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Wood Diaphragm Deflections. Part I: Generalizing Standard Equations Using Mechanics-Based Derivations for Panel Construction

By John Lawson S.E., M.ASCE[1], Scott Breneman Ph.D., M.ASCE[2] and Marco Lo Ricco Ph.D., M.ASCE[3]

Abstract

Horizontal wood diaphragm systems, whether decked with conventional or mass timber panels, transfer wind and seismic loads to vertical elements of the lateral force-resisting system (LFRS), in flexible, rigid, or semi-rigid fashion. Characterizing and calculating the resulting diaphragm deflections determines the distribution of forces to critically loaded components and a significant portion of lateral building translations and rotations. Deflection equations for sheathed wood structural panel (WSP) diaphragms are well established in U.S. design standards in a 4-term expression that models flexural, shear, and fastener-slip deformations and its full derivation using principles of mechanics is provided herein. Derivations of similar equations for cross-laminated timber (CLT) diaphragms have yet to unfold, despite growing industry consensus that CLT panels make efficient slabs and decks. In this first of two companion papers, the corrected full derivation of the current 4-term WSP diaphragm deflection expression is

[1] Professor, Department of Architectural Engineering, California Polytechnic State University, San Luis Obispo, CA, 93407, U.S.A., (corresponding author) E-mail: jwlawson@calpoly.edu; ORCiD: 0000-0002-3237-5929

[2] Senior Technical Director – Mass Timber Program Manager, WoodWorks – Wood Products Council, PO Box D, Deer Park, WA 99006, U.S.A., E-mail: Scott.Breneman@woodworks.org; ORCiD: 0000-0003-1189-1374

[3] Research General Engineer, USDA Forest Service, Forest Products Laboratory, One Gifford Pinchot Drive, Madison, WI, 53726 U.S.A., E-mail: marco.loricco@usda.gov; ORCiD: 0000-0001-9940-1602

provided and assessed, and two ways to quantify the cumulative contribution of fastener slip are presented in order to expand its usage to a wider variety of WSP and CLT configurations in current use. Building upon this generalized mechanics-based derivation, the authors are able to propose and assess in the companion paper a unified diaphragm deflection model to compute both WSP and CLT diaphragm deflections as implemented under current practice and guide further development.

Introduction

Engineered wood building diaphragms, using plywood structural panels, date back nearly three-quarters of a century. Since the 1970's other composite products, like oriented strand board (OSB), gained market share and gave rise to the general category of wood structural panel (WSP) sheathing (Peterson 1983), which typically measures 1220 x 2440 mm (4 x 8 ft) of rectilinear area and up to 28.5 mm (1 ⅛ in.) of thickness in nominal unitized dimensions. The WSP diaphragm type has extensively served conventionally framed residential and commercial wood buildings up to 6 stories tall. Given the importance of horizontal wood diaphragms in resisting a variety of loads (like earth, wind, and seismic) acting transversely to wall planes (Cobeen et al. 2014) design procedures for WSP diaphragms are well established in U.S. building code standards. Deflection equations for WSP diaphragms in the latest edition of the American Wood Council's *Special Design Provisions for Wind and Seismic* (SDPWS) (AWC 2021) were derived using principles of engineering mechanics, but past corrections and simplifications have obscured the origins over time.

Over the last decade, cross-laminated timber (CLT) has emerged as a new mass timber product, with a majority of engineers expressing they are likely to adopt CLT construction in a future project if given availability (Laguarda-Mallo and Espinoza 2018). The most common use

of CLT in building construction is as floor and roof decking, where direct use of the CLT deck as a diaphragm system is economically attractive. Like WSPs, CLT members are rectangular, layered-composite wood panels typically fastened around their perimeters, and designed to primarily span flat in one direction; however, CLT panels are much larger and have larger dimensional aspect ratios than those of WSPs. As with WSP diaphragms, CLT panel edges are fastened in order to provide stiffness and resist in-plane shears. For CLT diaphragms, SDPWS states that deflections shall be determined using principles of engineering mechanics (AWC 2021), yet no deflection equations are directly provided. The motivation to develop similarly established diaphragm deflection models for CLT panel diaphragms, therefore, leads us to document the principles of engineering mechanics underlying WSP diaphragm deflection equations.

Diaphragm deflections determine whether a diaphragm may be idealized as flexible (ASCE/SEI 2022; AWC 2021), or rigid (AWC 2021; ICC 2021), or what stiffness factor to use in a semi-rigid approach. Deformation compatibility, torsional irregularities, and building separations, are additionally influenced by this computation. Relative stiffness of the diaphragm and vertical elements, furthermore, calibrates the results of numerical earthquake response time-history analyses used for detailed evaluation of structures. Overestimation of diaphragm deflection may lead to conservative sizing of seismic joints, but underestimates period-based seismic forces due to diaphragm stiffness (Lawson 2019). Large single-story buildings for warehousing or "big-box" retailers in the United States are typically comprised of stiff in-plane concrete or masonry walls with a flexible diaphragm, resulting in a dynamic seismic response dominated more by the diaphragm's period than that of the shear walls. Because of the trend of using period-based seismic forces in the diaphragms of these building types (FEMA 2021;

ASCE/SEI 2017), the Building Seismic Safety Council identified the computation of period-based diaphragm stiffness as an important current research need (BSSC 2021). This paper reestablishes the derivation of the 4-term WSP deflection equations including two approaches that quantify the cumulative contribution of nail slip. Despite the size disparity between WSP and CLT panels, both systems share mechanical characteristics that make a unified approach to modeling diaphragm deflections adaptable to a wide variety of diaphragm configurations.

Standard Deflection Equations of Diaphragms Sheathed with Wood Structural Panels

Current U.S. practice of calculating wood-sheathed diaphragm deflections is based on modified forms of Equation (1), published in the SDPWS Commentary (AWC 2021). For definitions of the parameters appearing in the equations and figures, throughout this paper, see *Notation*.

$$\delta_{dia,US} = \frac{5vL^3}{8EAW} + \frac{vL}{4G_v t_v} + 0.188 L e_n + \frac{\sum(x\Delta_c)}{2W} \qquad \text{Equation (1).}$$

Equation (1), developed for customary units used by U.S. designers, includes implicit unit conversions in the first and third terms. This equation was developed specifically for uniformly loaded simple span diaphragms, to quantify the four sources of deflection schematically illustrated in **Fig. 1**.

In respective order of the terms, Equation (1) estimates the contributions from flexural deformation, shear-induced panel deformation, diaphragm deformation associated with nail slip, and the effects of the flexural chord connection slip. The next section reviews the chronological development of the 4-term deflection model to uncover the origins of these mechanical terms and track evolution of the equation from fundamental mechanics to the handy equations provided in building code references. Later in the paper, Equation (1) will be expanded for generalized use

