dot{K} = 0$) will depend on the form of the KPF. Analysis of the steady state conditions for (21) and (22) can provide valuable insight into the determinants of the steady state levels for IAS and knowledge.

$$g_n = r + h_n - \frac{h_x D_n}{C_x} \tag{24}$$

The steady state condition in (24) mirrors a familiar outcome in the management of biological resources such as fisheries (Anderson and Seijo, 2011), with the notable exception that the biological stock in this case is harmful rather than beneficial. g_n shows the marginal impact that the IAS stock has on population growth. A higher discount rate corresponds to a higher marginal growth rate, implying a lower steady state IAS stock. The interpretation of this result in a fisheries context is that when future benefits of the resource are discounted, then managers harvest more in the present leading to a smaller steady state population. In the present case where the biological stock is harmful, the discounted benefits are the damages avoided in the future as seen in (16). A larger discount rate reduces the discounted value of future harm, lowering the marginal social value of removal and leading to larger steady state populations. h_n reflects the marginal effect that the IAS population has on the productivity of control efforts. Sensibly, when IAS control is more effective (larger h_n) there is a lower steady state population. The final term represents the marginal damages avoided per dollar spent on population control. The fraction is positive, and thus its negative will lower the marginal growth rate, implying higher steady state stock. Again, this term is common to extraction problems concerning biological stocks and effectively this shows that as marginal costs of effort increase it puts upward pressure on the steady state IAS population. However, this term is particularly important to the present study as the marginal cost is a function of the endogenously determined knowledge stock ($C_x(x_s, K_s)$). Recall that $C_{xK} < 0$, implying that a larger steady state value of knowledge actually increases the optimal IAS population. At first blush this may seem counterintuitive, but the same result was found in a study examining the impact of endogenous technological change on CO_2 abatement (Goulder and Mathai, 2000). In what was termed the "shadow cost effect", we see that the ability of R&D to reduce the marginal cost of control makes the damage from IAS less worrisome and allows for a larger stock of IAS at the steady state.

Similarly, at a steady state (23) yields the following condition:

$$\eta_K - \frac{C_K \eta_I}{R_I} = r \tag{25}$$

The steady state condition presented in (25) shows that a steady state knowledge stock is achieved when the economic gains of investment are equal to the social discount rate. The terms on the left-hand-side of (25) represent the marginal economic benefits of R&D spending. The first term is simply the marginal productivity of the knowledge stock in producing new information, while the second term represents the marginal impact of research investment on control costs (recall that $C_K < 0$, by assumption). We see that at an optimal interior steady state, investment is suspended when the net marginal economic yield is equal to the social discount rate. This is an intuitive result as knowledge stock exhibits the positive characteristics of a conventional form of capital, unlike the invasive species stock that imposes social harm.

4. Numerical model

In this section we specify functional forms and parameter values, then solve our model numerically to illustrate the economic significance of R&D in IAS control. The optimal control model laid out in Section 3 is presented in continuous time, but does not lend itself to producing analytical solutions. We opt to approximate the continuous time solution by discretizing the problem using the first order necessary conditions as a set of ordinary differential equations (ODEs). This system of ODEs can be solved numerically using established methods programmed in *Matlab*.

We begin with a specific examination of brown tree snake management on Guam to measure the impact of technological advancement for a given species, followed by a sensitivity analysis using three species categories to assess the impact of research under varying biological and economic conditions. Invasive species, by definition, impose harm upon society and management often seeks to eliminate them from non-native ranges. However, the role of stock-dependence on control costs can make this outcome particularly challenging to model. Rather, we present a management goal with a positive but ambitious population target. The prospect of eradication is returned to in the discussion.

4.1. Brown tree snake management

The brown tree snake has been a persistent nuisance on Guam since they were introduced to the island following World War II by returning military vessels (Rodda et al., 1992). Since that time they have become a prime example of a catastrophic invasive species; causing the extirpation or extinction of 11 of the 13 native bird species on the island (Savidge, 1987) while also representing a significant economic threat to the island and its trade partners (Fritts, 2002; Shwiff et al., 2010). The Brown Tree Snake Eradication Act of 2004 formally targeted the elimination of the species on Guam (US Congress, 2004), an issue made more pressing by an ongoing U.S. military buildup on the island. To support this goal, several branches of the U.S. government are currently engaged in BTS research, namely USDA - Wildlife Services and the US Geological Survey. The specific eradication target, the island's geographic isolation, and the research emphasis create a precise management scenario that is aligned with the bioeconomic model we have constructed.

4.2. Functional forms and parameters

We briefly present functional forms for each model component, describe model parameters, and relevant parameter restrictions. Following exposition of the functional forms and generic parameters, we provide a summary of the parameter values and sources.

Total costs reflect the sum of population control costs, BTS damage, and research investment costs. The population control cost function is influenced by both the effort devoted to reducing BTS population (in hours) and the knowledge stock.

$$C(x_s, K_s) = \bar{w}\frac{x_s^{\delta_x}}{K_s^{\delta_K}}$$

In this function $\bar{w}$ is the baseline hourly cost of population control effort (captures all labor and capital costs), which is pre-multiplied by the ratio of control effort to knowledge stock. δ_x and δ_K represent the elasticity of control costs to control effort and knowledge stock, respectively. To be consistent with the assumptions made in the analytical section the elasticity parameters must satisfy certain conditions. To have convex costs with respect to control effort, we must specify $\delta_x > 1$. To satisfy the assumption that both marginal and total population control costs are decreasing in the stock of knowledge, we must have $\delta_K > 0$.

We employ a simple IAS damage function that is both flexible and common in the literature.

$$D(n_s) = dn_s^{\delta_n}$$

Damages are determined by a damage coefficient d, and an elasticity δ_n that determines the degree of non-linearity in damage as a function of the species stock. The only restriction on the characteristics of the damage function is $\delta_n \geq 1$, allowing for a high degree of flexibility in the damage function while remaining consistent with the demands of the second order conditions.

The research investment function employs a likewise simple and flexible form.

$$R(I_s) = \rho I_s^{\delta_I}$$

The cost of investment depends on a generic research cost coefficient ρ and the investment elasticity δ_I. Investment costs are assumed to increase in the level of research effort, and we do not expect the marginal costs to diminish, implying that $\delta_I \geq 1$.

We assume that biological IAS growth follows a logistic pattern with Allee effects (Sun, 2016) and the harvest function is represented by a Gordon-Schaefer production function (Gordon, 1953; Schaefer, 1957).

$$g(n_s) = \phi(n_s - n_{min})(1 - \frac{n_s - n_{min}}{M})$$

$$h(n_s, x_s) = \alpha(n_s - n_{min})x_s$$

In the biological growth function, ϕ is the instantaneous rate of growth in the IAS stock, while M reflects the natural carrying capacity of the species. Allee effects describe the notion that a population's growth may be dependent on a minimum stock. For instance, it may not be possible for growth to occur when there is only one animal and no available mate (Savidge et al., 2007). However, Burnett et al. (2012) point out that the BTS exhibit extremely strong allee effects, even $n_{min} = 2$ and as such a target of n_{min} would effectively be eradication. Within the harvest function α is the familiar Gordon-Schaefer catchability coefficient that describes the proportion of the species captured by one unit of effort. Due to the decision to make the control costs a function of knowledge, we do not treat catchability as being impacted directly by knowledge stock in order to minimize the risk of double-counting the effect of R&D in the model. Both of these functions are common within bioeconomic models and are used to capture the dynamics of the BTS population.

The KPF builds on work from Goulder and Mathai (2000), but is a common function in growth literature.

$$\eta(K_s, I_s) = \beta K_s + A I_s^{\theta} K_s^{\gamma}$$

Respectively, β and A represent the presence of autonomous knowledge growth and the ability of investment and knowledge to spur new technological development. β is left unrestricted to allow for knowledge to naturally grow, decay, or remain constant. $A \geq 0$ reflects an assumption that R&D investment will not reduce the stock of knowledge, though it does not guarantee an increase in the stock either. The parameters θ and γ describe the returns to research investment, and existing knowledge, respectively. The only restriction on these parameters is that $\theta \in (0, 1)$ to remain consistent with the demands of the second order conditions. As discussed in the construction of the analytical model, returns to knowledge stock are left flexible to explore how their values affect IAS management allow for nuances of stock-dependent knowledge growth. The KPF plays a prominent role in the analysis, and ideally we could rely on empirical foundations for these parameters, but as yet these do not exist for research on IAS species. Specification of an empirically grounded KPF is an important avenue for future research.