with any standard system of measure and application to a broader variety of WSP and CLT
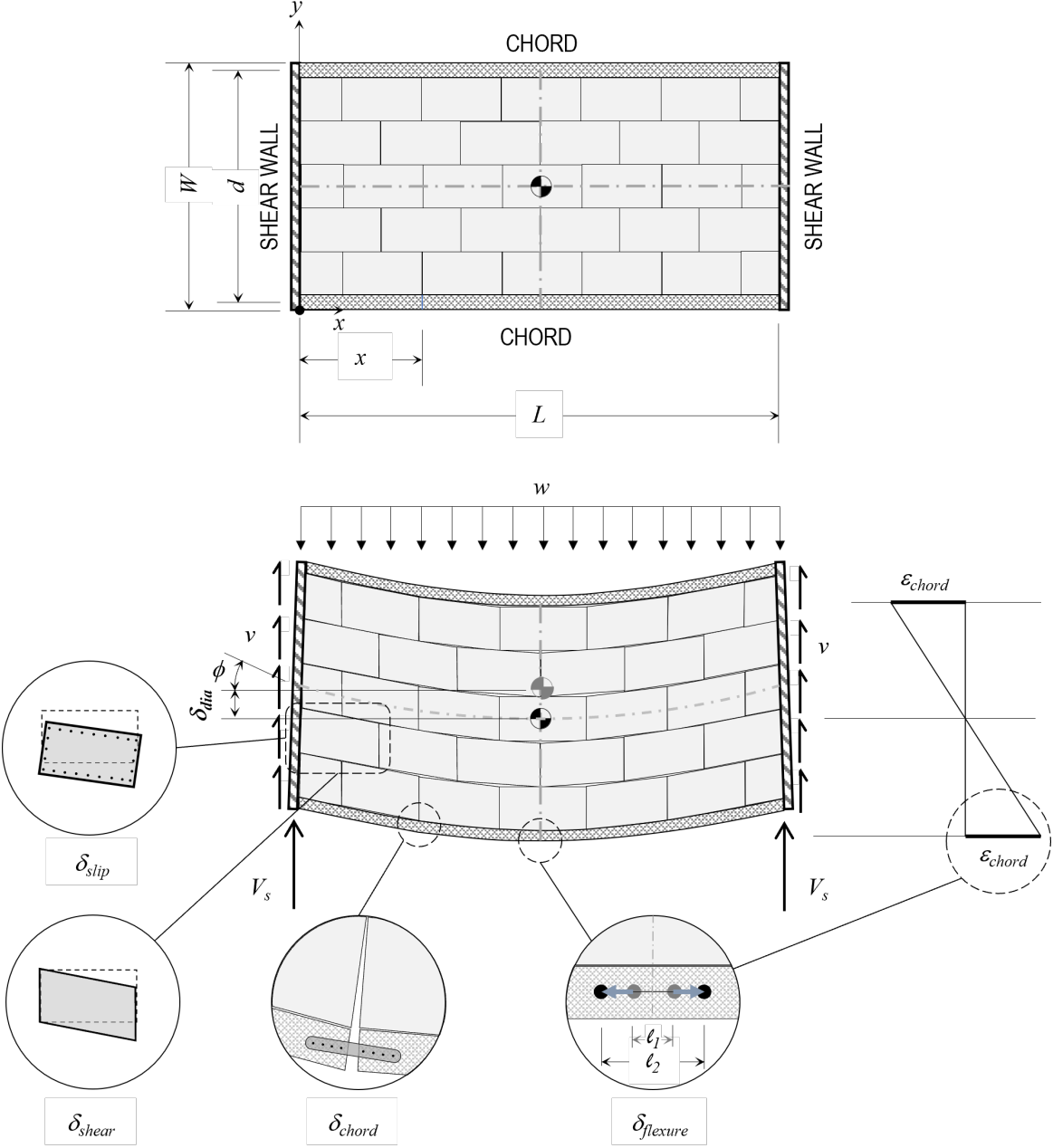diaphragms.

Fig. 1. Deflection parameters of a simple span diaphragm.

History of Diaphragm Deflection Modeling

The Douglas Fir Plywood Association developed an original deflection equation for wood sheathed diaphragms, based on testing of four full-scale and six quarter-scale diaphragms that respectively measured 12.19 m (40-ft) and 3.05 m (10-ft) in length. Countryman (1952) used principles of engineering mechanics to develop the 3 terms of Equation (2), which estimated the flexural, shear, and nail slip contributions to the observed cumulative deformations.

$$\delta_{dia,US} = \frac{5vL^3}{8EAW} + \frac{vL}{4G_v t_v} + 0.094Le_n \qquad \text{Equation (2).}$$

Though this 3-term deflection model was a pivotal development for analyzing panel-sheathed diaphragms, Equation (2), originally derived by Countryman (1952) and subsequently by ATC-7 (1981), erroneously underestimated the coefficient of the third term, by a factor of two. G.B. Walford, of the New Zealand Forest Research Institute, noted the error and provided a correction (Walford 1980; Walford 1981), which the USDA Forest Service Forest Products Laboratory (Liu 1981) and American Plywood Association (Tissell 1981) confirmed. Ever since, the nail-slip coefficient has been accepted as $0.188Le_n$, as written in Equation (1).

In addition to underestimating nail slip deflections, Equation (2) did not account for the deflection contributions of slip in the chord splices of the diaphragm. After testing long-span diaphragms, Tissell (1966) observed that diaphragms directly connected to the adhesively laminated and effectively continuous tension chords deflected less than diaphragms with mechanically spliced tension chords. To account for this additional source of deflection, APA added the fourth term of Equation (1) to the diaphragm deflection model (Bower 1974; Carney 1970). Based on testing of eleven diaphragms Tissell and Elliott (1977; 2004) developed design recommendations and compared a 4-term deflection model matching Equation (1) with observed deflections.

Many structural engineering documents, such as FEMA 273 (ATC 1997), the Uniform Building Code Standards (ICBO 1997), and International Building Code (ICC 2000), reference a version of this 4-term expression for diaphragm deflections of WSP systems. Additionally, the Canadian Standard *Engineering Design in Wood* (CSA Group 2019) contains an equivalent of Equation (1), and the New Zealand *Timber Structures Standard* commentary clauses (NZS 1993) provide a similar expression. The 4-term diaphragm deflection equation, rooted in the original work of Countryman (1952), therefore applies internationally.

Since the 2001 edition of SDPWS, a 3-term expression that combines the effects of panel shear deformation and fastener slip into one term has been the standardized deflection equation for WSP diaphragms.

$$\delta_{dia,US} = \frac{5vL^3}{8EAW} + \frac{0.25vL}{1000G_a} + \frac{\sum(x\Delta_c)}{2W} \qquad \text{Equation (3).}$$

SDPWS presents the 4-term Equation (1) and the approximation used to reduce it to the 3-term form in Commentary.

Equations (1) and (3) present terms published in U.S. design standards, which use U.S. customary units for each variable as a shortcut for common practice. For example, L is in feet and the deflection, $\delta_{dia,US}$ is in inches. This paper will use the subscript 'US' to distinguish when an equation uses U.S. customary units and implicit unit conversions. The U.S. customary units for each variable are listed in the *Notation* section of this paper. Equations without the subscript 'US' do not have implicit unit conversions and may be generally applied to any system of units.

To make derivations of deflection equations more accessible for engineers and researchers to build upon, this paper is the first publication to correctly and more fully document

the principles of mechanics, assumptions, and derivation of the 4-term diaphragm deflection model that accounts for flexural, shear, fastener slip, and chord slip deformations.

Fundamental Assumptions

The physical model underlying Equation (1) relies on several assumptions. First, distinct chords provide all resistance to flexural bending forces, and these chords may exhibit chord slip at specific locations that depend on connection detailing. Second, the shear stresses are distributed equally through the width, W or W', of the diaphragm and are resisted by the diaphragm panels and their fastening only. Third, classic beam theory applies to the diaphragm, to estimate the magnitude and direction of flexural and shear forces. According to these assumptions, the diaphragm acts as a beam spanning between supporting vertical elements, such as shear walls and braced frames, of the lateral force resisting system (LFRS). The main field of the diaphragm acts as an I-beam web providing shear resistance and chords independently act as I-beam flanges providing flexural resistance. This engineering model is often referred to as the "deep beam analogy" for diaphragm behavior. Out-of-plane forces on the diaphragms, such as from gravity loads, are ignored and assumed to not impact the diaphragm's in-plane behavior.

Because of common WSP construction detailing and practices, Equation (1) and other standard variations further assume that:

- Panels measure 1.22 m (4 ft) by 2.44 m (8 ft) in size, as viewed in plan,

- Shear fasteners used on all edges of all panels (e.g., nails) are the same type and size,

- Shear fastener spacing is equal around all edges of all panels,

- Detailing follows conventions of blocked diaphragms, and

- Axial chord forces act at the edges of the diaphragm.

Observing that construction practices have grown more diverse over the last 70 years, we revisit the fundamental diaphragm deformation equation to make the four terms readily adaptable to a wider variety of construction details and structural layouts.

Derivation of the 4-Term Diaphragm Deflection Equation for WSP Diaphragms

The following section provides in-depth review of each of the 4 terms of Equation (1). Presenting the derivation of each term, this paper provides generalized equations that address panelized WSP and CLT diaphragm configurations outside the assumptions listed above. The 4 terms of Equation (1) can be written as:

$$\delta_{dia} = \delta_{flex} + \delta_{shear} + \delta_{slip} + \delta_{chord} \qquad \text{Equation (4).}$$

For the uniformly loaded, simple span diaphragm in Figure 1, the following sections derive the 4 terms of Equation (1). The first two terms of Equation (1) address beam flexure and beam shear, which are assumed to act independently. The last two terms of Equation (1) consider additional sources of deformation inherent to panelized assembly. For WSP and CLT diaphragms, each of these sources of deflection may make significant contributions to the overall diaphragm deflection.

Flexural Deformation

For a uniformly loaded simple span diaphragm, the first term of Equation (4), δ_{flex}, may be derived using the following system of equations:

$$\begin{cases} \delta_{flex} = \dfrac{5wL^4}{384EI} & \text{(a)} \\[2mm] w = \dfrac{2vW}{L} & \text{(b)} \\[2mm] I = \dfrac{1}{2}Ad^2 & \text{(c)} \\[2mm] W = d & \text{(d)} \end{cases} \qquad \text{Equation (5).}$$

Equation (5a) provides the maximum bending deflection in a form commonly listed in mechanics of materials references. U.S. standards express the diaphragm deflection as a function of the unit shear force at the supports, so Equation (5b) converts the uniformly applied load (w) to terms of unit shear force (v), and diaphragm dimensions of width (W) and length (L).

Equation (5c) estimates the moment of inertia of the diaphragm using only cross-sectional area (A) of the chords and dimension (d) between the diaphragm chords. For conventional WSP diaphragms, the chords are typically located at the very edges of the diaphragm, using details like continuous rim joists or perimeter beams, so Equation (5d) equates width of the diaphragm and distance between chord forces for a practical approximation. Solving the system of Equation (5) leads to a general expression of maximum flexural deflection of a uniformly loaded simple span diaphragm:

$$\delta_{flex} = \frac{5vL^3}{96EAW}$$ Equation (6).

Converting to U.S. customary units results with:

$$\delta_{flex,US} = \left(\frac{12\ in}{ft}\right)\frac{5vL^3}{96EAW} = \frac{5vL^3}{8EAW}$$ Equation (7).

The flexural deformation to other load patterns may be addressed by beginning with the corresponding classic beam deflections and applying similar conversions. Table 1 of the Appendix provides several forms of the flexural deformation term resulting from the derivation above.

Shear Deformation of Wood Panels

Panel shear deformation contributes a significant amount towards the total deflection of a WSP diaphragm. For a uniformly loaded simple span beam, the cumulative shear deformation from one support to the midpoint may be expressed as the following:

$$\delta_{shear} = \int \frac{V(x)}{GA_w} dx = \int_0^{L/2} \frac{V_s(1 - \frac{2}{L}x)}{GA_w} dx = \frac{V_s L}{4GA_w}$$

Equation (8).

Using the relationships $V_s = vW$ and $GA_w = G_v t_v W$ results in

$$\delta_{shear} = \frac{vL}{4G_v t_v}$$

Equation (9).

where $G_v t_v$ is published for WSPs of various thicknesses, plies, and span ratings in the SDPWS

Commentary. The second term of Equation (8) generally applies to any load configuration.

Deformation due to Fastener Slip

The 3rd term of Equation (1) is specific to panelized diaphragms with the deformation due

to fastener slip around the panel edges. Countryman (1952) assumed a uniform distribution of

shear load across the diaphragm width and that all of shear load transfer is through the fasteners

with only negligible bearing contact between adjacent panels. Countryman's approach

recognizes that diaphragm deformations due to fastener slip δ_{slip} and panel shear δ_{shear} result

from the same diaphragm shear and are therefore directly related proportionally to the calculated

change in the panel's diagonal lengths for each respective behavior, e'_n and e'_s.

$$\frac{\delta_{slip}}{e'_n} = \frac{\delta_{shear}}{e'_s}$$

Equation (10).

To find the diagonal axis elongation associated with fastener slip, e'_n, Countryman

(1952) used the opposing corner nails as reference points. A single rectangular sheathing panel is

shown in equilibrium in **Fig. 2**. The parallel and perpendicular subscript notations define the

direction of parameters, with respect to direction of the externally applied diaphragm loads that

produce the shear. For WSP diaphragms, the edge distance from the nails to the closest panel

edge, S_e, is often specified as a minimum of 10 mm (⅜ in.). Because of the small magnitude of

the edge distance relative to the panel dimensions, for the diaphragm deflections the fasteners are assumed at the edges of the panels and S_e does not appear in calculations.

The panel aspect ratio causes longer panel edges to have a greater total shear, by a ratio of long-to-short panel edge lengths. In **Fig. 2**, the nail fasteners are equally spaced around all panel edges, and thus the shear per nail parallel to the adjacent panel edges, V_n, is assumed to have equal magnitude for all nails.

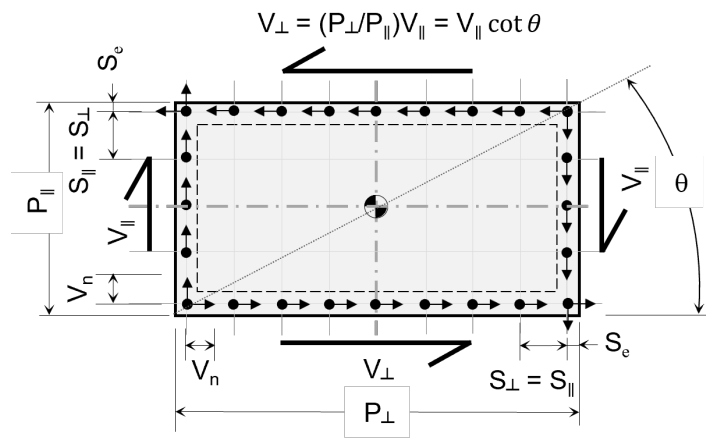$$V_\perp = (P_\perp/P_\parallel)V_\parallel = V_\parallel \cot\theta$$

Fig. 2. Panel dimensions, fastener spacings, and shear forces.

To investigate how the shear forces result in diaphragm deformation due to fasteners slip, **Fig. 3** shows fastener slip around the edges of a panel and transforms this slip to the panel diagonal axes. The enlarged vector diagram of **Fig. 3**(a) shows where orthogonal displacement components of equal magnitude, e_n, parallel to the panel edges, meet to produce a combined slip of $e_n\sqrt{2}$ at the panel corners, oriented at 45 degrees to each panel edge.
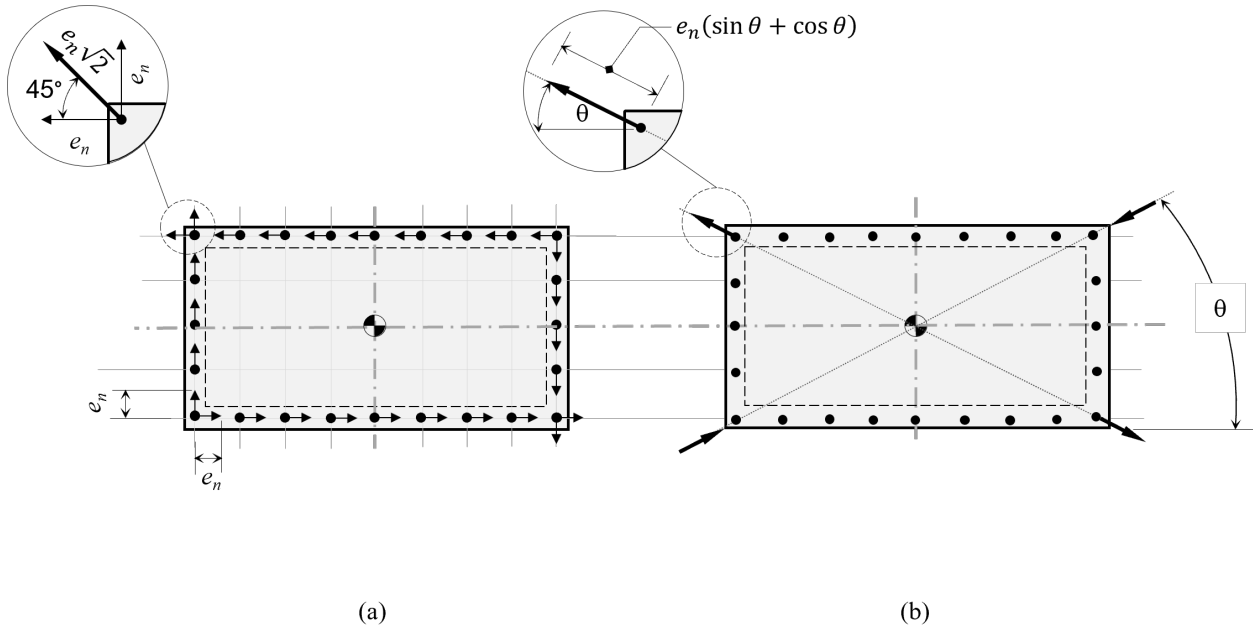
Fig. 3. Fastener slip (a) parallel to panel edges and (b) transformed to align with panel diagonals.