4.3. Solution overview

Using these functional forms, the necessary first order conditions of the dynamic optimization problem can be expressed as a system of ordinary differential equations presented in (27)–(30).

$$\dot{n} = \phi(n_s - n_{min})(1 - \frac{n_s - n_{min}}{M}) - \alpha n_s(\frac{\alpha}{\bar{w}\delta_x}\lambda_s n_s K_s^{\delta_K})^{(\frac{1}{\delta_x - 1})} \quad (26)$$

$$\dot{\lambda} = \lambda_s[r + \alpha(\frac{\alpha}{\bar{w}\delta_x}\lambda_s n_s K_s^{\delta_K})^{(\frac{1}{\delta_x - 1})} - \phi(1 - \frac{2(n_s - n_{min})}{M})] - d\delta_n n_s^{(\delta_n - 1)} \quad (27)$$

$$\dot{K} = \beta K_s + A(\frac{A\theta}{\delta_I \rho}\mu_s K_s^{\gamma})^{\frac{\theta}{(\delta_I - \theta)}} K_s^{\gamma} \quad (28)$$

$$\dot{\mu} = \mu_s[r - A\gamma(\frac{A\theta}{\delta_I \rho}\mu_s K_s^{\gamma})^{\frac{\theta}{(\delta_I - \theta)}} K_s^{\gamma - 1}] - \frac{\bar{w}\delta_K}{K_s^{\delta_K + 1}}(\frac{\alpha}{\bar{w}\delta_x}\lambda_s n_s K_s^{\delta_K})^{\frac{\delta_x}{\delta_x - 1}} \quad (29)$$

The solution of the dynamic system, which requires an appropriate transversality condition, yields solution paths for IAS population, knowledge stock, and the co-state variables. Transversality conditions determine the value of state, co-state, and time variables within the model and vary with the specific circumstances of management. In this analysis, we focus on a terminal population that represents 1% of total carrying capacity of the IAS population. This population target must be achieved by an exogenous time horizon T, which is identified by the planning authority. The corresponding transversality condition is the marginal social value of population reduction in the final planning period, $\lambda(T)$, is endogenously determined within the model. The optimal stock of knowledge, $K(T)$, may not be known prior to management, and is solved endogenously as part of the optimization problem. The transversality condition is thus $\mu(T) = 0$, implying that investment is suspended at the point where there is no longer any marginal social benefit to devote resources to R&D.

4.4. Brown tree snake management

Parameters for the model were selected from literature studying the biological characteristics and economic impacts of BTS, as well as studies on the effects of R&D on the provision of environmental public goods. Where possible, the parameters are also informed using empirical data and personal communication on BTS management received from USDA-Animal and Plant Health Inspection Services (APHIS). While a significant amount of research has focused on learning more about the BTS species and its impact as an invasive, many of these values are still uncertain and we must rely on point estimates. In selecting values for the analysis, we conducted thorough review of

Table 1
Parameter values.

Parameter	Value	Description
A	Range 0–0.02	KPF productivity (discussed below)
α	0.0049	Catchability coefficient (Schaefer, 1957, Calculated with BTS data)
β	0	Ruling out possibility of intrinsic knowledge growth
d	122.31	BTS damage per snake in $US (Burnett et al., 2008)
γ	0.5	Returns to knowledge stock in KPF (Goulder and Mathai, 2000)
M	2.6	BTS carrying capacity in millions of snakes (Rodda et al., 1999)
n_{min}	2	Minimum feasible population for BTS growth (Burnett et al., 2012)
ϕ	0.6	Annual intrinsic growth of BTS population (Burnett et al., 2008)
r	0.05	Discount rate (Epanchin-Niell and Liebhold, 2015)
ρ	117	Per-hour cost of investment in $US (discussed below)
θ	0.5	Returns to R&D investment (Goulder and Mathai, 2000)
$\bar{w}$	117	Baseline per-hour cost of control in $US (Unpublished USDA data)
δ_x	2	Cost elasticity of Pop. Control (Eiswerth and Johnson, 2002)
δ_I	2	Cost elasticity of investment
δ_n	1	Cost elasticity of IAS damage
δ_K	1	Cost elasticity of knowledge

the literature and worked directly with BTS managers to identify the most accurate values with the information available.

Each control variable is measured in hours of effort, all monetary values are expressed in $US. The time horizon used is informed by the scheduled military build up on Guam, which the Brown Tree Snake Eradication Act of 2004 was motivated by, at least in part. This buildup is ongoing, and will likely continue into the early 2020s. Using the legislation from congress as an approximate beginning point for management and the anticipated end of the buildup as the deadline, we solve the model over a 20-year time horizon with a time unit of 1-year.

As our analysis focuses on the effect of research on management, the KPF productivity given by A is the primary parameter of interest. We choose to employ a range of research productivities in order to comment on how the model responds to changes in technological development. The lower bound of zero describes research as completely ineffective, while the upper bound was selected based on productivity estimates in Goulder and Mathai (2000), as well as numerical tractability.

The catchability coefficient α is calculated using available data on BTS removal efforts from operations on the island recorded in the USDA Management Information System (MIS). This data included records of removal effort and snakes collected but the nuances of BTS management make the process difficult to measure with a high degree of accuracy. Specifically, there is a paucity of information on actual "removal" of species because the majority of efforts are in the application of acetaminophen baits which are highly toxic for the snakes but the snakes remain mobile after bait consumption (Savarie et al., 2001). Further, bait take rates have proven high and non-target impacts low suggesting effective BTS removal, despite absence of postmortem evidence (Clark and Savarie, 2012; Engeman et al., 2018). The result is that there is a record of bait distribution but not a clear link to number of snakes removed. The current status of BTS management provides some convenient context, however, as their focus on aerial dispersal yields some new findings that make our choice of α plausible although on the higher end of values.

The biological growth rate, ϕ, and carrying capacity, M, and minimum population, n_{min}, were taken from the existing literature on BTS growth and management.

The parameters selected for β, γ, and θ produce a constant-returns Cobb–Douglas knowledge production function, which follows Goulder and Mathai (2000), Jones (2005), and Abis and Veldkamp (2021). The structure of the KPF is further supported by the second order conditions, which show that if knowledge grows too rapidly relative to costs the solution may be unstable.

The damage coefficient d was taken from Burnett et al. (2008) and represents productivity losses, human health impacts, and loss of endangered species on the island. The control cost coefficient $\bar{w}$ is inclusive of all fixed and variable costs of BTS management on the island and was informed by data on BTS management provided by APHIS offices operating on Guam. As discussed above, research on BTS has been ongoing since the snake was targeted for eradication, however detailed research costs were more difficult to ascertain than costs for removal of the species. Consequently, we treat the research coefficient, ρ as equal to the control cost coefficient. This approach is taken for numerical simplicity, but highlights a demand for better research cost information. Given the uncertainties around discount rates with respect to natural rates of time preference, we chose to follow conventional choices from associated literature. The discount rate $r = 0.05$ was chosen based on observation of its use in dynamic optimization models of invasive species management (Sanchirico and Springborn, 2011; Epanchin-Niell and Wilen, 2012; Epanchin-Niell and Liebhold, 2015)). However, given the resources at stake this may be on the high end of the range of appropriate discount rates and could possibly lead to an understatement of the marginal social value of control contributing to the conservative nature of our estimates. In the event that this rate is higher than true time preference, the incentive to avoid future damages is weakened and there may be less control than would be seen otherwise. However, it is also possible that true discount rates could be higher than 5%, in which case managers might engage in more control.

Finally, the cost elasticities represent an important set of information in the model. Empirical estimates of these parameters are unavailable, so they were instead informed by the second order sufficiency conditions for optimality. We find that in order to satisfy the sufficiency conditions, it is imperative that control costs are strictly convex however damages and can be linear. The cost elasticity of knowledge stock being equal to one was found to fit within the second order conditions, which is consistent with Goulder and Mathai (2000).

4.5. R&D and IAS management

Using the BTS parameterization, we examine the effect of R&D on management by varying levels of research productivity. Our analysis focuses on three different measures, first and foremost being the ability of R&D to reduce the overall cost of a given management plan. We then investigate the specific investment behaviors, and then explore whether the benefit of research investment is worth the cost in a pseudo return on investment calculation.

The results in Table 2 show that the present value, discounted at 5% annually, of reaching the target population falls with greater levels of research productivity. While the present value of these management costs is strictly decreasing, the reductions at each level of productivity become smaller, suggesting diminishing returns to research productivity. We will return to this notion throughout the following discussion.

Fig. 1 shows the patterns of R&D spending that characterize the eradication solution. For all cases, investment is shifted toward the

Table 2
Management costs by research productivity.

Research productivity A	Present value of total management costs ($US Million)
0	29.4997
0.005	16.3842
0.01	13.7952
0.015	12.6345
0.02	11.8620

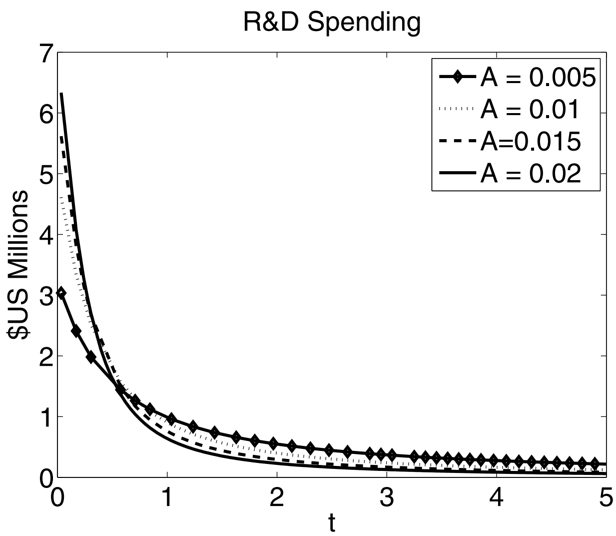


Fig. 1. Research investment for different levels of A.

earliest management periods, then falls over time. Such a result is intuitive, as it allows for lower control costs over a greater proportion of the management horizon. These investment paths are almost identical by year 5, with the exception of $A = 0$ where there is never any investment. For this reason, and to highlight the differences in initial research behavior, the figure demonstrates solutions for $s \in [1 : 5]$.