Fig. 3(b) transforms the nail slip at the panel corners to the diagonal axes of a panel. At each corner, fastener slip between the panel and supporting frame measured along the diagonal axes has a magnitude of $e_n(\sin \theta + \cos \theta)$.

Fig. 4 illustrates cumulative effects of fastener slip on the deformation of the supporting framing. The supporting framing of the diaphragm panel distorts into a parallelogram resulting from the fastener slip while the panel remains rectilinear. Measured along the panel diagonal, the frame deformation is twice the corner fastener slip in the same direction, so:

$$e_n' = 2e_n(\sin \theta + cos \theta) \hspace{3cm} \text{Equation (11).}$$

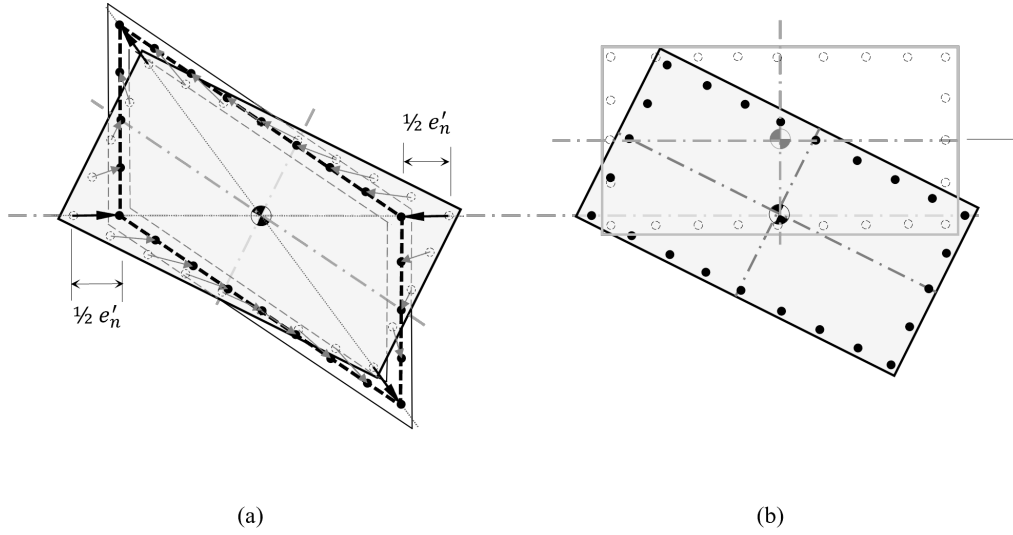<div align="center">(a)   (b)</div>

Fig. 4. Fastener-slip deformation of (a) supporting framing and of (b) sheathing panel with respect to original position.

To find the diagonal axis elongation associated with panel shear deformation, e'_s, Countryman (1952) used relationships published in *Technique of Plywood* (Norris 1943). Alternatively, when using principles of plane stress (Gere and Timoshenko 1990), the panel undergoes pure shear stress, τ_{xy}, from shear force per length, v, acting along the panel edges, which causes shear strain, γ_{xy}, calculated by:

$$\gamma_{xy} = \frac{\tau_{xy}}{G_v} = \frac{v}{G_v t_v} \qquad \text{Equation (12).}$$

The magnitude of net axial change in diagonal length, e'_s, elongation or shortening, may be found through the shear deformation relationships (Gere and Timoshenko 1990) of **Fig. 5**:

$$e'_s = \gamma_{xy} P_{\parallel} \cos \theta = \frac{v}{G_v t_v} P_{\parallel} \cos \theta \qquad \text{Equation (13).}$$
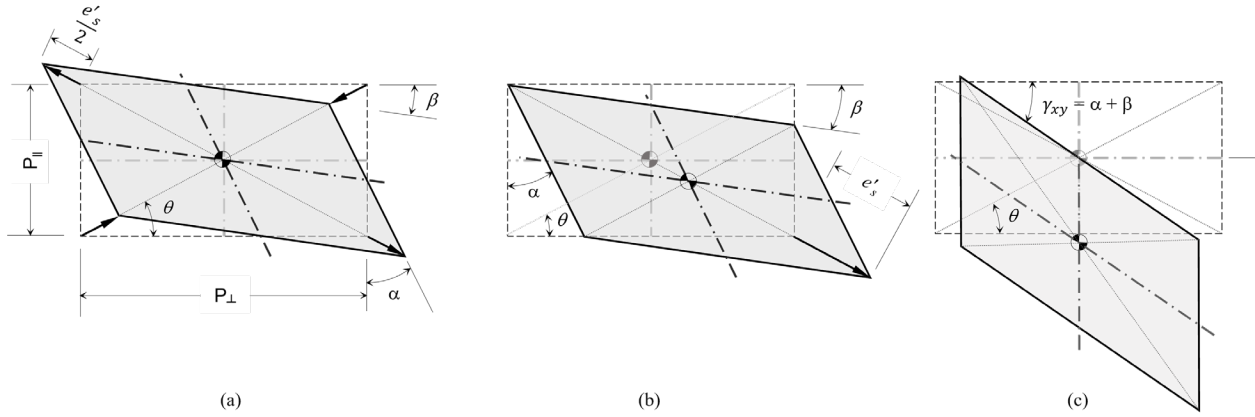
Fig. 5. Shear deformation of a panel shown with (a) strains at each corner, (b) cumulative diagonal elongation at lower right corner, and (c) net deformation relative to original position.

Therefore, using Equations (10), (11) and (13), the diaphragm deflection slip term for a uniformly loaded, simple span is:

$$\delta_{slip} = \frac{e'_n}{e'_s}\delta_{shear} = \frac{e_n L}{2P_{\parallel}}\frac{(sin\,\theta + cos\,\theta)}{cos\,\theta}$$

Equation (14).

Expressed in terms of panel dimensions, Equation (14) becomes:

$$\delta_{slip} = \frac{e_n L}{2}\left(\frac{1}{P_{\parallel}} + \frac{1}{P_{\perp}}\right)$$

Equation (15).

For the common WSP measuring 4-ft (1.22 m) by 8-ft (2.44m) in U.S. standard units, Equation (15) becomes:

$$\delta_{slip,US} = \frac{e_n L}{2}\left(\frac{1}{4\,ft} + \frac{1}{8\,ft}\right) = \frac{3}{16}\,e_n L \approx 0.188\,e_n L$$

Equation (16).

Equation (16) estimates the fastener slip contribution to diaphragm deflections and matches the third term of Equation (1) found in the SDPWS (AWC 2021). Equation (15) can more broadly be applied to any size and shape of panel. This derivation of the fastener slip term has generally followed the methods of Countryman (1952), but includes a correction (Walford 1980).

Deformation due to Chord Connections Slip

The last term in Equation (1) calculates the diaphragm deformation resulting from concentrated axial deformations, labelled chord slip Δ_c, at specific locations along the chords. Often this term addresses tension elongation of doweled splice connections, but similar principles apply to shortening of a compression chord splice. Skaggs and Martin (2004) reviewed the history of commonly used values for Δ_c in WSP diaphragms, which depend on details of the fasteners, construction assembly, and installation tolerances.