We see that when research is more productive, it prompts managers to invest more at the onset of management, but with each increase in A the shift in initial investment decreases. Further, as research productivity improves, the cost-minimizing investment paths do not shift in a parallel manner, but rather investment decreases over time at faster rate. These outcomes echo the result seen in Table 2, which implied that research productivity exhibits diminishing returns in terms of cost-savings.

A corresponding effect of this research activity is that it impacts the timing of population reduction, which is delayed to allow for lower control costs from R&D. The dynamics reflect the tradeoffs between managing damages, research costs, and control costs. The initial control effort is to drive down the population to a level where damages have fallen but the species is not at its terminal level. Focus then transitions to moderate population control while investing in economically valuable knowledge that lowers costs so that by the end of the period it is possible to engage in more control at a lower cost and meet the terminal population target. Fig. 2(a) demonstrates this effect, however we present the results of only three levels of research productivity for clarity of exposition. The general pattern is consistent across all models. In Fig. 2(b), it can be seen that despite the delay in activity, the effect is a more rapid reduction in BTS populations.

Table 2 showed that research investment lowers total management costs, albeit at a decreasing rate. To take this a step further, we calculate the cost-effectiveness ratio (CER) for research spending at each level of research productivity. By comparing the cost-savings of R&D relative to the baseline without any research investment, we can demonstrate these diminishing effects quickly and concisely as seen in Fig. 3

$$CER = \frac{TC_{BL} - TC_{R\&D|A}}{R(I)|A} \tag{30}$$

We see that at each level of research productivity, there are economically significant cost-savings relative to the expense of the research investment surpassing ratios of 5:1 in all cases. However, we see that the relative savings of research does not increase monotonically with the productivity of research. Table 2 and Fig. 1 present an intuitive explanation for why we see this pattern. Research productivity is able to lower the present value of management costs, but at a decreasing rate, implying that the numerator in (30) becomes smaller as A increases. However, when research is more effective it also prompts more research spending, which increases the size of the denominator.

The results above show two key points regarding the role of R&D in invasive species management. First, it is clear that, in the case of Brown Tree Snakes on Guam, there are real benefits to investing in control efforts in order to reduce the cost of population reduction. Second, the benefits of R&D productivity do not increase monotonically because total R&D costs increase while total IAS management costs fall with productivity.

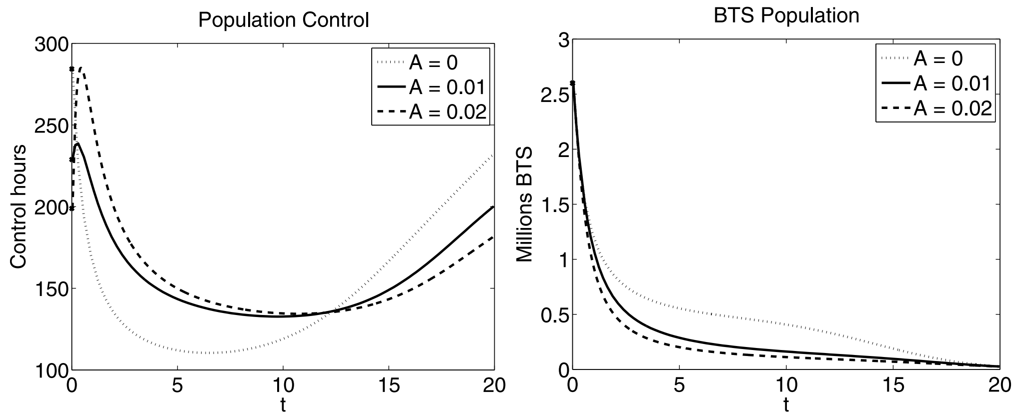
4.6. Sensitivity analysis: Varying w and d

The sensitivity analysis examines optimal species management under varying parameter values for control cost, w, and IAS damage, d. These parameters are of particular importance to effective IAS management, as they describe the central components of total cost. Comparative dynamic analysis based on these parameters offers additional insight to how management behavior depends on biological and economic characteristics in addition to providing an opportunity to validate the numerical model against the analytical foundations of the study.

Using "high" and "low" estimates of control cost and damage from the IAS literature (details in Section 4.7) to present four alternative scenarios of control and damage costs, while keeping all other parameters from Table 2 constant. The four scenarios characterize a range of IAS scenarios that might be considered low-impact (low damage, low control cost) to high-impact (high damage, high control cost) and their intermediates. Fig. 4 presents the optimized IAS population paths and knowledge accumulation for each case.

In Fig. 4(a), the x-axis has been restricted to showing the first ten management periods only to highlight the details of the solution paths in the early periods. There is practically no difference in the managed stock after $t = 10$. We can see that timepaths for population vary slightly across the four scenarios, and the effects can be broadly summarized into two key findings. First, high damage species are targeted more aggressively than low-damage ones. This effect is seen where population stock of both the high-damage species is driven down more rapidly than their low-damage counterparts. Second, there is a small effect of lower control costs leading to faster population reduction. These results correspond directly to the analytical foundations of the model, notably that there is greater incentive to reduce populations when the discounted sum of future damages is greater. This effect appears to be stronger with respect to damage than control cost.

While the optimal approach to population management exhibits moderate differences across parameterizations, 4(b) panel (b) in Fig. 4 shows substantial variation in knowledge accumulation. Again, the effects naturally coalesce into two findings. We see that optimizing managers respond strongly to high control costs by committing to immediate and significant knowledge accumulation, regardless of species damage. IAS damages appear to be responsible for the overall magnitude of knowledge accumulation with greater damages leading to larger values of K.



(a) Optimal Population Control Path   (b) Optimal Population Path

Fig. 2. Control and state variable solution paths.

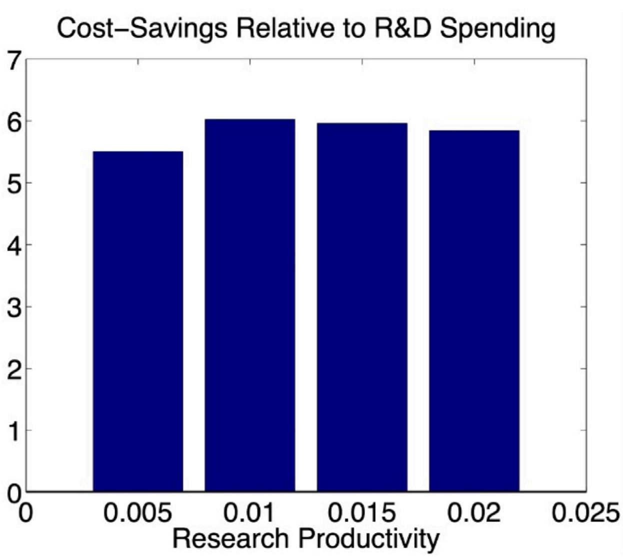


Fig. 3. Cost-effectiveness ratios.

4.7. Effect of research for varied species type

Results from the preceding analysis demonstrate the role of research investment in the eradication of the brown tree snake. To develop insight about the broader benefits of research in IAS management, we apply the model to three categories of invasive species determined by biological and economic characteristics. Our motivation for this exercise is to demonstrate that the model is not intended for the individual case but may be adopted in a more general assessment of resource management when targeting established invasive populations. By working with managers for a particular species control project or reviewing literature on the bioeconomic characteristics for some species of interest, this model can offer some preliminary insight on how to organize scarce funds and project paths for control and research activities. Additionally, by looking at a range of different impact categories, we can develop some broad intuition on what types of species management benefit from aggressive research plans as opposed to those where the research can be conducted more gradually.

The categories are framed in terms of the overall impact that a species may impose on the invaded ecosystem and economy. Based on the comprehensive review of invasive species provided by Pimentel et al. (2005), we select representative species that correspond to low- (e.g. Wild horses (*Equus caballus*)), medium-(e.g. Mongoose (*Herpestus auropunctatus*)), and high-impact (e.g. Feral pig (*Sus scrofa*)) categories. The model parameters distinguishing these categories are the damage coefficient (d), baseline cost of population reduction ($\bar{w}$), intrinsic growth rate (ϕ), the carrying capacity (M), and catchability coefficient (α). Table 3 summarizes the parameterizations for each category, solved at a constant research productivity $A = 0.01$, and all other model parameters are the same as in Table 1.