Values of Δ_c may be analytically or experimentally determined, and the resulting magnitude of diaphragm deflections are estimated using rigid-body mechanics. For small-angle rotations of rigid diaphragm sections A and B, about the point in the compression chord opposite to the tension chord splice shown in **Fig. 6**, the maximum diaphragm deflection at the location of the chord slip may be calculated as:

$$\delta_{c,max} = x\theta_A = (L - x)\,\theta_B \qquad\qquad \text{Equation (17).}$$

Solving for θ_A:

$$\theta_A = \left(\frac{L}{x} - 1\right)\theta_B \qquad\qquad \text{Equation (18).}$$

The total angle of rotation between the diaphragm areas is related to the chord slip by:

$$\theta_c = \frac{\Delta_c}{W} \qquad\qquad \text{Equation (19).}$$

The total angle of rotation is also:

$$\theta_c = \theta_A + \theta_B = \left(\frac{L}{x} - 1\right)\theta_B + \theta_B = \frac{L}{x}\theta_B \qquad\qquad \text{Equation (20).}$$

Equating these and solving for θ_B yields

$$\theta_B = \frac{x\Delta_c}{LW} \qquad\qquad \text{Equation (21).}$$

The deformation at the chord slip:

$$\delta_{c,max} = (L - x)\theta_B = (L - x)\frac{x\Delta_c}{LW} \qquad \text{Equation (22).}$$

The deformation mid-span due to chord slip:

$$\delta_c = (L/2)\theta_B = \left(\frac{L}{2}\right)\frac{x\Delta_c}{LW} = \frac{x\Delta_c}{2W} \qquad \text{Equation (23).}$$

Equation (23) accounts for a slip at a single chord splice location. The total diaphragm deflection attributable to chord slip is simply a summation of the values calculated at each chord splice:

$$\delta_{chord} = \frac{\Sigma(x\Delta_c)}{2W} \qquad \text{Equation (24).}$$

where x is the distance, or x ordinate, from the chord slip location to the nearest diaphragm support. **Fig. 6** and Equation (24) model the U.S. standard equation, which places the pivot point and tension chord at the extremities. If the pivot point and chords are located more interior, then d may be substituted for W in the preceding equations.
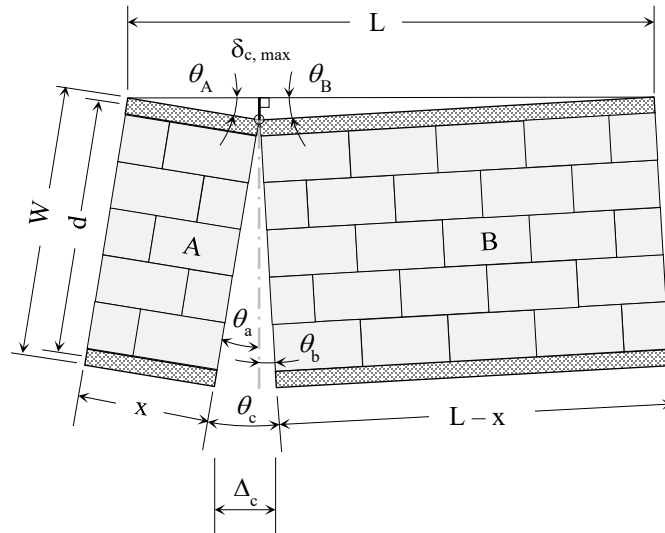

Fig. 6. Model of deformation caused by slip of a chord splice.

Alternative Derivation of Fastener Slip Term

The following provides a more general derivation of the fastener slip term similar to that presented in ATC 7's Appendix A (ATC 1981) and Lawson (2018), and provides a methodology

that can extend to a wider variety of construction configurations in both WSP and CLT assemblies. Note that the published ATC 7 derivation contains an error in the nail slip term, equivalent to that of Countryman (1952), but is correctly presented here.

This approach divides the fastener slip term into two separate contributions that are added as shown in **Fig. 7**. The translational contribution Δ_f' results from fastener slip on the panel edges parallel to the applied diaphragm load direction, and the rotational contribution Δ_f'' results from fastener slip on the panel edges perpendicular to the applied diaphragm load direction. For the superposition shown in **Fig. 7**(d), the two contributions are added in Equation (25).

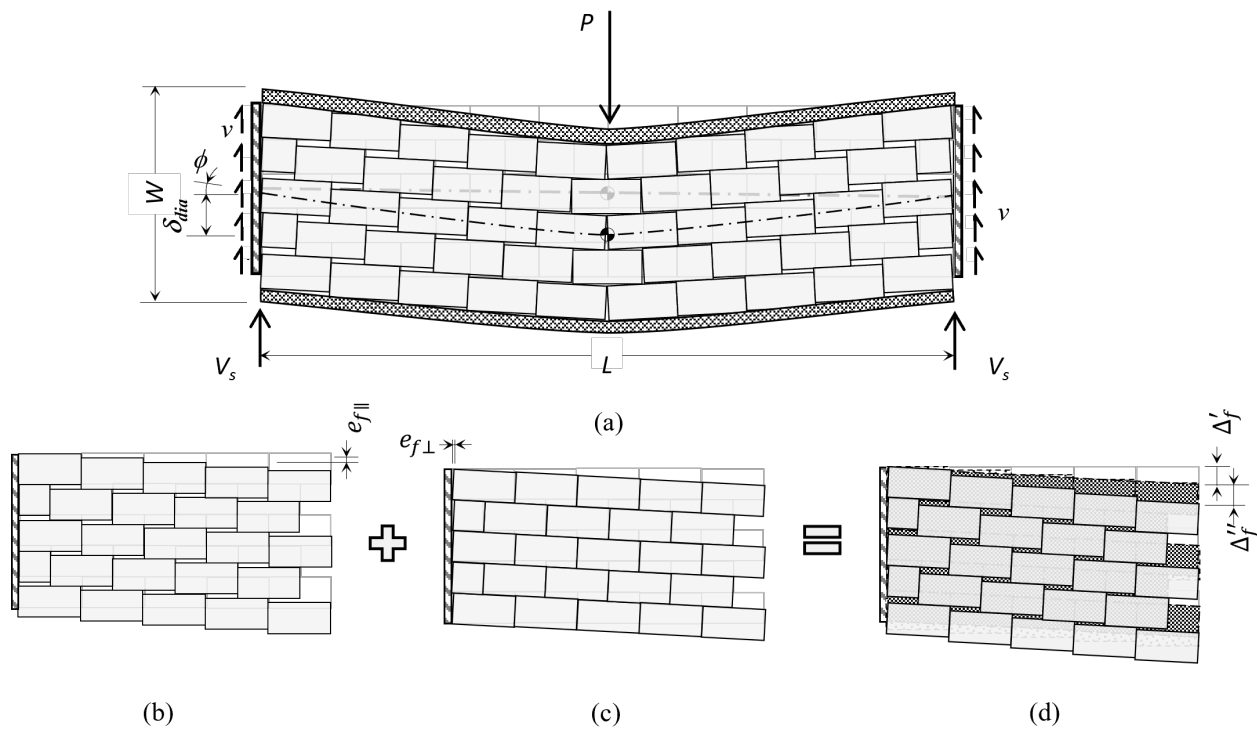$$\delta_{slip} = \Delta_f' + \Delta_f'' \hspace{4cm} \text{Equation (25).}$$

Fig. 7. Deformed (a) diaphragm, under constant shear, (b) translations, (c) rotations, and (d) superposition.

Translational Contribution Δ_f'

Fastener slip is caused by the shear experienced per fastener. Adjacent panel edges parallel to the diaphragm's loaded direction will slip $e_{f\parallel}$ in opposite directions, which opens gaps between panels as illustrated in **Fig. 7**(b). In a diaphragm configured like **Fig. 7**(a), with a concentrated load at midspan that produces constant shear of magnitude V_s on each side of the applied load, the gaps have uniform size. Considering two WSPs on a common framing member below, the connection joint parallel to the load contains two translational slip planes, one at the connection of one panel to the framing, and another at the connection of the framing to the adjacent panel. This $2e_{f\parallel}$ slip of translational movement will occur at every panel length from the support to midspan. For this situation, Δ_f' is the slip per panel multiplied by the number of panel lengths $P_\perp$ from the diaphragm boundary to midspan. The number of panel lengths to midspan equals:

$$n_{pl} = \frac{L/2}{P_\perp} \qquad\qquad \text{Equation (26).}$$

Thus, the translation contribution, dimensioned in **Fig. 7**(d), is:

$$\Delta_f' = 2e_{f\parallel}\frac{L/2}{P_\perp} = e_{f\parallel}\frac{L}{P_\perp} \qquad\qquad \text{Equation (27).}$$

Rotational Contribution Δ_f''

Fig. 7(c) dimensions the widest part of an open gap with distance $e_{f\perp}$ that is perpendicular to the applied load. The gaps open from rotation of the panels. The shear forces diagrammed in **Fig. 2**, produce the clockwise rotation shown in **Fig. 8**. Although the fasteners maintain force equilibrium, slip makes rotation inevitable and produces the beam curvature of the diaphragm.
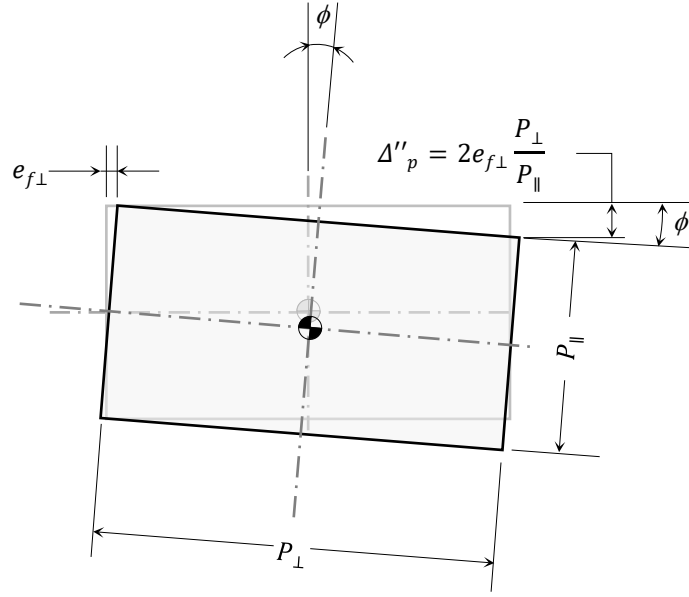
Fig. 8. Isolated diaphragm panel in rotation.

Using the theorem of similar triangles in **Fig. 8**, the rotational offset per panel Δ_p'' is:

$$\Delta_p'' = e_{f\perp} \frac{P_\perp}{P_\parallel / 2} = 2e_{f\perp} \frac{P_\perp}{P_\parallel}$$

Equation (28).

For the diaphragm of **Fig. 7**a that experiences constant shear on each side of the applied concentrated load, the rotation of every panel is constant ϕ, as illustrated in **Fig. 7**c. Therefore, Δ_f'' is the rotational offset per panel Δ_p'', Equation (28), multiplied by the number of panel lengths to midspan, Equation (26).

$$\Delta_f'' = \Delta_p'' \frac{L/2}{P_\perp} = 2e_{f\perp} \frac{P_\perp}{P_\parallel} \frac{L/2}{P_\perp} = e_{f\perp} \frac{L}{P_\parallel}$$

Equation (29).

Combining the two contributions of Equation (27) and Equation (29), as illustrated in **Fig. 7**(a) and (d), the total slip is obtained:

$$\delta_{slip} = \Delta_f' + \Delta_f'' = e_{f\parallel} \frac{L}{P_\perp} + e_{f\perp} \frac{L}{P_\parallel} = L\left(\frac{e_{f\parallel}}{P_\perp} + \frac{e_{f\perp}}{P_\parallel}\right)$$

Equation (30).

Equation (30) provides the fastener slip contribution specific to the **Fig. 7** configuration of a diaphragm placed in constant shear by a midspan concentrated load. Other loading conditions can be evaluated by proportioning to this case, using the Euler-Bernoulli beam relationship of the loading diagram to the shear diagram. For example, uniformly distributed loads on a simply supported diaphragm produce shear that varies linearly across the diaphragm from its peak at each boundary to zero at midspan. The average diaphragm shear, therefore, amounts to one-half the boundary shear, so Equation (30) may be factored by half to represent a uniformly distributed loaded diaphragm:

$$\delta_{slip} = \frac{L}{2}\left(\frac{e_{f\parallel}}{P_\perp} + \frac{e_{f\perp}}{P_\parallel}\right) \qquad \text{Equation (31).}$$

Equations (15) and (31) are similar except Equation (31) can investigate different fastener types and spacings on the parallel and perpendicular oriented sides. The terms $e_{f\parallel}$ and $e_{f\perp}$ are the fastener slip dimensions along the panel edges parallel and perpendicular to diaphragm load direction (respectively) determined from the load per fastener induced by the maximum diaphragm shear along the diaphragm boundary. The magnitudes of $e_{f\parallel}$ and $e_{f\perp}$ are based on the shear per fastener found from static equilibrium (Fig. 2) and the fastener's stiffness.

For CLT diaphragms, Spickler et al. (2015) presented a similar modification of the fastener slip term as in Equation (15) for panels other than the WSP standard 1.22 x 2.44m (4-ft x 8-ft) dimensions. Equation (31) is consistent with diaphragm connections where a separate shear-transfer element is provided at panel-to-panel shear connections, such as where adjacent panel edges are fastened to supporting framing members or where CLT panels are joined with surface spline connections (Mohammad et al. 2013). When a separate shear transfer element is employed, adjacent panel edges will slip e_f in opposite directions because each panel slips relative to the framing below or spline and result is the $2e_f$ value per panel identified earlier. In

Fig. 9, for example, the WSP and CLT-spline configurations each exhibit 2 slip planes. The half-lapped, or shiplap, panel edge connection of CLT in **Fig. 9**(b), however, provides only one slip plane, so total slip across the panel-to-panel connection is e_f. The total slip between panels can be generalized, by accounting for the number of slip planes in a connection, with parameter n. Replacing the panel slip, $2e_f$, with $n_\parallel e_{f\parallel}$ and $n_\perp e_{f\perp}$ for the number of slip planes corresponding to panel edges parallel and perpendicular to the applied load, Equation (31) becomes:

$$\delta_{slip} = \frac{L}{4}\left(\frac{n_\parallel e_{f\parallel}}{P_\perp} + \frac{n_\perp e_{f\perp}}{P_\parallel}\right) \qquad \text{Equation (32)}.$$

The generalization of Equation (32), therefore, makes it possible to estimate fastener slip for a wider variety of construction details by identifying the number of slip planes. Several recent studies address the behavior of these spline and half-lap connections in CLT (Sullivan et al. 2018; Taylor et al. 2021).
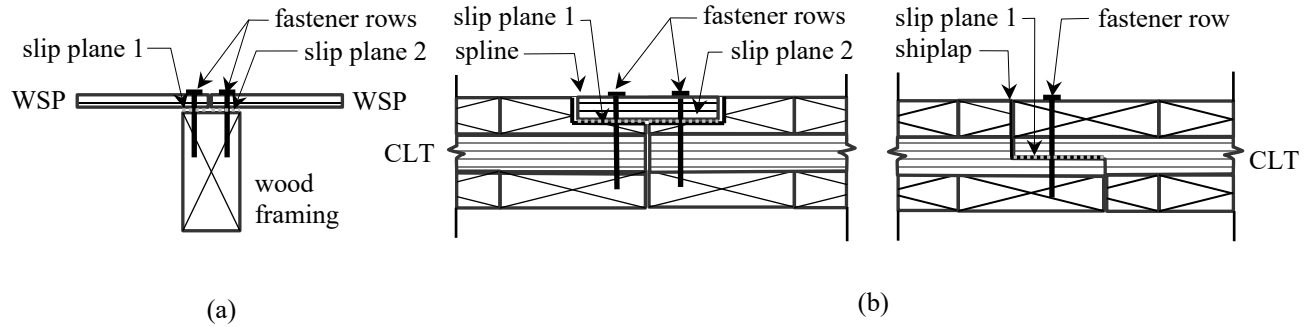

Fig. 9. Fastener-slip planes in typical panel joints of (a) WSP and (b) CLT diaphragms.