In order to inform all relevant parameters in Table 3, we supplement the Pimentel et al. (2005) review with species specific studies (Garrott and Taylor, 1990; Bieber and Ruf, 2005; Harper and Bunbury, 2015; Fukasawa et al., 2013; Johnson et al., 2016). Despite these additional resources, there is very little information regarding control efficacy of these species. Consequently, we specify the α parameters based on the impact category.

Table 4 shows the present value of total management costs for each species.

The values in Table 4 show the wide range of impacts that can be imposed by invasive species. The management costs of high impact species, such as feral pigs, can be extraordinary, reaching the billions of dollars. Given that current estimates of annual damage are approximately $800 million, we believe that this is a reasonable measure of potential costs for an ambitious population reduction program like the one we have modeled. The cost of addressing moderately harmful species are in the same range as that seen for our analysis of the Brown Tree Snake, and the least harmful species have fairly low management costs.

Research investment varies widely across species and is shown in Fig. 5. For the high- and medium-impact species, we truncate the x-axis due to most investment behavior occurring in the earliest stages of management, much like in the BTS specification. For the low-impact species, however, we see much different research behavior across the entire management period. The main takeaway from the examination is that for species with characteristics like low risk factors there is not only less incentive for knowledge generation, there may also be less urgency in the optimal research behavior.

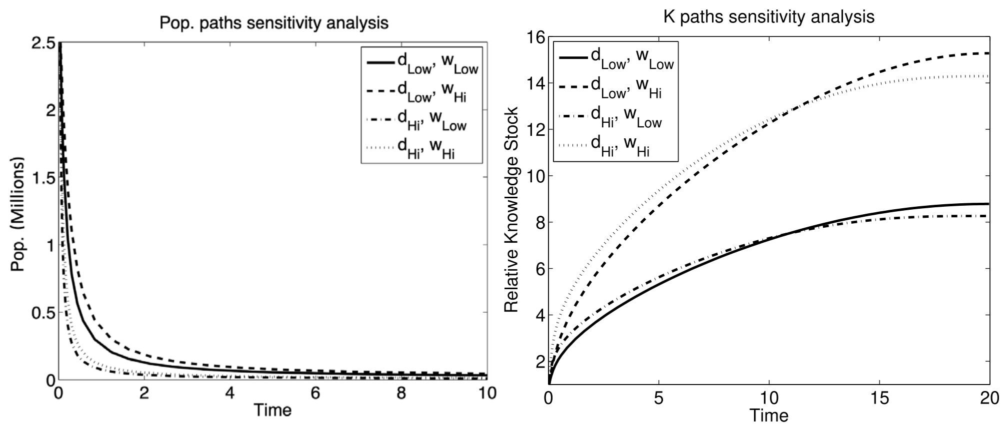
These results reinforce the notion that IAS management should not be approached as a one-size-fits-all endeavor. The nuances of each species may require significantly different distribution of resources across time as well as between population reduction and research into effective management.

5. Discussion and conclusion

This paper models IAS management decisions in the presence of R&D and shows that in addition to reducing total management costs, there are meaningful implications for the timing of population reduction versus research investment. Despite the growth in dynamic

Table 3
Parameter values for impact categories.

Category	d ($US per animal)	$\bar{w}$ ($US per hour)	ϕ	M (Million animals)	α
High impact	200	150	0.8	4	0.004
Medium impact	100	100	0.49	0.5	0.005
Low impact	20	50	0.18	0.05	0.007



(a) Pop Sensitivity Analysis

(b) K Sensitivity Analysis

Fig. 4. Comparative dynamics: w and d.

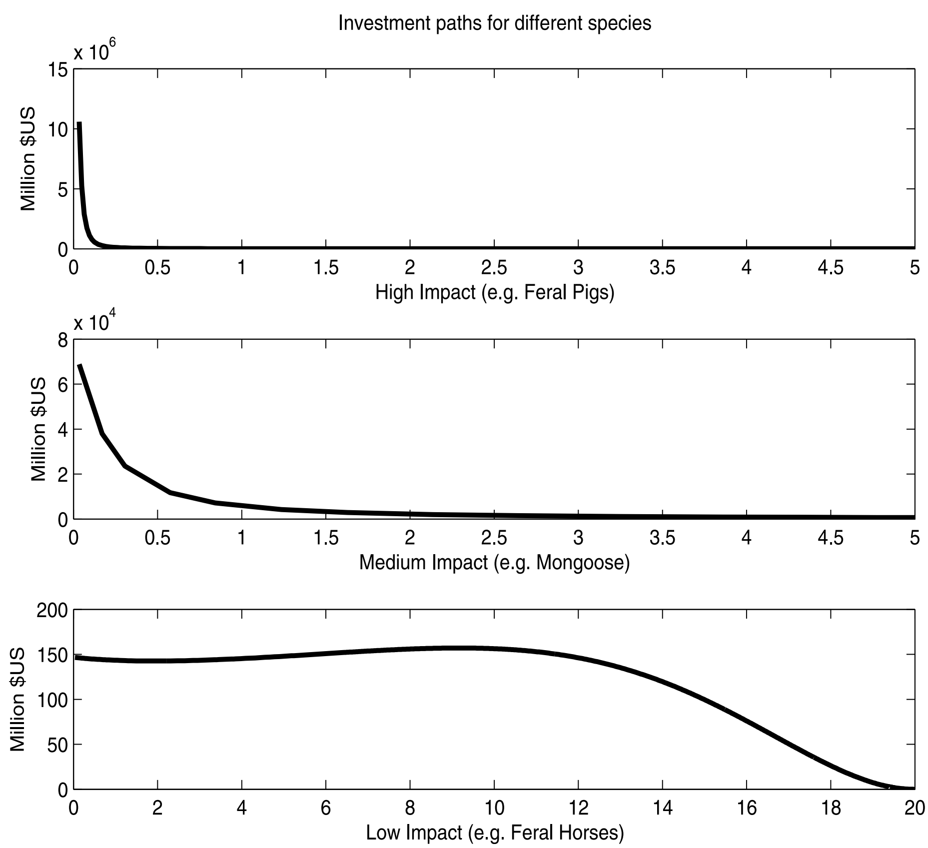


Fig. 5. Research investment for different species.

Table 4
Management costs for varied species.

Species	Present value of total management costs ($US Million)
Feral pig	4904
Mongoose	26.45
Feral horse	0.5867

optimization studies in IAS management strategies, there has been little to no examination of the role that research investment plays in managing an established population. We feel that the results of this study are a first step toward a more deliberate examination of how IAS managers navigate complex strategies with multiple decision variables.

By modeling a range of parameters for R&D productivity, we show that cost-savings are not dependent on high expectations for innovation. At every positive level of research productivity, the savings outweighed research spending by a factor of 5:1, at least. Diminishing returns in these net benefits may limit the ability of research to combat convex population control costs. When research is more productive, it prompts greater levels of investment, and a larger R&D expense, which is consistent with the theoretical characteristics established in the generalizable model (e.g (19)).

By modeling a range of parameters for R&D productivity, we show that cost-savings are attainable with modest expectations for innovation. Across all measures of R&D productivity in our selected range, benefits of research outweigh the expense by a factor of 5:1, at a minimum. There are, however, diminishing returns that represent limits to the ability of research to combat the convex population control costs that characterize IAS management programs. The diminishing returns are a consequence of the dual effect that R&D spending has on total costs: investing in population reduction technology has the potential to make reaching the target population cheaper, but it is accompanied by its own costs. Our numerical results are consistent with the theoretical characteristics established in the generalizable model (e.g (19)), which suggests that this pattern is not limited to the specific case study or parameterization central to our analysis.

We are able draw several conclusions about the effect of including R&D into the management scheme., First, Fig. 1 shows that when research is productive and pursued by managers, there is an inverse relationship with population reduction effort. After the initial research investments, the relationship reverses and population control is pursued at high rates without much additional research spending. This effect is magnified when research is more productive, with the peak levels of control effort being pushed higher but also delayed longer. One of the most notable outcomes is that, for a given target population, the control effort needed to reach this terminal value decreases with research productivity. This is especially interesting as it relates to the goal of eradication. The nature of stock-dependent harvest (and thus stock-dependent costs) demands higher removal effort at lower populations, but our findings suggest that research is able to lower the effort needed to achieve a given population target.

The model developed in this study represents a synthesis of several bodies of research within economics, applying models of technological change common in the climate change economics literature to invasive species issues. The key similarity between these two environmental issues is the management of a harmful environmental pollutant; the key difference is that in the case of IAS the stock exhibits biological growth as opposed to natural deterioration. The introduction of research-driven knowledge accumulation to the cost function is the primary contribution of this research. Knowledge reduces the costliness of population control, which can lead to higher levels of more cost effective control.