Combining the four generalized expressions for each source of deflection into a unified equation for a total deflection estimate of a uniformly loaded, simple span wood panelized diaphragm:

$$\delta_{dia} = \frac{5vL^3}{96EAW} + \frac{vL}{4G_v t_v} + \frac{L}{4}\left(\frac{n_\parallel e_{f\parallel}}{P_\perp} + \frac{n_\perp e_{f\perp}}{P_\parallel}\right) + \frac{\sum(x\Delta_c)}{2W} \qquad \text{Equation (33)}.$$

Extension to Cantilever Wood Diaphragms

Cantilevered diaphragm configurations commonly occur in construction practice. The SDPWS addresses cantilever diaphragms slightly differently than the simple span case, with notations of diaphragm width W' and length L' particular to the cantilever. The equations developed for the simple span diaphragm can be adapted to cantilever diaphragms recognizing the effects from their free-end and fixed-end supports. Combining the four adapted generalized expressions for each source of deflection provides a total deflection estimate of a uniformly loaded cantilever wood panel diaphragm:

$$\delta_{cant,u} = \frac{vL'^3}{4EAW'} + \frac{vL'}{2G_vt_v} + \frac{L'}{2}\left(\frac{n_\parallel e_{f\parallel}}{P_\perp} + \frac{n_\perp e_{f\perp}}{P_\parallel}\right) + \frac{\sum(x'\Delta_c)}{W'} \qquad \text{Equation (34).}$$

Experimental researchers will find value in a concentrated end-loaded diaphragm deflection equation. Given the different moments and constant shear, a general equation is:

$$\delta_{cant,p} = \frac{2vW'L'^3}{3EAd^2} + \frac{vL'}{G_vt_v} + L'\left(\frac{n_\parallel e_{f\parallel}}{P_\perp} + \frac{n_\perp e_{f\perp}}{P_\parallel}\right) + \frac{\sum(x'\Delta_c)}{W'} \qquad \text{Equation (35).}$$

Conclusions

Following straightforward principles of engineering mechanics on a deep beam analogy of diaphragm behavior, this paper reestablishes the derivation of the current 4-term WSP diaphragm deflection equation in the SDPWS (AWC 2021), and generalizes the derivation to target a wider range of WSP and CLT applications. A derivation updated to correct previous errors has not been readily available for U.S. and international designers, and so the steps shown in this paper offer a framework for designers who seek to adapt the standard equations to different loading conditions and frequently encountered construction details that evolve with changing practices over time. Accurate estimations of diaphragm deflections are routinely needed to characterize diaphragms as either flexible, rigid, or semi-rigid, as well as specifying

building separations, property line setbacks and evaluating deformation compatibility of the attached building elements.

Today's WSP diaphragms frequently do not match standard assumptions used in the original derivation of the diaphragm deflection model, first developed in the early 1950's. This paper has generalized a series of standard diaphragm deflection equations allowing for panel sizes other than 1220 x 2440 mm (4 x 8 ft). In addition, the generalized equations track the fastener slip e_f on the long and short panel edges separately, allowing investigation of different fastener types and spacings on the same panel as is often done in today's practice.

With the heightened interest in CLT floor and roof decks, designers need a diaphragm deflection model that addresses the unique aspect of this mass timber product. In line with SDPWS 2021 requirements that CLT diaphragm deflections be computed using principles of engineering mechanics, this paper derives a generalized tool that unifies these computations for both WSP and CLT diaphragms, based on their commonality of panelized construction. This paper's generalized standard diaphragm deflection equations address the high aspect ratios common with CLT panels, as well as half-lap connections with one instead of two slip planes per shear connection at panel joints.

Furthermore, variants of the deflection equations are shown which cover the uniformly loaded, simple span diaphragm, and the uniformly loaded and point loaded cantilever diaphragm, all of which have frequent applications in design and research. This generalized derivation of diaphragm deflections is based on a consistent set of engineering mechanics assumptions that may guide WSP and CLT diaphragm analysis and design in further research and construction practice. In the second of the two companion papers, equations are derived with specific variables for different fastener spacings and fastener stiffnesses on the short and long panel edges

of WSP and CLT diaphragms, as well as assessing the unique issues associated with chord and

shear deformations in current practice,

Appendix

For reference, **Table 1** shows the resulting diaphragm deformation components for a range of

assumptions presented in the derivations.

Table 1. Summary of diaphragm deflection contributions for various configurations

δ_{flex}	δ_{shear}	δ_{slip}	δ_{chord}
Simple Span, Uniform Loading			
General form		General Form	General Form
$\dfrac{5vWL^3}{192EI}$		$\dfrac{L}{4}\left(\dfrac{n_\parallel e_{f\parallel}}{P_\perp}+\dfrac{n_\perp e_{f\perp}}{P_\parallel}\right)$	$\dfrac{\sum(x\Delta_c)}{2d}$
Two equal chords	$\dfrac{vL}{4G_v t_v}$	Two slip planes at edges	Chords located at edge
$\dfrac{5vWL^3}{96EAd^2}$		$\dfrac{L}{2}\left(\dfrac{e_{f\parallel}}{P_\perp}+\dfrac{e_{f\perp}}{P_\parallel}\right)$	$\dfrac{\sum(x\Delta_c)}{2W}$
Chords located at edge		Equal fasteners and spacing	
$\dfrac{5vL^3}{96EAW}$		$\dfrac{e_f L}{2}\left(\dfrac{1}{P_\perp}+\dfrac{1}{P_\parallel}\right)$	
Cantilever Span, Uniform Loading			
General form		General Form	General Form
$\dfrac{vW'L'^3}{8EI}$		$\dfrac{L'}{2}\left(\dfrac{n_\parallel e_{f\parallel}}{P_\perp}+\dfrac{n_\perp e_{f\perp}}{P_\parallel}\right)$	$\dfrac{\sum(x'\Delta_c)}{d'}$
Two equal chords	$\dfrac{vL'}{2G_v t_v}$		Chords located at edge
$\dfrac{vW'L'^3}{4EAd^2}$		$L'\left(\dfrac{e_{f\parallel}}{P_\perp}+\dfrac{e_{f\perp}}{P_\parallel}\right)$	$\dfrac{\sum(x'\Delta_c)}{W'}$
Chords located at edge		Equal fasteners and spacing	
$\dfrac{vL'^3}{4EAW'}$		$e_f L\left(\dfrac{1}{P_\perp}+\dfrac{1}{P_\parallel}\right)$	
Cantilever Span, Point Loading			
General form		General Form	General Form
$\dfrac{vW'L'^3}{3EI}$		$L'\left(\dfrac{n_\parallel e_{f\parallel}}{P_\perp}+\dfrac{n_\perp e_{f\perp}}{P_\parallel}\right)$	$\dfrac{\sum(x'\Delta_c)}{d'}$
Two equal chords	$\dfrac{vL'}{G_v t_v}$	Two slip planes at edges	Chords located at edge
$\dfrac{2vW'L'^3}{3EAd^2}$		$2L'\left(\dfrac{e_{f\parallel}}{P_\perp}+\dfrac{e_{f\perp}}{P_\parallel}\right)$	$\dfrac{\sum(x'\Delta_c)}{W'}$
Chords located at edge		Equal fasteners and spacing	
$\dfrac{2vL'^3}{3EAW'}$		$2e_f L\left(\dfrac{1}{P_\perp}+\dfrac{1}{P_\parallel}\right)$	

Data Availability Statement

Acknowledgements

Notation

The following symbols are used in this paper. For parameters referenced in U.S. design standards, the following list provides U.S. customary units in parentheses. All parameters may be generally applied consistently with a standardized system of units

A = area of diaphragm chord (in^2);

A_w = cross-sectional web area of the diaphragm;

d = distance between axial tension and compression of diaphragm chords;

E = modulus of elasticity of diaphragm chords (lb$_f$/in^2);

$e_{f\parallel}$ = fastener slip along panel edge parallel to diaphragm load direction based on the maximum diaphragm shear along the diaphragm boundary;

$e_{f\perp}$ = fastener slip along panel edge perpendicular to diaphragm load direction based on the maximum diaphragm shear along the diaphragm boundary;

e_n = nail slip measured parallel to nearest panel edge based on shear along the diaphragm boundary (in);

e_n' = change in panel framing diagonal length due to nail slip (in);

e_s' = change in panel diagonal length due to shear strain (in);

G = panel shear modulus (psi);

G_a = apparent diaphragm shear stiffness (kips/in.);

$G_v t_v$ = panel in-plane, through-the-thickness, shear stiffness (lb$_f$/in. of depth) of WSP panels;

I = moment of inertia of diaphragm chords;

L = length of diaphragm span, perpendicular to diaphragm load direction (ft);

L' = length of cantilever diaphragm span, perpendicular to diaphragm load direction (ft);

n_{pl} = number of panel lengths from edge of diaphragm to midspan of diaphragm;

$n_{\parallel}$ = number of slip planes at panel-to-panel connections parallel to diaphragm load direction;

$n_{\perp}$ = number of slip planes at panel-to-panel connections perpendicular to diaphragm load direction;

$P_{\parallel}$ = panel dimension parallel to diaphragm load direction;

$P_{\perp}$ = panel dimension perpendicular to diaphragm load direction;

$S_{\perp}$ = fastener spacing on panel edges perpendicular to diaphragm load direction;

$S_{\parallel}$ = fastener spacing on panel edges parallel to diaphragm load direction;

V_{s} = total diaphragm shear at a line of support;

$V(x)$ = total shear over the width of diaphragm at location x;

V_{n} = shear force per nail;

$V_{\parallel}$ = total shear on panel edge parallel to diaphragm load direction;

$V_{\perp}$ = total shear on panel edge perpendicular to diaphragm load direction;

v = induced shear per unit length typically at the diaphragm boundary/support line (lbs/ft);

W = width of diaphragm, parallel to the diaphragm load direction (ft);

W' = width of cantilever diaphragm, parallel to the diaphragm load direction (ft);

w = uniformly distributed applied diaphragm load;

x = distance from nearest diaphragm support to the location of interest (ft);

x' = distance from the free end of the cantilever to the location of interest (ft);

α = angle of shear strain component measured with respect to panel edges oriented parallel to applied diaphragm load direction, in a geometric context, or

β = angle of shear strain component measured with respect to panel edges oriented perpendicular to applied diaphragm load direction;

Δ_c = diaphragm chord splice slip at the induced unit shear (in);

Δ_f' = diaphragm slip deformation resulting from panel translation;

Δ_f'' = diaphragm slip deformation resulting from panel rotation;

Δ_p'' = panel slip deformation resulting from panel rotation;

δ_c = diaphragm deformation at midspan due to chord slip at a single location;

$\delta_{c,max}$ = maximum diaphragm deformation due to chord slip at a single location;

δ_{chord} = diaphragm deformation component from chord slip;

δ_{dia} = diaphragm deformation;

δ_{flex} = diaphragm deformation component from bending;

δ_{shear} = diaphragm deformation component from panel shear deformation;

δ_{slip} = diaphragm deformation component from panel fastener slip;

θ = angle of diaphragm rotation due to chord slip;

γ_{xy} = shear strain in panel or supporting framing;

τ_{xy} = shear stress per unit area in panel;

References

ASCE/SEI. 2017. "Chapter 7: Analysis procedures and acceptance criteria." *Seismic Evaluation and Retrofit of Existing Buildings.* ASCE/SEI 41-17. Reston, VA: ASCE.

ASCE/SEI. 2022. "Chapter 12: Seismic design requirements for building structures." *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*. ASCE/SEI 7-22. Reston, VA: ASCE.

ATC. 1981. *Guidelines for the design of horizontal wood diaphragms,* ATC-7. Building Seismic Safety Council, ed., Berkeley, CA: Applied Technology Council.

ATC. 1997. *NEHRP guidelines for the seismic rehabilitation of buildings,* FEMA Publication 273. Building Seismic Safety Council, ed., Washington D.C.: Federal Emergency Management Agency (FEMA).

AWC. 2021. *Special Design Provisions for Wind and Seismic.* Leesburg, VA: American Wood Council.

Bower, W. H. 1974. "Lateral analysis of plywood diaphragms." *Journal of the Structural Division*, 100 (4), 759-772. Reston, VA: ASCE.

BSSC. 2021. *Recommended Future Issues and Research Needs – Identified During the Development of the 2020 NEHRP Recommended Seismic Provisions for New Buildings and Other Structures.* National Institute of Building Sciences' Building Seismic Safety Council (NIBS BSSC): Washington DC. September 2021.

Carney, J. M. 1970. *Plywood diaphragm construction*, Tacoma, WA: American Plywood Association.

Cobeen, K. E., J. D. Dolan, D. Thompson, and J. W. van de Lindt, 2014. *Seismic design of wood light-frame structural diaphragm systems, A guide for practicing engineers.* Applied Technology Council, ed., Gaithersburg, MD: National Institute of Standards and Technology.

Countryman, D. 1952. *Lateral tests on plywood sheathed diaphragms.* Tacoma, WA: Douglas Fir Plywood Association.

CSA Group. 2019. *Engineering Design in Wood*, CSA 086-14 (Reaffirmed 2019). Toronto, Ontario: Canadian Standards Association.

FEMA. 2021. *Seismic design of rigid wall-flexible diaphragm buildings: an alternative procedure.* 2nd edition, FEMA P-1026. Washington D.C.: Federal Emergency Management Agency (FEMA).

Gere, J. M., and S. P. Timoshenko. 1990. *Mechanics of Materials*, Boston, MA: PWS Publishing Company.

ICBO. 1997. *Uniform building code. Volume 2*, Whittier, CA: International Conference of Building Officials, Whittier, CA.

ICC. 2000. *International building code.* Country Club Hills, IL: International Code Council.

ICC. 2021. *International building code.* Country Club Hills, IL: International Code Council.

Laguarda-Mallo, M. F., and O. Espinoza. 2018. "Awareness, perceptions and willingness to adopt CLT by US engineering firms." *BioProducts Business*, 3 (1), 1-14.

Lawson, J. W. 2018. "Improving the accuracy of wood diaphragm deflection estimates." *Wood Design Focus*, 28 (2). LaGrange, GA: Forest Products Society.

Lawson, J. W. 2019. "Improving the accuracy of wood diaphragm deflection computations and its impact on ASCE 41 pseudo-lateral force estimates." In *Proceedings, Structural Engineers*

Association of California Convention (SEAOC 2019), Squaw Valley, CA: Curran Associates, Inc.

Liu, J. Y. 1981. [Letter from J. Y. Liu to G. B. Walford of New Zealand Forest Service, Forest Research Institute, Rotorua, New Zealand. May 13, 1981] Madison, WI: US Department of Agriculture, Forest Service, Forest Products Laboratory.

Mohammad, M., B. Douglas, D. Rammer, and S. E. Pryor. 2013. "Chapter 5: Connections - Connections in cross-laminated timber buildings." *In: CLT handbook: cross-laminated timber. US edition.* Edited by Erol Karacabeyli, Brad Douglas. Pointe-Claire, Quebec: FPInnovations.

Norris, C. B. 1943. *Technique of Plywood*, Seattle, WA: I.F. Laucks, Inc.

NZS. 1993. *Timber Structures Standard,* NZS 3603:1993. Wellington, New Zealand: Standards New Zealand.

Peterson, J. 1983. "Bibliography on Lumber and Wood Panel Diaphragms." *Journal of Structural Engineering*, 109 (12), 2838-2852. Reston, VA: ASCE.

Skaggs, T. D., and Z. A. Martin. 2004. "Estimating wood structural panel diaphragm and shear wall deflection." *Practice Periodical on Structural Design and Construction*, 9 (3), 136-141. Reston, VA: ASCE.

Spickler, K., M. Closen, P. Line, and M. Pohil. 2015. "Cross laminated timber: horizontal diaphragm design example." Revision 10. Accessed January 23, 2023. www.structurlam.com/resources/manufacturing-standards/, Penticton, B.C.: Structurlam.

Sullivan, K., T. H. Miller, and R. Gupta. 2018. "Behavior of cross-laminated timber diaphragm connections with self-tapping screws." *Engineering Structures*, 168, 505-524. Elsevier.

Taylor, B., A. R. Barbosa, and A. Sinha. 2021. "In-plane shear cyclic performance of spline cross-laminated timber-concrete composite diaphragms." *Journal of Structural Engineering*, 147 (10), 04021148. Reston, VA: ASCE.

Tissell, J. R. 1966. *1966 horizontal plywood diaphragm tests.* APA Laboratory Report 106, Tacoma, WA: American Plywood Association.

Tissell, J. R. 1981. "Diaphragm Deflection Equation," [Memorandum and attachments from John Tissell to Dan Brown, June 8, 1981]. Tacoma, WA: American Plywood Association.

Tissell, J. R., and J. R. Elliott. 1977. *Plywood diaphragms.* Report 138. Tacoma, WA: American Plywood Association.

Tissell, J. R., and J. R. Elliott. 2004. *Plywood diaphragms.* Report 138. Tacoma, WA: APA—The Engineered Wood Association.

Walford, G. B. 1980. "RE: Deflection of Plywood Diaphragms," [Letter G. B. Walford to Thomas E. Brassell of American Institute of Timber Construction, Englewood, CO, October 20,1980]. Rotorua, NZ: New Zealand Forest Service, Forest Research Institute, Forest Products Division.

Walford, G. B. 1981. [Letter from G. B. Walford to J. R. Tissell of American Plywood Association, Tacoma, WA, April 30, 1981]. Rotorua, NZ: New Zealand Forest Service, Forest Research Institute, Forest Products Division.