This research demonstrates the impacts that research investment and knowledge creation can have on IAS management. However, there are a number of important areas for future work. Results presented above suggest that research investment could possibly address some of the challenges associated with targeting eradication in the presence of stock-dependent costs. This is an important topic within the literature that demands additional exploration. Future research should look at the prospect of optimal eradication. In addition, the prominent role of knowledge creation in the model makes it important to improve the foundations of the KPF and its relevant parameters. Estimating an empirical KPF to inform the parameters for returns to investment and previous knowledge would build greatly on the current analysis. Finally, several simplifying assumptions were made to ensure model tractability. Future work could relax these assumptions by introducing more complex biological dynamics such as: allowing for secondary invasions that contribute to the established population, incorporating a density-dependent damage function (Yokomizo et al., 2009), and considering the role of knowledge spillovers from R&D.

We show that the ability for R&D to improve the efficacy of IAS control affects the timing and magnitude of control efforts. It also leads to economically significant reductions in the total cost of IAS management. Therefore, significant gains exist from the incorporation of R&D into IAS management plans and budgets.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgment

This research was funded by the United States Department of Agriculture Animal and Plant Health Inspection Service - National Wildlife Research Center.

Appendix A. Second order sufficiency conditions for optimality

The Pontryagin conditions provide the necessary requirements for an optimal solution, but do not ensure that the model will generate a cost-minimizing solution. The second-order sufficiency conditions (SOSC) for an optimum solution requires that the Hamiltonian functional be concave for all relevant variable combinations.

$$H = -[\bar{w}\frac{x_s^{\delta_x}}{K_s^{\delta_K}} + dn_s^{\delta_n} + \rho I_S^{\delta_I}]$$
$$+ \lambda_s[\phi(n_s - n_{min})(1 - \frac{n_s - n_{min}}{M}) - \alpha x_s(n_s - n_{min})] + \mu_s[\beta K_s + A I_s^{\theta} K_s^{\gamma}]$$

Determining the concavity of the above Hamiltonian requires an examination of its Hessian matrix (time indices have been suppressed to economize on space.): (see Box I).

If the Hessian is negative semi-definite, then we can state that the SOSC are met for a minimum[2] and examine the intuition of the conditions observed from that analysis.

Simon and Blume (1994) provide the necessary and sufficient conditions for negative semi-definite matrices. In the present context, the leading principal minors must alternate in sign with odd ones being ≤ 0.

$$|PM1| = -\frac{\delta_x(\delta_x - 1)\bar{w}x^{\delta_x - 2}}{K^{\delta_K}} < 0$$

The determinant of the first principal minor will always be negative when control costs are strictly convex.

The determinant of the second principal minor must be positive to fulfill the second order sufficiency conditions: (see Box II). The second

[2] Because we are modeling the cost-minimization problem by maximizing the negative of costs, we study the SOSC for identifying a maximum.

$$H = \begin{bmatrix} -\frac{\delta_x(\delta_x-1)\bar{w}x^{\delta_x-2}}{K^{\delta_K}} & 0 & -\alpha\lambda & \frac{\delta_x\delta_K\bar{w}x^{\delta_x-1}}{K^{\delta_K+1}} \\ 0 & \mu\theta(\theta-1)AI^{\theta-2}K^{\gamma} - \delta_I(\delta_I-1)\rho I^{\delta_I-2} & 0 & \mu\theta\gamma AI^{\theta-1}K^{\gamma-1} \\ -\alpha\lambda & 0 & -\frac{2\lambda\alpha\phi}{M} - \delta_n(\delta_n-1)Dn^{\delta_n-2} & 0 \\ \frac{\delta_x\delta_K\bar{w}x^{\delta_x-1}}{K^{\delta_K+1}} & \mu\theta\gamma AI^{\theta-1}K^{\gamma-1} & 0 & \mu\gamma(\gamma-1)AI^{\theta}K^{\gamma-2} - \frac{\delta_K(\delta_K+1)\bar{w}x^{\delta_x}}{K^{\delta_K+2}} \end{bmatrix}$$

Box I.

$$|PM2| = -\frac{\delta_x * w * x^{\delta_x} * (\delta_x - 1) * (I^{\delta_I} * \delta_I * \rho - I^{\delta_I} * \delta_I^2 * \rho + A * I^{\theta} * K^{\gamma} * \mu * \theta^2 - A * I^{\theta} * K^{\gamma} * \mu * \theta)}{(I^2 * K^{\delta_K} * x^2)} > 0$$

Box II.

principal minor will be positive whenever the following condition holds:

$$(I^{\delta_I} * \delta_I * \rho - I^{\delta_I} * \delta_I^2 * \rho + A * I^{\theta} * K^{\gamma} * \mu * \theta^2 - A * I^{\theta} * K^{\gamma} * \mu * \theta) < 0$$

This can be re-written as...

$$I^{\delta_I} * \delta_I * \rho * (1 - \delta_I) + A * I^{\theta} * K^{\gamma} * \mu * \theta * (\theta - 1) < 0$$

The condition depends on the rate of growth in research costs relative to knowledge growth from research. The principal minor can be positive if research costs are non-convex and knowledge is convex. Such a scenario would likely result in persistent spending and intense knowledge growth, but is unrealistic. It is much more likely that costs are strictly convex knowledge growth is diminishing or linear, at best.

The determinant of the third principal minor is given as:

$$|PM3| = -(I^{\delta_I}\delta_I\rho - I^{\delta_I}\delta_I^2\rho + AI^{\theta}K^{\gamma}\mu\theta^2 - AI^{\theta}K^{\gamma}\mu\theta) \frac{(2\delta_x\lambda n^2\phi\bar{w}x^{\delta_x} - 2\delta_x^2\lambda n^2\phi\bar{w}x^{\delta_x} + K^{\delta_K}M\alpha^2\lambda^2n^2x^2 - DM\delta_n^2\delta_x^2n^{\delta_n}wx^{\delta_x}}{(I^2K^{\delta_K}Mn^2x^2)} - \frac{DM\delta_n\delta_xn^{\delta_n}\bar{w}x^{\delta_x} + DM\delta_n\delta_x^2n_n^{\delta}\bar{w}x^{\delta_x} + DM\delta_n^2\delta_xn^{\delta_n}wx^{\delta_x})}{(I^2K^{\delta_K}Mn^2x^2)} < 0$$

The negative coefficient on the entire quotient, paired with the fact that the denominator must be positive means the sign of the principal minor depends on the product observed in the numerator. The first set of terms in parentheses is the same as that studied in determining the sign of the second principal minor, which must be negative to fit the SOSC. Thus, the third principal minor will be positive when the term in the numerator is negative:

Collecting like terms allows the condition to be written as :

$$2\delta_x\lambda n^2\phi\bar{w}x^{\delta_x}(1 - \delta_x) + K^{\delta_K}M\alpha^2\lambda^2n^2x^2 - DM\delta_n\delta_xn^{\delta_n}\bar{w}x^{\delta_x}(\delta_n\delta_x + \delta_x + \delta_n - 1) < 0$$

When control costs and damages are convex, the first term is negative and the third term is positive, which combined with the negative in front of the third term implies that the condition is met when the following condition is true:

$$2\delta_x\lambda n^2\phi\bar{w}x^{\delta_x}(1-\delta_x) - DM\delta_n\delta_xn^{\delta_n}\bar{w}x^{\delta_x}(\delta_n\delta_x+\delta_x+\delta_n-1) > K^{\delta_K}M\alpha^2\lambda^2n^2x^2$$

Dividing by $\frac{1}{x^2n^2}$ yields:

$$2\delta_x\lambda\phi\bar{w}x^{\delta_x-2}(1-\delta_x) - DM\delta_n\delta_xn^{\delta_n-2}\bar{w}x^{\delta_x-2}(\delta_n\delta_x+\delta_x+\delta_n-1) > K^{\delta_K}M\alpha^2\lambda^2$$

The determinant of the fourth principal minor has hundreds of terms and was solved using *Matlab's* symbolic toolbox. Determining $|PM4| > 0$ was approached by expressing the determinant as follows:

$$|PM4| = \frac{\frac{1}{K^{2\delta_K}}\psi}{(I^2K^2Mn^2x^2)}$$

ψ is an additively separable function of the model variables and parameters. The coefficient on ψ and the denominator will always be positive, so the sign of $|PM4|$ depends on the sign of ψ. This term is expanded below, with each additively separable term identified by a different line in the equation array below:

$$\begin{aligned}
\psi = \\
& 2I^{\delta_I}\delta_I\delta_K\delta_x\lambda n^2\phi\rho w^2x^{2\delta_x}(1 + \delta_K + \delta_I\delta_x - \delta_x - \delta_I - \delta_I\delta_K) \\
& + DI^{\delta_I}M\delta_I\delta_K\delta_n\delta_xn^{\delta_n}\rho w^2x^{(2\delta_x)}(\delta_x + \delta_n + \delta_I + \delta_K\delta_n + \delta_I\delta_K + \delta_I\delta_n\delta_x \\
& - \delta_K - \delta_n\delta_x - \delta_I\delta_x - \delta_I\delta_n - 1 - \delta_I\delta_K\delta_n) \\
& + I^{\delta_I}K^{\delta_K}M\alpha^2\delta_I\delta_K^2\lambda^2n^2\rho wx^{\delta_x}x^2(\delta_K - \delta_I) \\
& + 2AI^{\theta}K^{\gamma}\delta_K\delta_x^2\lambda\mu n^2\phi\theta^2w^2x^{2\delta_x}(\delta_K - \delta_x - \frac{1}{\theta}) \\
& + A^2I^{(2\theta)}K^{(2\delta_K)}K^{(2\gamma)}M\alpha^2\gamma\lambda^2\mu^2n^2\theta x^2(\theta + \gamma - 1) \\
& + I^{\delta_I}K^{\delta_K}M\alpha^2\delta_I\delta_K\lambda^2n^2\rho wx^{\delta_x}x^2(1 - \delta_I) \\
& + 2AI^{\theta}K^{\gamma}\delta_K\delta_x\lambda\mu n^2\phi\theta\bar{w}^2x^{(2\delta_x)}(\theta + \delta_x - \delta_K) \\
& - 2A^2I^{(2\theta)}K^{\delta_K}K^{(2\gamma)}\delta_x^2\gamma\lambda\mu^2n^2\phi\theta\bar{w}x^{\delta_x}(\theta + \gamma) \\
& + ADI^{\theta}K^{\gamma}M\delta_K\delta_n\delta_x\mu n^{\delta_n}\theta\bar{w}^2x^{(2\delta_x)} \\
& \times (1 - \delta_n\delta_x\theta + \delta_K\delta_n\theta - \theta - \delta_x - \delta_n + \delta_K) \\
& + 2A^2I^{(2\theta)}K^{\delta_K}K^{(2\gamma)}\delta_x\gamma\lambda\mu^2n^2\phi\theta\bar{w}x^{\delta_x}(\theta + \gamma + \delta_x - 1) \\
& + ADI^{\theta}K^{\gamma}M\delta_K\delta_n\delta_x\mu n^{\delta_n}\theta\bar{w}^2x^{(2\delta_x)}(\delta_x\theta + \delta_n\theta + \delta_n\delta_x - \delta_K\theta - \delta_K\delta_n) \\
& - A^2DI^{(2\theta)}K^{\delta_K}K^{(2\gamma)}M\delta_n\delta_x\gamma\mu^2n^{\delta_n}\theta\bar{w}x^{\delta_x} \\
& \times (\delta_n\delta_x\theta + \delta_n\delta_x\gamma + \theta + \gamma + \delta_x + \delta_n) \\
& + AI^{\theta}K^{\delta_K}K^{\gamma}M\alpha^2\delta_K\lambda^2\mu n^2\theta\bar{w}x^{\delta_x}x^2(\delta_K + \theta - 1) \\
& + AI^{\delta_I}I^{\theta}K^{\gamma}K^{(2\delta_K)}M\alpha^2\delta_I\gamma\lambda^2\mu n^2\rho x^2(\delta_I\gamma + 1) \\
& + A^2DI^{(2\theta)}K^{\delta_K}K^{(2\gamma)}M\delta_n\delta_x\gamma\mu^2n^{\delta_n}\theta\bar{w}x^{\delta_x}(\delta_x\theta + \delta_x\gamma + \delta_n\theta + \delta_n\gamma + \delta_n\delta_x) \\
& + AI^{\theta}K^{\delta_K}K^{\gamma}M\alpha^2\delta_K\lambda^2\mu n^2\theta^2wx^{\delta_x}x^2(\theta - \delta_K) \\
& - AI^{\delta_I}I^{\theta}K^{\gamma}K^{(2\delta_K)}M\alpha^2\delta_I^2\gamma\lambda^2\mu n^2\rho x^2(\gamma + \delta_I) \\
& + 2AI^{\delta_I}I^{\theta}K^{\delta_K}K^{\gamma}\delta_I\delta_x\gamma\lambda\mu n^2\phi\rho\bar{w}x^{\delta_x}(\delta_x\gamma + \delta_I\gamma \\
& + \delta_I\delta_x + 1 - \delta_I\delta_x\gamma - \gamma - \delta_x - \delta_I)
\end{aligned}$$

Each element of ψ includes a term that compares cost elasticities and/or the knowledge growth elasticities from the KPF. To identify the sign of ψ, we make use of the following conditions identified by signing the previous three principal minors:

- $\delta_x \geq 1$
- $\delta_n \geq 1$
- $\delta_I \geq 1$

Additionally, we assume a constant-returns Cobb–Douglas knowledge production function following Goulder and Mathai (2000), Jones (2005), and Abis and Veldkamp (2021, NBER working paper).

- $\delta_K = 1$
- $\theta = \gamma = 0.5$

Applying these values to the equations above, we then organize the positive and negative equations to provide an inequality assuring $\psi > 0$. (Note: Several terms in ψ become zero under the conditions referenced above.)

$$\begin{aligned}
&2AI^{\theta}K^{\gamma}\delta_K\delta_x\lambda\mu n^2\phi\theta\bar{w}^2x^{(2\delta_x)}(\theta+\delta_x-\delta_K)\\
&+2A^2I^{(2\theta)}K^{\delta_K}K^{(2\gamma)}\delta_x\gamma\lambda\mu^2n^2\phi\theta\bar{w}x^{\delta_x}(\theta+\gamma+\delta_x-1)\\
&+ADI^{\theta}K^{\gamma}M\delta_K\delta_n\delta_x\mu n^{\delta_n}\theta\bar{w}^2x^{(2\delta_x)}(\delta_x\theta+\delta_n\theta+\delta_n\delta_x-\delta_K\theta-\delta_K\delta_n)\\
&+AI^{\theta}K^{\delta_K}K^{\gamma}M\alpha^2\delta_K\lambda^2\mu n^2\theta\bar{w}x^{\delta_x}x^2(\delta_K+\theta-1)\\
&+AI^{\delta_I}I^{\theta}K^{\gamma}K^{(2\delta_K)}M\alpha^2\delta_I\gamma\lambda^2\mu n^2\rho x^2(\delta_I\gamma+1)\\
&+A^2DI^{(2\theta)}K^{\delta_K}K^{(2\gamma)}M\delta_n\delta_x\gamma\mu^2n^{\delta_n}\theta\bar{w}x^{\delta_x}(\delta_x\theta+\delta_x\gamma+\delta_n\theta+\delta_n\gamma+\delta_n\delta_x)\\
&+2AI^{\delta_I}I^{\theta}K^{\delta_K}K^{\gamma}\delta_I\delta_x\gamma\lambda\mu n^2\phi\rho\bar{w}x^{\delta_x}\\
&\times(\delta_x\gamma+\delta_I\gamma+\delta_I\delta_x+1-\delta_I\delta_x\gamma-\gamma-\delta_x-\delta_I)\\
&>\\
&I^{\delta_I}K^{\delta_K}M\alpha^2\delta_I\delta_K^2\lambda^2n^2\rho wx^{\delta_x}x^2(\delta_K-\delta_I)\\
&+2AI^{\theta}K^{\gamma}\delta_K\delta_x^2\lambda\mu n^2\phi\theta^2w^2x^{2\delta_x}(\delta_K-\delta_x-\frac{1}{\theta})\\
&+2AI^{\theta}K^{\gamma}\delta_K\delta_x^2\lambda\mu n^2\phi\theta^2w^2x^{2\delta_x}(\delta_K-\delta_x-\frac{1}{\theta})\\
&+2A^2I^{(2\theta)}K^{\delta_K}K^{(2\gamma)}\delta_x^2\gamma\lambda\mu^2n^2\phi\theta\bar{w}x^{\delta_x}(\theta+\gamma)\\
&+ADI^{\theta}K^{\gamma}M\delta_K\delta_n\delta_x\mu n^{\delta_n}\theta\bar{w}^2x^{(2\delta_x)}\\
&\times(1-\delta_n\delta_x\theta+\delta_K\delta_n\theta-\theta-\delta_x-\delta_n+\delta_K)\\
&+A^2DI^{(2\theta)}K^{\delta_K}K^{(2\gamma)}M\delta_n\delta_x\gamma\mu^2n^{\delta_n}\theta\bar{w}x^{\delta_x}\\
&\times(\delta_n\delta_x\theta+\delta_n\delta_x\gamma+\theta+\gamma+\delta_x+\delta_n)\\
&+AI^{\theta}K^{\delta_K}K^{\gamma}M\alpha^2\delta_K\lambda^2\mu n^2\theta^2wx^{\delta_x}x^2(\theta-\delta_K)\\
&-AI^{\delta_I}I^{\theta}K^{\gamma}K^{(2\delta_K)}M\alpha^2\delta_I^2\gamma\lambda^2\mu n^2\rho x^2(\gamma+\delta_I)5
\end{aligned}$$

Appendix B. Solving for λ_t

We derive Eq. (16): using an integrating factor methodology typical for these types of problems (Simon & Blume p. 639–640). The first step is to identify the integrating factor, identified as ψ_s. The selection of this term will become apparent momentarily, but for now it is given as:

$$\psi_s = e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau} \tag{31}$$

The parameter s represents some time period within the interval $[t,T]$ while τ is a constant of integration. Both functions g_{n_τ} and h_{n_τ} are defined for all $s \in [t,T]$. We will pre-multiply (14) by ψ_s:

$$e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}\dot{\lambda} + e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}\lambda_s[g_{n_s}-h_{n_s}-r] = e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}D_{n_s} \tag{32}$$

The integrating factor was constructed such that the LHS of (32) is equivalent to $\frac{d\psi_s\lambda_s}{ds}$.

$$\frac{d\psi_s\lambda_s}{ds} = \frac{de^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}\lambda_s}{ds}$$

$$\frac{de^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}\lambda_s}{ds} = e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}\dot{\lambda} + e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}\lambda_s\left(\frac{d\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}{ds}\right)$$

The first fundamental theorem of calculus states that $F(x)=\int_a^x [f(t)]dt$, then $\frac{d}{dx}F(x)=\frac{d}{dx}\int_a^x [f(t)]dt = f(x)$. Applying this to the final term in the last equation allows us to identify the LHS of (32)

$$\frac{d}{ds}\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau = g_{n_s}-h_{n_s}-r$$

We have shown that the LHS of (32) can be interpreted as the time derivative of $\psi\lambda_s$

$$\frac{de^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}\lambda_s}{ds} = e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}D_{n_s} \tag{33}$$

Moving forward, it will be convenient to separate the constant r from $e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}$. So (33) becomes:

$$\frac{de^{\int_t^s [g_{n_\tau}-h_{n_\tau}]d\tau}e^{-r(s-t)}\lambda_s}{ds} = e^{\int_t^s [g_{n_\tau}-h_{n_\tau}]d\tau}e^{-r(s-t)}D_{n_s} \tag{34}$$

Integrating both sides of (34) from t to T:

$$\int_t^T \left[\frac{d\lambda_s e^{\int_t^s [g_{n_\tau}-h_{n_\tau}-r]d\tau}e^{-r(s-t)}}{ds}\right]ds = \int_t^T \left[e^{\int_t^s [g_{n_\tau}-h_{n_\tau}]d\tau}e^{-r(s-t)}D_{n_s}\right]ds \tag{35}$$

Through familiar application of the second fundamental theorem of calculus, the LHS becomes:

$$\lambda_T e^{\int_t^T [g_{n_\tau}-h_{n_\tau}-r]d\tau}e^{-r(T-t)} - \lambda_t e^{\int_t^t [g_{n_\tau}-h_{n_\tau}-r]d\tau}e^{-r(t-t)}$$

Clearly, the exponential functions on the second term of the LHS collapse to $e^0 = 1$, so that we are left with

$$\lambda_T e^{\int_t^T [g_{n_\tau}-h_{n_\tau}-r]d\tau}e^{-r(T-t)} - \lambda_t = \int_t^T \left[e^{\int_t^s [g_{n_\tau}-h_{n_\tau}]d\tau}e^{-r(s-t)}D_{n_s}\right]ds \tag{36}$$

With some quick algebra we can now offer a qualitative statement for the co-state variable λ_t:

$$\lambda_t = \lambda_T e^{\int_t^T [g_{n_\tau}-h_{n_\tau}-r]d\tau}e^{-r(T-t)} - \int_t^T \left[e^{\int_t^s [g_{n_\tau}-h_{n_\tau}]d\tau}e^{-r(s-t)}D_{n_s}\right]ds \tag{37}$$

Appendix C. Solving for μ_t

$$\theta_s = e^{\int_t^s [\eta_{K_u}-r]du} \tag{38}$$

The function η_{K_u} is defined for all $s \in [t,T]$ and u is an integrating variable. Following the same process as above:

$$e^{\int_t^s [\eta_{K_u}-r]du}\dot{\mu} + e^{\int_t^s [\eta_{K_u}-r]du}\mu_s[\eta_{K_s}-r] = e^{\int_t^s [\eta_{K_u}-r]du}C_{K_s} \tag{39}$$

$$\frac{d\mu_s e^{\int_t^s [\eta_{K_u}-r]du}}{ds} = e^{\int_t^s [\eta_{K_u}-r]du}C_{K_s} \tag{40}$$

Integrating each side over the time horizon $[t,T]$ and applying the second fundamental theorem of calculus:

$$\int_t^T \left[\frac{d\mu_s e^{\int_t^s [\eta_{K_u}-r]du}}{ds}\right]ds = \int_t^T \left[e^{\int_t^s [\eta_{K_u}-r]du}C_{K_s}\right]ds$$

$$\mu_T e^{\int_t^T [\eta_{K_u}]du}e^{-r(T-t)} - \mu_t e^{\int_t^t [\eta_{K_u}]du}e^{-r(t-t)} = \int_t^T \left[e^{\int_t^s [\eta_{K_u}]du}e^{-r(s-t)}C_{K_s}\right]ds \tag{41}$$

Again, we see the exponential functions on μ_t collapse to 1, then rearrange to find:

$$\mu_t = \mu_T e^{\int_t^T [\eta_{K_u}]du}e^{-r(T-t)} - \int_t^T \left[e^{\int_t^s [\eta_{K_u}]du}e^{-r(s-t)}C_{K_s}\right]ds \tag{42}$$

References

Abis, Simona, Veldkamp, L., 2021. The Changing Economics of Knowledge Production. NBER Working Paper.

Acemoglu, Daron, et al., 2012. The environment and directed technical change. Am. Econ. Rev. 102 (1), 131–166.

Adams, Damian C., Lee, Donna J., 2012. Technology adoption and mitigation of invasive species damage and risk: application to zebra mussels. J. Bioecon. 14 (1), 21–40.

Akter, Sonia, Kompas, Tom, Ward, Michael B., 2015. Application of portfolio theory to asset-based biosecurity decision analysis. Ecol. Econom. 117, 73–85.

Anderson, Lee G., Seijo, Juan Carlos, 2011. Bioeconomics of Fisheries Management. John Wiley & Sons.

Babcock, Bruce A., Lichtenberg, Erik, Zilberman, David, 1992. Impact of damage control and quality of output: estimating pest control effectiveness. Am. J. Agric. Econ. 74 (1), 163–172.

Bartkowski, Bartosz, Lienhoop, Nele, Hansjürgens, Bernd, 2015. Capturing the complexity of biodiversity: A critical review of economic valuation studies of biological diversity. Ecol. Econom. 113, 1–14.

Bieber, Claudia, Ruf, Thomas, 2005. Population dynamics in wild boar sus scrofa: ecology, elasticity of growth rate and implications for the management of pulsed resource consumers. J. Appl. Ecol. 42 (6), 1203–1213.

Burnett, Kim M., D'Evelyn, Sean, Kaiser, Brooks M., Nantamanisikarn, Porntawee, Roumasset, James A., 2008. Beyond the lamppost: Optimal prevention and control of the Brown Tree Snake in Hawaii. Ecol. Econom. 67 (1), 66–74.

Burnett, Kim M., Kaiser, Brooks, Roumasset, James, 2007. Invasive species control over space and time: miconia calvescens on oahu, hawai'i. J. Ag. and Applied Econ. 39 (1), 125–132.

Burnett, Kim M., Pongkijvorasin, Sittidaj, Roumasset, James, 2012. Species invasion as catastrophe: the case of the brown tree snake. Environ. Resour. Econ. 51 (2), 241–254.

Chew, Matthew K, 2015. Ecologists, environmentalists, experts, and the invasion of the 'second greatest threat'. Int. Rev. Environ. Hist. 1, 7–40.

Clark, Larry, Savarie, Peter J, 2012. Efficacy of aerial broadcast baiting in reducing brown treesnake numbers. Human-Wildlife Interactions 6 (2), 212–221.

Eiswerth, Mark E., Johnson, Wayne S., 2002. Managing nonindigenous invasive species: insights from dynamic analysis. Environ. Resour. Econ. 23 (3), 319–342.

Engeman, Richard M., Shiels, Aaron B., Clark, Craig S., 2018. Objectives and integrated approaches for the control of brown tree snakes: An updated overview. J. Environ. Manag. 219, 115–124.

Epanchin-Niell, Rebecca S., 2017. Economics of invasive species policy and management. Biol. Invasions 19 (11), 3333–3354.

Epanchin-Niell, Rebecca S., Hastings, Alan, 2010. Controlling established invaders: integrating economics and spread dynamics to determine optimal management. Ecol. Lett. 13 (4), 528–541.

Epanchin-Niell, Rebecca S., Liebhold, Andrew M., 2015. Benefits of invasion prevention: effect of time lags, spread rates, and damage persistence. Ecol. Econom. 116, 146–153.

Epanchin-Niell, Rebecca S., Wilen, James E., 2012. Optimal spatial control of biological invasions. J. Environ. Econ. Manage. 63 (2), 260–270.

Epanchin-Niell, Rebecca S., et al., 2012. Optimal surveillance and eradication of invasive species in heterogeneous landscapes. Ecol. Lett. 15 (8), 803–812.

Fritts, Thomas H., 2002. Economic costs of electrical system instability and power outages caused by snakes on the island of guam. Int. Biodeterior. Biodegrad. 49 (2), 93–100.

Fukasawa, Keita, et al., 2013. Reconstruction and prediction of invasive mongoose population dynamics from history of introduction and management: a Bayesian state-space modelling approach. J. Appl. Ecol. 50 (2), 469–478.

Garrott, Robert A., Taylor, Lynne, 1990. Dynamics of a feral horse population in Montana. J. Wildl. Manage. 603–612.

Gordon, H. Scott, 1953. An economic approach to the optimum utilization of fishery resources. J. Fish. Board Canada 10 (7), 442–457.

Goulder, Lawrence H., Mathai, Koshy, 2000. Optimal CO 2 abatement in the presence of induced technological change. J. Environ. Econ. Manage. 39 (1), 1–38.

Griliches, Zvi, 1990. Patent Statistics As Economic Indicators: A Survey. No. W3301. National Bureau of Economic Research.

Grossman, Gene M., Helpman, Elhanan, 1994. Technology and Trade. No. W4926. National Bureau of Economic Research.

Haight, Robert G., Polasky, Stephen, 2010. Optimal control of an invasive species with imperfect information about the level of infestation. Resour. Energy Econ. 32 (4), 519–533.

Hall, Bronwyn H., Jaffe, Adam B., Trajtenberg, Manuel, 2001. The NBER Patent Citation Data File: Lessons, Insights and Methodological Tools. No. W8498. National Bureau of Economic Research.

Harper, Grant A., Bunbury, Nancy, 2015. Invasive rats on tropical islands: their population biology and impacts on native species. Glob. Ecol. Conserv. 3, 607–627.

Hellmann, Jessica J., et al., 2008. Five potential consequences of climate change for invasive species. Conserv. Biol. 22 (3), 534–543.

Hulme, Philip E., 2017. Climate change and biological invasions: evidence, expectations, and response options. Biol. Rev. 92 (3), 1297–1313.

Hyytiäinen, Kari, et al., 2013. An optimization framework for addressing aquatic invasive species. Ecol. Econom. 91, 69–79.

Jaffe, Adam B., Newell, Richard G., Stavins, Robert N., 2003. Technological change and the environment. Handb. Environ. Econ. 1, 461–516.

Jardine, Sunny L., Sanchirico, James N., 2018. Estimating the cost of invasive species control. J. Environ. Econ. Manage. 87, 242–257.

Johnson, Shylo R., et al., 2016. Estimates of small Indian mongoose densities: Implications for rabies management. J. Wildl. Manage. 80 (1), 37–47.

Jones, Charles I., 2005. The shape of production functions and the direction of technical change. Q. J. Econ. 120 (2), 517–549.

Kim, C.S, Aillery, M., Lewandrowski, J., Kim, S., 2012. Optimal policies for managing invasive species under uncertainty. Appl. Math. 10A, 1388–1394.

Kim, C.S, Schaible, Glenn D., Lewandrowski, Jan, Vasavada, Utpal, 2010. Managing invasive species in the presence of endogenous technological change with uncertainty. Risk Anal. 30 (2), 250–260.

Kling, David M., Sanchirico, James N., Fackler, Paul L., 2017. Optimal monitoring and control under state uncertainty: Application to lionfish management. J. Environ. Econ. Manage. 84, 223–245.

Leung, Brian, et al., 2002. An ounce of prevention or a pound of cure: bioeconomic risk analysis of invasive species. Proc. R. Soc. Lond. Ser. B: Biol. Sci. 269 (1508), 2407–2413.

Lodge, David M., et al., 2016. Risk analysis and bioeconomics of invasive species to inform policy and management. Annu. Rev. Environ. Resour. 41.

Lovell, Sabrina J., Stone, Susan F., Fernandez, Linda, 2006. The economic impacts of aquatic invasive species: a review of the literature. Agric. Resour. Econ. Rev. 35 (1), 195–208.

Marbuah, George, Gren, Ing-Marie, McKie, Brendan, 2014. Economics of harmful invasive species: a review. Diversity 6 (3), 500–523.

Margolis, Michael, Shogren, Jason F., Fischer, Carolyn, 2005. How trade politics affect invasive species control. Ecol. Econom. 52 (3), 305–313.

McAusland, Carol, Costello, Christopher, 2004. Avoiding invasives: trade-related policies for controlling unintentional exotic species introductions. J. Environ. Econ. Manage. 48 (2), 954–977.

Mehta, Shefali V., et al., 2007. Optimal detection and control strategies for invasive species management. Ecol. Econom. 61 (2), 237–245.

Olson, Lars J., 2006. The economics of terrestrial invasive species: a review of the literature. Agric. Resour. Econ. Rev. 35 (1), 178–194.

Olson, Lars J., Roy, Santanu, 2002. The economics of controlling a stochastic biological invasion. Am. J. Agric. Econ. 84 (5), 1311–1316.

Olson, Lars J., Roy, Santanu, 2008. Controlling a biological invasion: a non-classical dynamic economic model. Econom. Theory 36 (3), 453–469.

US Congress Office of Technology Assessment, 1993. Harmful Non-Indigenous Species in the United States. DC.

Parry, Ian WH., Pizer, William A., Fischer, Carolyn, 2000. How important is technological innovation in protecting the environment?.

Perrings, Charles, et al., 2002. Biological invasion risks and the public good: an economic perspective. Conserv. Ecol. 6 (1), 1.

Pimentel, David, Zuniga, Rodolfo, Morrison, Doug, 2005. Update on the environmental and economic costs associated with alien-invasive species in the United States. Ecol. Econom. 52 (3), 273–288.

Popp, David, 2004. ENTICE: endogenous technological change in the DICE model of global warming. J. Environ. Econ. Manage. 48 (1), 742–768.

Rodda, Gordon H., Fritts, Thomas H., Conry, Paul J., 1992. Origin and population growth of the brown tree snake, Boiga irregularis, on Guam. Pac. Sci..

Rodda, Gordon H., Fritts, Thomas H., McCoid, Michael J., III, Earl W. Campbell, 1999. An overview of the biology of the brown treesnake (boiga irregularis), a costly introduced pest on pacific islands. In: Rodda, G.H., Sawai, Y., Chizar, D., Tanaka, H. (Eds.), Problem snake management: The Habu and the Brown Treesnake. Cornell University Press, Ithaca, NY.

Romer, Paul M., 1990. Endogenous technological change. J. Polit. Econ. 98 (5), S71–S102, Part 2.

Sanchirico, James N., Springborn, Michael, 2011. How to get there from here: ecological and economic dynamics of ecosystem service provision. Environ. Resour. Econ. 48 (2), 243–267.

Sanchirico, James N., et al., 2010. Spatial management of invasive species: pathways and policy options. Environ. Resour. Econ. 45 (4), 517–535.

Savarie, Peter J., et al., 2001. Use of Acetaminophen for Large-Scale Control of Brown Treesnakes. USDA National Wildlife Research Center-Staff Publications, p. 584.

Savidge, Julie A., 1987. Extinction of an island forest avifauna by an introduced snake. Ecology 68 (3), 660–668.

Savidge, Julie A., Qualls, Fiona J., Rodda, Gordon H., 2007. Reproductive biology of the brown tree snake, boiga irregularis (Reptilia: Colubridae), during colonization of guam and comparison with that in their native range. Pac. Sci. 61 (2), 191–199.

Schaefer, Milner B., 1957. Some considerations of population dynamics and economics in relation to the management of the commercial marine fisheries. J. Fish. Board Canada 14 (5), 669–681.

Shwiff, Stephanie A., Holderieath, Jason J., Chomphosy, William K. Haden, anderson, Aaron M, 2017. Chapter 3. Economics of invasive species damage management. In: Pitt, William C., Beasley, James C., Witmer, Gary W. (Eds.), Ecology and Management of Terrestrial Vertebrate Invasive Species in the United States. Taylor and Francis Books, Inc., USA.

Shwiff, Sephanie A., et al., 2010. Potential economic damage from introduction of brown tree snakes, boiga irregularis (Reptilia: Colubridae), to the Islands of Hawai 'i. Pac. Sci. 64 (1), 1–10.

Simon, Carl P., Blume, Lawrence, 1994. Mathematics for Economists, Vol. 7. Norton, New York.

Sun, Gui-Quan, 2016. Mathematical modeling of population dynamics with Allee effect. Nonlinear Dynam. 85 (1), 1–12.

US Congress, 2004. Brown Tree Snake Control and Eradication Act of 2004. Pub. L, pp. 108–834.

Vogt-Schilb, Adrien, Meunier, Guy, Hallegatte, Stéphane, 2018. When starting with the most expensive option makes sense: optimal timing, cost and sectoral allocation of abatement investment. J. Environ. Econ. Manage. 88, 210–233.

Wilcove, David S., Rothstein, David, Dubow, Jason, Phillips, Ali, Losos, Elizabeth, 1998. Quantifying threats to imperiled species in the United States. BioScience 48 (8), 607–615.

Yokomizo, Hiroyuki, Possingham, Hugh P., Thomas, Matthew B., Buckley, Yvonne M., 2009. Managing the impact of invasive species: the value of knowing the density-impact curve. Ecol. Appl. 19 (2), 376–386.